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By **Clara Costa Duarte** and **Maria A. Cunha-e-Sá**, Faculdade de Economia, Universidade Nova de Lisboa

Renato Rosa, Fondazione Eni Enrico Mattei

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Keywords: Land Allocation Model, Forest Vintages, Carbon Sequestration, Carbon Accounting, Optimal Rotation, Transition/steady-state

JEL Classification: Q15, Q23

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Address for correspondence:

Renato Rosa
Fondazione Eni Enrico Mattei
Campo S. Maria Formosa 5252
30122 Venezia
Italy
E-mail: renato.nunesrosa@feem.it

The Role of Forests as Carbon Sinks: Land-Use and Carbon Accounting

Clara Costa Duarte, Maria A. Cunha-e-Sá, Renato Rosa ^{*†}

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^{*}Associate Professors, Faculdade de Economia, Universidade Nova de Lisboa, Campus de Campolide, P-1099-032, Lisbon, Portugal. Phone #:351-213801660; Fax #:351-213870933. Renato Rosa, Research Fellow at Fondazione Eni Enrico Mattei (FEEM), Venice, Italy. Email: mcunhasa@fe.unl.pt, mcduarte@fe.unl.pt, renato.nunesrosa@feem.it

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Abstract

The use of forests as carbon sinks is examined by introducing carbon sequestration benefits' accounting in a multi-vintage land allocation model. Following the IPCC, three carbon accounting methods are considered. We compare the results in each case with those without carbon sequestration, as well as the performances of the ton-year and the average methods (second-best) to the carbon flow (first-best) concerning optimal land allocation between forestry and alternative uses, total carbon sequestered, timber production and social welfare. A full proof of long-run optimality of steady state forest is provided. Numerical simulations are performed and results discussed illustrating the setup's potential.

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1 Introduction

Two-fifths of total global emissions arise from non-fossil fuel sources. In particular, almost 20% of total greenhouse gases (GHG) emissions are currently from deforestation and there is evidence suggesting that preventing it is relatively cheap when compared to other types of mitigation. Hence, acting now could originate immediate large-scale gains. The growing international carbon market represents a unique opportunity to also value forests as carbon sinks, giving rise to rapid reductions in CO₂ emissions. In order to make a standing forest more valuable than alternative uses of land, it is crucial to target land-use decisions as well as to promote sustainable forest management practices.

Forest carbon sequestration is widely recognized as both an ecological and economically viable instrument to help mitigating climate change.¹ Righelato and Spracklen [18] have recently shown that emissions avoided by the use of liquid biofuels over a 30-year period are two to nine times smaller than the amount of carbon sequestered by forestating an equivalent area of land. Moreover, a vast empirical literature attempting to estimate the economic costs of diverting land from other uses to forest carbon sinks concludes that carbon sequestration should play a major role in an overall portfolio of mitigation strategies. According to Lubowski, Plantinga and Stavins [10], about a third of the US target under the Kyoto Protocol could be achieved through the use of forest carbon sinks. Therefore, while emission reduction plans are still at an initial or experimental phase, in the near future it will be necessary to decide whether to consider carbon sequestration as a mitigation option. To correctly estimate and identify these costs is of utmost importance.

A recent review on this subject, for the US, was produced by Stavins and

¹According to the Third Assessment Report of the Intergovernmental Panel on Climate Change (IPCC), up to 20% of excessive emissions can be captured in forests and biological sinks over the next 50 years.

Richards [26]. The authors have identified several factors that may affect forest carbon sequestration costs: forestry practices utilized and related rates of carbon uptake over time, the opportunity cost of the land for alternative uses, the disposition of biomass through burning, harvesting, and forest product sinks, anticipated changes in forest and agricultural product prices, and the analytical methods to account for carbon flows over time, among others. According to these authors, three general approaches have been used to estimate marginal carbon sequestration costs: bottom-up engineering cost studies, optimization models that account for behavioral responses in the forest and agricultural sectors and econometric studies of the revealed preferences of agricultural land owners.²

The majority of the studies fall in the first category, where land and timber prices are taken as exogenous constants. To solve this problem, optimization models by Alig, Adams, McCarl, Callaway and Winnett [2] and Adams, Alig, McCarl, Callaway and Winnett [1] have addressed this issue using the Forestry and Agricultural Sector Model (FASOM). This model is a multi-period endogenous-price spatial-equilibrium model that links the forest and agricultural sectors in the US, where the welfare of producers and consumers in the two sectors is maximized. Besides, it also estimates where and how much conversion of land between forest and agricultural uses would be induced by a carbon sequestration program. Also, Sedjo and Sohngen [22] developed a world timber supply model to examine and assess the interactions between carbon sequestration forestry, particularly newly created carbon forests and the markets for timber.

Regarding the third approach, recent econometric studies by Stavins [25],

²Under a variety of settings, other studies strongly suggest the cost effectiveness of forest-based carbon sequestration, *inter alia*, Richards, Rosenthal, Edmonds and Wise [16], Sohngen and Mendelsohn [23] and more recently Tavoni, Bosetti and Sohngen [28]. Although the level of impact varies significantly depending on the model used, in all cases introducing forests as carbon sinks reduces costs relative to policies that consider only fossil-fuel emissions.

Newell and Stavins [13], Plantinga, Mauldin and Miller [14], Plantinga and Mauldin [15], Kerr, Pfaff and Sanchez [9] and Lubowski *et al.* [10], among others, have provided an alternative approach to modeling the potential costs of land for carbon sequestration in the US. These authors have analyzed how landowners have historically allocated land use between agriculture and forests in response to differences in land prices. Rather than assuming maximization of profits by landowners, a revealed-preference approach based on observed practices is used.

Yet, some of the factors identified above are not captured by this literature. As these models are designed to capture behavior concerning land-use change decisions, rather than changes in forestry practices, impacts on marginal carbon sequestration costs resulting from modified timber management (e.g., optimal rotation age) are not taken into account. However, forest management necessarily involves an intertemporal decision and changes in the harvesting regime will not be without consequences. These imply major adjustments over a large period of time, affecting timber and land markets and, more importantly, impacting the profile of the carbon sequestration time path. This should not be disregarded as it may not only significantly alter the assessment of marginal cost curves for carbon sequestration, but also it affects the potential of forests to obtain immediate carbon sequestration gains.

By sequestering and storing GHG from the atmosphere, forests can be used to compensate for GHG emissions. However, for this compensation to occur, the net effect of sequestration has to be comparable to that of avoided emissions. This issue raises an important question about how to incorporate the services provided by this activity when modeling forest management, which depends upon the choice of the carbon accounting method.

Following the IPCC Special Report on Land Use, Land Use Change and Forestry, we consider three different carbon accounting methods - the carbon flow method, the ton-year crediting regime and the average storage. In contrast

to the others, the carbon flow method is essentially a Pigouvian tax/subsidy on the carbon externality (first-best), as it fully internalizes at any point in time the carbon flows between forest and the atmosphere. However, the implementation of a tax on emissions and a subsidy on sequestration, as required under the carbon flow method, may imply prohibitive administrative costs. Therefore, from a social welfare perspective, we need to compare the performance of the other two methods to the first best solution.

This paper contributes to the current debate on the use of forests as carbon sinks by characterizing how several of the identified factors affecting the costs of forest-based carbon sequestration (cf. [26]) interact when both land use change and optimal timber management are addressed in a context of endogenous timber and land prices. By building upon the multiple vintage forest model developed in Salo and Tahvonen [19], [20] and [21], we solve the social planner's problem by maximizing the net benefits from timber, carbon and alternative land use. While the analytical study of incorporating carbon sequestration is already well understood in a constant timber price context, to the best of our knowledge this is the first paper to address this question in an endogenous timber and land prices' framework. We characterize and provide a complete proof of the new equilibrium conditions with respect to optimal forest management and land allocation, considering all carbon accounting methods.³ In particular, we analyze the effects of introducing carbon benefits on land and timber

³To the best of our knowledge, only the applied works by Sedjo and Sohngen [22] and Sohngen and Sedjo [24] tackle the issue of carbon sequestration in an endogenous timber and land price context. This paper differs from theirs in several respects. First, for the first time a theoretical discussion of the implications of introducing carbon sequestration with endogenous timber and land prices is provided, characterizing and providing a complete proof of the new equilibrium conditions, in particular, with respect to optimal forest management and land allocation. Second, in this paper no terminal conditions are imposed which have two major implications. As recognized in Sohngen and Sedjo [24], imposing terminal conditions on the forestry system may have important implications for forest species with large rotations. In addition, when land use change conversion costs are considered, it is not possible to demonstrate that the normal forest will be the new optimal solution, as imposed in these studies (see [21]). Finally, as the undertaken simulations aim to clarify the role of transition dynamics, the imposition of steady state terminal conditions is not appropriate, as it affects the optimal path towards the long run equilibrium.

markets, carbon sequestered and carbon stored in long term storage structures, welfare and the costs of carbon sequestration. Moreover, we identify the factors that are most likely to affect the choice of a carbon accounting method regime. Sensitivity analysis with respect to the most relevant parameters is performed. We perform numerical simulations of the model, which confirm the theoretical findings and provide an analysis of the transition to the new optimal equilibrium. Finally, carbon supply functions can be derived in a context of optimal land use, endogenous prices and timber management decisions.⁴

Both from the theoretical analysis and numerical simulations, we conclude that the adjustments are mainly driven by reallocation of land between alternative uses and changes in optimal timber management. In particular, adjustments on optimal timber management persist for a long period, contaminating timber markets and affecting the carbon supply sequestration path. Moreover, optimal rotation change is highly affected by the choice of the carbon accounting method. Therefore, studies analyzing the use of forest based carbon sequestration should accommodate both type of responses, at the cost of compromising marginal abatement cost estimates.

For a given carbon price, the transition paths to the steady state in the three accounting methods are rather similar in what concerns the dynamic behavior of forested area, as the optimal adjustment of land allocation is almost instantaneous, moving fast to cycle stabilization. In contrast, timber consumption performs rather distinctively between carbon accounting methods. Following the change in the optimal rotation period, the first-best case is characterized by major short-run adjustments in the timber markets under all considered carbon prices. In the other two accounting methods, however, adjustment is mainly driven by the allocation of land, as optimal rotations under the ton-year

⁴An important question regarding optimal timber management in the face of carbon sequestration policies is related to the optimal mix of forest species. On that question see Costa-Duarte, Cunha-e-Sá and Rosa [4].

method change only for sufficient high carbon prices and are never affected if the average is the considered accounting method. Moreover, when characterizing the intertemporal adjustment paths for the three accounting methods, we show that, for the performed numerical analysis, the first-best solution generates larger medium-term costs when compared to the less efficient ones.

In general, higher carbon prices increase both the optimal rotation period (for the flow and ton-year method) and land allocated to forest, increasing in all cases the amount of carbon sequestered relative to the case without carbon. However, the carbon sequestration time path differs between methods. In the flow method, the implementation of carbon policy always implies a new optimal harvesting time. As it takes approximately one rotation for carbon sequestered to completely stabilize, during this period the forest is a net contributor to carbon sequestered. In contrast, in the other two methods as optimal rotation remains unchanged for most considered carbon prices, total sequestered carbon stabilizes faster.

The choice between second-best alternatives is based on the behavior of the two decision variables, that is, the optimal harvest time and the amount of land allocated to forest. In general, we conclude that deviations of both the ton-year and the average storage with respect to the carbon flow (that is, requiring lower implementation costs) increase as carbon prices increase. However, for scenarios where different carbon prices determine only slight changes in the rotation period in the first-best, that is, for low carbon prices, the average storage method performs better than the ton-year. In contrast, for high carbon prices, it is the ton-year that is less distorting as it allows for adjustments in the optimal rotation. Hence, the performance of each carbon accounting method is intimately related to the observed induced adjustments in optimal timber management.

The use of forests as carbon sinks depends upon the species and, therefore,

on the amount of carbon sequestered into long-term structures, that is, not released at harvest. Interestingly enough, we show that while increases in the amount of carbon sequestered into long-term structures imply higher forested land profitability, and, consequently, more agricultural land being converted, it does not necessarily determine an increase in the amount of carbon sequestered in forest biomass as the optimal rotation period actually decreases as more carbon is captured in long-term structures. Ultimately, this is an empirical question.

The remainder of the paper is organized as follows. Section 2 presents the different accounting methods of carbon sequestration benefits. Section 3 extends the theoretical multiple vintage model to account for carbon sequestration benefits. Section 4 develops the model for the three carbon accounting methods considered: the carbon flow regime, the ton-year crediting and the average storage method. The results are compared to those obtained without carbon sequestration benefits. In Section 5 both transition and steady state results are discussed based on numerical simulations of the theoretical model. Section 6 concludes the paper. Technical details, figures and tables are presented in the Appendices.

2 Accounting Methods

By sequestering and storing GHG from the atmosphere, forests can generate carbon offsets, which may be used to compensate for GHG emissions. However, for this compensation to occur, the net effect of sequestration has to be comparable to that of avoided emissions. This issue raises two important questions: first, how to compare forest carbon sequestration with avoided emissions, examined in this section, and second, how to incorporate the services provided by this activity when modeling forest management, considered below.

The IPCC Special Report on Land Use, Land-Use Change and Forestry [8] considers different accounting methods to apply to forest or land use change investment projects, namely, the stock change method (carbon flow), the average stock method and the ton-yearly crediting. In this paper, these methods are adjusted in order to account for the time dimension of carbon sequestration and storage.

According to the carbon flow regime, as developed in Van Kooten, Binkley and G. Delcourt [29], social benefits are a function of the annual change in the forest carbon stock, as well as of the amount of carbon permanently stored in timber products and landfills. A net increase in the forest carbon stock over a year means that carbon has been removed from the atmosphere. Similarly, a fall in the forest carbon stock suggests that carbon has been released into the atmosphere. In this context, while carbon released at harvest is taxed, depending upon the timber use, sequestered carbon is subsidized yearly. Therefore, the carbon flow method is essentially a Pigouvian tax/subsidy on the carbon externality, representing a first-best solution.

An alternative approach is the ton-year crediting regime. The ton-year method consists of crediting a forestry project with a fraction of its total yearly GHG benefit, based on what is called an equivalence factor, E_f . This fraction is determined by the stock of carbon stored each year, which is then converted, using E_f , to its equivalent amount of preventing effect. In the context of this approach, two alternative calculations have been proposed by Moura-Costa and Wilson [12], and by Fearnside, Lashof and Moura-Costa [7], respectively. In both, they are based on the residence time and decay pattern of atmospheric CO₂, its Absolute Global Warming Potential (AGWP), taking explicitly into account the decay pattern of GHGs in the atmosphere. As a consequence, notice that this method does not require redemption of carbon credits upon harvest.

Moura Costa and Wilson [12] aim to determine the storing time of carbon

sequestered in biomass for which the carbon stored is equivalent to an amount of avoided emissions (equivalence time). It was found that keeping a megagram (Mg) of CO₂ out of the atmosphere for a full 100 years is equivalent to 55 Mg-year (or ton-year) equivalents, rather than the full 100 Mg-years if the CO₂ entering the atmosphere had no movement to the ocean or other sinks. The number obtained, in this case 55, is denoted by the equivalent time, T_e . In addition, assuming a linear relationship between the residence of CO₂ in the atmosphere and its radiative forcing effect, the effect of storing 1 ton of CO₂ in forest biomass for 1 year was derived. According to this rule, storing one ton of carbon for one year is equivalent to preventing the effect of 0.0182 tons of CO₂ emissions, which is denoted by the equivalence factor, $E_f = 1/T_e$. Therefore, to store one ton of carbon for one year is equivalent to receiving a subsidy for preventing the effect of $1/T_e$ of CO₂ emissions.

Also based on a Absolute Global Warming Potential (AGWP) function, Fearnside *et al.* [7] estimate the incremental credit that can be awarded for each additional year that carbon stocks remain sequestered. For this purpose these authors assume as the benchmark “keeping a Mg of CO₂ out of the atmosphere for a full 100 years”. If the stock remains intact for 100 years, the cumulative awarding of ton-year credits would equal the credits from a “permanent” emission reduction of the same magnitude. If the stock is released at any time prior to the 100-year time horizon, only the corresponding partial credit amount would be awarded.

The average carbon storage method consists of averaging the amount of carbon stored in a site over the long run, assuming an average cycle rotation period. As a result, the forest owner receives the corresponding subsidy.

Finally, notice that in contrast to the carbon flow method, the ton-year and the average carbon storage are second-best solutions.

3 The Model

The model used in this paper follows closely the multiple vintage forest model developed in Salo and Tahvonen [21], which can be summarized as follows. The model assumes multi vintages forest land, where $s = 1, \dots, n$ represents the age of trees, $x_{s,t}$ the area of forest land allocated to the age class s in period t , f_s the biomass content in timber per unit of land with trees of age class s , and $0 \leq f_1 \leq \dots \leq f_n$, and $f_0 = 0$. Land allocation must satisfy

$$0 \leq y_t = 1 - \sum_{s=1}^n x_{s,t} \quad (1)$$

that is, total land area equals 1, and y_t is the area of land allocated to an alternative use (agriculture or urban use).

Let us denote by $U(c_t) = \int d(c)dc$ the social utility from timber consumption, where $d(\cdot)$ is the inverse demand for timber, and assume $U(\cdot)$ is a continuous, twice differentiable, increasing and strictly concave function. Also, $W(y_t)$ represents the social utility of alternative land use, where $W(\cdot)$ is a continuous, twice differentiable, increasing and concave function and $W(y_t)$ is the opportunity cost of a unit of land. Finally, S_t represents the social value of carbon sequestration by forests ⁵ and depends on how the benefits from carbon sequestration are accounted for, as shown below.

