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**Localized Competition, Multimarket
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ABSTRACT

Localized Competition, Multimarket Operation and Collusive Behavior

by Frank Verboven^{*}

This paper studies collusive behavior in a repeated oligopoly model with localized competition. Private information about the rivals' past actions naturally arises from this product market structure. The resulting communication problems imply that firms should adopt strategies with sufficiently lenient punishments. Infinite grim punishments are too severe for large discount factors. The standard stick-and-carrot punishments from the perfect public information model are too severe for all discount factors. Modified stick-and-carrot punishments can be used, though for a smaller range of discount factors than the standard stick-and-carrot punishments under public information.

ZUSAMMENFASSUNG

Lokaler Wettbewerb, Multimarkt-Strategien und kollusives Verhalten

In diesem Beitrag wird das kollusive Verhalten in einem Oligopolmodell mit lokalem Wettbewerb untersucht. Private Information über die Aktionen des Konkurrenten resultieren normalerweise aus der Produkt-Marktstruktur. Das daraus folgende Kommunikationsproblem bedeutet, daß Unternehmen jene Strategien anwenden sollten, die moderat im Hinblick auf Bestrafung sind. Uneingeschränkt harte Strafen sind beim Vorliegen hoher Abzinsungsfaktoren zu schwerwiegend. Die übliche "Zuckerbrot und Peitsche"-Bestrafung des Modells mit vollständig öffentlicher Information sind für alle Abzinsungsfaktoren zu hart. Eine modifizierte "Zuckerbrot und Peitsche"-Bestrafung kann verwendet werden, jedoch nur für einen engeren Bereich von Abzinsungsfaktoren als dies für die Standard "Zuckerbrot und Peitsche"-Bestrafung gilt.

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1. Introduction

The essential problem of enforcing collusive agreements in an oligopoly consists of both detecting and punishing past cheating. This has become increasingly understood since Stigler's (1964) original contribution and the subsequent repeated game literature. Most contributions on collusion assume that cheating will be detected and punished with a one-period lag by *all* firms in the industry. Although this approach may be justified in some simple product markets, it is rather unrealistic in more complex economic environments. The present paper considers a product market with localized competition, and explores the enforcement problem in detail. Under localized competition different firms typically compete with different sets of rivals. Problems of *private information* about the firms' past actions endogenously arise. These information problems create serious difficulties in both detecting and punishing past cheating.

In the absence of an information exchange mechanism firms cannot easily observe their rivals' past actions. In simple product markets this is no serious problem since firms can still make *inferences* about these actions, based on their past realized profits. This is the case in a simple homogeneous goods oligopoly with certain demand. In such a market all firms compete directly with each other. As a result, an unexpectedly low price or high output by a cheating firm would be inferred by all other firms in the industry, because they would all suffer from a profit reduction. Most product markets, however, are more complex than that. They contain aspects of localized competition, with different firms competing directly with different sets of rivals. In such markets, cheating by one of the firms only affects a subset of the other firms in the industry, so that firms inevitably make different inferences about their rivals' past behavior. This creates the following *communication problem*. A firm who has privately observed cheating by one of its close competitors may consider to conceal this information by not punishing the cheater: such a strategy would avoid or delay a further punishment by the ignorant firms and make it possible to at least continue to collude with these firms. This communication problem applies when the firms who privately observed cheating are not able to impose selective punishments on the cheater through discriminatory actions. As will be argued below, the

presence of localized competition and the absence of discrimination between submarkets of competitors are reasonable and common phenomena.

In a repeated oligopoly model that captures the essential properties of localized competition, I analyze the communication problem in detail. I stress the importance of choosing sufficiently *lenient* punishments, in order to induce communication of privately observed cheating. The infinite grim-trigger punishments, as in Friedman (1971), are too severe for large discount factors. *Less severe* punishments are constructed to sustain collusion for this range of discount factors. In addition, the "standard" stick-and-carrot punishments, which are the optimal (symmetric) punishments under perfect public information¹, are too severe for *all* discount factors. This implies that the communication problem hinders collusion in a nontrivial sense. "Modified", less severe stick-and-carrot punishments can still be used, though for a much smaller range of discount factors than the standard stick-and-carrot punishments under public information.

Aspects of localized competition are present in many, if not most, product markets.² Producers of materials such as cement, steel or hardwood lumber tend to be scattered geographically throughout the country and are subject to significant transportation costs. Retailers of consumer products (e.g. gasoline stations) are located at different places in the city. Furthermore, many products are differentiated in non-geographical dimensions. To the extent that consumers are heterogeneous in that they value the characteristics of the products differently, competition can again be localized. Feenstra and Levinson (1995), for example, provide strong econometric evidence of localized competition in the car market. Schmalensee (1985) establishes the presence of localized competition in the breakfast cereal industry. At a more theoretical level, it has been shown that localized competition is present in a product market if the number of products is sufficiently large compared to the number of dimensions in which firms can differentiate themselves.³ As

¹ See Abreu (1986).

² The best known economic model of localized competition is Hotelling's (1929) famous linear city model. The general discussion on whether competition can be most adequately described as localized or nonlocalized dates back to Chamberlin (1933) and Kaldor (1935). See Anderson, de Palma and Thisse (1992) for a discussion.

³ See Anderson, de Palma and Thisse (1989). For example, in Hotelling's (1929) one-dimensional linear city there should be at least three firms for competition to be localized.

explained above, the communication problem applies to these examples provided that firms are not able to impose selective punishments through discriminatory actions that affect only the cheater. If localized competition is based on non-geographic differentiation, discriminatory actions that only affect the cheater are not likely to be feasible. In the automobile and breakfast cereal examples firms would need to obtain information about the consumers' brand preferences, i.e. their "locations" or ideal products and their "transportation costs" or costs of buying brands that differ from their ideal products. A straightforward mechanism to obtain this information does not seem to be readily available. Consequently, in these industries it seems impossible to selectively punish a cheater by offering discriminatory discounts to those consumers who would otherwise have purchased from the cheating brand. In contrast, if localized competition is based on geographic differentiation, then price discrimination to hurt a cheating firm may be easier, through the use of delivered pricing systems. In such systems firms accept to cover (part of) the -- verifiable -- consumers' transportation costs, so that buyers at different geographic locations pay different prices. Mund and Wolf (1971, p. 238-239) provide interesting evidence on delivered pricing practices in the cement industry, where firms charge low prices by covering transportation costs in areas where past cheaters are located, while maintaining high prices elsewhere. Although such discriminatory delivered pricing practices may alleviate the communication problem, they are frequently challenged and declared illegal by antitrust authorities, as discussed in detail in Scherer and Ross (1990).

Notice, finally, that localized competition may also be present in industries with *multimarket operation absent multimarket contact*, provided the multimarket firm is not able to follow a discriminatory policy between the markets in which it is active, as is the case under of small consumer arbitrage costs.⁴ Multimarket operation absent multimarket contact would then hinder collusion due to the communication problem analyzed in this paper, in sharp contrast with the established theoretical results on *multimarket contact* by Bernheim and Whinston (1990).

⁴ See Verboven (1995) for a detailed discussion on the relationship between localized competition and multimarket operation.

The results in the paper may help to better understand the potentially collusive role of information exchange mechanisms, such as trade associations. Trade associations collect detailed information on all firms' past prices and market shares, frequently under the supervision of a third party. Defendants argue that a more widespread dissemination of accurate information increases market efficiency, e.g. by reducing uncertainty about cost or demand conditions.⁵ Opponents, in contrast, argue that information exchange is anticompetitive, because it facilitates detection of cheating by reducing the detection lag.⁶ The usually unaddressed question is *how* information exchange could reduce the detection lag. If competition is nonlocalized, then all firms simultaneously make the *same* inferences about past cheating based on their own realized profits. The exchange of information on past prices and market shares would then come "too late" and be largely redundant in reducing the detection lag.⁷ In contrast, if competition is localized, then cheating is private information to some firms, and information exchange serves to make this information public.

Two antitrust cases may illustrate the use of trade associations as means of reducing the detection lag in the presence of localized competition. In the U.S. Hardwood case in 1921 the Court declared illegal a trade association of hardwood manufacturers. The association had adopted an "open-competition plan", collecting and providing detailed information on sales, production, inventories and prices. There were about 300 members, all of them being scattered widely throughout the U.S., located near the sources of supply and subject to high transportation costs. The industry defendants used this evidence on location as a proof that information was difficult to obtain, thereby causing a substantial risk of overproduction.⁸ However, this same evidence may be used to argue

⁵ There is a large theoretical literature on the incentives and welfare effects of information exchange in the presence of uncertainty about the rivals' cost or demand. This literature abstracts from collusive considerations. See, for example, Vives (1990).

⁶ See Salop (1985) for an overview of practices that -- in his view -- reduce the detection lag. Scherer and Ross (1990) take the view that information exchange is anticompetitive because it facilitates *ex ante* coordination to a collusive agreement.

