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## **Beyond the Home Market Effect: Market Size and Specialization in a Multi- Country World**

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By **Kristian Behrens**, Department of  
Economics, Université du Québec à  
Montréal, Canada, CORE, Université  
catholique de Louvain, Belgium,  
CIRPÉE, Canada and CEPR, UK

**Andrea R. Lamorgese**, Research  
Department, Bank of Italy

**Gianmarco I.P. Ottaviano**,  
Department of Economics and KITeS,  
Bocconi University, FEEM, Italy and  
CEPR, UK

**Takatoshi Tabuchi**, Faculty of  
Economics, University of Tokyo, Japan

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By Kristian Behrens, Department of Economics, Université du Québec à Montréal, Canada, CORE, Université catholique de Louvain, Belgium, CIRPÉE, Canada and CEPR, UK

Andrea R. Lamorgese, Research Department, Bank of Italy

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Takatoshi Tabuchi, Faculty of Economics, University of Tokyo, Japan

#### Summary

The standard two-country model of international trade with monopolistic competition predicts a more-than-proportional relationship between a country's share of world production of a good and its share of world demand for that same good, a result known as the "home market effect". We first show that this prediction does not generally carry through to the multi-country case, as production patterns are crucially affected by third country effects. We then derive an alternative prediction that holds whatever the number of countries considered. This new prediction takes into account important features of the real world such as comparative advantage due to cross-country technological differences and lack of factor price equalization.

**Keywords:** Comparative Advantage, Home Market Effect, Hub Effect, International Trade, Monopolistic Competition, Multi-country Models

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*Address for correspondence:*

Gianmarco I.P. Ottaviano

Department of Economics

Bocconi University

Via Roentgen 1, Floor 5, Room C1-09

20136 Milan

Italy

E-mail: gianmarco.ottaviano@unibocconi.it

The opinions expressed in this paper do not necessarily reflect the position of

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Corso Magenta, 63, 20123 Milano (I), web site: [www.feem.it](http://www.feem.it), e-mail: [working.papers@feem.it](mailto:working.papers@feem.it)

# Beyond the Home Market Effect: Market Size and Specialization in a Multi-Country World

Kristian Behrens\*    Andrea R. Lamorgese<sup>†</sup>  
Gianmarco I.P. Ottaviano<sup>‡</sup>    Takatoshi Tabuchi<sup>§</sup>

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## Abstract

The standard two-country model of international trade with monopolistic competition predicts a more-than-proportional relationship between a country's share of world production of a good and its share of world demand for that same good, a result known as the 'home market effect'. We first show that this prediction does not generally carry through to the multi-country case, as production patterns are crucially affected by third country effects. We then derive an alternative prediction that holds whatever the number of countries considered. This new prediction takes into account important features of the real world such as comparative advantage due to cross-country technological differences and lack of factor price equalization.

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\*Department of Economics, Université du Québec à Montréal (Canada); CORE, Université catholique de Louvain (Belgium); CIRPÉE (Canada); and CEPR (UK).

<sup>†</sup>Research Department, Bank of Italy (Italy).

<sup>‡</sup>Department of Economics and KITEs, Bocconi University (Italy); FEEM (Italy); and CEPR (UK). *Corresponding author:* Bocconi University, Department of Economics, Via Roentgen 1, Floor 5, Room C1-09, 20136 Milan, Italy. Phone: +39-02-58363385. Fax: +39-02-58363309. E-mail: gianmarco.ottaviano@unibocconi.it

<sup>§</sup>Faculty of Economics, University of Tokyo (Japan).

# 1 Introduction

Since Krugman (1980), general equilibrium models of international trade with increasing returns to scale and trade costs have been associated with what has come to be known as the ‘home market effect’ (henceforth, HME). This effect is generally defined as “a more-than-proportional relationship between a country’s share of world production of a good and its share of world demand for the same good” (Crozet and Trionfetti, 2008, p.2).<sup>1</sup> As a result, “countries will tend to export those kinds of products for which they have relatively large domestic demand” (Krugman, 1980, p.955).

The basic HME model is traditionally considered to be the one proposed by Helpman and Krugman (1985) in the wake of Krugman (1980). Their setup features two countries and two sectors employing labor as their only input. One sector supplies a freely-traded homogeneous good under constant returns to scale and perfect competition, whereas the other sector produces a horizontally differentiated good under increasing returns and monopolistic competition à la Dixit and Stiglitz (1977). Preferences are Cobb-Douglas across the two goods and symmetric CES across varieties of the differentiated good. For each variety of the differentiated good, fixed and marginal input requirements are constant and identical across countries. International trade in that good is hampered by frictional trade costs of the ‘iceberg’ type, whereas the homogenous good can be traded freely. The latter assumption leads to factor price equalization (henceforth, FPE) across countries, i.e., labor earns the same wage everywhere. When taken together, FPE, trade costs and a fixed input requirement imply that the larger country supports, in equilibrium, the production of a more than proportionate number of differentiated varieties. This makes the larger country a net exporter of the differentiated good as, due to symmetry, output per variety is identical across countries while demand is proportionate to country size.

The string of restrictive assumptions underlying the basic HME model is quite long. It concerns: (i) preferences; (ii) market structure; (iii) the existence of a freely traded good; (iv) factor price equalization; and (v) the focus on just two countries.<sup>2</sup> Given the central role played by the HME in new trade theory, a key issue has therefore become the extent to which this result survives changes in those assumptions. The literature has thus far made progress on the first four issues.

Concerning preferences, Helpman (1990) specifies the demand conditions under which the

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<sup>1</sup>There is an alternative definition of the HME that captures the impact of country size on wages when these are not equalized (Krugman, 1980). We discuss the issue of factor price equalization below. See also footnotes 4 and 6 for further details.

