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ABSTRACT

Incentives and Welfare Effect of Sharing Firm-Specific Information

by Jim Y. Jin*

This paper studies the incentives and the welfare effect of sharing firm-specific information in asymmetric Cournot and Bertrand oligopoly with mixed substitute and complement goods. Revealing firm-specific cost information is the dominant strategy in Cournot oligopoly, while concealing is so in Bertrand oligopoly. Such information sharing always hurts consumers. It increases social welfare in quantity competition and reduces social welfare in price competition. The results of sharing firm-specific cost information in Cournot oligopoly also apply to sharing firm-specific demand information in Cournot and Bertrand competition.

ZUSAMMENFASSUNG

Anreize und Wohlfahrtseffekte beim Austausch unternehmensspezifischer Information

In diesem Beitrag werden Anreize und Wohlfahrtseffekte des Austauschs von unternehmensspezifischer Information in einem asymmetrischen Cournot- und Bertrand-Oligopol mit einer Mischung von substitutiven und komplementären Gütern untersucht. Das Aufdecken unternehmensspezifischer Kosteninformation ist die dominante Strategie im Cournot-Oligopol, während im Bertrand-Oligopol diese Information vorenthalten wird. Derartiger Informationsaustausch geht immer zu Lasten der Konsumenten. Er erhöht die soziale Wohlfahrt im Mengenwettbewerb und verringert die soziale Wohlfahrt im Preiswettbewerb. Das Ergebnis des Austauschs unternehmensspezifischer Kosteninformation im Cournot-Oligopol läßt sich auch auf den unternehmensspezifischen Austausch von Nachfrageinformation im Cournot- und Bertrand-Wettbewerb anwenden.

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1. Introduction

Information sharing is a common practice in business. On one hand it improves the market efficiency by providing firms more accurate information. On the other hand, it may raise the chance of collusion among competitors. Thus the incentives and the welfare impact of information sharing have drawn considerably attention from economic research. The existing models can be divided into two groups according to the type of uncertainty they deal with: a common (demand) shock affecting all firms equally and private (cost) shocks affecting each individual firm differently. This paper examines the latter case, while extending the earlier models to asymmetric oligopoly with mixed substitute and complement products.

The first task of this paper is to find a general conclusion regarding firms' incentives to share firm-specific information. The information sharing literature started by studying firms' incentives to share information about a common demand shock. Novshek and Sonnenschein (1982) showed that such incentives do not exist in Cournot duopoly with homogeneous goods. The result was extended by Clarke (1983) to oligopoly. Further, Vives (1984) found that firms have incentives to share information in Cournot competition with complementary goods and in Bertrand competition with substitute goods. When private cost uncertainty is concerned, however, a very different picture emerges. Sakai (1985) considered various information structures about costs in Cournot duopoly and found that a firm is better off when its cost is known to its rival. In Cournot oligopoly, Li (1985) proved that revealing private uncertainty is the dominant strategy, and pointed out that the different result from the case of common demand uncertainty is due to "private values" vs. "common values", rather than cost vs. demand. The conclusion changed again as Gal-Or (1986) proved that concealing private costs is the dominant strategy for Bertrand duopolists, opposite to the Cournot case. Recently, Raith (1996) used a general Cournot oligopoly model to show that firms always have incentives to reveal private costs. He argued that the results "*are not due to cost uncertainty or 'private values' but are determined by the mere assumption of perfect signals*" (p. 276). In Bertrand oligopoly Raith found that Gal-Or's result may not hold, hence "*in general there do not even exist dominant revelation strategies*" (p. 279).

Extending Raith's symmetric model to asymmetric oligopoly with mixed complement and substitute products, we gain further insight which cannot be obtained in symmetric models. First, the example in the next section shows that mere assumption of perfect signal does not guarantee an incentive to reveal a private cost in asymmetric Cournot oligopoly. Such an incentive can be ensured if only firm-specific information is revealed, which means that the information only reveals the specific feature of the firm, but not anything about others. The same occlusion holds for sharing firm-specific demand information in both Cournot and Bertrand oligopoly. Secondly, Gal-Or's result can be generalized to asymmetric Bertrand oligopoly, i.e., no firm has an incentive to reveal firm-specific cost information. The strong results depends on neither "private values" nor "perfect signal", but are merely due to firm-specific information.

