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The KOF macro model in a time series perspective: Part one: the Swiss production function

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The KOF Macro Model in a Time Series Perspective

Part One: The Swiss Production Function

Comments welcome.

The KOF Macro Model in a Time Series Perspective

Part One: The Swiss Production Function

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Abstract

We use a vintage capital production function to model investment in Switzerland. A special feature is the inclusion of survey data to map observable data on latent variables. In addition a labour market is modelled. This results in a fairly complicated model structure and estimation requires ad hoc assumptions and calibration. In contrast this paper links the theoretical framework given by the structural model to a multivariate time series representation of the relationships, for which standard estimation procedures (Pesaran et al., 2000) and specification tests are available. The outcomes are improved forecasts and a zero-productivity-growth puzzle.

JEL classification: C32, C51, E17, E22

Keywords: vintage capital production, cointegrated time series, structural economic models

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1 Introduction

Economic theory very often implies highly non-linear relationships between the variables of interest. Generally speaking estimation therefore becomes complicated and as Laidler (1999) has noted, discussion often centres on where it is 'most fruitful to simplify the theoretical framework in order to bring it into contact with empirical evidence'. At a first glance, the KOF-macro-model provides yet another example of this kind. To begin with in this paper the real sector of that model will be scrutinized. The special focus will be on the time series properties of some central observable and non-observable variables as implied by the underlying theory. It will be shown that the model can be cast in a standard vector error-correction model (*VECM*) form with the long-run relationships being the backbones of the economic structure. Additional testable hypotheses can be formulated for the short-run parameters. For the corresponding tests standard inference can be used. Thus, at a second glance the econometric exercise becomes feasible without relinquishing much of the economics behind it.

The remainder of the paper is organized as follows. At first, the economic model is sketched, then time series properties of some of the variables are determined. Based on the economic theory equations are found which represent the observable variables in terms of their own and each other's past values. This produces a five dimensional multiple time series model which can be easily estimated.

2 A Vintage Capital Production Function Augmented by Survey Data

2.1 Goods Demand and Supply

This section is based on Stalder (1994). More recent adjustments are accounted for in the following section. Throughout the paper we use Latin letters to denote the coefficients on more or less reasonable ad-hoc explanatory variables of the model. Greek letters indicate either structural parameters or coefficients of the empirical

model.

The economy is characterized by monopolistic competition and its production function is assumed to be of a Cobb-Douglas type where capacity output $\nabla Y C_t$ is related to capacity labour input ∇L_t and gross investment I_t in the following manner

$$\nabla Y C_t = D \left(\nabla L_t e^{\theta t} \right)^\rho I_t^{1-\rho} \quad t = 1, 2, \dots \quad (1)$$

Here, ρ represents the labour share in the vintage production function and θ measures the rate of labour augmenting technical progress. Next, the optimal factor intensities are chosen by minimizing the present value of costs arising from the production of a certain output $\nabla Y C_t$ for a given lifetime T of the vintage over the inputs. Second, the optimal lifetime T_{opt} is determined by minimizing the present value of costs over an infinite horizon with replacements taking place at $t + T_{opt}, t + 2T_{opt}, t + 3T_{opt}, \dots$. Since this paper does not aim at discussing the theoretical model we focus on the main equations which help derive the econometric model. The interested reader is referred to the literature instead.¹

For the current purpose it is sufficient to collect the following variables definitions.² The price for labour is the nominal wage level W_t while the price of new capital goods is denoted V_t . The nominal interest rate and the rate by which labour productivity on existing machines deteriorates are signified by r and ϕ respectively. It is also convenient to define $Q_t = W_t/V_t$, and the average growth rate of the factor price ratio $q_t = w_t - v_t$ by \dot{q} where we make use of the convention that lower case letters denote the natural logarithm. It can then be shown that the optimal factor choice depends on G_t defined as

$$G_t(\Delta w_t + \phi - r, T) = \frac{1 - e^{-(r - \Delta w_t - \phi)T}}{r - \Delta w_t - \phi},$$

and allows to write the cost-minimizing technical productivities of labour A_t and capital B_t

$$A_t \equiv \frac{\nabla Y C_t}{\nabla L_t},$$

¹Stalder (1995) for example, quotes Sneesens (1990) as a guideline.

²See table 8 for all variable definitions and symbols used.

$$= D \left(\frac{1-\rho}{\rho} \right)^{1-\rho} \left(\frac{W_t G_t}{V_t} \right)^{1-\rho} e^{\theta \rho t}, \quad (2)$$

$$\begin{aligned} B_t &\equiv \frac{\nabla Y C_t}{\nabla I_t}, \\ &= D \left(\frac{1-\rho}{\rho} \right)^{-\rho} \left(\frac{W_t G_t}{V_t} \right)^{-\rho} e^{\theta \rho t}. \end{aligned} \quad (3)$$

Treating ρ, θ, ϕ and \dot{q} as constants, (2) and (3) can be re-written to yield

$$A_t = A_0 \left(\frac{W_t}{V_t} \right)^{1-\rho} e^{\theta \rho t} \quad (4)$$

$$B_t = B_0 \left(\frac{W_t}{V_t} \right)^{-\rho} e^{\theta \rho t}. \quad (5)$$

Furthermore, the normal mark-up price, \bar{P}_t is expressed as

$$\bar{P}_t = m W_t^\rho V_t^{1-\rho} e^{-\theta \rho t} \quad (6)$$

with m defining the firm specific elasticity of the demand curve and which is assumed constant over time. Production capacity is updated over time according to

$$Y C_t = S_t Y C_{t-1} + B_t I_t + U_{1,t} \quad (7)$$

where S_t is the share of equipment which is kept from one period to the next. In the steady state the optimal scrapping rate δ will be equal to $1/T_{opt}$ which is a consequence of having a constant \dot{q} . In the short-run however, S_t will fluctuate around \dot{q} . Stalder (1994) therefore suggests to specify

$$S_t = (1 - \delta) \left(\frac{W_t/V_t}{(W_{t-1}/V_{t-1}) e^{\dot{q}}} \right)^{-\xi} \quad (8)$$

with ξ being the scrapping elasticity with respect to the factor price ratio.

The firms expect the demand for their products to be

$$\begin{aligned} Y D_t^e &= Y D^e(p_t, \varpi_t) \\ &= p_t^{-\eta} f(\varpi_t) \end{aligned} \quad (9)$$

where the vector ϖ_t contains all those variables which are exogenous to the firms including the average price level in the relevant markets. Naturally, expected demand at the normal mark-up price determines the desired production capacity $Y C_t^*$

and the desired gross investment rate IR_t^* (see (7)). We follow Stalder (1995) by imposing the restriction that gross investment cannot become negative

$$\begin{aligned} IR_t^* &\equiv \frac{I_t^*}{YC_{t-1}} \\ &= S_t \varphi_0 e^{\varphi_1 \left(\frac{YD_t^e(\bar{p}_t, \varpi_t)}{YC_{t-1} S_t} - 1 \right)} B_t^{-1} \end{aligned} \quad (10)$$

This formulation allows to distinguish between two basic situations. Firms face either expected demand in excess of or below current production capacity. They therefore receive an incentive to raise prices and extend capacity or to stick to the actual equipment without replacements respectively. In the long-run, the expected expansion rate of capacities E defines the normal investment rate IR_t^0

$$IR_t^0 = (E - (1 - \delta)) B_t^{-1} \quad (11)$$

which however, will not hold in the short-run where investment is smoothed and where it has to account for variations in demand. This leads to the following actual investment rate

$$IR_t = \left(IR_t^* \lambda_1 IR_t^{0(1-\lambda_1)} \right)^{\lambda_2} IR_{t-1}^{1-\lambda_2} e^{u_{2,t}}, \quad 0 < \lambda_1, \lambda_2 \leq 1 \quad (12)$$

where IR_t is defined as

$$IR_t = I_t / YC_{t-1} \quad (13)$$

The following lines report the modifications necessary for the incorporation of survey data. Define the relative excess demand at the firm level z_j ,

$$z_j \equiv \frac{YD^e(\bar{p})_j}{YC}$$

where YC is the aggregate capacity, and assume that z_j is log-normally distributed with

$$\ln z_j \sim N(\mu_z, \sigma_z^2).$$

Then, the moments of the distribution can be related to survey data by forming the three categories: capacities too large, sufficient, and too small with respect to a threshold value c . The proportion of firms reporting either excess or sufficient

capacity at time t will be denoted $\pi_{z,t}$. Short-cutting again the outline, the following approximations will be used to map the observable variables $\pi_{z,t}$ and Y_t onto the unobservable variables

$$YD_t^e(\bar{p}_t) = Y_t(1 - \pi_{z,t})^{-\kappa} \quad (14)$$

$$YC_t = Y_t\pi_{z,t}^{-\kappa} \quad (15)$$

$$\bar{p}_t = p_t(1 - \pi_{z,t})^\tau. \quad (16)$$

This completes the theoretical part of the goods supply model. In the KOF-version three more equations are included which basically rest on ad hoc specifications and have been partly justified by conventions found in the literature. Among the latter is the data generating process (DGP) for the factor price ratio

$$q_t = q_{t-1} + \dot{q} + \varepsilon_t \quad (17)$$

It will turn out that this formulation is one of the main driving forces within the described economy. The remaining two equations describe the evolution of expected demand,³

$$\begin{aligned} \Delta \ln YD_t^e &= \alpha_{33} \left[\ln YD_{t-1}^e + \beta_{26} \ln YW_{t-1} + \beta_{27} \ln YNM_{t-1} + \beta_{28} \ln IE_{t-1} \right. \\ &\quad \left. + \beta_{212} + \beta_{214} \ln \left(\frac{\bar{P}_{t-1}}{PW_{t-1}} \right) \right] + \phi_{0,32} \Delta \ln YW_t + \phi_{0,33} \Delta \ln YNM_t \\ &\quad + \phi_{0,34} \Delta \ln IE_t + \phi_{0,38} \Delta \ln \left(\frac{\bar{P}_t}{PW_t} \right) + u_{3,t} \end{aligned} \quad (18)$$

and inflation,

$$\begin{aligned} \Delta \ln P_t &= \alpha_{44} \left[\ln P_{t-1} - \left(\ln \bar{P}_{t-1} + \beta_{32} \ln(1 - \pi_{z,t-1}) \right) \right] \\ &\quad + (\gamma_{0,41} - \phi_{0,45}) \Delta \ln W_t + \phi_{0,45} \Delta \ln V_t + \phi_{0,47} \Delta \ln(1 - \pi_{z,t}) \\ &\quad + u_{4,t}. \end{aligned} \quad (19)$$

These equations include a number of exogenous variables. The terms YW_t , YNM_t , IE_t and PW_t denote world activity in manufacturing, real value added in

³Subscripts will be used to distinguish between the various lags, equation ordering and variables entering the model. This proves helpful for writing the multivariate time series representation.

the Swiss economy outside manufacturing, total Swiss gross investment in machinery and equipment, and a world price index respectively.

It is worth pointing out that in this setup, the supply process is characterized by two innovation processes. One comes from the investment decision and the other from the law of motion for capacity output (see equation (7)). This could be viewed a rather generous assignment of innovations because the investment decision already aims at optimizing the production capacity given the actual expectations about aggregate demand and prices. Therefore, it appears sufficient to work with just one set of shocks which are supposed to hit the supply side of the economy. The next subsection will also provide some arguments as to why (7) is not exploitable for estimation purposes in general.

In the following, equations (1) to (17) will be used to express the observable variables in terms of their own pasts. All definition equations will consequently be rewritten in order obtain an econometric model which is accessible by standard estimation techniques.

2.2 Labour Demand and Supply

Stalder (1995) complemented the goods supply model by a labour market. One of its distinguishing feature is the differentiation between foreign and domestic labour. As in the previous section survey data is used to identify situations with and without pressure on the market.

