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Conference Paper

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22nd European Regional Conference of the International Telecommunications Society (ITS): "Innovative ICT Applications - Emerging Regulatory, Economic and Policy Issues", Budapest, Hungary, 18th-21st September, 2011

Provided in Cooperation with:

International Telecommunications Society (ITS)

Suggested Citation: Baranes, Edmond; Poudou, Jean-Christophe (2011) : Internet access and investment incentives for broadband service providers, 22nd European Regional Conference of the International Telecommunications Society (ITS): "Innovative ICT Applications - Emerging Regulatory, Economic and Policy Issues", Budapest, Hungary, 18th-21st September, 2011, International Telecommunications Society (ITS), Calgary

This Version is available at:
https://hdl.handle.net/10419/52196

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Internet access and investment incentives for broadband service providers

This paper studies a model of the Internet broadband market as a platform in order to show how different pricing schemes from the so-called "net neutrality " may increased economic efficiency by allowing more investment of access providers and enhancing consumers surplus and social welfare.

JEL Codes: L51, L86, L96
Keywords: Network neutrality, Flat rates, Termination fees

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Introduction

The debate over network neutrality has become a huge debate in Internet policy both in US and European Union. The question is whether the traffic management practices of network providers should be control by regulators and how should be defined rules allowing a better economic efficiency. In practice, network neutrality has been often interpreted as a set of rules allowing non-discrimination both between contents providers and end-users. For a long time and until networks could handle the traffic generated by uses, network neutrality has resulted in a free and open access to network for content providers. The main idea was that externalities from diversity of available contents to end-users on platforms justified an indirect mechanism of cross-subsidy through users' payments to access providers. Today the situation is different since traffics had sharply rose because of the increasing development of contents and thus, network investments by operators are became a necessity to support traffics. The question that arises directly from such an observation is twofold: How to give incentives to invest? How allocate investments financing between stakeholders? Beyond these questions analyzing the benefits and pitfalls of non-discrimination/discrimination rules for network access is a key component to evaluate the Internet policy.

Because of the diversity of practices and the complexity in the economic relations between network providers, content providers and Internet intermediaries, it is not easy to define what is exactly non-discrimination rules. However, one can consider that non-discrimination occurs whenever a network provider gives a same access to network traffics of the same characteristics. This does not imply that the network provider must treat different networks traffics from different contents providers in the same manner (same tariffs, same QoS...). So, the economic literature indicates that discrimination may improve economic efficiency whenever it does not lead to anti-competitive practices. The main goal of our paper is to show how different pricing schemes affect investment incentives of network providers, social welfare and its distribution between the stakeholders (network providers, contents providers and end-users). In particular, we focus in the following on the relevant impacts of termination fees.

A large strand of literature has recently developed on network neutrality. Most of papers were discussing legal issues of network neutrality and the expected consequences of its abolition. Economic analysis in this field is less developed although some recent theoretical research has been made in the field of two-sided market models. However, the analysis is not usually straightforward as network neutrality can be defined in several ways.

Economides and Tag (2007) model the Internet Broadband market as a two-sided platform in which broadband consumers stand on one side and content providers on the other side. Their results show that network neutrality regulation (that imposes zero fees on the opposite side of the market) generally increases industry surplus compared to the fully private optimum[1] at which the monopoly platform imposes positive fees on content

[1] These authors define the fully private optimum as the overall profit maximizing scheme for the monopolistic platform.

and applications providers. As platforms have incentives to attract more consumers to generate revenue from charging content provider, without network neutrality regulation they set a lower subscription fee, hence consumers' surplus increases. This positive impact is offset by the negative effect on content provision and finally, the welfare increases with network neutrality. In contrast, Hermalin and Katz (2007) consider network neutrality as a situation in which the broadband platform produces a single access quality (non discriminatory access quality). They assume both traditional markets and two-sided markets where platform providers offer services making a connection between consumers and Internet applicant providers. Network neutrality plays as a product-line restriction and as a direct effect low valuation applicant providers get ruled out of the market. Their results show that network neutrality regulation by product restriction may hinder both consumers' surplus and social welfare.

The effect of network neutrality regulation on investments incentives for network providers is analyzed by Choi and Kim (2008). They define network neutrality as non-discriminatory in the delivery of content through networks. The model developed is based on the queuing theory developed in operational research to take clearly account of bandwidth scarcity and the need for rationing as the main causes of the network neutrality regulation debate. In this setting, they show that the network providers' decision on the discrimination across content depends on a potential trade-off between access fee and the revenue from the trade of the first priority. Concerning the network providers' investment incentives, their results show that the growth in capacity affects the sale price of the priority right under the discriminatory regime. They conclude that as the relative merit of the first priority becomes relatively small for higher level capacity, under discrimination the network's incentives to invest may be smaller than that under network neutrality regulation where such rent extraction effects does not exist. Finally, the welfare effects of network neutrality regulation is ambiguous and depends largely on how capacity expansion affects the need to acquire the priority right and thus the ability to extract rent from content providers. Close to Choi and Kim (2008), Cheng et al. (2009) develop a game theoretic model to highlight gainers and losers of abolishing network neutrality and to analyze the broadband providers' incentives to expand capacity. They find that content providers are left worse off when network neutrality is abolished and consumer surplus either does not change or is higher in the short run. In the short run, social welfare increases whether one content provider pays for preferential treatment but remains unchanged whether both contents providers pay. Finally, they find that incentive to invest in capacity for broadband provider is generally higher under the neutrality regulation because the network owner incurs a lost from the content provider's side without net neutrality.

In this paper, we explicitly model the Internet broadband market as a platform consisting of end-users on one side and contents providers on the other side. We do not aim here to show how crossed externalities between both sides of the platform affects prices and social welfare. This question has been developed by a large literature. Our goal is to show how, in a simple model, different pricing schemes from the so-called "net neutrality " may increased economic efficiency by allowing more investment of access providers and enhancing consumers surplus and social welfare.

The outline of the paper is as follows. The next section sets up the model of network markets. Section 3 analyses the effects of network neutrality regulation on competition and social welfare and studies the impacts of introducing termination fees to reach end-users for content providers. Section 4 provides a complete analysis of the impacts of termination fees discrimination on investment incentives of network provider and stakeholders surplus. Section 5 offers extensions around the basis model examining the impact of termination fee discrimination on incentives to invest of the broadband provider, and how termination fee may induce more traffic management from content providers. Section 6 closes these analyses with concluding remarks. Most of the proofs for lemmas and propositions are relegated to the Appendix.

1. The model

We consider a model in which a monopolistic Internet service provider (ISP) sells broadband Internet access to consumers. Consumers can freely obtain the contents of two Content providers (CPs) from broadband Internet access. In that setting, we study how different pricing schemes may impact consumers surplus, firms' profits (ISP and CPs) and social welfare. This especially allows to analyze the ISP's incentives to invest to upgrade its network quality according to net neutrality is applied or not and how regulation may be relaxed. For the sake of analysis, we will define in the following net neutrality as non-discrimination in the delivery of contents. That is, the ISP provides access to content providers at a unique fixed fee, in order to make their content available across the Internet, and offers access network to consumers at a non discriminatory flat rate.

Content providers

We consider competition between two content providers differentiated *à la Hotelling*. Both content provider use the same technology and we normalize the marginal cost to *0*. For connecting to the ISP, the content provider i bears the fixed fee f_i and pays a unit price s_i for each users. We suppose that content provider i can exert an effort denoted θ_i in order to attract users towards their own content, where $\theta_i \geq 0$. Such effort θ_i can represent the quality of content that CP i offers to end-users. This quality represents a quality index that encompasses editorial design, web design and so on. In the following, we consider that the cost function is quadratic, $C_i(\theta_i) = \frac{\omega_i}{2}\theta_i^2$. Without loss of generality, we will assume that $\omega_1 = 1$ and $\omega_2 = \omega \geq 1$. That is, the CP2 is assumed to be less efficient than CP1. Furthermore, a content provider gets revenues from (exogenous) advertising related to its market's share, we denote a , the unit receipt for advertising.