Thus, the problem of optimal forest harvesting and allocation of land is obtained by maximizing the present value of social utility from the use of land, as follows:

$$v(x_{1,0}, \dots, x_{n,0})^i = \underset{\{x_{s,t+1}, s=1, \dots, n, t=0, \dots\}}{Max} \sum_{t=0}^{\infty} b^t [U(c_t) + S_t^i + W(y_t)] \quad (2)$$

⁵ S_t can alternatively be interpreted as the actual payment scheme given to forest owners to induce carbon sequestration. In this sense, the model can be used to evaluate and compare actual policy measures.

subject to

$$c_t = \sum_{s=1}^{n-1} f_s (x_{s,t} - x_{s+1,t+1}) + f_n x_{n,t} \quad (3)$$

$$y_t = 1 - \sum_{s=1}^n x_{s,t} \quad (4)$$

$$x_{s+1,t+1} \leq x_{s,t}, s = 1, \dots, n-1 \quad (5)$$

$$\sum_{s=1}^n x_{s,t+1} \leq 1 \quad (6)$$

$$x_{s,t} \geq 0, s = 1, \dots, n \quad (7)$$

for all $t = 0, 1, \dots$. Moreover, the initial land distribution satisfies

$$x_{s,0} \geq 0, s = 1, \dots, n, \sum_{s=1}^n x_{s,0} \leq 1 \quad (8)$$

Therefore, given the discount factor b , the problem consists of choosing the next period state, that is, the land allocation between different vintages and competing uses of land for all $t = 1, \dots$ ⁶

The necessary conditions for optimal solutions can be obtained from the following Lagrangian problem. For (2-8) it can be stated as

$$L^i = \sum_{t=0}^{\infty} b^t [U(c_t) + S_t^i + W(y_t)] + \lambda_t \left(1 - \sum_{s=1}^n x_{s,t+1} \right) + \sum_{s=1}^{n-1} [p_{s,t} (x_{s,t} - x_{s+1,t+1})] \quad (9)$$

where $p_{s,t}$ and λ_t are the Lagrangian multipliers, and $i = c, t, a$. While $p_{s,t}$ can be interpreted as the value of marginal changes in forest land area of vintage s

⁶In Salo and Tahvonen [21] no harvesting or plantation costs are considered nor any type of forest externalities. Under these conditions, m , as defined in (10), is the Faustmann rotation period in the one stand model. As consumption is constant in the steady-state, so is the marginal utility of consumption $U'(\cdot) = p$, the long-run market equilibrium price of timber. In addition, this condition also corresponds to the maximum value in the steady-state of a marginal unit of bare forest land.

at the beginning of period $t + 1$, λ_t represents the marginal value of forest land when all land is allocated to forestry.

Salo and Tahvonen [21] provide a full proof on the long-run optimality of the normal forest steady-state for the above problem, when $S_t = 0$. A forest is called an Optimal Faustmann Forest (OFF) if the age-class structure $x = (x_1, \dots, x_n)$ has the property $x_s = 0$ for $s = m + 1, \dots, n$ and if harvesting only trees of age m is the optimal solution for the above problem when $x_0 = x$. An OFF is an interior OFF if $x_s > 0$ for $s = 1, \dots, m$. In addition, an OFF with the normal forest structure is $x = (1/m, \dots, 1/m, 0, \dots, 0)$, and in each period it yields a constant consumption level of f_m/m . An OFF with consumption that is periodic with period length equal to m can be expressed as $x = (1/m + \phi_1, \dots, 1/m + \phi_m, 0, \dots, 0) \in S$, where ϕ^k represents the largest number ϕ that satisfies $x = (1/m + \phi_1, \dots, 1/m + \phi_m, 0, \dots, 0) \in K$ for all $|\phi_s| < \phi$, $s = 1, \dots, m$, $\sum_{s=1}^m \phi_s = 0$. The Faustmann rotation period, denoted by m , $1 \leq m \leq n$, is assumed to be unique and satisfies the following condition:⁷

$$b^m f_m / (1 - b^m) \geq b^s f_s / (1 - b^s), \quad s = 1, \dots, n. \quad (10)$$

Salo and Tahvonen [21] show that, if all land is allocated to forestry, optimal forest management can lead to optimal cyclical harvesting because smoothing an age class structure that deviates from the normal forest is not optimal. On the contrary, if it is optimal to allocate part of the land to alternative land use then optimal stationary cycles cannot exist.⁸

Using similar notation, let m^i , for $i = c, t, a$, denote the optimal rotation period with net carbon sequestration benefits for each accounting method. Assume that m^i is unique, for $i = c, t, a$. A forest is called an Optimal Carbon Forest (OCF) if the age-class structure $x = (x_1, \dots, x_n)$ is characterized by OFF for m^i , $i = c, t, a$, where m^i can be different from m .

⁷See Salo and Tahvonen [21], Proposition 1 and Corollary 1, pages 518-520.

⁸From now on, let i_∞ represent the stationary state level of variable i .

In this paper, the full proof on the long-run optimality of the normal forest steady state is extended to the case of carbon sequestration benefits.

4 Introducing Carbon Sequestration Benefits

In this section, for the three methods, the age-class and land allocation forestry decision problem of the social planner is presented and the necessary and sufficient optimality conditions are derived.⁹ It is shown how the optimal rotation period, the long run equilibrium and the optimal land allocation are affected by introducing carbon sequestration benefits in the three different cases. For each method, the comparison with the case without carbon sequestration benefits is provided.

When formalizing net carbon benefits, we assume in all cases that the social value of one unit of carbon removed from the atmosphere is constant and given by p_c .¹⁰ That is, the price of carbon is the value of the marginal damage of an additional unit of carbon added over to the atmosphere. Alternatively, if p_c is considered as a tax/subsidy to be paid to the forest owners, then the model can be used to estimate the cost of the policy in each case. Finally, S_t can be endogenized by imposing quantitative targets in terms of carbon sequestration amounts or afforestation areas. In these cases, the shadow price of carbon implicit in the constraint can be estimated for each method.

In what follows, we consider that the amount of carbon per cubic feet of timber biomass growing in forest land is constant and equal to β .

⁹While this same problem can be promptly modified to account for different timber market structures, where forest agents have market power, any policy design requires as a first step a thorough understanding of the social optimum of forest management and land use decisions.

¹⁰Assuming a constant price means that forests have only a partial (marginal) impact on carbon sequestration markets. In addition, since the time horizon is infinite in this model, it is not realistic to assume that prices increase indefinitely. However, in the numerical simulations, there is the possibility of considering increasing carbon prices for finite periods.

4.1 Carbon flow regime (first best)

The carbon flow regime measures the change of the carbon stock in the standing trees, as well as the amount of carbon that is assumed to remain as permanently stored in timber products and landfills. This last amount depends upon the different uses of timber. We introduce a parameter θ which measures the fraction of timber that is harvested but goes into long-term storage in structures and landfills. Notice that once carbon has been sequestered, no further carbon benefits will be obtained. Therefore, what is relevant is the change in the per period carbon uptake.¹¹

Since the carbon flow internalizes correctly the carbon externality, given that p_c is the social value of carbon, we may obtain an estimate of the efficient opportunity benefit of forests as carbon sinks.

The current net benefits from carbon sequestration at any period t , S_t^c , can be represented as follows:

$$S_t^c = \sum_{s=0}^{n-1} p_c \beta (f_{s+1} - f_s) x_{s+1,t+1} - p_c \beta (1 - \theta) c_t \quad (11)$$

where the first term represents the value of the carbon stock increase in forest standing biomass, for all the area of forest land, and the last term represents the value of the carbon released due to harvest at t , that is, the amount that is not permanently stored in timber products or landfills.¹²

By solving the problem (2-8) and taking S_t^c given by (11), as in Salo and

¹¹The carbon flow regime with $\theta = 0$ is similar to the rental approach that was proposed in the context of the Kyoto Protocol, namely, of the Clean Development Mechanism (CDM), as discussed in Marland, Fruit and Sedjo [11]. In the carbon flow regime presented in this paper, the forest owner is fully liable for the eventual carbon released and receives full credit for the amount of carbon sequestered for the whole duration period of the contract. This is similar to the rental approach, except that, in this case, the agent that receives full credit and the one that is liable are typically not the same, as well as the duration of the contract. Besides, as we assume that there is perfect information, and the ton of carbon is paid at its shadow price value, the problem is simplified in this case, as there are no issues of property rights, credibility, asymmetry of information, uncertainty or any other market imperfections that are present, for instance, in the CDM context. We are grateful to an anonymous referee that called our attention to this point.

¹²In alternative, we can write $S_t^c = p_c \beta \left[\sum_{s=0}^{n-1} f_s x_{s,t+1} - \sum_{s=0}^{n-1} f_s x_{s,t} \right] + p_c \beta \theta c_t$.

Tahvonen [21], we first study the existence of optimal stationary cycles in a regime where the oldest age class is clear-cut and immediately regenerated at the end of each period.

Denote the optimal rotation period by m^c , that satisfies $1 \leq m^c \leq n$ ¹³ and for which

$$(p - \beta p_c(1 - \theta)) \frac{b^{m^c} f_{m^c}}{1 - b^{m^c}} + \beta p_c \frac{\sum_{i=0}^{m^c-1} b^i (f_{i+1} - f_i)}{1 - b^{m^c}} \geq (p - \beta p_c(1 - \theta)) \frac{b^s f_s}{1 - b^s} + \beta p_c \frac{\sum_{i=0}^{s-1} b^i (f_{i+1} - f_i)}{1 - b^s} \quad (12)$$

for $s = 1, \dots, n$, holds. Assume that m^c is unique. We show in Appendix A that $m^c \geq m$ when all carbon is released at harvest ($\theta = 0$). When $\theta = 1$ and $\{f_i - f_{i-1}\}$ is a decreasing sequence, $m^c \leq m$. Otherwise, for $\theta = 1$, $m^c \geq m$.

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Proposition 1: *Given $g \equiv \frac{[U'(f_{m^c}/m^c) - \beta p_c(1-\theta)]b^{m^c} f_{m^c}}{1-b^{m^c}} + \frac{\beta p_c}{1-b^{m^c}} \sum_{i=0}^{m^c-1} b^i (f_{i+1} - f_i) - \frac{b}{1-b} W'(0) > 0$, $m^c \geq 2$, and $b < 1$, there exists a set of interior Optimal Carbon Forests with $\phi^k > 0$.*

Proof. The proof is in Appendix A. ■

In Proposition 1 it is shown that optimal stationary cycles exist when it is optimal to allocate all land to forestry. From (34) in Appendix A, we may conclude that when carbon sequestration benefits are accounted for the maximum cycle radius may either increase or decrease.¹⁵

Corollary 1: *If $g \equiv \frac{[U'(f_{m^c}/m^c) - \beta p_c(1-\theta)]b^{m^c} f_{m^c}}{1-b^{m^c}} + \frac{\beta p_c}{1-b^{m^c}} \sum_{i=0}^{m^c-1} b^i (f_{i+1} - f_i) - \frac{b}{1-b} W'(0) \leq 0$, optimal stationary cycles with $y_\infty \geq 0$ and y_∞ constant do*

¹³Since there are no carbon intakes after n , it is never optimal to postpone harvest after n .

¹⁴In fact, in the case of the typical Faustmann model discussed in the forestry literature (continuous-time model), when the timber growth function is strictly concave we can show that the optimal rotation increases for lower values of θ , while it decreases for values of θ close to 1. However, this is not the case when the timber growth function is only increasing, as shown in Appendix 1 for the discrete-time setting. Notice that, if the timber growth sequence is increasing, but its increments are decreasing with the age of the tree, s , the same result as in the continuous-time case applies. See Cunha-e-Sá and Rosa [5].

¹⁵For a more detailed explanation see Salo and Tahvonen [19], pages 8-9 and 15.

not exist.

Proof. The proof is in Appendix A. ■

When it is optimal to allocate land to alternative uses, Corollary 1 shows that optimal cycles are eliminated and the remaining equilibrium is the normal forest steady-state. Assuming that m^c is unique, for a stationary state, we have that $p_{s,t} = p_{s,\infty}$, $c_t = c_\infty$, $y_t = y_\infty$, $\lambda_t = 0$, and $x_{m,t} = x_\infty$, where $c_\infty, y_\infty, x_\infty$, and $p_{s,\infty}$, for $s = 1, \dots, n-1$, are constant. Direct substitution shows that in this case:

$$p_s = W'(y_\infty) \sum_{i=0}^{s-1} b^{-i} - f_s [U'(c_\infty) - \beta p_c (1 - \theta)] - \beta p_c \sum_{i=0}^{s-1} b^{i-s} (f_{i+1} - f_i) \quad (13)$$

where $\sum_{i=0}^{s-1} b^{-i} = -b \frac{1-b^{-s}}{1-b}$, for $s = 1, \dots, n$.

With some more algebra, we can write (13) for $s = m^c$ and as $p_{m^c,\infty} = 0$, we obtain:

$$W'(y_\infty) \frac{b}{1-b} - \frac{b^{m^c} f_{m^c}}{1-b^{m^c}} \left[U' \left(\frac{(1-y_\infty) f_{m^c}}{m^c} \right) - \beta p_c (1 - \theta) \right] - \frac{\beta p_c}{1-b^{m^c}} \sum_{i=0}^{m^c-1} b^i (f_{i+1} - f_i) = 0 \quad (14)$$

Solving for y_∞ , all the other steady-state variables are fully defined and, from (14), the allocation of land between forestry and the alternative use is optimal when the present value of output from a marginal unit of land equals the present value of a marginal unit of bare forest land, where both timber value and the net benefits from carbon sequestration are accounted for. From Appendix A, we conclude that the long-run optimal steady state will be characterized by an increase in the forest area and the opportunity cost of land when compared to the case without carbon benefits.

4.2 Ton-year crediting

The ton-year accounting method consists of crediting a forestry project with a fraction of its total yearly GHGs' benefit. This fraction is based on the stock

of carbon stored each year, which is then converted, using E_f , to its equivalent amount of preventing effect.¹⁶

In this case, S_t^t can be defined as follows:

$$S_t^t = p_c(\beta E_f \sum_{s=1}^{n-1} f_s x_{s+1,t+1}) \quad (15)$$

where the term in parenthesis represents the equivalent amount of emissions avoided in year t due to the amount of carbon stored during year t . By considering $f_s x_{s+1,t+1}$, this formalization excludes from benefits' accounting all possible harvesting of younger age classes, in period t . Notice also that there is no liability for carbon releases.

The necessary conditions for optimal solutions of problem (2-8) and S_t^t given by (15), are similar to the previous case and are presented in Appendix B.

Denote the optimal rotation period by m^t that satisfies $1 < m^t$ and for which¹⁷

$$p \frac{b^{m^t} f_{m^t}}{1 - b^{m^t}} + \beta p_c E_f \frac{\sum_{i=1}^{m^t-1} b^i f_i}{1 - b^{m^t}} \geq p \frac{b^s f_s}{1 - b^s} + \beta p_c E_f \frac{\sum_{i=1}^{s-1} b^i f_i}{1 - b^s}, \quad s = 1, \dots, n. \quad (16)$$

holds. Assume that m^t is unique. We show in Appendix B that $m^t \geq m$.

Proposition 2: *Given $g \equiv \frac{U'(f_{m^t}/m^t)b^{m^t} f_{m^t}}{1 - b^{m^t}} + \frac{\beta p_c E_f}{1 - b^{m^t}} (\sum_{i=1}^{m^t-1} b^i f_i) - \frac{b}{1-b} W'(0) > 0$, $m^t \geq 2$, and $b < 1$, there exists a set of interior Optimal Carbon Forests with $\phi^k > 0$.*

Proof. The proof is in Appendix B. ■

From Proposition 2, if all land is forested land, cyclical harvesting with consumption that is periodic with period length equal to $m^t \geq m$ is optimal.

¹⁶Here, we consider E_f constant. This assumption is consistent with Moura-Costa and Wilson' [12] approach, and also with Fearnside et al. [7], if in this last case we assume that the equivalence factor measures only the benefit of storing carbon in the forest for one additional year. To be fully consistent with Fearnside et al. [7], the equivalence factor should be different for each age class s , that is, $E_f(s)$. However, all the main results also apply. See section 2.

¹⁷Also, it may be optimal never to harvest the forest.

By inspection, from (64) in Appendix B, we observe that the maximum radius cycle can either increase or decrease.

Corollary 2: *If $g \equiv \frac{U'(f_{m^t}/m^t)b^{m^t}f_{m^t}}{1-b^{m^t}} + \frac{\beta p_c E_f}{1-b^{m^t}} \sum_{i=1}^{m^t-1} b^i f_i - \frac{b}{1-b} W'(0) \leq 0$, optimal stationary cycles with $y_\infty \geq 0$ and y_∞ constant do not exist.*

Proof. The proof is in Appendix B. ■

From Corollary 2 we conclude that the cycles are eliminated, and it is optimal to allocate land both in forestry and in an alternative use. Assuming again that m^t is unique, for a stationary state, we have that $p_{s,t} = p_{s,\infty}$, $c_t = c_\infty$, $y_t = y_\infty$, $\lambda_t = 0$, and $x_{m,t} = x_\infty$, where $c_\infty, y_\infty, x_\infty$, and $p_{s,\infty}$, for $s = 1, \dots, n-1$, are constant. Direct substitution shows that, for $s = 1, \dots, n$:

$$p_s = W'(y_\infty) \sum_{j=0}^{s-1} b^{-j} - f_s U'(c_\infty) - \beta p_c E_f \sum_{i=1}^{s-1} b^{i-s} f_i \quad (17)$$

With some more algebra, we can write (17) for $s = m^t$ and as $p_{m^t,\infty} = 0$, we obtain:

$$W'(y_\infty) \frac{b}{1-b} - \frac{b^{m^t} f_{m^t}}{1-b^{m^t}} U'\left(\frac{(1-y_\infty)f_{m^t}}{m^t}\right) - \frac{\beta p_c E_f}{1-b^{m^t}} \sum_{i=1}^{m^t-1} b^i f_i = 0 \quad (18)$$

In this case, the net benefits from carbon sequestration (third term of (18)) are the present value of “emissions equivalence reduction” of a marginal unit of forest bare land with a rotation period of dimension m^t . Also, as carbon sequestration benefits have always a positive net value, the present value of forest land increases and consequently more land will be put to forest reducing the area in alternative uses. Moreover, as the optimal rotation period may change, the steady-state timber consumption level, $\frac{(1-y_\infty)f_{m^t}}{m^t}$, will also change. In empirical terms it may increase or decrease compared to the case without carbon benefits.

Despite that at the one stand level it may be optimal to never harvest the forest, in a general equilibrium land allocation model this result is less likely and would require additional assumptions, namely, the existence of a choke price on

timber.

4.3 Average Storage Method

The average storage accounting method consists of yearly crediting a forestry project with the amount of carbon benefits that the land allocated to forest generates, on average, during a rotation period. Hence, S_t^a can be defined as follows:

$$S_t^a = p_c \beta C \sum_{s=1}^n x_{s,t} \quad (19)$$

where the term C is taken as a constant, representing the average carbon stored in each stand.¹⁸

The necessary conditions for optimal solutions of problem (2-8) are similar to the previous case and are presented in Appendix C.