<sup>2</sup>The basic HME model considers final goods only. However, it is homomorphic to a model in which the differentiated final good is replaced by a homogenous one and this is produced by assembling the varieties of a horizontally differentiated intermediate. See Ethier (1982).

HME materializes: the cross-elasticity between varieties of the differentiated good must exceed the overall price-elasticity of demand for the differentiated good as a whole. Replacing the upper-tier Cobb-Douglas preferences with a CES function, Yu (2005) finds that the value of the elasticity of substitution across the homogeneous and the differentiated goods matters for the existence of the HME. Head *et al.* (2002) show that, when goods are differentiated according to their place of production (as in Armington, 1969) rather than according to the firms producing them (as in Dixit and Stiglitz, 1977), the HME may also vanish. Finally, Ottaviano and van Ypersele (2005) show that CES preferences, leading to fixed markups over marginal cost, are not needed to generate a HME.

As for market structure, Feenstra *et al.* (2001) as well as Head *et al.* (2002) show that monopolistic competition per se is not crucial in that the HME can arise even in homogenous-good sectors with restricted entry and Cournot competition. All that matters is the presence of positive price-cost margins and trade costs.

The role of the freely traded homogeneous good produced by the perfectly competitive sector, the so-called ‘outside good’, has also been analyzed in detail. Its existence leads to FPE as long as the good is produced in both countries. The outside good also allows for international specialization as it absorbs the trade imbalances arising in the Dixit-Stiglitz sector. Extending previous insights by Davis (1998), Crozet and Trionfetti (2008) introduce Armington differentiation and ‘iceberg’ trade costs in the homogenous good sector, thus preventing FPE from holding in general.<sup>3</sup> Their set-up generates the results in Davis (1998) and Helpman and Krugman (1985) as special cases when, respectively, there is no Armington differentiation and there is neither Armington differentiation nor trade costs for the outside good. Through numerical analysis they show that the HME survives, with the qualification that it is stronger for countries whose demands deviate more significantly from the average. Accordingly, “the outside good assumption, although clearly at odds with reality, does not affect qualitatively the results concerning international specialization and the direction of trade [so that] its pervasive use is justifiable on the ground of algebraic convenience” (Crozet and Trionfetti, 2008, p.21).

The survival of the HME in a multi-country set-up is, instead, still a much neglected issue. This is surprising both because of its importance for empirical analysis (see, e.g., Davis and Weinstein, 1999 and 2003; Head and Mayer, 2004; Crozet and Trionfetti, 2008) and because of the early doubts on its theoretical robustness (Krugman, 1993). Our aim is to fill this important gap in the theoretical and empirical exploration of the predictions of international trade models with monopolistic competition. In so doing, we start by showing that the HME prediction does not generally carry through to the multi-country case, as production patterns are crucially affected

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<sup>3</sup>See Picard and Zeng (2005) for an analysis of the issue when utility is quasi-linear quadratic and the homogenous good incurs linear trade costs.

by third country effects. Then we derive an alternative prediction that holds whatever the number of countries considered. This prediction takes also into account other important features of the real world such as the cross-country variations in Ricardian comparative and absolute advantages leading to the violation of FPE. In particular, we show that the model predicts the existence of a more-than-proportional relationship between a country's share of world demand and its share of world production only after the impacts of third country effects and comparative advantage are controlled for, which can be achieved through a simple linear filter.

Two modelling choices make our results analytically neat. First, we maintain the assumption of a freely traded outside good. As argued by Crozet and Trionfetti (2008), this is not likely to substantively affect our results. Second, following Deardorff (1984) and Treffer (1995), we allow for the violation of FPE by introducing Ricardian differences in technology that generate international wage differences that are invariant to international sectoral specialization.<sup>4</sup>

The remainder of the paper is divided into four sections. Section 2 extends the model by Helpman and Krugman (1985) to a set-up with an arbitrary number of countries and Ricardian differences in technology. Section 3 characterizes the equilibrium of the extended model. Section 4 first shows that the HME is not a general property of the equilibrium. Then it explains how a more-than-proportional relationship between a country's share of world demand and its share of world production always emerges after controlling for third country effects and technological differences. Section 5 concludes.

## 2 An extended Helpman-Krugman model

The world economy consists of  $M$  countries indexed  $i = 1, 2, \dots, M$ . Country  $i$  hosts an exogenously given mass of  $L_i > 0$  consumers, each of whom supplies one unit of labor inelastically. Hence, both the world population and the world labor endowment are given by  $L = \sum_i L_i$ . Labor is the only factor of production, is assumed to be internationally immobile and its services are traded in perfectly competitive national labor markets.

Preferences are defined over a homogenous outside good ( $H$ ) and over a continuum of varieties of a horizontally differentiated good ( $D$ ). The preferences of a typical resident of country  $i$  are

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<sup>4</sup>When there is no freely traded outside good, factor prices react to changes in specialization, which requires analyzing the so-called 'wage equations'. These are transcendental and cannot be solved analytically (see, e.g., Fujita *et al.*, 1999, p.55). Hanson and Xiang (2004) have recently used the wage equations in a two-country setting to derive theoretical predictions about the HME when there is a continuum of industries that differ with respect to the degree of product differentiation and trade costs. Unfortunately, the analyses of Laussel and Paul (2007) and Crozet and Trionfetti (2008), again in the two-country case, suggest that general analytical results cannot be derived for an arbitrary number of countries.

represented by the following utility function:

$$U_i = H_i^{1-\mu} D_i^\mu, \quad 0 < \mu < 1. \quad (1)$$

In expression (1),  $D_i$  is a CES subutility defined over the varieties of the horizontally differentiated good as follows:

$$D_i = \left[ \sum_j \left( \int_{\Omega_j} d_{ji}(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right) \right]^{\frac{\sigma}{\sigma-1}},$$

where  $d_{ji}(\omega)$  is the consumption in country  $i$  of variety  $\omega$  produced in country  $j$ , and  $\Omega_j$  is the set of varieties produced in country  $j$  with  $j = 1, 2, \dots, M$ . The parameter  $\sigma > 1$  measures both the constant own-price elasticity of demand for any variety, and the elasticity of substitution between any two varieties.