On the supply side, foreign and domestic labour are denoted LS_t^F and LS_t^{CH} respectively. It is assumed that the supply of domestic labour depends on the real wage and the potentially active labour force (LPA_t).

$$LS_t^{CH} = C_0 LPA_t \left(\frac{W_t}{PC_t} \right)^{C_1} e^{c_2 t} \quad (20)$$

where PC_t stands for consumer price index and LPA_t is given by

$$LPA_t = LP_t^\omega \left(L^{CH} \right)^{1-\omega} \quad 0 \leq \omega \leq 1 \quad (21)$$

with LP_t being the number of citizen aged between 20 and 64. Stalder (1995)

assumes that the demand for foreign labour is a fixed fraction of overall demand for labour adjusted for the fraction of micro labour markets in excess demand, $\pi_{L,t}$.

$$l_t^F = f_0 + f_1 l_t + f_2 e^{f_3 t} \ln \left(\frac{\pi_{L,t}}{1 - \pi_{L,t}} \right) \quad (22)$$

In (22) the usual convention applies that lower case letter denote the natural logarithm of the variable defined in upper case letters. The aggregate labour supply is related to total employment by $\pi_{L,t}$ as

$$L_t \pi_{L,t}^{-\nu} = LS_t, \quad (23)$$

and under the assumption that (23) applies to domestic labour supply, the definition $L_t = L_t^{CH} + L_t^F$ gives rise to⁴

$$(L_t - L_t^F) \pi_{L,t}^{-\nu} = LS_t^{CH}. \quad (24)$$

The parameter ν measures the degree of mismatch on the labour market. Stalder (1995) suggests to model it as a time varying component as in (25)

$$\nu_t = \nu_0 (1 + \nu_1 t). \quad (25)$$

The next section discusses the implications of this formulation. Returning to (24) we can take logarithms, combine it with (20) and (21) to obtain

$$\begin{aligned} -\nu \ln(L_t - L_t^F) + \ln \pi_{L,t} &= c_0 + \omega LP_t + (1 - \omega) \ln(L_{t-1} - L_{t-1}^F) \\ &\quad + c_1(w_t - pc_t) + c_2 t \\ \Delta l_t^{CH} &= \gamma_0 + \omega^* l p_t + \gamma_1(w_t - pc_t) + \gamma_2 t \\ &\quad + \nu^* \ln \pi_{L,t} + \left(\frac{\omega - 1}{\nu} - 1 \right) l_{t-1}^{CH} \end{aligned} \quad (26)$$

where

$$\begin{aligned} \gamma_i &= -\frac{c_i}{\nu} \\ \omega^* &= -\frac{\omega}{\nu} \\ \nu^* &= \frac{1}{\nu} \end{aligned}$$

⁴Stalder (1995) justifies this by identical unemployment rates for both demographic groups ($LS_t/L_t = LS_t^{CH}/L_t^{CH}$).

Finally, the wage is assumed to be determined as

$$W_t = k_0 P_t^{K_1} \left(\frac{PC_t}{P_t} \right)^{K_2} \left(\frac{Y_t}{L_t} \right)^{K_3} \left(\frac{\pi_{L,t}}{1 - \pi_{L,t}} \right)^{K_4},$$

and after taking logs we find

$$w_t = k_0 + (k_1 - k_2)p_t + k_2 pc_t + k_3(y_t - l_t) + k_4 \ln \left(\frac{\pi_{L,t}}{1 - \pi_{L,t}} \right). \quad (27)$$

Equation (27) will be used to replace q_t in the above equations in order to derive parsimonious representations of the system equations.

2.3 Structural Equations and Their Implications for Univariate Time Series Properties

It has already been mentioned that (17) plays a central role in deriving the statistical properties of the model economy. It is therefore convenient to address the implied properties first. Note, that the moving average representation of (17) reveals that the variable q_t will be ruled by two trends, a stochastic trend and a linear time trend.

$$\begin{aligned} q_t &= \dot{q}t + \epsilon_t \\ \Delta \epsilon_t &= \varepsilon_t \end{aligned} \quad (28)$$

Since (17) is merely an assumption of how q_t evolves over time rather than an economically founded structure, to learn about its empirical justification is very important. Furthermore, one should also look at possible alternative formulations of (17). An alternative DGP for q_t could be

$$\begin{aligned} q_t &= \dot{q}t + \epsilon_t \\ a(L)\epsilon_t &= \varepsilon_t \end{aligned} \quad (29)$$

where $a(L) = (1 - L)b(L)$ and $b(L) = 1 - b_1L - b_2L^2 - \dots - b_{p-1}L^p$ is a polynomial in the lag-operator having all its roots outside the unit circle. Here L^i is defined to lag a variable x_t exactly i times: $L^i x_t = x_{t-i}$. In this setting, q_t would still be driven by time trend, but the autoregressive structure is much more flexible.

The model (29) would leave the long-run properties of q_t largely unchanged but could contribute to a better description of the time series in that it may cover some remaining autocorrelation in the error term as it may occur in the rather parsimonious specification of (17).

For the following algebraic exercises either (28) or (29) can be considered without changing the main conclusions.

The next lines will make use of the above by relating the various variables to q_t . To begin with, A_t and B_t will be reconsidered. Taking the natural logarithm and sticking to the convention that small letters denote the logarithm of the variable defined in capitals we obtain

$$a_t = a_0 + (1 - \rho)q_t + \theta\rho t,$$

and similarly for B_t one gets

$$b_t = b_0 - \rho q_t + \theta\rho t,$$

from which it is easy to see that both these variables are presumably driven by the same stochastic trend which is also present in q_t . In addition, the two linear time trends can be identified in each variable, where only one of them also originates in q_t . Against that, the logarithm of S_t as of (8) yields a representation without any trend

$$\begin{aligned} s_t &= \bar{\delta} - \xi\eta_t \\ \bar{\delta} &= \ln(1 - \delta) \end{aligned}$$

implying that s_t and hence S_t is a strictly stationary process. This conforms with its interpretation as the ratio of capital being kept from one period to another. If model (29) instead of (28) is assumed, the expression for $\bar{\delta}$ would read $\ln(1 - \delta) - \xi\dot{q}$ and affect the interpretation of the regression intercepts.

We use the definitions given in (14), (15) and the results above to substitute for the unknowns in the equations for IR^0 and IR^* . This allows to rewrite equations (11), (12), (10) in a straightforward manner.

$$ir_t^0 = \bar{E} - b_0 + \rho q_t - \theta\rho t \tag{30}$$

$$\begin{aligned}
\bar{E} &\equiv \ln[E - (1 - \delta)] \\
ir_t^* &= (1 - \phi_1)\bar{\delta} - (1 - \phi_1)\xi\varepsilon_t + \phi_1\Delta y_t + \phi_1\kappa\ln\left(\frac{\pi_{z,t-1}}{1 - \pi_{z,t}}\right) \\
&\quad - b_0 + \rho q_t - \theta\rho t
\end{aligned} \tag{31}$$

The calculation of ir_t requires some more effort. To save space, we will nevertheless make a short-cut by simply noting that (30) and (31) are inserted into (12), and combined with (7). By using the following definitions to keep the number of terms entering the equation in check

$$\begin{aligned}
\iota &\equiv \rho\lambda_2 + (1 - \varphi_1)\lambda_2\lambda_1\xi, \\
u_{2,t}^* &= \iota\varepsilon_t + u_{2,t}
\end{aligned}$$

replacing q_t by $w_t - v_t$ the error correction representation for investment can be given as

$$\begin{aligned}
\Delta i_t &= \gamma_{0,23}\Delta y_t + \phi_{0,27}\Delta\ln(1 - \pi_{z,t}) \\
&\quad + \alpha_{22}[i_{t-1} + \beta_{21}w_{t-1} + \beta_{23}y_{t-1} + \beta_{26}\ln\pi_{z,t-1} + \beta_{210}v_{t-1} \\
&\quad + \beta_{212}\ln(1 - \pi_{z,t-1}) + \beta_{218}(t - 1)] \\
&\quad + \gamma_{1,23}\Delta y_{t-1} + \phi_{1,21}\Delta\ln\pi_{z,t-1} + d_2 \\
&\quad + u_{2,t},
\end{aligned} \tag{32}$$

and theory implies that

$$\begin{aligned}
\gamma_{0,23} &= \varphi_1, \\
\gamma_{1,23} &= (1 - \lambda_2), \\
\alpha_{22} &= -\lambda_2, \\
\beta_{21} &= -\beta_{210} = -\rho, \\
\beta_{23} &= -1, \\
\beta_{26} &= \kappa(1 - \varphi_1\lambda_1), \\
\beta_{212} &= \varphi_1\kappa\lambda_1 \\
\beta_{218} &= \rho\theta, \\
\phi_{0,27} &= \lambda_1\lambda_2\kappa\varphi_1,
\end{aligned}$$

$$\begin{aligned}\phi_{1,21} &= \kappa(\lambda_2 - 1), \\ d_2 &= \lambda_2 \left(\lambda_1(\bar{\delta}(1 - \varphi_1)) + (1 - \lambda_1)\bar{E} - b_0 + \rho(\dot{q} - \theta) \right).\end{aligned}$$

Next, we will revise (19) by expressing \bar{P} in terms of the observable variables. This can be done by defining

$$\tilde{m} \equiv \ln m$$

and applying (6) to (19), yielding

$$\begin{aligned}\Delta p_t &= \gamma_{0,41}\Delta w_t \\ &+ \alpha_{44} [p_{t-1} + \beta_{41}w_{t-1} + \beta_{410}v_{t-1} \\ &+ \beta_{412}\ln(1 - \pi_{z,t-1}) + \beta_{418}(t - 1)] + \phi_{0,45}\Delta v_t + \phi_{0,47}\Delta \ln(1 - \pi_{z,t}) \\ &+ d_4 + u_{4,t},\end{aligned}\tag{33}$$

where, theoretically, the coefficients could be replaced by their following counterparts

$$\begin{aligned}\beta_{41} &= -\rho, \\ \beta_{410} &= \phi_{0,45} = -(1 - \rho), \\ \beta_{412} &= \tau, \\ \beta_{418} &= \theta\rho \\ d_4 &= \alpha_{43}(\theta\rho - \tilde{m}).\end{aligned}$$

The following equation gives an expression for real GDP. The starting point is the definition of aggregate demand in (14) which will be related to (18) by replacing the definition variables by their appropriate observable counterparts. The ultimate formula is specified ad hoc though. We follow Stalder (1994) in that respect in order to obtain a comparable model.

$$\begin{aligned}\Delta y_t &= \gamma_{0,31}\Delta w_t \\ &+ \alpha_{33} [y_{t-1} + \beta_{31}w_{t-1} + \beta_{37}yw_{t-1} + \beta_{38}ynm_{t-1} \\ &+ \beta_{39}ie_{t-1} + \beta_{310}v_{t-1} + \beta_{311}pw_{t-1} + \beta_{312}\ln(1 - \pi_{z,t}) + \beta_{318}(t - 1)] \\ &+ \phi_{0,32}\Delta yw_t + \phi_{0,33}\Delta ynm_t + \phi_{0,34}\Delta ie_t + \phi_{0,35}\Delta v_t + d_3 \\ &+ \phi_{0,36}\Delta pw_t + \phi_{0,37}\Delta \ln(1 - \pi_{z,t}) \\ &+ u_{3,t},\end{aligned}\tag{34}$$

where the preceding algebraic exercise implies the following relations between the coefficients

$$\begin{aligned}
\frac{\gamma_{0,31}}{\phi_{0,36}} &= \frac{\beta_{31}}{\beta_{311}} = -\rho, \\
\frac{\gamma_{0,31}}{\phi_{0,35}} &= \frac{\beta_{31}}{\beta_{310}} = \frac{\rho}{1-\rho}, \\
\beta_{318} &= -\beta_{31}\theta, \\
\phi_{0,37} &= -\beta_{312} = \kappa, \\
d_3 &= \alpha_{33}\beta_{311}(\tilde{m} - \theta\rho) + \phi_{0,36}\theta\rho.
\end{aligned}$$

The last exercise will produce error correction representations of (26) and (27). Noting that $x_t = \Delta x_t + x_{t-1}$ and adding an error term to (26) we find for Δw_t and Δl_t^{CH} ,

$$\begin{aligned}
\Delta w_t &= d_1 + \gamma_{0,13}\Delta y_t + \gamma_{0,14}\Delta p_t + \phi_{0,18}\Delta pc_t + \phi_{0,19}\Delta l_t \\
&\quad + \phi_{0,110}\Delta \ln\left(\frac{\pi_{L,t}}{1-\pi_{L,t}}\right) - \alpha_{11}[w_{t-1} + \beta_{14}p_{t-1} + \beta_{113}pc_{t-1} \\
&\quad + \beta_{13}y_{t-1} + \beta_{114}l_{t-1} + \beta_{115}\left(\frac{\pi_{L,t-1}}{1-\pi_{L,t-1}}\right)] + u_{1,t} \tag{35}
\end{aligned}$$

$$\begin{aligned}
\Delta l_t^{CH} &= d_5 + \gamma_{0,51}\Delta w_t + \phi_{0,58}\Delta pc_t + \phi_{0,111}\Delta \pi_{L,t} + \phi_{0,512}\Delta lp_t \\
&\quad + \alpha_{55}\left[l_{t-1}^{CH} + \beta_{51}w_{t-1} + \beta_{513}pc_{t-1}\right. \\
&\quad \left.+ \beta_{517}\ln\pi_{L,t-1} + \beta_{518}lp_{t-1} + \beta_{518}(t-1)\right] + u_{5,t} \tag{36}
\end{aligned}$$

The following list of coefficient definitions links (35) and (36) to the underlying theory

$$\begin{aligned}
d_1 &= k_0 & d_5 &= -\frac{c_0+c_2}{\nu}, \\
\gamma_0 &= k_0 & \gamma_{0,51} &= -\phi_{0,58} = -\frac{c_1}{\nu}, \\
\gamma_{0,13} &= \beta_{13} = k_3, & \phi_{0,510} &= \frac{1}{\nu}, \\
\gamma_{0,14} &= \beta_{14} = k_1 - k_2, & \phi_{0,511} &= -\frac{\omega}{\nu}, \\
\phi_{0,18} &= \beta_{113} = k_2, & \beta_{51} &= -\beta_{513} = -\frac{c_1}{\omega-1-\nu}, \\
\phi_{0,19} &= \beta_{114} = -k_3, & \beta_{516} &= \frac{1}{\omega-1-\nu}, \\
\phi_{0,110} &= \beta_{115} = k_4, & \beta_{517} &= -\frac{\omega}{\omega-1-\nu}, \\
\alpha_{11} &= -1, & \beta_{518} &= -\frac{c_2}{\omega-1-\nu}, \\
&& \alpha_{55} &= \frac{\omega-1-\nu}{\nu}.
\end{aligned}$$

Two remarks are in order. First considering (28) and (29), (35) provides a second explanation for w_t . To the extent that $u_{1,t}$ is linearly independent of v_t it can be regarded the innovation process driving q_t . That means a regression of $u_{1,t}$ on v_t will produce residuals which approximate ε_t . This gives then eventually rise to identification of ι in $u_{2,t}$. In other words, if Δv_t is added to the list of regressors the error $u_{1,t}$ will closely approximate ε_t . Therefore, in the empirical part we add Δv_t to the right hand side of (35).

