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## Efficiency and vertical networks: A note on demand uncertainty and separated markets

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**Beiträge zur angewandten Wirtschaftsforschung**

**Nr. 18 (2006)**

*Vorläufige Fassung! Nicht im Handel!*

**Efficiency and vertical networks:  
A note on demand uncertainty and separated markets**

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## 1. Introduction

Recent years have witnessed increasing interest in the determinants of firms' organizational choices. In particular questions relating to cooperation among firms have been frequently analyzed. New models of economic exchange namely networks have been developed recently. Vertical networks as supply structures vary across industries and are distinct from vertically integrated firms and markets.

By forming vertical collaboration structures, however, firms alter the competitive position of several firms and in turn influence market structure and performance. This two-way flow of influence is central in our analysis.

Therefore the goal in this paper is to identify a trade-off between stability and profitability possibly existing in vertical networks. We develop a model that captures demand uncertainty and separated markets.

The remaining part of the paper is organized as follows. Section 2 provides a brief overview of related literature in order to motivate our approach. The model is presented in section 3. In section 4 we explore the optimal distribution of prices for an input good in buyer- seller networks. Section 5 concludes.

## 2. Related Literature

This paper is a contribution to the literature of group formation and cooperation in oligopolies. Questions of group formation and cooperation have long been in focus of economic research especially in game theory. A central issue related to the more formal theory of group formation is the formulation of a proper coalition game which assigns cooperation rents to a given set of players and to every subset of players. A coalition game also specifies pay-off functions for every player and strategy. Stability and dimension of coalitions depending on different pay-off functions und cooperation rents are important aspects discussed in this context.<sup>1</sup> The described coalition approach analyses specific relationships between members of a coalition only indirect through the charac-

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<sup>1</sup> See MYERSON (1977), BLOCH (1995) or for a survey WIESE (2005), BLOCH (1997).

teristic function.<sup>2</sup> In this setup symmetric relationships are implicitly assumed: every firm who is part of a coalition cooperates with every firm who is member of the same coalition.

If we in turn allow for cooperative relationships that are nonexclusive, asymmetric structures of cooperation will be generated that are different from those studied in the coalition-formation literature. If there is cooperation between two firms we will call this relationship a “link”. A network can be defined as a set of firms related with a set of pair wise links between the firms.<sup>3</sup> In this context a star network is a structure of cooperation with a central firm directly linked to every firm while none of the other firms have a direct link with each other. To study concrete problems, industrial structures are often interpreted as networks in the sense above.<sup>4</sup> For example, recent years have witnessed a large body of literature regarding to buyer-seller networks.<sup>5</sup> Questions respecting to advantages of vertical networks in comparison to vertical integration and respecting the influence of different economic scenarios on the formation and optimality of buyer-seller networks have been studied recently. Importance and economic consequences of demand shocks in vertical buyer-seller networks have been an object of analysis too.

KRANTON and MINEHART (2000) show that networks can yield greater social welfare when manufactures experience large idiosyncratic demand shocks. They also highlight comparative advantages of networks: capacity sharing and flexibility.<sup>6</sup> Therefore incentives for the formation of vertical networks exist.

The paper of KRANTON and MINEHART (2000) is a refinement of Piore and Sabel’s (1984) work on „flexible specialists”. Piore and Sabel (1984) argue that networks emerge in times of greater economic uncertainty.<sup>7</sup> In addition to different kinds of un-

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<sup>2</sup> The characteristic function assigns to every subset  $S$  of the set of players  $N$  the payoff that can be realized through collaboration of the members of  $S$  independently from outside players  $N \setminus S$

<sup>3</sup> See JACKSON (2003) and VAN DEN NOUWELAND (2003) for a survey.

<sup>4</sup> For network models of special industrys see for example ECONOMIDES/ HIMMELBERG (1995), HENDRICKS/ PICCIONE/ TAN (1995), SMITH/ BACKERMAN/ RASSENTI (1996).

<sup>5</sup> For a more formal theory of buyer-seller networks see KRANTON/ MINEHART (2001) or HOLMSTRÖM/ ROBERTS (1998) for a survey.

<sup>6</sup> KRANTON/ MINEHART (2000) argue vertically integrated buyer that suffer large negative shocks regret having built costly unused productive capacity. In networks exists fewer units of productive capacity and buyers suffering the largest negative shock do not procure inputs. Inputs are allocated flexible to the buyers with the highest realisation of valuation for such an input.

<sup>7</sup> The connections between demand uncertainty and industry structure have also been in focus of earlier papers. See for example BARON (1971); HOLTHAUSEN (1976).

certainty, the competitive environment of the network firms is in focus of some scientific papers. GOYAL and MORAGA-GONZÁLEZ (2001) analyses the connection between competitive environment, incentives to invest in research und development (R&D) und the structure of the network.

They show that in absence of rivalry between the network firms in separated markets the complete network<sup>8</sup> is stable, profit maximizing und socially optimal.<sup>9</sup>

In case of strong rivalry in a model of Cournot competition in a homogenous product market GOYAL and MORAGA-GONZÁLEZ (2001) show that the complete network is stable, but intermediate levels of collaboration and asymmetric network structures maximizes industry profits and welfare.<sup>10</sup>

This paper develops a framework to marriage the idea of (idiosyncratic) demand shocks with the consideration of the competitive environment of the firms in a buyer-seller network as a vertical industry structure. We will analyze a vertical star network. The upstream level consists of one seller. The downstream level consists of several buyers who can procure inputs from the upstream firm.