The production of any variety of the differentiated good takes place under increasing returns to scale by a set of monopolistically competitive firms. This set is endogenously determined in equilibrium by free entry and exit. In what follows, we denote by  $n_i$  the mass of firms located in country  $i$ .

Production of each variety requires a fixed and a constant marginal labor requirements,  $f_i > 0$  and  $c_i > 0$  respectively, which may be country-specific. The ratio  $f_i/c_i$  measures the intensity of increasing returns to scale. These are assumed to be sector-specific as they depend on the state of technology and are common across countries. Increasing returns to scale and costless product differentiation yield a one-to-one relationship between firms and varieties, so we will use the two terms interchangeably. As to trade barriers, international shipments of any variety are subject to ‘iceberg’ trade costs:  $\tau_{ji} \geq 1$  units have to be shipped from country  $j$  to country  $i$  for one unit to reach its destination.

Given our assumptions, in equilibrium firms in each sector differ only by the country they are located in. Accordingly, to simplify notation, we drop the variety label  $\omega$  from now on. Then, the maximization of (1) subject to the budget constraint yields the following demand in country  $j$  for a variety produced in country  $i$ :

$$d_{ij} = \frac{p_{ij}^{-\sigma}}{P_j^{1-\sigma}} \mu E_j, \quad (2)$$

where  $p_{ij}$  is the delivered price of the variety,  $E_j$  is aggregate expenditure in country  $j$ , and  $P_j$  is the CES price index in country  $j$ , given by

$$P_j = \left[ \sum_k n_k p_{kj}^{1-\sigma} \right]^{1/(1-\sigma)}. \quad (3)$$

Because of the iceberg assumption, a typical firm established in country  $i$  has to produce  $x_{ij} = d_{ij}\tau_{ij}$  units to satisfy final demand  $d_{ij}$  in country  $j$ . The firm takes (2) into account when

maximizing its profit given by

$$\Pi_i = \sum_j (p_{ij}d_{ij} - w_i c_i x_{ij}) - w_i f_i = \sum_j (p_{ij} - w_i c_i \tau_{ij}) \frac{p_{ij}^{-\sigma}}{P_j^{1-\sigma}} \mu E_j - w_i f_i, \quad (4)$$

where  $w_i$  is the wage in country  $i$ . Profit maximization with respect to  $p_{ij}$ , taking  $P_j$  as given because of the continuum of varieties, then implies that the price per unit delivered is:

$$p_{ij} = \frac{\sigma}{\sigma - 1} w_i c_i \tau_{ij}. \quad (5)$$

Due to free entry and exit, profits must be non-positive in equilibrium. Then (4) and (5) imply that firms' equilibrium scale of operation in country  $i$  must satisfy:

$$\sum_j d_{ij} \tau_{ij} \leq \frac{f_i (\sigma - 1)}{c_i}. \quad (6)$$

In other words, total firm production inclusive of the amount of output lost in transit must be large enough for operating profits to cover the fixed costs of production. The fact that the ratio  $f_i/c_i$  determines the equilibrium scale of production justifies its choice as a measure of the intensity of increasing returns to scale.

Let  $\phi_{ij} \equiv \tau_{ij}^{1-\sigma}$  be a measure of trade freeness, valued one when trade is free and limiting zero when trade is prohibitively costly. Replacing (2) as well as (3) into (6), multiplying both sides by  $p_{ii} > 0$ , and using (5) as well as the income identity  $E_j \equiv L_j w_j$ , we then get:

$$\sum_j \frac{(c_i w_i)^{-\sigma} \phi_{ij} L_j w_j}{\sum_k n_k (c_k w_k)^{1-\sigma} \phi_{kj}} \leq \frac{\sigma f_i}{\mu c_i} \quad (7)$$

with equality if  $n_i > 0$ , for  $i = 1, 2, \dots, M$ .

Turning to the homogenous good  $H$ , this is produced by perfectly competitive firms under constant returns to scale with  $z_i$  denoting the corresponding unit labor requirement in country  $i$ . The ratio  $z_i/c_i$  measures the relative productivity (comparative advantage) of country  $i$  in the differentiated sector. Good  $H$  can be traded freely across countries and we choose it as numéraire. Hence, its price must be equalized to one across markets:  $p_i^H = 1$ . Marginal cost pricing then implies  $p_i^H = z_i w_i$ . Therefore,  $w_i = 1/z_i$  must hold in all countries, provided that some numéraire production takes place everywhere. We henceforth assume this to be the case.<sup>5</sup> This provides us with a simple way to account for international factor price differences driven by Ricardian variations in labor productivity (see Treffer, 1993 and 1995, for supportive evidence). Accordingly, compared with another country  $j$ , country  $i$  is said to exhibit an 'absolute advantage' in the differentiated good sector whenever  $z_i < z_j$ .

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<sup>5</sup>See Appendix A for the formal conditions.

### 3 Equilibrium

Given  $w_i = 1/z_i$ , we can rewrite the equilibrium conditions (7) as:

$$\frac{\mu(a_i)^\sigma}{\sigma r} \sum_j \frac{\phi_{ij} \frac{L_j}{z_j}}{\sum_k n_k (a_k)^{\sigma-1} \phi_{kj}} \leq 1, \quad (8)$$

where  $r \equiv f_i/c_i$  measures the intensity of increasing returns to scale, which is assumed to be the same across countries, and  $a_i \equiv z_i/c_i$  measures the relative productivity of country  $i$  in sector  $D$ . Accordingly, compared with another country  $j$ , country  $i$  is said to exhibit a ‘comparative advantage’ in the differentiated good sector whenever  $a_i > a_j$ .