⁷ The information would only be useful to identify the cheater, allowing the firms to impose asymmetric punishments.

⁸ See the view of Justice Brandeis as discussed in Posner (1976).

that competition was highly localized in the market for hardwood lumber, making cheating difficult to detect and punish in the absence of information exchange.

In the more recent 1992 U.K. agricultural tractor case the European Commission declared illegal an information exchange agreement between eight manufacturers and importers of agricultural tractors in the U.K. In its report the Commission objected against the high detail of exchanged information on retail sales, with breakdown by geographic territory (including counties, dealer territories and postcode sectors), by product (horsepower) group and by model. As argued by the Commission, "this therefore permits *each* member to follow the sales performance and market performance of *each* participating competitor [...] in respect of all the products, specific products and within the smallest geographic area."⁹

These cases illustrate the presence of information exchange in product markets with localized competition. In future research, it would be interesting to take a more systematic (econometric) approach in order to examine the role of localized competition in price-fixing cases involving information exchange.¹⁰

Much of the economic literature on collusive behavior has assumed *perfect public* information about the firms' past actions. The most famous exceptions are by Stigler (1964) and by Green and Porter (1984). Both contributions consider an industry with uncertain demand, in which firms face the problem of distinguishing between a negative demand shock and cheating, after observing a low realized sales level. Green and Porter analyze this inference problem in a repeated game framework. Crucially, in both contributions competition is nonlocalized, with firms competing symmetrically, so that an unexpectedly low profit is observed by *all* firms. As a result, the models by Stigler and

⁹ See OJ L68, 13.3.92, p 19, article (19).

¹⁰ Two interesting papers have collected detailed statistical evidence on price-fixing cases with trade associations and other characteristics. Hay and Kelley (1974) find that 29 percent of the U.S. price-fixing cases involve trade associations; Fraas and Greer (1977) find that 36 percent of the cases involve trade associations. Both studies also analyze the role of product differentiation. The interaction between the role of trade associations and product differentiation (and localized competition) is ignored. An empirically testable hypothesis is that trade associations are more likely to occur in complex, differentiated product markets.

Green and Porter, and the subsequent generalizations¹¹ are models of *imperfect public* information. In contrast, the present model with localized competition generates *private* information about the firms' past actions. To the best of my knowledge, this is the first economic oligopoly model in which such problems of private information arise naturally from the product market structure.

A few game-theoretic papers have addressed some aspects of private information about the players' actions in a repeated game. These papers only consider very patient players, i.e. with discount factors close to one. Fudenberg and Levine (1991) further assume "epsilon rational" players, playing in "informationally connected" games.¹² Ben-Porath and Kahneman (1993) and Kandori and Matsushima (1994) consider rational players. However, they assume that players can make explicit public announcements at certain periods in time about their obtained private information. In contrast with these previous contributions, the present paper considers rational players who are not able to make explicit public announcements about their own private information.

Section 2 presents the model of localized competition. Section 3 provides preliminary observations and section 4 obtains the main results. Section 5 discusses the generality of the results and the directions for future research.

2. The model of localized competition

2.1 Demand and profits

Consider a representative consumer with the following quadratic utility function:

$$U(q_0, q_1, q_2, q_3) = \alpha(q_1 + 2q_2 + q_3) - \frac{\beta}{2}(q_1^2 + 2q_2^2 + q_3^2) - \gamma(q_1q_2 + q_2q_3) + q_0 \quad (2.1)$$

¹¹ See especially Fudenberg, Levine and Maskin (1994).

¹² Epsilon-rational players do not mind behaving suboptimally for some time if they are very patient. In informationally connected games a message can always be passed from one player to another, regardless of which single player might try to interfere. With localized competition this property may not hold.

where q_i , $i = 0,1,2,3$, is the quantity consumed of good i . Assume $\alpha > 0$, $\beta \geq \gamma > 0$. Good 0 is the outside good: the utility from its consumption is independent of the consumption of all other goods. The utilities from the consumption of good 1 and good 3 are independent of each other. The utilities from the consumption of good 1 and 2, and of good 2 and 3 are, however, interdependent: good 1 and 2, and good 2 and 3, are substitutes for each other.¹³ One may think of good 2 as a good with *multiple independent uses*: with respect to its first use it is a substitute for good 1; with respect to its second use it is a substitute for good 3. Solving the representative consumer's utility maximization problem subject to the budget constraint $y \geq \sum_{i=0}^3 p_i q_i$, where y is income p_i is the price of good i , with $p_0 = 1$, yields the inverse demand equations for good 1, 2 and 3:

$$p_1 = \alpha - \beta q_1 - \gamma q_2, \quad i = 1,3$$

$$p_2 = 2[\alpha - \beta q_2 - \gamma(q_1 + q_3)/2].$$

There are three firms. Each firm i chooses to produce the quantity of a single good i , q_i , given the quantity chosen by the other firms. Normalizing marginal cost to zero and ignoring fixed costs, firm i 's one-shot profit function is given by

$$\pi_1(q_i, q_2) \equiv (\alpha - \beta q_i - \gamma q_2)q_i, \quad i = 1,3$$

(2.2)

$$\pi_2(q_i, q_2, q_3) \equiv 2[\alpha - \beta q_2 - \gamma(q_1 + q_3)/2]q_2.$$

These profit functions reveal that competition is localized in the following sense: firm i 's profit, $i = 1,3$, directly depends only on its own quantity choice and the quantity of firm 2, and not on the quantity choice of the third firm. In other words, firm 1 and firm 3 do not directly compete with each other; they only compete indirectly through the interaction with the common competitor firm 2.

¹³ The case in which good 1 and 3 are complements, $\gamma < 0$ is analogous. See Verboven (1995).

The model is admittedly somewhat specific, and there are several alternative ways to model localized competition. Alternative models capturing localized competition, are discussed in section 5. Appendix B analyzes a Hotelling triopoly model with price-setting firms. It is shown that the main conclusions continue to hold. At this point, it is worth emphasizing that the chosen model facilitates a close comparison with the results obtained in the literature on collusion with perfect public information.¹⁴ In particular, it is possible to make a comparison with Abreu's (1986) results in a homogeneous goods industry (using stick-and-carrot punishments) and with Deneckere's (1983) and Ross' (1992) results in a differentiated goods oligopoly (using grim-trigger punishments).

2.2 Information

In the one-shot game each firm i chooses a quantity independent of the past. In the infinite horizon game each firm i may condition its own quantity choice in every period t on the observed sequence of the firms' past quantity choices. Define *firm i 's history* h_i^t at the end of period t , $t \geq 1$, as the sequence of its own past quantity choices, $h_i^t \equiv (q_i^1, \dots, q_i^t)$. Define *firm i 's null-history* at the end of period 0 as h_i^0 . At the beginning of period t , each firm i observes its own history h_i^{t-1} and the sequence of its own realized profits $(\pi_i^1, \dots, \pi_i^{t-1})$. From this information each firm can make some inferences about the other firms' history. However, due to the localized structure of competition, these inferences are imperfect. Upon observing their own history and their own past realized profits, firm 1 and firm 3 can infer firm 2's history, with whom they directly compete, but they cannot infer each others' history. In contrast, firm 2 can infer a sequence $(q_1^1 + q_3^1, \dots, q_1^{t-1} + q_3^{t-1})$. Assume for simplicity that firm 2 can identify the quantity q_1^r from q_3^r .¹⁵ This then gives the following information for each firm i .

¹⁴ The close comparison is made possible due to some convenient symmetry properties of the model and due to the assumption of quantity-setting firms.

¹⁵ This can be justified if firm 2 can decide in each period whether or not to spend an amount ε upon observing a certain (unexpected) sequence $q_1^1 + q_3^1, \dots, q_1^{t-1} + q_3^{t-1}$ in order to identify q_1^r from

- At the beginning of period $t \geq 1$, firm i 's *information set*, $i=1,3$, is (h_i^{t-1}, h_2^{t-1}) .
- At the beginning of period $t \geq 1$, firm 2's *information set* is $(h_1^{t-1}, h_2^{t-1}, h_3^{t-1})$.

Hence, only firm 2 is perfectly informed. Firm 1 and 3 are imperfectly informed. Clearly, the localized structure of competition generates this private information structure.

If firm 1 and 3 could make a binding information exchange agreement at the start of the game (e.g. through the use of a third party), then a game of perfect public information results, yielding information sets $(h_1^{t-1}, h_2^{t-1}, h_3^{t-1})$ for each firm i at the beginning of each period t . It will be instructive in the analysis below to compare the results under private information with this standard case of public information.