The buyers operate in separated markets without the possibility of direct market interaction. But there exist the possibility to interact indirectly through the bundling of demand for the input which is produced by the upstream firm. Bundling of the demand for inputs enables the upstream firm to realise economies of scale for the network from a collective viewpoint.

The demand of every downstream firm for the homogenous input determines the cost savings per unit and therefore the cooperation rent in the network.

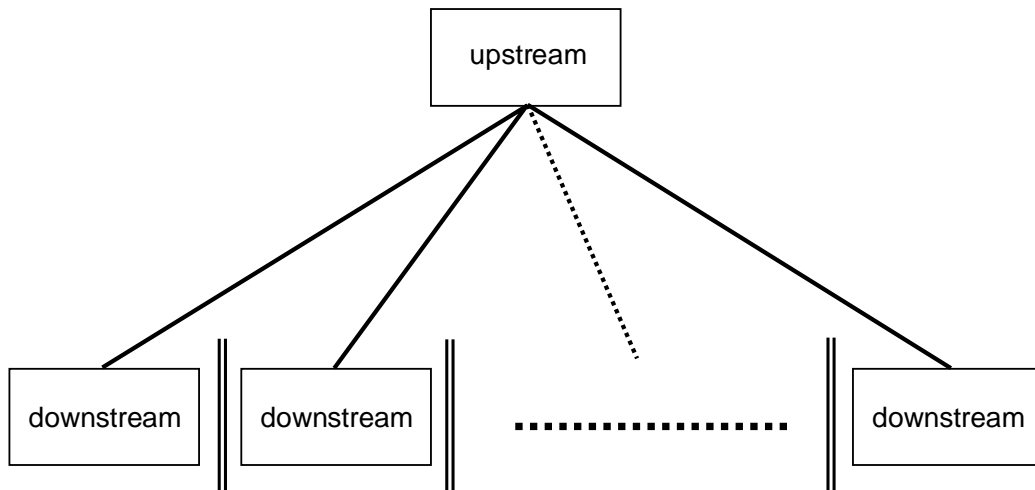
The figure below shows the described setup.

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<sup>8</sup> In complete networks there exists links between every member of the Network.

<sup>9</sup> See for more work on (R&D) networks in oligopol situations GOYAL/ JOSHI (1999) and LEAHY/ NEARY (1997).

<sup>10</sup> Especially they establish a trade off between the level of collaboration, the number of links in the network and the incentives to invest in research and development.



In this paper we assume zero-profits for the upstream firm. Therefore possible cost savings on the upstream level are completely redistributed to the downstream level.<sup>11</sup> In context of the literature above the upstream firm can be interpreted as a joint project of the downstream firms with the goal to reduce marginal cost of input production. Already without demand uncertainty questions regarding proper distributions of the cooperation rent in shape of cost savings arise. Each downstream firm face oligopolistic competition but its market is separated from the other downstream firms of the network. Demand for input which could be produced by the upstream firm is a function of the market success of the downstream firms in the network. Idiosyncratic demand shocks influence downstream markets randomly. Therefore demand for inputs possibly produced by the upstream firm is random too.

Is there a set of stable divisions of the cost savings? Can we identify divisions that firms would agree with ex ante? How should cost savings been allocated from a collective point of view?

In case of some downstream firms experience positive demand shocks while other downstream firms experience negative demand shocks or no demand shocks asymmetric demand for inputs arises in the network. In consequence asymmetric contributions to the realized economies of scale in shape of cost savings on the upstream level arise too. Should cost saving be allocated asymmetric in this situation from a network perspective?

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<sup>11</sup> Therefore problems of incomplete contracts are not in focus in this paper.

Should we strengthen differences in the network or should we balance between the asymmetric downstream markets with proper divisions of the cost savings? These questions will be studied in the submitted paper. At first we present the model.

### 3. Model

#### 3.1 Downstream profits without demand shocks

We consider a duopoly model with price competition and a given degree of product differentiation to model the  $N = \{1, \dots, n\}$  markets on the downstream level. In order to allow for price competition with the possibility of heterogeneity among firms and profits, a model of spatial competition called “linear city” is used.<sup>12</sup> The city consists of a street of length 1. There exist two firms and they sell their output on a single market and compete in prices (Bertrand-competition). We assume separated downstream markets. Therefore every firm on the downstream level face this setup and the competitor of every firm is not member of the network.

Consumers live along the street and are uniformly distributed with the density  $\omega$ .<sup>13</sup> Each consumer wants to buy exactly one unit of the good or nothing at all, if the price exceeds his surplus from consumption  $\bar{v} \in \mathbb{R}^+$ . Individuals only differ in taste specified by the spot, where the individual is situated which is labelled by  $q$ ,  $0 \leq q \leq 1$ . Consumer  $q$  buys at firm 1 if the total costs are lower than if buying at firm 2 and total expenses do not exceed his valuation  $\bar{v}$  for the good.

