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Strategic Voting over Strategic Proposals[1]

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Abstract

Prior research on "strategic voting" has reached the conclusion that unanimity rule is uniquely bad: it results in destruction of information, and hence makes voters worse off. We show that this conclusion depends critically on the assumption that the issue being voted on is exogenous, i.e., independent of the voting rule used. We depart from the existing literature by endogenizing the proposal that is put to a vote, and establish that under many circumstances unanimity rule makes voters better off. Moreover, in some cases unanimity rule *also* makes the proposer better off, even when he has diametrically opposing preferences. In this case, unanimity is the Pareto dominant voting rule. Voters prefer unanimity rule because it induces the proposing individual to make a more attractive proposal. The proposing individual prefers unanimity rule because the acceptance probabilities for moderate proposals are higher. We apply our results to jury trials and debt restructuring.

JEL classification: C7; D7; D8.

Keywords: Strategic voting; agenda setting; multilateral bargaining.

1 Introduction

Many collective decisions are made by holding a vote over an endogenous agenda. Examples include debt restructuring negotiations between creditors and a borrower; jury trials; congressional votes over presidential appointments in the U.S. and elsewhere; shareholder votes on executive compensation; and collective bargaining between a firm and union members. The voting rules used for different decisions differ, and the choice of voting rule has two consequences. First, the voting rule affects whether a given proposal is adopted. Second, the voting rule affects the proposal that is being voted over.

A large and influential recent literature analyzes voting when individuals have different information.[1] This "strategic voting" literature deals exclusively with the first consequence of the voting rule — whether a given proposal is adopted — and concludes that unanimity rule is inferior to majority rule.[2,3] Specifically, while majority rule aggregates information efficiently when the number of voters is large enough, unanimity rule results in mistaken decisions. As such, when the issue being voted over is exogenous, unanimity rule is a suboptimal voting rule, and reduces the expected payoff of voters.

Nevertheless, unanimity rule is used in many settings. For example, under the U.S. Trust Indenture Act of 1939 debt can be restructured only if all creditors agree. Likewise, unanimity is commonly required in jury trials. The results of the strategic voting literature suggest that a majority vote would be more efficient in such settings.

In this paper we show that the conclusion that majority rule is superior depends critically on the assumption that the proposal being voted over is exogenous. We do

[1] See, for example, Austen-Smith and Banks (1996), Feddersen and Pesendorfer (1996, 1997, 1998), McLennan (1998), Duggan and Martinelli (2001), Doraszelski *et al* (2003), Persico (2004), Yariv (2004), Martinelli (2005), Meirowitz (2005), Gerardi and Yariv (2007).

[2] By majority rule, we mean any threshold voting rule: that is, a proposal is accepted if the fraction voting to accept exceeds a pre-specified threshold.

[3] The main exception is Coughlan (2000), who shows that if pre-vote communication is possible and voter preferences are common knowledge and closely aligned, then both unanimity and majority rules may allow efficient aggregation of information. However, Austen-Smith and Feddersen (2006) show that if voter preferences are not common knowledge then unanimity is again the inferior voting rule from the perspective of information aggregation. Additionally, even Coughlan does not argue that unanimity rule is strictly superior to majority rule in the standard two-alternative voting game.

so by studying the second consequence of the voting rule mentioned above, namely that it affects the proposal being voted upon. We show that under many circumstances unanimity rule increases the expected utility of voters, because it induces the proposer to make a more attractive offer. Further, in a subset of such circumstances unanimity rule is Pareto superior, because it also increases the proposer's expected utility — even when his interests are diametrically opposed to those of the voters. At the end of the paper we analyze two specific applications: jury trials and debt restructuring.

We consider the following setting. One individual, the *proposer*, makes a take-it-or-leave-it offer to a group. The group members, the *voters*, must collectively decide whether to accept or reject the offer, and we assume they do so by holding a vote. The fraction of votes required to accept the proposer's offer is fixed prior to the offer (by, for example, law, contract, or the common consent of voters). As such, when the proposer makes his offer he takes the voting rule as given. We follow the strategic voting literature and focus on information aggregation when the number of voters is large.

As one would expect, and regardless of the voting rule, the acceptance probability is increasing in the attractiveness of the offer to voters. We assume there is sufficient conflict between the proposer and voters that the proposer faces a trade-off between a high offer that is accepted more often and a low offer that is accepted less often. Equilibrium offers are determined by this trade-off.

As in the prior literature, the group (asymptotically) makes the correct decision under majority rule but makes mistakes under unanimity rule. In particular, voters reject low offers more often than they should, and accept high offers more often than they should. Provided the proposer's payoff under disagreement is not too low, the mistakes that arise under unanimity rule benefit voters. In this case, when facing voters using majority rule, the proposer is not willing to make a high offer, but rather prefers a smaller offer accepted less often. Since voters make mistakes under unanimity rule by rejecting low offers more often than they should, the proposer responds by offering more than he would under

majority rule. Put slightly differently, voters would like to commit to reject low offers in order to increase the equilibrium offer. One way to accomplish this is to commit to have poor information via adopting a unanimity requirement.

Although under unanimity rule voters receive a better offer from the proposer (provided the proposer's payoff under disagreement is not too low), it is still not obvious whether they prefer unanimity rule or majority rule. The reason is that the better offer is made in response to voters' mistakes. However, a form of the envelope theorem holds in our voting environment. As such one can evaluate the effect of the higher offer simply by considering the direct effect, which is positive. It follows that *both* the equilibrium offer and voters' expected utility are higher under unanimity rule.

Moreover, and perhaps surprisingly, when the proposer's payoff under disagreement is neither too low nor too high, the proposer *also* prefers unanimity rule, making it the Pareto dominant voting rule. The key to this result is that against a group using unanimity the proposer is able to get a moderate offer accepted with very high probability, due to the mistakes of the voters. In contrast, as described above the proposer's best offer against majority rule is a lower offer that is accepted with a lower probability. In this case voters prefer unanimity rule because they get a higher offer than they would under majority rule. The proposer prefers unanimity rule because he can secure acceptance more often than he could under majority rule at a cost he is willing to bear.[4]

Overall, our results highlight the importance of the endogeneity of the proposal being voted over in the determination of optimal voting rules. While unanimity rule is inferior when the agenda is exogenous, it may Pareto dominate all other voting rules once the agenda is endogenous.

[4]An additional consequence of the voting rule relates to the proposer's scope for signaling his own information via his offer. Consider, for example, the case in which the proposer observes a relatively accurate signal, while each voter receives independent but relatively low quality signals. Majority voting rules efficiently aggregate voter information when the number of voters is large, and hence prevent the proposer from manipulating voter beliefs with his choice of offer. In contrast, unanimity rule fails to aggregate information efficiently, and enables a proposer who receives a very negative signal to pool with proposers who see positive signals (see Lemma 6).

Related Literature

As discussed above, our paper develops the strategic voting literature by endogenizing the issue being voted over. This literature has studied how differentially informed individuals vote over an exogenously specified agenda by explicitly taking into account that a vote only matters if it is pivotal, and so each voter should condition on the information implied by being pivotal. In particular, it is not an equilibrium for each voter to vote sincerely, i.e., purely according to his own information (Austen-Smith and Banks 1996). When the number of voters is large, in equilibrium information is nonetheless aggregated efficiently under majority rule. In contrast, unanimity rule does not lead to efficient information aggregation, and therefore results in mistakes (see Feddersen and Pesendorfer 1997, 1998, and also Duggan and Martinelli 2001). Given these results, one might be tempted to conclude that unanimity rule is inefficient, and in particular, hurts voters. Our results show that neither is true when the agenda being voted on is endogenous.

Our paper deals with a form of negotiation between a single proposer and a coalition. As such, it is related to studies of multilateral bargaining. In contrast to our paper, this literature focuses on proposals that can discriminate among individuals.[5] However, in many negotiations a proposal must treat all members of some group equally, either for feasibility reasons (e.g., jury trials), or for institutional/legal reasons (e.g., wage determination, debt restructuring). The literature analyzing this important class of bargaining problems is much smaller and focuses on complete information models.[6] Since agreement is always reached, there is no risk of breakdown of agreement from a "tougher" bargaining stance. In contrast, the possibility of failing to agree to a Pareto improving proposal is central to our analysis and results.

In our model, bargaining takes place under two-sided asymmetric information. The

[5]The classic paper is Baron and Ferejohn (1989).

[6]See Banks and Duggan (2001), Cho and Duggan (2003), Cardona and Ponsati (2007), and Manzini and Mariotti (2005). Separately, Chae and Moulin (2004) provide a family of solutions to group bargaining from an axiomatic viewpoint; and Elbittar *et al* (2004) provide experimental evidence that the choice of voting rule used by a group in bargaining affects outcomes.

literature on bargaining under asymmetric information is extensive.[7] We add to this literature by considering the effects of the internal organization of one of the parties.

Finally, in closely related independent work Henry (2006) studies equilibrium proposals in a legislative bargaining environment with asymmetric information. Specifically, he fixes the voting rule, and characterizes the proposer's best discriminatory offer.

Paper outline

The paper proceeds as follows. Section 2 describes the model. Section 3 establishes equilibrium existence and characterizes basic equilibrium properties. Section 4 bounds the equilibrium outcomes of the bargaining game when the group uses unanimity rule. Section 5 conducts the same exercise when the group adopts majority rule. Section 6 compares equilibrium outcomes across different voting rules, and explores two potential applications of our model. Section 7 concludes. All proofs are in the Appendix.

2 Model

There is a single proposer (agent 0), and a group of $n \geq 2$ voters, labelled $i = 1, \ldots, n$. The timing is as follows: (1) Each agent i privately observes a random variable $\sigma_i \in [\underline{\sigma}, \bar{\sigma}]$. (2) The proposer selects a proposal $x \in [0, 1]$. (3) Voters simultaneously cast ballots to accept or reject the proposal. (4) If at least a fraction α of the voters accept,[8] the proposal is implemented, while otherwise the status quo prevails. Common examples include simple majority, $\alpha = 1/2$; supermajority, e.g., $\alpha = 2/3$; and unanimity, $\alpha = 1$. The voting rule α is exogenously given:[9] in particular, it cannot be changed after the

[7] See Kennan and Wilson (1993) for a review. Of most relevance for our paper are Samuelson (1984), Evans (1989), Vincent (1989), Schweizer (1989), Deneckere and Liang (2006), and Dal Bo and Powell (2007), all of which study common values environments.

[8] Throughout, we ignore the issue of whether or not $n\alpha$ is an integer. This issue could easily be handled formally by replacing $n\alpha$ with $[n\alpha]$ everywhere, where $[n\alpha]$ denotes the smallest integer weakly greater than $n\alpha$. Since this formality has no impact on our results, we prefer to avoid the extra notation and instead proceed as if $n\alpha$ were an integer.

[9] Since our main results characterize the proposer's and voters' preferences over different voting rules, it would be straightforward to endogenize the choice of voting rule by having either the voters or proposer select it at an *ex ante* stage before $\{\sigma_i\}$ are realized.

proposer makes his offer.

PREFERENCES

Agent i's preferences over the proposal x and the status quo are determined by σ_i and an unobserved state variable $\omega \in \{L, H\}$. The realization of σ_i both provides agent i with information about ω, and affects his preferences over the offer and the status quo. Voter i's utility from the offer x is $U^\omega(x, \sigma_i, \lambda)$, where $\lambda \in [0, 1]$ is a parameter that describes the relative importance of ω and σ_i. We assume that $U^\omega(x, \sigma_i, \lambda)$ is independent of σ_i at $\lambda = 0$, and $U^L(\cdot, \cdot, \lambda) \equiv U^H(\cdot, \cdot, \lambda)$ at $\lambda = 1$. Likewise, we write $\bar{U}^\omega(\sigma_i, \lambda)$ for voter i's utility under the status quo, and make parallel assumptions for $\lambda = 0, 1$. As such, our framework includes pure *common values* ($\lambda = 0$) and pure *private values* ($\lambda = 1$) as special cases. (See footnote 12 below for a specific example.) For the most part, we focus on preferences close to common values: many existing strategic voting papers deal exclusively with pure common values,[10] and it is the natural benchmark in a variety of settings, e.g., debt restructuring, where the securities received trade *ex post*.

A key object in our analysis is the utility of a voter from the proposal above and beyond the status quo. Accordingly, we define $\Delta^\omega(x, \sigma_i, \lambda) \equiv U^\omega(x, \sigma_i, \lambda) - \bar{U}^\omega(\sigma_i, \lambda)$. Similarly, we write the proposer's utility from having his offer accepted as $V^\omega(x, \sigma_0)$, and his utility under the status quo as $\bar{V}^\omega(\sigma_0)$. Note that we do not require the relative weights of ω and σ_0 in determining the proposer's preferences to match the relative weights (given by λ) of ω and σ_i in determining voter i's preferences.

For all preferences $\lambda < 1$, the realization of σ_i provides voter i with useful (albeit noisy) information about the unobserved state variable ω. We assume the random variables $\{\sigma_i : i = 0, 1, \ldots, n\}$ are independent conditional on ω, and that except for σ_0 (observed by the proposer) are identically distributed. Let $F(\cdot|\omega)$ and $F_0(\cdot|\omega)$ denote the distribution functions for the voters and proposer respectively. We assume both distributions have associated continuous density functions, which we write $f(\cdot|\omega)$ and

[10] See, for example, Feddersen and Pesendorfer (1998), and Duggan and Martinelli (2001).

$f_0(\cdot|\omega)$. We let $\ell(\sigma)$ and $\ell_0(\sigma)$ denote the likelihood ratios $\frac{f(\sigma|H)}{f(\sigma|L)}$ and $\frac{f_0(\sigma|H)}{f_0(\sigma|L)}$. The realization of σ_i is informative about ω, in the sense that the monotone likelihood ratio property (MLRP) holds strictly;[11] but no realization is perfectly informative, i.e., $\ell(\underline{\sigma}) > 0$ and $\ell(\bar{\sigma}) < \infty$, with similar inequalities for ℓ_0. We denote the unconditional probability of state ω by p^ω, and its probability conditional on σ_i by $p^\omega(\sigma_i) = \frac{p^\omega f(\sigma_i|\omega)}{p^H f(\sigma_i|H)+p^L f(\sigma_i|L)}$.

Interpretations

In Section 6 we return to the first two of the following possible interpretations:

Debt restructuring: A debtor seeks to restructure debt claims held by n identical creditors. The offer terms are indexed by x, where $x = 0$ is a worthless offer and $x = 1$ is the highest feasible offer. If the offer is rejected the existing debt remains in place.[12]

Jury voting with endogenous charges: A prosecutor chooses a crime to charge, in turn determining a potential prison sentence. The status quo outcome is no punishment. The defendant is guilty in $\omega = H$ and innocent in $\omega = L$.[13]

Collective bargaining: An employer is in wage negotiations with its union workers, and offers a wage x.

Voting over endogenous policy in a presidential system: A president proposes a policy x.[14] The proposal is adopted only if passed by the legislature. This requires the support of a sufficient fraction of legislators from the opposing party to the president.

[11] That is, $\ell(\sigma)$ and $\ell_0(\sigma)$ are strictly increasing in σ.

[12] The following parameterization of corporate debt restructuring may help to fix ideas. A debtor offers creditors a fraction x of its equity. If creditors accept, they receive a payoff $U^\omega(x, \sigma_i, \lambda) = (1-\lambda)\, xR_\omega + \lambda\sigma_i$, where R_ω is the expected firm value and σ_i is a creditor-specific component of valuation (e.g., tax benefits). If creditors reject, they receive a liquidation payoff $\bar{U}^\omega(\sigma_i, \lambda) = \bar{U}$. Thus when $\omega = H$, it is both more likely that creditors believe the state is H, and their average private valuation of the offer is higher. Finally, the debtor receives $V^\omega(x, \sigma_0) = (1-x)\, R_\omega + B(\sigma_0)$ if the offer is accepted, where $B(\sigma_0)$ is his private benefits from running the firm, and $\bar{V}^\omega(\sigma_0) = 0$ if it is rejected and the firm is liquidated.

[13] In an alternative interpretation, the defendant is guilty of *some* crime in both $\omega = L, H$, but is guilty of a more severe crime in $\omega = H$.

[14] A judicial nominee, for example.

EQUILIBRIUM

We examine the sequential equilibria of the game just described. The proposer's strategy is a mapping from the set of possible signals, $[\underline{\sigma}, \bar{\sigma}]$, to probability distributions over the offer set $[0, 1]$. Conditional on the proposer's offer, and as is standard in the strategic voting literature on which we build, we restrict attention to equilibria in which the *ex ante* identical voters behave symmetrically.[15]

Voters are potentially able to infer information about the proposer's observation of σ_0 from his offer, and thus information about the state variable ω. Let $\beta_n(x; \lambda, \alpha)$ denote a voter's belief that $\omega = H$ after observing offer x, but ignoring his own signal σ_i, in the game with n voters using voting rule α, and preference parameter λ. (Equation (2) in the next section demonstrates how voter i combines the information revealed by the offer x with his own signal σ_i.) By belief consistency, β_n is the same for all voters.