Second, from (36) it is obvious that a linear time trend in ν would have a decisive effect on l_t^{CH} . In case of a positive linear time trend, the whole dynamic of the process would break down since $\lim_{t \rightarrow \infty} \alpha_{55} = -1$ for any given ω . At the same time all coefficients on the remaining variables would be squeezed to zero. Conversely, for $\lim_{t \rightarrow \infty} \nu = 0$ all coefficients except for α_{55} approach infinity which means that the whole process tends to explode. Therefore, imposing a linear time trend on ν does not seem to be a reasonable idea.

Summarizing the results obtained so far, it is worth mentioning that the observable variables y_t, i_t, p_t, q_t are all driven by stochastic trends where one of them stems from q_t which enters into (32),(33),(34),(35),(36) alike. Potentially, there are further, independent stochastic trends present which may arise from the ad-hoc specification of YD_t^e and P_t given in (18) and (19) respectively. It is less obvious that i_t also follows an independent stochastic trend. In this case, this feature can be derived from rewriting (12).

Second, building on the assumption that all endogenous observable variables are driven by stochastic trends the following long-run relationships should turn out stationary if the economic model was to prove true.

$$\begin{aligned}
w_t &= k_0 + (k_1 - k_2)p_t + k_2pc_t + k_3(y_t - l_t) + k_4 \ln \left(\frac{\pi_{L,t}}{1 - \pi_{L,t}} \right) + ec_{1,t} \\
i_t &= \rho w_t + y_t - \kappa(1 - \varphi_1 \lambda_1) \ln \pi_{z,t} - \rho v_t - \varphi_1 \kappa \lambda_1 \ln(1 - \pi_{z,t}) - \rho \theta t + ec_{2,t} \\
y_t &= \beta_{31} w_t + \beta_{38} yw_t + \beta_{39} ynm_t + \beta_{310} ie_t - \frac{1 - \rho}{\rho} \beta_{31} v_t + \frac{1}{\rho} \beta_{31} p w_t \\
&\quad + \kappa \ln(1 - \pi_{z,t}) + \beta_{31} \theta t + ec_{3,t} \\
p_t &= \rho w_t + v_t - \tau \ln(1 - \pi_{z,t}) + (1 - \rho)v_t - \rho \theta t + ec_{4,t},
\end{aligned}$$

$$\begin{aligned}
l_t^{CH} = & \frac{c_1}{\omega - 1 - \nu} w_t - \frac{c_1}{\omega - 1 - \nu} p c_t - \frac{1}{\omega - 1 - \nu} \ln \pi_{L,t-1} \\
& + \frac{\omega}{\omega - 1 - \nu} l p_{t-1} + \frac{c_2}{\omega - 1 - \nu} t + ec_{5,t}
\end{aligned}
\tag{37}$$

where the error correction terms $ec_{1,t}$, $ec_{2,t}$, $ec_{3,t}$, $ec_{4,t}$ and $ec_{5,t}$ are stationary processes. This said, it is not for all the relationships listed above, that the motivation for the long-run properties is equally well founded theoretically. In particular, only for i_t and for l_t^{CH} the structure follows more or less directly from the economic model. The remaining specifications can ultimately be considered ad hoc only. From that it follows that the focus will be on i_t and l_t^{CH} when it comes to identification of the structural parameters.

This closes the model. To sum up, five independent innovation processes, $U_{1,t}$, $u_{2,t}$, $u_{3,t}$, $u_{4,t}$, and ε_t rule the whole economy. The first two are related to the supply side because they determine the output capacity provided in each period. The error term $u_{3,t}$ describes shocks to the aggregate demand for domestic products. Since inflation is described by equation (19), the corresponding error term, $u_{4,t}$ can be regarded as inflationary shocks. The innovations ε_t are difficult to interpret. Technically they represent shocks to the price factor ratio. The source of unexpected changes to this ratio cannot be identified per se, though. A positive value of ε_t could either represent a wage increase beyond expectation, or mean a surprisingly low increase in the costs of capital equipment.

2.4 Estimation, Identification and Comparison to Previous Approaches

Before turning to the actual estimation problem, a few remarks are in order with respect to equation (7). In the current KOF model, it is used to identify the parameter ξ . The underlying equation is of a very special nature, however. To see this consider that (7) can be rewritten as

$$\Delta Y C_t = -(1 - S_t - B_t I R_t) Y C_{t-1} + U_{1,t}$$

where it has been made use of (13). It can easily be shown that for any value of λ_2 the product of B_t and IR_t follows a non-stationary process except for the borderline case of $\lambda_2 = 1$. Therefore, unless $\lambda_2 = 1$, it has to be made sure that B_t is stationary (which would imply that IR_t is also stationary) if standard estimation techniques like state space modelling (see e.g. Lütkepohl (1993)) are to be applied. Otherwise, equation (7) is not suitable for estimation. These considerations may explain the problems with the properties of the error term related to (7) reported in Stalder (1994) who found λ_2 to be significantly different from 1.

To cope with that problem, we assume that $U_{1,t} = 0$ for all t . We can now write the model in a compact form. We have six equations for the six endogenous variables q_t, w_t, i_t, y_t, p_t and l_t^{CH} . Since q_t can be written as $w_t - v_t$ treating v_t as exogenous the vector of dependent variables consists of w_t, i_t, y_t, p_t and l_t^{CH} . Furthermore, define the $(n_Y \times 1)$, $(n_X \times 1)$, and $(n_Y \times 1)$ vectors Υ_t, X_t , and D_t respectively

$$\begin{aligned}\Upsilon_t &= (w_t, i_t, y_t, p_t, l_t^{CH})', \\ X_t &= \left(\ln \pi_{z,t}, yw_t, ynm_t, ie_t, v_t, pw_t, \ln(1 - \pi_{z,t}), pc_t, l_t, \ln \left(\frac{\pi_{L,t}}{1 - \pi_{L,t}} \right), \right. \\ &\quad \left. \ln \pi_{L,t}, lp_t \right)',\end{aligned}$$

and D_t is an $(n_Y \times 1)$ vector of ones. Define also the coefficient matrices Γ_i ($n_Y \times n_Y$), \mathcal{D} ($n_Y \times 1$), α ($n_Y \times \varrho$), β ($(n_Y + n_X + 1) \times \varrho$), Φ_j ($n_Y \times n_X$). Then, the model can be given as in (38)

$$\begin{aligned}\Gamma_0 \Delta \Upsilon_t &= \alpha \beta' \left(\Upsilon'_{t-1} : X'_{t-1} : t - 1 \right)' + \sum_{i=1}^{l-1} \Gamma_i \Delta \Upsilon_{t-i} \\ &\quad + \sum_{j=0}^{h-1} \Phi_j \Delta X_{t-j} + \mathcal{D} D_t + \mathcal{E}_t\end{aligned}\tag{38}$$

where the $:$ indicates the concatenation of two matrices. It is convenient to decompose the $(n_Y \times 1)$ vector \mathcal{E}_t into $\Xi \mathcal{E}_t^*$ in the following way⁵

$$\begin{aligned}\mathcal{E}_t &= (\varepsilon_t, u_{2,t}^*, u_{3,t}, u_{4,t}, u_{5,t})' \\ &= \Xi \mathcal{E}_t^*\end{aligned}$$

⁵See also explanations on page 13.

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ \iota & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \varepsilon_t \\ u_{2,t} \\ u_{3,t} \\ u_{4,t} \\ u_{5,t} \end{bmatrix}$$

Finally, (38) will be pre-multiplied by Γ_0^{-1} to arrive at a reduced form of the model. Defining $\Gamma_i^* = \Gamma_0^{-1}\Gamma_i$, for $i > 0$, $\alpha^* = \Gamma_0^{-1}\alpha$, $\Phi_j^* = \Gamma_0^{-1}\Phi_j$, $\mathcal{D}^* = \Gamma_0^{-1}\mathcal{D}$, and $\Xi^* = \Gamma_0^{-1}\Xi$, we have

$$\begin{aligned} \Delta Y_t &= \alpha^* \beta' \left(Y'_{t-1} : X'_{t-1} : t-1 \right)' + \sum_{i=1}^{l-1} \Gamma_i^* \Delta Y_{t-i} \\ &\quad + \sum_{j=0}^{h-1} \Phi_j^* \Delta X_{t-j} + \mathcal{D}^* D_t + U_t^* \end{aligned} \quad (39)$$

In (39) the innovations are defined as $U_t^* = \Xi^* \mathcal{E}^*$. It is worth noting that (39) fits in the standard framework of non-stationary time series analysis with exogenous variables. Estimation of this kind of models can be handled with standard theory.

The general approach is as follows. In the first step the degree of integration of each of the variables involved is determined by unit root tests. Next, the number of cointegration relations, ϱ is estimated. To do so it seems inevitable to consider sub-system because otherwise, the dimension of the process would be too large. Moreover, since the tests designed for systems with exogenous variables require that no cointegration relation exist between the exogenous variables, this has to be checked in a separate investigation. To simplify life, the model could be estimated conditioning on y_t which is also justified by the poor economic grounds on which the respective equation rests. A further alternative provides the opportunity to assume the number of cointegration relations to be given and to estimate conditioning on that.

From the first step a number of parameters can already be obtained. These are all those contained in the long-run matrix β . It should be noted that writing the model in reduced form does not alter the coefficients of this matrix. Thus, estimates for $k_1, k_2, k_3, k_4, \rho, \theta, \kappa, \varphi_1, \tau, c_1, c_2, \omega, \nu$ result. Moreover, at this stage already, a

number of cross-equation restriction could be tested which allows to shed some light on the compliance of the model with the data. Interestingly, using the simplification $\lambda_1 = 1$ made by Stalder (1995) would enable us to identify κ and φ_1 even if one conditions on y_t .

In the second step, the information contained in the error correlation matrix could be used to identify the remaining parameters. To that aim, the structural innovations U_t would have to be recovered from their empirical counterparts, U_t^* (see the Appendix). Since the theoretical model delivers more restrictions than would be required for just identification, yet another tests for the quality of the model in relation to the observations can be conducted.

When comparing the previous approaches (Stalder (1994), Stalder (1995)) to the one which has been suggested in this paper, the following three advantages may be pointed out. First, the estimation procedure requires no iteration in order to identify all remaining parameters. To that aim, the innovations $U_{1,t}$ have to be skipped. However, the corresponding equation is not accessible for estimation anyway, because it involves combinations of integrated variables as coefficients. Thus, the trade-off seems to be acceptable. Likewise, in contrast to the previous procedure, no ad hoc guesses of certain parameter values have to be made.

Secondly, the time series approach can be considered more general in that it encompasses the traditional estimation technique. This becomes obvious when consulting the particular definitions of the parameter matrices provided in the appendix. The zero entries are the consequence of economic priors about the respective relationship, however almost all of those coefficients which have been set to zero might empirically be different from zero without contradicting the underlying economic theory. Therefore, the multiple time series approach is a more flexible tool for using economic theory which also has the potential for obtaining statistically as well as economically sensible results.

Finally, having cast the problem in the framework of standard multiple time series techniques, all well established means for time series econometrics are available. This includes for example the calculation of various kinds of impulse-responses and

their corresponding confidence bands, the provision of forecast statistics and so on.

The next section turns to the actual empirical exercise. Further details of estimation procedures and identification of parameters from the estimated coefficients are discussed in the appendix.

3 Empirical Application

The reduced form model will be analyzed in three ways. Doing so the principal consideration is that there exists a natural hierarchy in the restrictions implied by the structural model. The most important relationships are those which define the long-run equilibria given in (37). The estimation will focus on those at this stage of the analysis.

First, without further testing it will be assumed that all five relations as of (37) hold.

Second, foreshadowing some preliminary results appropriate testing will lead to a reduction of the relationships finding support while at the same time univariate analysis partly reveals tensions between basic assumptions made with respect to the maximum degree of integration and the model formulation. Therefore, as auxiliary measure, the model is projected into the appropriate $I(1)$ space which for the moment will lead to a loss in the interpretability of some of the parameter estimates.

Thirdly, a subsystem analysis which is both, consistent with the economic model and with the time series facts will be investigated.