For every price combination  $p_1$  and  $p_2$  we can find the consumer who is just indifferent from which store to buy. The marginal consumer is denoted by  $\hat{q} = \hat{q}(p_1, p_2)$ . He is located at the point where his total costs that include price und transportation costs are equal irrespectively of where he buys the good such that  $\delta T(\hat{q}) + p_1 = \delta T(1 - \hat{q}) + p_2$  holds, where  $T : [0, 1] \rightarrow \mathbb{R}_0^+$ ,  $q \mapsto T(q)$ ,  $T(0) = 0$ ,  $0 < T'$ ,  $\forall q \neq 0$ ,  $0 \leq T''$  is the common transportation cost function;  $\delta \in [0, 1]$  denotes a parameter that captures the degree of

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<sup>12</sup> The first model of spatial competition is attributed to LAUNHARDT (1885).

<sup>13</sup> Thus the total number of consumers is equal to  $\omega$ .



differentiation in the market.<sup>14</sup>  $p_j, j \in \{1,2\}$  is the price to be paid for the good chosen from firm 1 and firm 2. The marginal consumer is important to derive the two firm's market demand functions: All consumers located to the left of  $\hat{q}$  buy from firm 1 and all consumers located to the right of the marginal consumer buy from firm 2. Assume that the maximum willingness to pay  $\bar{v}$  is high enough that every individual buys in equilibrium. Recall that this setup holds for all of the  $N$  downstream markets. Therefore all firms at the downstream level face oligopolistic price competition with a given degree of product differentiation.

**Remark 3.1 (Shape of demand functions):** *If both companies serve some customers, the demand functions are strictly decreasing in the own and strictly increasing in the competitor's price. A priori it is impossible to identify the sign of their second derivatives. It is determined by the second and third derivatives of the transportation cost function. Note that a linear or quadratic transportation cost function implies that the demand functions are linearly decreasing (increasing) in the own (competitor's) price and consequently the second derivatives of the demand functions vanish:  $\frac{\partial^2 q_j}{\partial p_k \partial p_l} = 0; j, k, l \in \{1,2\}$ .*

**Proof:** An increase in the own price leads to a lower market share (and vice versa for a price increase of the competitor):

$$\frac{\partial q_1}{\partial p_1} = \frac{-1}{\delta [T'(q_1) + T'(q_2)]} = -\frac{\partial q_2}{\partial p_1} < 0, \quad \frac{\partial q_1}{\partial p_2} = \frac{1}{\delta [T'(q_1) + T'(q_2)]} = -\frac{\partial q_2}{\partial p_2} > 0.$$

$$\frac{\partial^2 q_1}{\partial p_1^2} = \frac{T''(q_1) - T''(q_2)}{\delta [T'(q_1) + T'(q_2)]^2} \frac{\partial q_1}{\partial p_1}, \quad \frac{\partial^2 q_1}{\partial p_1 \partial p_2} = \frac{T''(q_1) - T''(q_2)}{\delta [T'(q_1) + T'(q_2)]^2} \frac{\partial q_1}{\partial p_2}$$

$\frac{\partial^2 q_j}{\partial p_k \partial p_l} = 0; j, k, l \in \{1,2\}$  if  $T''$  is constant (which is the case if  $T$  is linear or quadratic).

The same holds for firm 2. ■

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<sup>14</sup> For  $\delta = 1$  the model can be interpreted as second stage of a Hotelling model with endogenous product choice that leads to maximal differentiation see HOTELLING (1929), D'ASPROMONT et al. (1979).

To derive demand, prices and profits in equilibrium, costs of production on the downstream level have to be specified. We assume the only costs occurring are expenses to produce or procure the input good. We further assume an relation of complementarity of 1:1 between the input good und the output that is sold on the market. In case of own input production only fix costs in shape of  $F \in \mathbb{R}^+$  arise. Procurement of the input good on (network) extern markets causes costs per unit denoted by  $p_M \in \mathbb{R}^+$ . Alternatively downstream firm  $i$  can procure the input from the (network) upstream firm with  $p_i'' \in \mathbb{R}^+$  per unit. Therefore the shape of marginal costs  $c_i$  of downstream firm  $i$  is characterized as follows:

$$c_i := \begin{cases} 0 & \text{own production of the input} \\ p_M & \text{procurement on (network) extern markets} \\ p_i'' & \text{procurement on (network) upstream firm} \end{cases}$$

The demand for input is a derivative of the market model described above. Average costs in case of own production of the input good are therefore  $k : (0, \infty) \rightarrow \mathbb{R}^+$ ;  $q_i \mapsto k(q_i) := \frac{F}{q_i}$ ;  $\frac{\partial k}{\partial q_i} < 0$ ;  $\frac{\partial^2 k}{\partial q_i^2} > 0$ .<sup>15</sup> We additionally assume that  $p_M \geq \frac{F}{q_i} \forall q_i$  holds. The condition implies that downstream firms, in comparison to procurement on (network) extern markets, are always better off with the own production of the input.

In this paper the frequently used examples of linear and quadratic transportation costs are applied to prove the existence of several results.