2.3 Strategies

In the infinite horizon game with private information a strategy for firm i is a sequence of quantity functions $\{f_i^t\}_{t=1}^{\infty}$. Each function f_i^t assigns a quantity q_i^t to every information set at every period t . In particular, for firm i , $i=1,3$, $f_i^t(h_i^{t-1}, h_2^{t-1})$ is the quantity to be chosen in period t when its information set is (h_i^{t-1}, h_2^{t-1}) . For firm 2, $f_2^t(h_1^{t-1}, h_2^{t-1}, h_3^{t-1})$ is the quantity to be chosen in period t when its information set is $(h_1^{t-1}, h_2^{t-1}, h_3^{t-1})$. Assuming firms discount the future at a common factor δ , the (normalized) present value of each firm i 's continuation profit from following its strategy, given the other firms follow their strategy, is at any period t at every information set:

$$(1 - \delta) \sum_{r=t}^{\infty} \delta^{r-t} \pi_i [f_i^r(h_i^r, h_2^r), f_2^r(h_1^r, h_2^r, h_3^r)], \quad i = 1, 3,$$

q_3^r for some r . In the Hotelling model of localized competition, presented in Appendix B, firm 2 (in the middle) may be assumed to be able to inspect the actual customers that he served in the past in order to distinguish between the prices charged by the left neighbour and the prices charged by the right neighbour.

$$(1 - \delta) \sum_{r=t}^{\infty} \delta^{r-t} \pi_2 [f_1^r(h_1^r, h_2^r), f_2^r(h_1^r, h_2^r, h_3^r), f_3^r(h_3^r, h_2^r)].$$

The normalization factor $(1-\delta)$ serves to measure the one-shot profits and the continuation profits in the same units.

There are several ways to describe strategies. In the present context, it is notationally most convenient to describe strategies using the language of machines.¹⁶ Firm i 's machine consists of three components: (i) a set of states S_i , with an initial state $s_i^0 \in S_i$; (ii) a quantity function $f_i: S_i \rightarrow \mathfrak{R}_+$, assigning a quantity to each state s_i ; and (iii) a transition function $g_i: S_i \times \mathfrak{R}_+^2 \rightarrow S_i$ for $i=1,3$, assigning a state to each vector (s_i, q_i, q_2) , and $g_2: S_2 \times \mathfrak{R}_+^3 \rightarrow S_2$ for firm 2, assigning a state to each vector (s_2, q_1, q_2, q_3) .

In principle, machines and the strategies they execute can be very complex, containing a large set of states, and sophisticated quantity and transition functions. To simplify the analysis, I impose some restrictions. The significance of these restrictions is discussed in section 5. Let c_i be firm i 's initial state and call it the "collusive" state. Furthermore, let m_i^i , $i=1,3$ be firm i 's "communication" state, and let m_2^1 , m_2^2 and m_2^3 be firm 2's first, second and third "communication" state, respectively. Finally, let p_i^j be firm i 's j -th "punishment" state. Abbreviate firm i 's quantity choice prescribed in the collusive state by $q_i^c \equiv f_i(c_i)$. The transitions *from* the collusive and communication states are restricted as follows, as illustrated on Figure 1 and explained intuitively below.

Transitions from c_i , for all i :

- Firm 2:
- move to p_2^1 if at least $q_2 \neq q_2^c$,
 - move to m_2^i if only $q_i \neq q_i^c$, $i = 1,3$
 - move to m_2^2 if only $q_1 \neq q_1^c$ and $q_3 \neq q_3^c$
 - stay in c_2 otherwise.

¹⁶ See Osborne and Rubinstein (1994) for an overview, and Verboven (1995) for a discussion in the present context.

- Firm $i, i=1,3$:
- move to p_i^1 if at least $q_2 \neq q_2^c$,
 - move to m_i^i if only $q_i \neq q_i^c$,
 - stay in c_i otherwise.

Transitions from m_i^i , for all i, j :

- Firm 2 in $m_2^i, i = 1,3$:
- move to p_2^1 if $q_2 \neq q_2^c$,
 - move to m_2^2 if $q_2 = q_2^c$ and $q_j \neq q_j^c, j \neq i, 2$,
 - move to m_2^i if $q_2 = q_2^c$ and $q_j = q_j^c, j \neq i, 2$.
- Firms i in $m_i^i, i = 1,2,3$:
- move to p_i^1 if $q_2 \neq q_2^c$,
 - stay in m_i^i otherwise.

The initial state is the collusive state. If firm 2 deviates from the quantity prescribed in its collusive state, q_2^c , then all firms move to the first punishment state, p_i^1 . This is because firm 2's action is publicly observed. In contrast, if only firm $i, i=1,3$ deviates from its quantity prescribed in the collusive state, then firms i and 2, who observed the deviation, move to an intermediate communication state.¹⁷ If firm 2 chooses a quantity different from its collusive quantity q_2^c in this intermediate communication state, then firm i 's past deviation becomes "communicated" and all firms can move to the first punishment state. Otherwise, play remains in a communication state.

If there would be perfect public information, say due to an information exchange agreement, then the communication states would be redundant. After a deviation from the collusive quantity q_i^c by *any* of the firms, play could immediately go to the first punishment state p_i^1 .

¹⁷ Firm 2 may go to three different communication states m_2^1, m_2^3 and m_2^2 , depending on whether firm 1, firm 3 or both have deviated in the past.

3. Preliminary observations

3.1 One-shot best-response functions

Important elements in the analysis are the firms' one-shot best-response functions. Using the profit functions (2.2), firm i 's *unilateral* one-shot best-response function is:

$$q_i = q_i^U(q_2) \equiv \frac{(\alpha - \gamma q_2)}{2\beta}, \quad i = 1, 3 \quad (3.1)$$

$$q_2 = q_2^U(q_1, q_3) \equiv \frac{\alpha - \gamma(q_1 + q_3)/2}{2\beta}.$$

Due to the localized structure of competition firm i 's best-response quantity, $i=1,3$, does only depend on firm 2's quantity. Firm 2's best-response quantity, in contrast, depends on both firm 1's and firm 3's quantity.

Firm i and firm 2's simultaneous, or *bilateral*, one-shot best-response quantities to the third firm j 's quantity q_j , $j \neq i, 2$, are given by the solution to the system $q_i = q_i^U(q_2)$, and $q_2 = q_2^U(q_1, q_3)$:

$$q_i = q_i^B(q_j) \equiv \frac{2\alpha(2\beta - \gamma) + \gamma^2 q_j}{8\beta^2 - \gamma^2}, \quad i = 1, 3, \quad j \neq i, 2 \quad (3.2)$$

$$q_2 = q_2^B(q_j) \equiv \frac{\alpha(4\beta - \gamma) - 2\gamma\beta q_j}{8\beta^2 - \gamma^2}, \quad j \neq i, 2.$$

3.2 Necessary equilibrium conditions

An appropriate solution concept for the infinite horizon game with private information is the concept of perfect Bayesian equilibrium, developed in Fudenberg and Tirole (1991). This solution concept requires a consistent specification of firm 1 and 3's beliefs about

each others' history, at every information set (h_i^{t-1}, h_2^{t-1}) at every period t . A specification of beliefs is consistent if it can be derived from the strategies using Bayes' rule whenever possible. The following simple specification of beliefs is consistent with the strategies in this paper, and will henceforth be used: at every information set at every period, firm 1 (firm 3) believes that firm 3 (firm 1) has chosen the quantity prescribed by its equilibrium strategy.¹⁸ A perfect Bayesian equilibrium then results if no firm i has an incentive to deviate from its strategy in any state s_i , given the other firms' strategies and given consistent beliefs.

Let w_i be firm i 's (normalized) continuation payoff once play has moved to the first punishment state p_i^1 , i.e. firm i 's *punishment payoff*. This payoff will be determined in the next section. For given values of w_i it is now possible to obtain the firms' no-deviation constraints in the collusive states c_i and the communication states m_i^j , using the transitions from c_i and m_i^j described above. Call these constraints briefly the collusion and communication constraints.

Observation 1 shows that the collusion and the communication constraints can hold only if the firms choose their one-shot best-response quantity in the communication state.

OBSERVATION 1. Given the transitions from c_i and m_i^j described above, a perfect Bayesian equilibrium obtains only if:

$$f_i(m_i^i) = q_i^B(q_j^c), i = 1,3, j \neq i, 2$$

$$f_2(m_2^i) = q_2^B(q_j^c), i = 1,3, j \neq i, 2; f_2(m_2^2) = q_2^U[q_1^B(q_3^c), q_3^B(q_1^c)].$$

¹⁸ Consider firm 1's period t beliefs after an observed history (h_i^{t-1}, h_2^{t-1}) . Suppose first that firm 1 did not observe a period $t-1$ defection by firm 2 from its equilibrium strategy. It can then use Bayes' rule to infer that firm 3's period $t-1$ quantity choice was according to its equilibrium strategy. Suppose next that firm 1 did observe a period $t-1$ defection by firm 2 from its equilibrium strategy. It can then have any belief about firm 3's period $t-1$ quantity choice. Specify these beliefs such that firm 1 believes firm 3 did not defect in period $t-1$. A similar specification of beliefs applies to firm 3.

PROOF: See Appendix.