Conditions (8) define a system of  $M$  conditions in  $M$  unknown  $n_i$  with exogenously given country characteristics, namely, sizes  $L_i$ , trade freeness measures  $\phi_{ij}$ , Ricardian coefficients  $a_i$  and  $z_i$ . Intuitively, consider the point of view of a firm based in country  $i$ . The ratio  $L_j/z_j$  represents the expenditures in country  $j$  where our firm competes with the all other firms based in the various countries  $k$ . Expenditures in the target country are ‘discounted’ twice. First, they are discounted by  $\phi_{ij}$  in order to account for the export costs from  $i$  to  $j$ . Second, they are also discounted by  $1/\sum_k n_k (a_k)^{\sigma-1} \phi_{kj}$ , which is a transformation of the price index in country  $j$  defined in (3). This second discounting factor captures the fact that the intensity of competition faced by our firm in country  $j$  increases with the number of competitors ( $n_k$ ) and their productivity ( $a_k$ ) while it decreases with the trade costs they incur to serve country  $j$ . The profits our firm makes on its sales to  $j$  are proportionate to  $L_j/z_j$  after such a double discounting. By repeating this calculation for all target markets  $j = 1, \dots, M$ , we are able to compute the overall operating profits of our firm. Then, conditions (8) tell us that, due to free entry and exit, the distribution of firms across countries adjusts so that in equilibrium operating profits do not exceed the fixed costs. In other words, the equilibrium distribution of firms across countries is such that no opportunity of profitable entry remains unexploited. Accordingly, conditions (8) state that in equilibrium *exogenous* cross-country differential advantages in terms of proximity to customers are exactly offset by *endogenous* differential disadvantages in terms of proximity to competitors: countries with better access to markets and a comparative advantage in the differentiated good host larger numbers of firms.<sup>6</sup>

To make the notation more compact, it is useful to turn to matrix form. In particular, we define the matrices of bilateral trade freeness  $\Phi$ , relative productivity in the differentiated good

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<sup>6</sup>In the absence of the freely traded outside good, better access to markets and a comparative advantage in the differentiated good would be also offset by higher wages per efficiency unit of labor. In this case, however, the linear representation of the equilibrium, on which all our ensuing results are based, would break down. See the discussion in footnote 4.

sector  $A$  and absolute productivity in the homogeneous good sector  $B$  respectively as

$$\Phi \equiv \begin{pmatrix} 1 & \phi_{12} & \cdots & \phi_{1M} \\ \phi_{21} & 1 & \cdots & \phi_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{M1} & \phi_{M2} & \cdots & 1 \end{pmatrix}, \quad A \equiv \text{diag} \begin{pmatrix} a_1 \\ \vdots \\ a_M \end{pmatrix}, \quad B \equiv \text{diag} \begin{pmatrix} 1/z_1 \\ \vdots \\ 1/z_M \end{pmatrix}$$

In equilibrium  $B$  is also the matrix of wages. Henceforth, we impose that trade is free within countries ( $\phi_{ii} \equiv 1$ ) and that trade flows between any given pair of countries are subject to the same frictions in both directions ( $\phi_{ij} = \phi_{ji}$ ). Although these assumptions on the freeness of trade are not strictly necessary for deriving our theoretical results, they simplify the analysis. Furthermore, we define the vector of labor endowments  $l \equiv (L_1 \ L_2 \ \dots \ L_M)$  and the vector of the numbers of firms  $n \equiv (n_1 \ n_2 \ \dots \ n_M)$ .

Then, letting  $\mathbf{1}$  stand for the  $M$ -dimensional vector whose components are all equal to one, the  $M$  equilibrium conditions (8) can be expressed in matrix notation as

$$\frac{\mu}{\sigma r} A^\sigma \Phi \text{diag}(\Phi A^{\sigma-1} n)^{-1} B l \leq \mathbf{1}, \quad (9)$$

The terms in (9) mirror those in (8). The first ‘numerator’ term  $A^\sigma$  stresses the role of each country’s marginal costs in the determination of its firms’ prices. The second ‘numerator’ term  $\Phi B l$  highlights the role of distance-weighted expenditures that can be served from each country.<sup>7</sup> The ‘denominator’ term  $\text{diag}(\Phi A^{\sigma-1} n)$  captures the role of distance-and-productivity weighted supply that can serve each national market, which is a measure of the intensity of local competition.

Let us call  $n^* = (n_1^* \ n_2^* \ \dots \ n_M^*)$  the vector satisfying conditions (9). This vector always exists and is unique for all admissible parameter values.<sup>8</sup> While for specific parameter values the vector may entail some  $n_i^*$ ’s equal to zero, in the literature the HME has been defined with reference to equilibria in which  $n_i^*$ ’s are strictly positive. For this reason in what follows we focus on interior equilibria in which  $n_i^* > 0$  for all countries  $i = 1, 2, \dots, M$  and condition (8) holds as an equality for all countries.

From (9), an interior spatial equilibrium  $n^*$  is such that:

$$B l = \frac{\sigma r}{\mu} A^{-\sigma} \text{diag}(\Phi^{-1} \mathbf{1}) \Phi A^{\sigma-1} n^*. \quad (10)$$

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<sup>7</sup>This measure is our counterpart to Davis and Weinstein’s (2003) IDIODEM index. This index is a heuristic measure of the ‘idiosyncratic demand’ firms face in a certain country that takes into account not only local demand but also demand originating from neighboring countries. By analogy with Krugman’s two-country case, Davis and Weinstein interpret a larger than one estimate of the elasticity of a country’s output to the IDIODEM index as evidence of the presence of the HME. In Appendix C we argue that the problem with this interpretation is that the analogy is not valid when there are more than two countries. See Behrens *et al.* (2004) for further details.

<sup>8</sup>See Appendix B.1 for a proof.