Let us denote the optimal rotation period by m^a , that satisfies $1 \leq m^a \leq n$, and for which:

$$p \frac{b^{m^a} f_{m^a}}{1 - b^{m^a}} + \beta p_c b \frac{C}{1 - b} \geq p \frac{b^s f_s}{1 - b^s} + \beta p_c b \frac{C}{1 - b}, \quad s = 1, \dots, n. \quad (20)$$

holds. By comparing (20) with (10), we conclude that $m^a = m$, implying that the optimal rotation period is the same as Faustmann's.

Proposition 3: *Given $g \equiv \frac{U'(f_{m^a}/m^a)b^{m^a}f_{m^a}}{1-b^{m^a}} + \frac{b}{1-b}D - \frac{b}{1-b}W'(0) > 0$, $m^a \geq 2$, and $b < 1$, there exists a set of interior Optimal Carbon Forests with $\phi^k > 0$, where $D = \beta p_c C$.*

Proof. The proof is in Appendix C. ■

According to Proposition 3, if all land is forested land, optimal forest management can lead to optimal cyclical harvesting, but here the maximum radius cycle is the same as without carbon sequestration benefits.

Corollary 3: *If $g \equiv \frac{U'(f_{m^a}/m^a)b^{m^a}f_{m^a}}{1-b^{m^a}} + \frac{b}{1-b}D - \frac{b}{1-b}W'(0) \leq 0$, optimal stationary cycles with $y_\infty \geq 0$ and y_∞ constant do not exist.*

¹⁸In particular, the average carbon stock stored can be given by $C = \frac{\sum_{s=1}^{m^a-1} f_s}{(m^a)^2}$.

Proof. The proof is in Appendix C. ■

Assuming again that m^a is unique, for a stationary state, we have that $p_{s,t} = p_{s,\infty}$, $c_t = c_\infty$, $y_t = y_\infty$, $\lambda_t = 0$, and $x_{m,t} = x_\infty$, where c_∞ , y_∞ , x_∞ , and $p_{s,\infty}$, for $s = 1, \dots, n-1$, are constant. Direct substitution shows that, for $s = 1, \dots, n$:

$$p_s = W'(y_\infty) \sum_{j=0}^{s-1} b^{-j} - f_s U'(c_\infty) - \sum_{j=0}^{s-1} b^{-j} D \quad (21)$$

With some more algebra, we can write (21) for $s = m^a$, and $p_{m^a,\infty} = 0$, we obtain:

$$W'(y_\infty) \frac{b}{1-b} - \frac{b^{m^a} f_{m^a}}{1-b^{m^a}} U'\left(\frac{(1-y_\infty) f_{m^a}}{m^a}\right) - \frac{b}{1-b} D = 0 \quad (22)$$

Here, the net benefits from carbon sequestration (third term of (22)) are the present value of the yearly constant payment to a marginal unit of forest land, D . Since $m^a = m$ is unique, it is clear from (22) that y_∞ has to decrease, when compared to the case without carbon benefits. As the optimal rotation period is the same as Faustmann's, steady-state timber consumption increases and market equilibrium price decreases. As well, more land will be put to forest when compared to the case without carbon benefits, and, at the steady-state, the incremental forest land area will be evenly distributed among the different vintages.

5 Discussion of Numerical Results

In this section, we follow the example in Salo and Tahvonen [21] to simulate the theoretical models developed in the previous sections, illustrating the potential use of this setting to applied empirical studies. We confirm our theoretical results and analyze the optimal transition towards the new long run equilibrium, which cannot be studied analytically. In particular, we show the relevance of

jointly considering land use and optimal timber management when addressing the cost implications for the use of forest based carbon sequestration.

The results obtained with the different accounting methods are analyzed with respect to the main aspects concerning the introduction of carbon sequestration benefits: optimal land use change, total carbon uptake, timber markets and social welfare.

The baseline scenario is the case without carbon sequestration benefits, i.e. business as usual. The ton-year and the average storage's performance with respect to the carbon flow are also examined, both at the steady-state and in the transition to the steady-state. Sensitivity analysis with respect to the most relevant parameters of the model is undertaken.

Henceforth, the following utility functions for consumption and non-forestry land are considered: $U(c) = \frac{c^{0.7}}{0.7}$, $W(y) = 0.5[\frac{y^{0.2}}{0.2}]$. The vector f_s containing the biomass content in timber per unit of land with age classes of trees, $s = 1, \dots, 24$, is given by

$$f_s = [0, 0, 0, 15, 22, 30, 39, 51, 65, 82, 101, 123, 148, \\ 175, 204, 234, 263, 293, 321, 346, 370, 390, 408, 423],$$

where $f_0 = 0$, and $b = 0.95$. All the main results of internalizing carbon benefits are presented in Tables 1, 2, 3, and Figures 1, 2, 3, 3A-3E, 4.¹⁹ The simulations assume the same “price of carbon” for all accounting methods, that is, $p_c = p_{ss} = 0.4368$, where p_{ss} is the steady-state timber price in the business as usual scenario,²⁰ and $\beta = 0.2$. The equivalence factor for the ton-year is $E_f = 0.0182$.

To compute numerically this problem we follow Salo and Tahvonen [21]. While transition dynamic analyses can only be performed through a numerical

¹⁹See tables and figures at the end of the paper.

²⁰Here it is assumed $p_c = 0.468$ as the benchmark for comparing the three accounting methods; however, any other hypotheses can be easily implemented. In all simulations, initial land distribution is the steady-state of the model without carbon benefits.

exercise, imposing a stationary state as a finite horizon endpoint condition would not be appropriate, as it affects the optimal path toward the long-run equilibrium. Therefore, we solve the model with free endpoint conditions and finite horizon. For a sufficient long horizon length the optimized age class structure for period 1 closely approximates the solution for the infinite horizon problem. Accordingly, we solve the problem using the solution for the first period as the initial steady state for the following iteration (for more details see Salo and Tahvonen [21]). In each iteration we use $t = 60$ as the period length. The number of iterations ranges from 200 to 500. The optimization algorithm used was developed by Andersen and Ye ([3]).²¹

Land markets

We examine first, for a given carbon price, the time paths for optimal land use change and the impacts felt on land markets following the internalization of carbon sequestration (see Figures 1 and 2). We conclude that the optimal land use evolves towards a stationary state where both the area dedicated to forest land and timber production increase. In all cases, an adjusted normal forest is also the long-run equilibrium, confirming the theoretical results.

Regarding the dynamic behavior of optimal land use change, we conclude that no major qualitative differences exist among the three accounting methods (see Figure 1). The optimal adjustment on land allocation is almost instantaneous, moving fast to cycle stabilization. The adjustment in the forested area consists of allocating more land to the area that is harvested each period. However, if the behavior is qualitatively similar, under the carbon flow accounting larger amounts of land are devoted to forest, implying that, for the considered

²¹Using this example, as in Salo and Tahvonen [21] without carbon, the solution reaches the saddle point path where only the oldest age class ($m = 19$) is harvested in period $t = 40$. After 120 periods the land allocation was approximately constant and the forest distribution was very close to the normal.

sensitivity analysis, the choice of this method imposes a higher pressure on the alternative use land.²² Therefore, as agricultural land prices increase by more in this case, it is likely that the same happens to the prices of agricultural products, *ceteris paribus*.

Timber markets

In contrast, the timber consumption paths perform rather distinctively among accounting methods, as a consequence of the adjustments taking place both on the optimal rotation period and on land allocation (see Figure 2). Consequently, timber market price adjustments are very sensitive to carbon policy, depending upon the method choice. While in the carbon flow method, adjustments both in the optimal rotation period and land allocation occur independently of the level of carbon prices (see Table 1), in the ton-year this is only observed for high carbon prices. In fact, under this accounting method, for low carbon prices only land allocation changes. Finally, in the average storage, the optimal rotation period never changes, implying that all the adjustments occur through land use changes. (see Figure 2).

For $p_c = p_{ss}$, timber prices increase significantly in the short-run only under the carbon flow accounting, as this is the only method in which rotation is adjusted. In fact, as optimal rotation changes, it is optimal to preserve a fraction of the age class previously harvested, creating, thus, a shortage of timber in the market. For the other two methods, the impact on timber consumption is only due to changes in the forested area. Therefore, timber market adjustments to the new carbon policy are postponed, as changes in the distribution of land between age classes have impact only a cycle ahead. As observed in Stavins

²²Note that while for a zero discount rate carbon sequestration still has value under both the ton-year and the average accounting methods, the same is not true for the carbon flow if $\theta = 0$. We decided, however, to focus our analysis on a 5% discount rate, as it is in line with most forest based carbon sequestration studies.

and Richards [26], anticipated changes in forest and agricultural product prices is one of the key factors affecting the costs of forest based carbon sequestration. Neglecting optimal timber harvest adjustments can, therefore, seriously compromise marginal sequestration costs estimates.

Carbon sequestered and carbon stored in long term storage structures

Our results confirm, for all accounting methods, the widespread claim that forests can play an important role in global policy to mitigate climate change, by giving rise to rapid reductions in CO₂ emissions (see Figure 4), and, therefore, buying time for the development and deployment of low carbon based technological innovations.

In what concerns the amount of carbon sequestered, for the same price of carbon and a discount rate of 5%, it is the carbon flow accounting method that contributes the most to the increase in the amount of carbon sequestered (see Figure 4). This is due both to an increase in the optimal rotation period and in forested land. In contrast, the other two have smaller impacts in terms of additional carbon sequestered, because, for most carbon prices, the adjustment only occurs on the amount of land dedicated to forest (see Table 1).

Moreover, the carbon sequestration time path differs between methods. In the flow method the implementation of carbon policy always implies a new optimal harvesting time. Since it takes approximately one rotation for sequestered carbon forest to completely stabilize, during this period forest is a net contributor to carbon sequestered. In contrast, in the other two methods as optimal rotation remains unchanged for most considered carbon prices, total sequestered carbon stabilizes faster.

In Table 1, the results of the sensitivity analysis to the price of carbon and to the value of θ are summarized. By inspection, we conclude that, in general, higher carbon prices increase both the optimal rotation period and land

allocated to forest, increasing in all cases the amount of carbon sequestered relative to business as usual. Notice, in addition, that, by changing carbon prices, a carbon supply function for each carbon accounting method can be derived.

An important issue that has been the subject of discussion in the context of the use of forests as carbon sinks in a Post-Kyoto world is related to the accounting of carbon sequestered in long term structures. In the carbon flow that is captured by considering different values for the parameter θ . We observe that higher values of this parameter are always associated with larger amounts of land dedicated to forests, thus contributing to an increase in the total amount of carbon sequestered. However, since the optimal rotation period also adjusts to these changes, varying inversely with θ , the total amount of carbon sequestered on forest biomass is ambiguous (see Table 1, net cumulative biomass carbon), depending upon which effect dominates.

Welfare analysis and the costs of carbon sequestration

Given the adjustments on land use change and timber consumption, it is possible to endogenously estimate the yearly value impact on timber and land markets as well as on the amount of carbon sequestered (see Figure 4). This allows for examining the impact of internalizing social benefits provided by forests as carbon sinks on social welfare, which can be undertaken either in terms of only steady-states' comparisons or also including the transition, as in Table 2.

When comparing only steady-state values, the impact is always positive and higher for the carbon flow method, as expected. The welfare gains reflect the increase in the social value of the timber market as well as the increase in carbon benefits (except for the case where $\theta = 0$, when the increase in total welfare is only due to increase in carbon benefits), net of the increase in the opportunity

cost of land use. However, note that the costs of the transition are higher in the carbon flow case (see Figure 3). This can be explained by the severe short run negative shock on timber markets due to the adjustment in the optimal rotation period (see Figures 3A-3C). In fact, the resulting sharp increase in timber prices implies a severe reduction on timber consumption benefits, increasing the costs of forest carbon sequestration. In contrast, in the other two cases, the impact on timber markets takes place smoothly and is delayed for a rotation cycle, decreasing costs of transition (see Figure 3, 3D, and 3E). Therefore, we conclude that transition dynamics play a major role determining the costs of carbon sequestration, while it is the adjustment on optimal timber management that is driving these results. Consequently, an analysis of forest based carbon sequestration that neglects the endogenous response of both timber markets and optimal timber management will likely fail to account for the most significant factors determining the costs of using forests as carbon sinks. This is the case in Van Kooten et al [29] and Cunha-e-Sá and Rosa [5], in the context of a private owner's perspective, where optimal rotation is changed to account for carbon policy but where timber prices are exogenous. Likewise, in econometric studies of the revealed preferences of agricultural land owners, optimal timber management's response is not accommodated, implying that the above mentioned effects are not captured by the respective carbon costs estimates.²³ Hence, we conclude that adjustments in optimal timber management have important intertemporal consequences. In particular, the first best solution presents the highest short/medium-run transition costs.

Second best method choice

From a policy perspective, the implementation of a tax on emissions and a subsidy on sequestration, as required under the carbon flow method, may imply

²³See Stavins [25].

prohibitive administrative costs. Therefore, from a social perspective we need to compare the performance of the other two methods to the first best solution.

First, we observe that an increase in carbon prices will always increase welfare gains. Given that the carbon flow is the first best solution, any welfare deviations (of both the ton-year and the average storage) from the optimal solution can be estimated, as in Table 2. In the carbon flow case, changes in carbon prices induce adjustments both in the optimal rotation period and the optimal allocation of land. Therefore, for low carbon prices, the average storage performs better than the ton-year, as the optimal rotation period is the same in both, while the average storage is closer to the carbon flow with respect to the optimal allocation of land (see Table 2, for $p_c = p_{ss}$ and $p_c = 2p_{ss}$). However, when carbon prices are high, the ton-year performs better, as it also allows for adjustments in the optimal rotation (see Table 2, for $p_c = 6p_{ss}$). The choice of the optimal method is, therefore, intimately related to the endogenous response to harvesting length.

Moreover, for each level of the price of carbon, the value of θ also matters. In particular, for high carbon prices, as θ increases, that is, the smaller is the amount of carbon released at harvest, the closer the average accounting benefits get to the ton-year ones, and, therefore, to the first-best solution. As the optimal rotation period adjusts, decreasing as θ increases, the observed differences in that variable between the three methods are reduced. Therefore, the amount of land allocated to forest becomes the main adjustment variable. This explains the results obtained.

6 Conclusion

In this paper we provide a framework where the impact of introducing forest carbon sequestration benefits can be addressed in an endogenous timber and

land prices' context without imposing steady state conditions. In contrast to the one stand version of the model, some of the main aspects regarding the introduction of carbon sequestration into forest management can be addressed, such as, effects on timber markets, optimal land use change, total forest carbon sequestration and welfare impact. In addition, only in this context, it is possible to study the transition path to the steady-state (the usual Faustmann framework only allows for comparative static analysis), and, more generally, comparative welfare analysis between the different accounting methods can be performed. Finally, timber and carbon supply functions can also be estimated.

Following the IPCC Special Report on Land Use, Land Use Change and Forestry, and to compare the net effect of sequestration to that of avoided emissions, three different carbon accounting methods are considered, namely, the carbon flow regime, the ton-year crediting and the average carbon storage. The carbon flow case is a first best solution as it fully internalizes at any point in time the carbon flows between forest and the atmosphere.

We characterize and provide a complete proof of the new equilibrium conditions with respect to optimal forest management and land allocation, considering all carbon accounting methods. In this context, we conclude that introducing carbon benefits is not without consequences. First, in the case all land is forested land, optimal harvest is cyclical and the maximum radius cycle changes when compared to the case without carbon benefits, except in the average carbon storage case. Second, the optimal area allocated to forest will, in general, increase, as the net value from accounting carbon sequestration benefits is positive although the impacts differ between the accounting method used. Third, the optimal forest rotation period may or may not change depending on the price of carbon and the accounting method considered. For all cases considered the changes in optimal rotation and in land allocation will determine the total amount of forest biomass carbon sequestration.

Formally, it is not possible to compare the impact of the different accounting methods both on the cycles dimension and on the optimal land allocation, because they are based on distinct parameters, θ , E_f , and C , respectively. However, numerically, depending on the values taken by the different parameters, comparisons can be undertaken. In particular, it is also possible to analyze the optimal transition towards the new long run equilibrium, which cannot be studied analytically.

Based on the numerical simulations, we conclude that the three accounting methods have distinct impacts on timber and land markets. Therefore, significant differences in social welfare paths are observed. One interesting result is that the carbon flow regime, a first best solution, is also the one for which the larger negative impact occurs in the transition period, namely, in the short/medium run. Moreover, welfare deviations of both the ton-year and the average storage from the first-best depend upon the carbon price level, increasing as carbon prices increase. Whenever carbon prices induce minor changes in the optimal rotation period, the average storage performs better than the ton-year; however, for high carbon prices, the ton-year performs better, as it allows for non negligible adjustments in the optimal rotation period. Also, it is shown that when sequestered carbon is considered in long-term storage structures, it is not necessarily the case that total carbon sequestered in forest biomass will be higher, as it depends upon how the optimal rotation period adjusts. So, from a second-best perspective, the costs of carbon released at harvest matter. Therefore, we conclude that the impact of carbon sequestration policies on optimal timber management cannot be neglected.

As shown, the effect of introducing carbon benefits can have a significant impact on agricultural land use, driving increases in land rents. Therefore, carbon offsets from carbon storage in forests will have implications on how land is used in the future, playing a substantial role in future climate policy. Our

results confirm for all accounting methods the widespread claim in the literature that forests can play an important role in global policy to mitigate climate change, by giving rise to immediate gains from rapid cost effective reduction in CO₂ emissions (see Figure 4) and, therefore, buying time for the development and deployment of low carbon based technological innovations. The theoretical model and the numerical simulations undertaken illustrate the potential of this framework to applied studies.

Therefore, we conclude that studies examining the use of forest based carbon sequestration should accommodate for both land use change and optimal timber harvesting adjustments, at the cost of compromising marginal abatement costs estimates. Moreover, this framework can be easily extended to account for different tree species and biodiversity, as well as to accommodate a game theoretical approach, where different market arrangements may determine timber and land use prices. In that case, not only the performance of different policy instruments could be examined for distinct targets, but also the timing of policy implementation could also be considered. These issues will be part of our future research agenda.