A sequential equilibrium thus consists of an offer strategy for the proposer, voter belief $\beta_n(\cdot; \lambda, \alpha)$, and a voting strategy $[\underline{\sigma}, \bar{\sigma}] \to \{\text{accept,reject}\}$ for each voter such that the proposer's strategy is a best response to voters' (identical) strategies; and each voter's strategy maximizes his expected payoff given belief $\beta_n(\cdot; \lambda, \alpha)$ and all other voters use the same strategy; and the belief itself is consistent. At a minimum, belief consistency requires that voters are never more (respectively, less) confident that $\omega = H$ than the proposer himself is after he sees the most (respectively, least) pro-H signal $\sigma_0 = \bar{\sigma}$ (respectively, $\sigma_0 = \underline{\sigma}$). That is, for all offers x, $\frac{\beta_n(x;\lambda,\alpha)}{1-\beta_n(x;\lambda,\alpha)} \in \left[\ell_0(\underline{\sigma})\frac{p^H}{p^L}, \ell_0(\bar{\sigma})\frac{p^H}{p^L}\right]$. Consequently consistency implies that $\beta_n(x; \lambda, \alpha) \in \left[\underline{b}, \bar{b}\right]$, for some $0 < \underline{b} < \bar{b} < 1$.

[15]For unanimity, Duggan and Martinelli (2001) prove that the symmetric voting equilibrium is the unique equilibrium if $\ell(\cdot) / ((1 - F(\cdot|H)) / (1 - F(\cdot|L)))$ is monotone. For majority, the key equilibrium property that we use in our analysis is that information is efficiently aggregated as the number of voters grows large. Existing strategic voting papers (see above) show that the symmetric voting equilibrium has this property. One way in which information aggregation could fail in an asymmetric equilibrium is if $n(1 - \alpha)$ voters always reject the offer, effectively transforming the voting rule into a unanimity vote involving the remaining $n\alpha$ voters. However, one can show that no such equilibrium exists if $\ell(\cdot) / (F(\cdot|H) / F(\cdot|L))$ is increasing and Duggan and Martinelli's condition holds. (A proof is available from the authors' webpages; both conditions are satisfied for the logistic distribution, among others.) We leave the remaining and more subtle question of whether information aggregation fails in some other asymmetric equilibrium for future research.

Preference assumptions

Assumption 1 *Δ^ω, V and $\bar{V}$ are twice continuously differentiable in their arguments.*

Assumption 2 *$\Delta^H \geq \Delta^L$ and Δ^ω is increasing in σ_i; both are strict for $x \in (0,1)$.*

Assumption 3 *$\min_{x\in[0,1]} \Delta^H(x,\bar{\sigma},\lambda) < 0 < \max_{x\in[0,1]} \Delta^H(x,\bar{\sigma},\lambda)$ for all λ.*

Assumptions 1 - 3 are straightforward. For future reference, observe that Assumption 1 implies that $|\Delta^\omega|$ is bounded since Δ^ω is continuous and has compact domain. Assumption 2 says voter i is more pro-acceptance when $\omega = H$ than $\omega = L$, and when the realization of σ_i is higher. Since higher values of σ_i are more likely when $\omega = H$ (by MLRP), the content of Assumption 2 (beyond being a normalization) is that the "private" and "common" components of voter utility act in the same direction. Combined with Assumption 2, Assumption 3 says that there exist offers that voters always reject, but other offers that they would like to accept under at least some conditions.

Our remaining two assumptions deal with how the offer x affects voter and proposer preferences, respectively:

Assumption 4 *For any σ, λ and $q \in [0,1]$, there exists a unique x such that $q\Delta^H(\cdot,\sigma,\lambda) + (1-q)\Delta^L(\cdot,\sigma,\lambda)$ is strictly negative over $[0,x)$ and strictly positive over $(x,1)$.*

Assumption 4 says that for any given belief about ω, a voter would accept an offer x if and only if the offer is sufficiently high.[16] It is satisfied if Δ^ω is strictly concave in the offer x, with $\Delta^\omega(x=1) \geq 0$; or if Δ^ω is strictly increasing in x. Of particular relevance for our analysis are the minimum offers that a fully informed voter (with $\sigma_i = \underline{\sigma}$) would accept, which we write as $x_L(\lambda)$ and $x_H(\lambda)$.[17] Formally, define $x_\omega(\lambda)$ by $\Delta^\omega(x_\omega(\lambda), \underline{\sigma}, \lambda) = 0$. We write $x_\omega(\lambda) = \infty$ if no solution exists. The minimum acceptable offer $x_H(0)$ is used in our final assumption, which concerns proposer preferences:

[16] If $q\Delta^H(\cdot,\sigma,\lambda) + (1-q)\Delta^L(\cdot,\sigma,\lambda) < 0$ for all $x \in [0,1)$, the cutoff value implied by Assumption 4 is $x = 1$. By Assumptions 2 and 3, $q\Delta^H(\cdot,\sigma,\lambda) + (1-q)\Delta^L(\cdot,\sigma,\lambda)$ cannot be everywhere positive.

[17] The uniqueness of $x_L(\lambda)$ and $x_H(\lambda)$ follows from Assumption 4 with $q = 0$ and $q = 1$.

Assumption 5 *(I) For all* σ_0, $V^\omega(x,\sigma_0) > \bar{V}^\omega(\sigma_0)$ *for* $x \in (0,1)$, *and* $V^\omega(1,\sigma_0) = \bar{V}^\omega(\sigma_0)$. *(II) There exists* $\underline{x} < x_H(0)$ *such that* V^ω *is decreasing in* x *for* $x \geq \underline{x}$.

The first part of Assumption 5 says that the proposer dislikes the status quo relative to the range of possible alternatives: regardless of the state, he would prefer to have any proposal $x \in (0,1)$ implemented.[18] The second part of Assumption 5 ensures that there is at least some conflict between the proposer's and voters' preferences, even if they are aligned over some regions of the offer space $[0,1]$. Specifically, in the pure common values case $\lambda = 0$, voters accept offers only if they are sufficiently far above $x_H(0)$, and Assumption 5 says that the proposer dislikes increasing the offer beyond $x_H(0)$.

Before turning to the analysis of our model, we note that in our framework voting is the only means by which voters can share their information. When voters are numerous and dispersed, as is often the case, this is a reasonably realistic assumption. We return to this issue in more detail in the conclusion. Somewhat related, we also take as given the information possessed by voters. Other authors have modelled strategic voting games with costly information acquisition,[19] but have done so under the assumption that the proposal being voted over is independent of the voting rule, i.e., is exogenous. We leave the simultaneous integration of costly information acquisition and endogenous proposals into strategic voting for future work.

3 Equilibrium existence and basic properties

In order to establish equilibrium existence we first look at the voting stage:

[18] In general, one can think of a larger set of proposals $[0,\infty)$, but with the proposer preferring the status quo to offers $x \in (1,\infty)$. The content of Assumption 5 is that the set of offers the proposer prefers to the status quo is independent of (ω,σ_0). For instance, consider the debt renegotiation example of footnote 12, where x is the fraction of the firm the debtor offers to creditors. Then Assumption 5 says that the debtor prefers being left with any fraction $1-x$ of the firm to liquidation (the status quo).

[19] See Persico (2004), Yariv (2004), and Martinelli (2005).

The voting stage

Fix a preference parameter λ and a number of voters n. A central insight of the existing strategic voting literature is that voter i's decision of how to vote depends on the expectation of his utility across states in which he is pivotal, since in other states his vote makes no difference. Let PIV_i denote the event in which voter i is pivotal. Thus voter i votes to accept offer x after observing σ_i if and only if

$$\sum_{\omega=L,H} \Pr\left(\omega, PIV_i|\sigma_i, x\right) U^{\omega}\left(x, \sigma_i, \lambda\right) \geq \sum_{\omega=L,H} \Pr\left(\omega, PIV_i|\sigma_i, x\right) \bar{U}^{\omega}\left(x, \sigma_i, \lambda\right). \quad (1)$$

Since σ_i are independent conditional on ω,

$$\Pr\left(\omega, PIV_i|\sigma_i, x\right) = \frac{\Pr\left(\omega, PIV_i, \sigma_i|x\right)}{\Pr\left(\sigma_i|x\right)} = \frac{\Pr\left(PIV_i|\omega\right)\Pr\left(\sigma_i|\omega\right)\Pr\left(\omega|x\right)}{\Pr\left(\sigma_i|x\right)}. \quad (2)$$

Write b for a voter's belief that $\omega = H$ based only on the offer x (and ignoring his own signal σ_i). Substituting b, Δ^{ω} and (2) into inequality (1) implies that voter i votes to accept proposal x after observing σ_i if and only if

$$\Delta^{H}\left(x, \sigma_i, \lambda\right)\Pr\left(PIV_i|H\right) f\left(\sigma_i|H\right) b + \Delta^{L}\left(x, \sigma_i, \lambda\right)\Pr\left(PIV_i|L\right) f\left(\sigma_i|L\right)(1-b) \geq 0. \quad (3)$$

By MLRP, it is immediate from (3) that in any equilibrium each voter i follows a cutoff strategy, i.e., votes to accept if and only if σ_i exceeds some critical level. As noted, throughout we focus on symmetric equilibria in which the *ex ante* identical voters follow the same voting strategy. Let $\sigma_n^*(x, b, \lambda, \alpha) \in [\underline{\sigma}, \bar{\sigma}]$ denote the common cutoff[20] when there are n voters, the offer is x, voters attach a probability b to $\omega = H$, and the preference parameter and voting rule are λ and α respectively. For clarity of exposition, we suppress the arguments n, x, b, λ and α unless needed. Evaluating, the probability that a voter is pivotal is given by $\Pr\left(PIV_i|\omega\right) = \binom{n-1}{n\alpha-1}\left(1 - F\left(\sigma^*|\omega\right)\right)^{n\alpha-1} F\left(\sigma^*|\omega\right)^{n-n\alpha}$. The acceptance condition (3) then rewrites to:

$$\begin{aligned} &\Delta^{H}\left(x, \sigma_i, \lambda\right)\left(1 - F\left(\sigma^*|H\right)\right)^{n\alpha-1} F\left(\sigma^*|H\right)^{n-n\alpha} f\left(\sigma_i|H\right) b \\ &+\Delta^{L}\left(x, \sigma_i, \lambda\right)\left(1 - F\left(\sigma^*|L\right)\right)^{n\alpha-1} F\left(\sigma^*|L\right)^{n-n\alpha} f\left(\sigma_i|L\right)(1-b) \geq 0. \end{aligned} \quad (4)$$

[20] As we show below, there exists a unique cutoff signal.

If there exists a $\sigma^* \in [\underline{\sigma}, \bar{\sigma}]$ such that voter i is indifferent between accepting and rejecting the offer x exactly when he observes the signal $\sigma_i = \sigma^*$, then the equilibrium is said to be a *responsive equilibrium*. Notationally, we represent a responsive equilibrium by its corresponding cutoff value $\sigma^* \in [\underline{\sigma}, \bar{\sigma}]$.

The following lemma establishes existence and uniqueness of cutoff strategies in the voting stage of the game. Part (1) extends Theorem 1 of Duggan and Martinelli (2001) to our more general preference framework. Parts (2) and (3) establish elementary properties of how the responsive equilibrium is related to the proposer's offer x.

Lemma 1 ***(Existence and uniqueness in the voting stage)*** *Fix belief b, a voting rule α and preferences λ. There exist $\underline{x}_n(b, \lambda, \alpha)$, $\bar{x}_n(b, \lambda, \alpha)$ such that:*
(1) For any n, a responsive equilibrium $\sigma^*(x) \in [\underline{\sigma}, \bar{\sigma}]$ exists if and only if $x \in [\underline{x}_n, \bar{x}_n]$. When a responsive equilibrium exists it is the unique symmetric responsive equilibrium.
(2) The equilibrium cutoff $\sigma^*(x)$ is decreasing and continuously differentiable over $(\underline{x}_n, \bar{x}_n)$, with $\sigma^*(\underline{x}_n) = \bar{\sigma}$ and $\sigma^*(\bar{x}_n) = \underline{\sigma}$. The equilibrium acceptance probability is increasing and continuously differentiable over $(\underline{x}_n, \bar{x}_n)$, and equals 0 and 1 at $\underline{x}_n$ and $\bar{x}_n$ respectively.
(3) If $\alpha < 1$ and x is such that $\Delta^H(x, \bar{\sigma}) > 0 > \Delta^L(x, \underline{\sigma})$, there exists N such that $x \in (\underline{x}_n, \bar{x}_n)$ for $n \geq N$.

In addition to responsive equilibria, non-responsive equilibria exist. Specifically, for any $\alpha > \frac{1}{n}$ there is an equilibrium in which each voter rejects regardless of his signal, $\sigma^* = \bar{\sigma}$. Likewise, for any $\alpha < \frac{n-1}{n}$ there is an equilibrium in which each voter accepts regardless of his signal, $\sigma^* = \underline{\sigma}$. We follow the literature and assume that if a responsive equilibrium exists, then it is played. From Lemma 1, as x increases over $(\underline{x}_n, \bar{x}_n)$ the acceptance probability increases continuously from 0 to 1. We thus assume that when $x \leq \underline{x}_n$ the rejection equilibrium is played, while for $x \geq \bar{x}_n$ the acceptance equilibrium is played. In addition to being intuitive and ensuring continuity, this rule selects the unique trembling-hand perfect equilibrium when $x \leq \underline{x}_n$.[21] Under this selection rule:

[21] Formally, for any b, λ, and any voting rule $\alpha > \frac{1}{2} + \frac{1}{2n}$, if $x \leq \underline{x}_n$ then the only trembling-hand perfect

Lemma 2 ***(Acceptance probability comparative statics)*** *The acceptance probability is increasing in the offer x, belief b, and from $\omega = L$ to $\omega = H$.*

The offer stage

In our environment, the proposer chooses an offer x from an infinite choice set $[0, 1]$. Additionally, the proposer "type" σ_0 is itself drawn from an infinite set $[\underline{\sigma}, \bar{\sigma}]$. It is well-known that sequential equilibria may fail to exist in infinite games, even when (as is the case here) payoff functions are continuous.

To establish equilibrium existence, we exploit Manelli's (1996) sufficient conditions for a canonical signaling game, in which a single "sender" of unknown type chooses an action, and a single uninformed "receiver" responds. To apply his results, we must first show that the aggregate behavior of the n partially informed voters in our model matches that of a single uninformed receiver endowed with suitable preferences.

At the extreme of pure common values ($\lambda = 0$) the voting stage is a game of common interest, and McLennan (1998) shows (see his Theorem 2) that if a symmetric voting strategy maximizes total voter welfare it is an equilibrium. Since there is at most one symmetric responsive equilibrium in our model, the converse is also true. That is, for an arbitrary symmetric profile of voter cutoff voting strategies $\hat{\sigma}$, define $u_i(x, \hat{\sigma}; b, \lambda, \alpha)$ as the expected utility of voter i given offer x. Then if the maximizer of $u_i(x, \cdot; \lambda = 0)$ lies in $(\underline{\sigma}, \bar{\sigma})$, it coincides with the (unique symmetric) responsive equilibrium. Consequently, under pure common values the equilibrium behavior of voters matches that of a single uninformed player who chooses a strategy $\hat{\sigma}$ to maximize $u_i(x, \cdot; \lambda = 0)$.

Away from the common values extreme our game is not one of common interest, and mapping equilibrium behavior of n voters to that of a single player is more complicated:[22]

equilibrium is the non-responsive equilibrium in which each voter always rejects. A proof is available from the authors' webpages. Moreover, although when $x \geq \bar{x}_n$ both the acceptance and rejection equilibria are trembling-hand perfect, the trembles required to support the rejection equilibrium do not satisfy the cutoff rule property we discussed earlier. Indeed, if tremble strategies were required to satisfy the mild monotonicity restriction that voting to accept is weakly more likely after a higher signal, then the acceptance equilibrium would be the *only* trembling-hand perfect equilibrium when $x \geq \bar{x}_n$.