**Example 3.1 (linear and quadratic transportation costs):** Assume that the transportation cost function is linear or quadratic  $T(q) := \alpha + \beta q + \gamma q^2$  the marginal consumer, demand and profits for the downstream firms is given with  $F = 0 \Leftrightarrow c_i > 0 \quad i \neq j \in \{1, 2\}$   
 $d \in \{d_q, d_l\}$ : *quadratic*:  $0 \leq \alpha, \beta, 0 < \gamma, \quad d_q := \beta + \gamma, \quad ; \quad \text{linear}$ :  $0 \leq \lambda, \quad 0 < \beta, \quad 0 = \gamma,$   
 $d_l := \beta,$

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<sup>15</sup> In case of procurement of the input good no fix costs  $F$  arise on the downstream level.

$$\hat{q} = \frac{p_2 - p_1 + d\delta}{2b\delta}; q_i = \left( \frac{p_j - p_i + d\delta}{2d\delta} \right) \omega; p_i = \frac{1}{3}c_j + \frac{2}{3}c_i + b\delta; \pi_i = \frac{\left( \frac{1}{3}c_j - \frac{1}{3}c_i + d\delta \right)^2}{2d\delta} \omega - F$$

**Proposition 3.1 (Comparative statics):** *If transportation costs are from quadratic type we can identify the following marginal effects:*

$$\frac{\partial p_i}{\partial c_k} \geq 0; \frac{\partial q_i}{\partial c_i} \leq 0; \frac{\partial q_i}{\partial c_j} \geq 0; \frac{\partial \pi_i}{\partial c_i} \leq 0; \frac{\partial \pi_i}{\partial c_j} \geq 0; \frac{\partial^2 p_i}{\partial c_k^2} = 0; \frac{\partial^2 q_i}{\partial c_k^2} = 0; \frac{\partial^2 \pi_i}{\partial c_k^2} \geq 0;$$

$$k \in \{i, j\}; i \neq j \in \{1, 2\}$$

### 3.2 Downstream profits with demand shocks

Market demand in the  $N = \{1, \dots, n\}$  downstream markets has a random size  $\tilde{\omega}_i = \omega + \tilde{\varepsilon}_i$ , where  $\tilde{\varepsilon}_i$  is an idiosyncratic shock. The shocks change the density  $\omega$  of consumers in the setup described in 3.1.<sup>16</sup> Assume the shocks are identically and independently distributed with mean zero  $E[\tilde{\varepsilon}_i] = 0$ . Therefore the random size of the  $i$ -th downstream market is identically and independently distributed with mean  $\omega$  or  $E[\tilde{\omega}_i] = \omega$  respectively. This approach is in contrast to the model established in KRANTON and MINEHART (2000). In absence of market environment they assume each buyer has a random valuation for such an input in a buyer-seller network. Therefore manufactures face idiosyncratic shocks<sup>17</sup> to their demand for inputs.

**Example 3.2 (demand shocks):** *Assume the existence of idiosyncratic demand shocks and that the transportation cost function is linear or quadratic. For the marginal consumer, the demand of each firm, prices and profits holds:  $T(q) := \alpha + \beta q + \gamma q^2$ ;  $d_q := \beta + \gamma$ ,  $0 \leq \alpha, \beta$ ,  $0 < \gamma$ ;  $d_i := \beta$ ,  $0 \leq \alpha$ ,  $0 < \beta$ ,  $0 = \gamma$ ;  $i \neq j \in \{1, 2\}$ ;  $d \in \{d_q, d_i\}$ :*

<sup>16</sup> Therefore shocks can be interpreted as random variations of the sum of consumers in the  $N$  downstream markets.

<sup>17</sup> In addition they assume aggregate shocks on the willingness to pay.

$$\hat{q} = \frac{p_2 - p_1 + \delta d}{2\delta d}; q_i = \left( \frac{p_j - p_i + \delta d}{2\delta d} \right) (\omega + \varepsilon_i);$$

$$p_i = \frac{1}{3}c_j + \frac{2}{3}c_i + \delta d; \pi_i = \frac{\left( \frac{1}{3}c_j - \frac{1}{3}c_i + \delta d \right)^2}{2\delta d} (\omega + \varepsilon_i)$$

It can be easily checked that comparative static's are the same as in the case without demand shocks. Up to now the model ignores the possibility of different costs situations faced by the firms in consequence of varying demands. Perhaps in case of positive demand shocks inputs can be produced or procured suffering lower costs per unit. Beyond the effect concerning both competitors, perhaps the participation in a vertical network can establish comparative cost advantages for a member of the network. Therefore next section is to specify cost functions explicitly.

### 3.3 Demand and costs for upstream firms

Procurement of the input from the (network) upstream firm causes fix costs  $F \in \mathbb{R}^+$  faced by the upstream firm.<sup>18</sup> The upstream firm is able to produce the input good for several downstream firms bundled. Therefore the average production costs per unit input good are in shape of  $K^U : (0, \infty) \rightarrow \mathbb{R}^+; Q \mapsto K^U(Q) := \frac{F}{Q}; \frac{\partial K^U}{\partial Q} < 0; \frac{\partial^2 K^U}{\partial Q^2} > 0$ . In this specification  $Q = \sum_{i=1}^n q_i^u; q_i^u \in \{0, q_i\}$  is the sum of demand for the input good from the  $N$  downstream firms derived from their individual (and separated) market situation. With this specification for all downstream firms it is implicitly assumed either the whole demand for input is passed to the upstream or zero units are passed.<sup>19</sup> For the upstream firm the zero-profit condition holds. Therefore all cost savings in shape of scale effects are given back to the downstream level. In the symmetric case  $c_i = p_i^u = \frac{F}{Q} = p_j^u = c_j$  for all downstream firms holds.

<sup>18</sup> In this framework fix costs can be widely interpreted. Examples are economies of scale or innovation technologies. The assumption of a convex innovation technology is in line with the approach of D'ASPREMONT/ JACQUEMIN (1988) and TIROLE (1992). Both papers assume increasing costs for the accumulation of further units of experience. Therefore the accumulated stock of experience is convex in investments in research and development.