Internet service provider

The monopolistic ISP, I , sells a network access to end-users and provides access to content providers. The ISP can invest β_i to increase the quality of network access for content i . This assumption take into account situations where the ISP can discriminate

between both content providers using the quality of access (i.e. access prioritization). An example of this kind of investment is an upgrade in network access with fibre optic cables to increase the capacity to deliver voice and data traffic. This in turn increases the end-users' utility when they get content from content providers and hence increases their willingness to pay for contents. We assume that the quality of network access affects the quality of content in a multiplicative form. In particular, we consider that when the ISP invests β_i , the gross utility for an end-user consuming the content of quality θ_i is $\beta_i(1+\theta_i)$. Focusing on incentives to invest only, we normalize the cost of this investment to zero, without loss of generality**.** We assume that the ISP charges an access fee p_i from connected end-users for content i . On the other side of the market, the ISP collects a fixed fee f_i for content provider i to allow access to its network and a unit price s_i for each user connected who consumes content i . This unit price corresponds here to a termination fees charged to content provider i to reach end-users. The ISP bears a fixed cost normalized to 0 to connect the two sides of the market, and a same marginal cost c for each unit of traffic coming from users and content providers. Under a network neutral regime, the ISP cannot price discriminate users, hence the access fee does not depends on which content is consumed, $p_1 = p_2 = p$. The same applies for content providers and we assume that they can get access to the network at a non discriminatory fixed fee, $f_1 = f_2 = f$, without paying a termination fee for each consumer connected, $s_i = 0$.

Consumers

Consumers are uniformly located on the segment $[0,\ 1]$. The two content providers are located at the two extremities of the segment, namely at $x_1 = 0$ and $x_2 = 1$ (respectively for CP 1 and CP 2). We assume that consumers single-home, that is each consumer buys Internet access from the ISP and consume one content only. Given the transportation unit cost t and the quality of network access $\beta_i \geq 0$, the utility for a consumer located at $x \in [0,\ 1]$ subscribing to the ISP is $\beta_1(1+\theta_1) - p_1 - tx$ if he gets content from CP 1 , and $\beta_2(1+\theta_2) - p_2 - t(1-x)$ if he gets contents from CP 2 . We consider here that the quality of network access affects positively end-user's utility whether he gets contents from CP 1 or CP 2 . That is, there may be asymmetry between content providers based on network's quality.

Let φ denote the difference in the quality of network access for content providers, that is, $\varphi = \beta_1 - \beta_2$.

We can interpret , φ as the degree of non price discrimination between both content providers. There is no discrimination in access quality between content providers when $\varphi = 0$, that is $\beta_1 = \beta_2$. This is because when the ISP does not discriminate content providers using the quality of network access (as management traffic), the two CPs are perceived as completely identical from the end-users point of view if the quality of contents are the same, $\theta_1 = \theta_2$.

We assume that $\varphi < \frac{t-\beta_1(\theta_1-\theta_2)}{(1+\theta_2)}$. This assumption limits the asymmetry in non price discrimination, and assumes away market cornering[2]. When the ISP does not discriminate CPs using access quality ($\beta_1 = \beta_2 = \beta$), this assumption becomes $\beta < \frac{t}{\theta_1-\theta_2}$, and it implies $\theta_1 > \theta_2$.

Surplus and welfare

We only consider full market coverage and we denote by α_1 and $\alpha_2 (= 1-\alpha_1)$ market shares for both content providers. Explicitly, the marginal consumer's is defined by:

$$\beta_1(1+\theta_1) - p_1 - tx = \beta_2(1+\theta_2) - p_2 - t(1-x)$$

Market shares are determined as follows:

$$\alpha_1 = \frac{1}{2} + \frac{p_2 - p_1 + (\beta_1 - \beta_2) + (\beta_1\theta_1 - \beta_2\theta_2)}{2t} \text{ and } \alpha_2 = 1-\alpha_1 \quad (1.1)$$

Under a discriminatory network regime, the profit for the ISP is:

$$\Pi_I = (p_1 + s_1 - 2c)\alpha_1 + (p_2 + s_2 - 2c)\alpha_2 + f_1 + f_2 \quad (1.2)$$

and the profit for CP i is given by:

$$(1.1)\,\Pi_i(\theta_i,\ \theta_j) = (a - s_i)\alpha_i - f_i - \frac{\omega_i}{2}\theta_i^2 \qquad \text{for } i,\ j = 1,\ 2 \text{ and } i \neq j \quad (1.3)$$

The impact of different pricing schemes can be evaluated by assessing consumer's surplus, profits of CPs and ISP, and social surplus. For regulatory purpose, it is possibly important to consider not only social surplus as a welfare measure but also consumer's surplus. Moreover, a regulator with the objective to encourage a broad diffusion of contents should take care of the benefit for ISP from different pricing schemes and especially from network neutrality.

Industry profit IP is equal to the sum of profits both for ISP and CPs, i.e. $IP = \Pi_I + \Pi_1 + \Pi_2$. Consumers' surplus, defined as the consumers ' aggregate net utility, is given by:

$$\begin{aligned} CS &= \int_0^{\alpha_1} \left(\beta_1(1+\theta_1) - p_1 - tx\right)\ dx + \int_{\alpha_1}^{1} \left(\beta_2(1+\theta_2) - p_2 - t(1-x)\right)\ dx \\ &= \alpha_1\left(\beta_1(1+\theta_1) - p_1\right) + \alpha_2\left(\beta_2(1+\theta_2) - p_2\right) - \frac{t}{2}\left(\alpha_1^2 + \alpha_2^2\right) \end{aligned} \quad (1.4)$$

Remark that the last term in (1.4) assesses the effect produced by the ISP's investment in access quality into the disutility that consumers incur getting content from a content providers. The total surplus is $TS = IP + CS$.

As mentioned above, we consider that under network neutrality, the access provider

[2]Market cornering happens, for example, when end-users can get access with a same price $p_1 = p_2 = p$, and all end-users consume only from the content provider with the high quality.

cannot price discriminate neither end-users nor content providers. The following analysis considers two cases according to the ISP can offer content providers a pricing scheme including a termination fee or not. We first consider that content providers pay a non discriminatory flat rate to get access from the ISP and secondly that content providers pay a unit price for each consumer they attract with their contents. We also assume that ISP does not discriminate CP's using its investment for access quality i.e. $\beta_1 = \beta_2 = \beta$.

For each case, we analyze a sequential game where in the first stage the ISP sets access prices for both side of the market (i.e. $p_i,\ s_i,\ f_i$) and, in the second stage content providers choose their quality of contents (i.e. θ_i). We solve this game by backward induction.

2. The benchmark analysis: flat rates

We consider first the network neutrality case. In this situation, end-users bear a fixed fee to get access from the broadband provider and content providers pay a fixed fee to reach end-users. Precisely, end-users pay an uniform fee i.e. $p_1 = p_2 = p$ for network access and content providers can get access to the network at a unique flat rate $f_1 = f_2 = f$ with $s_1 = s_2 = 0$. That is, ISP subscriptions for end-users are fixed rate and CPs charges are independent from the traffic their contents generate.