[22]The application of McLennan's Theorem 2 described above relies on total voter welfare being maximized at an interior voting strategy, $\sigma \in (\underline{\sigma}, \bar{\sigma})$. Moreover, in McLennan's model signals are drawn

Lemma 3 ***(Equivalent signaling game)*** *Fix n, λ, α. Suppose the proposer offers x and voters' beliefs about the proposer's observation σ_0 are given by the probability distribution φ on $[\underline{\sigma}, \bar{\sigma}]$. Then the equilibrium σ_n^* of the voting stage coincides with the best-response of a single fictitious receiver with the same belief and whose payoff depends on the offer x, sender action σ_0, and his own action σ' according to*

$$U_n\left(x, \sigma', \sigma_0; \lambda, \alpha\right) \equiv \int_{\underline{\sigma}}^{\sigma'} -Z\left(x, s, b = p^H(\sigma_0), \lambda, \alpha, n\right) ds, \tag{5}$$

where[23]

$$Z\left(x, \sigma, b, \lambda, \alpha, n\right) \equiv b\Delta^H\left(x, \sigma\right) \ell(\sigma) \left(\frac{F\left(\sigma|H\right)}{F\left(\sigma|L\right)}\right)^{n-n\alpha} \left(\frac{1-F\left(\sigma|H\right)}{1-F\left(\sigma|L\right)}\right)^{n\alpha-1} + (1-b)\,\Delta^L\left(x, \sigma\right).$$

From Lemma 3, Manelli's results imply:

Proposition 1 ***(Equilibrium existence)*** *An equilibrium exists.*

An additional implication of McLennan's results is that at the pure common values extreme the effect of an exogenous increase in the offer x on voter welfare can be evaluated by considering only the *direct* effect, i.e., is given by $\frac{\partial}{\partial x} u_i\left(x, \sigma^*\left(x\right)\right)$, where $\sigma^*\left(x\right)$ denotes the voting equilibrium given offer x. That is, even though voting behavior is determined as the outcome of an n-player non-cooperative game, the conclusion of the envelope theorem still holds and there is no need to consider how a change in the offer x affects the likelihood of acceptance (determined by the voting strategy $\sigma^*\left(x\right)$).

Under unanimity rule this argument can be generalized, and the direct effect $\frac{\partial}{\partial x} u_i\left(x, \sigma^*\left(x\right)\right)$ provides a lower bound for how the offer x affects voter welfare:

Lemma 4 ***(Effect of higher offers on voter payoffs)*** *If $\alpha = 1$ or $\lambda = 0$,*

$$\frac{d}{dx} u_i\left(x, \sigma^*\right) \geq \frac{\partial}{\partial x} u_i\left(x, \sigma^*\right). \tag{6}$$

from a finite distribution. Lemma 3 covers both formal difficulties.

[23]Note that $Z\left(x, \sigma, b, \lambda, \alpha, n\right) f(\sigma|L)(1-F\left(\sigma|L\right))^{n\alpha-1} F\left(\sigma|L\right)^{n-n\alpha}$ is equal to the left-hand side of (4) for $\sigma = \sigma_i = \sigma^*$.

4 Unanimity rule

An important property of the unanimity voting game is the existence of relatively moderate offers that voters *always* accept. Consider the offer $x_U(b,\lambda)$, defined implicitly by the unique[24] solution in $(0,1)$ to

$$\Delta^H(x,\underline{\sigma})\,\ell(\underline{\sigma})b + \Delta^L(x,\underline{\sigma})(1-b) = 0 \tag{7}$$

(we write $x_U(b,\lambda)=\infty$ if (7) has no solution). If voters hold belief b they always accept the offer $x_U(b,\lambda)$: under unanimity each voter is always pivotal, and so the vote-to-accept condition (3) is satisfied for all realizations of σ_i. Moreover, $\Delta^L(x_U(b,\lambda),\underline{\sigma})<0$ by Assumption 2, and so voters make themselves worse off by accepting this offer when $\omega=L$. For use below, note that $x_U(b,\lambda)$ is decreasing in b.[25]

Our next result deals with the acceptance probability for offers below $x_U(b,\lambda)$. We write $P_n^\omega(x,b,\lambda,\alpha)$ for the acceptance probability in state ω given offer x, along with $P^\omega(x,b,\lambda,\alpha)$ for the limit value as $n\to\infty$. Recall, moreover, that $x_\omega(\lambda)$ is the minimum offer acceptable to a fully informed voter (with $\sigma_i=\underline{\sigma}$) in state ω.

Lemma 5 ***(Limit acceptance probability under unanimity)*** *Suppose unanimity rule is in effect ($\alpha=1$). If the offer $x\geq x_U(b,\lambda)$ then $P_n^\omega(x,b,\lambda,1)=1$ for all n, for $\omega=L,H$. Moreover, there exists $\check{\lambda}>0$ such that for any $\varepsilon>0$, $P_n^\omega(\cdot)$ converges uniformly over $[0,1-\varepsilon]\times\left[\underline{b},\overline{b}\right]\times\left[0,\check{\lambda}\right]$ to*

$$P^\omega(x,b,\lambda,1)=\begin{cases}0 & \text{if } x\leq x_H(\lambda)\\ \left(-\frac{\Delta^H(x,\underline{\sigma},\lambda)}{\Delta^L(x,\underline{\sigma},\lambda)}\frac{b}{1-b}\ell(\underline{\sigma})\right)^{\frac{\ell(\underline{\sigma})}{1-\ell(\underline{\sigma})}+\mathbf{1}(\omega=L)} & \text{if } x\in(x_H(\lambda),\min\{1,x_U(b,\lambda)\})\\ 1 & \text{if } x\geq\min\{1,x_U(b,\lambda)\}\end{cases}.$$

The limit acceptance probability $P^\omega(x,b,\lambda,1)$ is continuous and increasing in x and b.

Lemma 5 is related to Duggan and Martinelli's (2001) Theorem 4, which characterizes limit acceptance probabilities for intermediate offers when preferences have no private

[24] Equation (7) has at most one solution by Assumption 4.
[25] This is easily shown to follow from Assumption 2.

value component ($\lambda = 0$). The main contribution of Lemma 5 relative to their result is to establish *uniform* convergence. We need this property because we use the limit acceptance probability to characterize the proposer's offer against a large number of voters, and this approach would be invalid without uniform convergence. Uniform convergence of the acceptance probability P_n^ω as a function of the offer x follows from the fact that P_n^ω is monotone in x (see Lemma 1) and has a continuous limit. This is established in Lemma A-3, stated and proved in the Appendix. Lemma A-4 then establishes uniform convergence of P_n^ω as a function of x, b, λ, where P_n^ω need not be monotone in λ.

The limit acceptance probabilities in Lemma 5 reflect the failure of information aggregation under unanimity rule. This leads to, on the one hand, offers above x_H being rejected when $\omega = H$; and on the other hand, offers below x_L being accepted when $\omega = L$. Because of the failure of information aggregation, the proposer's signal σ_0 may affect the acceptance probability, even when the number of voters is large. Consequently the proposer can try to signal his own information σ_0 with the offer he makes. However, σ_0 is only a noisy signal of ω. It follows that it is impossible for the offer to convey enough information to voters to persuade them to accept the offer x_H.[26] So regardless of the equilibrium played, the proposer must raise his offer a discrete amount above x_H in order to obtain a reasonable acceptance probability.

Proposition 2 ***(Equilibrium offer under unanimity)*** *Under unanimity rule ($\alpha = 1$) there exists $\check{\lambda} > 0$, $\kappa > 0$, and N such that for all σ_0, $\lambda \in \left[0, \check{\lambda}\right]$, in any equilibrium the proposer's offer always lies in $(x_H(\lambda) + \kappa, 1 - \kappa)$ when $n \geq N$, and is always less than $x_U(\underline{b}, \lambda)$ (regardless of n).*

Proposition 2 says that the mistakes voters make under unanimity rule force the proposer to offer strictly more than x_H. Combined with Lemma 5, this result implies

[26] Even if the proposer's signal σ_0 perfectly revealed ω, it is readily shown that there is no pure strategy separating equilibrium. Consequently, there is still no equilibrium in which the offer x_H is accepted with high probability under unanimity. A proof is available from the authors' webpages.

that the equilibrium acceptance probability under unanimity rule is bounded away from zero when the preferences are close to common values.

Possibly more surprising, when voters' information is sufficiently poor their mistakes under unanimity can lead to an equilibrium in which the proposer's offer is *always* accepted. To see this, note first that the acceptance probability is convex in the offer x up to x_U.[27] Provided each voter's information is not too informative (i.e., $\ell(\underline{\sigma})$ is not too low), the proposer's payoff is likewise convex. If the proposer is completely uninformed, it follows immediately that he finds it worthwhile to increase his offer all the way to x_U. More generally, our next result establishes that there is a pooling equilibrium in which the proposer always offers x_U and voters always accept.

Lemma 6 ***(Certain acceptance of equilibrium offer)*** *Suppose unanimity rule is in effect ($\alpha = 1$). If $x_U(b = p^H, \lambda) < 1$ then there exists $\underline{\ell}$ and N such that whenever voter information is sufficiently poor such that $\ell(\underline{\sigma}) \geq \underline{\ell}$, and $n \geq N$, there is a pooling equilibrium in which the offer is independent of σ_0 and lies within $1/n$ of $x_U(b = p^H, \lambda)$.*

5 Majority rule

We refer to any non-unanimity voting rule $\alpha < 1$ as a majority rule. To state our results, we need to generalize the $x_\omega(\lambda)$ notation introduced above. For $\omega = L, H$, define $\sigma_\omega(\alpha)$ and $x_\omega(\lambda; \alpha) < 1$ implicitly by $1 - F(\sigma_\omega(\alpha)|\omega) = \alpha$ and $\Delta^\omega(x_\omega(\lambda; \alpha), \sigma_\omega(\alpha), \lambda) = 0$. That is, conditional on ω there is a probability α that the realization of σ_i exceeds $\sigma_\omega(\alpha)$; and $x_\omega(\lambda; \alpha)$ is the proposal that gives a voter i the same payoff as the status quo,[28] given ω and $\sigma_i = \sigma_\omega(\alpha)$. As such, if the state ω were public information, then an offer just above $x_\omega(\lambda; \alpha)$ would be accepted with probability converging to 1 as the number of voters n grows large. Note that $x_\omega(\cdot; \alpha = 1) \equiv x_\omega(\cdot)$ since $\sigma_\omega(1) = \underline{\sigma}$, so that this notation contains the notation of prior sections as a special case. Moreover, under pure common values ($\lambda = 0$) the value $x_\omega(\lambda; \alpha)$ is independent of the voting rule α.

[27] See Lemma A-5 in the Appendix.

[28] If no such offer exists, we write $x_\omega(\lambda; \alpha) = \infty$.

The existing strategic voting literature establishes that majority rule perfectly aggregates information as the number of voters grows large. In terms of the above notation, this means that the limit acceptance probability given state ω is zero if the offer x is less than $x_\omega(\lambda;\alpha)$, and is one if the offer x exceeds $x_\omega(\lambda;\alpha)$. As such, the limit acceptance probability is discontinuous. In contrast, the acceptance probability for any finite number of voters n is continuous (see Lemma 1). An important consequence of these observations is that when majority rule is used the acceptance probabilities *do not* converge uniformly to their limit — in sharp contrast to the case of unanimity rule (Lemma 5).

Because of this failure of uniform convergence, it is not possible to first analyze the equilibrium of the limit game, and then to show that it is also the limit of equilibria of finite games. We deal with this complication by first extending the existing strategic voting literature to the case where the proposal varies with the number of voters. Since there is no reason to require the proposer's offers to have a well-defined limit, we state our result in terms of the limits infimum and supremum. We show that, as in Feddersen and Pesendorfer (1997) and Duggan and Martinelli (2001), the aggregate response of the voting group to an offer x matches that which would be obtained under full information.

Lemma 7 ***(Acceptance probabilities under majority)*** *Suppose a majority voting rule $\alpha < 1$ is in effect. Take any $\lambda \in [0,1]$, and consider a sequence of offers x_n. If $\liminf x_n > x_\omega(\lambda;\alpha)$ then $P_n^\omega(x_n) \to 1$. If $\limsup x_n < x_\omega(\lambda;\alpha)$ then $P_n^\omega(x_n) \to 0$.*

From Lemma 7, the acceptance probabilities are asymptotically constant over each of the ranges $[0, x_H)$, (x_H, x_L), and $(x_L, 1]$. Consequently, when facing a large number of voters using majority rule, the proposer never makes an offer that lies far from the lower ends of these ranges, i.e., 0, x_H, x_L. Since the offer 0 is always rejected when the number of voters is large, the proposer's choice boils down to x_H versus x_L. To determine the proposer's choice between the two, for any σ_0 define

$$W(\sigma_0;\lambda,\alpha) \equiv p^H(\sigma_0)V^H(x_H,\sigma_0) + p^L(\sigma_0)\bar{V}^L(\sigma_0) - E\left[V^\omega(x_L,\sigma_0)\,|\sigma_0\right]. \qquad (8)$$

Economically, $W(\sigma_0)$ is the proposer's gain to offering x_H instead of x_L after observing σ_0. Since the proposer vacuously prefers the offer x_H when $x_L = \infty$, set $W(\sigma_0) = \infty$ in this case. Our formal result is:[29]

Proposition 3 ***(Equilibrium payoffs under majority)*** *Suppose a majority voting rule $\alpha < 1$ is in effect and $x_H(\lambda;\alpha) \neq \infty$. If $W(\sigma_0) > 0$ then the proposer's offer converges to $x_H(\lambda;\alpha)$ and the acceptance probabilities in $\omega = L, H$ converge to 0 and 1 respectively. If $W(\sigma_0) < 0$ then the proposer's offer converges to $x_L(\lambda;\alpha)$ and the acceptance probabilities converge to 1 in both $\omega = L, H$. As $n \to \infty$, the payoffs of the voters and proposer respectively converge to:*

$$E_{\sigma_i,\omega}\left[\bar{U}^{\omega}(\sigma_i)\right] + \int_{\sigma_0 \text{ s.t. } W(\sigma_0)<0} E_{\sigma_i,\omega}\left[\Delta^{\omega}(x_L,\sigma_i,\lambda)\,|\sigma_0\right] dF_0(\sigma_0)$$
$$+ \int_{\sigma_0 \text{ s.t. } W(\sigma_0)>0} p^H(\sigma_0) E_{\sigma_i}\left[\Delta^{H}(x_H,\sigma_i,\lambda)\,|H\right] dF_0(\sigma_0),$$

and

$$\int_{\sigma_0 \text{ s.t. } W(\sigma_0)<0} E_{\omega}\left[V^{\omega}(x_L,\sigma_0)\,|\sigma_0\right] dF_0(\sigma_0)$$
$$+ \int_{\sigma_0 \text{ s.t. } W(\sigma_0)>0} \left(p^H(\sigma_0) V^H(x_H,\sigma_0) + p^L(\sigma_0)\bar{V}^L(\sigma_0)\right) dF_0(\sigma_0).$$

Note that because the true realization of ω is asymptotically revealed under majority voting, there is no scope for the proposer's offer to convey useful information. Consequently the signaling aspect of the bargaining game disappears. The equilibrium outcome is then asymptotically unique.[30]

Next, consider what economic circumstances lead W to be positive, and so to the proposer offering x_H against majority rule. First, W is increasing in x_L and decreasing in x_H. As such, the proposer is more likely to offer x_H if a voter's payoff relative to the status quo in state L is low (i.e., Δ^L low); or a voter's payoff relative to the status quo in

[29]The proof of Proposition 3 uses the very mild assumption that $W(\sigma_0;\lambda,\alpha) = 0$ for at most finitely many values of σ_0 when $x_L(\lambda;\alpha) \neq \infty \neq x_H(\lambda;\alpha)$. Whenever the effect of σ_0 on the proposer's preferences is weak enough, $W(\sigma_0) = 0$ for at most one value of σ_0, and so this assumption is satisfied.

[30]More accurately, the equilibrium is unique within the class of symmetric voter equilibria, and given our standard equilibrium selection rule that chooses a responsive equilibrium whenever one exists.

state H is high (i.e., Δ^H high). Second, turning to the proposer's own preferences, the proposer is more likely to offer x_H if his status quo payoff in state L (i.e., $\bar{V}^L$) is high; or the cost of increasing the offer in state H (i.e., $\left|\frac{\partial V^H}{\partial x}\right|$) is high; or the value of having an offer accepted in state L (i.e., V^L) is low.

The main difficulty in proving Proposition 3 lies in showing that when W is positive (respectively, negative) the proposer's offer converges uniformly to x_H (respectively, x_L) with respect to σ_0. Uniform convergence is necessary in order to evaluate the expected payoffs of the proposer and voters.

From the preceding results, although majority rule aggregates information efficiently independently of whether or not the proposer observes an informative signal, the proposer's signal *does* affect the offer he makes. *A priori*, one might conjecture that since voters and the proposer have opposing preferences, voters would prefer to deal with an uninformed proposer. However, there are circumstances under which this is not true. Specifically, consider the case in which proposer preferences are independent of σ_0, and the cutoff $\hat{\sigma}_0$ at which $W = 0$ is low. Here, the proposer makes the high offer x_L whenever $\sigma_0 < \hat{\sigma}_0$, and the low offer x_H otherwise. In contrast, consider the offer made by a completely uninformed proposer with the same preferences. The W function for this proposer is simply the integral of $W(\sigma_0)$ over all possible realizations of the informed proposer's signal σ_0. Because $\hat{\sigma}_0$ was assumed to be low, this integral is positive, and so the uninformed proposer makes the low offer x_H. It follows that voters prefer to deal with the informed proposer whenever they have close to common values preferences, since the informed proposer makes the high offer x_L at least sometimes.