<sup>19</sup> Otherwise duplication of fix costs on up- and downstream level occurs.

## 4. Optimal Prices

### 4.1 Normative

In this section we analyze the optimal distribution of realized economies of scales on the upstream level in case of bundling the demand for the input good. Recall that for the upstream level the zero-profit condition holds and therefore all cost savings have to be assigned to the downstream firms. Our purpose is to highlight the optimal pattern of constant prices for the input good that have to be paid by the downstream firms. The examples of chapter 3.2 imply that the profits of the  $N$  downstream firms are increasing in the market demand. For this reason we explore if it's possible to increase (aggregated) market demand (and therefore to increase aggregated profits) from a network perspective through a proper allocation of input prices to the  $N$  downstream firms. For simplicity and clearness we investigate and illustrate this important question for linear transportation costs  $T(q) := \alpha + \beta q$  in a symmetric setting.<sup>20</sup>

Consider two downstream firms  $i, j$  that faces Bertrand competition in separated markets as modelled above. Assume both markets are completely identical ex ante. In particular prices for the input good and market prices, demands and profits are the same in equilibrium. Now assume the  $i$ -th downstream market is affected by a (positive) demand shock  $(\omega + \varepsilon)$  and simultaneously the total number of consumers in the  $j$ -th downstream market is unchanged  $\omega$ .

**Remark 4.1:** *The positive demand shock in the  $i$ -th downstream market increases aggregated demand  $q = q_i + q_j$  and aggregated profits  $\pi = \pi_i + \pi_j$  from a network perspective. Constant input prices independently from the (positive) demand shock yield positive profits for the upstream firm  $\pi_u > 0$ :*

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<sup>20</sup> The results also emerge for quadratic transportation costs and can be shown for weaker assumptions of symmetry.

$$\Delta q = \frac{\frac{1}{3}c_2 - \frac{1}{3}c_1 + \delta b}{2\delta d} \varepsilon; \Delta \pi > \frac{\left(\frac{1}{3}c_2 - \frac{1}{3}c_1 + \delta b\right)^2}{2\delta d} \varepsilon; \pi_U = \frac{F\varepsilon}{Q(Q+\varepsilon)} \left(\omega + \frac{1}{2}\varepsilon\right) > 0$$

When in turn the zero-profit condition holds for the upstream firm, decreasing input prices for at least one downstream firm is an easy implication of positive demand shocks in our setup. In consequence the profit for all intramarginal unit increases. Furthermore additional demand can be served of at least one downstream firm in duopolistic price competition due to the better cost position. How should we design the input prices? Is it possible to increase the profit from network perspective with an additional asymmetric cost shock in presence of a demand shock?

Intuitively, one could recommend making the downstream firm  $i$  (with larger market demand due to a positive demand shock) better off at the expense of the downstream firm  $j$  with a market demand comparatively smaller. The reason could be seen in the more valuable market shares of downstream firm  $i$ . If this is true an optimal cost differentiation depending on different market demands would exist. Our results below show that this intuition does not hold in presence of the zero-profit condition for the upstream firm. Questioning optimal cost differentiation does not require explicit consideration of the height of the economies of scale on the upstream level changed due to demand shocks.

For simplicity assume identical marginal costs of the competitors in both markets  $i, j$  after the demand shock such that  $c_1 = c_2 = p_i^u = p_j^u > 0$  holds for downstream markets  $i, j$ .  $p_i^u, p_j^u$  denote the prices for the input good in the downstream markets  $i, j$  needed to guarantee zero-profits on the upstream level. Now assume downstream firm  $i$  (affected by a positive demand shock) get a cost reduction of  $\lambda_1$  what  $c_1^i = p_i^u - \lambda_1$  implies. To ensure zero-profits on the upstream level downstream firm  $j$  has to incur additional costs of  $\lambda_2$  such that  $c_1^j = p_j^u + \lambda_2$  holds.

**Proposition 4.1 (1):** *Asymmetric costs  $c_1^i = p_i^U - \lambda_1$ ;  $c_1^j = p_j^U + \lambda_2$  for downstream firms increase demand and profits from network perspective if and only if  $\lambda_2 < \lambda_1 + \lambda_1 \cdot \frac{\varepsilon}{\omega}$  holds.<sup>21</sup>*

**Proof:**  $c_1^i = p_i^U - \lambda_1$ ;  $c_1^j = c_2$   $\Rightarrow \Delta D_1^1 = \frac{\lambda_1(\omega + \varepsilon)}{6\delta d}$ ;  $c_1^j = p_j^U + \lambda_2$ ;  $c_1^j = c_2$   
 $\Rightarrow \Delta D_1^2 = -\frac{\lambda_2 \cdot \omega}{6\delta d}$   
 $\Delta D = \Delta D_1^1 + \Delta D_1^2 = \frac{1}{6\delta d}(\lambda_1\omega + \lambda_1\varepsilon - \lambda_2\omega) \Rightarrow >0$  with  $\lambda_2 < \lambda_1(1 + \frac{\varepsilon}{\omega})$ ;  $\leq 0$  with  $\lambda_2 \geq \lambda_1(1 + \frac{\varepsilon}{\omega})$

■

Note that it's possible to increase aggregated profits through cost reallocation if constellation of parameters  $\lambda_2 < \lambda_1(1 + \frac{\varepsilon}{\omega})$  does not harm the zero-profit condition for the upstream firm.