The definition of firm-specific information, i.e., not relevant to any other firms, also enables us to draw strong conclusions on welfare effects of sharing such information. The literature has so far only considered the problem assuming symmetry in demand, cost and information. Shapiro (1986) showed that exchange of cost information in Cournot oligopoly increases social welfare, but reduces consumer surplus. In Bertrand duopoly Sakai and Yamato (1990) found that both social welfare and consumer surplus fall if firms share cost information. It would be nice to generalize these results under more general conditions. Unfortunately, in symmetric Cournot oligopoly with differentiated products, Sakai and Yamato (1989) found an ambiguous impact of exchanging cost information on consumers. Also, the same ambiguity exists in Bertrand oligopoly with more than eight firms (Kühn and Vives [1995]). In these and other previous models, however, information shared among firms is not firm-specific¹. With firm-specific information, the ambiguity disappears even if in a general asymmetric oligopoly. We will show that exchanging firm-specific cost information hurts consumers in both Cournot and Bertrand oligopoly. It increases social welfare in quantity competition and reduces social welfare in price competition. The conclusions for sharing cost information in quantity competition also apply to sharing firm-specific demand information in both Cournot and Bertrand oligopoly.

¹ In their models costs are correlated in such a way that revealing firm i 's cost helps other firms to have better estimation on firm j 's cost.

This paper is organized as follows: the next section introduces the model; Section 3 examines firms' incentives to reveal firm-specific information; the welfare effect is analyzed in Section 4, followed by the closing remarks in the last section.

2. Asymmetric oligopoly with firm-specific information

We consider an oligopoly market with n differentiated products and a competitive market with a numeraire good x_0 whose price is normalized to 1. Denote firm i 's output by x_i for $i = 1, \dots, n$, and the $n \times 1$ output vector by \mathbf{x} . Firm i 's price is p_i and the price vector is \mathbf{p} . The representative consumer has a fixed wealth w and quadratic utility function $x_0 + \mathbf{a} \cdot \mathbf{x} - 0.5\mathbf{x}'\mathbf{B}\mathbf{x}$, where \mathbf{a} is an $n \times 1$ vector and \mathbf{B} is a symmetric $n \times n$ matrix. The elements of \mathbf{B} are denoted by b_{ij} . The utility function is strictly concave, so \mathbf{B} is positive definite, its inverse \mathbf{B}^{-1} exists and is also positive definite. Denote its elements by β_{ij} . Given the price vector \mathbf{p} the representative consumer chooses x_0 and \mathbf{x} to maximize her utility given her budget constraint $x_0 + \mathbf{p} \cdot \mathbf{x} \leq w$. Assume that her income is sufficiently large that an interior solution always exists. Since \mathbf{B} is positive definite, the first-order conditions $1 - \lambda = 0$ and $\mathbf{a} - \mathbf{B}\mathbf{x} - \lambda\mathbf{p} = \mathbf{0}$ give us the inverse demand function in quantity competition:

$$\mathbf{p} = \mathbf{a} - \mathbf{B}\mathbf{x} \tag{1}$$

Let $\boldsymbol{\alpha} \equiv \mathbf{B}^{-1}\mathbf{a}$. We have the demand function in price competition as

$$\mathbf{x} = \boldsymbol{\alpha} - \mathbf{B}^{-1}\mathbf{p} \tag{2}$$

When we consider firm-specific information, it can be about firms' private demand or cost uncertainty. As shown by aforementioned previous works, demand and cost uncertainties in Cournot competition are equivalent. Demand uncertainty in Bertrand competition has the same mathematical structure as in a Cournot case, with \mathbf{B} and \mathbf{a} replaced by \mathbf{B}^{-1} and $\boldsymbol{\alpha}$. These differences do not affect our results. Therefore we need only consider the cases of Cournot and Bertrand competition with private cost uncertainty. The conclusions from the former case automatically apply to demand uncertainty in Cournot and Bertrand oligopoly.

Assume that any firm i 's marginal cost c_i is a random variable with mean c_i° . The n firms' cost vector and its mean are \mathbf{c} and \mathbf{c}° . There may be some public information about \mathbf{c} . The estimation of \mathbf{c} based on public information is denoted by $\bar{\mathbf{c}}$. This implies $E[(c_i - \bar{c}_i)\bar{c}_i] = 0$ for all i . Firm i 's estimation of its own cost is denoted by \hat{c}_i and the corresponding vector is $\hat{\mathbf{c}}$. If firm i reveals its information, its private information becomes public, so \bar{c}_i and \hat{c}_i are identical². Otherwise they are different. We say that firm i 's private information is firm-specific when $E[(\hat{c}_i - \bar{c}_i)\bar{c}_j] = 0$ and $E[(\hat{c}_i - \bar{c}_i)(\hat{c}_j - \bar{c}_j)] = 0$ for all $j \neq i$. When firm i reveals its private information, it does not affect estimations, public or private, regarding any other firm's cost.