From (alpha) and considering that end-users pay a fixed fee to get access from the ISP, market shares for content providers are given by:

$$\alpha_1 = \frac{1}{2} + \frac{p_2 - p_1 + \beta(\theta_1 - \theta_2)}{2t} \text{ and } \alpha_2 = 1 - \alpha_1$$

Maximizing (prCP) with respect to θ_i and using (alpha), we can derive efforts both content providers exert to attract end-users:

$$\theta_1^N = \frac{a\beta}{2t} \quad \text{and} \quad \theta_2^N = \frac{a\beta}{2\omega t} \quad (1.5)$$

Profits derived from these efforts are then given by

$$\Pi_1(\theta_1^N,\ \theta_2^N) = \frac{a\left(4t^2\omega + a\omega\beta^2 - 2a\beta^2\right)}{8t^2\omega} - f$$

$$\Pi_2(\theta_2^N,\ \theta_1^N) = \frac{a(4t^2\omega - 2a\omega\beta^2 + a\beta^2)}{8t^2\omega} - f$$

Straightforwardly one can show that $\Pi_1(\theta_1^N,\ \theta_2^N) - \Pi_2(\theta_2^N,\ \theta_1^N) = \frac{3a^2\beta^2(\omega-1)}{8t^2\omega} \geq 0$ as $\omega \geq 1$.

Indeed, because of CP2' less efficiency in quality investment its net returns are lower than CP1 when uniform pricing applies. Using (1.1), (1.2), (1.3) and (1.5), the maximization problem for the ISP is thus given by:

$$\max_{p,\ f} \Pi_I = p - 2c + 2f \qquad \text{s.t.} \quad \Pi_2(\theta_2^N,\ \theta_1^N) \geq 0 \text{ and } \beta(1 + \theta_1^N) - p - t\alpha_1 \geq 0$$

where the two constraints are needed to ensure that the market is covered and both contents are offered at equilibrium. The equilibrium network access fee is then given by:

$$p^N = \frac{\beta^2 a(1+\omega) + 2t\omega(2\beta - t)}{4t\omega}$$

and the fixed fee for CPs:

$$f^N = \frac{a\left(4t^2\omega - 2\beta^2 a\omega + \beta^2 a\right)}{8t^2\omega}$$

Equilibrium market shares are:

$$\alpha_1^N = \frac{1}{2} + \frac{\beta^2 a(\omega - 1)}{4t^2\omega} \geq \frac{1}{2} \text{ and } \alpha_2^N = 1 - \alpha_1^N$$

they are admissible if $\beta \leq \overline{\beta} = \sqrt{\frac{2\omega}{a(\omega-1)}}t$.

Equilibrium profits write:

$$\Pi_I^N = \beta - 2c - \frac{t}{2} - \frac{\beta^2\left(2\omega - 1\right)}{4t^2\omega}a^2 + \frac{4t^2\omega + \beta^2\left(\omega + 1\right)\ t}{4t^2\omega}a \quad (1.6)$$

$$\Pi_1^N = \frac{3a^2\beta^2\left(\omega - 1\right)}{8t^2\omega} \quad \text{and} \quad \Pi_2^N = 0 \quad (1.7)$$

Expression (1.6) shows that the ISP profit is composed of two terms. The first ($\beta - 2c - \frac{t}{2}$) corresponds to the consumer's surplus created by the investment in access quality. The remaining term represents the consumer's surplus indirectly created by the content quality and captured by the ISP. In expressions (1.7) we see that CP1 earns a positive profit alone. This profit represents the consumer's surplus it obtains directly from the content quality it has created. As CP2 is the least efficient firm, the ISP gets its entire surplus choosing an appropriate flat rate.

The equilibrium consumer surplus is given by:

$$CS^N = \frac{\beta^4 a^2\left(\omega - 1\right)^2}{8t^3\omega^2}$$

Let us now consider the ISP's incentives for investing in access quality when flat rates applies:

$$\frac{\partial \Pi_I^*}{\partial \beta} = 1 + \frac{1}{2}a\beta\frac{t(\omega+1) - \left(2\omega - 1\right)\ a}{\omega t^2} \quad (1.8)$$

Proposition 1. *When flat rates apply, it exists a level of advertising profitability ($\overline{a}$) such that ISP's incentives for investing in access quality is negative whenever advertising returns are sufficiently high ($a > \overline{a}$) and conversely.*

This Proposition states that investment incentives of the ISP in access quality depends on the level of the unit receipt of advertising for the CPs. One can easily see in expression (1.8), that when $a = 0$ (i.e. advertising is free) the ISP's incentives for investing in access quality is unambiguously positive. On the other hand, when a takes very high values, this incentives are unambiguously negative. In the appendix, we show that there exists a level of advertising profitability (i.e. the threshold $\overline{a}$) under which the ISP's incentives positive for all parameter values. Therefore, when advertising is more

profitable (i.e. a is above the threshold $\overline{a}$) an incremental investment turns the ISP profit down. In appendix, we also show that the threshold $\overline{a}$ is a decreasing function of β.

The intuition is that there is a tradeoff between advertising and access quality that may occur at the equilibrium. When advertising is weakly profitable (i.e. low values of a), the CPs provide low quality for there contents and thus, consumers' valuations for their contents are low. In this case, to elicit consumers' demand for contents, the ISP has necessarily an incentive to increase its access quality. Hence, this investment allows the ISP to post a high access fee and get more surplus from consumers than without any additional investment. When advertising is very profitable (i.e. a above $\overline{a}$), consumers have high valuations for contents, reducing therefore the ISP incentives to increase its access quality. This situation is more likely when the additional investment is realized from an existing high access quality.

3. Termination fees

In the previous section, we have determined equilibrium outcomes and incentives to invest in access quality of the ISP when content providers pay a fixed fee to reach end-users. We now analyze how introducing termination fees could modify the surplus breakdown between stakeholders and enhance (or deteriorate) incentives to invest in access quality of the ISP.

Hence, we consider here that end-users pay an uniform fee p for network access and content providers can get access to the network at an uniform unit price, namely a termination fee, i.e. $s_1 = s_2 = s$, so that $f_i = 0$. We first give and analyze equilibrium outcomes and second we propose a comparison with the benchmark case.

Equilibrium

The main difference with the benchmark case is that with termination fee, quality contents and thus CP market shares depends directly on pricing conditions on the CP side of the platform, i.e. levels of termination fee denoted s. This creates additional effects on the strategic decisions of firms.

Now for a given termination fee, content qualities entail:

$$\theta_1 = \frac{\beta(a-s)}{2t} \quad \text{and} \quad \theta_2 = \frac{\beta(a-s)}{2t\omega} \quad (1.9)$$

Profits derived from these quality choices are

$$\Pi_1(\theta_1,\ \theta_2) = \frac{(a-s)\ \left(\beta^2 a\omega - \beta^2\omega s - 2\beta^2 a + 2\beta^2 s + 4t^2\omega\right)}{8t^2\omega}$$

$$\Pi_2(\theta_2,\ \theta_1) = \frac{(a-s)\ \left(4t^2\omega - 2\beta^2 a\omega + 2\beta^2\omega s + \beta^2 a - \beta^2 s\right)}{8t^2\omega}$$

Straightforwardly one can show again that

$$\Pi_1(\theta_1,\ \theta_2) - \Pi_2(\theta_2,\ \theta_1) = \frac{3(a-s)^2\,\beta^2\,(\omega-1)}{8t^2\omega} \geq 0 \text{ as } \omega \geq 1$$

Again, because of CP 2 ' less efficiency in quality investment its net returns are lower than CP . With termination fee, the ISP profit becomes $\Pi_I = p + s - 2c$ and the maximization problem for the ISP writes:

$$\max_{p,\, s} \Pi_I \quad \text{s.t. } \Pi_2(\theta_2,\ \theta_1) \geq 0 \text{ and } \beta(1+\theta_1) - p - t\alpha_1 \geq 0$$

Defining a value $\bar{t} = \frac{\beta^2(1+\omega)}{4\omega}$, the following Lemma gives termination fee equilibrium.