**Proposition 4.1 (2):** *If the zero-profit condition on the upstream level holds it is impossible to increase demand and profits from network perspective through cost reallocation.*

**Proof:** 
$$0 = (c_1^i - \lambda_1 - p_i^U) \overbrace{\left(\frac{\lambda_1}{6\delta d} + \frac{1}{2}\right)}^{D_1^1} (\omega + \varepsilon) + (c_1^j + \lambda_2 - p_j^U) \overbrace{\left(\frac{1}{2} - \frac{\lambda_2}{6\delta d}\right)}^{D_1^2} \omega \Leftrightarrow$$
  

$$\lambda_2 = \left(\lambda_1 + \lambda_1 \frac{\varepsilon}{\omega}\right) \cdot \left(\delta d + \frac{1}{3}\lambda_1\right) \Big/ \left(\delta d - \frac{1}{3}\lambda_2\right) \Rightarrow \lambda_2 > \lambda_1 \left(1 + \frac{\varepsilon}{\omega}\right)^{22}$$

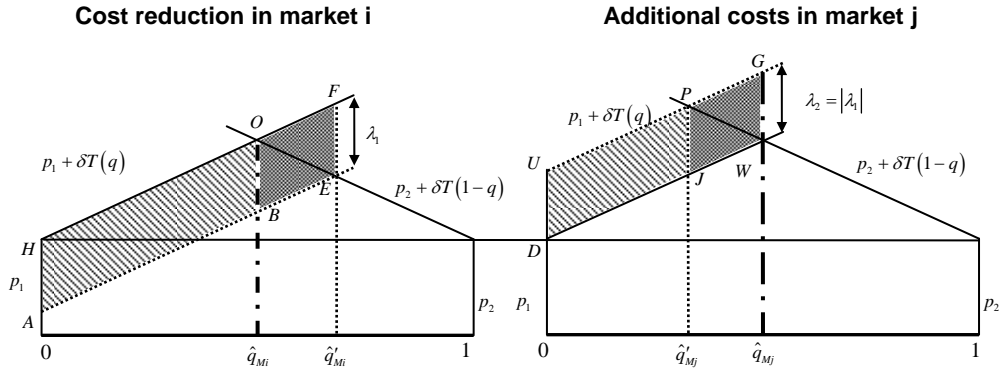
■

Note that in case of linear or quadratic transportation costs asymmetric cost allocations don't maximize profits from network perspective in a vertical star network. If we make a downstream firm better off through cost reduction of  $\lambda_1$  this cost advantage also holds for all intramarginal units. Due to the (positive) demand shock in downstream market  $i$

<sup>21</sup> Without demand shocks  $\lambda_2 < \lambda_1$  holds.

<sup>22</sup> Without demand shocks  $\lambda_2 > \lambda_1$  holds.

$\lambda_2$  hold for comparatively fewer units. We therefore have to establish additional costs in shape of  $\lambda_2$  such that  $\lambda_2 > |\lambda_1|$  holds to satisfy the zero-profit condition on the upstream level. The gain of demand in market  $i$  is overcompensated through the loss of demand in market  $j$ . Therefore in presence of asymmetric prices for the input good, demand and profit decreases from network perspective. The figure below shows that this result arises also for  $\varepsilon = 0$ .



Decreasing procurement costs in market  $i$  cause losses for the upstream firm for all in-tramarginal units of the input good demanded by downstream firm  $i$  before cost reduction. This is illustrated by the area ABOH. In consequence of the lower procurement costs downstream firm  $i$  chooses a lower optimal price and serves therefore additional demand  $\hat{q}'_{Mi} - \hat{q}_{Mi}$ .<sup>23</sup> Establishment of additional costs for the input good in market  $j$  such that  $\lambda_2 = |\lambda_1|$  holds implies a decline of served market demand  $\hat{q}_{Mj} - \hat{q}'_{Mj}$  by downstream firm  $j$ . Although changes in demand compensate each other the upstream firm incurs losses illustrated by the area BEFO. This harms the zero-profit condition for the upstream firm. Additional revenues in market  $j$  expressed by area DJPU are lower than missing revenues in market  $i$  represented by area AEFH. Therefore the condition of proposition 4.1 (1) can not be satisfied. In the next section we explore implications of demand shocks on negotiations about prices of inputs between the downstream firms in presence of the upstream zero-profit condition. We also discuss the relation of our results to the propositions stated above.

<sup>23</sup> See for a more general proof remark 3.1.



## 4.2 Nash Bargaining

In presence of (idiosyncratic) demand shocks the contribution of the  $N$  downstream firms to the economies of scale realized during production of the input good on the upstream level and gained through bundling demand for the input good differs. Assume that the downstream firms negotiate about the division of realized cost savings and the procurement prices for inputs respectively. Possible outcomes of this negotiations and the correspondence to the efficiency results derived in chapter 4.1 arise as important questions. Therefore we analyze a  $n$ -person bargaining game with the player set  $N = \{1, \dots, n\}$  consisting of the downstream firms. Among the various options to model<sup>24</sup> this situation we only consider a simple bargaining model where  $n$  players play for the NASH cooperative bargaining solution in the multilateral case.<sup>25</sup> First of all we have to determine cooperation surplus and individual outside options of the  $N$  firms on the downstream level.