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Appendix A (Not for Publication)

The necessary conditions for optimal solutions of the problem (2-8) and S_t^c given by (11), which can be derived from the Karush-Kuhn-Tucker conditions for all $t = 0, \dots$, are as follows:

$$b^t \frac{\partial L^c}{\partial x_{1,t+1}} = b f_1 U'(c_{t+1}) + f_1 p_c \beta - b f_1 p_c \beta (1 - \theta) -$$

$$-b W'(y_{t+1}) - \lambda_t + b p_{1,t+1} \leq 0 \quad (23)$$

$$b^t \frac{\partial L^c}{\partial x_{s+1,t+1}} = -f_s U'(c_t) + b f_{s+1} U'(c_{t+1}) + (f_{s+1} - f_s) p_c \beta + f_s p_c \beta (1 - \theta) -$$

$$-b f_{s+1} p_c \beta (1 - \theta) - b W'(y_{t+1}) - \lambda_t + b p_{s+1,t+1} - p_{s,t} \leq 0 \quad (24)$$

for $s = 1, \dots, n - 2$

$$b^t \frac{\partial L^c}{\partial x_{n,t+1}} = -f_{n-1} U'(c_t) + b f_n U'(c_{t+1}) + (f_n - f_{n-1}) p_c \beta + f_{n-1} p_c \beta (1 - \theta) -$$

$$-b f_n p_c \beta (1 - \theta) - b W'(y_{t+1}) - \lambda_t - p_{n-1,t} \leq 0 \quad (25)$$

$$x_{s,t+1} \geq 0, x_{s,t+1} \frac{\partial L^c}{\partial x_{s,t+1}} = 0, s = 1, \dots, n \quad (26)$$

$$p_{s,t} \geq 0, p_{s,t} (x_{s,t} - x_{s+1,t+1}) = 0, s = 1, \dots, n - 1 \quad (27)$$

$$\lambda_t \geq 0, \lambda_t (1 - \sum_{s=1}^n x_{s,t+1}) = 0 \quad (28)$$

The existence of optimal solutions for bounded utility and $b < 1$ follows from Theorem 4.6 in Stokey and Lucas (p. 79).

Proof of Proposition 1:

Proof. Following Salo and Tahvonen [21], by convexity of problem (2)-(8), if there exist multipliers $p_{s,t}$ satisfying conditions (23)-(28) under harvesting

at m^c , then the resulting age class structure is an interior OCF. The optimality follows since with harvesting at m^c , $\frac{\partial U}{\partial x_{s,t}}$ and $x_{s,t}$ remain bounded satisfying transversality conditions which, together with (23)-(28) are sufficient for optimality.

For $s = 1, \dots, m^c - 1$ using (23) to eliminate λ_t from (24) and (25), and to satisfy (26) we obtain a system of $m^c \times (m^c - 1)$ equality equations:

$$b(p_{s+1,t+1+k} - p_{1,t+1+k}) - p_{s,t+k} = -b[U(c_{t+k+1}) - \beta p_c(1 - \theta)](f_{s+1} - f_1) + \\ + [U(c_{t+k}) - \beta p_c(1 - \theta)]f_s - \beta p_c(f_{s+1} - f_s) + \beta p_c f_1 \quad (29)$$

$$-bp_{1,t+1+k} - p_{m^c-1,t+k} = -b[U(c_{t+k+1}) - \beta p_c(1 - \theta)](f_{m^c} - f_1) + \\ + [U(c_{t+k}) - \beta p_c(1 - \theta)]f_{m^c-1} - \beta p_c(f_{m^c} - f_{m^c-1}) + \beta p_c f_1 \quad (30)$$

where $s = 1, \dots, m^c - 2$, $k = 0, \dots, m^c - 1$.

This system is linear in the Lagrangian multipliers $p_{s,t+k}$, $s = 1, \dots, m^c - 1$, $k = 0, \dots, m^c - 1$ and solving for any multiplier yields:

$$p_{s,t} = \frac{b^{m^c} f_{m^c}}{1 - b^{m^c}} [b^{-s}(U'(c_{t+m^c-s}) - \beta p_c(1 - \theta)) - (U'(c_t) - \beta p_c(1 - \theta))] - \\ - f_s [U'(c_t) - \beta p_c(1 - \theta)] + A_s^c \quad (31)$$

where A_s^c is given by

$$A_s^c = \frac{\beta p_c}{1 - b^{m^c}} \left[(1 - b^s) \sum_{i=0}^{m^c-1} b^{i-s} (f_{i+1} - f_i) - (1 - b^{m^c}) \sum_{i=0}^{s-1} b^{i-s} (f_{i+1} - f_i) \right] \quad (32)$$

for $s = 1, \dots, m^c - 1$, $t = 0, \dots$, as can be verified by direct substitution into the two equations above. Moreover, from (32), we observe that $A_{m^c}^c = 0$, and that A_s^c decreases to zero as s increases to m^c . Condition (27) requires, for the

indefinitely repeated cycle, that $p_{s,t+k} \geq 0$ for $s = 1, \dots, m^c - 1, k = 0, \dots, m^c - 1$.

Thus, the fact that $x \in K$ implies by (31) that

$$\begin{aligned} \frac{U'(c_{t+k}) - \beta p_c(1 - \theta)}{U'(c_{t+k+m^c-j}) - \beta p_c(1 - \theta)} &\leq \frac{b^{m^c-j} f_{m^c}}{f_j + b^{m^c}(f_{m^c} - f_j)} + \\ &+ \frac{A_j^c(1 - b^{m^c})}{[U'(c_{t+k+m^c-j}) - \beta p_c(1 - \theta)][f_j + b^{m^c}(f_{m^c} - f_j)]} \end{aligned} \quad (33)$$

for $k = 0, \dots, m^c - 1, j = 1, \dots, m^c - 1$. Using (3) and the definition of optimal harvesting, we can write $c_{t+k} = f_{m^c} x_s$ and $c_{t+k+m^c-j} = f_{m^c} x_{s-m^c+j}$ where $s - m^c + j$ is understood as $s - j$, if $s - m^c + j \leq 0$. Equation (33) takes the form

$$\begin{aligned} \frac{U'(f_{m^c} x_s) - \beta p_c(1 - \theta)}{U'(f_{m^c} x_{s-m^c+j}) - \beta p_c(1 - \theta)} &\leq \frac{b^{m^c-j} f_{m^c}}{f_j + b^{m^c}(f_{m^c} - f_j)} + \\ &+ \frac{A_j^c(1 - b^{m^c})}{[U'(f_{m^c} x_{s-m^c+j}) - \beta p_c(1 - \theta)][f_j + b^{m^c}(f_{m^c} - f_j)]} \end{aligned} \quad (34)$$

for $s = 1, \dots, m^c, j = 1, \dots, m^c - 1$.

We now show that the right-hand side of (34) larger than one is equivalent to (12) for any $j < m^c$. By rearranging the right-hand side of (34) we obtain

$$\begin{aligned} &U'(f_{m^c} x_{s-m^c+j}) b^{m^c} f_{m^c} (1 - b^j) - U'(f_{m^c} x_{s-m^c+j}) b^j f_j (1 - b^{m^c}) + \\ &+ \beta p_c b^j \left[(1 - b^j) \sum_{i=0}^{m^c-1} b^{i-j} (f_{i+1} - f_i) - (1 - b^{m^c}) \sum_{i=0}^{j-1} b^{i-j} (f_{i+1} - f_i) \right] > 0 \end{aligned}$$

or

$$\begin{aligned} &U'(f_{m^c} x_{s-m^c+j}) b^{m^c} f_{m^c} (1 - b^j) - U'(f_{m^c} x_{s-m^c+j}) b^j f_j (1 - b^{m^c}) + \\ &+ \beta p_c \left[(1 - b^j) \sum_{i=0}^{m^c-1} b^i (f_{i+1} - f_i) - (1 - b^{m^c}) \sum_{i=0}^{j-1} b^i (f_{i+1} - f_i) \right] > 0 \end{aligned} \quad (35)$$

On the other hand, by reducing to the same denominator, (12) can be restated as (35). Therefore, if there exists a $m^c \stackrel{\geq}{\leq} m$ such that (12) holds, the right-hand side of (34) is larger than one. Then, by the strict concavity of U , there must exist a $\phi > 0$, such that (34) is satisfied if $x_s = 1/m^c + \phi_s$, $s = 1, \dots, m^c$, for all $|\phi_s| < \phi$, $\sum_{s=1}^{m^c} \phi_s = 0$, proving that optimal harvesting is cyclical harvesting and that it is not optimal to cut before m^c .

Similarly, for $s = m^c + 1, \dots, n$, and $k = 0, \dots, m^c - 1$, the optimality of the harvesting at m^c requires that land is not allocated to age classes $s = m^c + 1, \dots, n$. Since $x_{m^c, t} > 0$ and $x_{m^c+1, t+1} = 0$ in (27), we obtain $p_{m^c, t} = 0$, for $t = 0, \dots$, and $p_{st} \geq 0$, for $s = m^c + 1, \dots, n - 1$, $t = 0, \dots$, as can also be checked in (31). Using this and conditions (24) and (25), yields

$$b(p_{m^c+1, t+1+k} - p_{1, t+1+k}) - p_{m^c, t+k} \leq -b[U(c_{t+k+1}) - \beta p_c(1 - \theta)](f_{m^c+1} - f_1) + \\ + [U(c_{t+k}) - \beta p_c(1 - \theta)] f_{m^c} - \beta p_c(f_{m^c+1} - f_{m^c}) + \beta p_c f_1$$

$$b(p_{s+1, t+1+k} - p_{1, t+1+k}) - p_{s, t+k} \leq -b[U(c_{t+k+1}) - \beta p_c(1 - \theta)](f_{s+1} - f_1) + \\ + [U(c_{t+k}) - \beta p_c(1 - \theta)] f_s - \beta p_c(f_{s+1} - f_s) + \beta p_c f_1$$

for $s = m^c + 1, \dots, n - 2$, and

$$-bp_{1, t+1+k} - p_{n-1, t+k} \leq -b[U(c_{t+k+1}) - \beta p_c(1 - \theta)](f_n - f_1) + \\ + [U(c_{t+k}) - \beta p_c(1 - \theta)] f_{n-1} - \beta p_c(f_n - f_{n-1}) + \beta p_c f_1$$

where $k = 0, \dots, m^c - 1$. Using (31), by direct substitution we can show that the first two inequalities are satisfied as equalities. By eliminating $p_{1, t+1+k}$ and $p_{n-1, t+k}$ from the last inequality, using (31), and the facts that $c_{t+k+1} = f_{m^c} x_s$

and $c_{t+k+m^c-n+1} = f_{m^c} x_{s-m^c+n}$, we can write the last inequality above as follows:

$$\begin{aligned} \frac{U'(f_{m^c} x_s) - \beta p_c(1-\theta)}{U'(f_{m^c} x_{s-m^c+n}) - \beta p_c(1-\theta)} &\leq \frac{b^{m^c-n} f_{m^c}}{f_n + b^{m^c}(f_{m^c} - f_n)} + \\ &+ \frac{A_n^c(1-b^{m^c})}{[U'(c_{t+k+m^c-n}) - \beta p_c(1-\theta)][f_n + b^{m^c}(f_{m^c} - f_n)]} \end{aligned}$$

for $s = m^c + 1, \dots, n$.

The conditions $p_{s,t+k} \geq 0$ for $s = m^c + 1, \dots, n-1$, $k = 0, \dots, m^c - 1$ together with the last inequality yield

$$\begin{aligned} \frac{U'(f_{m^c} x_s) - \beta p_c(1-\theta)}{U'(f_{m^c} x_{s-m^c+j}) - \beta p_c(1-\theta)} &\leq \frac{b^{m^c-n} f_{m^c}}{f_n + b^{m^c}(f_{m^c} - f_n)} \\ &+ \frac{A_n^c(1-b^{m^c})}{[U'(f_{m^c} x_{s-m^c+j}) - \beta p_c(1-\theta)][f_n + b^{m^c}(f_{m^c} - f_n)]} \end{aligned} \quad (36)$$

for $s = 1, \dots, m^c$, and $j = m^c + 1, \dots, n$. Similarly, it is easy to show that the right-hand side of (36) larger than one is equivalent to (12).

Consequently, there exists a $\phi > 0$ such that (36) is satisfied if $x_s = 1/m^c + \phi_s$, $s = 1, \dots, m^c$, $x_s = 0$ for $s = m^c + 1, \dots, n$, for all $|\phi_s| < \phi$, $\sum_{s=1}^{m^c} \phi_s = 0$, and simultaneously it is never optimal to postpone harvest after m^c .

In addition, a stationary cycle with all land allocated to forestry must satisfy $\lambda_t \geq 0$, for $t = 0, \dots$. Solving (23) or (24) for λ_t , eliminating $p_{s,t}$, for $s = 1, \dots, m^c - 1$, $t = 0, \dots$, using (31), we obtain

$$\begin{aligned} \lambda_{t+k} &= \frac{[U'(c_{t+k}) - \beta p_c(1-\theta)] b^{m^c} f_{m^c}}{1 - b^{m^c}} - \frac{[U'(c_{t+1+k}) - \beta p_c(1-\theta)] b^{m^c+1} f_{m^c}}{1 - b^{m^c}} + \\ &+ \frac{\beta p_c}{1 - b^{m^c}} \sum_{i=0}^{m^c-1} b^i (1-b) (f_{i+1} - f_i) - bW'(0) \geq 0 \end{aligned} \quad (37)$$

for $s = 1, \dots, m^c$, where $c_{t+1+m^c} = c_{t+1}$. Writing $c_{t+k} = f_{m^c} x_s$ and $c_{t+1+k} = f_{m^c} x_{s-1}$, $s = 1, \dots, m^c$, where $x_0 = x_{m^c}$ yields

$$\lambda_s = \frac{[U'(f_{m^c} x_s) - \beta p_c(1-\theta)] b^{m^c} f_{m^c}}{1 - b^{m^c}} - \frac{[U'(f_{m^c} x_{s-1}) - \beta p_c(1-\theta)] b^{m^c+1} f_{m^c}}{1 - b^{m^c}} +$$

$$+\frac{\beta p_c}{1-b^{m^c}} \sum_{i=0}^{m^c-1} b^i(1-b)(f_{i+1}-f_i)-bW'(0) \geq 0 \quad (38)$$

for $s = 1, \dots, m^c$.

Given $g = \frac{[U'(f_{m^c}/m^c) - \beta p_c(1-\theta)]b^{m^c} f_{m^c}}{1-b^{m^c}} + \frac{\beta p_c}{1-b^{m^c}} \sum_{i=0}^{m^c-1} b^i (f_{i+1} - f_i) - \frac{b}{1-b} W'(0) > 0$, there must exist a $\phi > 0$ such that (38) is satisfied if $x_s = 1/m^c + \phi_s$, $s = 1, \dots, m^c$, for all $|\phi_s| < \phi$, $\sum_{s=1}^{m^c} \phi_s = 0$. ■

Let i_∞ represent the stationary state level of variable i .

We next show that if $g \leq 0$ there exists a stationary state that satisfies all the necessary conditions for optimality.²⁴

Proof of Corollary 1:

Proof. Given $g \leq 0$, no solutions for (38) exist. Thus, by letting $\lambda_t = 0$ in (23) or (24), eliminating $p_{s,t}$, $s = 1, \dots, m^c - 1$, $t = 0, \dots$, using (31), and writing (23) analogously to (38), we obtain for $s = 1, \dots, m^c$:

$$\begin{aligned} & \frac{[U'(f_{m^c} x_s) - \beta p_c(1-\theta)]b^{m^c} f_{m^c}}{1-b^{m^c}} - \frac{[U'(f_{m^c} x_{s-1}) - \beta p_c(1-\theta)]b^{m^c+1} f_{m^c}}{1-b^{m^c}} + \\ & + \frac{\beta p_c}{1-b^{m^c}} \sum_{i=0}^{m^c-1} b^i(1-b)(f_{i+1}-f_i) - bW'(y_\infty) \geq 0 \end{aligned} \quad (39)$$

This system is linear in $[U'(f_{m^c} x_s) - \beta p_c(1-\theta)]$, $s = 1, \dots, m^c$ and its solution is given by:

$$[U'(f_{m^c} x_s) - \beta p_c(1-\theta)] + \frac{\beta p_c}{b^{m^c} f_{m^c}} \sum_{i=0}^{m^c-1} b^i (f_{i+1} - f_i) = \frac{W'(y_\infty) \sum_{i=0}^{m^c-1} b^i}{b^{m^c-1} f_{m^c}}, s = 1, \dots, m^c \quad (40)$$

as can be verified by direct substitution. Thus, $x_s = (1 - y_\infty)/m^c$, $s = 1, \dots, m^c$ and optimal stationary cycles cannot exist. ■

²⁴The results obtained in Salo and Tahvonen [21] regarding convergence and stability of the stationary steady states (Lemma 1 and Lemma 2, pg. 523) still apply in the case of this paper, as the difference equation for x_{m^c} , for $i = c, t, a$, is similar to equation (34), pg. 522, in the paper. The additional terms that are present in our case are independent of x_{m^c} . Therefore, the marginal conditions yielding the corresponding characteristic polynomials turn out to be similar.

Impact on the optimal rotation period:

We now show that for $\theta = 0$, $m^c \geq m$:

At the steady-state, if there exists a $m^c \neq m$, for which

$$\frac{(p - p_c\beta)b^{m^c} f_{m^c}}{1 - b^{m^c}} + \frac{p_c\beta \sum_{i=0}^{m^c-1} b^i (f_{i+1} - f_i)}{1 - b^{m^c}} \geq \frac{(p - p_c\beta)b^m f_m}{1 - b^m} + \frac{p_c\beta \sum_{i=0}^{m-1} b^i (f_{i+1} - f_i)}{1 - b^m} \quad (41)$$

holds, then it is optimal to cut at m^c , where $m^c \geq m$. If (41), which is the same as (12), holds in particular for $m^c = m + 1$, then $m^c \geq m$, while if it holds in particular for $m^c = m - 1$, $m^c \leq m$. If (41) holds as an equality, $m^c = m$.

By making $m^c = m - 1$, we show below that (41) never holds, implying that $m^c \geq m$, that is, it is optimal to postpone harvest. Also, for some $m^c = m + 1$, (41) may be satisfied.