This can be written component by component as

$$\frac{L_i}{z_i} = \frac{\sigma r \varphi_i}{\mu (a_i)^\sigma} \sum_j \phi_{ij} n_j^* (a_j)^{\sigma-1}. \quad (11)$$

where  $\varphi_i$  is the  $i$ -th component of the vector  $\Phi^{-1}\mathbf{1}$ , which can be interpreted as an inverse measure of country  $i$ 's average centrality in the network of our  $M$  trading countries.<sup>9</sup>

A necessary condition for the existence of an interior solution can then be obtained by transforming expression (11) successively as follows:

$$\frac{L_i}{z_i} = \frac{\sigma r \varphi_i}{\mu (a_i)^\sigma} \sum_j \phi_{ij} n_j^* (a_j)^{\sigma-1} < \frac{\sigma r \varphi_i}{\mu (a_i)^\sigma} \sum_j n_j^* (a_j)^{\sigma-1} \iff L_i < \frac{\sigma}{\mu} f_i \varphi_i \sum_j n_j^* \left(\frac{a_j}{a_i}\right)^{\sigma-1}$$

where the inequality results from  $0 < \phi_{ij} < 1$  and we have used the definitions of  $r$  and  $a_i$ . Accordingly, an interior equilibrium cannot arise when country  $i$  is sufficiently large (large  $L_i$ ), has sufficiently strong comparative advantage (large  $a_i/a_j$ ), has sufficiently low fixed costs (small  $f_i$ ), or is sufficiently centrally located (small  $\varphi_i$ ). An interior equilibrium cannot arise either when product differentiation is sufficiently strong and the differentiated good absorbs a large share of expenditures (small  $\sigma/\mu$ ).

Assuming that an interior equilibrium obtains, the corresponding cross-country distribution of firms is given by

$$n^* = \frac{\mu}{\sigma r} A^{1-\sigma} \Phi^{-1} \text{diag}(\Phi^{-1} A^{-\sigma} \mathbf{1})^{-1} B l. \quad (12)$$

or, in share notation, by

$$\lambda^* = [\text{diag}(\Phi^{-1} A^{-\sigma} \mathbf{1}) \Phi A^\sigma]^{-1} \theta. \quad (13)$$

where

$$\theta \equiv \frac{B l}{B l \mathbf{1}} \quad \text{and} \quad \lambda^* \equiv \frac{F B n^*}{F B n^* \mathbf{1}} \quad (14)$$

where  $F = \text{diag}(f_1 \ f_2 \ \dots \ f_M)$  is the diagonal matrix of fixed input requirements. In (14),  $\theta$  and  $\lambda^*$  respectively denote the vector of countries' shares of world demand (as measured by aggregate expenditures) and the vector of countries' shares of world production (as measured by either aggregate fixed costs payments or, equivalently due to free entry, aggregate operating profits) in the differentiated good sector.<sup>10</sup>

The equilibrium condition (13) reveals that the relation between  $\lambda^*$  and  $\theta$  is linear at any interior solution.<sup>11</sup> This relation is parametrized by a matrix that depends itself on the trade

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<sup>9</sup>Behrens *et al.* (2004) provide sufficient conditions for the freeness of trade matrix  $\Phi$  to be invertible. See also Behrens *et al.* (2007) for additional interpretations of  $\varphi$  in terms of centrality measures.

<sup>10</sup>See Appendix B.2 for a proof.

<sup>11</sup>The labor share of the numéraire sector is computed as a residual. Of course, since wages are equalized in efficiency units, that share is strictly positive for all countries (see Appendix A for more details).

freeness matrix  $\Phi$  and the relative productivity matrix  $A$ . For equal shares of demand, countries with a relative advantage in terms of better centrality and higher productivity in the differentiated good sector attract larger shares of production in that sector.

## 4 Market size and specialization

As discussed in the introduction, the HME has been defined as a more-than-proportional relationship between a country's share of world production of a good and its share of world demand for the same good. Formally, the differentiated good sector exhibits a HME in country  $i$  at the expenditure distribution  $\theta$  if and only if

$$\theta_i \geq \theta_j \quad \Rightarrow \quad \frac{\lambda_i^*}{\theta_i} \geq \frac{\lambda_j^*}{\theta_j}, \quad j = 1, \dots, M \quad (15)$$

with  $\lambda_i^*/\theta_i > \lambda_j^*/\theta_j$  if and only if  $\theta_i > \theta_j$ . For the HME to be a general prediction of the model, (15) must hold for all countries  $i = 1, \dots, M$ . Hence, we may define the HME as follows:

**Definition 1 (Home Market Effect)** Assume, without loss of generality, that country labels are ordered such that  $\theta_1 \geq \theta_2 \geq \dots \geq \theta_M$ , then the extended model features a HME if and only if

$$\frac{\lambda_1^*}{\theta_1} \geq \frac{\lambda_2^*}{\theta_2} \geq \dots \geq \frac{\lambda_M^*}{\theta_M}. \quad (16)$$

Stated differently, there exists a HME whenever there is no 'industrial leap-frogging', in the sense that smaller countries always host a relatively smaller share of the differentiated good sector. This implies that the ordering in terms of sector shares reflects the 'natural' ordering in terms of countries' economic sizes.<sup>12</sup>

It is readily verified that condition (16) does not generally hold in the extended model. To see this, consider two simple counterexamples with  $M = 3$  countries. Suppose first that there are no Ricardian differences across countries that are evenly spaced at distance  $\phi$  on a line segment with trade costs measured by the simple Euclidian distance. Specifically assume:

$$\Phi = \begin{pmatrix} 1 & \phi & \phi^2 \\ \phi & 1 & \phi \\ \phi^2 & \phi & 1 \end{pmatrix}, \quad \phi = 0.4, \quad \theta = \begin{pmatrix} 0.45 \\ 0.30 \\ 0.25 \end{pmatrix}, \quad r = 1, \quad A^{-\sigma} \mathbf{1} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}. \quad (17)$$

---

<sup>12</sup>Appendix C presents an alternative definition that has been used in empirical analyses of the HME (see, e.g., Davis and Weinstein, 2003). This definition is equivalent to the one adopted in the main text only in the case of two countries. Anyway, even when defined according to such an alternative definition, the HME is not a general property of the extended model.

