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## Supporter influence on club governance in a sports league: A utility maximization model

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# Working Paper Series

## Supporter Influence on Club Governance in a Sports League: A “Utility Maximization” Model

Paul Madden  
Terry Robinson

Manchester Business School Working Paper No 598

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## **Abstract**

Motivated by increasing supporter involvement in club governance in English football, the paper presents a simple model of a sports league in which club objectives are utility functions defined over profits, win percentages and fan (=supporter) welfare, formalising a seminal suggestion of Sloane (1971). Consequences of increased utility weight on fan welfare (capturing increased supporter involvement in governance) are unambiguously higher attendances, with more nuanced affects on ticket prices and player expenditure. The attendance affect is consistent with some limited, anecdotal data currently available from clubs with existing supporter boardroom representation. Normatively, positive profits for club owners indicate social sub-optimality.

## **Keywords**

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# **SUPPORTER INFLUENCE ON CLUB GOVERNANCE IN A SPORTS LEAGUE; A “UTILITY MAXIMIZATION” MODEL**

**Paul Madden** \*      **and Terry Robinson** \*\*

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# 1. INTRODUCTION

Most existing theoretical analysis of the professional sports league industry has focused on leagues where the objective of individual clubs<sup>1</sup> is either profit maximization, or maximization of win percentage (equivalent to maximizing relative team quality) subject to a budget constraint<sup>2</sup>. The conventional view is that profit maximization may approximate reasonably well behaviour in the major North American sports leagues where clubs seem largely to have been run on the lines of businesses in other industries, but win maximization may be more prevalent in European football (soccer), where wealthy club owners have seemingly been prepared to forego profit to produce champion teams. In a recent paper Madden (2010) introduced a third alternative, namely fan welfare maximization, whereby (again subject to a budget constraint, and motivated by the observed members' clubs in European football) the fans or supporters of a club, who have a particular allegiance to the club and are the consumers of its products, also have direct control over club policies. In reality however one might reasonably expect that club objectives are more complicated, multi-dimensional objects, "utility functions" to use the term suggested in one of the early and now much-cited papers in the literature (Sloane (1971)). The current paper provides a first analysis of sports leagues with clubs whose objectives are utility functions defined over profits, win percentage and fan welfare<sup>3</sup>, thus combining (with varying weights) the three objectives studied separately elsewhere.

A particular motivation stems from the emerging and growing role for supporters' trusts in UK football<sup>4</sup>, where these associations of fans of a club are gradually acquiring increased representation on club boards, and so increased influence on the club decision processes. Whilst financial issues at various clubs in the English Premiership are currently creating significant pressure for supporter involvement in governance of clubs from this top tier of English soccer<sup>5</sup>, to date an actual supporter trust presence on club boards is restricted to lower league levels; section 2 of the paper will elaborate further details. The general tendency triggers questions, both

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<sup>1</sup> We use the following terms solely with their sporting meanings; club, team, match, player. However games refer to their usual meaning in economic models.

<sup>2</sup> Coverage of the literature can be found in the major surveys of Fort and Quirk (1995) and Szymanski (2003), the textbooks by Fort (2006) and Sandy et al. (2004), and in the materials for the increasing numbers of courses on Sports Economics being taught around the world. The book by Kesenne (2007) provides a full account of existing results on both profit and win maximization.

<sup>3</sup> Although some authors have used the term utility maximization to refer to the univariate win maximization objective, the only paper that has previously addressed a multi-variate objective is Rashcer (1997), where profits and win percentage were the two arguments. We add fan welfare, and argue later that this provides a close approximation to Sloane's (1971) original suggestion.

<sup>4</sup> Michie et al. (2006) provide information on governance of UK soccer clubs, and the role of supporters' trusts in particular.

<sup>5</sup> The reaction against the North American owners of Manchester United (the Glazer family), and the heavy debts and higher ticket prices that the club has acquired since the Glazer takeover has led (at the time of writing, April 2010) to the formation of a group of wealthy, influential supporters, who are attempting to launch a takeover, working with the Manchester United supporters' trust (MUST) whose membership has grown rapidly to over 100,000 in the last months. Negative reaction to the debts following a North American based takeover, and attempts to launch a supporter led alternative have also been seen at Liverpool. For the first time in history, a Premier League club (Portsmouth) has gone into receivership recently, immediately preceded by the launch and rapid growth of a supporters' trust for the club. Supporter ownership of football clubs has in fact become an item on the political agenda in the run-up to the 2010 UK General Election.

positive and normative, about the affects that the increasing power of supporters in club governance will have, questions which we aim to answer by investigating the consequences of increased weight on fan welfare in clubs' utility functions.

We take a simplified, basic framework that has been used in the previous literature for the study of profit and win maximization and in Madden (2010) for fan welfare maximization, and add the utility maximization analysis. With European football in mind and its relatively fierce inter-league competition for players, this established framework assumes a perfectly elastic supply of playing talent to the league, which consists of 2 clubs that play each other twice over the season, once at home and once away in stadiums of given large capacity. Clubs earn revenue from attendance by their fans at their home game and incur the costs of hiring playing talent, making decisions on ticket prices for entry to their home game and on their expenditure on playing talent. As suggested above, the focus on the positive economics side will be on the affects of changing the weights on the arguments of the utility function on ticket prices, player expenditures and the resulting match attendances. Normatively, the question to be answered will be whether these utility weight changes are a good thing in terms of welfare (aggregate surplus).

We start in section 2 with an account of the emergence of supporters' trusts in English football and their current influence on club governance and decisions. Section 3 sets out the model of a league with utility maximizing clubs, section 4 analyses individual club decisions in this framework and section 5 looks at league outcomes (Nash equilibria) and welfare. Section 6 concludes.

## **2. SUPPORTERS' TRUSTS IN ENGLISH FOOTBALL**

We first consider the mechanism by which supporters interests will be heard in English football. Like other companies operating in the UK, football clubs are owned by their shareholders and administered by a board of directors headed by a chairman (Michie et al. (2006)). Unlike other companies however, the chairman of a football club is often the major shareholder of the club or the outright owner. This means that the chairman has a major influence on the general strategy of the club. For fan welfare to become a priority, therefore, a club would require an altruistic chairman, or other influential board member, for whom this was a main concern, or supporter representation on the Board.

Historically supporters have had a significant influence on the policies of many football clubs in continental Europe (Szymanski (2008)). For example, at FC Barcelona the current membership of over 150,000 pay 150 euro each year and are represented by a randomly-selected group who meet with the board and vote on major decisions. These representatives have a major say on significant issues such as sponsorship, finances and sporting affairs, while the president of the club is elected in four-yearly polls. All German football clubs are required to have at least 51% member ownership<sup>6</sup>. These observed ownership structures of many clubs in continental Europe are predominantly a result of the fact that these clubs were set up as sporting clubs

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<sup>6</sup> German clubs are structured as *Verein*, which are associations with an independent legal status. (Franck (2009))



incorporating other activities apart from football (Szymanski (2008)). The associations governing these clubs have maintained strict guidelines on public share issues and have restrictions on ownership (Brown and Walsh (2000, p.89)).

Ironically, many English football clubs were initially founded as member's associations. Taylor (1992) provides an example of this in Leicester Fosse F.C (later to become Leicester City F.C.) which was formed in 1884 by members who elected a committee to run the club's affairs. Within a short time there were sixty-five members overseeing three teams. This reflects the originally simple democratic structure of most English clubs; one member, one vote to elect a committee to administer club affairs. However early pressure to create a winning team attracted wealthy backers, including banks, who loaned the club money. Financial problems ensued leading to clubs' applications for private limited company status. The original committee then became the board of directors.<sup>7</sup>

Once clubs became public limited companies, supporters had very little success in their attempts to acquire some effective representation on their board and were forced to be satisfied by forming supporters groups. This situation changed in 1985 with the formation of the Football Supporters Association (FSA)<sup>8</sup>, an independent body set up to represent the concerns of all supporters. The activities of the FSA, combined with the experiences of groups of supporters of lower-league clubs who had provided fund-raising and organisational assistance to clubs during severe financial difficulties, brought the active role that supporters can play to public prominence. This prompted the Labour Government to set up the Football Task Force in 1997 which was given a remit to produce three reports (the first two dealt with disability and racism) on how to improve the modern game. The Task Force's third report, *Investing in the Community* (Football Task Force (1999)), contained the suggestion that the government should assist supporters who want to acquire a meaningful stake in their club and led to the formation of Supporters Direct<sup>9</sup>.

Supporters Direct is an externally-funded non-governmental mutual organisation. Its stated aim is: "Through establishing and developing supporters' trusts, we aim to bring about responsible, democratic representation at spectator sports clubs, and so help promote the highest standards of governance, accountability and embed those clubs deeper into their communities."([www.supporters-direct.org](http://www.supporters-direct.org). Visited 15 Oct 2009). It would be uncontroversial to say that Supporters Direct has been very successful. Supporters' trusts have now been established at over 150 League and Non-League clubs with over 120,000 members. [www.supporters-direct.org](http://www.supporters-direct.org). Visited 15 Oct 2009)<sup>10</sup>.

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<sup>7</sup> Szymanski (2008) provides a historical survey of the development of current observed governance structures in football.

<sup>8</sup> In 2002 the Football Supporters Association merged with National Federation of Football Supporters' Clubs to become the Football Supporter's Federation

<sup>9</sup> The impetus behind supporter involvement in governance goes beyond England and the UK. Indeed, in the wake of UEFA's 10-year strategy statement (UEFA (2004)), the EU commissioned an independent report (Arnaut (2005)) to tackle specific issues faced by European soccer, and a broad recommendation for greater supporter involvement in governance emerged; "The authors of the Review believe that properly structured supporter involvement will help to contribute to improved governance and financial stability (as well as other benefits)", Arnaut (2005, p.81).

<sup>10</sup> This does not include the recent growth at MUST, referred to earlier.