Let $m^c = m - 1$ in (41). By rearranging and collecting terms we obtain

$$pb^{m-1} f_{m-1}(1 - b^m) - pb^m f_m(1 - b^{m-1}) >$$

$$p_c\beta(b^m - b^{m-1}) \sum_{i=0}^{m-2} b^i (f_{i+1} - f_i) - p_c\beta b^m f_m(1 - b^{m-1}) + p_c\beta b^{m-1}(f_m - f_{m-1})(1 - b^{m-1}) + p_c\beta b^{m-1} f_{m-1} \quad (42)$$

which can be restated as

$$p_c\beta(b^m - b^{m-1}) \sum_{i=0}^{m-2} b^i (f_{i+1} - f_i) - p_c\beta f_m(b^m - b^{m-1})(1 - b^{m-1}) - p_c\beta f_{m-1} b^{m-1}(b^m - b^{m-1}) \quad (43)$$

and, finally, as

$$p_c\beta(b^m - b^{m-1}) \left[\sum_{i=0}^{m-1} b^i f_{i+1} - \sum_{i=0}^{m-1} b^i f_i - f_m \right] \quad (44)$$

In (42) the left-hand side is negative. Since

$$\sum_{i=0}^{m-1} b^i f_{i+1} = \sum_{i=0}^{m-1} b^i (f_{i+1} - f_m) + \sum_{i=0}^{m-1} b^i f_m$$

substituting above we obtain

$$p_c \beta (b^m - b^{m-1}) \left[\sum_{i=0}^{m-1} b^i (f_{i+1} - f_m) - f_m + \sum_{i=0}^{m-1} b^i (f_m - f_i) \right]$$

Given that $\{f_i\}$, for $i = 1, \dots, m-1$, is an increasing sequence, and $b < 1$, we may conclude this expression is positive, as the sign of the algebraic sum inside the square brackets is negative. Consequently, the right-hand side of (42) is positive, implying that (42) never holds for any $m^c < m$. Also, we can show that it may hold for some $m^c > m$. Therefore, when $\theta = 0$, it is never optimal to cut earlier, that is, $m^c \geq m$.

In the case $\theta = 1$, we will consider two cases. In case (i) we assume that the sequence $\{f_i - f_{i-1}\}$ is a decreasing sequence. In case (ii) we only assume that the sequence $\{f_i\}$ is an increasing sequence.

(i) Let $m^c = m + 1$ in (41). By rearranging and collecting terms we obtain

$$pb^{m+1} f_{m+1} (1 - b^m) - pb^m f_m (1 - b^{m+1}) >$$

$$p_c \beta \left[-(f_{m+1} - f_m) (1 - b^m) b^m + (b^m - b^{m+1}) \sum_{i=0}^{m-1} b^i (f_{i+1} - f_i) \right] \quad (45)$$

In (42) the left-hand side is negative. Dividing (42) by $(1 - b^m) b^m$, the right-hand side can be stated as follows:

$$p_c \beta \left[-(f_{m+1} - f_m) + \frac{b^m - b^{m+1}}{(1 - b^m) b^m} \sum_{i=0}^{m-1} b^i (f_{i+1} - f_i) \right]$$

Since

$$\sum_{i=0}^{m-1} b^i (f_{i+1} - f_i) = \sum_{i=0}^{m-1} b^i [(f_{i+1} - f_i) - (f_{m+1} - f_m)] + \sum_{i=0}^{m-1} b^i (f_{m+1} - f_m)$$

implying that

$$\sum_{i=0}^{m-1} b^i (f_{i+1} - f_i) = \sum_{i=0}^{m-1} b^i [(f_{i+1} - f_i) - (f_{m+1} - f_m)] + \frac{1 - b^m}{1 - b} (f_{m+1} - f_m)$$

Substituting above we obtain

$$p_c \beta \left[\frac{b^m - b^{m+1}}{(1 - b^m)b^m} \left(\sum_{i=0}^{m-1} b^i [(f_{i+1} - f_i) - (f_{m+1} - f_m)] + \frac{1 - b^m}{1 - b} (f_{m+1} - f_m) \right) - (f_{m+1} - f_m) \right]$$

which can be restated as

$$p_c \beta \left[\frac{b^m - b^{m+1}}{(1 - b^m)b^m} \left(\sum_{i=0}^{m-1} b^i [(f_{i+1} - f_i) - (f_{m+1} - f_m)] \right) + \left(\frac{b^m - b^{m+1}}{(1 - b^m)b^m} \frac{1 - b^m}{1 - b} - 1 \right) (f_{m+1} - f_m) \right]$$

Given that $\{f_i - f_{i-1}\}$, for $i = 1, \dots, m$, is a decreasing sequence, we may conclude that this expression is positive, as the term that multiplies $(f_{m+1} - f_m)$ vanishes. Consequently, the right-hand side of (42) is positive, implying that (42) never holds for any $m^c > m$. By inspection, we observe that it may hold for some $m^c < m$. Therefore, for $\theta = 1$, it is never optimal to postpone harvest, that is, $m^c \leq m$.

(ii) Let $m^c = m + 1$ in (41). By rearranging and collecting terms we obtain

$$pb^{m+1}f_{m+1}(1 - b^m) - pb^m f_m(1 - b^{m+1}) >$$

$$p_c \beta \left[- (f_{m+1} - f_m) (1 - b^m)b^m + (b^m - b^{m+1}) \sum_{i=0}^{m-1} b^i (f_{i+1} - f_i) \right] \quad (46)$$

Since

$$\sum_{i=0}^{m-1} b^i f_i = \sum_{i=0}^{m-1} b^i (f_i - f_{m+1}) + \sum_{i=0}^{m-1} b^i f_{m+1}$$

which can be rewritten as

$$\sum_{i=0}^{m-1} b^i f_i = \sum_{i=0}^{m-1} b^i (f_i - f_{m+1}) + \frac{1-b^m}{1-b} f_{m+1}$$

By substituting above, we obtain

$$p_c \beta [-(f_{m+1} - f_m)(1 - b^m)b^m] + \\ + p_c \beta (b^m - b^{m+1}) \left(\sum_{i=0}^{m-1} b^i f_i - \sum_{i=0}^{m-1} b^i (f_i - f_{m+1}) - \frac{1-b^m}{1-b} f_{m+1} \right)$$

Finally, by collecting terms, we get

$$p_c \beta \left(-(1-b^m)b^m - (b^m - b^{m+1}) \frac{1-b^m}{1-b} \right) f_{m+1} + p_c \beta f_m (1-b^m)b^m$$

$$+ p_c \beta (b^m - b^{m+1}) \sum_{i=0}^{m-1} b^i f_i - p_c \beta (b^m - b^{m+1}) \sum_{i=0}^{m-1} b^i (f_i - f_{m+1})$$

where the first-term is negative and the other three are positive. In particular, the last term is positive as long as $\{f_i\}$ is an increasing sequence for $i = 1, \dots, m+1$. Therefore, the sign of this expression, that is, the right-hand side of (46) can be either positive or negative. Since the left-hand side of (46) is negative, it may be optimal to postpone harvest. This is in contrast to case (i), in which by imposing a more restrictive assumption, namely, that the sequence $\{f_i - f_{i-1}\}$ is decreasing, it is never optimal to postpone harvest.

Let now $m^c = m - 1$ in (41). By rearranging and collecting terms we obtain

$$pb^{m-1}f_{m-1}(1-b^m) - pb^m f_m(1-b^{m-1}) >$$

$$p_c \beta \left[(f_m - f_{m-1})(1-b^{m-1})b^{m-1} + (b^m - b^{m-1}) \sum_{i=0}^{m-1} b^i (f_{i+1} - f_i) \right] \quad (47)$$

which can be rewritten as

$$p_c \beta \left[(f_m - f_{m-1}) (1 - b^{m-1}) b^{m-1} + (b^m - b^{m-1}) \left(\sum_{i=0}^{m-2} b^i (f_{i+1} - f_i) \right) \right]$$

Using the same procedure as before, we may write

$$\sum_{i=0}^{m-2} b^i [(f_{i+1} - f_i) - (f_m - f_{m-1})] = \sum_{i=0}^{m-2} b^i + \sum_{i=0}^{m-2} b^i (f_m - f_{m-1})$$

that is,

$$\sum_{i=0}^{m-2} b^i [(f_{i+1} - f_i) - (f_m - f_{m-1})] = \sum_{i=0}^{m-2} b^i [(f_{i+1} - f_i) - (f_m - f_{m-1})] + \frac{1 - b^{m-1}}{1 - b} (f_m - f_{m-1})$$

Substituting above and collecting terms, we obtain

$$\begin{aligned} & p_c \beta (b^m - b^{m-1}) \left(\sum_{i=0}^{m-2} b^i [(f_{i+1} - f_i) - (f_m - f_{m-1})] \right) + \\ & + p_c \beta \left((b^m - b^{m-1}) \frac{1 - b^{m-1}}{1 - b} + (1 - b^{m-1}) b^{m-1} \right) (f_m - f_{m-1}) \end{aligned}$$

Therefore, the right-hand side of (47) can be rewritten as the algebraic sum of the two above terms. The first term is negative, as long as $\{f_i - f_{i-1}\}$ is a decreasing sequence for $i = 1, \dots, m-1$, while the second one is positive, as the term that multiplies $(f_m - f_{m-1})$ is positive. Therefore, it may be optimal to cut earlier than m . The same result is obtained if, instead, we consider a less restrictive assumption such that $\{f_i\}$ is an increasing sequence for $i = 1, \dots, m+1$.

Appendix B (Not for Publication)

The Karush-Kuhn-Tucker conditions for optimal solutions of problem (2-8) and S_t^t given by (15) for all $t = 0, \dots$, as follows:

$$b^t \frac{\partial L^t}{\partial x_{1,t+1}} = b f_1 U'(c_{t+1}) - b W'(y_{t+1}) - \lambda_t + b p_{1,t+1} \leq 0 \quad (48)$$

$$b^t \frac{\partial L^t}{\partial x_{s+1,t+1}} = -f_s U(c_t) + b f_{s+1} U(c_{t+1}) + f_s p_c \beta E_f -$$

$$-bW'(y_{t+1}) - \lambda_t + b p_{s+1,t+1} - p_{s,t} \leq 0 \quad (49)$$

for $s = 1, \dots, n-2$,

$$b^t \frac{\partial L^t}{\partial x_{n,t+1}} = -f_{n-1} U(c_t) + b f_n U(c_{t+1}) + f_{n-1} p_c \beta E_f - bW'(y_{t+1}) - \lambda_t - p_{n-1,t} \leq 0 \quad (50)$$

$$x_{s,t+1} \geq 0, x_{s,t+1} \frac{\partial L^t}{\partial x_{s,t+1}} = 0, s = 1, \dots, n \quad (51)$$

$$p_{s,t} \geq 0, p_{s,t}(x_{s,t} - x_{s+1,t+1}) = 0, s = 1, \dots, n-1 \quad (52)$$

$$\lambda_t \geq 0, \lambda_t \left(1 - \sum_{s=1}^n x_{s,t+1}\right) = 0 \quad (53)$$

The existence of optimal solutions for bounded utility and $b < 1$ follows from Theorem 4.6 in Stokey and Lucas (p.79).

Proof of Proposition 2:

Proof. Based on this new formulation, using a similar procedure as used to prove Proposition 1, for $s = 1, \dots, m^t - 1$, using (48) to eliminate λ_t from (49) and (50), and to satisfy (51) we obtain a system of $m^t \times (m^t - 1)$ equality equations:

$$b(p_{s+1,t+1+k} - p_{1,t+1+k}) - p_{s,t+k} = -bU'(c_{t+k+1})(f_{s+1} - f_1) + [U(c_{t+k}) - \beta p_c E_f] f_s \quad (54)$$

$$-b p_{1,t+1+k} - p_{m^t-1,t+k} = -bU'(c_{t+k+1})(f_{m^t} - f_1) + [U(c_{t+k}) - \beta p_c E_f] f_{m^t-1} \quad (55)$$

where $s = 1, \dots, m^t - 2$, $k = 0, \dots, m^t - 1$. This system is linear in the Lagrangian multipliers $p_{s,t+k}$, $s = 1, \dots, m^t - 1$, $k = 0, \dots, m^t - 1$. Solving for any multiplier yields

$$p_{s,t} = \frac{b^{m^t} f_{m^t}}{1 - b^{m^t}} [b^{-s} U'(c_{t+m^t-s}) - U'(c_t)] - f_s U'(c_t) + A_s^t \quad (56)$$

where A_s^t is given by

$$A_s^t = \frac{\beta p_c E_f}{1 - b^{m^t}} \left[(1 - b^s) \sum_{i=1}^{m^t-1} b^{i-s} f_i - (1 - b^{m^t}) \sum_{i=1}^{s-1} b^{i-s} f_i \right] \quad (57)$$

for $s = 1, \dots, m^t - 1$, $t = 0, \dots$, as can be verified by direct substitution into the two equations above. Moreover, from (57), we observe that $A_{m^t}^t = 0$.

Condition (52) requires, for the indefinitely repeated cycle, that $p_{s,t+k} \geq 0$ for $s = 1, \dots, m^t - 1$, $k = 0, \dots, m^t - 1$. Thus, the fact that $x \in K$ implies by (56) that

$$\frac{U'(c_{t+k})}{U'(c_{t+k+m^t-j})} \leq \frac{b^{m^t-j} f_{m^t}}{f_j + b^{m^t}(f_{m^t} - f_j)} + \frac{A_j^t(1 - b^{m^t})}{[f_j + b^{m^t}(f_{m^t} - f_j)] U'(c_{t+k+m^t-j})} \quad (58)$$

for $k = 0, \dots, m^t - 1$, $j = 1, \dots, m^t - 1$, where A_j^t is given by (57). Using (3) and the definition of carbon harvesting, we can write $c_{t+k} = f_{m^t} x_s$ and $c_{t+k+m^t-j} = f_{m^t} x_{s-m^t+j}$ where $s-m^t+j$ is understood as $s-j$, if $s-m^t+j \leq 0$. Equation (58) takes the form

$$\frac{U'(f_{m^t} x_s)}{U'(f_{m^t} x_{s-m^t+j})} \leq \frac{b^{m^t-j} f_{m^t}}{f_j + b^{m^t}(f_{m^t} - f_j)} + \frac{A_j^t(1 - b^{m^t})}{[f_j + b^{m^t}(f_{m^t} - f_j)] U'(f_{m^t} x_{s-m^t+j})} \quad (59)$$

or, alternatively,

$$\frac{U'(f_{m^t} x_s)}{U'(f_{m^t} x_{s-m^t+j})} \leq \frac{b^{m^t-j} f_{m^t}}{f_j + b^{m^t}(f_{m^t} - f_j)} + \frac{A_j^t(1 - b^{m^t})}{U'(f_{m^t} x_{s-m^t+j}) b^{m^t-j} f_{m^t}} \quad (60)$$

for $s = 1, \dots, m^t$, $j = 1, \dots, m^t - 1$.

We now show that the right-hand side of (60) larger than one is equivalent to (16) for any $j < m^t$. By rearranging the right-hand side of (60), we obtain

$$U'(f_{m^t} x_{s-m^t+j}) b^{m^t} f_{m^t} (1 - b^j) - U'(f_{m^t} x_{s-m^t+j}) b^j f_j (1 - b^{m^t}) +$$

$$+\beta p_c E_f b^j \left[(1 - b^j) \sum_{i=1}^{m^t-1} b^{i-j} f_i - (1 - b^{m^t}) \sum_{i=1}^{j-1} b^{i-j} f_i \right] > 0$$

or

$$\begin{aligned}
& U'(f_{m^t} x_{s-m^t+j}) b^{m^t} f_{m^t} (1-b^j) - U'(f_{m^t} x_{s-m^t+j}) b^j f_j (1-b^{m^t}) + \\
& + \beta p_c E_f \left[(1-b^j) \sum_{i=1}^{m^t-1} b^i f_i - (1-b^{m^t}) \sum_{i=1}^{j-1} b^i f_i \right] > 0 \quad (61)
\end{aligned}$$

On the other hand, by reducing to the same denominator, (16) can be restated as (61). Therefore, using a similar reasoning as in the previous case, we conclude that not only it is not optimal to cut earlier than m^t , but also, by the strict concavity of U , there must exist a $\phi > 0$, such that (59) is satisfied if $x_s = 1/m^t + \phi_s$, $s = 1, \dots, m^t$, for all $|\phi_s| < \phi$, $\sum_{s=1}^{m^t} \phi_s = 0$.

Similarly, we now derive the results for $s = m^t + 1, \dots, n$, and $k = 0, \dots, m^t - 1$. For the cases $m^t < n$, the optimality of the carbon harvesting requires that land is not allocated to age classes $s = m^t + 1, \dots, n$. Since $x_{m^t} > 0$ and $x_{m^t+1, t+1} = 0$ in (52), we obtain $p_{m^t} = 0$, for $t = 0, \dots$, as can be checked in (56). Using this and conditions (49), (50), and $p_{st} \geq 0$, for $s = m^t + 1, \dots, n - 1$, $t = 0, \dots$, yields

$$b(p_{m^t+1, t+1+k} - p_{1, t+1+k}) - p_{m^t, t+k} \leq -bU'(c_{t+k+1})(f_{m^t+1} - f_1) + [U'(c_{t+k}) - \beta p_c E_f] f_{m^t} \quad (62)$$

$$b(p_{s+1, t+1+k} - p_{1, t+1+k}) - p_{s, t+k} \leq -bU'(c_{t+k+1})(f_{s+1} - f_1) + [U'(c_{t+k}) - \beta p_c E_f] f_s$$

for $s = m^t + 1, \dots, n - 2$, and

$$-bp_{1, t+1+k} - p_{n-1, t+k} \leq -bU'(c_{t+k+1})(f_n - f_1) + [U'(c_{t+k}) - \beta p_c E_f] f_{n-1} \quad (63)$$

where $k = 0, \dots, m^t - 1$. Using (56), by direct substitution we can show that the first two inequalities are satisfied as equalities. By eliminating $p_{1, t+1+k}$ and $p_{n-1, t+k}$ from the last inequality, using (56), and the facts that $c_{t+k+1} = f_{m^t} x_s$ and $c_{t+k+m^t-n+1} = f_{m^t} x_{s-m^t+n}$, we can write the last inequality above as follows:

$$\frac{U'(f_{m^t} x_s)}{U'(f_{m^t} x_{s-m^t+n})} \leq \frac{b^{m^t-n} f_{m^t}}{f_n + b^{m^t}(f_{m^t} - f_n)} + \frac{(1-b^{m^t}) A_n^t}{[f_n + b^{m^t}(f_{m^t} - f_n)] U'(f_{m^t} x_{s-m^t+n})}$$

for $s = 1, \dots, n$.