Plugging (17) into (13) gives  $(\lambda_1^*, \lambda_2^*, \lambda_3^*) = (0.42, 0.50, 0.08)$ . This implies  $\lambda_2^*/\theta_2 > \lambda_1^*/\theta_1 > \lambda_3^*/\theta_3$ , thus violating again (16). Hence, although the expenditure share in the central country 2 is smaller than that in the peripheral country 1, country 2 attracts a more than proportionate share of firms ('hub effect').

Consider next a situation in which there are no cross-country differences in centrality because countries are evenly spaced around a circle with radius  $\sqrt{\bar{\phi}}$  and shipping between any two locations takes place through the center. Specifically, assume:

$$\Phi = \begin{pmatrix} 1 & \phi & \phi \\ \phi & 1 & \phi \\ \phi & \phi & 1 \end{pmatrix}, \quad \phi = 0.4, \quad \theta = \begin{pmatrix} 0.45 \\ 0.30 \\ 0.25 \end{pmatrix}, \quad r = 1, \quad A^{-\sigma} \mathbf{1} = \begin{pmatrix} 1.4 \\ 1.2 \\ 1.1 \end{pmatrix}. \quad (18)$$

Plugging (18) into (13) gives  $(\lambda_1^*, \lambda_2^*, \lambda_3^*) = (0.32, 0.29, 0.39)$ . This implies  $\lambda_3^*/\theta_3 > \lambda_2^*/\theta_2 > \lambda_1^*/\theta_1$ , thus violating (16). Hence, although its demand share is the smallest, country 3 attracts a more than proportionate production share thanks to its higher relative productivity in the differentiated good sector ('comparative advantage effect'). The fact that Ricardian differences interact with market size to affect the equilibrium location of industry is not surprising but it is important to keep that in mind in applied work as Ricardian differences are the rule rather than the exception in the real world.

These examples prove that the HME does not generally arise in the extended model because in (13) countries' equilibrium production shares  $\lambda^*$  are affected not only by their demand shares  $\theta$  but also by relative centrality and comparative advantage in the differentiated good sector. We now show: (i) how to define an alternative production measure whose country shares always magnify the cross-country variation in demand shares  $\theta$ ; (ii) how to recover such measure from the actual production shares  $\lambda^*$ .

The key issue is to find a way to separate the impact of relative centrality and comparative advantage on the one side from the impact of relative demand driven by relative size (i.e. relative labor endowments) and relative wages (i.e. absolute advantage) on the other side. Consider first the production shares that would prevail without comparative advantage ( $a_i = a$  for all  $i$ 's) and without centrality advantage ( $\phi_{ij} = \bar{\phi}$  for all  $i \neq j$ , where  $\bar{\phi}$  is the average bilateral freeness of trade). In this case, size and absolute advantage alone determine the cross-country variation of production shares so that (13) can be expressed component by component as:

$$\lambda_i^{\text{SA}} = \frac{1 + (M-1)\bar{\phi}}{1 - \bar{\phi}} \theta_i - \frac{\bar{\phi}}{1 - \bar{\phi}}, \quad (19)$$

for  $i = 1, \dots, M$ . In (19) the label SA is a mnemonic for "size and absolute advantage". It is readily verified that (16) holds with  $\lambda^*$  replaced by  $\lambda^{\text{SA}}$ . Hence, the extended model predicts a HME when countries are evenly spaced and in the absence of comparative advantage.

Now remove, instead, absolute advantage ( $z_i = z$  for all  $i$ 's) so that  $\theta_i = 1/M$  for all  $i = 1, \dots, M$ . In this case centrality and comparative advantage alone determine the cross-country variation of production shares and expression (13) simplifies to:

$$\lambda^{\text{CC}} = \frac{1}{M} [\text{diag}(\Phi^{-1}A^{-\sigma}\mathbf{1})\Phi A^\sigma]^{-1}\mathbf{1} \quad (20)$$

where CC is a mnemonic for ‘‘centrality and comparative advantage’’. Note that (20) does not generally satisfy (16).

Interestingly, (13), (19) and (20) allow us to linearly decompose  $\lambda^*$  as follows:  $\lambda^* = \beta W \lambda^{\text{SA}} + (1 - \beta)\lambda^{\text{CC}}$ , with  $W \equiv [\text{diag}(\Phi^{-1}A^{-\sigma}\mathbf{1})\Phi A^\sigma]^{-1}$  and  $\beta \equiv (1 - \bar{\phi}) / [1 + (M - 1)\bar{\phi}] \in (0, 1)$ . Inverting this expression gives:

$$\lambda^{\text{SA}} = (\beta W)^{-1} [\lambda^* - (1 - \beta)\lambda^{\text{CC}}]. \quad (21)$$

By construction, (16) holds with  $\lambda^*$  replaced by  $\lambda^{\text{SA}}$ . Hence, we have a *general prediction of the extended model*: a more-than-proportional relationship between a country’s share of world demand and its share of world production only obtains after the influence of centrality and comparative advantage on the latter is filtered out through (21).

The working of the linear filter in (21) can be clarified by its application to the two counterexamples discussed above. Consider the first counterexample, described by (17), with no comparative advantage and countries evenly spaced along a line segment. Applying the filter (21) to the corresponding  $(\lambda_1^*, \lambda_2^*, \lambda_3^*)$  yields the filtered production shares  $(\lambda_1^{\text{SA}}, \lambda_2^{\text{SA}}, \lambda_3^{\text{SA}}) = (0.61, 0.25, 0.13)$ , which satisfy (16) as  $\lambda_1^{\text{SA}}/\theta_1 > \lambda_2^{\text{SA}}/\theta_2 > \lambda_3^{\text{SA}}/\theta_3$ .