6 Comparing majority and unanimity voting rules

We are now ready to compare equilibrium payoffs under majority and unanimity rule. We focus on voter preferences that are not too far from pure common values (i.e., λ close enough to 0).

From Section 5, under majority rule α the proposer either makes a low offer $x_H(\alpha)$ that is accepted only in the good state H, or else makes a higher offer $x_L(\alpha)$ that is accepted in both states. His choice depends on the additional cost of making the higher offer (i.e., the distance $x_L(\alpha) - x_H(\alpha)$) and the extent to which he dislikes the status quo, and is summarized by the function W.

In contrast, under unanimity the proposer makes an offer that is (A) strictly greater than $x_H(\alpha)$, but (B) lower than $x_U(\underline{b})$. Close to common values $x_U(\underline{b}) < x_L(\alpha)$ whenever $x_L(\alpha) \neq \infty$. Comparison (A) means that when the proposer would offer $x_H(\alpha)$ against majority, voters' mistaken reluctance to accept offers close to $x_H(\alpha)$ under unanimity helps them by inducing the proposer to offer more. Conversely, comparison (B) means that if the proposer would offer $x_L(\alpha)$ against majority, voters' mistakes under unanimity instead help the proposer, since voters accept x_U too readily.

The above comparisons relate to the proposer's offer, but the effects on voter welfare may be different because of the offer's effect on acceptance probabilities. However, Lemma 4 is enough to ensure that the direct effect of changing the offer dominates, and voter welfare is higher if the offer is higher:

Proposition 4 ***(Voter welfare and voting rules)*** *Fix a majority rule $\alpha < 1$. Then:*

(I) If $W(\cdot; \lambda = 0, \alpha)$ is strictly positive for all σ_0, then for all λ sufficiently small voters strictly prefer unanimity to the majority rule α when n is sufficiently large.

(II) If $W(\cdot; \lambda = 0, \alpha)$ is strictly negative for all σ_0, and additionally Δ^ω is strictly increasing in the offer x,[31] *then for all λ sufficiently small, voters strictly prefer the majority rule α to unanimity rule when n is sufficiently large.*

[31] As discussed prior to Proposition 4, when W is positive the proposer offers more against unanimity, while when W is negative the proposer offers less. In the former case Assumption 4 alone guarantees that voters are better off from the higher offer, because the lower offer against majority drives their utility all the way down to the status quo level. In contrast, in the latter case we need an additional assumption on preferences to ensure that unanimity hurts voters in addition to helping the proposer.

JURY VOTING WITH ENDOGENOUS CHARGES

Feddersen and Pesendorfer (1998) have argued that the unanimity requirement that is common in jury trials leads to too many convictions of innocent defendants and too few convictions of guilty defendants, relative to the alternative of a majority requirement. Proposition 4 identifies a potentially important countervailing advantage of unanimity: it reins in overly aggressive prosecutors, leading them to propose more modest punishments.

Specifically, consider again the jury interpretation of our model (see page 7). The proposer is a prosecutor who chooses a crime to charge, in turn determining a potential prison sentence $1 - x$ to be imposed on a defendant. Exactly as in Feddersen and Pesendorfer (1998), a jury then votes to convict or acquit. We make the following mild assumptions on social preferences, which we assume are shared by jury members: acquitting an innocent defendant is better than imposing any penalty; imposing a small penalty on a guilty defendant is better than acquittal; and the worst penalty ($x = 0$) is so harsh that acquital is preferable, even if the defendant is guilty. That is, $\Delta^L(x) < 0$ for all $x < 1$; and $\Delta^H(x) > 0$ for x close enough to 1; and $\Delta^H(0) < 0$.

Unanimity has the potential to improve overall welfare when there are prosecutors whose interests diverge from society's. Using contemporary data, Boylan (2005) presents evidence that prosecutors seek to maximize the total sentence imposed on defendants: in our notation, $V^\omega(x)$ is decreasing in x. Additionally, many historical accounts emphasize prosecutor aggressiveness (e.g., Beccaria 1775), and modern legal systems may well reflect such antecedents.

Under the above assumptions, the most severe punishment that a fully informed jury would impose on a guilty defendant is given by $x_H \in (0, 1)$, while $x_L = \infty$ since a fully informed jury would always acquit an innocent defendant. From Section 5, an aggressive[32] prosecutor would propose x_H when dealing with a majority jury. Unanimity makes it harder to successfully impose a tough penalty on a defendant, and leads the prosecutor to

[32]That is, a prosecutor for whom V^ω is decreasing in x, for $\omega = L, H$.

moderate the proposed penalty away from x_H (see Proposition 2). Hence although unanimity has the cost of wrongful convictions, it also has the benefit of shielding defendants from overzealous prosecutors. By Proposition 4 the latter effect dominates.

Debt restructuring: creditor rights and issuance yields

A second application of Proposition 4 is restructuring negotiations between a debtor and creditors. To analyze our results' implications, we assume both the debtor and creditors are risk-neutral; creditors like higher offers, i.e., Δ^ω is increasing in x; and that the total surplus gained or lost from restructuring, $S^\omega \equiv U^\omega(x) - \bar{U}^\omega + V^\omega(x) - \bar{V}^\omega$, is independent of the terms of the offer x. That is, x simply determines how the gain/loss from restructuring is shared. Assumptions 3 and 5 imply that the restructuring surplus S^H is positive. For use below, note that if $x_L \neq \infty$ then $U^L(x_L) = \bar{U}^L$, $S^L = V^L(x_L) - \bar{V}^L$, and $W(\sigma_0) = p^H(\sigma_0)\left(V^H(x_H) - V^H(x_L)\right) - p^L(\sigma_0)S^L$. Moreover, $x_L \neq \infty$ if and only if $S^L \geq 0$.[33]

In the U.S., the Trust Indenture Act (TIA) mandates that all bondholders must agree to a given restructuring, i.e., unanimity is required. Our analysis provides a rationale for this requirement, as follows. Consider a firm that cannot meet its debt obligations. Suppose that in $\omega = H$ the firm is nonetheless viable, in the sense that its future profits exceed the proceeds from immediate liquidation; but in $\omega = L$ the firm is not viable in this sense, and so the surplus from restructuring is negative, $S^L < 0$. Since $x_L = \infty$, the indebted firm would offer x_H when negotiating with creditors bound by majority. In contrast, if the unanimous consent of creditors is required, the debtor improves his offer, and the welfare of creditors is increased (Propostion 4).

Until recently, sovereign debt issued in the U.S. followed the TIA and required bondholders to agree unanimously to any restructuring. In contrast, sovereign debt issues in other jurisdictions, including the U.K., commonly include collective action clauses whereby debt can be restructured if more than a pre-specified fraction (e.g., 75%) of

[33] If $x_L \neq \infty$ then $S^L = V^L(x_L) - \bar{V}^L \geq 0$ by Assumption 5. Conversely, if $x_L = \infty$ then $U^L(1) < \bar{U}^L$ by Assumption 4, and so $S^L = U^L(1) - \bar{U}^L + V^L(1) - \bar{V}^L < 0$ by Assumption 5.

bondholders agree. This cross-sectional variation in voting rules used by otherwise comparable securities provides a natural venue to check the predictions of our analysis. To recap, Proposition 4 implies that bondholders fare better (worse) in restructuring under U.S. law, and hence yields at issuance are lower (higher), if W is positive (negative).

Eichengreen and Mody (2004) have empirically examined the relationship between bond yields, issue location (i.e., voting rule), and issuer credit quality.[34] They find that issuance yields in the U.S. are lower (respectively, higher) than in the U.K. for less (respectively, more) credit-worthy borrowers. Provided that surplus from restructuring in state L is increasing in the credit-worthiness of the borrower, this finding is exactly as our analysis predicts, since W is decreasing in the restructuring surplus S^L.

Debt restructuring: choice of governing law

Eichengreen and Mody also empirically examine where sovereigns issue in the first place. If the issuing country is not credit-constrained (so that side-payments are possible), one would expect debt to be issued subject to the voting rule that maximizes overall surplus.[35] At first glance, it might seem that this criterion favours majority voting rules, because of their efficient aggregation of information. However, this neglects the debtor's choice of offer in restructuring negotiations, as we now show.

Consider values of restructuring surplus S^L in $\left(0, \frac{p^H(\bar{\sigma})}{p^L(\bar{\sigma})}\left(V^H(x_H) - V^H(x_L)\right)\right)$. Suppose, moreover, that creditor information quality is sufficiently poor that there is an equilibrum under unanimity in which the debtor's offer is always accepted (Lemma 6).[36] Because restructuring is efficient in both states, total surplus is maximized. In contrast, the surplus S^L is low enough that when the debtor is sufficiently confident the state is

[34] See also Eichengreen, Kletzer and Mody (2003) and the citations therein.

[35] If instead the debtor is credit constrained the voting rule that maximizes creditor payoffs in restructuring may be used in order to raise borrowing capacity, even at the cost of total surplus.

[36] For the comparison that follows, the key property of the equilibrium outcome under unanimity is that agreement is reached with high probability. Focusing on the equilibrium in which proposers pool at the offer x_U guarantees this. Moreover, under pure common values, at the extreme cases of an uninformed proposer and a fully informed proposer, there is no pure strategy separating equilibrium. This is vacuously true in the former case; for the latter case a proof is available on the authors' webpages.

H (i.e., σ_0 high enough) he reduces his offer to x_H against majority rule,[37] leading to rejection if the state is in fact L. Hence total surplus is strictly lower under majority.

Conversely, if restructuring surplus S^L falls outside the above range, total surplus is instead maximized under majority rule. This is easily seen by considering equilibrium outcomes under majority. If S^L is negative, there is no offer that would be accepted in state L (i.e., $x_L = \infty$), and so the debtor offers x_H. Agreement is reached precisely in state H, which maximizes surplus. At the other extreme, if S^L exceeds $\frac{p^H(\bar{\sigma})}{p^L(\bar{\sigma})}\left(V^H(x_H) - V^H(x_L)\right)$ he prefers to raise his offer to x_L to guarantee successful restructuring in both states L and H — and again, surplus is maximized.[38]

To conclude, our analysis implies that unanimity maximizes total surplus when S^L is neither too high nor too low. Empirically, Eichengreen and Mody find that countries with intermediate levels of credit-worthiness are more likely to issue in the U.S. (i.e., choose unanimity), while countries with either higher or lower levels of credit-worthiness are more likely to issue in the U.K. Provided that S^L is monotone in the creditworthiness of the issuing debtor, this finding is again consistent with our model.

Pareto dominance of unanimity rule

Identifying the surplus-maximizing voting rule is relevant when utility is fully transferable — as it is when agents are risk-neutral and side-payments are possible. But even when these assumptions are not satisfied, there still exist circumstances under which unanimity passes the more demanding criterion of Pareto dominance:

Proposition 5 ***(Pareto dominance of unanimity)*** *There exist economies such that for any majority rule $\alpha < 1$, both the proposer and voters strictly prefer unanimity rule when n is sufficiently large (regardless of the equilibrium played).*

Unanimity rule can Pareto dominate majority, in spite of the latter's superior information aggregation properties, because unanimity can lead to better offers and corre-

[37] Formally, $W(\sigma_0) > 0$.

[38] When $S^L > \frac{p^H(\bar{\sigma})}{p^L(\bar{\sigma})}\left(V^H(x_H) - V^H(x_L)\right)$, total surplus under majority rule and in the pooling equilibrium at x_U under unanimity rule is the same.

sponding higher acceptance probabilities. As discussed above, this effect can increase total surplus. To establish Pareto dominance, one then simply needs to check that the cost to the proposer of the higher offer is not too great.

In the special case of pure common values and a completely uninformed proposer, it is straightforward to construct an economy in which unanimity Pareto dominates, as follows. First, construct an economy in which $W = 0$,[39] and so the proposer is indifferent between the offer x_H being accepted in state H, and the offer x_L being accepted always. From Section 5, as n grows large the proposer's utility converges to $E_\omega [V^\omega (x_L)]$. However, under unanimity the offer $x_U < x_L$ is accepted always, and so the proposer's utility is *at least* $E_\omega [V^\omega (x_U)] > E_\omega [V^\omega (x_L)]$, and the proposer strictly prefers unanimity for n large enough. By continuity the same is true if the economy is perturbed slightly by increasing $\bar{V}^L$. Under this perturbation, W is strictly positive, and Proposition 4 implies that voters *also* prefer unanimity.

We close with three remarks:

Remark 1 *An immediate corollary of Proposition 4 is that unanimity Pareto dominates majority* only if $W(\sigma_0) \geq 0$ *for some* σ_0*, or equivalently, the proposer offers* x_H *against majority rule with positive probability.*

Remark 2 *For an uninformed proposer and pure common values, the converse of Proposition 5 is easily proved: there is no economy in which majority Pareto dominates unanimity for large* n*. To see this, simply note that by Proposition 4 voters prefer majority rule only if* $W \leq 0$*; but then the argument immediately prior to Remark 1 implies that the proposer strictly prefers unanimity.*

Remark 3 *Although we have proved Proposition 5 using an economy with pure common values and an uninformed proposer, neither feature is essential. A proof is contained in an earlier working paper.*

[39] Given pure common values and an uninformed proposer, W is independent of σ_0.

7 Concluding remarks

In this paper we analyze a strategic voting game in which the agenda is set endogenously. We show that unanimity rule may be the preferred voting rule not only of the voting group, but also of the opposing party. These results contrast sharply with the results of the existing strategic voting literature that has analyzed voting over exogenous agendas.

Inevitably our analysis has neglected some important issues. We focus almost exclusively on equilibrium payoffs as the group size grows large. The chief reason for this focus is that it allows us to establish our results with fewer assumptions on preferences and the distributional properties of agents' information. Numerical simulations[40] suggest that the group size needed for our asymptotic results to apply is not large — in many cases the equilibrium with twelve agents is very close to the limiting equilibrium.

Our analysis has focused primarily on common values environments in which voters' preferences are aligned. Of course, when preferences are close to pure private values agreement is very hard to obtain under unanimity rule. Related, to ensure that our results do not depend on complete preference alignment, we have established all our main results for the case in which voter preferences are not perfectly aligned, but instead are "sufficiently close" to common values. An alternative robustness check would be to consider the case in which a fraction $1-\varepsilon$ voters have pure common values preferences, while the remaining fraction ε have extreme private valuations. In such circumstances, unanimous agreement would be impossible to obtain asymptotically. However, a version of our results should still hold when the number of voters is not too large. As we discussed above, acceptance probabilities converge relatively quickly to their limiting expressions.

We conclude with a discussion of implications our analysis has for pre-vote communication, i.e., deliberation. In our analysis, the role of voting is to aggregate information, and no communication is permitted. As is well-known, when voters have biases, full information sharing during communication is not always possible (see Coughlan 2000,

[40] Available from the the authors' webpages.

Meirowitz 2005, Austen-Smith and Feddersen, 2006).[41] In contrast, when there are no biases, as in the pure common values case, voters would share their information truthfully when voting is over an exogenous agenda. The same is true when voting is over an endogenous agenda and voters are worse off under unanimity rule due to mistakes. Note however that the mistakes often benefit voters by inducing the proposer to make a better offer. In this case, voters would want to *ex ante* commit not to communicate *ex post* (i.e., after the offer is made).[42] Of course, *ex post* they still wish to change their minds and communicate, but when the number of voters is large such communication will be hard to achieve without pre-existing arrangements. As such, our analysis complements Austen-Smith and Feddersen's (2006) result that when the agenda is exogenous voters may not communicate truthfully under unanimity rule when their interests are imperfectly aligned. Our analysis implies that even when interests are perfectly aligned, voters may still not communicate truthfully, because by refraining from communication they generate a better (endogenous) offer.

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[41] See also Doraszelski *et al* (2003), and Gerardi and Yariv (2007) for communication in strategic voting games. See also Caillaud and Tirole (2007) which adopts a mechanism design approach to communication, and emphasizes the need to distill information selectively to create persuasion cascades.

[42] An additional reason why voters may not communicate truthfully is that the proposer may be able to observe their messages. In this case, voters have an incentive to convince the proposer that the state is L, and that, as such, he needs to make an attractive offer.

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Appendix

We repeatedly use the following straightforward result. The proof is available on request.

Lemma A-1 *$F(\sigma|H)/F(\sigma|L)$ is increasing in σ, and is bounded above by 1. Consequently, $F(\sigma|H) \leq F(\sigma|L)$, and is strict if $\sigma \in (\underline{\sigma}, \bar{\sigma})$. Moreover, $(1-F(\sigma|H))/(1-F(\sigma|L))$ is increasing in σ, and is bounded above by $\ell(\bar{\sigma}) > 1$.*

Proof of Lemma 1

The proof of Lemma 1 uses the Theorem of the Maximum (see Berge 1963). For completeness we state the relevant half of the Theorem here:

Theorem of the Maximum: *Let $C \subset \mathbb{R}^k$ and $D \subset \mathbb{R}^l$, $g : C \times D \to \mathbb{R}$ a continuous function, and $h : C \to D$ a compact-valued and continuous correspondence. Then $\max_{d \in h(c)} g(c,d)$ is continuous in c.*

We start by defining the values $\underline{x}_n(b,\lambda,\alpha)$, $\bar{x}_n(b,\lambda,\alpha)$ named in the Lemma's statement. Take Z as defined in the statement of Lemma 3. Observe that if $Z(x,\sigma)$ is positive (negative), and all but one of the voters use a cutoff strategy σ, then the remaining voter i is better off voting to accept (reject) the proposal x if he observes $\sigma_i = \sigma$. Similarly, if $Z(x,\sigma) = 0$ then there is a responsive equilibrium in which all voters use the cutoff strategy σ.