For example, we can let $c_i = c_i^\circ + \varepsilon + \varepsilon_i$. ε represent a common uncertainty, such as GDP and the oil price and ε_i is private. Public information about ε such as government policies, media reports leads to a common estimation of ε . Although this estimation is useful to predict c_i , another firm j 's private information may be totally irrelevant to ε and ε_i . Then, c_i and c_j can be correlated, while private information is firm-specific.

We write firm i 's profit functions $\pi_i(\mathbf{x})$ in quantity and $\pi_i(\mathbf{p})$ in price competition as:

$$\pi_i(\mathbf{x}) = (a_i - c_i - \sum_{j=1}^n b_{ij}x_j)x_i \quad (3)$$

$$\pi_i(\mathbf{p}) = (\alpha_i - \sum_{j=1}^n \beta_{ij}p_j)(p_i - c_i) \quad (4)$$

Firms play a two-stage game. In the first stage, they decide whether to reveal firm-specific information about their marginal costs. In the second stage, costs are realized and revealed according to the first stage decisions. Then firms choose quantities or prices to maximize their expected profits (3) or (4). Assume that for any possible cost estimations, firms always get interior solutions for their equilibrium quantities or prices.

Our model is an extension of Raith's (1996). Setting $b_{ii} = b$ and $b_{ij} = r$ for all i and $j \neq i$, we get his symmetric Cournot oligopoly model. This asymmetric model allows us to better understand why the previous conclusions hold and investigate whether they can be generalized. The model also allows correlated costs and firms to have imperfect signals about their own costs. Actually, as special cases we can obtain Raith's "private values" by

² We can also allow firms to partially reveal their firm-specific information. In that case conclusions obtained in this paper still hold.

adding a condition $E[(c_i - c_i^o)(c_j - c_j^o)] = 0$ for all $j \neq i$. We can also have his "perfect signals" by requiring $\hat{c} \equiv c$. The generality is necessary first because our research also covers the case of demand uncertainty where demand is often correlated and signals are normally imperfect. More importantly, one can see that the general results can be obtained without either "private values" or "perfect signals", but mainly due to firm-specific information.

The following is an example of our asymmetric oligopoly. It shows that, different from Raith' conclusion in symmetric Cournot oligopoly, a firm may not want to reveal its uncertainty even though it has perfect signal. Assume that there are three firms with zero cost and asymmetric demand functions:

$$p_1 = 1 - x_1 - x_2 + \varepsilon, \quad p_2 = 2 - x_1 - 2x_2 - x_3 + \varepsilon, \quad p_3 = 1 - x_2 - x_3$$

where ε has a zero mean and a variance σ^2 , and is known by firms 1 and 2. The equilibrium outputs are $x_1 = 1/3 + 3\varepsilon/7$, $x_2 = 1/3 + \varepsilon/7$ and $x_3 = 1/3$. Firm 1's expected profit is $1/9 + \sigma^2(3/7)^2$. If firm 1 reveals ε to firm 3, the strategies become $x_1 = 1/3 + 5\varepsilon/12$, $x_2 = 1/3 + \varepsilon/6$ and $x_3 = 1/3 - \varepsilon/12$. Then, firm 1's profit falls to $1/9 + \sigma^2(5/12)^2$. This example shows that perfect signals are not sufficient for firms' willingness to reveal their uncertainties. The current paper will show that it is also not necessary. Rather, it is firm-specific information that guarantees the incentive even if we allow mixed products in Cournot oligopoly.

Another unsolved problem is whether one can generalize Gal-Or's result of no incentive for cost revelation from Bertrand duopoly to oligopoly. Raith argued that this is impossible even in his symmetric case where every firm i faces a demand function $x_i = \alpha - \beta p_i - \gamma \sum_{j \neq i} p_j$. He derived the following condition for information concealing: $4(2\beta - \gamma) + (n-1)\gamma(4\beta - 3\gamma) > 0$. Actually, the first term should be multiplied by β . Then, if one uses his assumption on page 266, $-\beta/(n-1) < \gamma < \beta$, the inequality is guaranteed. Thus, Gal-Or's result probably can be extended. We will show that this is indeed the case if firms share firm-specific information.