There is by now a group of clubs where supporters have some direct influence on club governance operating alongside a larger population of clubs with a traditional governance structure. Firstly, we require a more precise definition of “supporter involvement”. We have obtained data from Supporters Direct on three methods by which supporters trusts have involvement in their clubs. Firstly, some clubs are fully owned by supporters trusts. Secondly, there are clubs with a supporters trust director on the board and finally, clubs for which the supporters trust shareholding is greater than ten per cent. Many of the clubs that fall into these categories are non-league clubs (below tier 4 in the hierarchy), and none are from the top tier Premier League. We concentrate on clubs from the Football League divisions 1 and 2 (tiers 3 and 4) to avoid the problem of comparing professional and non-professional clubs. Table 1 shows details of fourteen Football League clubs that comply with our above definition of supporter involvement, and have operated in these two divisions. As can be seen from the Table, some clubs meet all three criteria, and some clubs only two. For example, Brentford Football Club is owned by the supporters trust, and so obviously has a director on the club board. The supporters trust also has a shareholding of sixty per cent in this club. However, the supporters trust at Sheffield Wednesday FC, whilst having a member on the club board, only has a shareholding of ten per cent. Hence supporters will have a different level of involvement in these clubs.

Table 1 English Football League Clubs with Supporter Involvement

<b>Club Name</b>	<b>Owned by Supporters Trust</b>	<b>Supporters Trust Director on Club’s Board</b>	<b>Supporters Trust Shareholding</b>
Brentford	Yes 2006	Yes	60%
Bournemouth	No	Yes since 1997	No
Bristol City	No	Yes since beginning of the 2006/7 season (non-voting)	No
Bury	No	Yes since July 2002	Yes 11% since July 2002
Carlisle United	No	Yes since April 2003	Yes 25% since April 2003
Chesterfield	No	Yes since 2001	No
Exeter	Yes since 2007	Yes since 2001	Majority since 2007
Lincoln City	No	Yes since 2002	Yes 26% since 2002
Luton Town	No	Yes since 2003	No

Northampton Town	No	Yes since 1992	No
Oldham Athletic	No	Yes since 2004	No
Swansea City	No	Yes since 2002	Yes 17% since 2002
Wycombe Wanderers	No	Yes since 2005	No
Sheffield Wednesday	No	Yes since 2005	Yes 10% since 2002

*Source: Supporters Direct*

We can reasonably make the assumption that the weight on fan welfare is greater for the clubs in Table 1 than for other league clubs. Ideally, we would proceed by comparing the key variables of ticket prices, attendances and wage expenditure for these two groups of clubs to see if interesting patterns emerge. We are faced with two stumbling blocks, however. Firstly, there is the paucity of the data available. Historical data on ticket prices was unobtainable although we do provide information on current ticket prices below. Moreover, many of the clubs in our sample did not file accounts during the period of interest meaning that data on wage expenditure was unavailable. The second stumbling block when making comparisons is the issue of promotion and relegation between divisions. Wage expenditure, attendances and ticket prices will vary depending on which division of a league a club is operating in. All these variables will tend to be higher the further up the league hierarchy the club climbs.

In an attempt to obviate these problems, we focus on two clubs whose league position was relatively stable around the period in which they made the transition to supporter involvement. This will enable us to assess the extent to which the characteristics of these clubs changed over time once supporter welfare became more important<sup>11</sup>. Tables 2 and 3 provide data on league position, real wage expenditure, and attendances for Brentford F. C. and Lincoln City F.C for four seasons before and after the transition to effective supporter involvement. Brentford F.C. supporters trust is called Bees United and was formed in April 2001 with the objective of giving ordinary football supporters greater involvement in the future of the club. The trust acquired the majority shareholding (60%) in Brentford F. C. on January 20th 2006. Bees United has loaned over £1.4m to Brentford F. C. and now has four director

<sup>11</sup> A static comparison of average attendances is worthless since this takes no account of a club's fan base. Despite only having a sample of 14 SI teams we estimate a regression of a simple model of attendances to evaluate whether there is any systematic difference between SI and non SI teams:  $ATT_{i,t} = -1573.7 + 0.002 \text{MKT SIZE}_{i,t} + 13143.7 \text{WINPERCENT}_{i,t} + 22.37SI$ . The variables are defined as follows. *ATT* is the seasonal average attendance of each club in Football Leagues 1 and 2 in seasons 2006-7, 2007-8 and 2008-9. *MKT SIZE* is (log) population (millions) in the inner/outer zones of a club's catchment area. *WINPERCENT*, is the percentage of games won by team *i* in season *t*, where a draw is counted as half a win. *SI* = 1 if a club is an SI club and 0 otherwise. The coefficients on *MKT SIZE* and *WINPERCENT* are positive, as one would expect. The estimated coefficient on the dummy variable, *SI*, shows that, on average for our sample, that SI clubs attracted 22 more supporters than non-SI clubs. The number is small, but statistically significant. Obviously, this model also ignores the issue of divisional change over this period.

places on the eight-seat football club board. The Lincoln City supporters' trust or Imps Trust was formed in 2001 and holds nearly 25% of the club's shares in addition to two trust board members on the club board. The trust has over 1,400 members, and between 2001 and 2008 donated over a quarter of a million pounds to the club. Both of these clubs remained in the bottom two tiers of the English league both before and after SI. Brentford operated mainly in the third tier (League 1) despite suffering a relegation in 2005-6, and Lincoln City mainly in the fourth tier (League 2) with one season in the third tier in 1998-99. As can be seen from pre- and post-SI mean values real wage expenditure fell and average seasonal attendances rose after the weight on supporter welfare increased.

Table 2 Brentford F.C: League Position, Real Wage Expenditure, and Attendances

Season	League position	Real wage expenditure (£000s)	Average attendance
2001-2	49	1,291	6,713
2002-3	62	918	5,759
2003-4	63	746	5,541
2004-5	50	1,057	5,477
2005-6	49	1,175	6,774
2006-7	70	1,010	5,599
2007-8	84	919	4,465
2008-9	71	1,000	5,707
2009-10	57	NA	6,141
PRE-SI MEAN		1,174	6,095
POST-SI MEAN		1,087	6,457

Table 3 Lincoln City F.C: League Position, Real Wage Expenditure, and Attendances

Season	League position	Real wage expenditure (£000s)	Average attendance
1997-8	73	881	3,968
1998-9	69	1,051	4,654
1999-00	84	NA	3,405
2000-01	87	933	3,194
2001-02	91	758	3,223
2002-03	76	734	3,923
2003-04	77	814	4,910
2004-05	76	840	4,483
2005-06	77	861	4,739
PRE-SI MEAN		907	3,581
POST-SI MEAN		809	3,981

Notes: League Position is calculated as position in entire league hierarchy (top of premier league = 1), Real wages were deflated using RPI: 1987 = 100

Finally, we compare the current ticket prices of the two groups of clubs. If fan welfare is an important argument in the club's utility function then a priori we may expect admission prices for clubs with supporters involvement (SI Clubs) to be lower than conventionally owned clubs (Non SI clubs). Table 4 displays average ticket prices for the two groups for the season 2009-10<sup>12</sup>; the clubs used for the comparison can be found in the Appendix. It can be seen that the mean ticket price is lower for SI clubs than for non-SI clubs for that season, although this was not statistically significant even at the 10% level.

Table 4: Comparison of Admission Prices between SI teams and non SI teams in Leagues One and Two (2009-10 prices)

	Average ticket price (£)
SI Clubs	19.18
Non SI clubs	19.74

*Source: Clubs' official websites*

### 3. LEAGUES WITH UTILITY MAXIMIZING CLUBS: A MODEL

We now present a simple economic model of a league with utility maximizing clubs; the framework is similar to those found in previous literature, e.g. Madden (2010).

Two clubs and their teams comprise the league; each team plays the other twice over the season, once at home and once away. Club  $i=1,2$  has a stadium where its team plays its home match; the stadium has a given capacity, sufficiently large so as to be never binding on match attendance<sup>13</sup>. Clubs hire players and  $Q_i \geq 0$  denotes the expenditure on playing talent by team  $i$ . Following the established treatment for a European football league, talent is in perfectly elastic supply at a wage normalised to 1, so  $Q_i$  is also the quantity of playing talent (and alternatively a measure of the quality of team  $i$ ). Player expenditures are the only club costs, abstracting (e.g.) from stadium costs, as is usual in the literature.

Club  $i$  sets the ticket price  $p_i$  for admission to its home match and receives all gate revenue from this match; no price discrimination is possible. There are disjoint sets of fans of each club  $i$ , who feel an (exogenously given) affinity to club  $i$ . In a terminology used in the literature, our fans are core rather than floating fans – a fan of club  $i$  could not switch allegiance to the rival club<sup>14</sup>. To simplify, it is assumed that

<sup>12</sup> It may be instructive to make an inter-season comparison of ticket prices, but historical data for individual English clubs is notoriously difficult to obtain, unlike current prices which are publicly available.

<sup>13</sup> In the context of Football Leagues 1 and 2, this assumption is entirely appropriate, where attendances are almost always significantly below capacity. The average attendance as a percentage of capacity for League One (3<sup>rd</sup> tier) was 53% for the 2006-7 season and 54% for the 2007-8 season. The same figures for League Two (4<sup>th</sup> tier) are 39% and 38% respectively (Deloitte and Touche 2008, 2009).

<sup>14</sup> Partisanship is an often assumed characteristic of football supporters. However there is evidence that certainly not all fans are core in this sense – see Robinson (2009).

fans do not attend away matches<sup>15</sup>. Fans of  $i$  are heterogeneous in their willingness to pay for tickets, denoted  $v(Q_i, Q_j) - x$  where the heterogeneity parameter is  $x \geq 0$  and  $v(Q_i, Q_j)$  is the maximum willingness to pay<sup>16</sup>. It is assumed that  $x$  is uniformly distributed over  $[0,1]$  with density  $\mu_i$ ;  $\mu_i$  is a measure of the fanbase of club  $i$ , assumed to exceed stadium capacity. It is also assumed that  $v(Q_i, Q_j)$  is  $C^2$  and strictly increasing in both arguments, reflecting the desire of fans to see better quality matches. Since  $v(Q_i, Q_j)^2$  appears in the objective function of many of the subsequent optimization problems we assume that it (and hence  $v(Q_i, Q_j)$  itself) is strictly concave and satisfies the Inada conditions. If  $v(Q_i, Q_j)$  is symmetric, fans are non-partisan and would divide a given amount of talent equally between the 2 teams for their optimal match. In our context, with core fans in mind with their wish to see their team win, an asymmetry leading to more talent going to the home team for a fan's optimal match is appropriate. This fan bias is captured by the assumption that  $v(Q_i, Q_j) > v(Q_j, Q_i)$  if  $Q_i > Q_j$ ; in the extreme limiting case of completely home partisan fans  $v(Q_i, Q_j)$  depends only on  $Q_i$ <sup>17</sup>, with symmetric  $v(Q_i, Q_j)$  and non-partisan fans at the other extreme<sup>18</sup>.