By the Theorem of the Maximum, $\max_{\sigma \in [\underline{\sigma}, \bar{\sigma}]} Z(x,\sigma)$ and $\min_{\sigma \in [\underline{\sigma}, \bar{\sigma}]} Z(x,\sigma)$ are both continuous in x. So we can define $\underline{x}_n(b,\lambda,\alpha)$ and $\bar{x}_n(b,\lambda,\alpha)$ that describe the range of offers for which a responsive equilibrium exists:

$$\underline{x}_n(b,\lambda,\alpha) = \begin{cases} \min\{x | \max_\sigma Z(x,\sigma) \geq 0\} & \text{if } \{x | \max_\sigma Z(x,\sigma) \geq 0\} \neq \varnothing \\ 1 & \text{otherwise} \end{cases} \quad \text{(A-1)}$$

$$\bar{x}_n(b,\lambda,\alpha) = \begin{cases} \max\{x | \min_\sigma Z(x,\sigma) \leq 0\} & \text{if } \{x | \min_\sigma Z(x,\sigma) \leq 0\} \neq \varnothing \\ 0 & \text{otherwise} \end{cases} \quad \text{(A-2)}$$

That is, $\underline{x}_n(b,\lambda,\alpha)$ is the lowest offer that is ever accepted in a responsive equilibrium: if $x < \underline{x}_n(b,\lambda,\alpha)$, then $Z(x,\sigma) < 0$ for all σ. Similarly, $\bar{x}_n(b,\lambda,\alpha)$ is the highest offer

that is ever rejected in a responsive equilibrium.

Next, and for use below, note that if $Z(x,\sigma) \geq 0$, then it must be the case that $\Delta^H(x,\sigma) > 0$ by Assumption 2. This implies that $Z(x,\sigma)$ is strictly increasing in σ whenever $Z(x,\sigma) \geq 0$. In turn, $Z(x,\sigma') < 0$ for all $\sigma' < \sigma$ if $Z(x,\sigma) = 0$.

Part 1: By definition, if $x < \underline{x}_n$ then $Z(x,\cdot) < 0$, while if $x > \bar{x}_n$ then $Z(x,\cdot) > 0$. For $x \in [\underline{x}_n, \bar{x}_n]$ we claim that $Z(x,\sigma) = 0$ for some unique σ, which we write as $\sigma^*(x)$. Existence is immediate, since $\max_\sigma Z(x,\sigma) \geq 0 \geq \min_\sigma Z(x,\sigma)$, and $Z(x,\sigma)$ is continuous in σ. Uniqueness follows from the result we have just shown that $Z(x,\sigma)$ is strictly increasing in σ whenever $Z(x,\sigma) \geq 0$.

Part 2: To see that $\sigma^*(x)$ is decreasing, consider x and $x' \in (x,1)$ in $(\underline{x}_n, \bar{x}_n)$. Since $Z(x,\sigma^*(x)) = 0$, it follows from Assumption 4 that $Z(x',\sigma^*(x)) > 0$. Since $Z(x',\sigma)$ is increasing in σ it must be the case that $\sigma^*(x') < \sigma^*(x)$. By the Implicit Function Theorem, $\sigma(x)$ is continuously differentiable over $(\underline{x}_n, \bar{x}_n)$. To see $\sigma^*(\underline{x}_n) = \bar{\sigma}$, suppose to the contrary that $\sigma^*(\underline{x}_n) < \bar{\sigma}$. By definition $Z(\underline{x}_n, \sigma^*(\underline{x}_n)) = 0$, and so $Z(\underline{x}_n, \bar{\sigma}) > 0$. By continuity there exists an $x < \underline{x}_n$ such that $Z(x,\bar{\sigma}) > 0$ as well. This contradicts the definition of $\underline{x}_n$. Likewise, to see $\sigma^*(\bar{x}_n) = \underline{\sigma}$ suppose to the contrary that $\sigma^*(\bar{x}_n) > \underline{\sigma}$. By definition $Z(\bar{x}_n, \sigma^*(\bar{x}_n)) = 0$ which implies that $Z(\bar{x}_n, \underline{\sigma}) < 0$. By continuity there exists an x such that $x > \bar{x}_n$ and $Z(x,\sigma) < 0$, contradicting the definition of $\bar{x}_n$.

Part 3: Immediate from the observation that as $n \to \infty$,

$$\ell(\sigma) \left(\frac{F(\sigma|H)}{F(\sigma|L)} \right)^{n-n\alpha} \left(\frac{1-F(\sigma|H)}{1-F(\sigma|L)} \right)^{n\alpha-1}$$

converges to 0 and ∞ respectively for $\sigma = \underline{\sigma}, \bar{\sigma}$. ∎

Proof of Lemma 2

Given the equilibrium selection rule, the comparative static in x is immediate from Lemma 1. For the comparative static in b, observe that (by Assumption 2) if $Z(x,\sigma,b) \geq 0$ then $Z(x,\sigma,b') > Z(x,\sigma,b)$ for $b' > b$. Hence if $x \in (\underline{x}_n(b), \bar{x}_n(b))$ the equilibrium σ^* is decreasing in b, so the acceptance probability is increasing in b. If $x \leq \underline{x}_n(b)$

the statement is vacuously true. Finally, if $x \geq \bar{x}_n(b)$ it suffices to show that $x \geq \bar{x}_n(b')$ for any $b' > b$. To prove this, we must show that $\{x | \min_\sigma Z(x,\sigma,b') \leq 0\} \subset \{x | \min_\sigma Z(x,\sigma,b) \leq 0\}$, for which it is sufficient to show that $Z(x,\sigma,b) \leq 0$ whenever $Z(x,\sigma,b') \leq 0$. To see this, suppose to the contrary that $Z(x,\sigma,b) > 0 \geq Z(x,\sigma,b')$ — in contradiction to above. ∎

Proof of Lemma 3

Define $b_\varphi = \int p^H(\sigma_0)\varphi(d\sigma_0)$. The equilibrium σ_n^* of the voting stage of the game is the unique solution to $Z(x,\sigma,b_\varphi) = 0$, provided a solution exists; is $\underline{\sigma}$ if $Z(x,\sigma,b_\varphi) > 0$ for all $\sigma \in [\underline{\sigma},\bar{\sigma}]$; and is $\bar{\sigma}$ if $Z(x,\sigma,b_\varphi) < 0$ for all $\sigma \in [\underline{\sigma},\bar{\sigma}]$.

The fictitious player chooses σ' to maximize

$$\begin{aligned}\int_{\underline{\sigma}}^{\bar{\sigma}} U_n(x,\sigma',\sigma_0)\,\varphi(d\sigma_0) &= \int_{\underline{\sigma}}^{\bar{\sigma}} \left(\int_{\underline{\sigma}}^{\sigma'} -Z\left(x,s,b=p^H(\sigma_0)\right) ds \right) d\varphi(d\sigma_0) \\ &= \int_{\underline{\sigma}}^{\sigma'} \left(\int_{\underline{\sigma}}^{\bar{\sigma}} -Z\left(x,s,b=p^H(\sigma_0)\right) d\varphi(d\sigma_0) \right) ds \\ &= \int_{\underline{\sigma}}^{\sigma'} -Z(x,s,b_\varphi)\,ds.\end{aligned}$$

(The change of integration order in the second equality follows from standard arguments, while the third equality follows from the linearity of Z in b.)

By prior arguments (see the proof of Lemma 1) we know that if $Z(x,\hat{\sigma},b_\varphi) = 0$ for some $\hat{\sigma}$, then $Z(x,\sigma,b_\varphi) < 0$ for $\sigma < \hat{\sigma}$ and $Z(x,\sigma,b_\varphi) > 0$ for $\sigma > \hat{\sigma}$. It follows that if $Z(x,\sigma_n^*,b_\varphi) = 0$ then σ_n^* is the unique maximizer of $\int_{\underline{\sigma}}^{\sigma'} -Z(x,s,b_\varphi)\,ds$; if $Z(x,\sigma,b_\varphi) > 0$ for all $\sigma \in [\underline{\sigma},\bar{\sigma}]$ then the unique maximizer of $\int_{\underline{\sigma}}^{\sigma'} -Z(x,s,b_\varphi)\,ds$ is $\underline{\sigma}$; and finally, if $Z(x,\sigma,b_\varphi) < 0$ for all $\sigma \in [\underline{\sigma},\bar{\sigma}]$ then the unique maximizer of $\int_{\underline{\sigma}}^{\sigma'} -Z(x,s,b_\varphi)\,ds$ is $\bar{\sigma}$. This completes the proof. ∎

The proof of Proposition 1 uses Manelli's (1996) Corollary 3, stated below:

Corollary 3 (Manelli): *Consider a signaling game in which the sender's type σ_0 is drawn from $[\underline{\sigma},\bar{\sigma}]$; the sender's action x is drawn from a compact subset $X \subset [0,1]$; the receiver chooses an action $\sigma' \in [\underline{\sigma},\bar{\sigma}]$; and both the sender and receiver payoffs are*

continuous in (σ_0, x, σ')*. Suppose moreover that given his belief, the receiver has a unique best response to any offer* x*; and the game is strongly monotonic, in the sense that if the receiver weakly prefers response* σ' *to* σ'' *when his type is* σ_0*, with the opposite true when his type is* $\sigma_0 \neq \sigma_0'$*, then* $\sigma' = \sigma''$*. Then the game has a sequential equilibrium.*

Proof of Proposition 1: As established in Lemma 3, it is possible to replace the n voters with a single uninformed fictitious agent with preferences defined in (5). The proposer strictly prefers more acceptance (lower values of σ_n^*), regardless of his "type" σ_0 and offer x. Moreover, for any belief the best-response of the fictitious agent is a pure-strategy. As such, the game is strongly monotonic, and so possesses a sequential equilibrium (Corollary 3, Manelli). ∎

Proof of Lemma 4

Differentiability follows directly from Lemma 1. If $x < \underline{x}_n$ or $x > \bar{x}_n$, it is immediate that (6) holds at equality, since over these regions $\sigma_n^*(x)$ is constant (and equal to $\bar{\sigma}$ and $\underline{\sigma}$ respectively). For the intermediate case $x \in (\underline{x}_n, \bar{x}_n)$ we also need to account for the effect changing x has on the equilibrium voting strategies. We use a generalized version of the notation in the main text: for an arbitrary profile of voter cutoff voting strategies $\hat{\sigma} = (\hat{\sigma}_1, \ldots, \hat{\sigma}_n)$, define $u_i(x, \hat{\sigma}; b, \lambda, \alpha)$ as the expected utility of voter i given offer x. Since $x \in (\underline{x}_n, \bar{x}_n)$ the voting equilibrium is responsive. Let $\sigma^* = (\sigma_n^*(x), \ldots, \sigma_n^*(x))$. By definition, in equilibrium no voter can increase his utility by adopting a different voting strategy: $\frac{\partial}{\partial \hat{\sigma}_i} u_i(x, \sigma^*) = 0$. The total derivative of voter utility with respect to the offer x is

$$\frac{d}{dx} u_i(x, \sigma^*) = \frac{\partial}{\partial x} u_i(x, \sigma^*) + \sum_{j=1}^{n} \frac{\partial \sigma_n^*(x)}{\partial x} \frac{\partial}{\partial \hat{\sigma}_j} u_i(x, \sigma^*), \tag{A-3}$$

where the first term captures the direct effect, and the second term reflects how a change in the offer affects the acceptance probability. By the envelope theorem, the fact that voter j chooses $\sigma_n^*(x)$ to maximize his utility implies that $\frac{\partial}{\partial \hat{\sigma}_j} u_j(x, \sigma^*) = 0$ for all j, x.

Pure common values case ($\lambda = 0$): Here $u_j \equiv u_i$ for all voters i, j, and so $\frac{\partial}{\partial \hat{\sigma}_j} u_i(x, \sigma^*) = 0$ for all i, j, x. Hence $\frac{d}{dx} u_i(x, \sigma^*) = \frac{\partial}{\partial x} u_i(x, \sigma^*)$, from (A-3).

UNANIMITY CASE ($\alpha = 1$): For any common cutoff strategy $\hat{\sigma}$

$$\begin{aligned}\frac{\partial}{\partial \hat{\sigma}_i} u_i(x, \hat{\sigma}, b, \lambda, \alpha) &= E_\omega\left[-f(\hat{\sigma}|\omega)(1 - F(\hat{\sigma}|\omega))^{n-1} \Delta^\omega(x, \hat{\sigma})\right] \\ \frac{\partial}{\partial \hat{\sigma}_j} u_i(x, \hat{\sigma}, b, \lambda, \alpha) &= E_\omega\left[-f(\hat{\sigma}|\omega)(1 - F(\hat{\sigma}|\omega))^{n-1} E\left[\Delta^\omega(x, \sigma_i)|\sigma_i \geq \hat{\sigma}\right]\right].\end{aligned}$$

Observe that $\frac{\partial}{\partial \hat{\sigma}_i} u_i(x, \hat{\sigma}, b, \lambda, \alpha) \geq \frac{\partial}{\partial \hat{\sigma}_j} u_i(x, \hat{\sigma}, b, \lambda, \alpha)$ since $E\left[\Delta^\omega(x, \sigma_i)|\sigma_i \geq \hat{\sigma}\right] \geq \Delta^\omega(x, \hat{\sigma})$ by Assumption 2. In equilibrium $\frac{\partial}{\partial \hat{\sigma}_i} u_i(x, \hat{\sigma}, b, \lambda, \alpha) = 0$. Since $\partial \sigma_n^* / \partial x < 0$ (see Lemma 1), the result then follows from (A-3). ∎

PROOF OF LEMMA 5 (PRELIMINARIES)

To prove the uniform convergence half of Lemma 5, we make use of the following three results, which we state separately for clarity.

Lemma A-2 *$-\frac{\Delta^H(x, \underline{\sigma}, \lambda)}{\Delta^L(x, \underline{\sigma}, \lambda)}$ is strictly increasing over $[x_H(\lambda), \min\{1, x_L(\lambda)\}]$. Moreover, there exists $\varepsilon > 0$ such that $-\frac{\Delta^H(x, \sigma, \lambda=0)}{\Delta^L(x, \sigma, \lambda=0)}$ is strictly increasing over $[x_H(0) - \varepsilon, \min\{1, x_L(0)\}]$.*

Proof of Lemma A-2: First, note that $-\frac{\Delta^H(x, \underline{\sigma}, \lambda)}{\Delta^L(x, \underline{\sigma}, \lambda)}$ is strictly increasing over $(x_H(\lambda), \min\{1, x_L(\lambda)\}$ To see this, note that over this interval $\Delta^H(x, \underline{\sigma}, \lambda)$ is strictly positive and $\Delta^L(x, \underline{\sigma}, \lambda)$ is strictly negative, and suppose to the contrary that there exist x_1, $x_2 > x_1$, and $K > 0$ such that $-\frac{\Delta^H(x_1, \underline{\sigma}, \lambda)}{\Delta^L(x_1, \underline{\sigma}, \lambda)} \geq K \geq -\frac{\Delta^H(x_2, \underline{\sigma}, \lambda)}{\Delta^L(x_2, \underline{\sigma}, \lambda)}$. Then

$$\Delta^H(x_1, \underline{\sigma}, \lambda) + K\Delta^L(x_1, \underline{\sigma}, \lambda) \geq 0 \geq \Delta^H(x_2, \underline{\sigma}, \lambda) + K\Delta^L(x_2, \underline{\sigma}, \lambda),$$

contradicting Assumption 4.

Second, note that by Assumptions 2 and 4, $\Delta^\omega(x, \sigma, \lambda = 0)$ is strictly negative for $x < x_H(0)$, $\omega = L, H$, and so $-\frac{\Delta^H(x, \sigma, \lambda=0)}{\Delta^L(x, \sigma, \lambda=0)}$ is likewise strictly negative. So there exists $\varepsilon > 0$ such that $-\frac{\Delta^H(x, \sigma, \lambda=0)}{\Delta^L(x, \sigma, \lambda=0)}$ is strictly increasing over $(x_H(0) - \varepsilon, x_H(0))$. ∎

Lemma A-3 *If $\{g_n : [0,1] \to [0,1]\}$ is a sequence of increasing functions that converge pointwise to a continuous function g, then convergence is uniform.*

Proof of Lemma A-3: Observe that the limit function g is increasing. Take $\varepsilon > 0$. By the continuity of g, choose a finite set of points $0 = x_1 \leq \ldots \leq x_m = 1$ such that

for any $x', x'' \in [x_i, x_{i+1}]$, $|g(x') - g(x'')| \leq \frac{\varepsilon}{2}$. Consequently, $g(x) - \frac{\varepsilon}{2} \leq g(x_i)$ and $g(x) + \frac{\varepsilon}{2} \geq g(x_{i+1})$ for any $x \in [x_i, x_{i+1}]$.