Lemma 1. *When ISP can charge a termination fee, at the equilibrium they are given by:*

(1) $s^T = a - \dfrac{4t^2\omega}{\beta^2(2\omega-1)} < a$ *and* $p^T = \beta + \dfrac{3t}{2(2\omega-1)}$ *if* $t \leq \bar{t}$

(2) $s^T = a$ *and* $p^T = \beta - \dfrac{t}{2}$ *if* $t > \bar{t}$

The intuition behind this result is that both equilibrium prices are related to degree of substitutability between contents 1 or 2 , i.e. parameter t . When contents are highly differentiated ($t > \bar{t}$), competition is relaxed between CPs so that consumers are sufficiently captive and the ISP has no need to provide price incentives to promote CPs quality investments. Therefore it can set a high termination fee (a) which leads to a minimal level of content's quality ($\theta_i = 0$). Conversely, when contents are weakly differentiated ($t \leq \bar{t}$), some investments in content's quality are needed to increase consumer's valuations and for the ISP to extract more surplus. To do that it chooses a low termination fee ($a - \frac{4t^2\omega}{\beta^2(2\omega-1)}$) which give some rents to the more efficient CP.

From (1.9) and Lemma 1, the equilibrium quality contents depend on the degree of substitutability level : $\theta_1^T = \frac{2t\omega}{\beta(2\omega-1)}$ and $\theta_2^T = \frac{2t}{\beta(2\omega-1)}$ if $t \leq \bar{t}$; $\theta_1^T = \theta_2^T = 0$ otherwise.

With termination fee, again the least efficient content provider (CP2) earns zero profit ($\Pi_2^T = 0$) and equilibrium profits for other firms now write:

$$\Pi_I^T = \beta + a - 2c + t\frac{3\beta^2 - 8t\omega}{2\beta^2(2\omega-1)} \ ; \ \Pi_1^T = \frac{6t^2\omega(\omega-1)}{\beta^2(2\omega-1)^2} \qquad \text{if } t \leq \bar{t}$$

$$\Pi_I^T = \beta + a - 2c - \frac{t}{2} \text{ and } \Pi_i^T = 0 \qquad \text{if } t > \bar{t}$$

As previously mentioned, we address now the issue of investment incentives of the ISP by investigating the ISP's marginal change in its profit with respect to the level of access quality parameter, β. In the line of the previous Lemma, the ISP's incentive to invest depends on the substitutability between contents providers, that is

$$\begin{aligned} \frac{\partial \Pi_I^T}{\partial \beta} &= 1 + \frac{8\omega t^2}{(2\omega-1)\beta^3} \quad \text{if } t \leq \bar{t} \\ \frac{\partial \Pi_I^T}{\partial \beta} &= 1 \qquad\qquad\qquad\quad \text{if } t > \bar{t} \end{aligned} \tag{1.10}$$

From (1.10) one can see that ISP's incentives to invest are always positive for all values of the quality access. However, when contents are strong substitutes (i.e. $t \leq \bar{t}$), the

ISP's incentives is greater than when contents are weak substitutes. Remark that when contents are strong substitutes, an increase in the access quality (β) reduces the ISP's incentives to invest.
The following Proposition sum up the above discussion

Proposition 2. *Allowing a termination fee always gives positive incentives for the ISP to invest in access quality. Theses incentives are greater when contents are strong substitutes.*

The termination fee have as main property to fully internalize the effects produced by the unit receipt of advertising on qualities of contents. Contrarily to the benchmark case, at the equilibrium, the tradeoff between advertising and access quality does not occur and thus a marginal investment of the ISP is always profitable.

A related question that may be addressed is that does termination fees lead the ISP is to abuse of his market power on the platform. In other words, are the ISP's profit maximizing termination fees may exceed the termination fees that a benevolent planner could choose taking into account the state of competition on the platform. To explore this issue, lets now consider the welfare maximization problem. Hence, mimicking the ISP problem, the welfare maximizing scheme ($s,\ p$) solves

$$\max_{p,\ s}\ TP \ \text{ s.t. } \ \Pi_I \geq 0 \ ; \Pi_2(\theta_2,\ \theta_1) \geq 0 \text{ and } \beta\left(1+\theta_1\right) - p - t\alpha_1 \geq 0$$

From the solution of this problem (given in Appendix), one can derive the following Proposition.

Proposition 3. *Whenever* $\omega \leq \bar{\omega}$ *and* $t \leq \bar{t}$ *, the ISP profit maximizing termination fee never exceeds the welfare maximizing termination fee.*

The result shows that when competition between CPs (similarly efficient i.e. ω lower than $\bar{\omega}$) is sufficiently fierce (t low), optimal termination fees are never lower than the ISP termination fees. In this situation, one can consider that the ISP does not abuse of its market power on the content side of the platform fixing a high termination fee. This results from the fact, the ISP can capture rents on both sides of the platform whereas the benevolent (utilitarist) regulator cannot achieve this trade-off. As shown in Lemma 1, when contents are strong substitutes ($t < \bar{t}$), the ISP chooses a relatively low termination fee to give incentives for CPs to invest in quality in order to retrieve rents on the consumer's side.

Flat rate vs. termination fee

This part is devoted to comparisons between flat rate and termination fee regimes. In the following we then study how introducing termination fees affect all equilibrium outcomes, the surplus breakdown and ISP's investment incentives in access quality.

Prices and content qualities

We first compare unit fees for consumers and content qualities in both flat rate and termination fee pricing configurations. Define a value t denoted $\hat{t} = \frac{\beta\sqrt{a\omega(2\omega-1)}}{2\omega}$ and state the following lemma.

Lemma 2 *(i) When* $t \leq \bar{t}$ *and* $t > (<)\hat{t}$ *then* $p^T > (<)p^N$ *and* $\theta_i^T > (<)\theta_i^N$
(ii) When $t > \bar{t}$ *then* $p^T < p^N$ *and for* $i = 1,\ 2 : \theta_i^T < \theta_i^N$

Consider the case *(ii)* where contents are weak substitutes. In this situation, introducing a termination fee reduce both unit access price and content qualities compared to flat rates. The intuition is that, as we have already seen, offering an access to a platform, the ISP can capture rents on both sides. Using a termination fee instead of a flat rate alleviate the incentives for the ISP to post a high unit access price on consumer's side because he can get surplus directly from the CPs choosing a termination fee that fully internalizes the effect of advertising (see Lemma 1). Consequently, content qualities turn down. These results also happen when contents are strong substitutes (i.e. $t \leq \hat{t} < \bar{t}$) because the termination fee is closely set to the fully internalizing level. For intermediate values of the degree of substitutability between contents (i.e. $\hat{t} < t < \bar{t}$), the equilibrium termination fee does not fully internalize the effect of advertising. As a result, the ISP must choose a higher unit access price than the flat rate configuration. Remark that this intermediate case does not appear when advertising is very profitable (a is high)[3].

Profits and Surplus

We now compare profits and surplus in both flat rate and termination fee. Define another value of t denoted $\underline{t}$ and given in the Appendix. We can state the following proposition.

Proposition 4. *Assume that the degree of substitutability between contents is not too high* ($\underline{t} < t < \bar{t}$) *and advertising is not so profitable* ($a < \hat{a}$). *Compared to flat rate, termination fee (i) increase the ISP profit ; (ii) do not decrease CPs profits; (iii) and yield a higher consumer surplus.*

These results show that a termination fee may be profitable both for firms (ISP and CPs) and consumers. In appendix we precisely show that these results occur when contents are not too strong substitutes ($\underline{t} < t < \bar{t}$) and the unit receipt of advertising takes low values. Hence, the ISP benefits from using a termination fee instead of a flat rate (i) and this benefits to content providers (ii). The intuition behind this result is closely related to the one discussed in the previous Lemma. In Lemma 2, we show when contents are not too strong substitutes, the unit access price for consumers is relatively high but a termination fee push up the level of content qualities. Result (iii) shows that the later effect dominate the former and the termination fee finally increases the consumer

[3] Indeed $\hat{t}$ is higher than the bound $\frac{\beta^2(1+\omega)}{4\omega}$ if a is high (i.e. if $a > \hat{a} = \frac{\beta^2(\omega+1)^2}{4\omega(2\omega-1)}$) and conversely.

surplus.