The conditions $p_{s,t+k} \geq 0$ for $s = m^t + 1, \dots, n - 1$, $k = 0, \dots, m^t - 1$ together with the last inequality yield

$$\frac{U'(f_{m^t} x_s)}{U'(f_{m^t} x_{s-m^t+j})} \leq \frac{b^{m^t-j} f_{m^t}}{f_j + b^{m^t}(f_{m^t} - f_j)} + \frac{(1 - b^{m^t})A_j^t}{[f_j + b^{m^t}(f_{m^t} - f_j)]U'(f_{m^t} x_{s-m^t+j})} \quad (64)$$

for $s = 1, \dots, m^t$, and $j = m^t + 1, \dots, n$. Similarly, it is easy to show that the right-hand side of (64) larger than one is equivalent to (16).

Consequently, there exists a $\phi > 0$ such that (64) is satisfied if $x_s = 1/m^t + \phi_s$, $s = 1, \dots, m^t$, $x_s = 0$ for $s = m^t + 1, \dots, n$, for all $|\phi_s| < \phi$, $\sum_{s=1}^{m^t} \phi_s = 0$, and simultaneously it is never optimal to postpone harvest after m^t .

In addition, a stationary cycle with all land allocated to forestry must satisfy $\lambda_t \geq 0$, for $t = 0, \dots$. Solving (48) or (49) for λ_t , eliminating $p_{s,t}$, $s = 1, \dots, m^t - 1$, $t = 0, \dots$, using (56), we obtain

$$\lambda_{t+k} = \frac{U'(c_{t+k})b^{m^t} f_{m^t}}{1 - b^{m^t}} - \frac{U'(c_{t+1+k})b^{m^t+1} f_{m^t}}{1 - b^{m^t}} + \frac{\beta p_c E_f}{1 - b^{m^t}} \sum_{i=1}^{m^t-1} b^i (1 - b) f_i - bW'(0) \geq 0 \quad (65)$$

for $s = 1, \dots, m^t$, where $c_{t+1+m^t} = c_{t+1}$. Writing $c_{t+k} = f_{m^t} x_s$ and $c_{t+1+k} = f_{m^t} x_{s-1}$, $s = 1, \dots, m^t$, where $x_0 = x_{m^t}$ yields

$$\lambda_s = \frac{U'(f_{m^t} x_s)b^{m^t} f_{m^t}}{1 - b^{m^t}} - \frac{U'(f_{m^t} x_{s-1})b^{m^t+1} f_{m^t}}{1 - b^{m^t}} + \frac{\beta p_c E_f}{1 - b^{m^t}} \sum_{i=1}^{m^t-1} b^i (1 - b) f_i - bW'(0) \geq 0 \quad (66)$$

for $s = 1, \dots, m^t$.

Given $g = \frac{U'(f_{m^t}/m^t)b^{m^t} f_{m^t}}{1 - b^{m^t}} + \frac{\beta p_c E_f}{1 - b^{m^t}} \sum_{i=1}^{m^t-1} b^i f_i - \frac{b}{1-b} W'(0) > 0$, there must exist a $\phi > 0$ such that (66) is satisfied if $x_s = 1/m^t + \phi_s$, $s = 1, \dots, m^t$, for all $|\phi_s| < \phi$, $\sum_{s=1}^{m^t} \phi_s = 0$. ■

Proof of Corollary 2:

Proof. Given $g \leq 0$, no solutions for (66) exist. Thus, by letting $\lambda_t = 0$ in (48) or (49), eliminating $p_{s,t}$, $s = 1, \dots, m^t - 1$, $t = 0, \dots$, using (56), and writing

(48) analogously to (66), we obtain, for $s = 1, \dots, m^t$:

$$\frac{U'(f_{m^t}x_s)b^{m^t}f_{m^t}}{1-b^{m^t}} - \frac{U'(f_{m^t}x_{s-1})b^{m^t+1}f_m}{1-b^{m^t}} + \frac{\beta p_c E_f}{1-b^{m^t}} \sum_{i=1}^{m^t-1} b^i(1-b)f_i - bW'(y_\infty) \geq 0 \quad (67)$$

This system is linear in $U'(f_{m^t}x_s)$, $s = 1, \dots, m^t$. Its solution is given by

$$U'(f_{m^t}x_s) + \frac{p_c \beta E_f \sum_{i=1}^{m^t-1} b^i f_i}{b^{m^t} f_{m^t}} = \frac{W'(y_\infty) \sum_{i=0}^{m^t-1} b^i}{b^{m^t-1} f_{m^t}}, \quad s = 1, \dots, m^t \quad (68)$$

as can be verified by direct substitution. Thus, $x_s = (1 - y_\infty)/m^t$, $s = 1, \dots, m^t$ and optimal stationary cycles cannot exist. ■

Impact on the optimal rotation period:

We now show that $m^t \geq m$:

At the steady-state, if there exists a $m^t \neq m$, for which

$$\frac{pb^{m^t}f_{m^t}}{1-b^{m^t}} + \frac{p_c \beta E_f \sum_{i=1}^{m^t-1} b^i f_i}{1-b^{m^t}} \geq \frac{pb^m f_m}{1-b^m} + \frac{p_c \beta E_f \sum_{i=1}^{m-1} b^i f_i}{1-b^m} \quad (69)$$

holds, then it is optimal to cut at m^t , where $m^t \geq m$. If (69), which is the same as (16), holds in particular for $m^t = m + 1$, then $m^t \geq m$, while if it holds in particular for $m^t = m - 1$, $m^t \leq m$. If (69) holds as an equality, $m^t = m$.

By making $m^t = m - 1$, we show below that (69) never holds, implying that $m^t \geq m$, that is, it is optimal to postpone harvest. Also, for $m^t = m + 1$, (69) can be satisfied.

Let $m^t = m - 1$ in (69). By rearranging and collecting terms we obtain

$$pb^{m-1}f_{m-1}(1-b^m) - pb^m f_m(1-b^{m-1}) > p_c \beta E_f \left[(b^m - b^{m-1}) \sum_{i=1}^{m-2} b^i f_i + (1-b^{m-1})b^{m-1}f_{m-1} \right] \quad (70)$$

In (70) the left-hand side is negative. Dividing (70) by $b^{m-1}(1-b^{m-1})$, and since $b = \frac{1}{1+r}$, we have that

$$\frac{1}{b^{m-1}} \frac{b^m - b^{m-1}}{1 - b^{m-1}} = b \left[-\frac{r}{1 - b^{m-1}} \right]$$

where the term in square brackets is the equivalent in discrete time to the discounting term $\frac{r}{1-e^{-rT}}$ in the continuous time, as long as $e^r \cong 1 + r$, and $T = m - 1$. Therefore, the right-hand side of (70) becomes

$$P_c \beta E_f \left[b \left[\frac{r}{1 - b^{m-1}} \right] \sum_{i=1}^{m-2} -b^i f_i + f_{m-1} \right]$$

Since

$$\sum_{i=1}^{m-2} b^i f_i = -b \frac{1 - b^{m-1}}{r} f_{m-2} + \sum_{i=1}^{m-2} b^i (f_i - f_{m-2})$$

we obtain

$$P_c \beta E_f \left[\frac{-br}{1 - b^{m-1}} \sum_{i=1}^{m-2} b^i (f_i - f_{m-2}) + \left(f_{m-1} - \frac{b - b^{m-1}}{1 - b^{m-1}} f_{m-2} \right) \right] \quad (71)$$

Since $\{f_i\}$ is an increasing sequence, we may conclude that $f_i - f_{m-2} \leq 0$ for $i = 1, \dots, m - 2$. Since $\frac{b - b^{m-1}}{1 - b^{m-1}} < 1$, the second-term in the expression above is positive. This implies that the first term of the right-hand side of (70) is positive. Notice that if $\{f_i\}$ is strictly increasing and for $m \geq 2$, then it is strictly positive. Since the second term is also positive, the inequality never holds. Moreover, since (70) can be satisfied for $m^t = m + 1$, this implies that $m^t \geq m$. This is also similar to the result obtained in the continuous version of the one stand model, as shown in Costa-Duarte, Cunha-e-Sá and Rosa [6].

Appendix C (Not for Publication)

The Karush-Kuhn-Tucker conditions for optimal solutions of problem (2-8), S_t^a given by (19), for all $t = 0, \dots$, as follows:

$$b^t \frac{\partial L^a}{\partial x_{1,t+1}} = b f_1 U'(c_{t+1}) - b W'(y_{t+1}) - \lambda_t + b p_{1,t+1} + b D \leq 0 \quad (72)$$

$$b^t \frac{\partial L^a}{\partial x_{s+1,t+1}} = -f_s U(c_t) + b f_{s+1} U(c_{t+1}) + bD -$$

$$-bW'(y_{t+1}) - \lambda_t + b p_{s+1,t+1} - p_{s,t} \leq 0 \quad (73)$$

for $s = 1, \dots, n-2$,

$$b^t \frac{\partial L^a}{\partial x_{n,t+1}} = -f_{n-1} U(c_t) + b f_n U(c_{t+1}) + bD - bW'(y_{t+1}) - \lambda_t - p_{n-1,t} \leq 0 \quad (74)$$

$$x_{s,t+1} \geq 0, x_{s,t+1} \frac{\partial L^a}{\partial x_{s,t+1}} = 0, s = 1, \dots, n \quad (75)$$

$$p_{s,t} \geq 0, p_{s,t}(x_{s,t} - x_{s+1,t+1}) = 0, s = 1, \dots, n-1 \quad (76)$$

$$\lambda_t \geq 0, \lambda_t \left(1 - \sum_{s=1}^n x_{s,t+1}\right) = 0 \quad (77)$$

The existence of optimal solutions for bounded utility and $b < 1$ follows from Theorem 4.6 in Stokey and Lucas (p.79).

Proof of Proposition 3:

Proof. Following Salo and Tahvonen [21], for $s = 1, \dots, m^a - 1$ using (72) to eliminate λ_t from (73) and (74), and to satisfy (75) we obtain a system of $m^a \times (m^a - 1)$ equality equations

$$b(p_{s+1,t+1+k} - p_{1,t+1+k}) - p_{s,t+k} = -bU(c_{t+k+1})(f_{s+1} - f_1) + f_s U(c_{t+k}) \quad (78)$$

$$-b p_{1,t+1+k} - p_{m^a-1,t+k} = -bU(c_{t+k+1})(f_{m^a} - f_1) + f_{m^a-1} U(c_{t+k}) \quad (79)$$

where $s = 1, \dots, m^a - 2, k = 0, \dots, m^a - 1,$

This system is linear in the Lagrangian multipliers $p_{s,t+k}, s = 1, \dots, m^a - 1, k = 0, \dots, m^a - 1$. Solving for any multiplier yields

$$p_{s,t} = \frac{b^{m^a} f_{m^a}}{1 - b^{m^a}} [b^{-s} U'(c_{t+m^a-s}) - U'(c_t)] - f_s U'(c_t) \quad (80)$$

for $s = 1, \dots, m^a - 1$, $t = 0, \dots$, as can be verified by direct substitution into the two equations above. condition (76) requires, for the indefinitely repeated cycle, that $p_{s,t+k} \geq 0$ for $s = 1, \dots, m^a - 1$, $k = 0, \dots, m^a - 1$. Thus, the fact that $x \in K$ implies by (80) that

$$\frac{U'(c_{t+k})}{U'(c_{t+k+m^a-j})} \leq \frac{b^{m^a-j} f_m}{f_j + b^{m^a} (f_{m^a} - f_j)} \quad (81)$$

for $k = 0, \dots, m^a - 1$, $j = 1, \dots, m^a - 1$. Using (3) and the definition of carbon harvesting, we can write $c_{t+k} = f_{m^a} x_s$ and $c_{t+k+m^a-j} = f_{m^a} x_{s-m^a+j}$ where $s - m^a + j$ is understood as $s - j$, if $s - m^a + j \leq 0$. Equation (81) takes the form

$$\frac{U'(f_{m^a} x_s)}{U'(f_{m^a} x_{s-1})} \leq \eta_j^a \equiv \frac{b^{m^a-j} f_{m^a}}{f_j + b^{m^a} (f_{m^a} - f_j)} \quad (82)$$

for $s = 1, \dots, m^a$, $j = 1, \dots, m^a - 1$. Since $\eta_j^a > 1$, the right-hand side of (82) is larger than one. Moreover, this is equivalent to (20), as can be easily checked. Then, by the strict concavity of U , there must exist a $\phi > 0$, such that (82) is satisfied if $x_s = 1/m^a + \phi_s$, $s = 1, \dots, m^a$, for all $|\phi_s| < \phi$, $\sum_{s=1}^{m^a} \phi_s = 0$.

Similarly, results can be derived for $s = m^a + 1, \dots, n$, and $k = 0, \dots, m^a - 1$. Following the previous cases, we can show that a similar condition to (82) can be obtained for $s = m^a, \dots, n$, $j = m^a + 1, \dots, n$,

$$\frac{U'(f_{m^a} x_s)}{U'(f_{m^a} x_{s-m^a+j})} \leq \eta_j^a \equiv \frac{b^{m^a-j} f_{m^a}}{f_j + b^{m^a} (f_{m^a} - f_j)} \quad (83)$$

to which all we have shown above for $s = 1, \dots, m^a$, $j = 1, \dots, m^a - 1$, still applies. Consequently, there exists a $\phi > 0$ such that (83) is satisfied if $x_s = 1/m^a + \phi_s$, $s = 1, \dots, m^a$, $x_s = 0$ for $s = m^a + 1, \dots, n$, for all $|\phi_s| < \phi$, $\sum_{s=1}^{m^a} \phi_s = 0$. Moreover, $m^a = m$, as it can be easily observed by comparing (20) with (10).

In addition, a stationary cycle with all land allocated to forestry must satisfy $\lambda_t \geq 0$, for $t = 0, \dots$. Solving (72) or (73) for λ_t , eliminating $p_{s,t}$, $s = 1, \dots, m^a - 1$,

$t = 0, \dots$, using (80), we obtain

$$\lambda_{t+k} = \frac{U'(c_{t+k})b^{m^a} f_{m^a}}{1 - b^{m^a}} - \frac{U'(c_{t+1+k})b^{m^a+1} f_{m^a}}{1 - b^{m^a}} + bD - bW'(0) \geq 0 \quad (84)$$

for $s = 1, \dots, m^a$, where $c_{t+1+m^a} = c_{t+1}$. Writing $c_{t+k} = f_{m^a} x_s$ and $c_{t+1+k} = f_{m^a} x_{s-1}$, $s = 1, \dots, m^a$, where $x_0 = x_{m^a}$ yields

$$\lambda_s = \frac{U'(f_{m^a} x_s) b^{m^a} f_{m^a}}{1 - b^{m^a}} - \frac{U'(f_{m^a} x_{s-1}) b^{m^a+1} f_{m^a}}{1 - b^{m^a}} + bD - bW'(0) \geq 0 \quad (85)$$

for $s = 1, \dots, m^a$.

Given $g = \frac{U'(f_{m^a}/m^a) b^{m^a} f_{m^a}}{1 - b^{m^a}} + \frac{b}{1-b} D - \frac{b}{1-b} W'(0) > 0$, there must exist a $\phi > 0$ such that (85) is satisfied if $x_s = 1/m^a + \phi_s$, $s = 1, \dots, m^a$, for all $|\phi_s| < \phi$, $\sum_{s=1}^{m^a} \phi_s = 0$. ■

Proof of Corollary 3:

Proof. Given $g \leq 0$, no solutions for (85) exist. Thus, by letting $\lambda_t = 0$ in (72) or (73), eliminating $p_{s,t}$, $s = 1, \dots, m^a - 1$, $t = 0, \dots$, using (80), and writing (72) analogously to (85), obtaining:

$$\frac{U'(f_{m^a} x_s) b^{m^a} f_{m^a}}{1 - b^{m^a}} - \frac{U'(f_{m^a} x_{s-1}) b^{m^a+1} f_{m^a}}{1 - b^{m^a}} + bD - bW'(y_\infty) \geq 0 \quad (86)$$

for $s = 1, \dots, m^a$.

This system is linear in $U'(f_{m^a} x_s)$, $s = 1, \dots, m^a$. Its solution is given by

$$U'(f_{m^a} x_s) + \frac{\sum_{i=0}^{m^a-1} b^i}{b^{m^a-1} f_{m^a}} D = \frac{W'(y_\infty) \sum_{i=0}^{m^a-1} b^i}{b^{m^a-1} f_{m^a}}, s = 1, \dots, m^a \quad (87)$$

as can be verified by direct substitution. Thus, $x_s = (1 - y_\infty)/m^a$, $s = 1, \dots, m^a$ and optimal stationary cycles cannot exist. ■