The intuition is simple. First, there can be no equilibrium in which firm 2 chooses q_2^c in its communication state, because then firm i , $i = 1, 3$, would always have an incentive to deviate in its collusive state (as this deviation would go "uncommunicated"). Next, given that firm 2 chooses a quantity different from q_2^c in its communication states, all firms in their communication states expect the punishment state to obtain independent of their own action. As a result, the one-shot best-response quantity remains the single candidate equilibrium quantity in the communication state. Given observation 1, it is possible to write the collusion and communication constraints as follows, after some rearrangements (see Appendix).

Collusion constraints for firm 1, 2 and 3:

$$\begin{aligned}
 c_2: \quad \pi_2(q_1^c, q_2^c, q_3^c) &\geq (1-\delta)\pi_2[q_1^c, q_2^U(q_1^c, q_3^c), q_3^c] + \delta w_2 \\
 c_i, i = 1, 3: \quad \pi_i(q_i^c, q_2^c) &\geq (1-\delta)\pi_i[q_i^U(q_2^c), q_2^c] + \delta(1-\delta)\pi_i[q_i^B(q_j^c), q_2^B(q_j^c)] + \delta^2 w_i
 \end{aligned} \tag{3.3}$$

Communication constraints for firm 2:¹⁹

$$m_2^1: \quad (1-\delta)\pi_2[q_1^B(q_3^c), q_2^B(q_3^c), q_3^c] + \delta w_2 \geq \pi_2[q_1^B(q_3^c), q_2^c, q_3^c] \tag{3.4}$$

$$m_2^3: \quad (1-\delta)\pi_2[q_1^c, q_2^B(q_1^c), q_3^B(q_1^c)] + \delta w_2 \geq \pi_2[q_1^c, q_2^c, q_3^B(q_1^c)]$$

Firm 2's collusion constraint states that the (normalized) present value from colluding forever should be at least as high as the profit from (optimally) deviating once and then going to the first punishment state. Firm i 's collusion constraint, $i = 1, 3$, is slightly

¹⁹ The communication constraints for firm 1 and 3 (states m_1^1 and m_3^3) and for firm 2 (state m_2^2) are trivially satisfied.

different: firm i realizes that, after deviating from its collusive quantity q_i^c , there is an intermediate state before the first punishment state, in which the quantities from observation 1 are produced. Firm 2's communication constraints say that firm 2's (normalized) profit from communicating a past deviation by firm i , $i = 1, 3$, to the third firm, and then going to the first punishment state, should be at least as high as the present value of the payoff from *never* communicating firm i 's defection to the third firm, and hence at least keeping collusion with firm 3.

These constraints may be contrasted with the case of perfect public information due to an information exchange agreement. Recall that, under public information, the communication states are redundant and play can immediately go to the first punishment state after a deviation from the collusive quantity by any of the firms. Consequently, there are no communication constraints. Furthermore, firm i 's collusion constraint, $i=1,3$, needs an appropriate modification as it does no longer obtain an intermediate communication payoff before it is punished. Firm 2's collusion constraint retains its form.

Generally speaking, the collusion constraints are more likely to be satisfied the lower the punishment payoffs w_i . Public information models have emphasized one bound on the severity of the punishments: the punishment should not be so severe that firms have an incentive to deviate from their strategy in any of their punishment states. The present model with private information now identifies a new potential bound on the severity of the punishment: the punishment should not be so severe that firm 2 has an incentive not to communicate a privately observed deviation. From (3.4), if the punishment is too severe, firm 2 may prefer not to communicate a deviation by one of the other firms. This allows him to at least continue to collude with the other firm, who did not observe the deviation.

3.3 Symmetric collusive outcomes, symmetric punishments

The static Cournot-Nash outcome is given by the solution to firm 1, 2, and 3's one-shot best-response functions (3.1). This solution turns out to be symmetric and is given by:²⁰

$$q_i^n = q^n \equiv \frac{\alpha}{2\beta + \gamma}, \quad i = 1, 2, 3.$$

In the infinite horizon game attention will be restricted to the set of *symmetric collusive outcomes*, defined as the set of symmetric outcomes (q, q, q) that give all firms strictly greater profits than the Cournot-Nash profits. This restriction can be defended as follows. As is easily seen from the profit function (2.2), for all symmetric outcomes (q, q, q) , the firms' profits are in the same, constant proportion, i.e. $\pi_1(q, q) = \pi_2(q, q, q)/2 = \pi_3(q, q)$. Consequently, because the one-shot Cournot-Nash outcome (q^n, q^n, q^n) is symmetric, any symmetric collusive outcome (q, q, q) increases the firms' profits proportionally, a property frequently imposed in the literature.²¹

Attention is further restricted to *symmetric punishments*, defined as punishments such that all firms choose the same quantity in a given j -th communication state, i.e. $f_1(p_1^j) = f_2(p_2^j) = f_3(p_3^j)$ for all j , implying that $w_1 = w_2/2 = w_3$. Symmetric punishments reduce all firms' profits proportionally after a publicly observed deviation. This generalizes Abreu's (1986) notion of symmetric punishments.

It is now possible to make the following observation, determining which of the collusion and communication constraints is nonbinding.

OBSERVATION 2. Given a symmetric collusive outcome (q, q, q) and a symmetric punishment, firm 1 and 3's collusion constraints coincide. Moreover, firm 2's

²⁰ The intuition for the symmetry between firm 1 and firm 3 is straightforward. The additional symmetry with firm 2 follows from an exact balance of two forces. On the one hand, firm 2 has a higher "intrinsic" demand, inducing it to set higher quantities. On the other hand, firm 2 has two competitors instead of just one, inducing it to set lower quantities.}

²¹ Friedman and Thisse (1993), for example, define the collusive solution as the Pareto-optimal solution that yields profits that are in the same proportion as in the one-shot equilibrium.

communication constraint in m_2^1 and in m_2^3 coincide. Finally, firm i 's collusion constraint, $i=1, 3$, is nonbinding if $\pi_i[q_i^B(q), q_2^B(q)] - w_i \leq 0$; otherwise, firm 2's collusion constraint is nonbinding.

PROOF: Check the collusion and communication constraints (3.3) and (3.4), noting that

$$\begin{aligned} \pi_1(q, q) &= \pi_2(q, q, q)/2 = \pi_3(q, q), \quad \pi_1[q_1^U(q), q_2^B, q] = \pi_2[q, q_2^U(q, q), q]/2 = \pi_3[q, q_3^U(q)], \\ \pi_2[q_1^B(q), q_2^B(q), q] &= \pi_2[q, q_2^B(q), q_3^B(q)], \quad \pi_2[q_1^B(q), q, q] = \pi_2[q, q, q_3^B(q)] \quad \text{and} \quad w_1 = \\ w_2/2 &= w_3. \end{aligned}$$

Using observation 2, it suffices to consider only (i) either firm 1 or firm 2's collusion constraint depending on the sign of $\pi_i[q_i^B(q), q_2^B(q)] - w_i$, (ii) firm 2's first communication constraint, and (iii) the no-deviation constraints in the punishment states, or shortly, the punishment constraints. These punishment constraints are derived in the next section for alternative punishments.

The *joint-profit maximizing outcome*, i.e. the outcome that maximizes the sum of the firms' profits, is given by:

$$q_i^c = q^c \equiv \frac{\alpha}{2(\beta + \gamma)} \quad \text{for each } i. \quad (3.6)$$

This outcome (q^c, q^c, q^c) belongs to the set of symmetric collusive outcomes. Furthermore, it is Pareto-optimal. The analysis below focuses attention on the sustainability of this particular outcome. Given that observations 1 and 2 hold for *all* symmetric collusive outcomes, it would be straightforward to use the results to also compute the symmetric *most* collusive outcome from the relevant incentive constraints, whenever the joint-profit maximizing outcome is not sustainable.

4. Alternative punishments

4.1 Grim-trigger punishments

First consider *infinite* grim-trigger punishments, as in Friedman (1971). These are defined as follows. For each firm i , there is only one punishment state, p_i^1 . In p_i^1 firm i produces its Cournot-Nash quantity q^n . Transitions from p_i^1 are very simple: once in p_i^1 , firms always stay in p_i^1 .

With infinite grim punishments, the punishment constraints are trivially satisfied for each firm i , given that its rivals produce their Cournot-Nash quantity. From observation 1, necessary and sufficient conditions for a perfect Bayesian equilibrium are then firm 2's first communication constraint in (3.4), and either firm 1 or firm 2's collusion constraints (3.3), where $w_1=w_2/2$ is substituted by $\pi_1(q^n, q^n) = \pi_2(q^n, q^n, q^n)/2$. Using the joint-profit maximizing outcome as the collusive solution, straightforward calculations show that firm 1's collusion constraint is satisfied whenever firm 2's collusion constraint is satisfied; firm 2's collusion constraint is satisfied if and only if $\delta \geq \underline{\delta}_g$; and firm 2's communication constraint is satisfied if and only if $\delta \leq \overline{\delta}_g$, where

$$\underline{\delta}_g \equiv \frac{(2+\sigma)^2}{(2+\sigma)^2 + 4(1+\sigma)}$$

$$\overline{\delta}_g \equiv \frac{(2+\sigma)^2}{(2+\sigma)^2 + \sigma(4+\sigma)(8-\sigma^2)/(4-\sigma)^2},$$

and where $\sigma \equiv \gamma/\beta$ is the substitution parameter.²² These findings yield:

²² The calculations follow from substituting the collusive and Cournot-Nash quantities in the payoffs, and from substituting the obtained payoffs in the various constraints. Mathematica is used to perform some of these calculations. The case of complements, $\gamma < 0$, follows analogous reasoning and is presented in Verboven (1995).