Incentives to invest for the ISP

Now, we study how both price regimes, namely flat rate and termination fee, on the CPs side of the platform affect the ISP's incentives to expand its network access quality. Broadband operators usually claim that content providers act as free-riders since with a flat rate they do not support the intensive use of network. In turn, this alleviate the ISP's investment incentives and finally harms end-users. We give here some explanations about these potential effects.
Comparing incentives to invest in both price regimes leads to the following Proposition.

Proposition 5. *Introducing the termination fee increases incentives to invest for the ISP (i) if advertising is sufficiently profitable ($a \geq \bar{a}$) or (ii) if contents are weakly differentiated ($t \leq \bar{t}$).*

As can be seen from Proposition 1, the ISP has no incentive to invest in access quality when the unit receipt for advertising is high. On the other hand, Proposition 2 shows that the termination fee give always ISP incentives to invest whatever the level of the unit receipt for advertising. From these two results, we obtain directly (i) in Proposition. Furthermore, with the termination fee and when contents are strong substitutes ($t \leq \bar{t}$), the ISP's incentives to invest in access quality is the greater (see (1.10)). Although the ISP's incentives are positive with the flat rate when advertising is not so high, it does not exceed incentives produce by the termination fee regime. This results because the tradeoff between advertising and access quality, discussed following Proposition 1, does not occur in the termination fee regime. Hence, the effect of advertising is fully internalize and ISP can get entirely the return of its investment in access quality.

An implication of last Propositions is the following.

Corollary 1. *Assume that the degree of substitutability between contents is not too high ($\underline{t} < t < \bar{t}$) and advertising is not so profitable ($a < \hat{a}$). Then, the termination fee increases both the ISP profit and its incentives to invest, and benefit to consumers.*

This corollary results directly from Proposition 4 and Proposition 5. Results highlight that there are parameter values for which allowing to charge a termination fee provides positive incentives for the ISP to marginally increase its access quality, and this benefits to consumers. It is particularly the case when both contents are of an intermediary degree of substitutability and the unit receipt of advertising is low. The intuition derives directly from that developed for last propositions.

5. Extensions

We focus briefly in the following on two main extensions. The first extension consists to depart from the linear price for termination fees considering that the ISP can now engage in price discrimination. The second extension introduces the possibility for CPs to control traffic and we analyze in this setting how termination fees may give CPs better

incentives to management traffic.

Termination fee discrimination

Allowing a monopolist, the ISP, to engage in price discrimination may be welfare improving compared to linear price. In the following, we first analyze whether the ISP has an incentive to charge discriminating termination fees to content provider, and how it could affect the consumer surplus.

We assume that the ISP can use termination fees s_1 and s_2 to price discriminate between content providers. We now consider that end-users pay an uniform fee p for network access and each content provider can get access to the network at a unit price s_i i.e. s_1 and s_2 can be now different, so that $f_i = 0$.

From (prCP), we obtain the equilibrium content qualities:

$$\theta_1 = \frac{\beta(a - s_1)}{2t} \quad \text{and} \quad \theta_2 = \frac{\beta(a - s_2)}{2t\omega} \quad (1.11)$$

Engaging in termination fee discrimination, ISP is able to capture more of the CPs' surplus. However, it remains the well known trade off: a high termination fee alleviates ISP to charge high unit access price to consumers, and a lower termination fee induces high content qualities increasing the consumers' valuation and allowing ISP to charge a higher access unit price. Discrimination could be more efficient as it gives additional tool for the ISP to solve this trade off.

Using (1.11), the CPs' profits become:

$$\begin{aligned} \Pi_1(\theta_1,\ \theta_2) &= \frac{(a - s_1)\ \left(\beta^2\omega a - \beta^2\omega s_1 - 2\beta^2 a + 2\beta^2 s_2 + 4t^2\omega\right)}{8t^2\omega} \\ \Pi_2(\theta_2,\ \theta_1) &= \frac{(a - s_2)\ \left(4t^2\omega - 2\beta^2\omega a + 2\beta^2\omega s_1 + \beta^2 a - \beta^2 s_2\right)}{8t^2\omega} \end{aligned} \quad (1.12)$$

We now assume that s_1 is fixed to a given level, denoted $s_1 = \bar{s}$, and we analyze how the investment incentives of the ISP is maximized by controlling the termination fee s_2 . The following Proposition shows how departing from uniform pricing (no termination fee discrimination) the ISP's investment incentives may move with price discrimination.

Proposition 6. *Suppose that a give termination fee is set to* $\bar{s}$ *(here* $s_1 = \bar{s}$ *). Allowing price discrimination leads to a higher investment incentives of the ISP. When the CP* 1 *' termination fee is set at* $\bar{s}$ *, then there exists a termination fee* $s_2^* < \bar{s}$ *that gives the best incentive to invest for the ISP.*

When termination fee discrimination is assumed, the ISP can improve his incentives to invest choosing a lower termination fee for the less efficient content provider (CP2). This a standard rent extraction result. One can remark that discrimination is stronger (i.e. $\bar{s} - s_2^*$ is large) when content are weak substitutes (t large) or when efficiency gap between CPs is more important (i.e. ω is larger over 1).

CPs incentive to manage traffic

To investigate this issue, we need to introduce a new parameter, $\lambda_i \in [0,\ 1]$, which represents the effort produces by CPs to control the traffic that they pass through the ISP's network for delivering their contents to end-users. In the following, we come back to the case of no price discrimination. Hence, the ISP charge a same termination fee for both CPs.

The CPs profits write now:

$$\Pi_i(\theta_i,\ \theta_j) = (a - (1-\lambda_i)\ \ s_i)\alpha_i - f_i - \frac{\omega_i}{2}\theta_i^2 \qquad \text{for } i,\ \ j = 1,\ \ 2 \text{ and } i \neq j \quad (1.13)$$

Expression (1.13) shows how the parameter that represents the effort of traffic management of CPs affects their profits. Hence, a high value for λ_i means that CP i carefully manages its traffic: for each consumer, the CP i earns an unit receipt, a, from advertisers whereas he pays $(1-\lambda_i)$ unit of access to the ISP, which is less than unity. Therefore, increasing λ_i mitigates the impact of the termination fee for the CP i.

The question here is how terminations fee impacts on incentives of CPs to marginally increase λ_i, and in turn what could be the best regime for consumers.

Considering a termination fee s, we deduce from (1.13) the equilibrium content qualities as:

$$\theta_1 = \frac{\beta(a - s(1-\lambda_1))}{2t} \quad \text{and} \quad \theta_2 = \frac{\beta(a - s(1-\lambda_2))}{2t\omega} \quad (1.14)$$

Using (1.14), the CPs' profits write:

$$\begin{aligned} \Pi_1(\theta_1,\ \theta_2) &= \frac{(a - s(1-\lambda_1))\ \left(\beta^2\omega a - \beta^2\omega s(1-\lambda_1) + 2\beta^2 s(1-\lambda_2) - 2\beta^2 a + 4t^2\omega\right)}{8t^2\omega} \\ \Pi_2(\theta_2,\ \theta_1) &= \frac{(a - s(1-\lambda_2))\ \left(\beta^2 a - \beta^2 s(1-\lambda_2) + 2\beta^2\omega s(1-\lambda_1) - 2\beta^2\omega a + 4t^2\omega\right)}{8t^2\omega} \end{aligned} \quad (1.15)$$

Proceeding as section 4, we derive, from the maximization problem of the ISP, outcomes equilibrium. From the same reason than previously, as CP 2 is the least efficient firm, the ISP can get its entire surplus with the termination fees.