At the final stage, all models will be compared with respect to the parameters they estimate, residual properties and the forecasting performance of the endogenous variables. The results obtained by the KOF model estimated in structural form serve as a benchmark.

3.1 Univariate Time Series Properties

The times series for the variables entering the model are tested for unit root. To this end, the augmented Dickey-Fuller (Dickey and Fuller (1979)) test is employed.

Table 1: Univariate Properties of the Data

Variable	lag order			specification and test statistics*					
	AIC	FPE	HQ	spec.	D.F.	c. v.	KPSS	c.v.	decision
w_t	2	2	1	$c, t, 2$	-1.56	-3.41			$I(2)$
Δw_t	5	5	1	$c, 5$	-3.79	-2.86	.73	.463	$I(1)$
$\Delta\Delta w_t$				$c, 5$.09	.463	$I(0)$
v_t	0	0	0	$c, t, 2$	-1.82	-3.41			$I(1)$
Δv_t	4	0	0	$c, 3$	-3.79	-2.86			$I(0)$
y_t	2	2	1	$c, t, 2$	-1.96	-3.41			$I(1)$
Δy_t	1	1	0	$c, 1$	-4.35	-2.86			$I(0)$
i_t	5	3	1	$c, t, 3$	-1.58	-3.41			$I(1)$
Δi_t	5	2	0	$c, 1$	-5.00	-2.86			$I(0)$
p_t	4	3	0	$c, t, 4$	-.93	-3.41			$I(2)$
Δp_t	2	2	2	$c, 2$	-2.35	-2.86	.91	.463	$I(1)$
$\Delta\Delta p_t$				$c, 5$.03	.463	$I(0)$
$\pi_{z,t}$	3	3	0	$c, 3$	-2.79	-2.86	.35	.463	$I(0)$
$\Delta\pi_{z,t}$	2	2	2	$c, 2$	-5.51	-2.86			$I(0)$
pc_t	4	4	4	$c, t, 4$	-1.48	-3.41			$I(2)$
Δpc_t	3	3	3	$c, 3$	-2.07	-2.86			$I(1)$
$\Delta\Delta pc_t$	2	2	2	$c, 2$	-21.59	-2.86			$I(0)$
ime_t	7	5	3	$c, t, 5$	-2.52	-3.41			$I(1)$
Δime_t	4	2	0	$c, 2$	-4.40	-2.86			$I(0)$
lp_t	7	6	1	$c, t, 6$	-1.14	-3.41			$I(2)$
Δlp_t	6	5	1	$c, 6$	-2.35	-2.86	.97	.463	$I(1)$
$\Delta\Delta lp_t$	5	5	0	$c, 5$	-2.87	-2.86			$I(0)$

* D.F. is short for the augmented Dickey-Fuller test, the column spec. reports the number of lagged endogenous variables used for the D.F., c.v. abbreviates critical value (at the 5% level of significance), KPSS denotes the Kwiatkowski, Phillips, Schmidt and Shin (1992) test. The columns headed by AIC, FPE, HQ give the optimal lag lengths according to the commonly used model selection criteria Akaike information criterion, Final Prediction Error, and Hennon-Quinn criteria respectively.

The appropriate lag order is determined by lag order selection criteria with the final judgement being based on the lag length that ensures no significant autocorrelation in the residuals. If the test result does not appear clear without ambiguity the KPSS (see Kwiatkowski et al. (1992)) test with a inverted hypothesis will also be used.

Due to the large data set, table 1 collects only the most important test results. Generally speaking, all variables seem to have a unit root. Some important exceptions have to be acknowledged, though. These are the price deflator of the GDP, the consumer price index, the labour force potential and the nominal wage. In the latter case the evidence is not clear cut which could be attributed to the fact that the statistical office needs to produce quarterly data from the annually measured wage level. This procedure might introduce effects which could be difficult to distinguish from true $I(2)$ behaviour. Nevertheless, both endogenous variables wages and prices, will be considered $I(2)$ and that will be taken into account in the second way of analysis. Finally and as expected, the survey data turns out stationary.

3.2 Multivariate Time Series Properties

For the $VECM$ in (39) contains exogenous variables the standard so-called Johansen trace tests (see Johansen (1991), Johansen (1992)) cannot be applied. Instead, the approach by Pesaran, Shin and Smith (2000) solves the problem. In that setting efficient estimation and hypothesis testing requires that the exogenous variables entering the long-run regressor matrix X_{t-1} are not cointegrated with each other. Thats why, several $VECM$ are estimated which include the components of X_t as endogenous variables in order to find combinations which are free of cointegration relationships. We refrain from reporting the details of the test results here, but provide them on request. Finally, two groups of exogenous variables ($X_{1,t}, X_{2,t}$) can be identified which fulfill the requirement. We have

$$\begin{aligned} X_{1,t} &= (yw_t, ynm_t, \Delta lp_t)', \\ X_{2,t} &= (l_t, \Delta pc_t, \Delta lp_t)', \end{aligned}$$

where the $I(2)$ property of the consumer price index and the labour force potential have already been taken into account. Table 2 lists the test results of various systems and subsystems all being cast in the framework of (39).

Table 2 provides the test statistics and critical values for the whole system, the full system but Δw_t being replaced by v_t , and subsystems that are made of all those variables which should form cointegration relationships according to (37). Where an alternative set of exogenous variables could be used, the corresponding results are given in Table 9 in the Appendix. Fortunately, there is no contradiction between these two sets.

The cointegration tests point to the existence of only three stationary relationships. Within the full sample, this result holds independently of what set of exogenous variables is used. The only case where there is indication of a further stationary linear combination within the full system is when q_t replaces Δw_t and the set $X_{1,t}$ of exogenous variables is used. However, turning to the theoretically reasonable relationships, it seems that only three of them do find support.⁶ Therefore, the conclusion is that the system is cointegrated with β having rank three. Potentially, investment, nominal wage growth, and labour supply are forming stationary relationships with explanatory variables as given in (37).

However, the restricted reduced form has not yet been derived for the nominal wage, the GDP deflator, the consumer price index and the potential labour force to enter in second differences. Therefore, when it comes to identification of the long-run relationships the focus will be on investment within the full system. Moreover, in order to save some interpretability the variable w_t will be replaced by q_t which is equivalent to imposing one of the restrictions implied by the long-run investment relation from the very beginning.

⁶For the investment equation whether to include q_t or Δw_t seems to matter. In the following, we are obliged to continue with q_t however, for which the result is consistent with the overall picture.

Table 2: Cointegration Tests

No.	$H_0 : rank_0 = r$	Pesaran et al. (2000)		
		test statistic	5% c.v.	10% c.v.
1	Full System (1) $\Upsilon_t = (\Delta w_t, i_t, y_t, \Delta p_t, l_t^{CH})'$, $X_t = X_{1,t}$			
	0	163.11**	120.0	114.7
	1	106.88**	90.02	85.59
	2	68.05**	63.54	59.39
	3	33.37	40.37	37.07
	4	14.81	20.47	18.19
2	Full System (1a) $\Upsilon_t = (q_t, i_t, y_t, \Delta p_t, l_t^{CH})'$, $X_t = X_{1,t}$			
	0	178.44**	120.0	114.7
	1	114.94**	90.02	85.59
	2	72.42**	63.54	59.39
	3	38.28*	40.37	37.07
	4	10.31	20.47	18.19
3	Investment (1) $\Upsilon_t = (\Delta w_t, i_t, y_t)'$, $X_t = v_t$			
	0	62.08**	49.36	46.00
	1	32.63**	30.77	27.96
	2	9.74	15.44	13.31
4	Wages (1) $\Upsilon_t = (y_t, \Delta w_t, \Delta p_t)'$, $X_t = \Delta p_t$			
	0	56.02**	49.36	46.00
	1	22.94	30.77	27.96
	2	3.62	15.44	13.31
5	Labour $\Upsilon_t = (\Delta w_t, l_t)'$, $X_t = (\Delta p_t, \Delta l_t)'$			
	0	33.29*	35.37	32.51
	1	6.63	18.08	15.82
6	Income $\Upsilon_t = (\Delta w_t, l_t)'$, $X_t = (\Delta p_t, \Delta l_t)'$			
	0	21.78	35.37	32.51
	1	6.70	18.08	15.82
7	Prices $\Upsilon_t = (\Delta w_t, l_t)'$, $X_t = (\Delta p_t, \Delta l_t)'$			
	0	31.78	35.37	32.51
	1	4.94	18.08	15.82

* and ** indicate significance at the 10 and 5 percent levels respectively.

+ see Table T.4 * of Pesaran et al. (2000).

3.3 Estimating and Identifying Cointegration Relationships

The estimation focusses on the coefficients of $\beta' = \beta'_{Upsilon} : \beta'_X$ which should theoretically be representing the following parameters:

$$\beta'_Y = \begin{bmatrix} 1 & 0 & k_3 & k_1 - k_2 & 0 \\ -\rho & 1 & -1 & 0 & 0 \\ \beta_{31} & 0 & 1 & 0 & 0 \\ -\rho & 0 & 0 & 1 & 0 \\ -\frac{c_1}{\omega-1-\nu} & 0 & 0 & 0 & 1 \end{bmatrix}$$

and

$$\beta'_X = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & k_2 & -k_2 k_4 & 0 & 0 & 0 \\ \kappa(1 - \varphi_1 \lambda_1) & 0 & 0 & 0 & \rho & 0 & \varphi_1 \kappa \lambda_1 & 0 & 0 & 0 & 0 & \rho \theta \\ 0 & \beta_{37} & \beta_{38} & \beta_{39} & \beta_{310} & \frac{1-\rho}{\rho} \beta_{31} & -\frac{1}{\rho} \beta_{31} & 0 & 0 & 0 & 0 & -\beta_{31} \theta \\ 0 & 0 & 0 & 0 & \rho - 1 & 0 & \tau & 0 & 0 & 0 & 0 & \theta \rho \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_{513} & 0 & 0 & \frac{-1}{\omega-1-\nu} & \frac{-\omega}{\omega-1-\nu} & \frac{-c_2}{\omega-1-\nu} \end{bmatrix} \quad (40)$$

Each row of β corresponds from top to bottom to one of the long-run equilibrium relationships defined in (39). We will refer to this model by the term *Full model* or equivalently *model with $rk(\beta) = 5$* . In the partial analysis for the three cointegration relationships indicated by the testing procedure, rows two and three will not be considered. Moreover, due to the univariate properties of w_t , w_t will be replaced by q_t in the subsystem analysis which implies that

$$\Upsilon_t = (q_t, i_t, y_t, p_t, l_t^{CH})',$$

$$X_t = \left(\ln \pi_{z,t}, v_t, \ln(1 - \pi_{z,t}), p_{c,t}, l_t, \ln \left(\frac{\pi_{L,t}}{1 - \pi_{L,t}} \right), \ln \pi_{L,t}, l_{p,t} \right)',$$

and

$$\beta'_Y = \begin{bmatrix} 1 & 0 & k_3 & k_1 - k_2 & 0 \\ -\rho & 1 & -1 & 0 & 0 \\ -\frac{c_1}{\omega-1-\nu} & 0 & 0 & 0 & 1 \end{bmatrix}$$

and

$$\beta'_X = \begin{bmatrix} 0 & 1 & 0 & k_2 & -k_2 & k_4 & 0 & 0 & 0 \\ \kappa(1 - \varphi_1 \lambda_1) & 0 & \varphi_1 \kappa \lambda_1 & 0 & 0 & 0 & 0 & 0 & \rho \theta \\ 0 & \frac{-c_1}{\omega-1-\nu} & 0 & \beta_{513} & 0 & 0 & \frac{-1}{\omega-1-\nu} & \frac{-\omega}{\omega-1-\nu} & \frac{-c_2}{\omega-1-\nu} \end{bmatrix}$$

where only the three relationships for wages, investment and labour supply enter. We will refer to this model by the term *model with $rk(\beta) = 3$* . In the special case when only the relationship for investment is investigated, the relations defined in the second row of the above β'_Y, β'_X will be used. The regressor matrix X_t will be adjusted accordingly. That case we will refer to by the term *model for investment only*.