PROPOSITION 1. Suppose firms use infinite grim-trigger punishments. The joint-profit maximizing outcome is sustainable as a perfect Bayesian equilibrium if and only if $\delta \in [\underline{\delta}_g, \overline{\delta}_g]$.

Proposition 1 is illustrated on Figure 2, plotting the range of feasible discount factors, delineated by the critical discount factors $\underline{\delta}_g$, and $\overline{\delta}_g$, as a function of the substitution parameter σ . It is straightforward to show that $\partial \underline{\delta}_g / \partial \sigma > 0$ and $\partial \overline{\delta}_g / \partial \sigma < 0$, as drawn: the closer the goods become substitutes, the smaller the range of discount factors for which the joint-profit maximizing solution is sustainable.

These findings may be compared with the case in which there would be perfect public information, say due to an information exchange agreement between firm 1 and 3 prior to the game. In this case only firm 2's collusion constraint would be relevant,²³ so that the joint-profit maximizing outcome would be sustainable if and only if $\delta \geq \underline{\delta}_g$. This is exactly the same condition as in Deneckere (1983), in a related model without localized competition.²⁴ The analysis then stresses that when there is private information due to localized competition, there exists a binding *upper bound* on the discount factor, $\overline{\delta}_g$, in addition to the standard lower bound from the public information model, $\underline{\delta}_g$. This upper bound arises from firm 2's incentive problem to communicate privately observed deviations by firm 1 or firm 3. If firm 2 is very patient, then it prefers to continue to collude with even just one firm, rather than to communicate and then to be punished. This suggests that the infinite grim-trigger punishment may sometimes be *too severe*, rather than insufficiently severe as argued in the literature on public information [e.g.

²³ To see this, recall that under public information, there are no communication constraints and firm i 's collusion constraints, $i = 1, 3$, need modification, because there is no intermediate communication state. In fact, in the present symmetric model, firm i 's collusion constraint, $i = 1, 3$, turns out to coincide with firm 2's collusion constraint if there is public information.

²⁴ The results on product differentiation and collusion differ somewhat when firms set prices rather than quantities. See Deneckere (1983), Chang (1991) and Ross (1992).

Abreu (1986)]. The question now arises whether the upper bound may be relaxed if firms follow alternative, less severe punishments.²⁵

Consider therefore the following less severe punishments: finite grim punishments. These punishments specify T punishment states, i.e. $\{p_i^1, \dots, p_i^T\}$ for each firm i . In each state p_i^j firm i produces its Cournot-Nash quantity q^n . The transition from p_i^j , $j=1, \dots, T-1$, is always to p_i^{j+1} ; the transition from p_i^T is to c_i . In other words, after a deviation by firm 2, each firm i reverts to the Cournot quantity for only a *finite* number of periods, T , and then goes back to the collusive state.

The punishment constraints in all states p_i^j , $j=1, \dots, T$, are trivially satisfied for each firm i , given that its rivals produce their Cournot quantity. Necessary and sufficient conditions for a perfect Bayesian equilibrium are then the collusion and communication constraints (3.3) and (3.4), where $w_1=w_2/2$ is now substituted by $\pi_1(q^n, q^n) + \delta^T [\pi_1(q^c, q^c) - \pi_1(q^n, q^n)]$. Clearly, as $T \rightarrow \infty$ the case of infinite grim punishments obtains; as T decreases, the punishment payoff increases and hence the punishment becomes less severe. This generates a great deal of flexibility on the punishments. Figure 3 plots the range of discount factors that sustain the joint-profit maximizing outcome for alternative punishment lengths, $T \rightarrow \infty$, $T=50$ and $T=10$. As could be expected, reducing the punishment length, and thus softening the punishment, has two effects: the collusion constraint is tightened and the communication constraint is relaxed. For high discount factors, the collusion constraint is nonbinding and the communication constraint is binding, so that a reduction of the punishment length is in fact appropriate. Is it possible to vary the punishment length such that the joint-profit maximizing outcome becomes sustainable for *all* discount factors above $\bar{\delta}_g$? The following proposition establishes that this is indeed the case.

²⁵ An alternative way to relax the upper bound may seem to be to increase firm 2's profit from communicating. However firm 2 is already producing a one-shot best-response strategy against the other firms. Hence, the only way to increase firm 2's communication profit is by changing firm i , $i \neq 2$ quantity when it is in the communication state. By observation 1 this is not possible.

PROPOSITION 2. There are grim punishments, finite or infinite, such that the joint-profit maximizing outcome is sustainable as a perfect Bayesian equilibrium for $\delta \in [\underline{\delta}_g, 1]$.

PROOF. First, $T \rightarrow \infty$ supports collusion for $\delta \in [\underline{\delta}_g, \bar{\delta}_g]$ by proposition 1. Now consider $T=2$. Straightforward algebra using the collusion and communication constraints (3.3) and (3.4) with the appropriate substitutions shows that a grim punishment with length $T=2$ supports collusion if $\delta \in [\underline{\delta}_g^*, 1]$, where

$$\underline{\delta}_g^* \equiv \frac{1}{2} \left[\sqrt{1 + \frac{(\sigma + 2)^2}{\sigma + 1}} - 1 \right].$$

Furthermore, $\partial \underline{\delta}_g^* / \partial \sigma > 0$, $\partial \bar{\delta}_g / \partial \sigma < 0$; and at $\sigma = 1$, $\underline{\delta}_g^* = .673 < \bar{\delta}_g = .698$. Therefore, for any $1 \geq \sigma > 0$, $\underline{\delta}_g^* < \bar{\delta}_g$. This implies that the regions $[\underline{\delta}_g, \bar{\delta}_g]$ and $[\underline{\delta}_g^*, 1]$ overlap so that the range $[\underline{\delta}_g, 1]$ is covered by either infinite grim punishments or grim punishments with length $T=2$. This suffices to show the proposition.

This analysis shows that *less severe* punishments than infinite grim-trigger punishments can and should be used to sustain collusion for discount factors above $\bar{\delta}_g$. This contrasts with the literature on public information [Abreu(1986)], which criticizes infinite grim-trigger punishments as being not sufficiently severe.

4.2 Stick-and-carrot punishments

Do there exist punishments such that collusion is sustainable for some discount factors *below* $\underline{\delta}_g$? This requires relaxing the collusion constraints (3.3), which should be done by making the punishments *more severe* than the infinite grim punishments considered above. To find the "true" lower bound on the discount factor, the most severe credible

punishments should be found. In a perfect public information model Abreu (1986, Theorem 14) has shown that the most severe punishments, within the class of symmetric punishments, take the unique simple form of stick-and-carrot punishments: a very severe one-period punishment after which collusion is restored. In Abreu's public information model, the severity of the punishment is limited solely by a simple no-deviation constraint in the punishment state. In the present model with private information the severity of the punishment may be further limited due to communication constraints.²⁶

As in the previous subsection, the focus is on the joint profit maximizing outcome (q^c, q^c, q^c) . An analysis of alternative symmetric collusive outcomes (q, q, q) would follow similar arguments.

Stick-and-carrot punishments are defined as follows. For each firm i , there is one punishment state, p_i^1 . In this punishment state firm i chooses a symmetric punishment quantity $q^p \equiv f_i(p_i^1)$ yet to be determined. Transitions from the punishment state are analogous to transitions from the collusive state as described above:

Firm 2:

- stay in p_2^1 if at least $q_2 \neq q^p$
- move to m_2^i if only $q_i \neq q^p$
- move to m_2^2 if only $q_1 \neq q^p$ and $q_3 \neq q^p$
- move to c_2 otherwise.

Firm i , $i=1, 3$:

- stay in p_i^1 if at least $q_2 \neq q^p$
- move to m_i^i if only $q_i \neq q^p$
- move to c_i otherwise.

In contrast to Abreu's model with public information, firms cannot always immediately move back to the first punishment state p_i^1 in the event one of the firms has deviated

²⁶ I will assume here that firms operate at a sufficiently large marginal cost c , $\alpha > c > 0$. This rules out cases in which the punishment quantities cause prices to be negative. In this section, the parameter α should therefore be interpreted net of (a sufficiently large) marginal cost.

from the prescribed punishment quantity q^p . This is only possible if it was firm 2 who deviated. If firm 1 or 3 deviated, an intermediate communication state is required.