Turning to the second counterexample, described by (18), in which there are no cross-country differences in centrality because all countries are evenly spaced around a circle and all trade flows go through the center. Applying (21) to the corresponding  $(\lambda_1^*, \lambda_2^*, \lambda_3^*)$  yields the filtered production shares  $(\lambda_1^{\text{SA}}, \lambda_2^{\text{SA}}, \lambda_3^{\text{SA}}) = (0.68, 0.23, 0.08)$ , which again satisfy (16) as  $\lambda_1^{\text{SA}}/\theta_1 > \lambda_2^{\text{SA}}/\theta_2 > \lambda_3^{\text{SA}}/\theta_3$ .

## 5 Conclusion

In the two-country case the standard model of international trade with monopolistic competition predicts a more-than-proportional relationship between a country’s share of world production of a good and its share of world demand for the same good, a result known as the ‘home market effect’. We have shown that this prediction does not generally carry through to the empirically relevant case in which there are several trading countries differing in terms of centrality and technology. We have then derived a new prediction of the model that does hold for any number of trading

countries and any pattern of technological differences. In particular, we have shown that the model predicts a more-than-proportional relationship between a country's share of world demand and its share of world production only after the influence of centrality and comparative advantage on the latter has been controlled for through a simple linear filter. As this prediction also takes into account technology-driven differences in factor prices across countries, it may prove useful for better identifying home market effects empirically. We keep this for future work.

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## Appendix A: Incomplete Specialization

Some numéraire production takes place everywhere only if any  $M - 1$  dimensional subset of countries is unable to satisfy world demand (see, e.g., Baldwin *et al.*, 2003). This is the case if the total mass of workers in each country is greater than the total labor requirement in the

differentiated good sector:  $L_i > n_i \ell_i$  for all  $i$ , where  $\ell_i$  is the amount of labor employed by a representative sector  $D$  firm in country  $i$ . It is readily verified that

$$n_i^* \ell_i = n_i^* \left( f_i + c_i \sum_j x_{ij} \right) = n_i^* \left[ f_i + c_i \frac{f_i(\sigma - 1)}{c_i} \right] = n_i^* \sigma f_i$$

so that, in equilibrium, some numéraire production takes place everywhere if:

$$L_i > n_i^* \sigma f_i \quad i = 1, \dots, M$$

where  $n_i^*$  is given by (12). As can be seen from (12), the equilibrium mass of firms is proportional to  $\mu$  for all countries  $i$ . Thus, the expenditure share  $\mu$  must be small enough for the numéraire good to be produced everywhere. Alternatively, the expenditure share  $1 - \mu$  on the numéraire good must be large enough.

## Appendix B: Equilibrium Properties

**B.1. Existence and Uniqueness** Since each component of the left hand side vector in (9) is a continuous function of  $n$ , Proposition 1 in Ginsburgh *et al.* (1985) shows that an equilibrium always exists.

Now assume that firms relocate in response to profit differentials, so that  $n_i$  increases (resp. decreases) if  $\Pi_i(n) > 0$  (resp.  $< 0$ ) where we have made the dependence of the profit function (4) on  $n$  explicit. The dynamics of the relocation process is given by

$$\dot{n}_i = \xi_i \Pi_i(n), \quad (\text{B.1})$$

where  $\dot{n}_i \equiv dn_i/dt$  and where  $\xi_i > 0$  stands for the speed of the adjustment in country  $i$ . Denote the Jacobian of  $\Pi$  by  $J$ . Its generic element is given by  $\xi_i \partial \Pi_i(n) / \partial n_j$  with

$$\frac{\partial \Pi_i(n)}{\partial n_j} = -\frac{\mu}{\sigma} \left( \frac{c_j c_i}{z_j z_i} \right)^{1-\sigma} \sum_l \frac{\phi_{jl} \phi_{il} L_l \frac{1}{z_l}}{\left[ \sum_k \phi_{kl} n_k \left( \frac{c_k}{z_k} \right)^{1-\sigma} \right]^2} < 0,$$

so that, by symmetry of the  $\phi_{ij}$ 's, the matrix  $J$  is symmetric. Then, for any nonzero vector  $x$ , we have

$$x^T J x = -\frac{\mu}{\sigma} \sum_l \frac{L_l}{z_l} \frac{\left[ \sum_i \xi_i x_i \left( \frac{c_i}{z_i} \right)^{1-\sigma} \phi_{il} \right]^2}{\left[ \sum_k \phi_{kl} n_k \left( \frac{c_k}{z_k} \right)^{1-\sigma} \right]^2} < 0$$

thus implying that  $J$  is negative definite.

Finally, let  $\Delta$  stand for the unit simplex of  $\mathbb{R}^n$ . According to Rosen (1965, Theorem 8), if  $J$  is negative definite for every  $\lambda \in \Delta$ , the system (B.1) is globally stable on  $\Delta$ .

Because existence and global stability of an equilibrium implies uniqueness, the extended model always admits one and only one equilibrium.

**B.2. Interior Equilibrium** To derive the expression for an interior equilibrium, note that conditions (9) can be successively rewritten as follows:

$$\begin{aligned} \text{diag} [\Phi A^{\sigma-1} n]^{-1} B l &= \frac{\sigma}{\mu} \Phi^{-1} A^{-\sigma} \mathbf{1} = \frac{\sigma}{\mu} \text{diag} [\Phi^{-1} A^{-\sigma} \mathbf{1}] \mathbf{1} \\ \iff \text{diag} [\Phi^{-1} A^{-\sigma} \mathbf{1}]^{-1} B l &= \frac{\sigma}{\mu} \text{diag} [\Phi A^{\sigma-1} n] \mathbf{1} = \frac{\sigma}{\mu} \Phi A^{\sigma-1} n, \end{aligned}$$

where we have used the commutativity property of the diagonal matrix product and used the fact that the freeness of trade matrix  $\Phi$  is invertible (see Behrens *et al.*, 2004, for sufficient conditions). Hence, the equilibrium distribution of firms is given by (12).