The rent to be divided  $R: [0, \infty) \rightarrow \mathbb{R}_0^+$ ;  $Q \mapsto R(Q)$  can be calculated as the difference between the market price for the input good and the average costs occurring on the upstream level in case of producing the input good bundled. Furthermore this difference is multiplied by the aggregated market demand from network perspective to determine absolute cooperation surplus:

$$R(Q) := \left( p_M - \frac{F}{Q} \right) \cdot Q \quad \text{with} \quad Q = \sum_{i=1}^n q_i^u; \quad q_i^u = q_i; \quad \frac{\partial R}{\partial q_i} > 0; \quad \frac{\partial^2 R}{\partial q_i^2} = 0$$

Note that in our setup a downstream firm is able to procure input goods by own production. Therefore in case of no cooperation the outside option  $A_i: [0, \infty) \rightarrow \mathbb{R}_0^+$ ;  $q_i \mapsto A_i(q_i)$  of the  $i$ -th downstream firm can be calculated as difference between market price for the input good and the average costs of own production:

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<sup>24</sup> See for other revenue rules, in particular for the shapely value WIESE (2005) and HOLLER/ ILLING (1993).

<sup>25</sup> See for example HOLLER (1992).

$$A_i(q_i) := \left( p_M - \frac{F}{q_i} \right) \cdot q_i \quad \text{with } i \in N; \quad \frac{\partial A}{\partial q_i} > 0; \quad \frac{\partial^2 A}{\partial q_i^2} = 0$$

In this paper  $\theta_i, i \in N$  denote the share of the downstream firm  $i$  of  $R$ . The common approach of maximizing the NASH-product under constraints leads to the shares  $\theta_i$  on  $R$  negotiated between the  $N$  downstream firms:

$$\max_{\theta_1, \dots, \theta_n} \prod_{i=1}^n (\theta_i - A_i) \quad \text{s.t.} \quad \sum_{i=1}^n \theta_i \leq R \quad \Rightarrow \quad \theta_i = \frac{1}{n} \cdot R + \frac{\sum_{j \neq i \in N} (A_i - A_j)}{n} \quad \text{with} \quad \sum_{i=1}^n \theta_i = R$$

It is easily checked that in absence and in case of identical outside options the cooperation surplus is divided completely symmetric. Demand shocks as introduced in section 3.2 in turn lead to different market demands  $q_i$  and therefore heterogeneous alternatives or outside options respectively. Note that in this situation downstream firm  $i$  get the sum of the  $n$ -th share of the cooperation surplus und the  $n$ -th share of the aggregated differences to the outside options of the other downstream firms. Therefore the Nash-solution makes parties better off with comparatively valuable alternatives.<sup>26</sup> In our model downstream firms realizing the highest positive (idiosyncratic) demand shocks dispose of relatively valuable outside options.<sup>27</sup>

Negotiated shares of  $R$  now can calculated as individual input prices per unit  $p_i^u$  that have to be paid by the  $N$  downstream firms respectively. Calculation has to take into consideration the zero-profit condition for the upstream firm.

$$\theta_i = \frac{1}{n} \cdot R + \frac{\sum_{j \neq i \in N} (A_i - A_j)}{n} = (p_M - p_i^u) q_i \quad \Leftrightarrow \quad p_i^u = p_M - \frac{R}{n \cdot q_i} - \frac{1}{n} \cdot \frac{\sum_{j \neq i \in N} (A_i - A_j)}{q_i}$$

<sup>26</sup> Therefore parties with comparatively less valuable alternatives receive less than the  $n$ -th share of cooperation surplus.

<sup>27</sup> For reasons of clearness this derivation does not consider that parties calculate cooperation surplus and outside options in anticipation of the result of the bargaining game. Therefore no perfect foresight on the implications of bargaining results in particular on costs, prices und resulting demand is assumed. This is in contrast with the strong assumptions of rationality and information relating to the Nash-solution.

**Proposition 4.2 (1) (Comparative static's):** Assume that downstream firm  $i$  disposes of a complementarily valuable outside option in consequence of idiosyncratic demand shocks. The  $n$ -person Nash bargaining game ensures downstream firm  $i$  a complementarily larger share of the cooperation surplus in shape of a lower prices for the input good.

**Proof:**

$$\frac{\partial \theta_i}{\partial q_i} = p_M > 0 \Rightarrow \theta_i = p_M q_i - p_i^u q_i \Leftrightarrow \frac{\partial \theta_i}{\partial q_i} = \frac{\partial (p_M q_i - p_i^u q_i)}{\partial q_i} \Leftrightarrow \frac{\partial p_i^u}{\partial q_i} = -\frac{p_i^u}{q_i} < 0$$

■

Note that this asymmetric distribution of procurement costs among the downstream firms is in contrast to the efficiency results of chapter 4.1. In particular the outcome of the  $n$ -person Nash bargaining game does not maximize aggregate profits from network perspective.<sup>28</sup>

**Proposition 4.2 (2) (stability):** The NASH cooperative bargaining solution in the multi-lateral case implies stability of the network. The network is stable independently from concrete realisation of the idiosyncratic demand shocks. Firms can never be better off with own production for example in case of positive demand shocks.