A fan with heterogeneity parameter  $x$  will demand a ticket if  $x \leq v(Q_i, Q_j) - p_i$  so that  $i$ 's (linear in price) ticket demand or match attendance (given large stadium capacity) is  $A_i(Q_i, Q_j, p_i) = \mu_i [v(Q_i, Q_j) - p_i]$  yielding revenues<sup>19</sup>  $p_i A_i(Q_i, Q_j, p_i)$ , and profits  $\Pi_i(Q_i, Q_j, p_i) = p_i A_i(Q_i, Q_j, p_i) - Q_i$ , which is the first club utility function argument.

Once talent has been hired and tickets sold, matches are played and a winner emerges. Ex ante, before the play of matches, the probability that  $i$  is the winner is some  $C^2$  contest success function  $W(Q_i, Q_j)$ , increasing and strictly concave in  $Q_i$  and decreasing in  $Q_j$  with values in  $[0,1]$ , whose exact specification is irrelevant for most of our purposes. Following established usage  $W(Q_i, Q_j)$  is referred to as the win percentage and is the second component of the club utility function.

The final club utility function argument is their fan welfare, defined to be the following aggregate surplus accruing to their fans;

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<sup>15</sup> Given that away fan attendance is always a small fraction of attendance, the assumption is plausible.

<sup>16</sup> Implicitly we are assuming that the full fan utility function is quasi-linear, defined over a numeraire (endowment  $y$  and large) and the match ticket. Full utility is then  $y$  without the ticket and  $y - p_i + v(Q_i, Q_j) - x$  with the ticket.

<sup>17</sup> One could strengthen the fan "affinity" by assuming that  $v(Q_i, Q_j)$  is strictly increasing only on some large enough cone in the  $(Q_i, Q_j)$  plane, but this adds nothing to the analysis and is not pursued.

<sup>18</sup> The fan utility microfoundation essentially generalises that of Falconieri et al. (2004) who assume the specific Cobb-Douglas form  $v(Q_i, Q_j) = Q_i^\alpha Q_j^\beta$  with non-partisan fans for their TV audience ( $\alpha = \beta$ ).

<sup>19</sup> Home gate revenues are the only revenue source in the model – merchandising and broadcasting provide relatively little extra in Football Leagues 1 and 2.

$$F_i(Q_i, Q_j, p_i) = \int_0^{v(Q_i, Q_j) - p_i} \mu_i [v(Q_i, Q_j) - p_i - x] dx$$

$$= \mu_i [v(Q_i, Q_j) - p_i]^2 / 2.$$

Notice that fan welfare is a monotone transformation of attendance.

A club's utility function is assumed to be a weighted average of profits, win percentage and fan welfare, where  $\lambda_{iX}, X = \Pi, W, F$  are the non-negative weights<sup>20</sup>:

$$U_i(Q_i, Q_j, p_i) = \lambda_{i\Pi} \Pi_i(Q_i, Q_j, p_i) + \lambda_{iW} W(Q_i, Q_j) + \lambda_{iF} F_i(Q_i, Q_j, p_i) \quad (3.1)$$

The clubs make independent decisions about ticket prices ( $p_i$ ) and player expenditure ( $Q_i$ ) to maximize utility subject to a budget constraint, which is taken for simplicity to be the non-negative profit requirement. So the decision problem for a utility maximizing club is:

$$\max_{p_i, Q_i} U_i(Q_i, Q_j, p_i) \text{ subject to } \Pi_i(Q_i, Q_j, p_i) \geq 0 \quad (\text{UMAX})$$

#### 4. ANALYSIS OF CLUB DECISIONS

We consider the ticket price ( $p_i$ ) and player expenditure ( $Q_i$ ) best responses of a club with fanbase  $\mu_i$  to the  $Q_j$  chosen by the other team in the league, and how these (and the resulting match attendance) vary with the club's utility function weights. The other team's price ( $p_j$ ) does not affect any of the answers ( $p_j$  has no affect on  $i$ 's payoff or constraints – each club sells tickets only to its own fans). Best responses are denoted  $p_{iW}(Q_j)$ ,  $Q_{iW}(Q_j)$ , and the resulting attendances are  $A_{iW}(Q_j)$ .

Consider first the nature of the budget constraint. The zero profit contour is  $p_i^2 - p_i v(Q_i, Q_j) + Q_i / \mu_i = 0$ , with roots of the quadratic;

$$p_{iL}(Q_i, Q_j) = \frac{1}{2} v(Q_i, Q_j) - \frac{1}{2} \sqrt{v(Q_i, Q_j)^2 - 4Q_i / \mu_i}$$

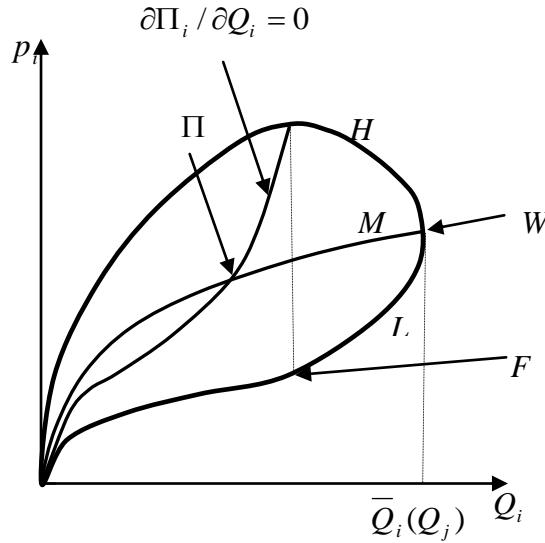
$$p_{iH}(Q_i, Q_j) = \frac{1}{2} v(Q_i, Q_j) + \frac{1}{2} \sqrt{v(Q_i, Q_j)^2 - 4Q_i / \mu_i}$$

The roots are real if  $Q_i \in [0, \bar{Q}_i(Q_j)]$ , where  $\bar{Q}_i(Q_j)$  is the unique positive solution in  $Q_i$  (given the strict concavity and Inada properties of  $v$ ) to  $v(Q_i, Q_j)^2 = 4Q_i / \mu_i$ . We refer to  $p_{iL}(Q_i, Q_j)$  as *the low break-even price*, and  $p_{iH}(Q_i, Q_j)$  as *the high break-*

<sup>20</sup> We thus effectively pick up 3 of the 4 utility function arguments suggested originally by Sloane (1971, p. 136). In his terminology these are playing success (equates to win percentage), profits, and attendance (equates to fan welfare). The fourth argument suggested by Sloane is the health of the league; “utility is derived from the health of the league because it is better to win a keenly fought competition than to win easily”, (Sloane (1971, p.136)). We suggest that this might anyway be captured by our first 2 arguments. The previous literature has attempted to capture the Sloane idea more crudely than here. Kesenne (2007) equates utility maximization to win maximization whilst Rascher (1997) uses a weighted average of just profits and win percentage.

even price, with dependence on  $Q_i$  as shown by L,H respectively in Figure 4.1. Notice that the roots sum to  $v(Q_i, Q_j)$ , and that  $p_{iH}(Q_i, Q_j)$  is strictly concave under our assumptions. Between the L,H branches in Figure 4.1, labelled as M, we have the monopoly price  $p_M(Q_i, Q_j) = v(Q_i, Q_j)/2$  which maximizes gate revenue (given  $Q_i, Q_j$ ). The feasible set for (UMAX) is bounded by the bubble Figure 4.1.

To find optimal ticket prices and player expenditures we proceed in 2 stages, solving first for optimal prices given both  $Q_j$  and  $Q_i$ , and then solving for the optimal  $Q_i$ . For univariate or “pure” objectives (with a positive weight on only one utility argument), the optimal prices are obvious – monopoly prices for the pure profit-maximizer, low break-even prices for the pure fan welfare maximizer, with indifference to all prices between high and low break-even levels for the pure win maximizer where price does not affect the objective. Using this, Madden (2010) shows that the resulting optimal player expenditures lead to a point  $\Pi$  in Figure 4.1 under pure profit maximization, to  $W$  with a pure win maximizer, and to  $F$  under pure fan welfare maximization. Precisely, and unambiguously, player expenditure is highest at  $W$  and lowest at  $\Pi$ ; match attendance is highest at  $F$  and lowest at  $\Pi$ ; and ticket price is highest at  $W$ . Or, with obvious notation,  $Q_{iW}(Q_j) > Q_{iF}(Q_j) > Q_{i\Pi}(Q_j)$ ,  $A_{iF}(Q_j) > A_{iW}(Q_j) > A_{i\Pi}(Q_j)$ ,  $p_{iW}(Q_j) > p_{iF}(Q_j)$  and  $p_{iW}(Q_j) > p_{i\Pi}(Q_j)$ .