The following Proposition states how termination fees affect the most efficient CP's incentives to manage traffic.

Proposition 7. *Suppose that both CPs produce the same effort to manage their traffic ($\lambda_1 = \lambda_2$) and contents are strong substitutes ($t < \widehat{t}$): (i) the most efficient CP has incentives to manage its traffic in a better way; (ii) this incentive increases with the network access quality (β) offered by the ISP.*

The economics behind Proposition 7 is as follows. Consumers value more for contents

that have a high quality. As content quality is an increasing function of λ_i , CP1 has then an incentive to provide a better traffic management. This, in turn, leads the ISP to choose a relatively high termination fee (as shown in Appendix) and this hurts the CP 1 's profit. Thus, for CP1 it appears a tradeoff between managing traffic (increasing λ_1) to attract more consumers, and supporting a low termination fee (decreasing λ_1) to directly increase its profit. When contents are strong substitutes, the ISP chooses a low termination fee to induce a higher content quality from CPs. In this case, CP 1 can increase its effort to manage traffic without inducing a too high termination fee. Result (ii) shows that a better access quality gives more incentives to content providers to control their traffic. This highlights complementarity between access quality and traffic management from CPs.

6. Concluding remarks

Are more sophisticated pricing schemes than the so-called "net neutrality " may increase economic efficiency on telecommunication platforms? How these pricing scheme are able to enhance incentives to invest from Internet service providers? Broadband operators usually claim that content providers act as free-riders since with a flat rate they do not support the intensive use of network. In turn, this alleviate the ISP's investment incentives and finally harms end-users. In this paper, we aimed to tackle these questions analyzing the benefits and pitfalls of non-discrimination/discrimination rules for network access using a simple model of two-sided platform.

Our results can be summarized as follows. First departing from the "net neutrality" status-quo where flat rates are used, introducing termination fees may increases incentives to invest for the ISP, more precisely when one-sided revenues from the platform for content providers (i.e. advertising) are sufficiently high or when contents are weakly differentiated. Moreover if the degree of substitutability between contents is not too high and advertising is profitable but not extremely, termination fees increase both the ISP profit and benefit to consumers. In some sense, our results support the idea that termination fees can be an appropriate instrument to regulate a too intensive use of network by content providers and therefore to alleviate the Internet service providers incentives to invest in network extension or enhancement.

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Appendix

Proposition 1

$$\frac{\partial \Pi_I^*}{\partial \beta} = \frac{\beta a^2(1-2\omega)+\beta t a(1+\omega)+2t^2\omega}{2t^2\omega} > (<)0 \text{ iff } a < (>)\bar{a}$$

We analyze $\beta a^2(1-2\omega)+\beta t a(1+\omega)+2t^2\omega = 0$.

Two roots are found $\bar{a} = \frac{\left(\beta(\omega+1)+\sqrt{\beta^2\omega^2+2\beta^2\omega+\beta^2-8\beta\omega+16\beta\omega^2}\right)t}{2\beta(2\omega-1)} > 0$

and $\underline{a} = \frac{\left(\beta(\omega+1)-\sqrt{\beta^2\omega^2+2\beta^2\omega+\beta^2-8\beta\omega+16\beta\omega^2}\right)t}{2\beta(2\omega-1)} < 0$.

Remark that $\frac{\partial \bar{a}}{\partial \beta} < 0$; $\bar{a} \to \infty$ if $\beta \to 0^+$ and $\bar{a} = \frac{(\omega+1)t}{2\omega-1}$ if $\beta \to \infty$. One can also directly see that it exists $\hat{a} = \frac{\omega+1}{2\omega-1}t$ where $\frac{1}{2}t < \hat{a} < 2t$, such that if $a \le \hat{a}$ then $\frac{\partial \Pi_I^*}{\partial \beta} \ge 1$ for all β . However whenever $a > \hat{a}$ then $\frac{\partial \Pi_I^*}{\partial \beta} < 1$ and it exists a value of β i.e. $\hat{\beta} = \frac{2t^2\omega}{a\left((2\omega-1)\,a-t(\omega+1)\right)}$ such that $\frac{\partial \Pi_I^*}{\partial \beta} < 0$, for $\beta > \hat{\beta}$.

Lemma 1

As Π_I is linearly increasing in p , the last constraint is necessarily binding and using it in Π_I yields a linear function of s for which

$$\frac{\partial \Pi_I}{\partial s} ? 0 \Leftrightarrow t ? \frac{\beta^2(1+\omega)}{4\omega}$$

We denote $\bar{t} = \frac{\beta^2(1+\omega)}{4\omega}$ Therefore, as CP 2 constraint has two zeros namely $s'' = a$ and $s' = a - \frac{4t^2\omega}{\beta^2(2\omega-1)} < a$, these prices are chosen in turns according to

$$\text{if } t \le \bar{t} \text{ then } s^* = s' \text{ and } p^* = \frac{-2\beta+4\beta\omega+3t}{2(2\omega-1)}$$

$$\text{if } t > \bar{t} \text{ then } s^* = s'' \text{ and } p^* = \beta - \frac{t}{2}$$

- if $t \le \bar{t}$ then
$$\Pi_I^* = \frac{4\beta^3\omega - 8t^2\omega + 4\beta^2 a\omega - 2\beta^2 a - 2\beta^3 + 3\beta^2 t + 4c\beta^2 - 8c\beta^2\omega}{2\beta^2(2\omega-1)}$$
- $\Pi_1(\theta_1^*,\ \theta_2^*) = \frac{6t^2\omega(\omega-1)}{\beta^2(2\omega-1)^2}$

  $\Pi_2(\theta_2^*,\ \theta_1^*) = 0$
-
- if $t > \bar{t}$ then $\Pi_I^* = \beta + a - 2c - \frac{t}{2}$ and $\Pi_i(\theta_i^*,\ \theta_j^*) = 0$.

The ISP investment incentive is:

$$\text{if } t \le \bar{t} \text{ then } \frac{\partial \Pi_I^*}{\partial \beta} = \frac{2\beta^3(2\omega-1) + 8t^2\omega}{\beta^3(2\omega-1)} > 0$$

$$\text{if } t > \bar{t} \text{ then } \frac{\partial \Pi_I^*}{\partial \beta} = 1$$

Proposition 3

Considering a concave shape in s for TP leads to verify the following parametric condition: $t \ge t_c = \frac{\beta^2(\omega-1)^2}{2\omega(\omega+1)}$. Here TP does not respond to p so the consumer's and ISP constraints are not necessarily binding. However, it always exist a value of $p \ge 0$ such that both constraints are not violated.
The unconstrained solution is defined by $s^W = a - \frac{2t^2\omega(\omega+1)}{2t\omega(\omega+1) - \beta^2(\omega-1)^2} < a$ and is admissible while $\Pi_2^W(\theta_2,\ \theta_1) \ge 0$ (i.e. evaluated for $s = s^W$). One can see that this is the case whenever $t \ge t_w = \frac{\beta^2(4\omega^2 - 3\omega + 4)}{4\omega(\omega+1)}$ with $t_w > t_c$. For situations where $t < t_w$ then the welfare maximizing termination fee is defined by Lemma 1. Moreover $t_w \le \bar{t}$ if $\omega \le \bar{\omega} = \frac{5}{3}$ and in that case $s^W \ge s^T$.

Proposition 4.