3. Dominant revelation strategy

To simplify the mathematical expression, we introduce some notations. Let D and Λ be diagonal matrices whose every i th diagonal element is identical to that in B and B^{-1} respectively. Let \mathbf{x}^* and \mathbf{p}^* be the equilibrium output and price vectors when all costs are known and equal to zero. Now we can give our first result.

Proposition 1: Given firm-specific cost uncertainty, the unique equilibrium strategies in Cournot and Bertrand competition are given as follows

$$\mathbf{x} = \mathbf{x}^* - (D+B)^{-1}\bar{\mathbf{c}} - 0.5D^{-1}(\hat{\mathbf{c}}-\bar{\mathbf{c}}) \quad (5)$$

$$\mathbf{p} = \mathbf{p}^* + (\Lambda+B^{-1})^{-1}\Lambda\bar{\mathbf{c}} + 0.5(\hat{\mathbf{c}}-\bar{\mathbf{c}}) \quad (6)$$

Proof: see Appendix A.

Given (5) and (6), we can evaluate the change in firm i 's expected profit when it reveals its firm-specific cost information. Firm i 's revelation makes other firms' estimation of firm i 's cost, \bar{c}_i , equal to firm i 's own estimation \hat{c}_i . Since the information is firm-specific, estimations about other firms' costs remain unchanged. Given (5), firm i 's output changes by $[0.5/b_{ii}-(D+B)^{-1}]_i(\hat{c}_i-\bar{c}_i)$ in Cournot competition. Similarly, given (6), if firm i reveals its cost, its price changes by $[(\Lambda+B^{-1})^{-1}\beta_{ii}-0.5]_i(\hat{c}_i-\bar{c}_i)$ in Bertrand competition. In Cournot competition, firm i 's first-order condition requires that $b_{ii}x_i = E_i(p_i-c_i)$. Thus, its expected profit $E[x_i(p_i-c_i)] = b_{ii}E(x_i^2)$. Likewise, in Bertrand competition, firm i 's expected profit can be written as $\beta_{ii}E[(p_i-c_i)^2]$. Therefore, a Cournot firm wants to reveal its private cost if its expected squared output rises, and a Bertrand firm will do so if its expected squared mark-up rises. Then, we can find a general conclusion regarding firms' incentives to reveal firm-specific information.

Proposition 2: Sharing firm-specific cost information is the dominant strategy for Cournot firms, but concealing information is so for Bertrand firms.

Proof: see Appendix B.

The intuitive explanation is given as follows. When a firm reveals its firm-specific information, e.g. a low cost, other firms' payoffs do not change directly and they respond

to the revelation only because they anticipate a change in revealing firm's action, e.g. a higher output in Cournot competition. Consequently, the production of substitute goods decreases and that of complement goods rises. In return, these responses induce the revealing firm further raise its output than it would do without cost revelation. Similarly, other firms' responses make its output decreases more when it reveals a high cost. Thus, its expected squared output becomes larger, which means a higher expected profit in Cournot competition. In Bertrand competition, revealing a high cost signals a higher price. The substitute producers respond by raising prices and the complement produces by lowering prices. Therefore, the revealing firm's price varies more due to cost revelation. However, a Bertrand firm's profit depends on the expected squared mark-up. A higher price variation means a lower variation of mark-up and a lower expected profit. This is why we have opposite incentives for revealing firm-specific cost information in quantity and price competition. The result in Cournot competition applies to revealing firm-specific demand information in Bertrand competition because a higher price variation implies a higher variation of mark-up given a constant marginal cost.

4. Welfare effect

In this section we evaluate the impact of revealing firm-specific information on the expected consumer surplus and social welfare. In Cournot competition, using the demand function (1) we can write consumer surplus and social welfare as

$$CS(\mathbf{x}) = 0.5\mathbf{x}'\mathbf{B}\mathbf{x}$$

$$SW(\mathbf{x}) = (\mathbf{a}-\mathbf{c})\cdot\mathbf{x} - 0.5\mathbf{x}'\mathbf{B}\mathbf{x}$$

When any firm reveals its firm-specific cost information, we can show that $E(\mathbf{x}'\mathbf{B}\mathbf{x})$ decreases and $E[(\mathbf{a}-\mathbf{c})\cdot\mathbf{x}]$ increases. This is independent of whether other firms reveal their firm-specific information. Hence the welfare effect of sharing firm-specific information is unambiguous.