**Figure 4.1; the bubble-shaped non-negative profit region**

For (UMAX) consider first the simplest (non-pure) case,  $\lambda_{i\Pi} = 0$  but  $\lambda_{iF}, \lambda_{iW} > 0$ . Optimal prices will be low break-even, since  $\lambda_{iF} > 0$  means there is always a benefit to utility from lowering price and thereby increasing attendance and fan welfare, exactly as with the pure fan welfare maximizer, and this has no effect on win percentage. Utility is  $U_i = \frac{1}{2} \lambda_{iF} \mu_i p_{iH}(Q_i, Q_j)^2 + \lambda_{iW} W(Q_i, Q_j)$ ,  $Q_i \in [0, \bar{Q}_i(Q_j)]$ , which has the following feature ensuring that its stationary point is a global maximum;

**Lemma 4.1** For  $Q_i \in [0, \bar{Q}_i(Q_j)]$ , the function  $p_{iH}(Q_i, Q_j)^2$  is strictly increasing in  $Q_i$  if  $v(Q_i, Q_j)v_i(Q_i, Q_j) > 2/\mu_i$  and strictly concave in  $Q_i$  if  $v(Q_i, Q_j)v_i(Q_i, Q_j) \leq 2/\mu_i$ .



Proof See appendix.

The optimal player expenditure is then defined by the stationary point condition;

$$\frac{1}{2} \lambda_{iF} \mu_i \frac{\partial p_{iH}^2}{\partial Q_i}(Q_i, Q_j) + \lambda_{iW} \frac{\partial W}{\partial Q_i}(Q_i, Q_j) = 0 \quad (4.1)$$

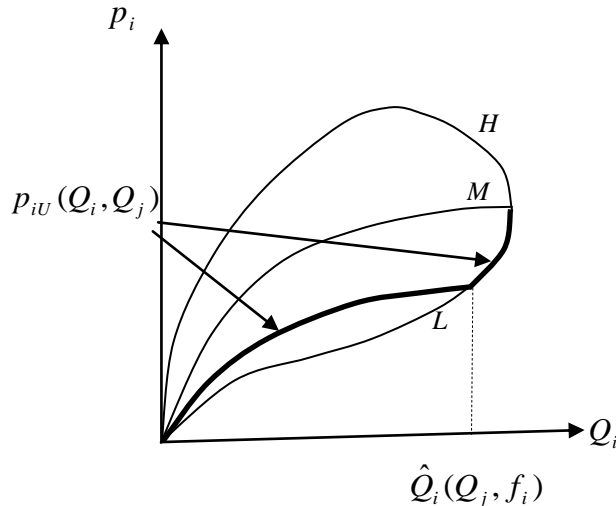
It is easily shown that the optimal  $Q_i$  increases with  $\lambda_{iW} / \lambda_{iF}$ , ranging from  $Q_{iF}(Q_j)$  when  $\lambda_{iW} / \lambda_{iF} \rightarrow 0$ , up to  $Q_{iW}(Q_j)$  as  $\lambda_{iW} / \lambda_{iF} \rightarrow \infty$ ; in Figure 4.1 the best response locus follows  $L$  from  $F$  to  $W$  as  $\lambda_{iW} / \lambda_{iF}$  increases from 0 to  $\infty$ . Optimal prices are not so obvious when  $\lambda_{iH} > 0$ , and are given by the following Lemma where  $f_i = \lambda_{iF} / \lambda_{iH}$  and  $w_i = \lambda_{iW} / \lambda_{iH}$  (see appendix for proof):

**Lemma 4.2** Assume a given  $Q_i, Q_j$  where  $Q_i \in [0, \bar{Q}_i(Q_j)]$ , and assume  $\lambda_{iH} > 0$ . The utility maximizing ticket price is:

- (a)  $p_{iU}(Q_i, Q_j) = p_{iL}(Q_i, Q_j)$  if  $f_i \geq 1$  or if  $f_i < 1$  and  $Q_i \geq \hat{Q}_i(Q_j, f_i)$  where  $\hat{Q}_i(Q_j, f_i)$  is defined by  $(2 - f_i)^2 Q_i = \mu_i (1 - f_i) v(Q_i, Q_j)^2$ ;
- (b)  $p_{iU}(Q_i, Q_j) = \frac{1-f_i}{2-f_i} v(Q_i, Q_j)$  if  $f_i < 1$  and  $Q_i < \hat{Q}_i(Q_j, f_i)$ .

Proof See appendix.

The content is as follows, illustrated in Figure 4.2. First, if there is no weight on fan welfare ( $f_i = 0$ ), utility maximizing pricing is just monopoly pricing, along  $M$  in Figure 4.1 and 4.2. Second, if  $f_i \in (0,1)$  the pricing rule is as shown in bold in Figure 4.2, with low break-even pricing at high values of  $Q_i$  ( $\geq \hat{Q}_i(Q_j, f_i)$  where  $\hat{Q}_i(Q_j, f_i)$  decreases as  $f_i$  increases), and with prices between the low break-even and monopoly levels at low values of  $Q_i$  ( $< \hat{Q}_i(Q_j, f_i)$ ). Finally the low break-even pricing becomes universal when  $f_i \geq 1$ . Notice that, as one would expect, for given weights the ticket price always increases with home team player expenditure (=home team quality), and, for given home and away team qualities (i.e. for a given match quality), the price always falls as the fan welfare weight increases.



**Figure 4.2; The pricing rule**

For the second stage  $Q_i$  solution with  $\lambda_{\text{II}} > 0$ , substitution of the pricing rule from Lemma 4.2 into utility gives;

$$U_i = \begin{cases} \frac{1}{2} \mu_i v(Q_i, Q_j)^2 \frac{\lambda_{\text{II}}}{2-f_i} - \lambda_{\text{II}} Q_i + \lambda_{iW} W(Q_i, Q_j), & Q_i < \hat{Q}_i(Q_j, f_i), f_i < 1 \\ \frac{1}{2} \mu_i \lambda_{iF} p_{iH}(Q_i, Q_j)^2 + \lambda_{iW} W(Q_i, Q_j), & f_i \geq 1, \text{ or } Q_i \geq \hat{Q}_i(Q_j, w_i), f_i < 1 \end{cases} \quad (4.2)$$

Some preliminary points about this function are the following.

- (i) Under our assumptions both top and bottom branches of (4.2) define differentiable functions of  $Q_i$ . But in fact (after some tedious calculations – details omitted) derivatives of both branches are the same at  $Q_i = \hat{Q}_i(Q_j, f_i)$ , and  $U_i$  is globally differentiable.
- (ii) Using Lemma 4.1, it is easy to show that  $U_i$  is strictly increasing in  $Q_i$  if  $v(Q_i, Q_j)v_i(Q_i, Q_j) > 2/\mu_i$ , which is equivalent to  $Q_i < Q_{\text{II}}(Q_j)$ . Thus the optimal player expenditure for a utility maximizer is always at least as large as that of a pure profit maximizer.
- (iii) Again from Lemma 4.1, it follows that  $U_i$  is strictly concave in  $Q_i$  if  $v(Q_i, Q_j)v_i(Q_i, Q_j) \leq 2/\mu_i$  or  $Q_i \geq Q_{\text{II}}(Q_j)$ . Thus stationary points of  $U_i$  characterise best responses – that of the top (bottom) branch provides the best response if  $Q_i < \hat{Q}_i(Q_j, f_i)$  and  $f_i < 1$  (otherwise), with strictly positive (zero) profits. The stationary point calculation produces:

**Lemma 4.3** Suppose that  $\lambda_{\text{II}} > 0$ . Then:

(a)  $(p_i, Q_i)$  is a best response to  $Q_j$  with strictly positive profits iff  $f_i < 1$ ,

$$p_i = \frac{1-f_i}{2-f_i} v(Q_i, Q_j), Q_i < \hat{Q}_i(Q_j, f_i), \text{ and}$$

$$\mu_i v(Q_i, Q_j)v_i(Q_i, Q_j)/(2-f_i) + w_i \frac{\partial W}{\partial Q_i}(Q_i, Q_j) = 1;$$

(b)  $(p_i, Q_i)$  is a best response to  $Q_j$  with zero profits iff  $p_i = p_{iL}(Q_i, Q_j)$  and,

$$\text{either } f_i < 1, Q_i \geq \hat{Q}_i(Q_j, f_i), \frac{1}{2} f_i \mu_i \frac{\partial p_{iH}^2}{\partial Q_i}(Q_i, Q_j) + w_i \frac{\partial W}{\partial Q_i}(Q_i, Q_j) = 0,$$

$$\text{or, } f_i \geq 1, \frac{1}{2} f_i \mu_i \frac{\partial p_{iH}^2}{\partial Q_i}(Q_i, Q_j) + w_i \frac{\partial W}{\partial Q_i}(Q_i, Q_j) = 0.$$

The question of main interest in this section is now addressed; how do optimal prices, player expenditures and the implied match attendances change as the utility function weights change? Local comparative static answers are provided. Let  $Q_{iU}(\lambda_{\text{II}}, f_i, w_i)$  and  $p_{iU}(\lambda_{\text{II}}, f_i, w_i)$  denote the best responses to (a suppressed)  $Q_j$ , and let  $A_{iU}(\lambda_{\text{II}}, f_i, w_i)$  denote the resulting match attendance.

**Theorem 4.1** Suppose  $\lambda_{\text{II}} > 0$  and suppose profits are strictly positive at the best response. Then;

$$(a) \partial Q_{iU} / \partial \lambda_{\text{II}} < 0, \partial Q_{iU} / \partial f_i > 0, \partial Q_{iU} / \partial w_i > 0;$$

$$(b) \partial A_{iU} / \partial \lambda_{\text{II}} < 0, \partial A_{iU} / \partial f_i > 0, \partial A_{iU} / \partial w_i > 0;$$

$$(c) \partial p_{iU} / \partial w_i > 0.$$

**Proof** See appendix.

**Theorem 4.2** Suppose  $\lambda_{\Pi} > 0$  and suppose profits are zero at the best response. Then;

(a)  $\partial Q_{iU} / \partial \lambda_{\Pi} = 0$ ,  $\partial Q_{iU} / \partial f_i < 0$ ,  $\partial Q_{iU} / \partial w_i > 0$ ;

(b)  $\partial A_{iU} / \partial \lambda_{\Pi} = 0$ ,  $\partial A_{iU} / \partial f_i > 0$ ,  $\partial A_{iU} / \partial w_i < 0$ ;

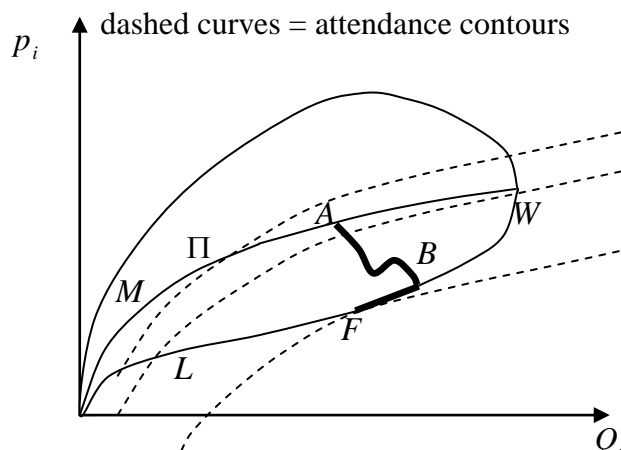
(c)  $\partial p_{iU} / \partial \lambda_{\Pi} = 0$ ,  $\partial p_{iU} / \partial f_i < 0$ ,  $\partial p_{iU} / \partial w_i > 0$ .