To determine the punishment quantity q^p in the stick-and-carrot punishment, note that firm i 's (normalized) continuation profit at the start of the punishment state, w_i , is now given by

$$w_i = w_2/2 = w_3 \equiv (1 - \delta)\pi_1(q^p, q^p) + \delta\pi_1(q^c, q^c)$$

and that the punishment constraints for firms 1, 2 and 3 are:²⁷

$$w_i \geq (1 - \delta)\pi_i[q_i^U(q^p), q^p] + (1 - \delta)\delta\pi_i[q_i^B(q^c), q^c] + \delta^2 w_i, \quad i=1, 3$$

$$w_2 \geq (1 - \delta)\pi_2[q^p, q_2^U(q^p, q^p), q^p] + \delta w_2.$$

The punishment quantity q^p in the stick-and-carrot punishment is then determined as follows: q^p minimizes the punishment payoff (4.1) subject to *all* no-deviation constraints. The following observation is useful to determine which of the punishment constraints is nonbinding.

OBSERVATION 3. Given a symmetric collusive outcome and symmetric stick-and-carrot punishments, firm 1 and 3's punishment constraints coincide. Furthermore, firm i 's, $i=1, 3$, punishment constraint is nonbinding if $\pi_i[q_i^B(q^c), q_2^B(q^c)] - w_i \leq 0$; otherwise firm 2's punishment constraint is nonbinding.

PROOF: Check (4.2), noting that $\pi_1[q_1^U(q^p), q^p] = \pi_2[q^p, q_2^U(q^p, q^p), q^p] / 2 = \pi_3[q_3^U(q^p), q^p]$, $\pi_1[q_1^B(q^c), q^c] = \pi_3[q_3^B(q^c), q^c]$ and $w_1 = w_2/2 = w_3$.

From observations 2 and 3, it suffices to consider only (i) firm 2's first communication constraint, and (ii) either firm 1's collusion and punishment constraints or firm 2's

²⁷ In contrast to grim-trigger punishments, these constraints are not trivially satisfied.

collusion and punishment constraints depending on the sign of $\pi_i[q_i^B(q^c), q_2^B(q^c)] - w_i$. Using observations 2 and 3, it is straightforward to show that q^p is the *largest* quantity that solves *either* firm 1's punishment constraint in (4.2), *or* firm 2's punishment constraint in (4.2) *or* firm 2's communication constraint (3.4) as an equality without violating any of the other no-deviation constraints. Call these solutions respectively \hat{q}^p , \bar{q}^p and \tilde{q}^p .

If there would be public information, say due to an information exchange agreement, then communication constraint (3.4) would be absent and firm 1's punishment constraint in (4.2) would look the same as firm 2's punishment constraint. As in Abreu (1986), q^p is then the largest solution to firm 2's punishment constraint in (4.2), i.e.

$$q^p = \bar{q}^p \equiv \frac{\alpha}{\beta + \gamma} \left(\frac{\beta + \gamma - \sqrt{\beta(\beta + \gamma)\delta}}{2\beta + \gamma - 2\sqrt{\beta(\beta + \gamma)\delta}} \right).$$

Call the stick-and-carrot punishment using this quantity \bar{q}^p the *standard* stick-and-carrot punishment (referring to Abreu), and the stick-and-carrot punishments using either \hat{q}^p or \tilde{q}^p the *modified* stick-and-carrot punishments. Under public information the standard stick-and-carrot punishment is the unique most severe, and hence optimal, punishment in the class of symmetric punishments²⁸. The corresponding lower bound on the discount factor for the joint-profit maximizing outcome to be sustainable is found from firm 2's collusion constraint (3.3), after substituting w_2 given in (4.1) with $q^p = \bar{q}^p$:

$$\delta \geq \underline{\delta}_s \equiv \frac{(2 + \sigma)^2}{16(1 + \sigma)}.$$

This lower bound is always below the grim punishment lower bound $\underline{\delta}_g$, as could be expected (see Figure 4).

²⁸ This was shown by Abreu for $\gamma = \beta$; it immediately generalizes to $\gamma < \beta$.

The first question is whether the standard stick-and-carrot punishment, which is optimal in the public information model, is still feasible under private information. This question is answered in the following proposition:

PROPOSITION 3. When firms use the standard stick-and-carrot punishment ($q^p = \bar{q}^p$), the joint-profit maximizing outcome is not sustainable as a perfect Bayesian equilibrium.

PROOF: See Appendix.

Intuitively, under private information the standard stick-and-carrot punishment is too severe to induce firms 1 and 3 to carry out the punishment, or to induce firm 2 to communicate deviations by firm 1 or 3. More specifically, for all $\sigma \in (0,1]$, the standard stick-and-carrot punishment violates firm 1 and 3's punishment constraint. (Deviating gives them an intermediate communication payoff.) Furthermore, for $\sigma \in (0,0.795]$, the standard stick-and-carrot punishment violates firm 2's communication constraints. Given Abreu's result that the standard stick-and-carrot punishment is the *unique* optimal (symmetric) punishment under public information, Proposition 3 implies that problems of private information hinder collusion in a nontrivial sense. This then helps to explain the presence of information exchange agreements, designed to make private information public in order to facilitate collusion.

The next question is for which range of discount factors the modified stick-and-carrot punishments, with either $q^p = \hat{q}^p$ or $q^p = \tilde{q}^p$, can sustain the joint-profit maximizing outcome. An answer to this question will give more concreteness about the firms' reduced collusive possibilities. Let the value of the discount factor for which firm 1's collusion constraint (3.3) is just satisfied with equality be $\hat{\delta}_s$ if $q^p = \hat{q}^p$, and $\tilde{\delta}_s$ if $q^p = \tilde{q}^p$. We then have:

PROPOSITION 4. Suppose firms use modified stick-and-carrot punishments. (i) For $\sigma \in (0,0.117]$, the joint-profit maximizing outcome is sustainable as a perfect Bayesian equilibrium if and only if $\delta \geq \hat{\delta}_s$, where $\underline{\delta}_s < \hat{\delta}_s < \underline{\delta}_g$.

(ii) For $\sigma \in (0.117, 1]$, the joint-profit maximizing outcome is sustainable as a perfect Bayesian equilibrium if and only if $\delta \geq \tilde{\delta}_s$, where $\underline{\delta}_s < \tilde{\delta}_s < \underline{\delta}_g$.

PROOF: See Appendix.

Intuitively, for $\sigma \in (0, 0.117]$ firm 1 and 3's punishment constraints are the binding constraints, so that firms produce \hat{q}^p in the punishment state, i.e. the largest quantity such that firm 1 and 3 are just indifferent between punishing and deviating in their punishment state. For $\sigma \in (0.117, 1]$, firm 2's communication constraint is the binding constraint, so that firms produce \tilde{q}^p , i.e. the largest quantity such that firm 2 is indifferent between communicating and not communicating in its communication states.²⁹

Propositions 3 and 4 are illustrated on Figure 4. The lower bounds $\hat{\delta}_s$ and $\tilde{\delta}_s$ are much below $\underline{\delta}_g$, the lower bound when grim punishments are used. They are also, however, much above $\underline{\delta}_s$, the lower bound when the standard stick-and-carrot punishments from the public information case would have been feasible.

5. Suggested extensions

This paper has studied the stability of collusive behavior when cheating is privately observed due to localized competition. Several extensions are desirable, both at a game-theoretic and at an economic level.

The paper has focused attention on a restricted class of strategies, in particular on symmetric punishments.³⁰ Removing these restrictions would make it possible to

²⁹ Although for $\sigma \in (0, 0.117]$, \tilde{q}^p is more severe than \hat{q}^p , this would violate firm 1's punishment constraint for discount factors close to the lower bound $\hat{\delta}_s$ in this range of σ . Furthermore, although for $\sigma \in (0.117, 1]$ \hat{q}^p is more severe than \tilde{q}^p , this would violate either firm 1's collusion constraint or firm 2's communication constraint for any discount factor in this range of σ .

³⁰ Propositions 1-4 focused on the sustainability of the joint-profit maximizing outcome. However, Observations 1-3 apply to the whole set of symmetric collusive outcomes, as defined in the text.

characterize the full set of perfect Bayesian equilibria. Unfortunately, as illustrated by Abreu's (1986) analysis of a simple homogeneous goods model, the analysis of asymmetric punishments quickly becomes intractable. Furthermore, new theoretical work would be required, in particular an extension of Abreu's (1988) results to games with privately observed actions. In a public information model with discounting Abreu (1988) shows that any subgame perfect equilibrium can be generated by a strategy profile that specifies that play switches to the worst equilibrium for player i , $i=1 \dots n$, whenever i has deviated from its prescribed action. A strategy profile then simply consists of an initial path, and n asymmetric, player-specific punishments. However, with private information, player-specific punishments are only possible if the firms with private information on the identity of the cheater can credibly signal the identity of the cheating firm to the uninformed firms. In the model adopted in this paper, cheating by, say, firm 1 is observed by firm 1 and 2, and not by firm 3. A punishment, specific for firm 1, would require that firm 2 be able to credibly signal firm 1's past deviation to firm 3. *If* it can be shown that this is not possible, then the restriction to symmetric punishments would become natural, in contrast with the public information models. A detailed analysis on the possibility and the properties of asymmetric, player-specific punishments in oligopoly with private information is an extremely interesting and promising topic for future research.