3.3.1 Full System Result

Estimating the complete system with all restrictions according to (39) means ignoring the possible $I(2)$ properties of some of the variables as well as imposing more or less arbitrary, economically not well founded restrictions especially with respect to the income equation. These two factors might be the final reason as to why calculating the log-likelihood with the complete set of restrictions fails. Therefore, the following two adjustment was made. All restrictions corresponding to the income equation were relaxed except for the zero restrictions. Finally, with $l = h = 3$ the following estimates are obtained (standard errors in parentheses).⁷

$$\beta'_Y = \begin{bmatrix} 1 & 0 & -.27 & -.89 & 0 \\ & & (.06) & (.03) & \\ -1.00 & 1 & -1 & 0 & 0 \\ (.04) & & & & \\ .86 & 0 & 1 & 0 & 0 \\ (.12) & & & & \\ -1.00 & 0 & 0 & 1 & 0 \\ (-) & & & & \\ 24.28 & 0 & 0 & 0 & 1 \\ (2.22) & & & & \end{bmatrix}$$

and

$$\beta'_X = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & -.26 & .26 & -.02 & 0 & 0 & 0 \\ & & & & & & (.06) & (-) & (.06) & & & \\ -.11 & 0 & 0 & 1.00 & 0 & -2.75 & 0 & 0 & 0 & 0 & 0 & .002 \\ (.06) & & & (-) & & (.53) & & & & & & (.0003) \\ 0 & -1.38 & .45 & -.53 & -.09 & .76 & 0 & 0 & 0 & 0 & 0 & -.002 \\ & (.23) & (.22) & (.07) & (.04) & (.15) & & & & & & (.002) \\ 0 & 0 & 0 & .004 & 0 & .52 & 0 & 0 & 0 & 0 & 0 & .002 \\ & & & (-) & & (.10) & & & & & & (-) \\ 0 & 0 & 0 & 0 & 0 & 0 & -9.39 & 0 & 0 & -.33 & -52 & -.01 \\ & & & & & & (1.76) & & & (.15) & (5.86) & (.004) \end{bmatrix}$$

⁷For all estimation outcomes reported in section 3.3 the sample is 1983-2001. The model selection has been made on information for the sample 1983-1999.

Here, we removed ie_t from the list of exogenous regressors because i_t is a sub-aggregate of ie_t and therefore causes problems due to non-informative correlation with the dependent variables. Thus, the vector X_t reads

$$X_t = \begin{pmatrix} \ln(\pi_{z,t}), yw_t, ynm_t, v_t, pw_t, \ln(1 - \pi_{z,t}), pc_t, l_t, \ln\left(\frac{\pi_{L,t}}{1 - \pi_{L,t}}\right), \\ \ln(\pi_{L,t}), lp_t \end{pmatrix}'.$$

All those entries in matrices β for which no numerical standard error is provided, the coefficient estimates are subject to constraints. A discussion of the results follows below jointly with the remaining competing approaches.

3.3.2 Partial System Analysis Using the Empirical Rank Restriction

It has been discussed above that the appropriate formulation of the model would have to be in second differences due to evidence of a second unit root in certain variables. Doing so, the appropriate rank is three. Unfortunately, after taking second differences not all coefficients possess a straightforward interpretation as parameters of the underlying structural model. Therefore, we focus on the relationship which is least affected by this problem. This relationship is the long-run investment equilibrium. In fact, using second differences of p_t , pc_t , and lp_t does not alter the relation between coefficients of the empirical and parameters of the economic model. To conclude, we impose zero restrictions according to rows one, two, and three of β'_Y and β'_X as of (40). Moreover, in order to gain efficiency with respect to estimating the long-run relationships we drop those variables from the list of potential cointegrating variables for which the theoretical support ceases. In the first relationship (first row, related to $q_t = w_t - v_t$) the coefficient on v_t is forced to be one in order to model a long-run relationship for nominal wages.

$$\beta'_Y = \begin{bmatrix} 1 & 0 & -5.65 & -98 & 0 \\ & & (1.38) & (14) & \\ -.54 & 1 & -1 & 0 & 0 \\ (.14) & & & & \\ -.79 & 0 & 0 & 0 & 1 \\ (.07) & & & & \end{bmatrix}$$

and

$$\beta'_X = \begin{bmatrix} 0 & 1 & 0 & -49 & 5.87 & -.61 & 0 & 0 & 0 \\ & & & (12) & (2.14) & (.16) & & & \\ .01 & 0 & .17 & 0 & 0 & 0 & 0 & 0 & .0004 \\ (.05) & & (.06) & & & & & & (.001) \\ 0 & -.79 & 0 & .47 & 0 & 0 & .06 & 5.87 & .005 \\ & (-) & & (.67) & & & (.02) & (6.33) & (.001) \end{bmatrix}$$

As before, we defer the interpretation of coefficients to the discussion below.

3.3.3 Partial System Analysis for Investment Only

Following the arguments raised in the previous paragraph we now formulate a partial system that is only made up of those variables relevant for the long-run investment relationship. This is in principal equivalent to the other approaches while conditioning on those variables now dropped. This partial system can be described by (39) with $\Upsilon_t = (q_t, i_t, y_t)'$, $X_t = (\ln(\pi_{z,t}), \ln(1 - \pi_{z,t}))'$. As before the optimal values for l and h have been found with lag order selection criteria and studying the residual properties. Finally, the following long-run relationship resulted:

$$\beta' = \begin{bmatrix} -.57 & 1 & -1 & .38 & .70 & 0 \\ (.08) & & & (.19) & (.21) & \end{bmatrix} \quad (41)$$

This model is the only one for which the restrictions imposed on the coefficients could not be rejected. Further discussion of the results are provided in the next section.

3.4 Evaluation of the Estimation Results

We now present a number of statistics that should help assess the quality of the various model variants estimated. Of course, having conducted cointegration tests and having found that the full system with cointegrating rank five is at odds with the data, already provides some information. In addition, the following aspects will be investigated. First, all models are specified and evaluated for the sample 1983q1 to 1999q4. That leaves eight pre-sample values and allows to perform out-of sample forecasts for the years 2000 and 2001.

The model selection process will be documented which includes reports of model selection criteria, LR tests on the restrictions imposed on the cointegration matrix

Table 3: Lag Order Selection

No.	$l-1$	$h-1$	SC	HQ	AIC
$rank(\beta) = 5, \Upsilon_t = (\Delta w_t, i_t, y_t, \Delta p_t, l_t^{CH})', X_t =$					
1	2	2	-37.524	-42.076	-45.064
2	2	1	nc	nc	nc
3	1	2	-37.606	-41.666	-44.330
4	1	1	-38.255	-41.526	-43.673
$rank(\beta) = 3, \Upsilon_t = (q_t, i_t, y_t, \Delta p_t, l_t^{CH})', X_t = X_{1,t}$					
5	2	2	-36.745	-40.962	-43.730
6	2	1	-36.657	-40.086	-42.336
7	1	2	-35.857	-39.582	-42.026
8	1	1	-37.229	-40.166	-42.093
$rank(\beta) = 1, \Upsilon_t = (\Delta q_t, i_t, y_t)', \text{ no } X_t$					
9	2	3	-13.487	-15.792	-17.306
10	2	2	-13.835	-15.786	-17.067
11	2	1	-14.342	-15.938	-16.986
12	1	3	-13.517	-15.645	-17.042
13	1	2	-13.997	-15.770	-16.934
14	1	1	-14.482	-15.901	-16.832

The model: $\Delta \Upsilon_t = \alpha^* \beta' (\Upsilon_{t-1}' : X_{t-1}' : t-1)' + \sum_{i=1}^{l-1} \Gamma_i^* \Delta \Upsilon_{t-i} + \sum_{j=0}^{h-1} \Phi_j^* \Delta X_{t-j} + \mathcal{D}^* D_t + U_t^*$.

The symbols SC, HQ, and AIC stand for the Bayesian Schwartz, the Hennon-Quinn and the Akaike information criteria respectively. The table entry nc indicates that the reduced rank regression procedure did not converge.

Bold face indicates the minimum of the respective column.

β and tests for the residual properties. With respect to testing restrictions on the cointegrating space the procedure is as follows. Imposing the respective rank all prospective cointegration relations are tested one by one. That means the theoretical restrictions on the coefficients of the cointegrating matrix will be enforced. In most of the cases these are zero restrictions as can be conceived by checking the estimated matrices above. Each of those restrictions on single cointegration vectors is reflected in single rows of these matrices. Finally, all theoretically reasonable restrictions are imposed jointly.

The model selection criteria are calculated with the theoretical restrictions on β being imposed.

For each of the models SC selects the most restricted version while HQ and AIC

chose the more generously parametrised. Since the parsimonious specifications result in model versions with undesirable residual properties, we follow HQ and AIC in the first two cases. The picture is less clear for the third case. Here, three different possibilities emerge. Again, the suggestion by SC has to be rejected on grounds of unfavourable residual properties. It is worth noting though, that model 9 nests models 10 and 11, and model 10 nests model 11. That enables us to perform F -tests on those regressors which have to be deleted when moving to the more restricted model. It turns out that neither a reduction from 9 to 10 ($F(18, 76) = 1.33$, p -value .19) nor from 9 to 11 ($F(36, 80) = 1.33$, p -value .15) can be rejected at conventional levels. The same holds true for an F -test on 10 versus 11 ($F(18, 93) = 1.26$, p -value .24). Model 11 however, results in residuals of the investment and the factor price ratio equation for which the hypothesis of normality has to be rejected at the ten and five percent levels of significance respectively. We therefore continue with version 10.

Having specified the models, we can check whether or not the restrictions on the cointegrating vectors are empirically acceptable. For simplicity, we refer to the cointegrating vectors in question in terms of their economic motivation. We test in the order given by the rows in each cointegrating matrix from top to bottom. Together with a test on all restrictions in all vectors this leads to six, four and two tests for the three models respectively.

Obviously, imposing the theoretical restrictions on the cointegrating vectors does not receive much support from the data. However, as can be seen from a comparison between the test results for the long-run investment relation this conclusion crucially depends on whether or not one conditions on the variables that theoretically do not matter for the relation under consideration. Thus, it appears worthwhile to investigate what particular restrictions cause the test to reject the null hypothesis. Moreover, since the theoretical restrictions for the long-run investment relation cannot be rejected when conditioning on some of the variables, the same procedure could be applied to the remaining relations which found support from the cointegration tests. This however, is beyond the scope of the paper and will be left for

Table 4: Likelihood Ratio Tests of Restrictions on the Cointegrating Vectors

vector	χ^2 statistic	d. f.*	p -value
<i>rank</i> (β) = 5			
wage	28.48	8	.00
investment	20.83	8	.01
GDP	16.95	5	.00
prices	30.93	9	.00
Swiss Labour	nc	-	-
ALL	293.7	39	.00
<i>rank</i> (β) = 3			
wage	17.06	6	.01
investment	19.61	7	.01
Swiss Labour	19.72	6	.00
ALL	84.03	19	.00
<i>rank</i> (β) = 1			
investment	.53	2	.77

* Degrees of freedom of the χ^2 distribution.

For definitions of the models please refer to Table 3.

further research.

Even though the restrictions on the β matrices are rejected, the residual properties appear satisfying in each of the model. To check this consider Table 5.

Overall, the tests do not indicate any significant problem except, maybe in the equation for l_t^{CH} in the subsystem with three cointegration relations. The null hypothesis of normality of the residuals can be rejected at the 6 percent marginal level of significance. A visual impression of all the residuals is provided in Figures 2 to 4 in the Appendix which include estimated residual autocorrelation functions.

3.5 Forecast Evaluation

The ultimate goal of the econometric model is to obtain the best possible forecasts. All three models can be used to forecast the level of investment and the GDP. When considering only the first two models, the demand for Swiss labour can also be considered. We therefore focus on the forecasting performance of investment and GDP income. In addition to these three models we also look at the forecasting

Table 5: Residual Properties

Test	Dependent variable	Model								
		$rk(\beta) = 5$			$rk(\beta) = 3$			$rk(\beta) = 1$		
		stat.	d.f.	prob.	stat.	d.f.	prob.	stat.	d.f.	prob.
Portman- teau AR(12)	w_t	15.83			-			-		
	q_t	-			14.43			5.44		
	i_t	14.02			9.82			9.80		
	y_t	10.47			7.19			7.23		
	p_t	10.85			-			-		
	Δp_t	-			22.56			-		
	l_t^{CH}	8.13			11.67			-		
	ALL	296.26			300.24			99.36		
Normality (χ^2)	w_t	.09	2	.96	-			-		
	q_t	-			.40	2	.82	.43	2	.81
	i_t	.89	2	.64	3.93	2	.14	2.37	2	.31
	y_t	1.82	2	.40	.60	2	.74	3.74	2	.15
	p_t	2.32	2	.31	-			-		
	Δp_t	-			.22	2	.89	-		
	l_t^{CH}	1.20	2	.55	5.57	2	.06	-		
	ALL	4.61	10	.92	12.78	10	.23	5.03	6	.54

The model definitions are given in Table 3.

Portmanteau AR(12) tests for autocorrelation within the residuals of up to order 12. Normality checks whether or not the null hypothesis of normality of the residuals can be rejected. The abbreviations stat., d.f., prob. are short for value of the test statistic, degree of freedom, and probability respectively.

performance of the KOF structural model. In contrast to the modelling strategy used so far, we will use the specification of the KOF model derived with all information up to 2002. This should generally work in favour of the purely structural approach.

The forecast period extends over the eight quarters following the last one of 1999. Table 6 below reports the root mean squared (RMSE), the mean absolute percentage (MAPE), and the mean forecast (MFE) errors.