**Proof** See appendix.

Figure 4.3 illustrates possible consequences of Theorems 4.1 and 4.2 for the affect of increasing the weight on fan welfare from zero to infinity, with given  $\lambda_{\Pi}, w_i > 0$ .

When  $f_i = 0$ , the best response would produce positive profits, somewhere on  $M$  between the pure profit ( $\Pi$ ) and pure win maximizer ( $W$ ) solutions, such as the  $A$  shown. As  $f_i$  increases player expenditure and attendance increase (as shown, price need not monotonically decrease) until a point on  $L$  is reached such as  $B$ , when profits become zero and low break-even pricing takes over. Further increases in  $f_i$  continue to increase attendance but now with falling player expenditure and prices, along  $BF$ , converging to  $F$  as  $f_i \rightarrow \infty$ .

These theoretical results are consistent with the indication from Section 2 earlier, namely that higher levels of supporter involvement in club governance have produced higher match attendances. The theoretical effects of increases in  $f_i$  on ticket prices and player expenditures are more nuanced and depend on club profitability. But if profits are zero the model clearly predicts that the outcome would be a lowering of both  $p_i$  and  $Q_i$ , consistent again with Section 2's findings, and the available profit data which certainly is more consistent with zero rather than positive club profits<sup>21</sup>.



**Figure 4.3; Optimal price-quality locus**

<sup>21</sup> A sample of 36 clubs operating in Leagues 1 and 2 in the seasons 2001-2 to 2007-8 yielded 194 annual observations for pre-tax profits. The sample mean level of profit was -£0.32 million. Data was collected from the FAME database

## 5. LEAGUE EQUILIBRIA AND AGGREGATE SURPLUS

We now turn to the Nash equilibria of the normal form game where the players are the 2 utility maximizing clubs in the league (called the U-league), strategy sets are  $Q_i \geq 0, i=1,2$  and payoffs are either  $U_i(Q_i, Q_j, p_{iU}(Q_i, Q_j)), i=1,2$  if  $\Pi_i(Q_i, Q_j, p_{iU}(Q_i, Q_j)) \geq 0$  or 0 otherwise. The formulation allows utility function weights to differ between clubs, as well as their fanbases, and has the special cases studied in Madden (2010) of the  $\Pi$ -league (2 pure profit maximizers), the F-league (2 pure fan welfare maximizers) and the W-league (2 pure win maximizers). The previous section provided information on club best responses in the general U-league game, which will be the basis for the U-league Nash equilibrium analysis to follow. There is essentially no loss of generality in assuming that  $\lambda_{\Pi} > 0, i=1,2$  throughout this section<sup>22</sup> and U-league equilibrium team qualities will be denoted  $Q_{iU}, i=1,2$  with the resulting prices and attendances denoted  $p_{iU}$  and  $A_{iU}, i=1,2$ .

We focus on welfare evaluation of the league outcomes, via the following surplus definition. For each club  $i=1,2$  (with  $j \neq i$ ) define the sum of consumer (fan) and producer surplus for that club to be:

$$S_i(Q_i, Q_j, p_i) = \int_0^{v(Q_i, Q_j) - p_i} \mu_i [v(Q_i, Q_j) - p_i - x] dx + p_i \mu_i [v(Q_i, Q_j) - p_i] - Q_i \quad (5.1)$$

The usual sum of these surpluses is our welfare measure. This aggregate surplus is denoted  $S(Q_1, Q_2, p_1, p_2) = S_1(Q_1, Q_2, p_1) + S_2(Q_2, Q_1, p_2)$  where  $p_i = p_{iU}(Q_i, Q_j), i=1,2$ <sup>23</sup>.  $S_{1U}, S_{2U}$  and  $S_U = S_{1U} + S_{2U}$  will denote surplus values in U-league equilibria.

A driving force behind all the results in Section 5 will be the elementary inefficiency of monopoly. The core nature of our fans, who cannot switch allegiance to the rival club, endows clubs with monopoly power in the sale of tickets to their fans. Positive weight on profits in club objectives can lead to some of this elementary inefficiency. Thus, loosely at the moment, positive profits are not a good thing from the aggregate surplus viewpoint.

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<sup>22</sup> Any Nash equilibrium in which, for some  $i, \lambda_{\Pi} = 0, \lambda_{iF} > 0$  can also be generated as a Nash equilibrium with, for the same  $i, \lambda_{iF} > \lambda_{\Pi} > 0$ . And any Nash equilibrium in which, for some  $i, \lambda_{\Pi} = \lambda_{iF} = 0$ , the (pure win maximizing) behaviour of club  $i$  can be approximated with  $\lambda_{\Pi} > 0, \lambda_{iW} > 0, \lambda_{\Pi} \rightarrow 0$ .

<sup>23</sup> Notice that our assumption of a perfectly elastic supply of playing talent means that players gain no extra surplus from playing in our league, and so do not enter the social welfare evaluation. Given the supply assumption, this seems appropriate, but differs from the social welfare specifications analysed in Falconieri et al. (2004) who also have the perfectly elastic supply assumption. Also, with quasi-linear utility for fans (footnote 11), maximization of this aggregate surplus equates in the usual way to Pareto efficiency, legitimising the use of  $S_1(Q_1, Q_2, p_1) + S_2(Q_2, Q_1, p_2)$  as the appropriate welfare criterion.

A first precise (and surprisingly general) result is quite immediate. Suppose there is a U-league equilibrium in which for some club  $\Pi_i > 0$  and  $w_i > 0$ . Then, from Lemma 4.3(a),  $f_i < 1$  and;

$$p_i = \frac{1-f_i}{2-f_i} v(Q_i, Q_j) \quad (5.2)$$

$$\mu_i v(Q_i, Q_j) v_i(Q_i, Q_j) / (2 - f_i) + w_i \frac{\partial W}{\partial Q_i}(Q_i, Q_j) = 1 \quad (5.3)$$

But with  $(Q_i, Q_j)$  unchanged (5.3) can be maintained with equality at a slightly lower  $w_i$  and slightly higher  $f_i$ . Thus  $Q_i$  remains a best response to  $Q_j$  at these adjusted values of  $w_i$  and  $f_i$ , and from (5.2)  $p_i$  goes down. Leaving utility weights unchanged for club  $j$  means  $Q_j$  remains a best response to  $Q_i$  with unchanged  $p_j$ , and  $(Q_i, Q_j)$  is still a Nash equilibrium. The lower  $p_i$  with unchanged  $(Q_i, Q_j)$  means a higher attendance at club  $i$ 's home match and an increase in  $S_{iU}$ , whilst  $S_{jU}$  is unchanged. Thus aggregate surplus increases;

**Theorem 5.1** Suppose there is a U-league equilibrium in which for some club  $\Pi_i > 0$  and  $w_i > 0$ . Then, some small decrease in  $w_i$  and increase in  $f_i$ , other utility weights unchanged, produces a U-league equilibrium with higher aggregate surplus.

If one thinks of an aggregate surplus maximizing planner with the power to influence the nature of club governance via the utility weights, then the optimum for this planner will imply that for each club either the weight on win percentage is zero, or profits are zero in the resulting U-league equilibrium.

To go beyond this first and general result requires more structure. The following formulae, found by substituting utility maximizing prices from Lemma 4.1 into (5.1), are useful in the next 2 sub-sections;

$$S_i(Q_i, Q_j, p_{iU}(Q_i, Q_j)) = \frac{3-2f_i}{2(2-f_i)^2} \mu_i v(Q_i, Q_j)^2 - Q_i \text{ if } \Pi_i > 0 \quad (5.4)$$

$$S_i(Q_i, Q_j, p_{iU}(Q_i, Q_j)) = \frac{1}{2} \mu_i p_{iH}(Q_i, Q_j)^2 \text{ if } \Pi_i = 0 \quad (5.5)$$

## **5.1 Leagues with strategic independence**

A useful benchmark is the special case where the game entails global strategic independence, and the best response results of Section 4 provide immediate information on U-league equilibria. Naturally the conditions required are quite restrictive. Specifically we assume first that fans are completely home partisan in that  $v(Q_i, Q_j)$  depends only on  $Q_i$ ; dividing a given amount of talent between their own team and the rival for their optimal match, a fan would allocate all talent to their own team. Secondly, the win percentage or contest success function is assumed to possess globally the property that<sup>24</sup>  $\frac{\partial^2 W}{\partial Q_i \partial Q_j} = 0$ , so that  $\frac{\partial W}{\partial Q_i}$  depends only on  $Q_i$ ; to reflect these

<sup>24</sup> An example of a contest success function with the desired properties is a difference form (Skaperdas (1996)), with  $W(Q_i, Q_j) = g(Q_i) - g(Q_j) + \frac{1}{2}$  where  $g : \mathfrak{R}_+ \rightarrow [0, \frac{1}{2}]$  is increasing and concave with  $g(0) = 0$  and  $g(Q_i) \rightarrow \frac{1}{2}$  as  $Q_i \rightarrow \infty$ .

assumptions we use the single variable notation  $v'(Q_i)$  and  $W'(Q_i)$  for derivatives. Since best responses always exist and are unique under our assumptions, it follows that there is always a unique Nash equilibrium in this section, characterised by (5.6) for clubs with  $\Pi_i > 0$  and (5.7) when  $\Pi_i = 0$ ;

$$\mu_i v(Q_i) v'(Q_i) / (2 - f_i) + w_i W'(Q_i) = 1 \quad (5.6)$$

$$\frac{1}{2} f_i \mu_i \frac{\partial p_{iH}^2}{\partial Q_i}(Q_i) + w_i W'(Q_i) = 0 \quad (5.7)$$

A much stronger result than Theorem 5.1 is now available<sup>25</sup>;

**Theorem 5.2** In a U-league with strategic independence, the value of aggregate surplus in the F-league equilibrium (with 2 pure fan welfare maximizers) is greater than its value in any other U-league equilibrium.