Proposition 3: In Cournot competition sharing firm-specific cost information reduces consumer surplus but increases social welfare.

Proof: see Appendix C.

The result applies to sharing firm-specific demand information in Cournot oligopoly. In Bertrand oligopoly, given the demand function (2) we write consumer surplus and social welfare as functions of prices,

$$CS(\mathbf{p}) = 0.5\boldsymbol{\alpha}'\mathbf{B}\boldsymbol{\alpha} - \boldsymbol{\alpha}\cdot\mathbf{p} + 0.5\mathbf{p}'\mathbf{B}^{-1}\mathbf{p}$$

$$SW(\mathbf{p}) = 0.5\boldsymbol{\alpha}'\mathbf{B}\boldsymbol{\alpha} - \boldsymbol{\alpha}\cdot\mathbf{c} + \mathbf{c}'\mathbf{B}^{-1}\mathbf{p} - 0.5\mathbf{p}'\mathbf{B}^{-1}\mathbf{p}$$

The expected values of the first term in consumer surplus and the first three terms in social welfare are not affected by information sharing. The other two terms in consumer surplus have the same mathematical form as the social welfare function in Cournot competition with \mathbf{p} replacing \mathbf{x} and an opposite sign. The last term in social welfare has the same functional form as the consumer surplus function in Cournot competition, with \mathbf{p} replacing \mathbf{x} and an opposite sign.

One can find the identically corresponding terms in the Bertrand equilibrium strategy (6) to (5), with \mathbf{p} for \mathbf{x} , \mathbf{B}^{-1} for \mathbf{B} , $\boldsymbol{\Lambda}$ for \mathbf{D} and $\boldsymbol{\Lambda}\mathbf{c}$ for \mathbf{c} . Given the similarity of equilibrium strategies and welfare functions in price and quantity competition, an opposite effect on consumer surplus in quantity competition can be applied to social welfare in price competition, and an opposite social welfare effect in quantity competition is applicable to consumer surplus in price competition. Therefore, we conclude that sharing firm-specific demand information in price competition must lower consumer surplus and raise social welfare.

When firms share cost information in Bertrand oligopoly, the expected values of the first two terms in consumer surplus and social welfare, namely $0.5\boldsymbol{\alpha}'\mathbf{B}\boldsymbol{\alpha} - \boldsymbol{\alpha}\cdot\mathbf{p}$ and $0.5\boldsymbol{\alpha}'\mathbf{B}\boldsymbol{\alpha} - \boldsymbol{\alpha}\cdot\mathbf{c}$, are not affected. Hence, consumer surplus must fall. We can show that the expected value of $\mathbf{c}'\mathbf{B}^{-1}\mathbf{p} - 0.5\mathbf{p}'\mathbf{B}^{-1}\mathbf{p}$ in social welfare also falls.

Proposition 4: In price competition both consumers and society are worse off when firms share firm-specific cost information.

Proof: see Appendix D.

The findings bear some policy implications regarding exchange of firm-specific information. When firms exchange firm-specific demand information or exchange cost information in quantity competition, their incentives are justified by non-collusive

behavior and the social welfare increases provided firms do not collude. Hence restricting such information exchange may not be necessary when there is not evidence of collusion. In contrast, when firms exchange firm-specific cost information in price competition, their incentives can not be justified by non-collusive behavior and the social welfare decreases even if firms do not collude. Then an anti-trust authority should pay close attention to such information sharing activities. If the policy goal is merely to benefit consumers, sharing any firm-specific information should be closely monitored in both Cournot and Bertrand industries.

5. Closing Remarks

This paper examines firms' incentive to exchange firm-specific information and the welfare effect in asymmetric oligopoly with any mixture of substitute and complement goods. We found that revealing demand information is the dominant strategy in both Cournot and Bertrand competition. Revealing cost is dominant in Cournot competition, but concealing cost is so in Bertrand competition. While exchange of firm-specific demand or cost information always hurts consumers, sharing firm-specific demand information is socially desirable. Exchange of firm-specific cost information is socially desirable in quantity competition, but undesirable in price competition.