Judging on grounds of the forecast performances, the decision as to what model is the best depends on what variable one focusses. The full model with cointegration rank five does best with respect to forecasting aggregate income. However, the advantage over the model which had been restricted to yield $rk(\beta) = 3$ is almost negligible. Considering that the most important difference between these two model and the smaller subsystem for investment mainly lays in the fact that the latter

Table 6: Forecasting Performance

Dependent variable	criterion	Model			KOF 2002
		$rk(\beta) = 5$	$rk(\beta) = 3$	$rk(\beta) = 1$	
i_t	RMSE×100	10.265	7.905	6.972	11.766
	MAPE×100	89.786	67.796	54.486	95.517
	MFE×100	-8.159	-6.366	-3.890	-8.399
y_t	RMSE×100	.253	.264	.565	3.114
	MAPE×100	1.728	1.582	4.050	20.270
	MFE×100	.019	.172	-.368	-2.269

The model definitions are given in Table 3. KOF 2002 stands for the structural model of the KOF (Institute for Business Cycle Research) used for the autumn 2002 economic report.

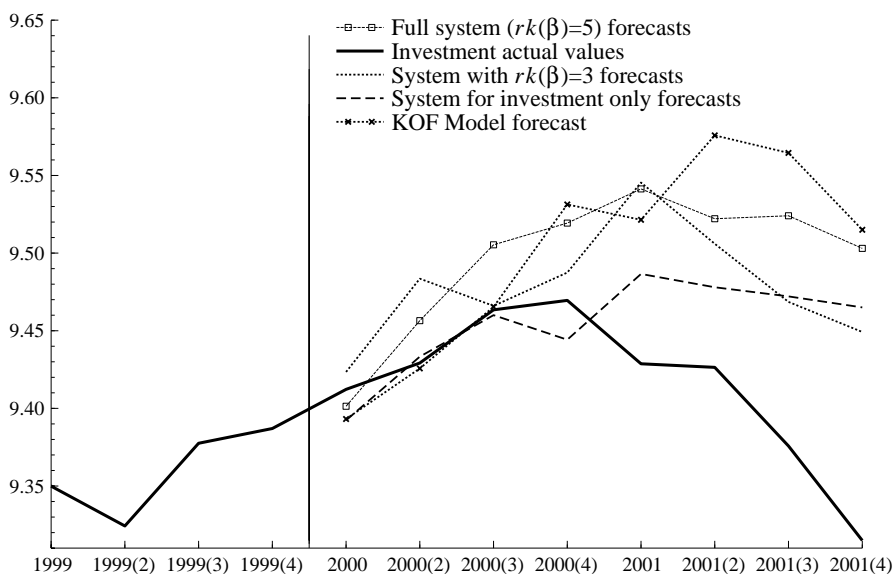
Bold face signifies the minimum of the absolute values in the respective column.

comprises fewer exogenous variables, the better performance of the larger model can certainly be attributed to exactly that reason.

Looking at the performances paying particular attention to investment leaves no doubt though that the smallest model definition does best. This is true on all accounts. Moreover, given the extraordinary economic development between 1999 and 2001 where the economy went from a huge boom into a recession like situation, the ability to mimic the moves in investment appears impressive. Figures 5 to 7 in the Appendix underline this notion. Comparing the forecasting power of all three models (See Figure 1.) the visual impression is that the differences in forecasting GDP are less pronounced than for any other variable. It should also be noticed that the model with the single cointegration relation for investment is very parsimoniously specified but still outperforms the other two in that respect.

Finally, the structural model does worst on all accounts. Interestingly though, it comes very close to the model where all restrictions, theoretical and the inherited ad-hoc ones, have been imposed in the multivariate time series estimation approach (model with $rk(\beta) = 5$). This can be explained by the thereby imposed similarity between these models. It underlines again the necessity to carefully check whether or not the assumed long-run relationships can be justified empirically.

Figure 1: Forecasting Investment - Performances of All Models



3.6 Recovering the Structural Parameters

The tests for restrictions on the cointegration relations revealed that the theoretical model is partially at odds with the recent data. At the same time an important part of it could be shown to be in line with the observations, though. This is the long-run relationship for the economy wide investment in machinery and equipment. Therefore, when it comes to identifying the structural parameters of the model it seems natural to look at those parts of the model that have found support empirically. Doing so restricts the number of parameters that can be recovered, though. On the other hand, it appears not reasonable to report parameter of a structural model that is not supported by the data.

In the following we use the estimates of (41) for identification. Remember that the long-run relationship has been found to be:

$$i_t = \underset{(.08)}{.57} q_t + y_t - \underset{(.19)}{.38} \ln(\pi_{z,t}) - \underset{(.21)}{.70} \ln(1 - \pi_{z,t}) + 0t + ec_{2,t}$$

with $ec_{2,t}$ being a stationary process. All coefficients are significant and the restrictions imposed on the trend and on the income variable have been accepted.

Table 7: Comparison of Estimates

parameter	Model reference					relevance
	$rk(\beta) = 5$ 1983-2001	$rk(\beta) = 3$ 1983-2001	i_t only 1983-2001	KOF 2002 1980-2001	Stalder '95 1970-1994	
ρ	1 (.04)	.54 (.14)	.57 (.08)	.65	.7	labour share
κ	2.64 (-)	-.16 (-)	.32 (-)	.062	.042	mismatch on goods market
θ	0 (0)	.007 (-)	0 (0)	.0017	.007	labour aug- menting techni- cal progress
φ_1	1.04 (-)	1.06 (-)	2.20 (-)	1	3.82 (.45)	

Moreover, although not reported here, these restrictions are not rejected even for smaller sample sizes and the point estimates are pretty stable over time too. The corresponding theoretical relationship is of the structure

$$i_t = \rho(w_t - v_t) + y_t - \kappa(1 - \varphi_1 \lambda_1) \ln \pi_{z,t} - \varphi_1 \kappa \lambda_1 \ln(1 - \pi_{z,t}) - \rho \theta t + ec_{2,t}$$

Using the same restriction as in the KOF model, $\lambda_1 = 1$, this provides us with the necessary tools to identify the parameters ρ , θ , κ , and φ_1 .

We compare the estimates for these coefficients to the parameters used in the KOF macro model in the following table and add those parameter values that can be obtained by a similar procedure from the other two models considered in the previous analysis.

The parameter values obviously differ quite substantially in some cases. Among them the value of .65 for ρ appears to be the most robust against various estimation procedures. Thus, even the very crude OLS estimate on (17) which is used to derive it in the structural approach seems to deliver reasonable values. It is worthwhile remembering that taking simply the wage share of the GDP might be a poor representative of the labour share in the economy since it neglects all labour income of the self-employed for example.

With respect to κ the difference is more pronounced. It measures the miss-

match on the goods markets. Following Stalder (1995) in an equilibrium on the markets ($\pi_{z,t} = .5$) the capacity utilisation can be computed as $(\pi_{z,t})^\kappa$ which would be equivalent to approximately 80 percent with the new estimates ($\kappa = .32$) and 96 percent for $\kappa = .064$ as in the current version of the KOF model. Stalder (1995) notes that a value of 97 percent which he found seemed to be rather high in an equilibrium situation. Therefore the recent result might well yield a reasonable value. Moreover, during the sample period the reported value of $\pi_{z,t}$ has never had a mean value of .5 when estimating the sample average recursively. That means on average, the equilibrium value has never been observed for a significant period of time. What is therefore more important for the new estimate is the implication for the dispersion of the capacity constrained firms. The larger κ the more firms produce at the given price above or below their capacity. In the limit $\kappa = 0$ means that all firms face a demand such that they produce at their normal capacity.

The next parameter of interest is θ . According to (1) it measures the labour augmenting technical process in the economy. Therefore a value of zero implies a puzzling picture of the Swiss economy to say the least. In comparison to the alternative estimates, we note that Stalder (1995) already has pointed to the fact that $\theta = .004$ has been estimated with large standard error from which we conjecture that the hypothesis $\theta = 0$ could hardly be rejected.

Finally, φ_1 measures the extent to which the investment rate changes due to changes in the ratio between expected demand and production capacity. Its sign should be positive, which actually is the case.

At this stage we refrain from attempting to recover the remaining parameters. In most of the cases they are linear combinations of the short-run coefficients for which we do not yet have results regarding their stability over time. The latter requirement would be desirable however for qualifying them as structural model parameters. On the other hand, all those parameters defining the price, income, and wage relationships will not be identified because the implicit time series models have not found support.

4 Conclusions

We looked at an elaborate structural economic model and turned it into a multivariate time series representation by linearising the underlying partly non-linear relationships. This could be done without having to render the steady state non-identified as is often the case when economic models are linearised *around* the steady state.

The multivariate time series representation being more general than the pure structural form enabled us to test whether or not the economically plausible relationships are in line with the observed data. It turned out that this is not the case in all respects. In particular, the hypothesised price and wage setting behaviour as it has been suggested by the assumption of monopolistic competition did not find support. This finding however hinges to some extent on the fact that the derived multivariate time series representation cannot map the empirically $I(2)$ variables into the $I(0)$ space. This will therefore be the subject of future research.

With respect to the labour market, the empirical results were not yet promising. However, given the fact that in the case of long-run investment modelling the economically and empirically reasonable relationship could only be recovered from a partial model, the same might be true for the labour market. It is therefore desirable to obtain a more parsimoniously specified system which is free of potentially slack regressors which introduce statistical noise and may distort the statistical inference.

Finally, the most robust part of the model appeared to be the investment equation. Built on a vintage capital production function the optimal investment decisions can be derived theoretically and a long-run equilibrium between investment, economy wide income, and the factor price ratio results. Taking into account that production capacity and actual output of firms may differ offers the scope for introducing the outcome of data generated by business surveys. Although not being strictly speaking part of the long-run solution they could be shown to play an important role for explaining the path of investment. Despite the observed and unexplained productivity puzzle, we therefore conclude that the vintage capital production function

fairly well approximates the behaviour of economic agents in Switzerland.

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A Appendix: Definitions of Coefficient Matrices

A.1 Matrix Definitions

The complete model can be written as in (38) and in (39). With

$$\Upsilon_t = (w_t, i_t, y_t, p_t, l_t^{CH})'$$

and

$$X_t = \begin{pmatrix} \ln(\pi_{z,t}), yw_t, ynm_t, ie_t, v_t, pw_t, \ln(1 - \pi_{z,t}), pc_t, l_t, \ln\left(\frac{\pi_{L,t}}{1 - \pi_{L,t}}\right), \\ \ln(\pi_{L,t}), lp_t \end{pmatrix}'$$

the corresponding coefficient matrices are defined as follows:

$$\Gamma_0 = \begin{bmatrix} 1 & 0 & -\gamma_{0,13} & -\gamma_{0,14} & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -\gamma_{0,31} & 0 & 1 & 0 & 0 \\ -\gamma_{0,41} & 0 & 0 & 1 & 0 \\ -\gamma_{0,51} & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Gamma_0 = \begin{bmatrix} 1 & 0 & -\gamma_{0,13} & -\gamma_{0,14} & 0 \\ 0 & 1 & -\gamma_{0,23} & 0 & 0 \\ -\gamma_{0,31} & 0 & 1 & 0 & 0 \\ -\gamma_{0,41} & 0 & 0 & 1 & 0 \\ -\gamma_{0,51} & 0 & 0 & 0 & 1 \end{bmatrix}$$

Letting $z^* = \frac{1}{1-\gamma_{0,13}\gamma_{0,31}}$ and $z^{**} = \frac{z^*}{1-\gamma_{0,14}\gamma_{0,41}z^*}$ we find

$$\Gamma_0^{-1} = \begin{bmatrix} z^{**} & 0 & \gamma_{0,13}z^{**} & \gamma_{0,14}z^{**} & 0 \\ 0 & 1 & 0 & 0 & 0 \\ \gamma_{0,31}z^{**} & 0 & (1 - \gamma_{0,14}\gamma_{0,41})z^{**} & \gamma_{0,14}\gamma_{0,31}z^{**} & 0 \\ \gamma_{0,41}z^{**} & 0 & \gamma_{0,13}\gamma_{0,41}z^{**} & \frac{z^{**}}{z^*} & 0 \\ \gamma_{0,51}z^{**} & 0 & \gamma_{0,13}\gamma_{0,51}z^{**} & \gamma_{0,51}\gamma_{0,14}z^{**} & 1 \end{bmatrix},$$

and,

$$\Phi_0 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & \phi_{0,18} & \phi_{0,19} & \phi_{0,110} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \phi_{0,32} & \phi_{0,33} & \phi_{0,34} & \phi_{0,35} & \phi_{0,36} & \phi_{0,37} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \phi_{0,45} & 0 & \phi_{0,47} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \phi_{0,58} & 0 & 0 & \phi_{0,511} & \phi_{0,512} \end{bmatrix}$$

$$\Phi_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \phi_{1,21} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

as well as,

$$\beta' = \beta'_Y : \beta'_X \tag{42}$$

with the $(\varrho \times n_Y)$ matrix

$$\beta'_Y = \begin{bmatrix} 1 & 0 & \beta_{13} & \beta_{14} & 0 \\ \beta_{21} & 1 & \beta_{23} & 0 & 0 \\ \beta_{31} & 0 & 1 & 0 & 0 \\ \beta_{41} & 0 & 0 & 1 & 0 \\ \beta_{51} & 0 & 0 & 0 & 1 \end{bmatrix}$$

and the $((n_X + 1) \times \varrho)$ matrix

$$\beta'_X = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_{114} & \beta_{115} & 0 & 0 & 0 \\ \beta_{26} & 0 & 0 & 0 & \beta_{210} & 0 & \beta_{212} & 0 & 0 & 0 & 0 & 0 & 0 & \beta_{218} \\ 0 & \beta_{37} & \beta_{38} & \beta_{39} & \beta_{310} & \beta_{311} & \beta_{312} & 0 & 0 & 0 & 0 & 0 & 0 & \beta_{318} \\ 0 & 0 & 0 & 0 & \beta_{410} & 0 & \beta_{412} & 0 & 0 & 0 & 0 & 0 & 0 & \beta_{418} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_{513} & 0 & 0 & 0 & \beta_{516} & \beta_{517} & \beta_{518} \end{bmatrix}$$