Proof See appendix.

The social welfare maximising planner referred to earlier, would now want all clubs to place positive weight only on fan welfare in their objectives. All clubs should be supporter owned, as is currently the case only at Brentford and Exeter in Football Leagues 1 and 2. But the assumptions that lead to these conclusions are strong; so we relax them next.

## **5.2 Strategic complementarity**

In this section we weaken the strategic independence assumption to allow also strategic complementarity, making the following first assumptions;

Assumption 1 (A1)  $v_{ij}(Q_i, Q_j) \geq 0$ .

This seems a natural assumption on fan preferences – increases in rival team quality increase the amount a fan is willing to pay for an increase in the quality of their team. We also assume;

Assumption 2 (A2)  $\partial^2 W(Q_i, Q_j) / \partial Q_i \partial Q_j \geq 0$

A useful Lemma is:

Lemma 5.1 Assume (A1) and suppose  $Q_i \in [0, \overline{Q_i}(Q_j)]$ ,  $v(Q_i, Q_j) v_i(Q_i, Q_j) > 2 / \mu_i$ .

Then  $\partial^2 p_{iH}(Q_i, Q_j)^2 / \partial Q_i \partial Q_j > 0$ .

Proof See appendix.

The best response problems faced by utility maximizing clubs always generate continuous reaction functions under the general assumptions made here. (A1) and (A2) ensure that these functions are (weakly) upward sloping whenever they are differentiable which ensures (weak) global strategic complementarity;

<sup>25</sup> A weaker version of Theorem 5.2 can be found in Madden (2010), where it is shown that, under strategic independence, the F-league equilibrium produces greater aggregate surplus than the W-league or the  $\Pi$  – league.

Lemma 5.2 Assume (A1) and (A2). Then  $dQ_{iU} / dQ_j \geq 0$  whenever  $Q_{iU}(Q_j)$  is differentiable.

Proof See appendix.

The welfare conclusion is now<sup>26</sup>:

Theorem 5.3 In a U-league with strategic complementarity, suppose there is an equilibrium in which for some club  $\Pi_i > 0$ . Then some small change in clubs' utility weights, including an increase in  $f_i$ , produces a U-league equilibrium with higher aggregate surplus.

Proof See appendix.

This final result is not as definitive as Theorem 5.2, but the assumptions are reasonable. If our planner observed positive profits being taken from a club, then this indicates a social sub-optimality; increasing the weight on fan welfare in such a club's objective (perhaps by increasing supporter representation on the board) is needed to effect an improvement. The lack of positive profits in Football Leagues 1 and 2 is not an indication of a poorly performing industry – quite the opposite.

## 6. CONCLUSIONS

Formalising one of the seminal ideas in the literature on the economics of professional sports leagues (Sloane (1971)), the paper has provided a model of club decisions on player expenditure and match ticket prices in such a league, where club objectives (utility functions in Sloane's terminology) are a weighted average of profits, win percentages and the welfare of the club's fans. The effects of changes in utility function weights have been traced, for player expenditure, match ticket prices and attendances, and for the resulting aggregate surplus. Particular attention has been given to the effects of changing the weight on fan welfare, to capture the recent and increasing influence of supporters' trusts on club governance in the English Football Leagues 1 and 2 (tiers 3 and 4 in the hierarchy).

The local comparative static effects of an increase in the fan welfare utility weight depend in general on club profitability. If a club is making positive profits, the increase will definitely also increase player expenditure (equated here to team quality), although the effect on ticket prices is more ambiguous – holding team quality fixed would certainly produce a fall in the match ticket price, but an increase in ticket price to accompany the increased team quality is also a possibility. For an unprofitable club (zero profit in the model) the effects are a (now definite) fall in ticket price, but team quality typically also decreases. However, irrespective of profitability, the effects on match attendances are clear and unambiguous - increase in the fan welfare utility weight will increase attendances.

The emerging influence of supporters' trusts on club governance in the Football League is too short-lived to provide definitive data and tests. However the theoretical

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<sup>26</sup> A rough parallel to Theorem 5.3 in Madden (2010) is that, under strategic complementarity, the F-league aggregate surplus always exceeds that of the  $\Pi$  – league.

results are consistent with the limited data we have been able to collect for clubs in Football Leagues 1 and 2, namely that increased supporter trust involvement in club governance (interpreted as an increase in the fan welfare utility weight) has led to an increase in attendances. The data also indicate zero (rather than positive) profits, and lower ticket prices and player expenditure when there is supporter involvement in governance, again consistent with the comparative static results<sup>27</sup>.

Normatively, in an extreme (strategic independence) case, it should be the case that clubs' objectives reflect only the interests of their fans, with zero profits; club ownership by fans (as currently at Brentford and Exeter in the Football League) would be a good thing. Under more reasonable (strategic complementarity) assumptions, the theoretical results still point clearly towards the conclusion that the observation of positive profits accruing to owners would not be a good thing, and increased supporter involvement in governance (e.g. increased supporter trust representation on club boards) is needed to improve performance. For such leagues, zero rather than positive profits are a better indication of a socially healthy league.

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<sup>27</sup> A natural experiment is in progress – as supporter involvement in governance expands (and maybe reaches the highest Premier League level, e.g. if MUST have their way), further data will emerge for refinement of the conclusions



## APPENDIX

### Clubs used for Table 4

*SI clubs:* Brentford, Carlisle, Exeter, Oldham, Wycombe, Bournemouth, Bury, Chesterfield, Lincoln, Northampton.

*Non SI clubs:* Brighton, Bristol Rovers, Charlton, Colchester, Gillingham, Hartlepool, Huddersfield, Leeds, Leyton Orient, Millwall, MK Dons, Norwich, Southampton, Southend, Stockport, Swindon, Tranmere, Walsall, Yeovil, Accrington, Aldershot, Barnet, Bradford, Burton, Cheltenham, Crewe, Dag & Red, Darlington, Grimsby, Hereford, Macclesfield, Morecambe, Notts Co, Port Vale, Rochdale, Rotherham, Shrewsbury, Torquay.

Proof of Lemma 4.1 For notation convenience we omit arguments of functions

$(Q_i, Q_j)$  and write  $\rho_i = 2p_{iH}^2 = v^2 - 2\frac{Q_i}{\mu_i} + v\sqrt{v^2 - 4\frac{Q_i}{\mu_i}}$  and  $A = \sqrt{v^2 - 4\frac{Q_i}{\mu_i}}$ ;

$\frac{\partial \rho_i}{\partial Q_i} = 2vv_i - \frac{2}{\mu_i} + v_i A + \frac{v}{A}(vv_i - \frac{2}{\mu_i})$  which has the sign of

$B = (vv_i - \frac{1}{\mu_i})(v + A) - \frac{1}{\mu_i}v + \frac{1}{\mu_i}(v - 2Q_i v_i)$  whose last term is positive as  $v^2$  is strictly concave. The remaining terms are also positive if  $v(vv_i - \frac{2}{\mu_i}) + A(vv_i - \frac{1}{\mu_i}) > 0$  which is true if  $vv_i - \frac{2}{\mu_i} > 0$ .

$$\begin{aligned} \frac{\partial^2 \rho_i}{\partial Q_i^2} &= 2vv_{ii} + 2v_i^2 + v_{ii}A + \frac{v_i}{A}(vv_i - \frac{2}{\mu_i}) + \frac{2vv_i^2 + v^2 v_{ii}}{A} - v^2 v_i \frac{vv_i - \frac{2}{\mu_i}}{A^3} - \frac{2v_i}{\mu_i A} + \frac{2}{\mu_i} v \frac{vv_i - \frac{2}{\mu_i}}{A^3} \\ &= \frac{2}{\mu_i} (vv_i - \frac{2}{\mu_i}) \left( \frac{v^2 - 2Q_i v_i}{A^3} \right) + 2vv_{ii} + 2v_i^2 + v_{ii}A + \frac{2vv_i^2}{A} + \frac{v^2 v_{ii}}{A} - \frac{2v}{\mu_i A} \end{aligned}$$

The first term on the right hand side is negative since  $vv_i > 2/\mu_i$  is assumed, and  $v^2/Q_i > 2vv_i$  since  $v^2$  is strictly concave. Also because  $v^2$  is strictly concave  $vv_{ii} < -v_i^2$ . The result follows if  $B = -2v_i^2 + 2v_i^2 - \frac{v_i^2}{v}A + 2\frac{vv_i^2}{A} - \frac{v_i^2}{A} - \frac{2v_i}{\mu_i A} < 0$ . But  $B = \frac{v_i}{A}[-\frac{v_i}{v}(v^2 - 4\frac{Q_i}{\mu_i}) + vv_i - \frac{2}{\mu_i}] = -\frac{2v_i}{\mu_i A}(1 - 2\frac{Q_i v_i}{v}) < 0$ , since  $v/Q_i > 2v_i$  as  $v^2$  is strictly concave.