The assumption of firm-specific information plays an essential role in obtaining robust results. Since our model allows "independent values" and "perfect signals" as special cases, it proposes firm-specific information as an alternative explanation for the incentive problem. A possible extension is to consider other economic games with firm-specific information. For instance, companies may reveal firm-specific features, such as the product quality, before R&D or advertisement decisions. It is not known yet whether there always exists a dominant revelation strategy, and if so, how it is determined. These questions may deserve future research.

APPENDIX

Appendix A: (i) Prove that (5) is the unique equilibrium strategy. Let $(D+B)_i$ be the i th row of $D + B$. The first order condition for firm i 's quantity is

$$a_i - \hat{c}_i = E_i[(D+B)_i \mathbf{x}] \quad (\text{A})$$

Substituting (5) into (A) and using $E_i(\hat{c}_j - \bar{c}_j) = 0$ for $j \neq i$, we have

$$E_i[(D+B)_i \mathbf{x}] = (D+B)_i \mathbf{x}^* - \bar{c}_i - (\hat{c}_i - \bar{c}_i)$$

(A) becomes $a_i = (D+B)_i \mathbf{x}^*$. It holds given the definition of \mathbf{x}^* . So (5) is an equilibrium.

Suppose there is another equilibrium strategy $\mathbf{x} + \mathbf{y}$ where \mathbf{x} is given by (5). The first-order condition (A) implies $(D+B)_i E_i(\mathbf{y}) = 0$ for every i . Let \mathbf{y}_{-i} be the sub-vector of \mathbf{y} without y_i . Another firm j has all the information about \mathbf{y}_{-i} that firm i has. So firm j knows $E_i(\mathbf{y}_{-i})$ and can infer y_i from $(D+B)_i E_i(\mathbf{y}) = 0$. Hence, all firms know \mathbf{y} and we must have $(D+B)\mathbf{y} = \mathbf{0}$, which implies $\mathbf{y} = \mathbf{0}$.

(ii) The proof of (6) easily follows the same procedure and will not be repeated. \parallel

Appendix B: (i) Prove $E(x_i^2)$ increases when \hat{c}_i replaces \bar{c}_i . We need to show that

$$E[\Delta x_i (2x_i + \Delta x_i)] > 0 \quad (\text{B1})$$

where Δx_i is the change in x_i due to revelation of \hat{c} . As $x_i = x_i^* - (D+B)_i^{-1} \bar{c} - 0.5(\hat{c}_i - \bar{c}_i)/b_{ii}$, $\Delta x_i = [0.5/b_{ii} - (D+B)_{ii}^{-1}](\hat{c}_i - \bar{c}_i)$, and $E[(\hat{c}_i - \bar{c}_i)\bar{c}] = \mathbf{0}$, (B1) holds if $(D+B)_{ii}^{-2} - 1/(2b_{ii})^2 > 0$, or

$$2b_{ii}[(D+B)_{ii}^{-1}] > 1 \quad (\text{B2})$$

Let F_i be the sub-matrix of $D + B$ without its i th row and column, \mathbf{f}_i be the sub-vector of $(D+B)_i$ without $2b_{ii}$, and ϕ_i be the sub-vector of $(D+B)_i^{-1}$ without $(D+B)_{ii}^{-1}$. Then we have

$$2b_{ii}(D+B)_{ii}^{-1} + \mathbf{f}_i \cdot \phi_i = 1 \quad (\text{B3})$$

$$(D+B)_{ii}^{-1} \mathbf{f}_i + F_i \phi_i = \mathbf{0} \quad (\text{B4})$$

Pre-multiply (B4) by ϕ_i , we get $(D+B)_{ii}^{-1} \phi_i \cdot \mathbf{f}_i + \phi_i F_i \phi_i = 0$. $\phi_i F_i \phi_i > 0$ because F_i is positive definite. So $\phi_i \cdot \mathbf{f}_i < 0$. (B3) implies $2b_{ii}(D+B)_{ii}^{-1} = 1 - \phi_i \cdot \mathbf{f}_i > 1$, (B2) holds.