The chosen economic model made it possible to formalize the information problems generated by the product market structure in a relatively simple way. The model allowed for sharp results and a close comparison with the existing collusion literature under perfect public information. Nevertheless, the chosen model describes a somewhat specific oligopolistic industry. In the Introduction it was argued that the phenomenon of localized competition is widespread; different economic models are required to describe different industries. Do the results extend to alternative economic models where similar private information problems arise from the product market structure?

A first alternative worth investigating is a triopoly model with price-setting firms, rather than quantity-setting firms. One possibility is the well-known Hotelling model,

Consequently, the analysis could be easily extended to the symmetric *most* collusive outcomes when the joint-profit maximizing outcome is not sustainable, i.e. for low discount factors.

with three firms located on a line representing a one-dimensional product space.³¹ This model is studied in greater detail in Appendix B. It yields localized competition, with exactly the same information structure as the representative consumer model studied in the text: the firm located in the middle of the line can infer the prices set by its left and right neighbours from its realized past sales; the left and right firms can only infer the prices set by the firm in the middle and not the prices set by the third firm. One may therefore view the analysis of the Hotelling triopoly model as a test for the sensitivity of the results to the specific functional forms, given the same information structure. Observations 1 and 2 can be immediately applied. It is shown that results similar to Propositions 1 and 2 hold.

Second, it would be interesting to analyze more realistic oligopoly models with an information structure such that firms can only *delay* a punishment by not communicating past cheating, but not fully *prevent* it as in the present model. One possibility is Salop's (1979) model, with four or more firms located on a circle, rather than on a line. In this model, a firm realizes that he cannot unilaterally "block" its private information about past cheating: in choosing whether or not to communicate a past deviation, he realizes that independent of his action the whole industry will eventually become informed about the deviation anyway, through the equilibrium communications taking place at the other side of the deviator. A similar possibility of delay, rather than full prevention of punishment, may occur in more realistic product markets in which products are differentiated in *multiple* dimensions.

Third, it is worthwhile to consider industries with more than three firms. Most existing economic models predict that collusion is harder to sustain as the number of firms increases. However, none of these models capture the conventional antitrust wisdom that detection becomes more difficult as the number of firms increases.³² Models with localized competition may better capture this conventional antitrust intuition.

³¹ Note that the representative consumer model adopted in this paper would no longer yield localized competition if firms set prices rather than quantities. This can easily be seen from inverting the demand system (??).

³² The intuition is instead based on a change in the relative payoffs from collusion, deviation and punishment as n changes.

Consider first Salop's circular model with one-dimensional product differentiation, as described above. With two or three firms, all firms can detect cheating with a one period lag. With four firms an extra period is required before cheating is "communicated" to all firms in the industry. The detection lag further increases as the number of firms increases. More generally, consider product markets with multidimensional product differentiation. As the number of differentiating dimensions increases, a larger number of firms is required for competition to be localized, see Anderson, de Palma and Thisse (1989).³³ In complex product markets with multidimensional product differentiation it may therefore be expected that information problems due to localized competition are absent when there is a small number of firms, and emerge when the number of firms becomes sufficiently large.

Finally, it would be interesting to extend the model of localized competition to allow for random demand shocks, as in Stigler (1964) or Green and Porter (1984). The model in this paper assumes that the goods produced by firm 1 and 3 are fully independent of each other. This assumption may be viewed as the opposite extreme of most other repeated oligopoly models with quantity-setting firms, in which firms compete fully symmetrically.³⁴ It is possible that results similar to those obtained in the present paper hold in a model with an intermediate level of interdependence between firm 1 and 3 (different from the level of interdependence between firm 1 and 2, and firm 3 and 2), provided that some random noise is added.

³³ See also footnote 3.

³⁴ See, for example, Green and Porter (1984), Abreu (1986), Deneckere (1983) and Ross (1992).

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A. Appendix

Proof of Observation 1. First I show that $f_2(m_2^i) \neq q_2^c$. Given $f_2(m_2^i) \neq q_2^c$. I then show that firms should produce their one-shot best-response quantity in the communication states.

Suppose $f_2(m_2^i) = q_2^c$. Then firm i 's, $i=1, 3$, collusion constraint (state c_i) is given by

$$\pi_i(q_i^c, q_2^c) \geq (1 - \delta)\pi_i(q_i^U(q_2^c), q_2^c) + \delta\pi_i[f_i(m_2^i), q_2^c].$$

For this to hold it is necessary, given $\pi_i(q_i^U(q_2^c), q_2^c) > \pi_i(q_i^c, q_2^c)$ for $q_i^c \neq q_i^U(q_2^c)$, that

$$\pi_i(q_i^U(q_2^c), q_2^c) > \pi_i[f_i(m_2^i), q_2^c] \quad (\text{A.1})$$

Moreover, if $f_2(m_2^i) = q_2^c$, then firm i 's, $i=1, 3$, communication constraint (state m_2^i) is:

$$(1-\delta)\pi_i[f_i(m_2^i), q_2^c] + \delta\pi_i[f_i(m_2^i), q_2^c] \geq (1 - \delta)\pi_i(q_i^U(q_2^c), q_2^c) + \delta\pi_i[f_i(m_2^i), q_2^c]$$

so that it is necessary that $\pi_i[f_i(m_2^i), q_2^c] \geq \pi_i(q_i^U(q_2^c), q_2^c)$, which contradicts (A.1). This demonstrates that $f_2(m_2^i) \neq q_2^c$. Given that $f_2(m_2^i) \neq q_2^c$, the communication constraints for firm 1 and 3, and for firm 2 in m_2^1 are as follows:

$$m_2^i, i = 1, 3: \quad (1-\delta)\pi_i[f_i(m_2^i), f_2(m_2^i)] + \delta w_i \geq (1-\delta)\pi_i[q_i^U(f_2(m_2^i)), f_2(m_2^i)] + \delta w_i$$

$$m_2^1: \quad (1-\delta)\pi_2[f_1(m_2^1), f_2(m_2^1), q_3^c] + \delta w_2 \geq$$

$$\max \{ (1-\delta)\pi_2[f_1(m_2^1), q_2^U(f_1(m_2^1), q_3^c), q_3^c] + \delta w_2,$$

$$(1-\delta)\pi_2[f_1(m_2^1), q_2^c, q_3^c] + \delta(1-\delta)\pi_2[f_1(m_2^1), f_2(m_2^1), q_3^c] + \delta^2 w_2 \}$$

For these inequalities to hold it is necessary that $\pi_i[f_i(m_i^i), f_2(m_2^i)] \geq \pi_i[q_i^U(f_2(m_2^i)), f_2(m_2^i)]$ and $\pi_2[f_1(m_1^1), f_2(m_2^1), q_3^c] \geq \pi_2[f_1(m_1^1), q_2^U(f_1(m_1^1), q_3^c), q_3^c]$. A similar argument holds for the communication states m_2^2 and m_2^3 . This shows Observation 1.

Proof of Proposition 3. By definition of the standard stick-and-carrot punishment firm 2's punishment constraint in (4.2) is just satisfied with equality. It may be verified that firm 1's punishment constraint is satisfied if and only if $\pi_1[q_1^U(q^p), q^p] \geq \pi_1[q_1^B(q^c), q_2^B(q^c)]$, which holds after some calculations if and only if

$$\delta \leq \frac{\sigma^2}{(4 - \sigma)^2(1 + \sigma)}.$$

It can be easily verified that this condition is violated for $1 \geq \sigma > 0$, given that (3.3) should hold for firm 2's collusion constraint to be satisfied. This is sufficient to prove the proposition. (Note that it can analogously be checked that firm 2's communication constraint (3.4) is violated for $.795 \geq \sigma > 0$, given that (3.3) should hold for firm 2's collusion constraint to be satisfied.)