Proof of Lemma 4.2 The constraint in (UMAX) is alternatively written as  $p_i \in [p_{iL}(Q_i, Q_j), p_{iH}(Q_i, Q_j)]$ . When  $\lambda_{iF} \geq \lambda_{iH}$  (so  $\lambda_{iF} > 0$ ),  $U_i(Q_i, Q_j, p_i)$  is decreasing in  $p_i$  for all  $Q_i, Q_j$  where  $Q_i \in [0, \overline{Q_i}(Q_j)]$ , so  $p_{iU}(Q_i, Q_j) = p_{iL}(Q_i, Q_j)$ . When  $\lambda_{iF} < \lambda_{iH}$  (so  $\lambda_{iH} > 0$ ), the utility function is strictly concave (quadratic) in  $p_i$  for  $Q_i, Q_j$  where  $Q_i \in [0, \overline{Q_i}(Q_j)]$ , with stationary point  $p_i = \frac{\lambda_{iH} - \lambda_{iF}}{2\lambda_{iH} - \lambda_{iF}} v(Q_i, Q_j)$ . Since the stationary point involves a price less than the monopoly (and so the high break-even) price, it will be the utility maximizing price if it exceeds  $p_{iL}(Q_i, Q_j)$ ; otherwise  $p_{iU}(Q_i, Q_j) = p_{iL}(Q_i, Q_j)$  again, completing the proof of (a). For (b)

$\frac{\lambda_{i\Pi} - \lambda_{iF}}{2\lambda_{i\Pi} - \lambda_{iF}} v(Q_i, Q_j) \} > p_{iL}(Q_i, Q_j)$  if (rearranging)  $(2 - f_i)^2 Q_i / (1 - f_i) < \mu_i v(Q_i)^2$ ; (b) follows.

**Proof of Theorem 4.1** (a) From Lemma 4.3(a), positive profit best quality responses are characterised by the condition;

$$\mu_i v(Q_i, Q_j) v_i(Q_i, Q_j) / (2 - f_i) + w_i \frac{\partial W}{\partial Q_i}(Q_i, Q_j) = 1 \quad (A1)$$

Differentiating with respect to  $f_i$  and treating  $Q_i$  as a function of  $f_i$  ( $Q_j$  fixed) gives, suppressing function arguments;

$$\frac{\partial Q_{iU}}{\partial f_i} [\mu_i (v_i^2 + v v_{ii}) + w_i (2 - f_i) \frac{\partial^2 W}{\partial Q_i^2}] = -1 - w_i \frac{\partial W}{\partial Q_i} = -\frac{\mu_i v v_i}{2 - f_i} < 0$$

Since  $v^2$  is strictly concave and  $\frac{\partial^2 W}{\partial Q_i^2} < 0$ , the square bracket on the left hand side is negative which ensures  $\partial Q_{iU} / \partial f_i > 0$ . Differentiating (A1) similarly with respect to  $w_i$  gives;

$$\frac{\partial Q_{iU}}{\partial w_i} [\mu_i (v_i^2 + v v_{ii}) + w_i (2 - f_i) \frac{\partial^2 W}{\partial Q_i^2}] = -(2 - f_i) \frac{\partial W}{\partial Q_i} < 0 \text{ and } \partial Q_{iU} / \partial w_i > 0 \text{ follows. The}$$

sign of the effect of a change in  $\lambda_{i\Pi}$  is that of  $-(\frac{\partial Q_{iU}}{\partial f_i} + \frac{\partial Q_{iU}}{\partial w_i})$ , and so  $\partial Q_{iU} / \partial \lambda_{i\Pi} < 0$ .

(b) Given the best response price in Lemma 4.3(a), match attendance is;

$$A_{iU} = \mu_i [v - p_{iU}] = \mu_i v / (2 - f_i) \Rightarrow \frac{\partial A_i}{\partial f_i} = \frac{\mu_i v}{(2 - f_i)^2} + \frac{\mu_i v_i}{(2 - f_i)} \frac{\partial Q_{iU}}{\partial f_i} > 0.$$

Similarly,  $\frac{\partial A_i}{\partial w_i} = \frac{\mu_i v_i}{(2 - f_i)} \frac{\partial Q_{iU}}{\partial w_i} > 0$ , and  $\partial A_{iU} / \partial \lambda_{i\Pi} < 0$  follows analogous to (a).

(c) Given the best response price in Lemma 4.3(a),  $\frac{\partial p_{iU}}{\partial w_i} = \frac{(1 - f_i) v_i}{2 - f_i} \frac{\partial Q_{iU}}{\partial w_i} > 0$ .

**Proof of Theorem 4.2** (a) From Lemma 4.3 zero profit best quality responses are characterised by the condition;

$$\frac{1}{2} f_i \mu_i \frac{\partial p_{iH}}{\partial Q_i}(Q_i, Q_j) + w_i \frac{\partial W}{\partial Q_i}(Q_i, Q_j) = 0 \quad (A2)$$

$\lambda_{i\Pi}$  has no influence locally, giving the zero derivatives in parts (a), (b) and (c). Differentiating (A2) with respect to  $f_i$  and treating  $Q_i$  as a function of  $f_i$  ( $Q_j$  fixed) gives, suppressing function arguments;

$$\frac{\partial Q_{iU}}{\partial f_i} [\mu_i f_i \frac{\partial^2 p_{iH}}{\partial Q_i^2} + w_i \frac{\partial^2 W}{\partial Q_i^2}] = -\mu_i \frac{\partial p_{iH}}{\partial Q_i} > 0, \text{ where the inequality follows from (A2). The}$$

square bracket is negative from Lemma 4.3 and  $\frac{\partial^2 W}{\partial Q_i^2} < 0$ . Thus  $\partial Q_{iU} / \partial f_i < 0$ .

Differentiating (A2) similarly with respect to  $w_i$  gives;

$$\frac{\partial Q_{iU}}{\partial w_i} [\mu_i f_i \frac{\partial^2 p_{iH}}{\partial Q_i^2} + w_i \frac{\partial^2 W}{\partial Q_i^2}] = -\frac{\partial W}{\partial Q_i} < 0, \text{ which implies } \partial Q_{iU} / \partial w_i > 0, \text{ and completes (a).}$$

Given the best response price in Lemma 4.3, match attendance is;

$$A_{iU} = \mu_i [v - p_{iU}] = \mu_i p_{iH} \Rightarrow \frac{\partial A_i}{\partial f_i} = \mu_i \frac{\partial p_{iH}}{\partial Q_i} \frac{\partial Q_{iU}}{\partial f_i} > 0, \text{ where the inequality follows from}$$

part (a) and since (A2) implies  $\frac{\partial p_{iH}}{\partial Q_i} < 0$ . Similarly  $\frac{\partial A_i}{\partial w_i} = \mu_i \frac{\partial p_{iH}}{\partial Q_i} \frac{\partial Q_{iU}}{\partial w_i} < 0$ , completing (b).

Given the best response price and  $\frac{\partial p_{iL}}{\partial Q_i} > 0$ ,  $\frac{\partial p_{iU}}{\partial f_i} = \frac{\partial p_{iL}}{\partial Q_i} \frac{\partial Q_{iU}}{\partial f_i} < 0$  and  $\frac{\partial p_{iU}}{\partial w_i} = \frac{\partial p_{iL}}{\partial Q_i} \frac{\partial Q_{iU}}{\partial w_i} > 0$ , completing (c).

**Proof of Theorem 5.2** Consider first an equilibrium of a U-league where for some club  $\Pi_i > 0$ ,  $w_i = 0$ . From Lemma 4.3(a) and (5.6), omitting function arguments;

$$f_i < 1, \mu_i v v' = 2 - f_i \quad (A3)$$

From Theorem 4.1(a) and (5.4);

$$\partial Q_{iU} / \partial f_i > 0 \text{ and } \partial S_{iU} / \partial f_i = \frac{1-f_i}{(2-f_i)^3} \mu_i v^2 + \left[ \frac{3-2f_i}{(2-f_i)^2} \mu_i v v' - 1 \right] \frac{\partial Q_{iU}}{\partial f_i}$$

From (A3) it follows that  $\partial S_{iU} / \partial f_i = \frac{1-f_i}{(2-f_i)^3} \mu_i v^2 + \frac{1-f_i}{2-f_i} \frac{\partial Q_{iU}}{\partial f_i} > 0$ . Thus in any such U-league equilibrium the value of aggregate surplus can be increased by increasing  $f_i$ .

Consider next a U-league equilibrium in which for some club  $\Pi_i = 0, w_i > 0$ . From Lemma 4.3(b) and (5.7);

$$0 < f_i < 1, \frac{1}{2} f_i \mu_i \frac{\partial p_{iH}^2}{\partial Q_i}(Q_i) + w_i W'(Q_i) = 0 \quad (\text{A4})$$

From Theorem 4.2(a) and (5.5);

$$\partial Q_{iU} / \partial f_i < 0 \text{ and } \partial S_{iU} / \partial f_i = \frac{1}{2} \mu_i \frac{\partial p_{iH}^2}{\partial Q_i} \frac{\partial Q_{iU}}{\partial f_i}$$

From (A4) it follows that  $\frac{\partial p_{iH}^2}{\partial Q_i} < 0$  and so  $\partial S_{iU} / \partial f_i > 0$ . Similarly,  $\partial S_{iU} / \partial w_i < 0$ .

Invoking also Theorem 5.1, it follows that from any  $f_i \geq 0, w_i \geq 0$  with  $\Pi_i > 0$ , one can construct a path with  $f_i$  increasing and  $w_i$  decreasing leading to  $f_i \geq 1, w_i = 0$  (a pure fan welfare maximizer) along which  $S_{iU}$  is everywhere increasing. The result follows.