(ii) Prove $E[(p_i + \Delta p_i - c_i)^2] < E[(p_i - c_i)^2]$, where Δp_i is due to revelation of \hat{c}_i , i.e.,

$$E[\Delta p_i(2p_i - 2c_i + \Delta p_i)] < 0 \quad (\text{B5})$$

As $p_i = p_i^* + (\Lambda + B^{-1})^{-1} \Lambda \bar{c} + 0.5(\hat{c}_i - \bar{c}_i)$, $\Delta p_i = [(\Lambda + B^{-1})^{-1} \beta_{ii} - 0.5](\hat{c}_i - \bar{c}_i)$, $E[(\hat{c}_i - \bar{c}_i) \bar{c}] = \mathbf{0}$ and $E[c_i(\hat{c}_i - \bar{c}_i)] = E[(\hat{c}_i - \bar{c}_i)^2]$, (B5) holds if

$$\{(\Lambda + B^{-1})^{-1} \beta_{ii} - 0.5\} \{(\Lambda + B^{-1})^{-1} \beta_{ii} - 1.5\} < 0 \quad (\text{B6})$$

(B6) holds if $0.5 < (\Lambda + B^{-1})^{-1} \beta_{ii} < 1$. $1 - (\Lambda + B^{-1})^{-1} \beta_{ii}$ is the i th diagonal of $I - (\Lambda + B^{-1})^{-1} \Lambda$, which equals $\Lambda^{-1}(\Lambda + B^{-1})^{-1}$. The diagonals of this matrix are positive, so $(\Lambda + B^{-1})^{-1} \beta_{ii} < 1$. $(\Lambda + B^{-1})^{-1} \beta_{ii} > 0.5$ is implied by (B2) given the similarity of B and B^{-1} . \parallel

Appendix C: (i) Prove $E(\mathbf{x}'\mathbf{B}\mathbf{x})$ falls if \bar{c}_i becomes \hat{c}_i . Using (5), $E[(\hat{c}_j - \bar{c}_j) \bar{c}] = \mathbf{0}$ for all j , $E[(\hat{c}_j - \bar{c}_j)(\hat{c}_k - \bar{c}_k)] = 0$ for all $k \neq j$, we get

$$E(\mathbf{x}'\mathbf{B}\mathbf{x}) = k_1 + E[\bar{c}(D+B)^{-1}B(D+B)^{-1}\bar{c}] + 0.25\sum_{j=1}^n E[(\hat{c}_j - \bar{c}_j)^2]/b_{jj} \quad (\text{C1})$$

where k_1 is constant. If \bar{c}_i changes to \hat{c}_i , the last term of (C1) loses $0.25E[(\hat{c}_i - \bar{c}_i)^2]/b_{ii}$, the second term gains by $\{(D+B)^{-1}B(D+B)^{-1}\}_{ii}E[(\hat{c}_i - \bar{c}_i)^2]$. Thus, (C1) falls if

$$\{(D+B)^{-1}B(D+B)^{-1}\}_{ii} - \frac{1}{4b_{ii}} < 0 \quad (\text{C2})$$

Since $(D+B)^{-1}B(D+B)^{-1} = (D+B)^{-1} - (D+B)^{-1}D(D+B)^{-1}$, $\{(D+B)^{-1}B(D+B)^{-1}\}_{ii} = (D+B)^{-1}_{ii} - \sum_{i=1}^n b_{ij}[(D+B)^{-1}_{ij}]^2$. Then, (C2) holds because it becomes

$$-\frac{1}{4b_{ii}}[2b_{ii}(D+B)^{-1}_{ii} - 1]^2 - \sum_{j \neq i}^n b_{jj}[(D+B)^{-1}_{ij}]^2 < 0$$

(ii) Prove $E[(\mathbf{a} - \mathbf{c}) \cdot \mathbf{x}]$ rises as \bar{c}_i becomes \hat{c}_i . As $E(\mathbf{c}\bar{c}_j) = E(\bar{c}\bar{c}_j)$, $E[(c_j(\hat{c}_j - \bar{c}_j))] = E[(\hat{c}_j - \bar{c}_j)^2]$ for all j , and $E[(c_k(\hat{c}_j - \bar{c}_j))] = 0$ for all $k \neq j$, we have

$$E[(\mathbf{a} - \mathbf{c}) \cdot \mathbf{x}] = k_2 + E[\bar{c}(D+B)^{-1}\bar{c}] + \sum_{j=1}^n \frac{E[(\hat{c}_j - \bar{c}_j)^2]}{2b_{jj}} \quad (\text{C3})$$

k_2 is fixed. As \bar{c}_i changes to \hat{c}_i , the last term of (C3) loses $0.5E[(\hat{c}_i - \bar{c}_i)^2]/b_{ii}$, the second term gains $(D+B)^{-1}_{ii}E[(\hat{c}_i - \bar{c}_i)^2]$. Given (B) the change is positive as $b_{ii}(D+B)^{-1}_{ii} > 0.5$. \parallel