Let us denote $\Pi_I^N = \Pi_I^*$ when network neutrality applies and $\Pi_I^T = \Pi_I^*$ when termination fees are used.
(1)

- If $t \le \bar{t}$ then $\Pi_I^N = \beta - 2c - \frac{t}{2} - \frac{\beta^2(2\omega-1)}{4t^2\omega}a^2 + \frac{4t^2\omega + \beta^2(\omega+1)\ t}{4t^2\omega}a$

  and $\Pi_I^T = a - 2c + \frac{4\beta^3\omega - 8t^2\omega - 2\beta^3 + 3\beta^2 t}{2\beta^2(2\omega-1)}$ so that

$$\Pi_I^N - \Pi_I^T = \frac{\left(-2\beta^2 a\omega + \beta^2 a + \beta^2 t + \beta^2 \omega t - 4t^2\omega\right)\ \left(-4t^2\omega + 2\beta^2 a\omega - \beta^2 a\right)}{4t^2\omega\beta^2\left(2\omega - 1\right)}\ .$$

(i) We show that $A = -4t^2\omega + 2\beta^2 a\omega - \beta^2 a$ is a decreasing function of t and, takes a positive value ($\beta^2 a(2\omega - 1)$) when $t = 0$ and negative value when $t = \bar{t}$. Hence, there exists a value of t denoted $\hat{t}$ such that A is positive when $t \le \hat{t}$ and negative when $t \ge \hat{t} = \frac{\beta\sqrt{a\omega(2\omega-1)}}{2\omega}$.

(ii) We show that $-2\beta^2 a\omega + \beta^2 a + \beta^2 t + \beta^2 \omega t - 4t^2\omega$ is always negative when $a \ge \hat{a} = \frac{\beta^2(\omega+1)^2}{4\omega(2\omega-1)}$ and has 2 roots when $a < \hat{a}$:

$t' = \frac{\left(\beta\omega + \beta - \sqrt{\beta^2\omega^2 + 2\beta^2\omega + \beta^2 - 32a\omega^2 + 16a\omega}\right)\beta}{8\omega}$ and $t'' = \frac{\left(\beta\omega + \beta + \sqrt{\beta^2\omega^2 + 2\beta^2\omega + \beta^2 - 32a\omega^2 + 16a\omega}\right)\beta}{8\omega}$

Remark that $t' > t''$ where $t'' > 0$ and $t' < \bar{t}$, we note $t' = \hat{t}$. Hence, $-2\beta^2 a\omega + \beta^2 a + \beta^2 t + \beta^2 \omega t - 4t^2\omega$ is positive when $t \le \hat{t}$ and negative otherwise. We can also show that $\hat{t} - \hat{\bar{t}} \ge 0$ if $a \le \hat{\hat{a}} = \frac{\beta^2(\omega+1)^2}{32\omega(2\omega-1)}$ where $\hat{\hat{a}} < \hat{a}$

Finally, we conclude that:

$\Pi_I^N < \Pi_I^T$ if $a > \hat{a}$

$\Pi_I^N > \Pi_I^T$ if $a < \hat{a}$ and $\inf\{\hat{t},\ \hat{\bar{t}}\} < t < \sup\{\hat{t},\ \hat{\bar{t}}\}$; and $\Pi_I^N < \Pi_I^T$ otherwise

- If $t > \bar{t}$ then $\Pi_I^N - \Pi_I^T = \frac{\beta^2 a(t(\omega+1) - a(2\omega-1))}{4t^2\omega}$

  We can show that $\Pi_I^N - \Pi_I^T < 0$ if $t < \frac{a(2\omega+1)}{\omega+1}$ and $\Pi_I^N - \Pi_I^T > 0$ otherwise. We also show that $\frac{a(2\omega+1)}{\omega+1} > \bar{t}$ if $a > \hat{a}$

  Finally: $\Pi_I^T > \Pi_I^N$ if $t < \frac{a(2\omega+1)}{\omega+1}$ and $a > \hat{a}$; and $\Pi_I^T < \Pi_I^N$ otherwise.

(2)

- If $t \le \bar{t}$ then $\Pi_1^T - \Pi_1^N = -\frac{3(\omega-1)\left(2\beta^2 a\omega - \beta^2 a + 4t^2\omega\right)\left(2\beta^2 a\omega - \beta^2 a - 4t^2\omega\right)}{8t^2\omega\beta^2(2\omega-1)^2}$,

  where $\left(2\beta^2 a\omega - \beta^2 a + 4t^2\omega\right) > 0$ and $\left(2\beta^2 a\omega - \beta^2 a - 4t^2\omega\right) > 0$ iff $t < \hat{t} = \frac{\beta\sqrt{a\omega(2\omega-1)}}{2\omega}$. Finally, $\Pi_1^T - \Pi_1^N \,?\, 0$ iff $t \,?\, \hat{t}$. Remark that $\hat{t} \,?\, \bar{t}$ iff $a \,?\, \hat{a} = \frac{\beta^2(\omega+1)^2}{4\omega(2\omega-1)}$. Then, $\Pi_1^T - \Pi_1^N > 0$ iff $\hat{t} < t < \bar{t}$ and $a < \hat{a}$ and, $\Pi_1^T - \Pi_1^N < 0$ otherwise.

- If $t > \bar{t}$ then $\Pi_1^T - \Pi_1^N = 0 - \frac{3a^2\beta^2(\omega-1)}{8t^2\omega} < 0$. Then, $\Pi_1^N > \Pi_1^T$.

  Finally, $\Pi_1^T > \Pi_1^N$ iff $\hat{t} < t < \bar{t}$ and $a < \hat{a}$.

(3) $\Pi_2^T = \Pi_2^N$, $\forall t,\ \beta,\ \omega,\ a,\ c$

(4)

- If $t \le \bar{t}$ then $CS^T - CS^N = -\frac{(\omega-1)^2\left(2\beta^2 a\omega-\beta^2 a-4t^2\omega\right)\left(2\beta^2 a\omega-\beta^2 a+4t^2\omega\right)}{16t^3\omega^2(2\omega-1)^2} = \left(\Pi_1^T - \Pi_1^N\right)\ \frac{\beta^2(\omega-1)}{6t\omega}$. The sign of $CS^T - CS^N$ is the same as $\left(\Pi_1^T - \Pi_1^N\right)$.
- If $t > \bar{t}$ then $CS^T - CS^N = -\frac{\beta^4 a^2(\omega-1)^2}{16t^3\omega^2} < 0$
- Finally, $CS^T > CS^N$ iff $\hat{t} < t < \bar{t}$ and $a < \hat{a}$.

Hence we can conclude that

(1) When $t \le \bar{t}$ then $\Pi_I^N > \Pi_I^T$ if $a < \hat{a}$ and $\underline{\underline{t}} < t < \underline{t}$, where $\underline{\underline{t}} = \inf\{\hat{t},\ \hat{t}\}$ and $\underline{t} = \sup\{\hat{t},\ \hat{t}\}$

When $t > \bar{t}$ then $\Pi_I^T > \Pi_I^N$ if $t < \frac{a(2\omega+1)}{\omega+1}$ and $a > \hat{a}$

(2) When $t \le \bar{t}$ then $\Pi_1^T > \Pi_1^N$ iff $\hat{t} < t < \bar{t}$ and $a < \hat{a}$

(3) $\Pi_2^T = \Pi_2^N$, $\forall t,\ \beta,\ \omega,\ a,\ c$

(4) When $t \le \bar{t}$ then $CS^T > CS^N$ iff $\hat{t} < t < \bar{t}$ and $a < \hat{a}$

Proposition 5.