Appendix D

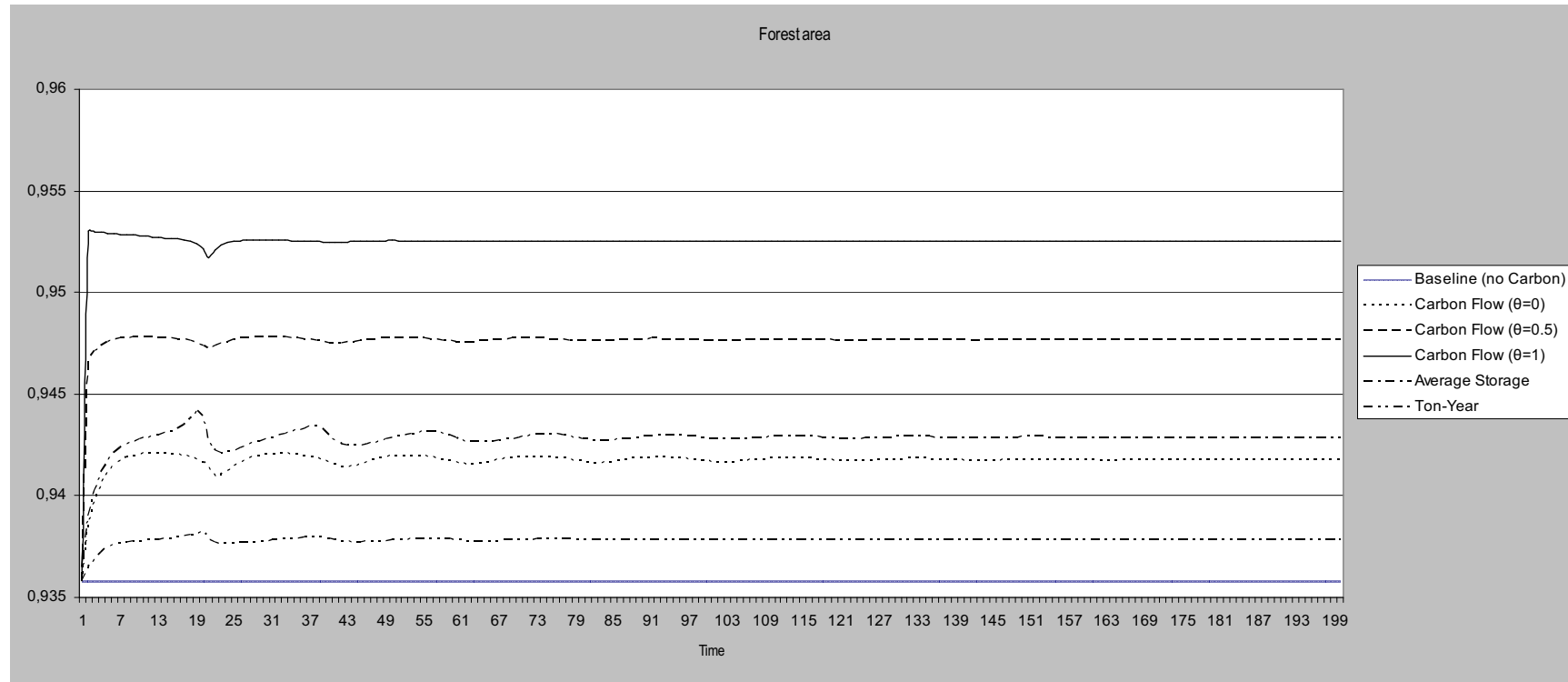


Figure 1. Forest Area

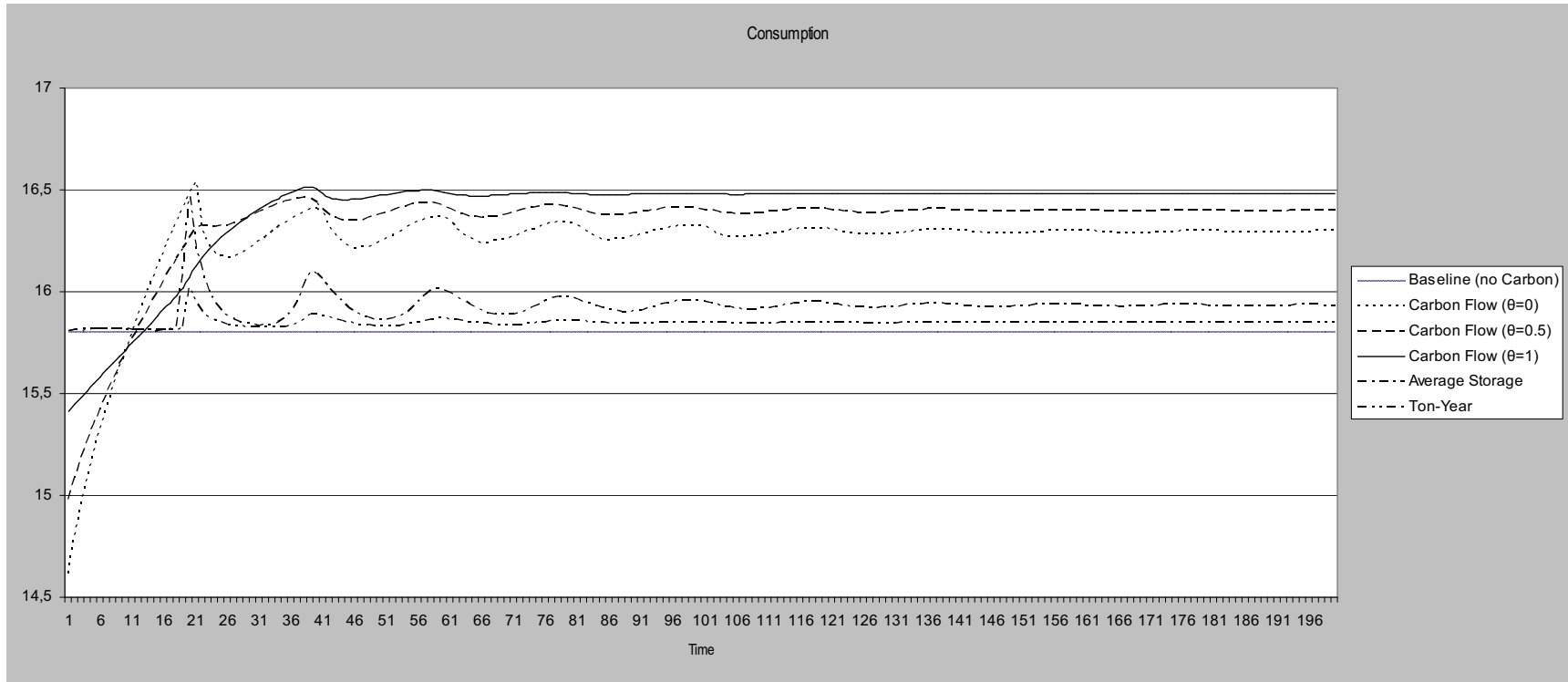


Figure 2. Timber Consumption

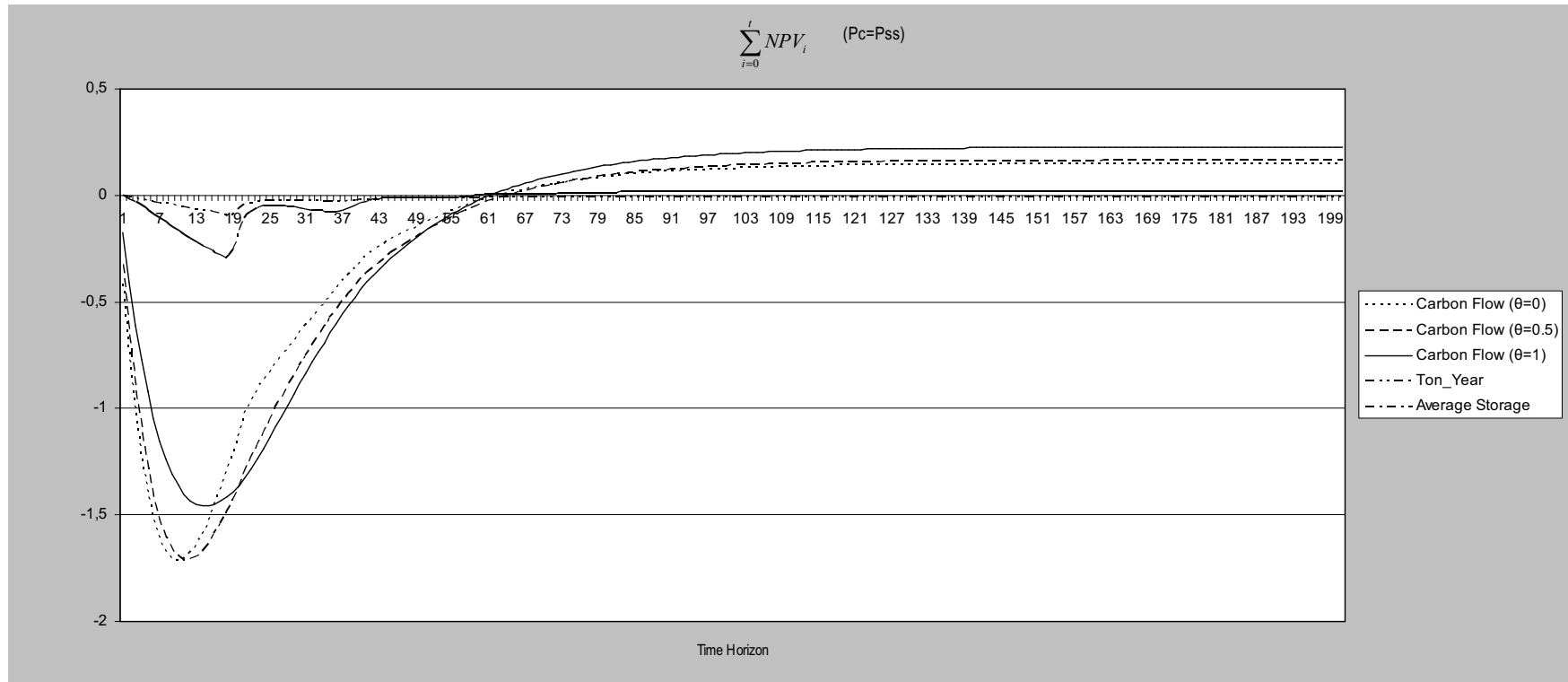


Figure 3. Cumulative Net Present Value

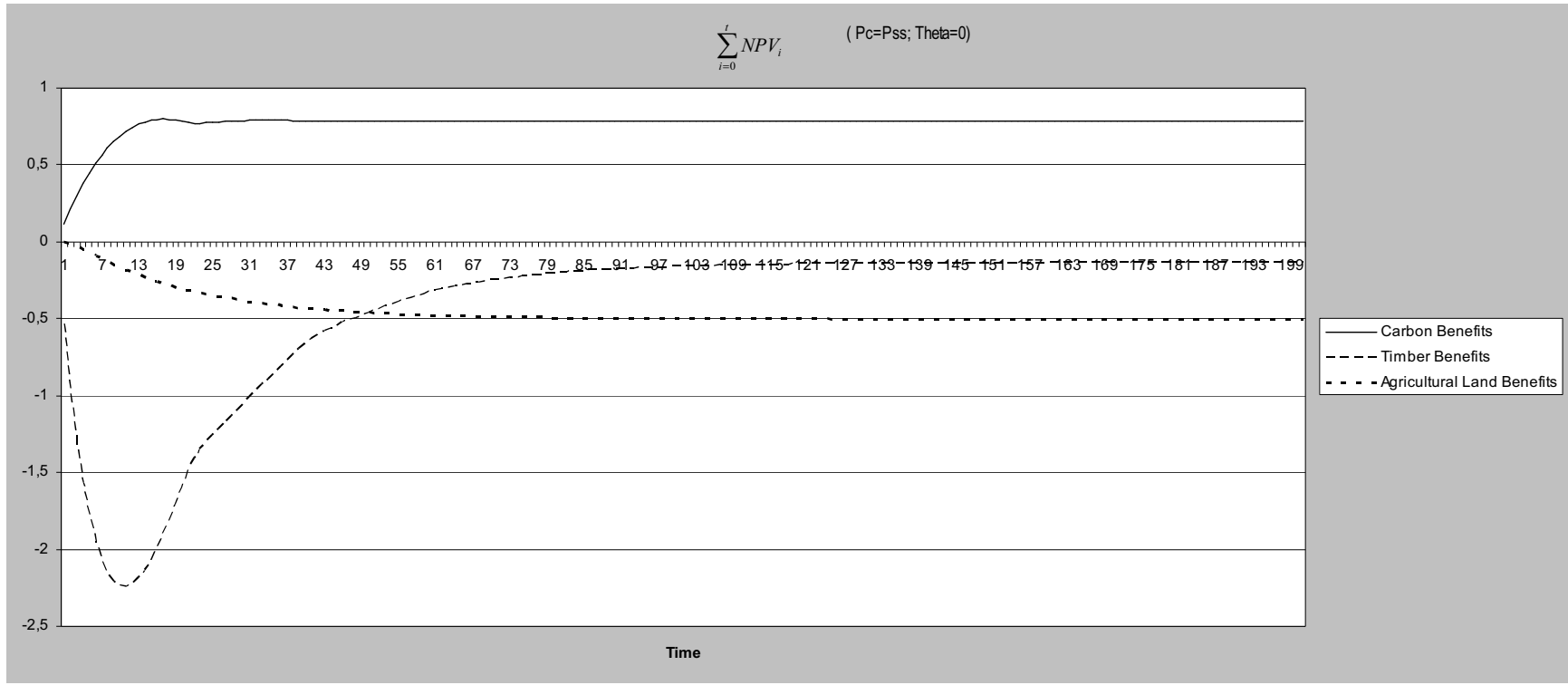


Figure 3a. Cumulative Net Present Value (components) – Carbon Flow Method

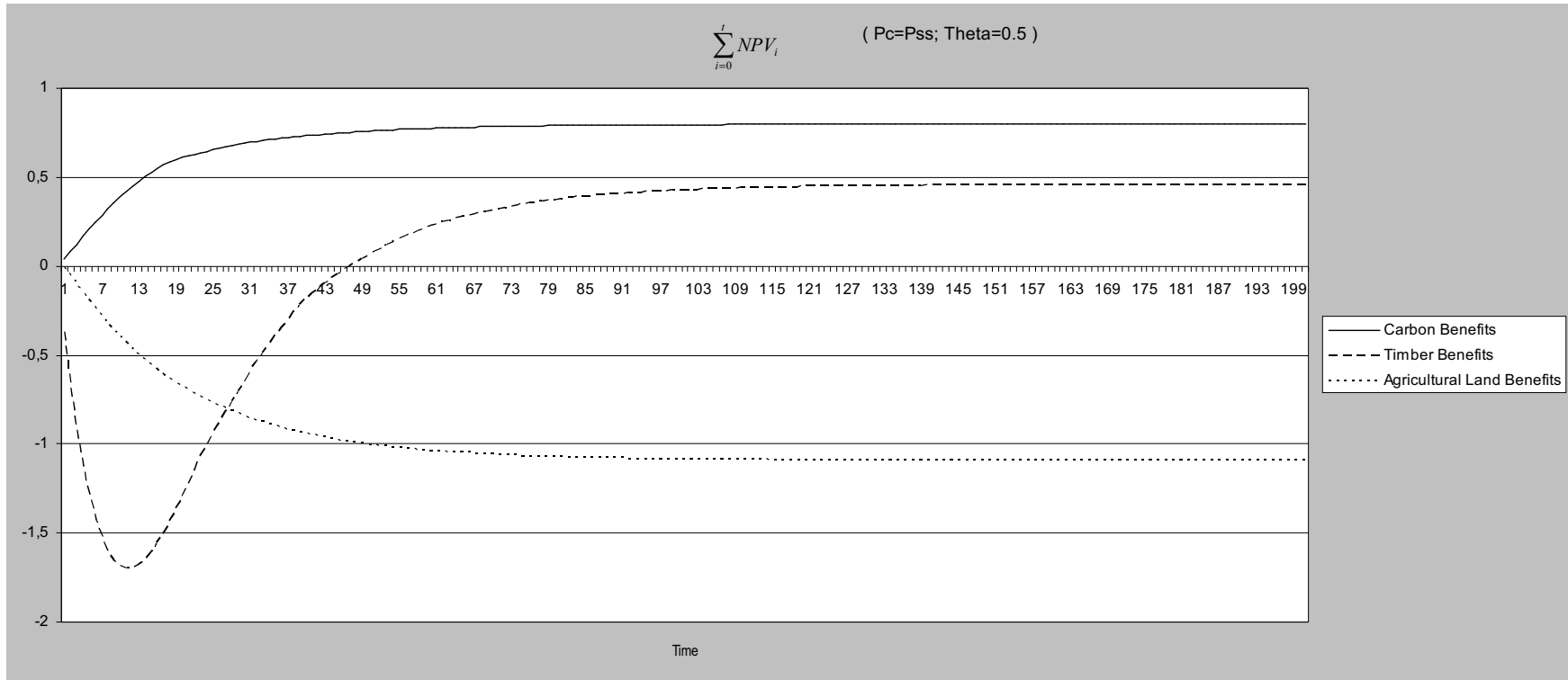


Figure 3b. Cumulative Net Present Value (components) – Carbon Flow Method

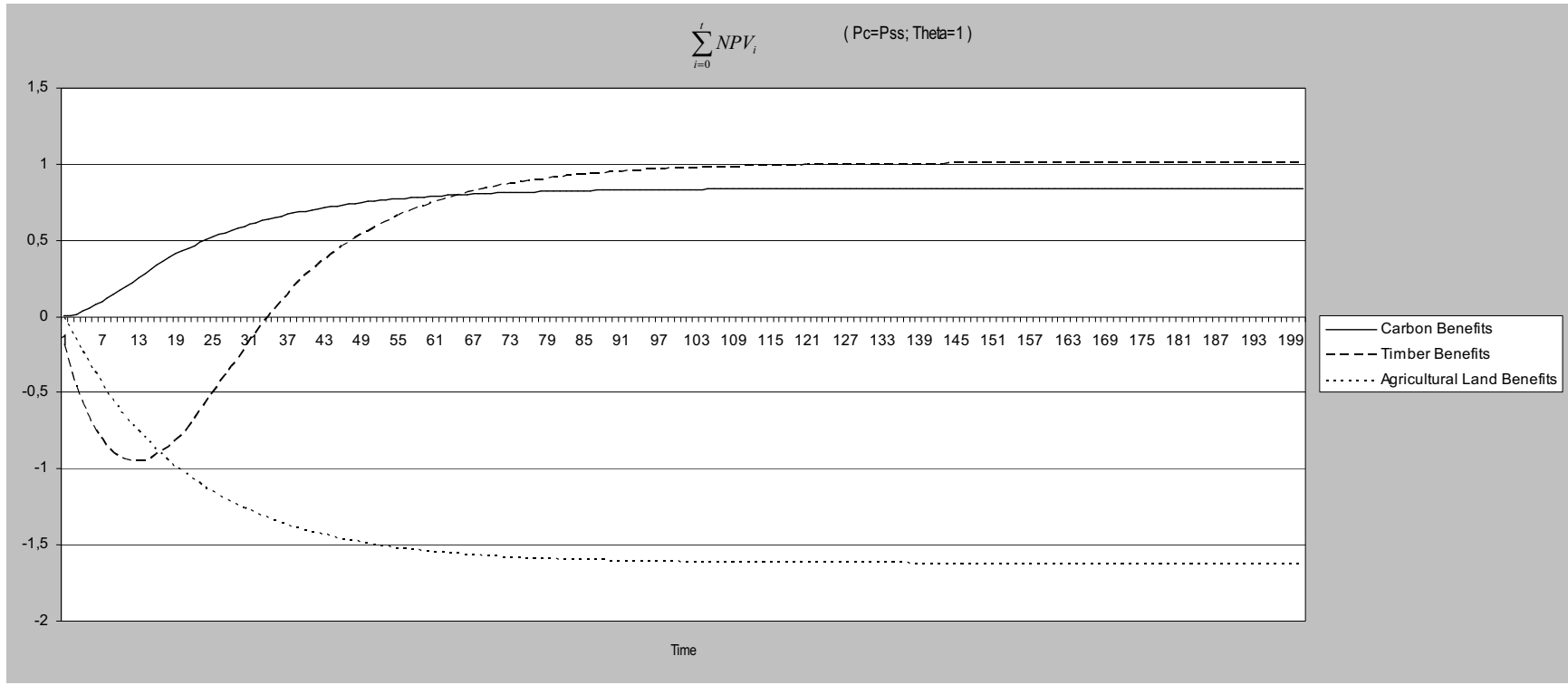


Figure 3c. Cumulative Net Present Value (components) – Carbon Flow Method

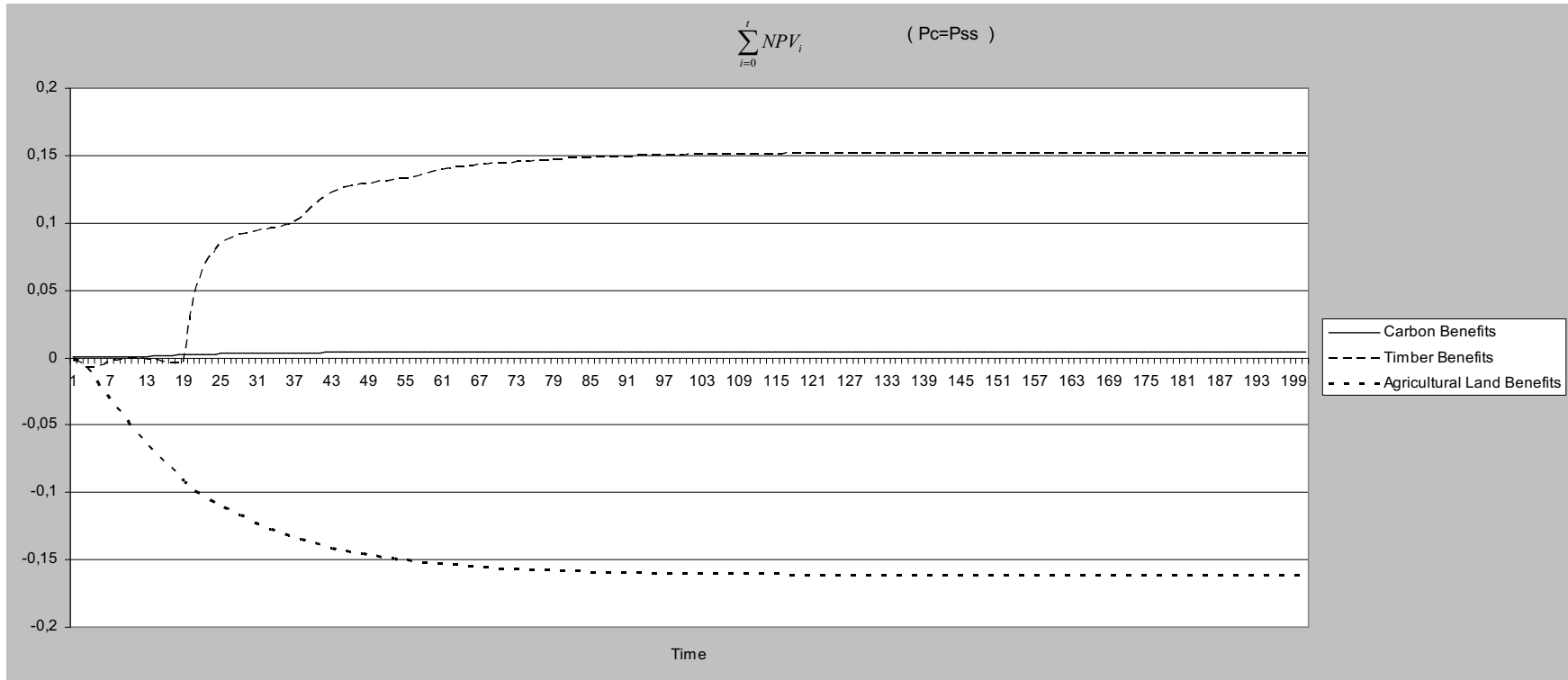


Figure 3d. Cumulative Net Present Value (components) – Ton-Year Method

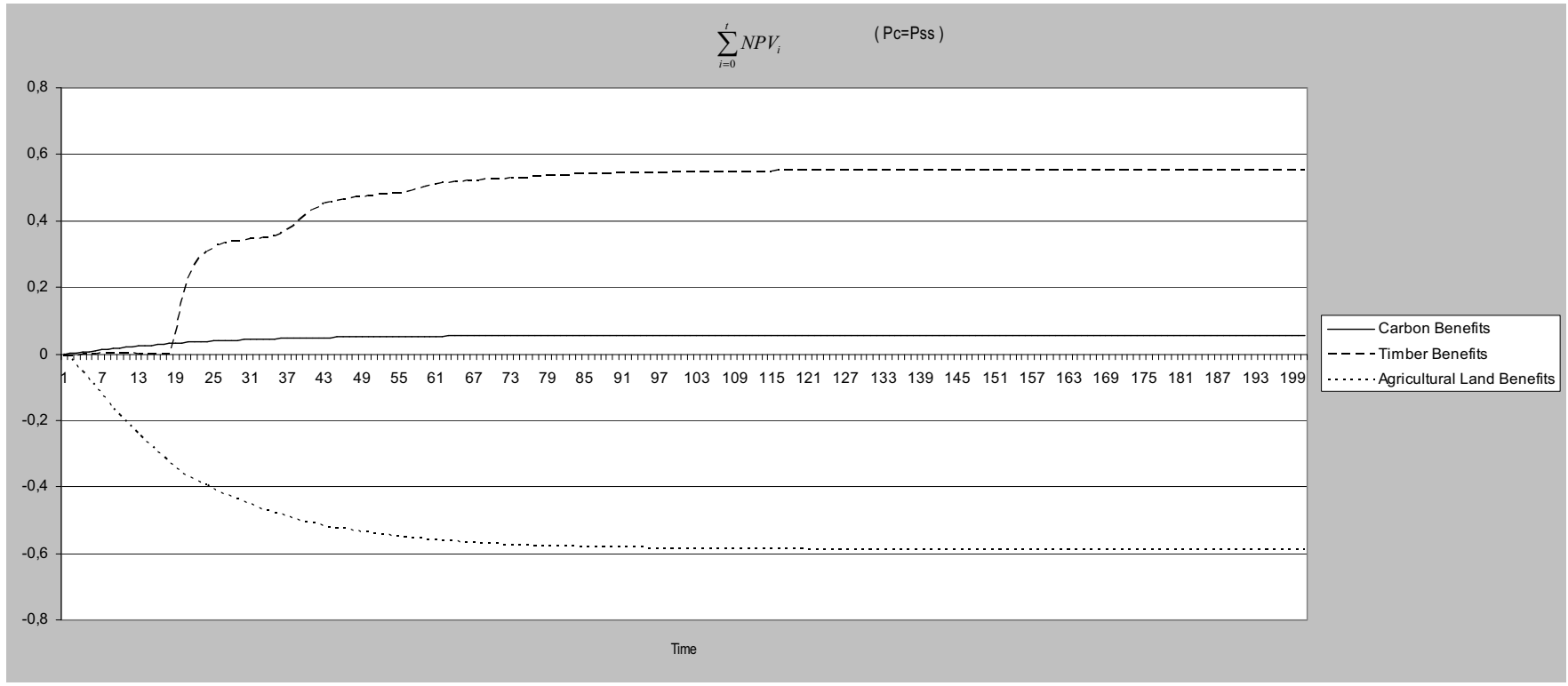


Figure 3e. Cumulative Net Present Value (components) – Average Storage Method

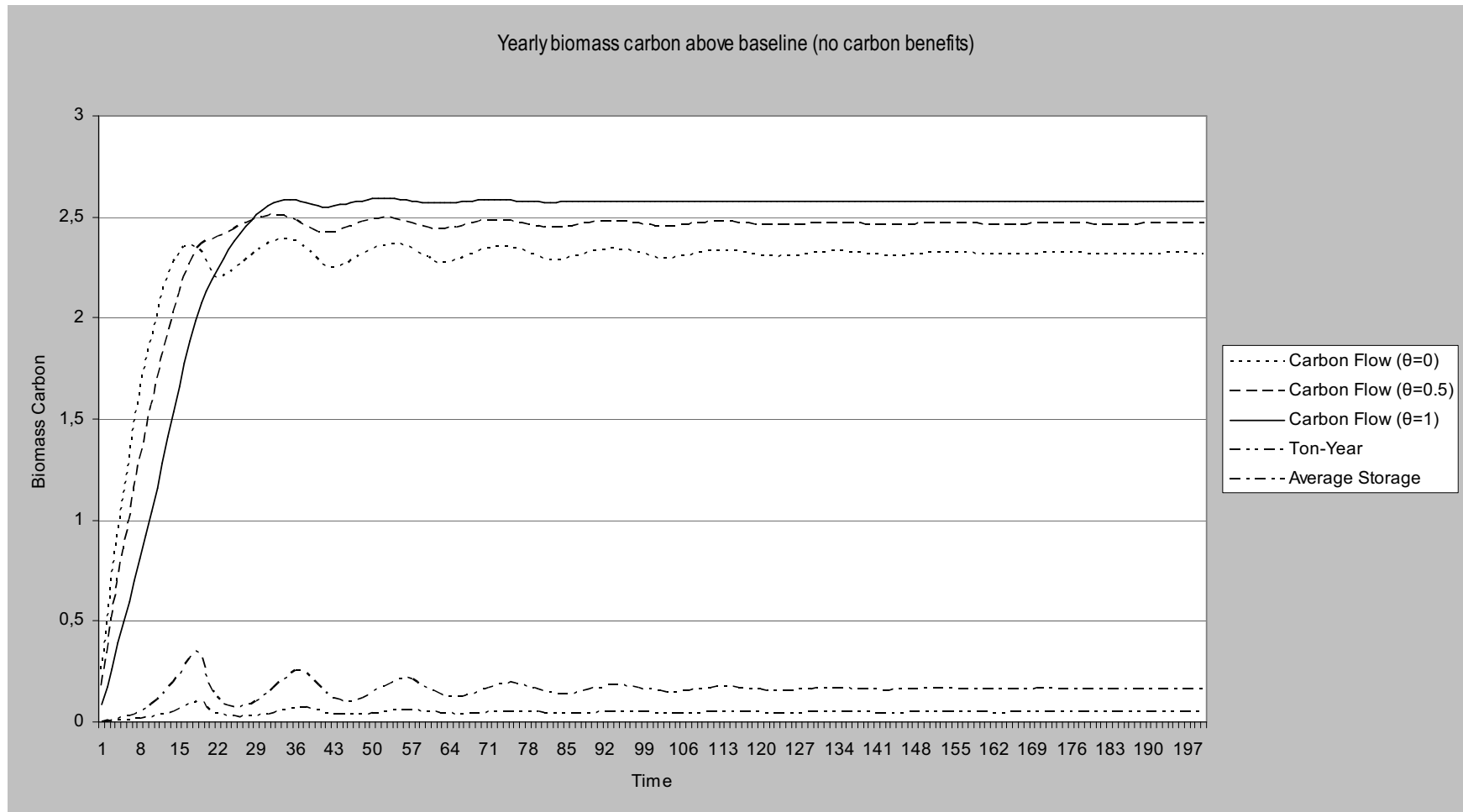


Figure 4. Yearly biomass carbon

Ton-Year					Average				
	No carbon benefits	Pc_Pss	Pc_2Pss	Pc_6Pss	No carbon benefits	Pc_Pss	Pc_2Pss	Pc_6Pss	
Forested area (1 - y^∞)	0,9358	0,93783	0,93945	0,94616	0,9358	0,94286	0,94864	0,96395	
Optimal rotation	19	19	19	21	19	19	19	19	
Area by age x_s, ∞	0,04925	0,04936	0,04944	0,0451	0,04925	0,049623	0,049928	0,050736	
Price of timber	0,4368	0,4365	0,4363	0,43003	0,4368	0,43582	0,43504	0,43336	
Unit cost of land $W'(y^\infty)$	4,4971	4,6143	4,713	5,18	4,4971	4,9363	5,3763	7,136	
Timber Consumption	15,814	15,848	15,875	16,67	15,814	15,933	16,029	16,237	
Net Cumulative Biomass Carbon	0	9,279	16,932	909,93	0	31,308	56,899	124,42	

Carbon Flow

Pc=Pss					Pc=2Pss				
	No carbon benefits	θ			No carbon benefits	θ			
		0	0,5	1		0	0,5	1	
Forested area (1 - y^∞)	0,9358	0,94177	0,94767	0,95252	0,9358	0,94748	0,95666	0,96333	
Optimal rotation	19	20	20	20	19	21	21	20	
Area by age x_s, ∞	0,04925	0,04709	0,04738	0,04762	0,04925	0,045119	0,045554	0,048156	
Price of timber	0,4368	0,43288	0,43207	0,43143	0,4368	0,42985	0,42853	0,42984	
Unit cost of land $W'(y^\infty)$	4,4971	4,8631	5,2964	5,7252	4,4971	5,2831	6,1578	7,0389	
Timber Consumption	15,814	16,297	16,399	16,481	15,814	16,683	16,854	16,684	
Net Cumulative Biomass Carbon	0	451,19	473,85	484,41	0	916,74	953,72	575,47	

Pc=4Pss*					Pc=6Pss*				
	No carbon benefits	θ			No carbon benefits	θ			
		0	0,5	1		0	0,5	1	
Forested area (1 - y^∞)	0,9358	0,95694	0,96851	0,97582	0,9358	0,96439	0,97573	0,98218	
Optimal rotation	19	23	21	21	19	24	22	21	
Area by age x_s, ∞	0,04925	0,04161	0,04612	0,04647	0,04925	0,04	0,044348	0,046766	
Price of timber	0,4368	0,42758	0,42742	0,42726	0,4368	0,4273	0,42524	0,42344	
Unit cost of land $W'(y^\infty)$	4,4971	6,1898	7,9521	9,8215	4,4971	7,2058	9,7921	12,541	
Timber Consumption	15,814	16,98	17,002	17,022	15,814	17,031	17,294	17,54	
Net Cumulative Biomass Carbon	0	1807,9	1054,3	1082,7	0	2293,3	1540,3	1127,1	

*These values are obtained after 300 iterations

*These values are obtained after 500 iterations

The Net Cumulative Biomass Carbon is calculated as the sum of the yearly sequestered biomass carbon above the one resulting from the baseline scenario(no carbon benefits). The considered time horizon is of 200 years.

Table 1. Simulation Results