**Proof (per contradiction):** Assume that downstream firm  $i$  is better off due own production of the input good which implies the existence of a  $q_i$  such that  $p_i^u > \frac{F}{q_i}$  holds.<sup>29</sup>

Then

$$p_M q_i - F > \frac{1}{n} R + \frac{1}{n} \sum_{j \neq i \in N} (A_i - A_j) \Leftrightarrow$$

$$p_M q_i n - F n > (n-1) [q_j p_M + q_i p_M + F - q_j p_M - F] + q_i p_M - F$$

holds. Finally this leads to  $0 > (n-1)F$  what contradicts with  $F \in \mathbb{R}^+$ .

■

<sup>28</sup> In particular it would be possible to over compensate firms which are worse of in comparison to the Nash bargaining solution through side payments.

<sup>29</sup> We assume symmetry for the other firms.

The result is very intuitive because if Nash solution is applied the parties with comparatively valuable outside options can not attain the aggregated differences to the outside options of the other downstream firms. Therefore all parties profits from positive realizations of idiosyncratic demand shocks. In case of negative realizations of idiosyncratic demand shock all downstream firms' profits from bundling the demand for the input good. From proposition 4.1 (1) together with proposition 4.2 (2) follows directly theorem 4.2:

**Theorem 4.2:** *In case of idiosyncratic demand shocks negotiations on the prices for the input good leads to a stable but inefficient (complete) network.*<sup>30</sup>

Seen together, the results obtained for a vertical star network as defined above yield a number of observations.

First we note that firms generally have an incentive to collaborate in shape of bundling the demand for an input good, so the empty network is never incentive compatible.

Second, (idiosyncratic) demand shocks lead to an asymmetric distribution of bargaining power in the negotiations for input prices between the downstream firms.

Third, this difference in firms outside options does not threat the stability of the network but has negative consequences for efficiency from network perspective. Individual considerations lead firms to a distribution of input prices that does not maximize aggregated profits.

This problem becomes more relevant if the strong assumptions  $p_M \leq \frac{F}{q_i}$  and Nash-cooperative bargaining solution are softened. Then it could feasibly happen that in presence of strong demand shocks the bargaining solution leads to instability of the network.<sup>31</sup> In the next section we briefly discuss some possibilities to soften or solve this kind of problems.

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<sup>30</sup> Recall the assumption that maximum willingness to pay  $\bar{v} \in \mathbb{R}^+$  is high enough that every individual buys in equilibrium in the  $N$  downstream markets. Therefore we don't analyze questions relating to overall social welfare.

<sup>31</sup> See ROTEMBERG/ SALONER (1986) for the possibility of deviation from cooperation and collusion in consequence of large positive demand shocks modelled in a Bertrand setup.

### 4.3 Additional comments

In our approach efficiency problems of the Nash cooperative bargaining solution arise because downstream firms realizing comparatively large positive demand shocks doesn't internalise the external effect of their strong bargaining power due valuable outside options. In particular they don't consider demand effects of higher input prices for downstream firms confronted by a lower density of consumers. In consequence inefficiencies arise from network perspective. Therefore solution concepts have to take into consideration possibilities of internalising these extern effects. On the one hand we could establish a system of side payments inspired by tax and transfer systems. In general it has to be guaranteed that values of the alternatives equate each other after realization of the demand shocks.

Starting from the ex ante expected market demand  $\omega$  additional demand could be taxed per unit. This implies increasing procurement costs of the input good for all units exceeding expected market demand  $\omega$ . These tax revenues can be used to subsidize the weaker downstream parties until the outside options equate each other. Note that the expected realization of the demand shock is zero for all firms from ex ante perspective. Therefore all downstream firms would agree with these non linear prices ex ante. In this case the question arises how strong are incentives to deviate from ex ante agreement and to renegotiate the input prices ex post. This question becomes much more interesting if we generalize our setup. In our paper the ex ante agreement is renegotiation proof because in case of cooperation all downstream firms reach at least the same costs that would occur in case of own production which in turn are never higher than network extern market prices for the input good. If the downstream firms are not able to write complete conditional contracts from ex ante perspective perhaps a proper structure of control- and governance mechanisms can be implemented to soften the described problem of externalities. However these possibilities have to be analyzed carefully. For example the vertical control problem inherent in delegation is essentially that of double marginalization of rents. Furthermore establishment of a governance structure regularly implies that the zero-profit condition and the implicit assumed productive efficiency on

upstream level vanishes. Therefore additional questions related to problems of delegation- and incentive constraints arise.<sup>32</sup>

## 5. Concluding Remarks

Our goal in this paper was to explore the existence of a trade-off between profitability and stability in vertical networks in presence of demand uncertainty. For the chosen setup we showed bargaining on cooperation surplus leads to inefficient allocations from network perspective. In case of weaker assumptions with respect to the outside option or other revenue rules stability of cooperation and network is expected to be endangered. Some solution concepts are shortly introduced and discussed in the section above. Our results may suggest at least two avenues for future research. First, to analyze proper governance structures of vertical networks the relation of this paper to the literature of incomplete contracts in particular to the literature of the theory of the firm has to be investigated carefully. Secondly, implications of heterogeneity among firms in vertical networks could be studied from a more practical point.

Varying demand among the downstream firms could be the result of different business strategies. In this case heterogeneity would be endogenous and incentive mechanisms could be studied from network perspective.

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<sup>32</sup> See for an overview of problems in hierarchical structures of production MOOKHERJEE (2006).

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