Appendix D: Prove that $E(\mathbf{c}\mathbf{B}^{-1}\mathbf{p}-0.5\mathbf{p}\mathbf{B}^{-1}\mathbf{p})$ falls when \bar{c}_i changes to \hat{c}_i . Given (6), $E(\mathbf{c}\bar{c}_j) = E(\bar{c}_j)$, $E[(c_j(\hat{c}_j-\bar{c}_j))] = E[(\hat{c}_j-\bar{c}_j)^2]$ for all j , and $E[(c_k(\hat{c}_j-\bar{c}_j))] = 0$ for all $k \neq j$, we get

$$\begin{aligned} E(\mathbf{c}\mathbf{B}^{-1}\mathbf{p}-0.5\mathbf{p}\mathbf{B}^{-1}\mathbf{p}) &= k_3 + E[\bar{\mathbf{c}}\mathbf{B}^{-1}(\Lambda+\mathbf{B}^{-1})^{-1}\Lambda\bar{\mathbf{c}}] + 0.5\sum_{j=1}^n\beta_{jj}E[(\hat{c}_j-\bar{c}_j)^2] \\ &\quad - 0.5E[\bar{\mathbf{c}}\Lambda(\Lambda+\mathbf{B}^{-1})^{-1}\mathbf{B}^{-1}(\Lambda+\mathbf{B}^{-1})^{-1}\Lambda\bar{\mathbf{c}}] - \sum_{j=1}^n\beta_{jj}E[(\hat{c}_j-\bar{c}_j)^2]/8 \end{aligned} \quad (\text{D1})$$

As \hat{c}_i is revealed, the change in (D1) is $\{\mathbf{B}^{-1}(\Lambda+\mathbf{B}^{-1})^{-1}\Lambda\}_{ii} - 0.5\beta_{ii}^2(\mathbf{B}^{-1}+\Lambda)_i^{-1}\mathbf{B}^{-1}(\mathbf{B}^{-1}+\Lambda)_i^{-1} - 0.5\beta_{ii} - \beta_{ii}/8$ multiplied by $E[(\hat{c}_i-\bar{c}_i)^2]$.

Since $\mathbf{B}^{-1}(\Lambda+\mathbf{B}^{-1})^{-1}\Lambda = \Lambda - \Lambda(\Lambda+\mathbf{B}^{-1})^{-1}\Lambda$, we get $\{\mathbf{B}^{-1}(\Lambda+\mathbf{B}^{-1})^{-1}\Lambda\}_{ii} = \beta_{ii} - \beta_{ii}^2[\beta_{ii}(\Lambda+\mathbf{B}^{-1})_{ii}^{-1}]$.

So (D1) falls due to revelation of \hat{c}_i if

$$2\beta_{ii}(\mathbf{B}^{-1}+\Lambda)_{ii}^{-1} - 1.25 + \beta_{ii}(\mathbf{B}^{-1}+\Lambda)_i^{-1}\mathbf{B}^{-1}(\mathbf{B}^{-1}+\Lambda)_i^{-1} > 0 \quad (\text{D2})$$

If $2\beta_{ii}(\mathbf{B}^{-1}+\Lambda)_{ii}^{-1} > 1.25$, (D2) holds. Otherwise, write \mathbf{B}^{-1} as $\mathbf{A} + \mathbf{C}$, with its i th column and row in \mathbf{A} and the rest elements in \mathbf{C} . Since \mathbf{C} is positive semi-definite,

$$\begin{aligned} (\mathbf{B}^{-1}+\Lambda)_i^{-1}\mathbf{B}^{-1}(\mathbf{B}^{-1}+\Lambda)_i^{-1} &\geq (\mathbf{B}^{-1}+\Lambda)_i^{-1}\mathbf{A}(\mathbf{B}^{-1}+\Lambda)_i^{-1} \\ &= 2(\mathbf{B}^{-1}+\Lambda)_{ii}^{-1} - 3\beta_{ii}[(\mathbf{B}^{-1}+\Lambda)_{ii}^{-1}]^2 \end{aligned}$$

Hence (D2) always holds given $1 < 2\beta_{ii}(\mathbf{B}^{-1}+\Lambda)_{ii}^{-1} < 1.25$. $E(\mathbf{c}\mathbf{B}^{-1}\mathbf{p}-0.5\mathbf{p}\mathbf{B}^{-1}\mathbf{p})$ falls. \parallel

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