For the adjustment to deviations from the long-run relations and for \mathcal{D} we find

$$\alpha = \begin{bmatrix} \alpha_{11} & 0 & 0 & 0 & 0 \\ 0 & \alpha_{22} & 0 & 0 & 0 \\ 0 & 0 & \alpha_{33} & 0 & 0 \\ 0 & 0 & 0 & \alpha_{44} & 0 \\ 0 & 0 & 0 & 0 & \alpha_{55} \end{bmatrix}$$

$$\mathcal{D} = \begin{bmatrix} d_{11} \\ d_{21} \\ d_{31} \\ d_{41} \\ d_{51} \end{bmatrix} = \begin{bmatrix} k_0 \\ \lambda_2 \left(\lambda_1 (\bar{\delta} (1 - \varphi_1)) + (1 - \lambda_1) \bar{E} - b_0 + \rho (\dot{q} - \theta) \right) \\ \alpha_{33} \beta_{311} (\tilde{m} - \theta \rho) + \phi_{0,36} \theta \rho \\ \alpha_{43} (\theta \rho - \tilde{m}) \\ -\frac{c_0 + c_2}{\nu} \end{bmatrix}.$$

For the identification of coefficients and the impact of changes in the model structure the theoretical definition of the matrix \mathcal{D} is of special interest. We call these intercepts the structural means. Some of the structural coefficients of the model turn up in these means only. Therefore, identification of the innovations \mathcal{E} is a pre-condition of recovering all those parameters when estimating the whole model in reduced form. On the other hand, writing the model in reduced form reveals that a number of shifts in parameter values are equivalent to shifts in the mean of the respective endogenous variable. This information can be used to simulate variations in the parameter value of e.g. the scrapping elasticity δ .

The easiest way to identify all parameters is to first estimate the β coefficients, to fix them in turn and proceed by imposing the coincident restrictions on the matrix Γ_0 by assuming an diagonal variance/covariance matrix of the residuals. For that appropriate standard software tools exist.

A.2 Further Output

Estimation result for the sample 1983-1999 which has been used for the forecasting comparisons.

- The model with $rk(\beta) = 5$, $\Upsilon_t = (w_t, i_t, y_t, p_t, l_t^{CH})'$, and

$$X_t = \begin{pmatrix} \ln(\pi_{z,t}), yw_t, ynm_t, v_t, pw_t, \ln(1 - \pi_{z,t}), pc_t, l_t, \ln\left(\frac{\pi_{L,t}}{1 - \pi_{L,t}}\right), \\ \ln(\pi_{L,t}), lp_t \end{pmatrix}'$$

$$\beta'_\Upsilon = \begin{bmatrix} 1 & 0 & -.33 & -1.42 & 0 \\ & (.02) & (.01) & & \\ -.97 & 1 & -1 & 0 & 0 \\ (.02) & & & & \\ -1.52 & 0 & 1 & 0 & 0 \\ (.06) & & & & \\ -.97 & 0 & 0 & 1 & 0 \\ (-) & & & & \\ -6.51 & 0 & 0 & 0 & 1 \\ (3.32) & & & & \end{bmatrix}$$

and

$$\beta'_X = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & .04 & .04 & 0 & 0 & 0 & 0 \\ & & & & & & (.01) & (-) & (0) & & & \\ -.06 & 0 & 0 & .96 & 0 & -1.52 & 0 & 0 & 0 & 0 & 0 & .001 \\ (.05) & & & (-) & & (.03) & & & & & & (.001) \\ 0 & .16 & .32 & .09 & -.02 & -1.01 & 0 & 0 & 0 & 0 & 0 & -.009 \\ & (.06) & (.05) & (.11) & (.007) & (.23) & & & & & & (.001) \\ 0 & 0 & 0 & -.03 & 0 & .20 & 0 & 0 & 0 & 0 & 0 & .001 \\ & & & (-) & & (.05) & & & & & & (-) \\ 0 & 0 & 0 & 0 & 0 & 0 & .77 & 0 & 0 & .13 & -.34 & .05 \\ & & & & & & (1.66) & & & (.15) & (8.53) & (.008) \end{bmatrix}$$

- The model with $rk(\beta) = 3$, $\Upsilon_t = (q_t, i_t, y_t, \Delta p_t, l_t^{CH})'$, and

$$X_t = \left(\ln(\pi_{z,t}), \ln(1 - \pi_{z,t}), \Delta pc_t, l_t, \ln\left(\frac{\pi_{L,t}}{1 - \pi_{L,t}}\right), \ln(\pi_{L,t}), \Delta lp_t \right)'$$

$$\beta'_\Upsilon = \begin{bmatrix} 1 & 0 & .12 & 2.82 & 0 \\ & & (.20) & (.54) & \\ -.57 & 1 & -1 & 0 & 0 \\ (.14) & & & & \\ -.26 & 0 & 0 & 0 & 1 \\ (.05) & & & & \end{bmatrix}$$

and

$$\beta'_X = \begin{bmatrix} 0 & 1 & 0 & -25 & -3.76 & -.06 & 0 & 0 & 0 \\ & & & (6.75) & (.35) & (.03) & & & \\ .12 & 0 & .10 & 0 & 0 & 0 & 0 & 0 & -.001 \\ (.05) & & (.05) & & & & & & (.001) \\ 0 & -.26 & 0 & 5.63 & 0 & 0 & .01 & -1.91 & 0 \\ & (-) & & (1.56) & & & (.01) & (1.61) & (0) \end{bmatrix}$$

- Model for investment only with $\Upsilon_t = (q_t, i_t, y_t)'$, and $X_t = (\ln(\pi_{z,t}), \ln(1 - \pi_{z,t}))'$

$$\beta' = \begin{bmatrix} -.91 & 1 & -1 & 1.12 & 1.56 & 0 \\ (.19) & & & (.35) & (.38) & \end{bmatrix}$$

A.3 Tables and Figures

Figure 2: Residual Properties of the Full System ($rk(\beta) = 5$)

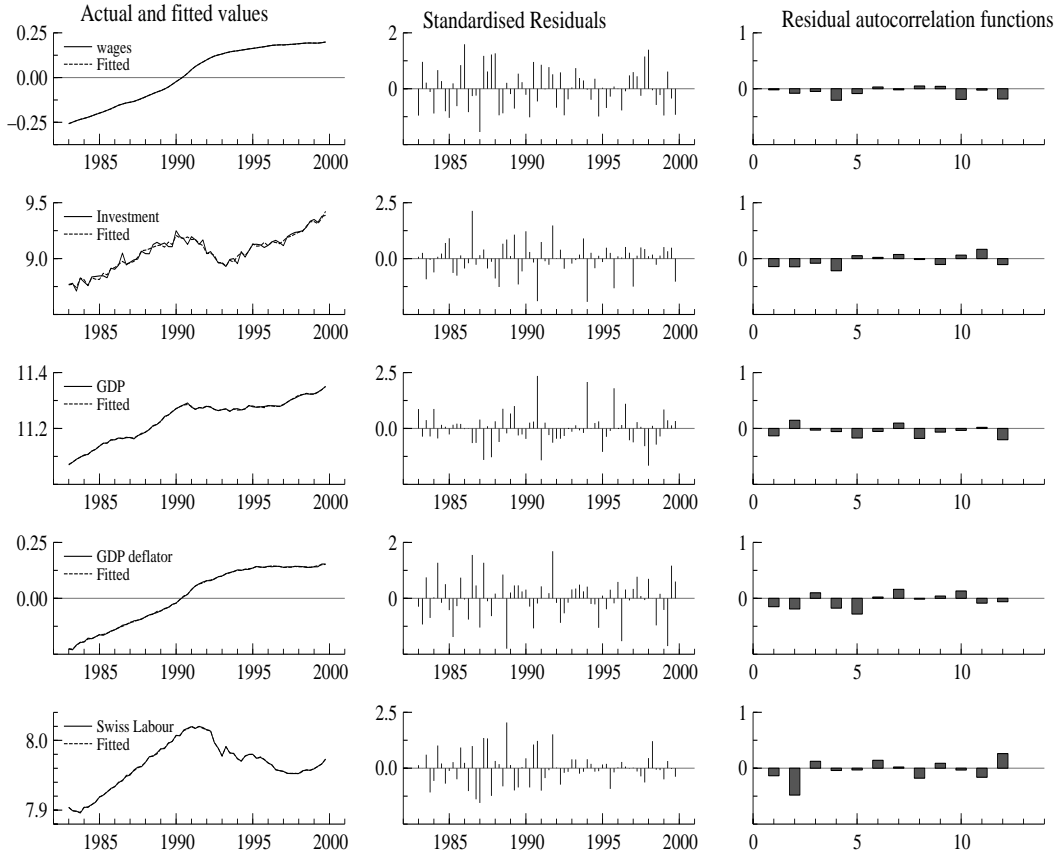


Figure 3: Residual Properties of the Modified System ($rk(\beta) = 3$)