Proof of Proposition 4. Given that the standard stick-and-carrot punishment, in which $q^p = \bar{q}^p$, is not a perfect Bayesian equilibrium by Proposition 3, only the stick-and-carrot punishments in which $q^p = \hat{q}^p$ or $q^p = \tilde{q}^p$ remain to be considered. The punishment quantity \hat{q}^p is the maximum quantity such that firm 1's punishment constraint (4.2) is just satisfied with equality; \tilde{q}^p is the maximum quantity such that firm 2's communication constraint (3.4) is just satisfied with equality. For both $q^p = \hat{q}^p$ and $q^p = \tilde{q}^p$, tedious calculations show that if firm 1's collusion constraint is satisfied, then $\pi_1[q_1^B(q^c), q_2^B(q^c)] - w_l \geq 0$. This implies, by observations 2 and 3, that firm 2's collusion and punishment constraints are nonbinding. (a) For $q^p = \hat{q}^p$, it then remains to check firm 1's collusion constraint and 2's communication constraint. These two inequality

constraints are tedious, but depend on only two parameters: δ and σ . Hence numerical simulations can be used to determine when the two constraints are simultaneously satisfied. These simulations show that for $\sigma \geq 0.117$ these constraints cannot be simultaneously satisfied, and that for $\sigma < 0.117$ these constraints are satisfied if and only if $\delta \in [\hat{\delta}_s, \hat{\delta}'_s]$, where $\underline{\delta}_s < \hat{\delta}_s < \underline{\delta}_g$, and where $\hat{\delta}'_s$ is the upper bound on the discount factor for firm 2's communication constraint to be satisfied. (b) For $q^p = \tilde{q}^p$, it remains to check firm 1's collusion and punishment constraints. Simulations show that for $\sigma \geq 0.117$ the constraints are satisfied if and only if $\delta \geq \tilde{\delta}_s$ where $\underline{\delta}_s < \tilde{\delta}_s < \underline{\delta}_g$, and for $\sigma < 0.117$ these constraints are satisfied if and only if $\delta \geq \tilde{\delta}'_s$, where $\tilde{\delta}'_s$ is the lower bound on the discount factor for firm 1's punishment constraint to be satisfied. Combining (a) and (b), part (ii) of Proposition 4 immediately follows. Further simulations show that, for $\sigma \leq .117$, $\hat{\delta}_s < \tilde{\delta}'_s < \hat{\delta}'_s$, from which part (i) of Proposition 4 follows.

B. Appendix

This Appendix considers a Hotelling triopoly model with price-setting firms. The model generates the same type of information structure and similar symmetry properties as the representative consumer model developed in the text. As a result, Observations 1 and 2 immediately apply. One may view the exercise in this Appendix as a test to evaluate the sensitivity of the results to the specific assumptions about the functional forms, *given* the assumed information structure. A discussion on the sensitivity of the results to the assumed information structure is found in the concluding section (section 5).

Consider three price-setting firms located on a line of length $2L$. Firm 1 is located at the extreme left (at 0); firm 2 is located in the middle (at L); firm 3 is located at the extreme right (at $2L$). Consumers are distributed uniformly on the line. A consumer located at a distance y from firm i , purchasing one unit of good i , obtains a utility of:

$$v - ty - p_i,$$

where t is a unit transportation cost. Note that transportation costs are linear in distance y . Each consumer chooses the good that yields the highest utility. Aggregate demand for each good i is found in the standard way by calculating the location, x^* , of the consumer indifferent between good 1 and 2; and the location, x^{**} , of the consumer indifferent between good 2 and 3. Assuming that the market is fully covered, these locations define the borders of the firms' market areas. These locations are given by:

$$x^* = \frac{p_2 - p_1}{2t} + \frac{L}{2} \text{ and } x^{**} = \frac{p_3 - p_2}{2t} + \frac{3L}{2}. \quad (\text{B.1})$$

Assuming the market is fully covered, demand for good 1 is given by $D_1(p_1, p_2) = x^*$; demand for good 2 is given by $D_2(p_1, p_2, p_3) = x^{**} - x^*$; demand for good 3 is given by $D_3(p_3, p_2) = 2L - x^*$, or, using (B.1):

$$D_i(p_i, p_2) = \frac{L}{2} + \frac{p_2 - p_i}{2t}, \quad i = 1, 3$$

$$D_2(p_1, p_2, p_3) = L - \frac{p_2 - p_1}{2t} - \frac{p_2 - p_3}{2t}.$$

Normalize marginal costs to zero, so that the firms' profits just equal their revenues. The Bertrand-Nash equilibrium prices are straightforwardly derived from solving the system of first-order conditions from revenue maximization, given the demand functions (B.2). It turns out that this solution is a symmetric solution (p^n, p^n, p^n) , where

$$p^n = tL.$$

The corresponding Bertrand equilibrium demands are $L/2$ for firm 1 and firm 3, and L for firm 2. The Bertrand equilibrium profits are given by $\pi_1^n = \pi_2^n / 2 = \pi_3^n = tL^2 / 2$. It can be verified that the market is indeed fully covered, as assumed, if products are sufficiently close substitutes, i.e. $v \geq (3/2)tL$. If this condition is violated, the problem of collusion becomes trivial since firms are local monopolies.

Candidate collusive prices are the prices such that the market is just covered. These are the prices such that the consumer indifferent between 1 and 2, and the consumer indifferent between 3 and 2, obtain zero surplus. This yields a symmetric solution (p^c, p^c, p^c) , where

$$p^c = v - \frac{tL}{2},$$

with corresponding collusive profits $\pi_1^c = \pi_2^c / 2 = \pi_3^c = (v - tL/2)L/2$. It can be verified that the candidate collusive prices are in fact the actual collusive prices, i.e. firms cannot do better by charging even higher prices so that the market would no longer be fully covered.³⁵

The information structure is similar to the representative consumer model presented in the text: firm 2 can infer firm 1 and firm 3's past prices by inspecting the customers it served; firm 1 and 3 can only infer firm 2's past prices and not those of the third firm. Furthermore, the Bertrand-Nash and collusive prices are symmetric, and firm 2 obtains

³⁵ See Chang (1991) for a similar observation for the case of quadratic transportation costs.

twice the profits of firm 1 and 3 at symmetric prices. These facts make it possible to immediately apply Observations 1 and 2. One can easily calculate the unilateral and bilateral best-response functions, as defined in the text. For simplicity, only the cases with an interior solution are considered, requiring some additional restrictions on the parameters, as given below. The firms' unilateral best-response prices to the collusive prices p^c , given above, are:

$$p_1^U(p^c) = p_2^U(p^c, p^c) = p_3^U(p^c) = \frac{v}{2} + \frac{tL}{4},$$

assuming $v \leq (7/2)tL$. The firms' bilateral best-response prices to the collusive price p^c of the third firm, are given by:

$$p_i^B(p^c) = \frac{1}{7}(v + (11/2)tL), \quad i = 1, 3 \tag{B.2}$$

$$p_2^B(p^c) = \frac{2}{7}(v + 2tL),$$

assuming $v \leq (29/10)tL$.

Consider infinite grim punishments, restricting attention to the range of parameters $v \in [(3/2)tL, (29/10)tL]$. The punishment payoffs w_i are then simply given by the Bertrand-Nash payoffs computed above, and the punishment constraints are trivially satisfied. From Observation 2, it remains to check only firm 2's communication constraint, which has the form analogous to (3.4), and either firm 1 or firm 2's collusion constraint, similar to (3.3). It can be shown that $\pi_1(p_1^B(p^c), p_2^B(p^c)) - \pi_1(p^n, p^n) < 0$ for $v < 5.116 tL$, which is satisfied, so that from Observation 2 firm 1's collusion constraint is nonbinding. Calculations show that firm 2's collusion constraint is satisfied if and only if $\delta \geq \underline{\delta}_h$; and firm 2's communication constraint is satisfied if and only if $\delta \leq \overline{\delta}_h$, where

$$\underline{\delta}_h \equiv \frac{2r-3}{2r+5}$$

$$\bar{\delta}_h \equiv \frac{2r-3}{2r+11} \frac{25}{4},$$

and where $r \equiv v/tL$ is the substitution parameter. These findings yield a proposition similar to Proposition 1 in the text. A proposition similar to Proposition 2 can also be shown, i.e. less severe, finite grim punishments can be constructed to also support collusion for the range of discount factors $[\bar{\delta}_h, 1]$. To see this, consider a grim punishment with length $T=1$, i.e. after a deviation by firm 2, firms set the Bertrand-Nash price for one single period, after which they go back to the collusive prices. It can be verified that firm 2's communication constraint is now always satisfied; firm 1's collusion constraint is nonbinding and firm 2's collusion constraint is satisfied if and only if $\delta \geq \underline{\delta}_h^*$, where

$$\underline{\delta}_h^* \equiv \frac{2r-3}{8}.$$

It can be verified that $\underline{\delta}_h^* < \bar{\delta}_h$ for $v < (39/2)tL$, so the regions $[\underline{\delta}_h^*, \bar{\delta}_h]$ and $[\underline{\delta}_h^*, 1]$ overlap, analogous to the proof in Proposition 2. An analysis of the more severe stick-and-carrot punishments, to check the sustainability of collusion for discount factors below $\underline{\delta}_h$, is more involved. The problem is that the action space is bounded: prices cannot be negative. The worst punishment price is therefore zero and the sales do not become arbitrarily high at this price in the Hotelling model. Consequently, payoffs cannot be made arbitrarily low and the punishment phase may have to last for more than one period.