By pointwise convergence, there exists some N such that when $n \geq N$, $|g_n(x) - g(x)| < \frac{\varepsilon}{2}$ for all $x \in \{x_1, \ldots, x_m\}$. Hence $g_n(x_i) \geq g(x_i) - \frac{\varepsilon}{2}$ and $g_n(x_{i+1}) \leq g(x_{i+1}) + \frac{\varepsilon}{2}$.

Take $n \geq N$, and $x \in [0,1]$. Let i be such that $x \in [x_i, x_{i+1}]$. By monotonicity, $g_n(x) \in [g_n(x_i), g_n(x_{i+1})] \subset \left[g(x_i) - \frac{\varepsilon}{2}, g(x_{i+1}) + \frac{\varepsilon}{2}\right] \subset [g(x) - \varepsilon, g(x) + \varepsilon]$, completing the proof. ∎

Lemma A-3 concerns the uniform convergence of monotone functions on a compact subset of the real-line. Next, Lemma A-4 shows that under certain conditions monotonicity in one variable is enough to imply uniform convergence for functions of higher-dimensional Euclidean spaces:

Lemma A-4 *Let $C = [\underline{c}, \bar{c}] \times D$, where D is a compact subset of a Euclidian space, and let $g_n : C \to \mathbb{R}$ be a sequence of continuous functions that are monotone in their first argument and converge pointwise to a continuous function $g : C \to \mathbb{R}$. Suppose that there exists a compact Euclidean set S, a sequence of functions $s_n : C \to S$, and a continuous function $h : C \times S \to \mathbb{R}$ that is strictly monotone and continuously differentiable in its first argument such that, for all n, $g_n(z) = g_n(z')$ whenever $h(z, s_n(z)) = h(z', s_n(z))$. Then for any $\kappa > 0$, g_n converges uniformly to g over $[\underline{c} + \kappa, \bar{c} - \kappa] \times D$.*

Proof of Lemma A-4: Given $\kappa > 0$, write $\hat{C} = [\underline{c} + \kappa, \bar{c} - \kappa] \times D$. Fix $\varepsilon > 0$ and choose $\mu < \kappa$ such that $|g(z) - g(z')| < \frac{\varepsilon}{4}$ whenever $|z - z'| < \mu$ and $z, z' \in C$. Define $\psi = \min_{(z,s) \in C \times S} \left| \frac{\partial h(z,s)}{\partial z_1} \right|$, and choose $\delta \in (0, \mu)$ such that $\delta < \psi\mu$. Choose $\gamma \in (0, \mu)$ such that $|h(z,s) - h(z',s')| < \delta$ whenever $|(z,s) - (z',s')| < \gamma$ and $(z,s), (z',s') \in C \times S$. Select a finite set $D^* \subset D$ such that for all $z_{-1} \in D$, there exists $z'_{-1} \in D^*$ such that $\left|z_{-1} - z'_{-1}\right| < \gamma$.

By Lemma A-3, for any $z_{-1} \in D$ the function sequence $g_n(\cdot, z_{-1})$ converges uniformly to $g(\cdot; z_{-1})$. So there exists some N such that $\left|g_n\left(z_1, z'_{-1}\right) - g\left(z_1, z'_{-1}\right)\right| < \frac{\varepsilon}{2}$ for any $\left(z_1, z'_{-1}\right) \in [\underline{c}, \bar{c}] \times D^*$ whenever $n \geq N$.

Fix $z \in \hat{C}$ and $n \geq N$. Let $z'_{-1} \in D^*$ be such that $\left|z_{-1} - z'_{-1}\right| < \gamma$. Note that

$$h\left(z, s_n\left(z\right)\right) \in \left(h\left(z_1, z'_{-1}, s_n\left(z\right)\right) - \delta, h\left(z_1, z'_{-1}, s_n\left(z\right)\right) + \delta\right).$$

Moreover, as $\tilde{z}_1$ ranges over $(z_1 - \mu, z_1 + \mu)$ the value of $h\left(\tilde{z}_1, z'_{-1}, s_n\left(z\right)\right)$ ranges continuously over a superset of

$$\left(h\left(z_1, z'_{-1}, s_n\left(y, z\right)\right) - \psi\mu, h\left(z_1, z'_{-1}, s_n\left(y, z\right)\right) + \psi\mu\right).$$

Since $\delta < \psi\mu$, there exists some $z'_1 \in (z_1 - \mu, z_1 + \mu)$ such that $h\left(z', s_n\left(z\right)\right) = h\left(z, s_n\left(z\right)\right)$, where $z' = \left(z'_1, z'_{-1}\right) \in [\underline{c}, \bar{c}] \times D^*$. By assumption it follows that $g_n\left(z\right) = g_n\left(z'\right)$, and so

$$\begin{aligned} \left|g_n\left(z\right) - g\left(z\right)\right| &\leq \left|g_n\left(z'\right) - g\left(z'\right)\right| + \left|g\left(z'\right) - g\left(z_1, z'_{-1}\right)\right| + \left|g\left(z_1, z'_{-1}\right) - g\left(z\right)\right| \\ &\leq \frac{\varepsilon}{2} + \frac{\varepsilon}{4} + \frac{\varepsilon}{4} = \varepsilon. \end{aligned}$$

∎

Proof of Lemma 5 (pointwise limits)

Take Z as defined in the statement of Lemma 3. First, consider offers $x \geq x_U\left(b, \lambda\right)$. We claim that if $x_U\left(b, \lambda\right) \neq \infty$ then $\bar{x}_n\left(b, \lambda, \alpha = 1\right) = x_U\left(b, \lambda\right)$ for all n. To see this, note that when $\alpha = 1$, $Z\left(x, \underline{\sigma}\right)$ coincides with the lefthand side of (7), regardless of n. As such, $Z\left(x_U, \underline{\sigma}\right) = 0$, and so $Z\left(x_U, \sigma\right) > 0$ for $\sigma > \underline{\sigma}$. So $\bar{x}_n \geq x_U$. Moreover, it follows that $Z\left(x, \sigma\right) > 0$ for all σ if $x > x_U$, so that $\bar{x}_n = x_U$. Given this, the equilibrium for $x \geq x_U$ is the non-responsive acceptance equilibrium.

Second, consider any offer $x \in \left(x_H\left(\lambda\right), x_U\left(b, \lambda\right)\right)$. There is a responsive equilibrium when n is large enough, as follows. By definition, $\Delta^H\left(x_H\left(\lambda\right), \underline{\sigma}\right) = 0$, and so $\Delta^H\left(\frac{x_H(\lambda)+x}{2}, \underline{\sigma}\right) > 0$. The term $\ell(\sigma)\left(\frac{1-F(\sigma|H)}{1-F(\sigma|L)}\right)^{n-1}$ grows without bound for any $\sigma > \underline{\sigma}$. So by the definition of $\underline{x}_n$ (see (A-1)), $x > \underline{x}_n$ for all n sufficiently large. Moreover, by definition $Z\left(x_U, \underline{\sigma}\right) = 0$. So $Z\left(x, \underline{\sigma}\right) < 0$, and so $x < \bar{x}_n$ (see (A-2)). Hence a responsive equilibrium exists, and the equilibrium condition is

$$-\frac{\Delta^H\left(x, \sigma_n^*, \lambda\right)}{\Delta^L\left(x, \sigma_n^*, \lambda\right)} \frac{b}{1-b} \ell(\sigma_n^*) \left(\frac{1 - F\left(\sigma_n^*|H\right)}{1 - F\left(\sigma_n^*|L\right)}\right)^{n-1} = 1. \tag{A-4}$$

Since $\frac{1-F(\sigma|H)}{1-F(\sigma|L)} > 1$ for $\sigma > \underline{\sigma}$, it follows that

$$\sigma_n^* \to \underline{\sigma} \text{ as } n \to \infty. \tag{A-5}$$

In the proof of their Theorem 4, Duggan and Martinelli (2001) show that (A-4) and (A-5) together imply that

$$\lim P_n^H = \lim \left(1 - F\left(\sigma_n^* | H\right)\right)^n = \left(-\frac{\Delta^H\left(x, \underline{\sigma}, \lambda\right)}{\Delta^L\left(x, \underline{\sigma}, \lambda\right)} \frac{b}{1-b} \ell(\underline{\sigma})\right)^{\frac{f(\underline{\sigma}|H)}{f(\underline{\sigma}|L) - f(\underline{\sigma}|H)}}. \tag{A-6}$$

The limit acceptance probability for $\omega = L$ is then immediate from (A-4) and (A-5).

Third, consider the offer $x = x_H(\lambda)$, and suppose that contrary to the claimed result $\lim \left(1 - F\left(\sigma_n^* | H\right)\right)^n \neq 0$. As such, there exists a subsequence of (σ_n^*), $\left(\sigma_{n_m}^*\right)$ say, such that $\lim_{m\to\infty} \left(1 - F\left(\sigma_{n_m}^* | H\right)\right)^{n_m} > 0$. Since $x_H(\lambda) < x_U(b, \lambda)$, for m large $\sigma_{n_m}^*$ is a responsive equilibrium, and so by (A-5) $\sigma_{n_m}^* \to \underline{\sigma}$. It follows that $\lim_{m\to\infty} \left(1 - F\left(\sigma_{n_m}^* | H\right)\right)^{n_m}$ equals the righthand side of equation (A-6) evaluated at $x = x_H(\lambda)$. However, $-\frac{\Delta^H(x_H(\lambda), \underline{\sigma}, \lambda)}{\Delta^L(x_H(\lambda), \underline{\sigma}, \lambda)} = 0$, and so this contradicts the hypothesis that $\lim_{m\to\infty} \left(1 - F\left(\sigma_{n_m}^* | H\right)\right)^{n_m} > 0$.

Fourth, and finally, if $x < x_H(\lambda)$ then from Lemma 2, $P_n^\omega(x) \leq P_n^\omega(x_H(\lambda))$ for all n. As such, $\lim P_n^\omega(x) = 0$.

For continuity and monotonicity in x, note that the limiting acceptance probability is zero at $x_H(\lambda)$. Moreover, by Lemma A-2 the term inside parentheses in (A-6) increases between $x_H(\lambda)$ and $\min\{1, x_U(b, \lambda)\}$; and if $x_U(b, \lambda) < 1$, it equals 1 at $x = x_U(b, \lambda)$.

For continuity and monotonicity in b, there are four cases to consider. If $x \leq x_H(\lambda)$, the limit acceptance probability is 0 for all b. If $x \in \left(x_H(\lambda), \min_{b \in [\underline{b}, \bar{b}]} x_U(b, \lambda)\right)$ the result is immediate. If $x \in \left[\min_{b \in [\underline{b}, \bar{b}]} x_U(b, \lambda), \max_{b \in [\underline{b}, \bar{b}]} x_U(b, \lambda)\right)$, then since $x_U(b, \lambda)$ is continuous and decreasing in b, there exists $\hat{b} \in [\underline{b}, \bar{b}]$ such that $x = x_U\left(\hat{b}, \lambda\right)$, $x < x_U(b, \lambda)$ if $b < \hat{b}$, and $x > x_U(b, \lambda)$ if $b > \hat{b}$. Monotonicity is then immediate, while continuity follows from $-\frac{\Delta^H(x, \underline{\sigma}, \lambda)}{\Delta^L(x, \underline{\sigma}, \lambda)} \frac{b}{1-b} \ell(\underline{\sigma}) = 1$ at $x = x_U(b, \lambda)$. Finally, if $x \geq \max_{b \in [\underline{b}, \bar{b}]} x_U(b, \lambda)$ then the limit acceptance probability is 1 for all b. ∎

Proof of Lemma 5 (uniform convergence)

From Lemma A-2, $-\frac{\Delta^H(\cdot, \sigma, \lambda=0)}{\Delta^L(\cdot, \sigma, \lambda=0)}$ is strictly increasing from a point below $x_H(\lambda = 0)$ to $\min\{1, x_L(\lambda = 0)\}$, for all σ. Consequently, the same is true for all λ sufficiently

small. Given this, if $x_U(\underline{b}, \lambda = 0) < 1$, choose $\check{\lambda} > 0$ and $\delta \in (0, 2\varepsilon)$ such that $\max_{\lambda \in [0,\check{\lambda}]} x_U(\underline{b}, \lambda) + \delta < \min\left\{1, \min_{\lambda \in [0,\check{\lambda}]} x_L(\lambda)\right\}$ and $-\frac{\Delta^H(x,\sigma,\lambda)}{\Delta^L(x,\sigma,\lambda)}$ is strictly increasing over $[\underline{c}, \bar{c}] \equiv \left[\min_{\lambda \in [0,\check{\lambda}]} x_H(\lambda) - \delta, \max_{\lambda \in [0,\check{\lambda}]} x_U(\underline{b}, \lambda) + \delta\right]$ for $\lambda < \check{\lambda}$. Similarly, if $x_U(\underline{b}, \lambda = 0) \geq 1$, choose $\check{\lambda} > 0$ and $\delta > 0$ such that $-\frac{\Delta^H(x,\sigma,\lambda)}{\Delta^L(x,\sigma,\lambda)}$ is strictly increasing over $[\underline{c}, \bar{c}] \equiv \left[\min_{\lambda \in [0,\check{\lambda}]} x_H(\lambda) - \delta, 1\right]$.

By Assumption 4, the term $\Delta^H(x, \underline{\sigma}, \lambda)$ is strictly negative for $x \in \left[0, \min_{\lambda \in [0,\check{\lambda}]} x_H(\lambda) - \frac{\delta}{2}\right]$ and $\lambda \in \left[0, \check{\lambda}\right]$. Hence there exists $\check{\sigma} > \underline{\sigma}$ such that $\Delta^H(x, \sigma, \lambda)$ is strictly negative for $\sigma \in [\underline{\sigma}, \check{\sigma}]$ and the same values of x and λ. So when $\lambda \in \left[0, \check{\lambda}\right]$, any voter i observing $\sigma_i \leq \check{\sigma}$ would reject any offer $x \in \left[0, \min_{\lambda \in [0,\check{\lambda}]} x_H(\lambda) - \frac{\delta}{2}\right]$. It follows that the acceptance probability converges uniformly to zero over $\left[0, \underline{c} + \frac{\delta}{2}\right] \times \left[\underline{b}, \bar{b}\right] \times \left[0, \check{\lambda}\right]$.

Next, we establish uniform convergence over $\left[\underline{c} + \frac{\delta}{2}, \bar{c} - \frac{\delta}{2}\right] \times \left[\underline{b}, \bar{b}\right] \times \left[0, \check{\lambda}\right]$. To do so, we apply Lemma A-4. Define

$$D = \left[\underline{b}, \bar{b}\right] \times \left[0, \check{\lambda}\right], \quad C = [\underline{c}, \bar{c}] \times D, \quad S = [\underline{\sigma}, \bar{\sigma}] \times \left[0, \check{\lambda}\right],$$

$$h\left(x, b, \lambda, s^1, s^2\right) = -b\frac{\Delta^H(x, s^1, \lambda)}{\Delta^L(x, s^1, \lambda)}\ell(s^1)s^2 + (1 - b)$$

$$s_n(x, b, \lambda) = \left(\sigma_n^*(x, b, \lambda), \left(\frac{1 - F(\sigma_n^*(x, b, \lambda)|H)}{1 - F(\sigma_n^*(x, b, \lambda)|L)}\right)^{n-1}\right),$$

$$g_n(x, b, \lambda) = P_n^\omega(x, b, \lambda).$$

By construction, h is strictly monotone and continuously differentiable in x. It remains to show that if $h(z, s_n(z)) = h(z', s_n(z))$, then $g_n(z) = g_n(z')$.

There are three cases to consider. First, suppose $\sigma_n^*(x, b, \lambda) \in (\underline{\sigma}, \bar{\sigma})$. By construction, $h(z, s_n(z)) = 0$. Suppose that $h(z', s_n(z)) = 0$, Then, by the uniqueness of responsive equilibrium, $\sigma_n^*(x', b', \lambda') = \sigma_n^*(x, b, \lambda)$.