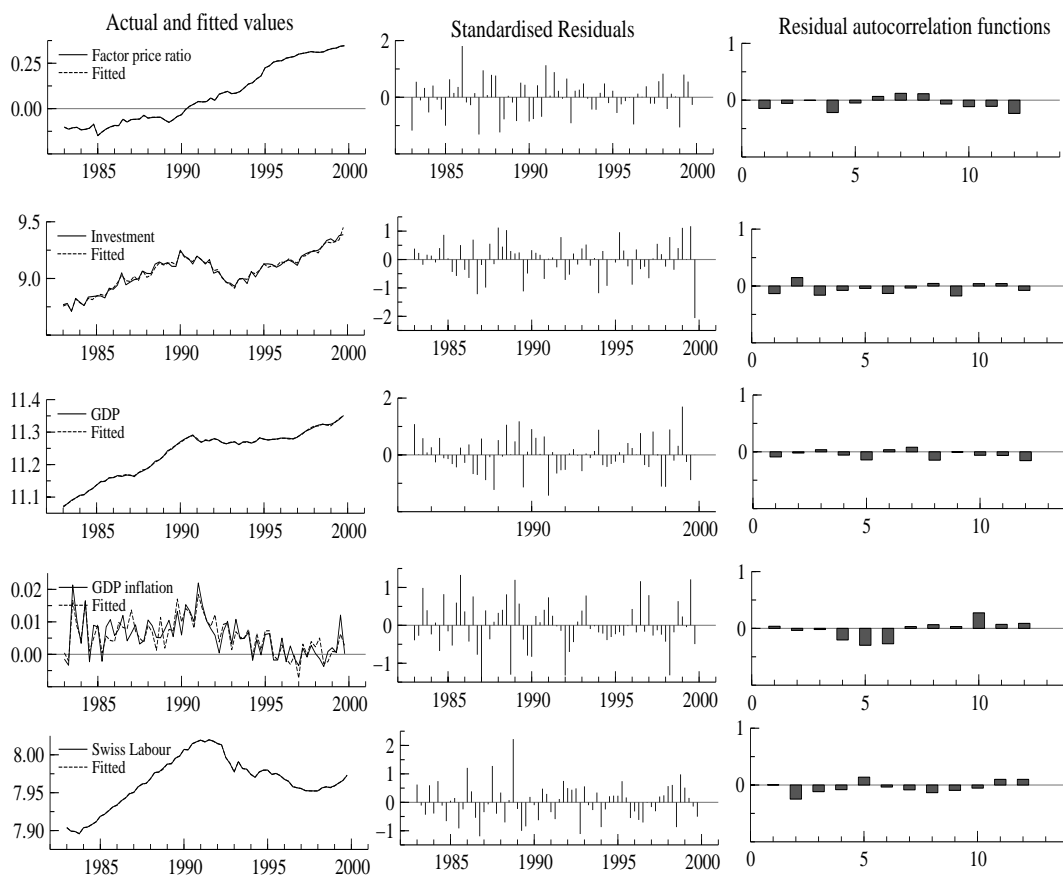


Figure 4: Residual Properties of Subsystem for Investment

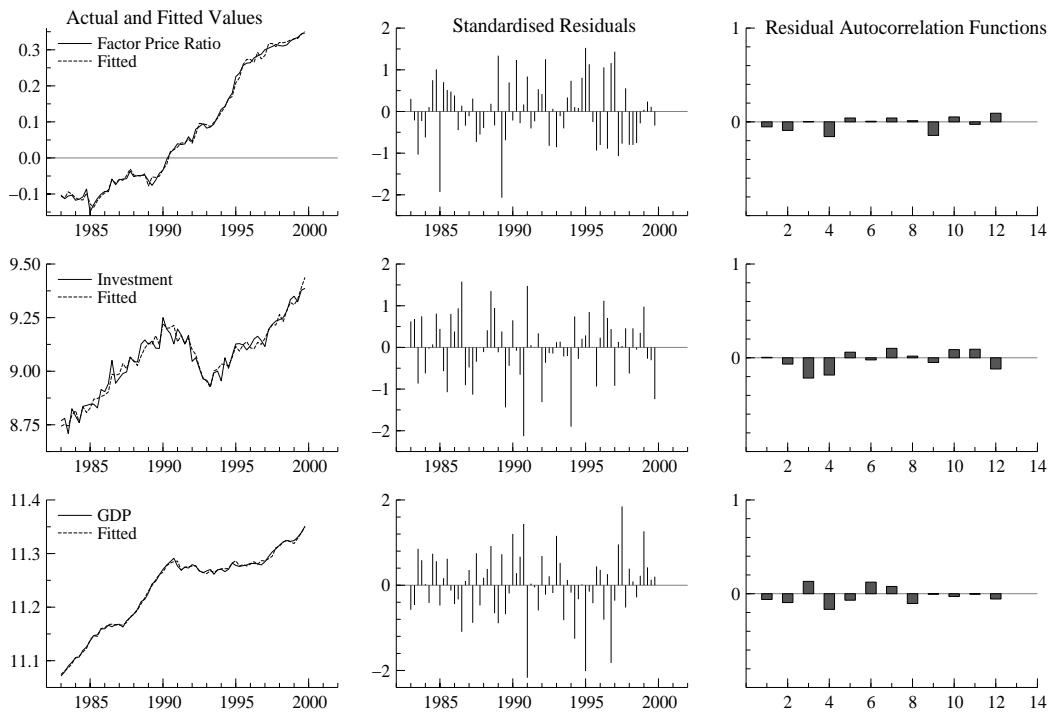


Figure 5: Forecasting Performance of the Full System ($rk(\beta) = 5$)

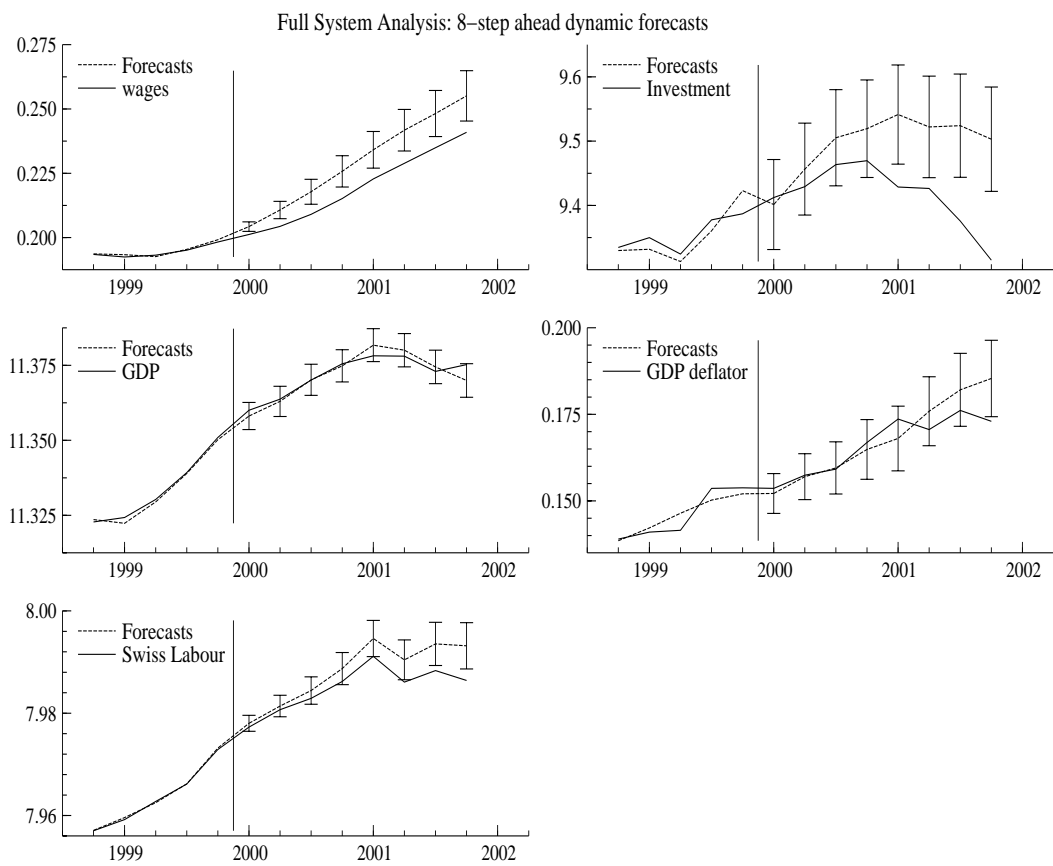


Figure 6: Forecasting Performance of the Modified System ($rk(\beta) = 3$)

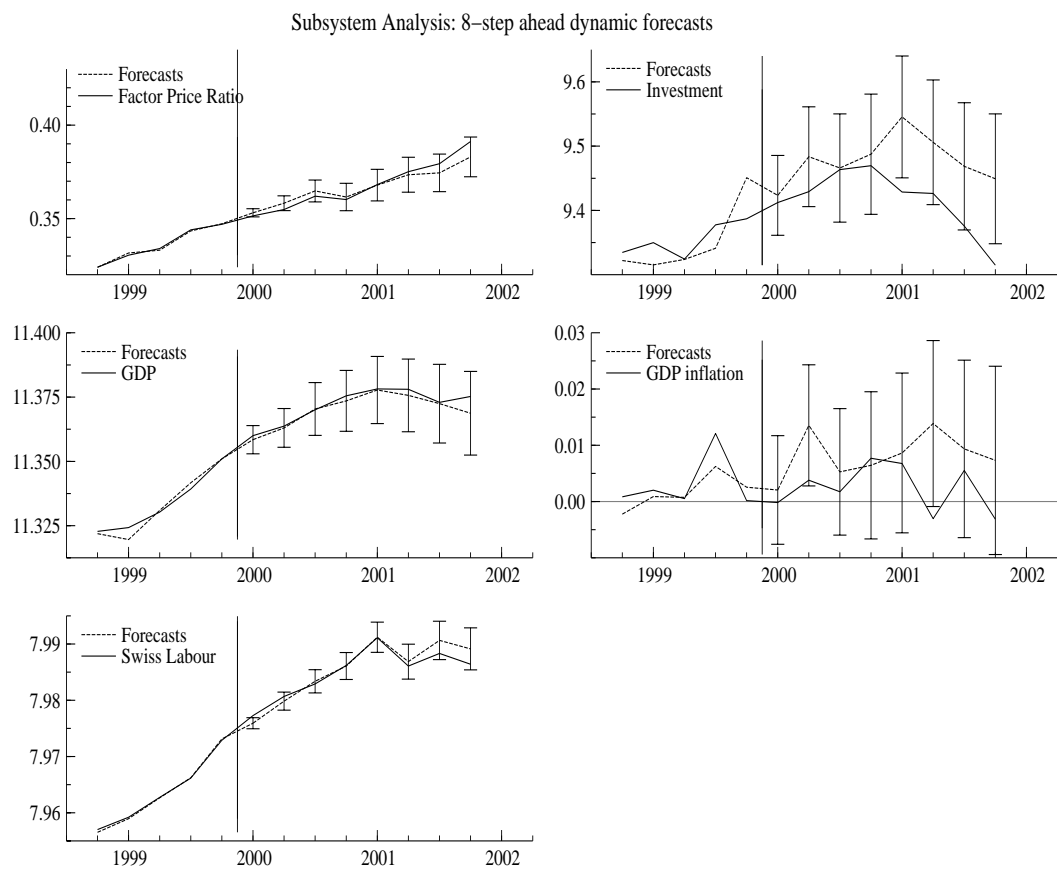


Figure 7: Forecasting Performance of Subsystem for Investment

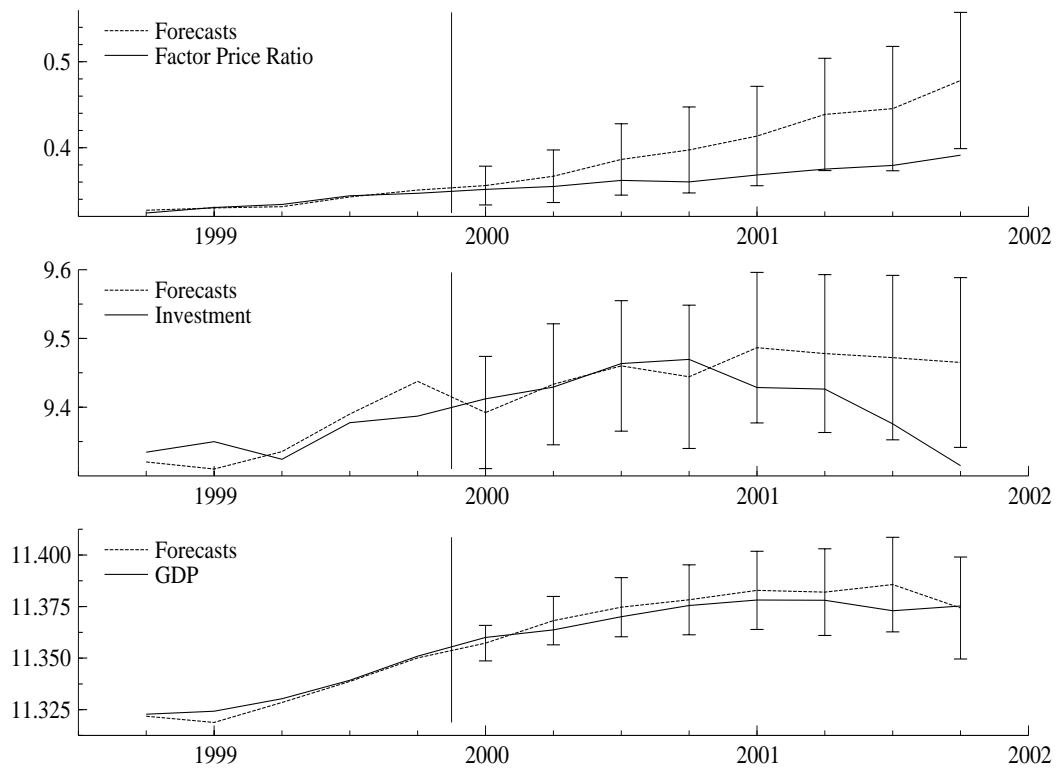


Table 8: Data Description

Data description and variable symbols

symbol	description
w_t	nominal wages, Federal Statistical Office compilation
$icnstr_t$	real gross fixed capital formation in construction
i_t	real investment in machinery and equipment
ie_t	real investment in machinery, equipment, and build- ings ($ie_t = i_t + icnstr_t$)
y_t	real GDP
p_t	deflator of GDP (base year 1995)
l_t^{CH}	labour force (full time equivalent), Swiss residents
$\pi_{z,t}$	share of firms reporting capacity utilisation above or on the limit
v_t	price of i_t at 1995 prices
pc_t	consumer price index (1995 prices)
pw_t	price index of the rest of the world (1995 prices)
$ltotv_t$	total labour force (full time equivalent)
$\pi_{L,t}$	proportion of firms reporting too few or just enough (bottleneck) labour supply
lp_t	potential labour force (total number of permanent residents in the age range 20 to 64 years)

Table 9: Cointegration Tests with Alternative Sets of Exogenous Variables

No.	$H_0 : rank_0 = r$	Pesaran et al. (2000)			
		r	test statistic	5% c.v.	10% c.v.
1	Full System (2) $\Upsilon_t = (\Delta w_t, i_t, y_t, \Delta p_t, l_t^{CH})'$, $X_t = X_{2,t}$	0	166.53**	120.0	114.7
		1	112.50**	90.02	85.59
		2	67.87**	63.54	59.39
		3	33.18	40.37	37.07
		4	13.62	20.47	18.19
2	Full System (2a) $\Upsilon_t = (q_t, i_t, y_t, \Delta p_t, l_t^{CH})'$, $X_t = X_{1,t}$	0	160.71**	120.0	114.7
		1	109.42**	90.02	85.59
		2	63.38**	63.54	59.39
		3	28.72	40.37	37.07
		4	6.41	20.47	18.19
3	Investment (2) $\Upsilon_t = (\Delta q_t, i_t, y_t)'$, no X_t	0	40.90*	42.34	39.34
		1	14.46	25.77	23.08
		2	6.80	12.39	10.55
4	Wages (2) $\Upsilon_t = (y_t, \Delta w_t, \Delta p_t)'$, $X_t = l_t$	0	48.35*	49.36	46.00
		1	18.11	30.77	27.96
		2	4.08	15.44	13.31

* and ** indicate significance at the 10 and 5 percent levels respectively.

+ see Table T.4 * of Pesaran et al. (2000).