$\frac{\partial \Pi_I^N}{\partial \beta} = 1 + \frac{1}{2}a\beta\frac{(2\omega-1)\ \ a-t(\omega+1)}{\omega t^2}$

$\frac{\partial \Pi_I^T}{\partial \beta} = 1 + \frac{8\omega t^2}{(2\omega-1)\beta^3}$ if $t \le \bar{t}$ and $\frac{\partial \Pi_I^T}{\partial \beta} = 1$ otherwise

- If $t \le \bar{t}$,
- $\frac{\partial \Pi_I^T}{\partial \beta} - \frac{\partial \Pi_I^N}{\partial \beta} = \frac{8\omega t^2}{(2\omega-1)\beta^3} + \frac{1}{2}a\beta\frac{(2\omega-1)\ \ a-t(\omega+1)}{\omega t^2}$
- First we see that $\frac{\partial \Pi_I^T}{\partial \beta} > \frac{\partial \Pi_I^N}{\partial \beta}$ if $a > \bar{a}$ for all β admissible ($\beta \ge \beta_* \equiv 2\sqrt{\frac{\omega t}{1+\omega}}$). If $a \le \bar{a}$, lets us study $\Delta(\beta,\ a) = \frac{\partial \Pi_I^T}{\partial \beta} - \frac{\partial \Pi_I^N}{\partial \beta}$. Since
- $\frac{\partial \Delta(\beta,\ a)}{\partial \beta} = \frac{2\omega-1}{2\omega t^2}a^2 - \frac{\omega+1}{2\omega t}a - \frac{24t^2\omega}{\beta^4(2\omega-1)}$,
- then we see that $\frac{\partial \Delta(\beta,\ 0)}{\partial \beta} = \frac{\partial \Delta(\beta,\ \bar{a})}{\partial \beta} = -\frac{24t^2\omega}{\beta^4(2\omega-1)} < 0$ for all β and
- $\frac{\partial \Delta(\beta,\ a)}{\partial \beta \partial a} = \frac{2(2\omega-1)\ \ a-t(\omega+1) > 0}{2\omega t^2 \le 0}$ if $\begin{matrix} a > \frac{1}{2}\bar{a} \\ a \le \frac{1}{2}\bar{a} \end{matrix}$
- Moreover, $\frac{\partial \Delta(\beta,\ 0)}{\partial \beta} > 0$, $\frac{\partial \Delta\left(\beta,\ \frac{\bar{a}}{2}\right)}{\partial \beta} < 0$, $\Delta(\beta,\ 0) > 0$ and $\Delta\left(\beta,\ \bar{a}\right) > 0$ then $\Delta(\beta,\ a) > 0$ for $a \le \bar{a}$
- If $t > \bar{t}$
- $\frac{\partial \Pi_I^T}{\partial \beta} - \frac{\partial \Pi_I^N}{\partial \beta} = \frac{1}{2}a\beta\frac{(2\omega-1)\ \ a-t(\omega+1) > 0}{\omega t^2 \le 0}$ if $\begin{matrix} a > \bar{a} \\ a \le \bar{a} \end{matrix}$

- Conclusion is:
  (i) When $t \le \bar{t}$ then $\frac{\partial \Pi_I^T}{\partial \beta} > \frac{\partial \Pi_I^N}{\partial \beta}$
  (ii) When $t > \bar{t}$ then $\frac{\partial \Pi_I^T}{\partial \beta} > \frac{\partial \Pi_I^N}{\partial \beta}$ if $a > \bar{a}$

Proposition 6

Using (prISP), (alpha) and (thetadiscrim) gives:

$$\Pi_I(p,\ s_1,\ s_2) = p + \frac{a\beta^2\omega s_1 - s_2\beta^2\omega a + s_2\beta^2\omega s_1 - \beta^2 s_2^2 - \beta^2\omega s_1^2 + a\beta^2 s_2 - s_1\beta^2 a + s_1\beta^2 s_2 + 2s_1 t^2\omega + 2s_2 t^2\omega - 8ct^2\omega}{4t^2\omega}$$

with

$$p^* = \frac{4\beta\omega t + \beta^2\omega a - \beta^2\omega s_1 + \beta^2 a - \beta^2 s_2 - 2t^2\omega}{4t\omega}$$

$$\frac{\partial \Pi_I(p^*,\ s_1,\ s_2)}{\partial \beta} = \frac{\beta\omega at - \beta\omega st + \beta at + 2t^2\omega - \beta s_2 t - \beta\omega s^2 - \beta s_2^2 + s\beta\omega a - s\beta a + s\beta s_2 - s_2\beta\omega a + s_2\beta\omega s + s_2\beta a}{2t^2\omega}$$

$$\frac{\partial \Pi_I(p^*,\ s_1,\ s_2)}{\partial \beta} = \frac{\beta at - \beta st + 2t^2 - \beta s^2 + s\beta a - s_2\beta a + s_2\beta s}{2t^2} + \frac{\beta(a - s_2)(t - s + s_2)}{2t^2\omega}$$

$$\frac{\partial \Pi_I(p^*,\ s_1,\ s_2)}{\partial \beta} = 1 + \frac{\beta(a - s)(t + s - s_2)}{2t^2} + \frac{\beta(a - s_2)(t - s + s_2)}{2t^2\omega}$$

Remark that $\frac{\partial \Pi_I(p^*,\ s_1,\ s_2)}{\partial \beta} = f(s_2)$ where $f'(s_2) = -\frac{\beta(a - s_1)}{2t^2} + \frac{\beta(a - t + s_1 - 2s_2)}{2t^2\omega}$ and $f''(s_2) = -\frac{2\beta}{2t^2\omega} < 0$. Suppose that $s_1 = \bar{s},$ ISP's investment incentives reach a maximum at s_2^* given by $f'(s_2^*) = 0$: $s_2^* = \frac{a - t + \bar{s} + \omega(\bar{s} - a)}{2}$.

Finally, we show that $\bar{s} - s_2^* = \frac{t + (a - \bar{s})(\omega - 1)}{2} > 0$ with $\omega > 1$

Proposition 7

The proof is in the same line that of Lemma 1. After calculus we can show:

- $t \le \bar{\bar{t}} = \frac{\beta^2(\omega(1-\lambda_1)+1-\lambda_2)}{4\omega}$
- $\Pi_1^M = \frac{3\left(-\beta^2 a\lambda_2 - 4t^2\omega + 4\lambda_1 t^2\omega + \beta^2 a\lambda_1\right)\left(\beta^2 a\lambda_2 - 4t^2\omega + 4\lambda_1 t^2\omega - \beta^2 a\lambda_1 + 4t^2 - 4\lambda_2 t^2\right)}{8\beta^2\left(-2\omega + 2\omega\lambda_1 + 1 - \lambda_2\right)^2 t^2}$.
- Assuming $\lambda_2 = \lambda$:

  $$\left.\frac{\partial \Pi_1^M}{\partial \lambda_1}\right|_{\lambda_1 = \lambda} = \frac{3\left(\beta^2 a(2\omega - 1) - 4t^2\omega\right)}{2\beta^2(2\omega - 1)^3(1 - \lambda)} ? 0 \quad \text{iff} \quad t ? \hat{t} = \frac{\beta\sqrt{a\omega(2\omega - 1)}}{2\omega}$$

  $$\left.\frac{\partial^2 \Pi_1^M}{\partial \beta \partial \lambda_1}\right|_{\lambda_1 = \lambda} = \frac{12t^2\omega}{(2\omega - 1)^3(1 - \lambda)\beta^3} \ge 0$$

  In this case, the equilibrium termination fee is given by $s = \frac{\beta^2 a(2\omega - 1) - 4t^2\omega}{\beta^2(1-\lambda)(2\ \omega - 1)}$ which is increasing w.r.t. λ .

- $t \geq \bar{t} : \Pi_1^M = \dfrac{a\left(-\lambda_2 + \lambda_1\right)\ \left(-a\beta^2\lambda_2 + \lambda_1\beta^2 a + 4t^2 - 4\lambda_2 t^2\right)}{8\left(-1+\lambda_2\right)^2 t^2}$.

  Assuming again $\lambda_2 = \lambda$:

  $$\left.\frac{\partial \Pi_1^M}{\partial \lambda_1}\right|_{\lambda_1=\lambda} = \frac{a}{2(1-\lambda)} \geq 0$$

  The equilibrium termination fee is now $s = \dfrac{a}{1-\lambda}$ which is increasing w.r.t. λ.