Multiplying both sides of (7) by  $c_i w_i = c_i / z_i$  and by the positive  $n_i$ 's, and summing across countries, we get

$$B l \mathbf{1} = \frac{\sigma}{\mu} F B n \mathbf{1}. \quad (\text{B.2})$$

Using (B.2), (12) implies (13).

## Appendix C: The ‘home market shadow’

Davis and Weinstein (2003) adopt a definition of the HME in terms of ‘comparative statics’ that is different from the one in terms of ‘rankings’ we use in the main text. Specifically, they define the HME as “a more than one-for-one movement of production in response to idiosyncratic demand” (Davis and Weinstein, 2003, p.7). Whereas the two definitions are equivalent in the case of two countries (Ottaviano and Thisse, 2004, p.2582), they are not necessarily so in a multi-country setup. Nonetheless, we show here that in such setup also the ‘comparative statics’ HME is not generally predicted by the extended Helpman-Krugman model.

Formally, assume that country  $i$  hosts a sector share at period  $t$  that is proportionate to its demand share, which can be expressed as  $(\lambda_i^*)^t = \kappa^t \theta_i^t$ . Assume that in period  $t + 1$ , all  $\theta_j$ 's have changed such that  $\theta_i^{t+1} - \theta_i^t > 0$  and  $\sum_j (\theta_j^{t+1} - \theta_j^t) = 0$ , so that the new equilibrium production share is given by  $(\lambda_i^*)^{t+1} = \kappa^{t+1} \theta_i^{t+1}$ . In the presence of a HME, the disproportionate positive causation from demand to supply requires that  $\kappa^{t+1} > \kappa^t$  whenever  $\theta_i^{t+1} > \theta_i^t$ . Hence,

$$\frac{(\lambda_i^*)^{t+1}}{\theta_i^{t+1}} = \kappa^{t+1}, \quad \frac{(\lambda_i^*)^t}{\theta_i^t} = \kappa^t \quad \text{and} \quad \kappa^{t+1} > \kappa^t \quad \Rightarrow \quad \frac{(\lambda_i^*)^{t+1}}{\theta_i^{t+1}} > \frac{(\lambda_i^*)^t}{\theta_i^t}.$$

Switching to differential notation, the last condition can be expressed as

$$\frac{\lambda_i^* + d\lambda_i^*}{\theta_i + d\theta_i} > \frac{\lambda_i^*}{\theta_i} \quad \Rightarrow \quad \frac{d\lambda_i^*}{d\theta_i} \frac{\theta_i}{\lambda_i^*} > 1.$$

This result suggests, quite naturally, the following definition for the HME:

**Definition 2** *A monopolistically competitive sector exhibits a HME in country  $i$  at the demand distribution  $\theta$  and for the perturbation  $d\theta$  if and only if*

$$\frac{d\lambda_i^*}{d\theta_i} \frac{\theta_i}{\lambda_i^*} > 1, \tag{C.1}$$

where  $d\theta$  is a small variation satisfying  $d\theta_i > 0$  and  $\sum_j d\theta_j = 0$ .

Unfortunately condition (C.1) need not hold at the equilibrium (13). In particular, we have:

**Proposition 1** *Assume that trade costs are not pairwise symmetric. Then, there exists a perturbation  $d\theta$ , with  $d\theta_i > 0$  and  $\sum_j d\theta_j = 0$ , such that the disproportionate causation from demand to supply does not hold.*

**Proof.** Because  $\lambda_i^* > 0$ ,  $\theta_i > 0$ , and  $d\theta_i > 0$ , a necessary condition for (C.1) to hold requires  $d\lambda_i^*$  to be strictly positive. However, by linearity,

$$d\lambda_i^* = \lambda_i^*(\theta + d\theta) - \lambda_i^*(\theta) = \sum_j g_{ij} d\theta_j = \sum_{j \neq i} (g_{ij} - g_{ii}) d\theta_j \tag{C.2}$$

where the  $g_{ij}$ 's are the coefficients implied by (13), and where the last equality stems from the constraint that the perturbations sum up to zero. When trade costs are not pairwise symmetric, we can always find perturbations  $d\theta_j$  such that (C.2) is negative, in which case (C.1) does not hold for all perturbations satisfying  $d\theta_i > 0$  and  $\sum_j d\theta_j = 0$ . It is sufficient to note that in the general asymmetric case  $\min_j \{g_{ij}\} < \max_j \{g_{ij}\}$  and that at least one  $d\theta_j$ ,  $j \neq i$ , must be strictly negative. ■

Proposition 1 shows that (C.1) need not hold for some variations  $d\theta$  unless trade costs are pairwise symmetric across all countries (i.e.,  $\phi_{ij} = \phi$ ,  $\forall i \neq j$ ). Hence, the disproportionate causation from demand to supply does not generally hold.

For example, as demand shares change between two periods, a ‘HME shadow’ may arise, in the sense that, even though the demand share of country  $i$  increases, its production share may increase less than proportionately if also the demand share of another country  $j$  increases. In some cases, this effect may be so strong that country  $i$  simply loses some of the differentiated sector, despite the increase in its demand share. As in the case of the definition in terms of ‘rankings’, the reason is that the appeal of a country as a production site depends not only on the relative size of its domestic market, but also on its relative proximity to all other foreign markets as well as on technology and factor price differences.

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