$\theta=0$				$Pc=Pss$ $\theta=0,5$				$\theta=1$			
	v^i	forest area	rotation		v^i	forest area	rotation		v^i	forest area	rotation
Carbon Flow	226,3744	0,94177	20	Carbon Flow	240,2064	0,94767	20	Carbon Flow	254,0965	0,95252	20
Ton-year	226,2331	0,93783	19	Ton-year	240,0632	0,93783	19	Ton-year	253,8933	0,93783	19
Average	226,2407	0,94286	19	Average	240,111	0,94286	19	Average	253,9813	0,94286	19
Deviations from optimal solution											
		Ton-year	Average			Ton-year	Average			Ton-year	Average
Welfare Loss				Welfare Loss				Welfare Loss			
$[(v^i - v^{cf})/v^{cf}]$		-0,06%	-0,06%	$[(v^i - v^{cf})/v^{cf}]$		-0,06%	-0,04%	$[(v^i - v^{cf})/v^{cf}]$		-0,08%	-0,05%
$(\text{forest area}^i - \text{forest area}^{cf})/\text{forest area}^{cf}$		-0,39%	0,11%	$(\text{forest area}^i - \text{forest area}^{cf})/\text{forest area}^{cf}$		-0,98%	-0,48%	$(\text{forest area}^i - \text{forest area}^{cf})/\text{forest area}^{cf}$		-1,47%	-0,97%

$\theta=0$				$Pc=2Pss$ $\theta=0,5$				$\theta=1$			
	v^i	forest area	rotation		v^i	forest area	rotation		v^i	forest area	rotation
Carbon Flow	227,6339	0,94748	21	Carbon Flow	255,2039	0,95666	21	Carbon Flow	282,998	0,96333	20
Ton-year	226,2678	0,93945	19	Ton-year	253,9539	0,93945	19	Ton-year	281,64	0,93945	19
Average	226,2992	0,94864	19	Average	254,1353	0,94864	19	Average	281,9713	0,94864	19
Deviations from optimal solution											
		Ton-year	Average			Ton-year	Average			Ton-year	Average
Welfare Loss				Welfare Loss				Welfare Loss			
$[(v^i - v^{cf})/v^{cf}]$		-0,60%	-0,59%	$[(v^i - v^{cf})/v^{cf}]$		-0,49%	-0,42%	$[(v^i - v^{cf})/v^{cf}]$		-0,48%	-0,36%
$(\text{forest area}^i - \text{forest area}^{cf})/\text{forest area}^{cf}$		-0,80%	0,12%	$(\text{forest area}^i - \text{forest area}^{cf})/\text{forest area}^{cf}$		-1,72%	-0,80%	$(\text{forest area}^i - \text{forest area}^{cf})/\text{forest area}^{cf}$		-2,39%	-1,47%

$\theta=0$				$Pc=6Pss$ $\theta=0,5$				$\theta=1$			
	v^i	forest area	rotation		v^i	forest area	rotation		v^i	forest area	rotation
Carbon Flow	240,1809	0,96439	24	Carbon Flow	319,9694	0,97573	22	Carbon Flow	402,9598	0,98218	21
Ton-year	234,4237	0,94616	21	Ton-year	316,7785	0,94616	21	Ton-year	399,1327	0,94616	21
Average	226,8387	0,96395	19	Average	311,1256	0,96395	19	Average	395,4119	0,96395	19
Deviations from optimal solution											
		Ton-year	Average			Ton-year	Average			Ton-year	Average
Welfare Loss				Welfare Loss				Welfare Loss			
$[(v^i - v^{cf})/v^{cf}]$		-2,40%	-5,56%	$[(v^i - v^{cf})/v^{cf}]$		-1,00%	-2,76%	$[(v^i - v^{cf})/v^{cf}]$		-0,95%	-1,87%
$(\text{forest area}^i - \text{forest area}^{cf})/\text{forest area}^{cf}$		-1,82%	-0,04%	$(\text{forest area}^i - \text{forest area}^{cf})/\text{forest area}^{cf}$		-2,96%	-1,18%	$(\text{forest area}^i - \text{forest area}^{cf})/\text{forest area}^{cf}$		-3,60%	-1,82%

* The welfare values obtained for the ton-year and average storage method are calculated using the carbon flow accounting to obtain the benefits of the internalization of the carbon externality (S).

Table 2. Welfare deviations: Ton-year vs Carbon Flow / Average Storage vs Carbon Flow

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