Proof of Lemma 5.1 For notation convenience we omit arguments of functions

$(Q_i, Q_j)$  and write  $\rho_i = 2p_{iH}^2 = v^2 - 2\frac{Q_i}{\mu_i} + v\sqrt{v^2 - 4\frac{Q_i}{\mu_i}}$  and  $A = \sqrt{v^2 - 4\frac{Q_i}{\mu_i}}$ ;

$\frac{\partial \rho_i}{\partial Q_i} = 2v v_i - \frac{2}{\mu_i} + v_i A + \frac{v}{A}(v v_i - \frac{2}{\mu_i})$  and  $\frac{\partial^2 \rho_i}{\partial Q_i \partial Q_j} = 2v_i v_j + 2v v_{ij} + v_{ij} A + C$ , where

$C = \frac{v v_i v_j}{A} + \frac{2v v_i v_j + v^2 v_{ij}}{A} - v^3 \frac{v_i v_j}{A^3} - \frac{2v_j}{\mu_i A} + \frac{2}{\mu_i} v^2 \frac{v_j}{A^3}$ . Hence;

$$\begin{aligned} A^3 C &= v v_i v_j (v^2 - 4\frac{Q_i}{\mu_i}) + (2v v_i v_j + v^2 v_{ij})(v^2 - 4\frac{Q_i}{\mu_i}) - v^3 v_i v_j - \frac{2}{\mu_i} v_j (v^2 - 4\frac{Q_i}{\mu_i}) + \frac{2}{\mu_i} v^2 v_j \\ &= 2v_j [4\frac{Q_i}{\mu_i^2} - 6\frac{Q_i}{\mu_i} v v_i + v^3 v_i] + v^2 v_{ij} (v^2 - 4\frac{Q_i}{\mu_i}) \end{aligned}$$

Since  $v v_i > 2/\mu_i$  the square bracket exceeds  $v^3 v_i - 4\frac{Q_i}{\mu_i} = v v_i (v^2 - 4\frac{Q_i}{\mu_i}) > 0$ . Using (A1),  $C > 0$  and so  $\frac{\partial^2 \rho_i}{\partial Q_i \partial Q_j} > 0$ , as required.

Proof of Lemma 5.2 Suppose first that  $\Pi_i > 0$  at  $Q_{iU}(Q_j)$ . Then from Lemma 4.3,

$f_i < 1, Q_i < \hat{Q}_i(Q_j, f_i)$  and;

$$\mu_i v(Q_i, Q_j) v_i(Q_i, Q_j) / (2 - f_i) + w_i \frac{\partial W}{\partial Q_i}(Q_i, Q_j) = 1.$$

Differentiating with respect to  $Q_j$ , treating  $Q_i$  as a function of  $Q_j$ , gives;

$$\frac{dQ_{iU}}{dQ_j} [\mu_i (v_i^2 + v v_{ii}) + w_i (2 - f_i) \frac{\partial^2 W}{\partial Q_i^2}] = -\mu_i (v v_{ij} + v_i v_j) - w_i (2 - f_i) \frac{\partial^2 W}{\partial Q_i \partial Q_j}$$

The right hand side is non-positive from (A1) and (A2). The square bracket on the left hand side is strictly negative from the strict concavity of  $v^2$  and  $W$ . Thus  $\frac{dQ_{iU}}{dQ_j} \geq 0$ .

Suppose now that  $\Pi_i = 0$  at  $Q_{iU}(Q_j)$ . Then from Lemma 4.3,

$$\frac{1}{2} f_i \mu_i \frac{\partial p_{iH}^2}{\partial Q_i}(Q_i, Q_j) + w_i \frac{\partial W}{\partial Q_i}(Q_i, Q_j) = 0$$

Differentiating with respect to  $Q_j$ , treating  $Q_i$  as a function of  $Q_j$ , gives;

$$\frac{dQ_U}{dQ_j} \left[ \frac{1}{2} \mu_i f_i \frac{\partial^2 p_{iH}}{\partial Q_i^2} + w_i \frac{\partial^2 W}{\partial Q_i^2} \right] = -\frac{1}{2} f_i \mu_i \frac{\partial p_{iH}}{\partial Q_i \partial Q_j} - w_i \frac{\partial^2 W}{\partial Q_i \partial Q_j}$$

The right hand side is non-positive from Lemma 5.1 and (A2). The square bracket on the left hand side is strictly negative from the strict concavity of  $W$  and Lemma 4.2. Thus  $\frac{dQ_U}{dQ_j} \geq 0$ .

### Proof of Theorem 5.3

Suppose there is an equilibrium in which  $\Pi_1 > 0$  and  $\Pi_2 > 0$ .

From Lemma 4.3 and from (5.4), for  $i=1,2$ ,  $f_i < 1$ ,  $Q_i < \hat{Q}_i(Q_j, f_i)$  and;

$$\mu_i v(Q_i, Q_j) v_i(Q_i, Q_j) / (2 - f_i) + w_i \frac{\partial W}{\partial Q_i}(Q_i, Q_j) = 1 \quad (\text{B1})$$

$$S_i(Q_i, Q_j, p_{iU}(Q_i, Q_j)) = \frac{3-2f_i}{2(2-f_i)^2} \mu_i v(Q_i, Q_j)^2 - Q_i \quad (\text{B2})$$

If  $w_1 > 0$  or  $w_2 > 0$ , Theorem 5.1 ensures the result, so assume that  $w_1 = w_2 = 0$ . A small enough increase in  $f_1$  (say), other weights unchanged, will continue to produce an equilibrium characterised by the same (B1), (B2) conditions, and;

$$\frac{\partial S_1}{\partial f_1} = \frac{1-f_1}{(2-f_1)^3} \mu_1 v^2 + \frac{\partial Q_1}{\partial f_1} \left[ \frac{3-2f_1}{(2-f_1)^2} \mu_1 v v_1 - 1 \right] + \frac{3-2f_1}{2(2-f_1)^2} \mu_1 v v_2 \frac{\partial Q_2}{\partial f_1} \quad (\text{B3})$$

Since  $w_1 = 0$ ,  $\mu_1 v v_1 = 2 - f_1$ , and the square bracket above is positive. From the strategic complementarity assumptions,  $\frac{\partial Q_1}{\partial f_1} > 0$ ,  $\frac{\partial Q_2}{\partial f_1} \geq 0$ , and so  $\frac{\partial S_1}{\partial f_1} > 0$ . Also;

$$\frac{\partial S_2}{\partial f_1} = \frac{\partial Q_2}{\partial f_1} \left[ \frac{3-2f_2}{(2-f_2)^2} \mu_2 v v_2 - 1 \right] + \frac{3-2f_2}{2(2-f_2)^2} \mu_2 v v_1 \frac{\partial Q_1}{\partial f_1} > 0$$

for analogous reasons to above. Thus the change in  $f_1$  increases aggregate surplus.

Suppose there is an equilibrium in which one club makes strictly positive profits and the other zero profits, say  $\Pi_1 > 0$  and  $\Pi_2 = 0$ . Again we assume  $w_1 = 0$ , otherwise Theorem 5.1 completes the proof. From Lemma 4.3 for club 2 we know:

either  $f_2 < 1$  and  $Q_2 \geq \hat{Q}_2(Q_1, f_2)$  or  $f_2 \geq 1$  and, in both cases;

$$\frac{1}{2} f_2 \mu_2 \frac{\partial p_{2H}}{\partial Q_2}(Q_2, Q_1) + w_2 \frac{\partial W}{\partial Q_2}(Q_2, Q_1) = 0 \quad (\text{B4})$$

$$S_2(Q_2, Q_1, p_{2U}(Q_2, Q_1)) = \frac{1}{2} \mu_2 p_{2H}(Q_2, Q_1)^2 \quad (\text{B5})$$

Suppose first that  $w_2 = 0$  also. If either  $f_2 < 1$  and  $Q_2 > \hat{Q}_2(Q_1, f_2)$  or  $f_2 \geq 1$ , then (B4) continues to define the best response and (B5) continues to define the surplus generated by club 2 after small enough changes in  $Q_1$ . Then;

$$\frac{\partial S_2}{\partial f_1} = \mu_2 \left[ \frac{\partial p_{2H}}{\partial Q_2} \frac{\partial Q_2}{\partial f_1} + \frac{\partial p_{2H}}{\partial Q_1} \frac{\partial Q_1}{\partial f_1} \right] = \mu_2 \frac{\partial p_{2H}}{\partial Q_1} \frac{\partial Q_1}{\partial f_1} > 0 \quad (\text{B6})$$

where the sign follows since generally  $\frac{\partial p_{2H}}{\partial Q_1} > 0$ , and  $\frac{\partial Q_1}{\partial f_1} > 0$  from strategic complementarity. As in the first part of the proof,  $\frac{\partial S_1}{\partial f_1} > 0$ , and again aggregate surplus increases after a small increase in  $f_1$ . If  $f_2 < 1$  and  $Q_2 = \hat{Q}_2(Q_1, f_2)$  then small increases in  $f_1$  and hence  $Q_1$  imply  $Q_2 < \hat{Q}_2(Q_1, f_2)$ , since  $\partial \hat{Q}_2 / \partial Q_1 > 0$ , and club 2's best response and contribution to surplus are now described by (B1) and (B2). The argument of the first part of the proof then ensures that aggregate surplus increases after a small increase in  $f_1$ , again.

Now suppose that  $w_2 > 0$ . We now construct a nearby dominating equilibrium by increasing  $f_1$  again, but also by changing  $f_2$  and  $w_2$  so that  $Q_2$  remains unchanged.

The effect of such a change on  $Q_1$  is described locally by (B1), equivalent locally to a function  $Q_1(f_1)$  say with  $dQ_1/df_1 > 0$ . The effect of such a change on  $S_1$  is given by the right hand side of (B3), deleting the final term and replacing  $\partial Q_1/\partial f_1$  by  $dQ_1/df_1$ ; the effect is certainly positive again. For club 2,  $f_2/w_2$  is adjusted to maintain (B4) with an unchanged  $Q_2$  and with  $Q_1 = Q_1(f_1)$ ; in order to do this it follows from (A1) and (A2) that  $f_2/w_2$  increases and so  $\hat{Q}_2(Q_1, f_2)$  decreases and club 2's best response continues to be described by (B4), even when originally  $Q_2 = \hat{Q}_2(Q_1, f_2)$ . The effect on  $S_2$  is therefore given by  $\mu_2 \frac{\partial p_{2H}^2}{\partial Q_1} \frac{dQ_1}{df_1} > 0$ , as in (B6), now since  $Q_2$  is unchanged. Thus the changes in  $f_1$ ,  $f_2$  and  $w_2$  lead to an increase in aggregate surplus, completing the proof.

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