For the next two cases, we claim that if $h_n\left(x, b, \lambda, \sigma, \left(\frac{1-F(\sigma|H)}{1-F(\sigma|L)}\right)^{n-1}\right) \geq 0$ at $\sigma = \underline{\sigma}$, the same is true for all $\sigma \in [\underline{\sigma}, \bar{\sigma}]$; and likewise, if $h_n\left(x, b, \lambda, \sigma, \left(\frac{1-F(\sigma|H)}{1-F(\sigma|L)}\right)^{n-1}\right) \leq 0$ at $\sigma = \bar{\sigma}$, the same is true for all $\sigma \in [\underline{\sigma}, \bar{\sigma}]$. Both these statements follow since $h_n\left(x, b, \lambda, \sigma, \left(\frac{1-F(\sigma|H)}{1-F(\sigma|L)}\right)^{n-1}\right)$ equals $\frac{Z(x,\sigma,b,\lambda,\alpha,n)}{-\Delta^L(x,\sigma,\lambda)}$, and if $Z(x, \sigma, b, \lambda, \alpha, n) \geq 0$ for some σ, the same is true for all higher σ (see proof of Lemma 1).

Now consider the case $\sigma_n^*(x,b,\lambda)=\underline{\sigma}$. By definition, $h_n\left(x,b,\lambda,\sigma,\left(\frac{1-F(\sigma|H)}{1-F(\sigma|L)}\right)^{n-1}\right)\geq 0$ for all σ, and so in particular $h_n\left(x,b,\lambda,\underline{\sigma},\left(\frac{1-F(\underline{\sigma}|H)}{1-F(\underline{\sigma}|L)}\right)^{n-1}\right)\geq 0$. So if

$$h_n\left(x',b',\lambda',\underline{\sigma},\left(\frac{1-F(\underline{\sigma}|H)}{1-F(\underline{\sigma}|L)}\right)^{n-1}\right)=h_n\left(x,b,\lambda,\underline{\sigma},\left(\frac{1-F(\underline{\sigma}|H)}{1-F(\underline{\sigma}|L)}\right)^{n-1}\right),$$

it follows (by above claim) that $h_n\left(x',b',\lambda',\sigma,\left(\frac{1-F(\sigma|H)}{1-F(\sigma|L)}\right)^{n-1}\right)\geq 0$ for all σ, and hence $\sigma_n^*(x',b',\lambda')=\underline{\sigma}$.

Finally, consider the case $\sigma_n^*(x,b,\lambda)=\bar{\sigma}$. By definition, $h_n\left(x,b,\lambda,\sigma,\left(\frac{1-F(\sigma|H)}{1-F(\sigma|L)}\right)^{n-1}\right)\leq 0$ for all σ, and so in particular $h_n\left(x,b,\lambda,\bar{\sigma},\left(\frac{1-F(\bar{\sigma}|H)}{1-F(\bar{\sigma}|L)}\right)^{n-1}\right)\leq 0$. So if

$$h_n\left(x',b',\lambda',\bar{\sigma},\left(\frac{1-F(\bar{\sigma}|H)}{1-F(\bar{\sigma}|L)}\right)^{n-1}\right)=h_n\left(x,b,\lambda,\bar{\sigma},\left(\frac{1-F(\bar{\sigma}|H)}{1-F(\bar{\sigma}|L)}\right)^{n-1}\right),$$

it follows (by above claim) that $h_n\left(x',b',\lambda',\sigma,\left(\frac{1-F(\sigma|H)}{1-F(\sigma|L)}\right)^{n-1}\right)\leq 0$ for all σ, and hence $\sigma_n^*(x',b',\lambda')=\bar{\sigma}$.

The proof is complete if $\bar{c}=1$. For the case $\bar{c}<1$, we establish uniform convergence over $\left[\bar{c}-\frac{\delta}{2},1\right]\times\left[\underline{b},\bar{b}\right]\times\left[0,\check{\lambda}\right]$. This is immediate: if (x,b,λ) is in this range, $x>x_U(b,\lambda)$, and so the acceptance probability equals 1 for all n. ∎

Proof of Proposition 2

For clarity, we suppress $\alpha=1$ throughout. Let $v_n(x,\sigma_0,b,\lambda)$ denote the proposer's expected payoff from an offer x when he has observed σ_0, the voters attach belief b to offer x, and have preferences λ, i.e.,

$$v_n(x,\sigma_0,b,\lambda)\equiv E_\omega\left[\bar{V}^\omega(\sigma_0)+P_n^\omega(x,b,\lambda)\left(V^\omega(x,\sigma_0)-\bar{V}^\omega(\sigma_0)\right)|\sigma_0\right]. \tag{A-7}$$

Define v as the pointwise limit of v_n.

We first show that the proposer offers less than $x_U(\underline{b},\lambda)$. If $x_U(\underline{b},\lambda)=\infty$ this is vacuously true. If instead $x_U(\underline{b},\lambda)\leq 1$, then $x_U(b,\lambda)<x_U(\underline{b},\lambda)$ for all $b>\underline{b}$. From Lemma 5, the offer $x_U(\underline{b},\lambda)$ is always accepted, no matter what belief voters assign to it. So there is no equilibrium in which the offer is strictly more than $x_U(\underline{b},\lambda)$.

Next, we establish the lower bound on the proposer's equilibrium offer. Choose $\check{\lambda} > 0$, $\varepsilon_x > 0$ and $\check{x}$ such that Lemma 5 holds and

$$\max_{\lambda \in \left[0,\check{\lambda}\right]} x_H(\lambda) < \check{x} < \min_{\lambda \in \left[0,\check{\lambda}\right]} \min\left\{x_U(b,\lambda), 1-\varepsilon_x\right\}.$$

From Lemma 5, for any $(b,\lambda) \in \left[\underline{b},\bar{b}\right] \times \left[0,\check{\lambda}\right]$ the limit acceptance probabilities $P^{\omega}(\check{x},b,\lambda)$ are strictly positive for $\omega = L, H$. By continuity and compactness it follows that there exists $\varepsilon > 0$ such that for any $(\sigma_0,b,\lambda) \in [\underline{\sigma},\bar{\sigma}] \times \left[\underline{b},\bar{b}\right] \times \left[0,\check{\lambda}\right]$,

$$v(\check{x},\sigma_0,b,\lambda) > E_{\omega}\left[\bar{V}^{\omega}(\sigma_0)\,|\sigma_0\right] + 4\varepsilon.$$

From Lemma 5, v_n converges uniformly to v over $[0,1-\varepsilon_x] \times [\underline{\sigma},\bar{\sigma}] \times \left[\underline{b},\bar{b}\right] \times \left[0,\check{\lambda}\right]$. This implies that there exists N_1 such that whenever $n \geq N_1$,

$$v_n(\check{x},\sigma_0,b,\lambda) > E_{\omega}\left[\bar{V}^{\omega}(\sigma_0)\,|\sigma_0\right] + 3\varepsilon.$$

Let $\varpi = \max_{\omega,\sigma_0,x} V^{\omega}(x,\sigma_0) - \bar{V}^{\omega}(\sigma_0)$. From Lemma 5, there exists $\kappa > 0$ such that $P^{\omega}(x,b,\lambda) \leq \frac{\varepsilon}{\varpi}$ if $x \in [0, x_H(\lambda) + \kappa]$. Uniform convergence of $P_n^{\omega}(\cdot)$ to $P^{\omega}(\cdot)$ (Lemma 5) implies that there exists N_2 such that $P_n^{\omega}(x_H(\lambda)+\kappa,b,\lambda) \leq \frac{2\varepsilon}{\varpi}$ whenever $n \geq N_2$. Combined with monotonicity of the acceptance probability in the offer x, it follows that if $x \leq x_H(\lambda) + \kappa$ and $n \geq N_2$,

$$v_n(x,\sigma_0,b,\lambda) \leq E_{\omega}\left[\bar{V}^{\omega}(\sigma_0)\,|\sigma_0\right] + 2\varepsilon.$$

Hence for $n \geq \max\{N_1,N_2\}$, for any $(\sigma_0,\lambda) \in [\underline{\sigma},\bar{\sigma}] \times \left[0,\check{\lambda}\right]$ it cannot be an equilibrium for the proposer to offer $x \in [0, x_H(\lambda)+\kappa]$: doing so generates at most 2ε over the status quo payoff, while offering $\check{x}$ generates at least 3ε over the status quo payoff, regardless of beliefs. This completes the proof. ∎

Proof of Lemma 6

To show this result, we first show that whenever the information quality of voters is low, the proposer's limit expected payoff is convex in the offer x. As in the proof of

Proposition 2, let v_n be the proposer's payoff from an offer x when he has observed σ_0, and v the pointwise limit of v_n.

Lemma A-5 *Fix preferences λ and suppose unanimity rule is in effect ($\alpha = 1$). Then there exists $\underline{\ell}$ such that whenever $\ell(\underline{\sigma}) > \underline{\ell}$, the proposer's limit payoff v is convex in x over $[0, \min\{1, x_U(b,\lambda)\}]$ for all σ_0, b. As a consequence, v is maximized at $\min\{1, x_U(b,\lambda)\}$; and if $x_U(b,\lambda) < 1$ the maximizer is unique.*

Proof of Lemma A-5: For use throughout the proof, write $\check{x} = \min\{1, x_U(b,\lambda)\}$. The limit acceptance probability is identically equal to zero over $[0, x_H(\lambda)]$, and is increasing thereafter. So to prove the result, it suffices to show that for $\omega = H, L$ there exists $\underline{\ell}$ such that whenever $\ell(\underline{\sigma}) > \underline{\ell}$, the second derivative of $P^\omega(x,b,\lambda)\left(V^\omega(x,\sigma_0) - \bar{V}^\omega(\sigma_0)\right)$ is positive for $x \in (x_H(\lambda), \check{x}]$, for all σ_0, b.

Let $\ell(\underline{\sigma})$ denote the likelihood ratio at $\underline{\sigma}$. Write $R(x,\sigma,b,\lambda) = -\frac{\Delta^H(x,\sigma,\lambda)}{\Delta^L(x,\sigma,\lambda)}\frac{b}{1-b}$, $\gamma_H = \ell(\underline{\sigma})$ and $\gamma_L = 1$, so that $P^\omega(x,b,\lambda) = [R(x,\underline{\sigma},b,\lambda)\ell(\underline{\sigma})]^{\frac{\gamma_\omega}{1-\ell(\underline{\sigma})}}$ for $x \in (x_H(\lambda), \check{x})$. The second derivative of $P^\omega(x,b,\lambda)\left(V^\omega(x,\sigma_0) - \bar{V}^\omega(\sigma_0)\right)$ with respect to x is given by

$$
\begin{aligned}
&P^\omega(x,b,\lambda)\frac{\partial^2}{\partial x^2}V^\omega(x,\sigma_0)\\
&+2\frac{\partial}{\partial x}V^\omega(x,\sigma_0)\frac{\gamma_\omega}{1-\ell(\underline{\sigma})}[R(x,\underline{\sigma},b,\lambda)\ell(\underline{\sigma})]^{\frac{\gamma_\omega}{1-\ell(\underline{\sigma})}-1}\frac{\partial}{\partial x}R(x,\underline{\sigma},b,\lambda)\\
&+\left(V^\omega(x,\sigma_0) - \bar{V}^\omega(\sigma_0)\right)\frac{\gamma_\omega}{1-\ell(\underline{\sigma})}[R(x,\underline{\sigma},b,\lambda)\ell(\underline{\sigma})]^{\frac{\gamma_\omega}{1-\ell(\underline{\sigma})}-1}\frac{\partial^2}{\partial x^2}R(x,\underline{\sigma},b,\lambda)\\
&+\left(V^\omega(x,\sigma_0) - \bar{V}^\omega(\sigma_0)\right)\frac{\gamma_\omega}{1-\ell(\underline{\sigma})}(\frac{\gamma_\omega}{1-\ell(\underline{\sigma})}-1)[R(x,\underline{\sigma},b,\lambda)\ell(\underline{\sigma})]^{\frac{\gamma_\omega}{1-\ell(\underline{\sigma})}-2}\left(\frac{\partial}{\partial x}R(x,\underline{\sigma},b,\lambda)\right)^2,
\end{aligned}
$$

which can be written as $P^\omega(x,b,\lambda)\frac{\gamma_\omega}{1-\ell(\underline{\sigma})}K$, where

$$
\begin{aligned}
K &= \frac{1-\ell(\underline{\sigma})}{\gamma_\omega}\frac{\partial^2}{\partial x^2}V^\omega(x,\sigma_0) + 2\frac{\partial}{\partial x}V^\omega(x,\sigma_0)\,\ell(\underline{\sigma})^{-1}\frac{\partial}{\partial x}\ln R(x,\underline{\sigma},b,\lambda)\\
&\quad+\left(V^\omega(x,\sigma_0) - \bar{V}^\omega(\sigma_0)\right)R(x,\underline{\sigma},b,\lambda)^{-1}\ell(\underline{\sigma})^{-1}\frac{\partial^2}{\partial x^2}R(x,\underline{\sigma},b,\lambda)\\
&\quad+\left(V^\omega(x,\sigma_0) - \bar{V}^\omega(\sigma_0)\right)\frac{\gamma_\omega - 1 + \ell(\underline{\sigma})}{1-\ell(\underline{\sigma})}R(x,\underline{\sigma},b,\lambda)^{-1}\ell(\underline{\sigma})^{-1}\frac{\partial}{\partial x}\ln R(x,\underline{\sigma},b,\lambda).
\end{aligned}
$$

Observe that $\gamma_L - 1 + \ell(\underline{\sigma}) = \ell(\underline{\sigma})$ and $\gamma_H - 1 + \ell(\underline{\sigma}) = 2\ell(\underline{\sigma}) - 1$. The term $V^\omega(x,\sigma_0) - \bar{V}^\omega(\sigma_0)$ is bounded away from zero over $[0,\check{x}]$. The term $R(x,\underline{\sigma},b,\lambda)^{-1}$ is bounded away

from zero over $(x_H(\lambda), \check{x}]$. From Lemma A-2, the term $R(x, \underline{\sigma}, b, \lambda)$ is strictly increasing over $[x_H(\lambda), \check{x}]$, and so $\frac{\partial}{\partial x} \ln R(x, \underline{\sigma}, b, \lambda)$ is positive and bounded away from zero over $[x_H(\lambda), \check{x}]$. From these observations, it is immediate that there exists $\underline{\ell}$ such that $K > 0$ whenever $\ell(\underline{\sigma}) > \underline{\ell}$ and $x \in (x_H(\lambda), \check{x}]$, and for all σ_0, b, completing the proof. ∎

Given Lemma A-5, we are now ready to establish the existence of a pooling equilibrium. Fix voter preferences λ, and suppose that $x_U(b = p^H, \lambda) < 1$ and that voter information is sufficiently poor such that $\ell(\underline{\sigma}) \geq \underline{\ell}$. If the proposer's signal is completely uninformative (i.e., $\ell_0(\cdot) \equiv 1$), the result is immediate from the uniform convergence of acceptance probabilities (Lemma 5) and the proposer payoff convexity (Lemma A-5). The remainder of the proof deals with the case in which the proposer's signal is informative, and so $\underline{b} < p^H < \bar{b}$. We claim that for all n large enough, there is a pooling equilibrium in which the proposer offers $x_U(b = p^H, \lambda)$ independent of his signal σ_0; the voters accept with probability one; and the voters assign belief $\underline{b}$ to any downwards deviation by the proposer.

If the proposer offers $x_U\left(b = p^H, \lambda\right)$ and the voters believe $b = p^H$, the acceptance probability is one for all n. So the proposer's payoff for any finite n in the claimed equilibrium equals the limit payoff $v\left(x_U\left(b = p^H, \lambda\right), \sigma_0, b = p^H, \lambda\right)$, which for the remainder of the proof we write as v^*.

Fix $\delta > 0$. From the payoff convexity result Lemma A-5, there exists $\varepsilon > 0$ such that $v\left(x, \sigma_0, b = p^H, \lambda\right) < v^* - \varepsilon$ whenever $x < x_U\left(b = p^H, \lambda\right) - \delta$. *A fortiori*, $v(x, \sigma_0, \underline{b}, \lambda) < v^* - \varepsilon$ also. By uniform convergence (Lemma 5) it follows that there exists N_1 such that $v_n(x, \sigma_0, \underline{b}, \lambda) < v^* - \varepsilon$ whenever $n \geq N_1$.

It remains only to rule out downwards deviations between $x_U\left(b = p^H, \lambda\right) - \delta$ and $x_U\left(b = p^H, \lambda\right)$. From Lemma 5, there exists some $\varphi > 0$ such that the limit acceptance probabilities satisfy $P^\omega(x, \underline{b}, \lambda) < P^\omega\left(x, b = p^H, \lambda\right) - \varphi$ for offers x in this interval. So there exists $\hat{\varphi} > 0$ such that the limit proposer payoffs similarly satisfy $v(x, \sigma_0, \underline{b}, \lambda) < v\left(x, \sigma_0, b = p^H, \lambda\right) - \hat{\varphi}$ for offers x in this same interval. By payoff convexity (Lemma

A-5), $v\left(x, \sigma_0, b = p^H, \lambda\right) < v^*$. So by uniform convergence, it follows that there exists N_2 such that $v_n\left(x, \sigma_0, \underline{b}, \lambda\right) < v^*$ whenever $n \geq N_2$. So for all $n \geq \max\{N_1, N_2\}$ the proposer has no profitable deviation, completing the proof. ∎

Proof of Lemma 7

We prove the lemma in four steps. For clarity, we suppress λ and α and write x_ω in place of $x_\omega(\lambda; \alpha)$ throughout. Take Z as defined in the statement of Lemma 3.

Claim 1 *If* $\limsup x_n < x_H$ *then* $P_n^H(x_n) \to 0$.

Proof: It suffices to show that $\liminf \sigma_n^* > \sigma_H$, since in this case the acceptance probability of each voter is bounded away from $1 - F(\sigma_H|H) = \alpha$ from below, and so the law of large numbers implies that $P_n^H(x_n) \to 0$.

Suppose to the contrary that $\liminf \sigma_n^* \leq \sigma_H$. So for any $\delta > 0$, there exists a subsequence of σ_n^* such that $\sigma_n^* \leq \sigma_H + \delta$. By hypothesis, there exists ε such that $x_n \leq x_H - \varepsilon$ for all n large enough. By definition $\Delta^H(x_H, \sigma_H, \lambda) = 0$; so for δ small enough, there exists $\hat{\varepsilon}$ such that $\Delta^H(x_n, \sigma_n^*, \lambda) < -\hat{\varepsilon}$. Moreover, $\Delta^L(x_n, \sigma_n^*, \lambda) \leq \Delta^H(x_n, \sigma_n^*, \lambda)$. Consequently $Z(x_n, \sigma_n^*) < 0$. As such, σ_n^* is not a responsive equilibrium; and since $x_n \leq \bar{x}_n$ then σ_n^* is not an acceptance equilibrium either. The only remaining possibility is that σ_n^* is a rejection equilibrium — but then $\sigma_n^* = \bar{\sigma}$, which gives a contradiction when δ is chosen small enough.

Claim 2 *If* $\limsup x_n < x_L$ *then* $P_n^L(x_n) \to 0$.

Proof: Parallel to Claim 1, it suffices to show that $\liminf \sigma_n^* > \sigma_L$. Suppose to the contrary that $\liminf \sigma_n^* \leq \sigma_L$. So for any $\delta > 0$, there exists a subsequence of σ_n^* such that $\sigma_n^* \leq \sigma_L + \delta$. By hypothesis, there exists ε such that $x_n \leq x_L - \varepsilon$ for all n large enough. By definition $\Delta^L(x_L, \sigma_L, \lambda) = 0$; so for δ small enough, there exists $\hat{\varepsilon}$ such that $\Delta^L(x_n, \sigma_n^*, \lambda) < -\hat{\varepsilon}$. Next, define

$$\phi = \max_{\sigma \in [\underline{\sigma}, \sigma_L + \delta]} \frac{(1 - F(\sigma|H))^\alpha F(\sigma|H)^{1-\alpha}}{(1 - F(\sigma|L))^\alpha F(\sigma|L)^{1-\alpha}}$$

Note that the function $(1-q)^{\alpha}q^{1-\alpha}$ is increasing for $q \in (0, 1-\alpha)$ and decreasing for $q \in (1-\alpha, 1)$. Recall that by definition $F(\sigma_L|L) = 1-\alpha$, and by Lemma A-1 $F(\sigma|H) < F(\sigma|L)$ for all $\sigma \in (\underline{\sigma}, \bar{\sigma})$. It follows that $\phi < 1$ for δ chosen small enough, and so

$$\left(\frac{(1-F(\sigma^*|H))^{\alpha}F(\sigma^*|H)^{1-\alpha}}{(1-F(\sigma^*|L))^{\alpha}F(\sigma^*|L)^{1-\alpha}}\right)^n \leq \phi^n \to 0.$$

Since σ_n^* is bounded away from $\bar{\sigma}$, then $1-F(\sigma_n^*|H)$ is bounded away from 0. By belief consistency, $\beta_n(x_n)$ is bounded away from 1. Consequently $Z(x_n, \sigma_n^*) < 0$ for n sufficiently large. A contradiction then follows as in Claim 1.

Claim 3 *If* $\liminf x_n > x_L$ *then* $P_n^L(x_n) \to 1$.

Claim 4 *If* $\liminf x_n > x_H$ *then* $P_n^H(x_n) \to 1$.

Proofs of Claims 3 and 4: Exactly parallel to those of Claims 1 and 2.[43] ∎

Proof of Proposition 3

Proposition 3 follows immediately from the following result, which establishes that the limiting behavior of the proposer's offers stated in the result is uniform with respect to the proposer's signal σ_0.

Lemma A-6 ***(Equilibrium offer under majority)*** *Suppose a majority voting rule* $\alpha < 1$ *is in effect. Then:*

(1) If $x_L(\lambda;\alpha) \neq \infty \neq x_H(\lambda;\alpha)$*, then for any* $\varepsilon, \delta > 0$ *there exists* $N(\varepsilon, \delta)$ *such that*

(a) If $W(\sigma_0) > \varepsilon$ *and* $n \geq N(\varepsilon, \delta)$ *then for any equilibrium offer* x, $|x - x_H(\lambda;\alpha)| < \delta$; $P_n^H(x|\sigma_0) > 1-\delta$; *and* $P_n^L(x|\sigma_0) < \delta$.

(b) If $W(\sigma_0) < -\varepsilon$ *and* $n \geq N(\varepsilon, \delta)$ *then for any equilibrium offer* x, $|x - x_L(\lambda,\alpha)| < \delta$; $P_n^H(x|\sigma_0) > 1-\delta$; *and* $P_n^L(x|\sigma_0) > 1-\delta$.

(2) If $x_H(\lambda;\alpha) \neq \infty$ *and* $x_L(\lambda;\alpha) = \infty$ *then for any* $\delta > 0$ *there exists* $N(\delta)$ *such that for any equilibrium offer* x, $|x - x_H(\lambda;\alpha)| < \delta$ *and* $P_n^H(\cdot|\sigma_0) > 1-\delta$ *for all* σ_0 *when* $n \geq N(\delta)$.

[43] Full details are available in an online supplement on the authors' webpages.

(3) If $x_L(\lambda;\alpha) = x_H(\lambda;\alpha) = \infty$*, for any* $\delta > 0$ *there exists* $N(\delta)$ *such that for any equilibrium offer* x*,* $P_n^\omega(x|\sigma_0) < \delta$ *for all* $\sigma_0, \omega = L, H$ *when* $n \geq N(\delta)$*.*

Proof of Lemma A-6: We focus on Part 1a. Part 1b is proved by similar arguments.[44] Part 2 is essentially a special case of Part 1a, as we explain below. Part 3 is an immediate consequence of Lemma 7.

The main idea is straightforward: for any σ_0 such that $W(\sigma_0) > 0$, the proposer prefers offering $x_H(\lambda;\alpha)$ and gaining acceptance if and only if $\omega = H$ to offering $x_L(\lambda,\alpha)$ and gaining acceptance all the time. Given the limiting behavior of voters established in Lemma 7, intuitively it follows that the proposer's offer converges to $x_H(\lambda;\alpha)$ as the number of voters grows large. The main difficulty encountered in the formal proof is establishing uniform convergence: for any $\varepsilon, \delta > 0$, there is some $N(\varepsilon,\delta)$ such that when $n \geq N(\varepsilon,\delta)$, the proposer's offer lies within δ of $x_H(\lambda;\alpha)$ for all σ_0 such that $W(\sigma_0) > \varepsilon$.

Take any $\varepsilon, \delta > 0$. Throughout the proof, we omit all λ and α arguments for readability. We define $\Delta_0^\omega(x,\sigma_0) \equiv V^\omega(x,\sigma_0) - \bar{V}^\omega(\sigma_0)$, the proposer's gain to offer x being accepted conditional on ω.

Preliminaries: The first part of the proof consists of defining bounds which we will use to establish uniform convergence below. Choose μ, δ_1, δ_2, $\delta_3 \in (0,\delta]$ such that $x_H + \mu < x_L - \mu$, and for all σ_0 for which $W(\sigma_0) > \varepsilon$,

$$p^H(\sigma_0)V^H\left(x_H + \frac{\mu}{2}, \sigma_0\right) + p^L(\sigma_0)\bar{V}^L(\sigma_0) \geq E\left[V^\omega(x_L - \mu, \sigma_0)\,|\sigma_0\right] + \frac{\varepsilon}{2}, \tag{A-8}$$

$$\delta_1 \Delta_0^H\left(x_H + \frac{\mu}{2}, \sigma_0\right) \leq \frac{\varepsilon}{4}, \tag{A-9}$$

$$\delta_2\left(p^H(\sigma_0)\Delta_0^H(0,\sigma_0) + p^L(\sigma_0)\Delta_0^L(0,\sigma_0)\right) < (1-\delta_1)\,p^H(\sigma_0)\Delta_0^H\left(x_H + \frac{\mu}{2}, \sigma_0\right), \tag{A-10}$$

$$p^H(\sigma_0)\left((1-\delta_1)\,\Delta_0^H\left(x_H + \frac{\mu}{2}, \sigma_0\right) - \Delta_0^H(x_H + \mu, \sigma_0)\right) > p^L(\sigma_0)\delta_3\Delta_0^L(x_H + \mu, \sigma_0), \tag{A-11}$$

[44] Full details are available in an online supplement on the authors' webpages.

$$
\begin{aligned}
& p^H(\sigma_0)\left(\left(1-\delta_1\right)V^H\left(x_H+\frac{\mu}{2},\sigma_0\right)+\delta_1\bar{V}^H\left(\sigma_0\right)\right)+p^L(\sigma_0)\bar{V}^L\left(\sigma_0\right) \\
> \; & p^H(\sigma_0)\left(\left(1-\delta\right)V^H\left(x_H-\mu,\sigma_0\right)+\delta\bar{V}^H\left(\sigma_0\right)\right) \\
+ \; & p^L(\sigma_0)\left(\delta_3 V^L\left(x_H-\mu,\sigma_0\right)+\left(1-\delta_3\right)\bar{V}^L\left(\sigma_0\right)\right). \qquad \text{(A-12)}
\end{aligned}
$$

A choice of μ, δ_1, δ_2, δ_3 exists such that (A-8), (A-9), (A-10), (A-11), and (A-12) hold as follows. First, choose μ such that (A-8) holds, along with

$$
V^H\left(x_H+\frac{\mu}{2},\sigma_0\right)>\left(1-\delta\right)V^H\left(x_H-\mu,\sigma_0\right)+\delta\bar{V}^H\left(\sigma_0\right). \qquad \text{(A-13)}
$$

It is possible to choose $\mu>0$ that satisfies these two inequalities for all σ_0 since $|V_x^\omega|$ is bounded. The same argument applies in choosing $\delta_1,\delta_2,\delta_3$ below. Second, choose δ_1 such that (A-9) holds, along with

$$
\left(1-\delta_1\right)\Delta_0^H\left(x_H+\frac{\mu}{2},\sigma_0\right)-\Delta_0^H\left(x_H+\mu,\sigma_0\right)>0, \qquad \text{(A-14)}
$$

$$
\left(\left(1-\delta_1\right)V^H\left(x_H+\frac{\mu}{2},\sigma_0\right)+\delta_1\bar{V}^H\left(\sigma_0\right)\right)-\left(\left(1-\delta\right)V^H\left(x_H-\mu,\sigma_0\right)+\delta\bar{V}^H\left(\sigma_0\right)\right)>0, \qquad \text{(A-15)}
$$

where (A-15) is possible by (A-13). Third, choose δ_2 such that (A-10) holds. Finally, by (A-14) and (A-15) respectively, choose δ_3 such that (A-11) and (A-12) hold.

Fix a realization of σ_0 such that $W\left(\sigma_0\right)\geq\varepsilon$. Define the following offer sequences, which we use throughout the proof:

$$
x_n^{H+}\equiv x_H+\frac{\mu}{2},\quad x_n^{H-}\equiv x_H-\mu,\quad x_n^{L-}\equiv x_L-\mu.
$$

By Lemma 7, $P_n^H\left(x_n^{H+},\underline{b}\right)\to 1$ and $P_n^L\left(x_n^{H+},\bar{b}\right)\to 0$; $P_n^\omega\left(x_n^{H-},\bar{b}\right)\to 0$ for $\omega=L,H$; and $P_n^L\left(x_n^{L-},\bar{b}\right)\to 0$. Thus there exists $N\left(\varepsilon,\delta\right)$ such that for $n\geq N\left(\varepsilon,\delta\right)$, $P_n^H\left(x_n^{H+},\underline{b}\right)\geq 1-\delta_1$, $P_n^L\left(x_n^{H+},\bar{b}\right)\leq\delta_1$, $P_n^\omega\left(x_n^{H-},\bar{b}\right)\leq\delta_2$ for $\omega=L,H$, and $P_n^L\left(x_n^{L-},\bar{b}\right)\leq\delta_3$.

Fix σ_0 such that $W\left(\sigma_0\right)\geq\varepsilon$, along with the associated equilibrium offers x_n.

Part A: $P_n^L\left(x_n\right)\leq\delta_3\leq\delta$ for $n\geq N\left(\varepsilon,\delta\right)$.

Proof: If $x_n\leq x_n^{L-}$ then $P_n^L\left(x_n\right)\leq P_n^L\left(x^{L-},\bar{b}\right)\leq\delta_3$ for $n\geq N\left(\varepsilon,\delta\right)$. Consequently it suffices to show that $x_n\leq x_n^{L-}$ for all $n\geq N\left(\varepsilon,\delta\right)$. Suppose to the contrary that $x_n>$

x_n^{L-} for some $n \geq N(\varepsilon, \delta)$. By Assumption 5 the proposer is better off when his offer is accepted, and so the proposer's payoff from x_n is bounded above by $E\left[V^{\omega}\left(x_n^{L-}, \sigma_0\right)|\sigma_0\right]$. In contrast, the proposer's payoff from the offer x_n^{H+} is bounded below by

$$\begin{aligned} & p^H(\sigma_0)\left((1-\delta_1)V^H\left(x_n^{H+}, \sigma_0\right) + \delta_1 \bar{V}^H(\sigma_0)\right) + p^L(\sigma_0)\bar{V}^L(\sigma_0) \\ = \; & p^H(\sigma_0)\left(V^H\left(x_n^{H+}, \sigma_0\right) - \delta_1 \Delta_0^H\left(x_n^{H+}, \sigma_0\right)\right) + p^L(\sigma_0)\bar{V}^L(\sigma_0) \\ \geq \; & p^H(\sigma_0)V^H\left(x_n^{H+}, \sigma_0\right) + p^L(\sigma_0)\bar{V}^L(\sigma_0) - \frac{\varepsilon}{4} \end{aligned}$$

where the inequality follows by (A-9) (and the fact that $p^H(\sigma_0) \leq 1$). By (A-8) this lower bound exceeds $E\left[V^{\omega}\left(x_n^{L-}, \sigma_0\right)|\sigma_0\right]$, contradicting the optimality of x_n.

Part B: $|x_n - x_H| \leq \mu \leq \delta$ for $n \geq N(\varepsilon, \delta)$.

Proof: First, suppose that $x_n < x_n^{H-}$ for some $n \geq N(\varepsilon, \delta)$. The acceptance probability of x_n given ω is consequently less than that of x_n^{H-} under the most pro-acceptance belief $\bar{b}$, which is in turn less than δ_2. The acceptance probability of x_n^{H+} given H is at least $1 - \delta_1$. It follows from (A-10) that the proposer's payoff is higher under x_n^{H+} than under x_n. But this contradicts the optimality of the proposer's offer x_n.

Second, suppose that $x_n > x_H + \mu$ for some $n \geq N(\varepsilon, \delta)$. By Part A, the proposer's payoff under x_n is bounded above by

$$p^H(\sigma_0)\left(\Delta_0^H(x_H + \mu, \sigma_0) + \bar{V}^H(\sigma_0)\right) + p^L(\sigma_0)\left(\delta_3 \Delta_0^L(x_H + \mu, \sigma_0) + \bar{V}^L(\sigma_0)\right).$$

In contrast, the proposer's payoff from the offer x_n^{H+} is bounded below by

$$p^H(\sigma_0)\left((1-\delta_1)\Delta_0^H\left(x_n^{H+}, \sigma_0\right) + \bar{V}^H(\sigma_0)\right) + p^L(\sigma_0)\bar{V}^L(\sigma_0),$$

which exceeds the payoff from the offer x_n by (A-11), contradicting optimality of x_n.

Part C: $P_n^H(x_n) \geq 1 - \delta$ for $n \geq N(\varepsilon, \delta)$.

Proof: Suppose not. By Part A, $P_n^L\left(x_n^{L-}, \bar{b}\right) \leq \delta_3$, and by Part B, $x_n \geq x_n^{H-}$, and hence the proposer's payoff is bounded above by

$$p^H(\sigma_0)\left((1-\delta)V^H\left(x_n^{H-}, \sigma_0\right) + \delta \bar{V}^H(\sigma_0)\right) + p^L(\sigma_0)\left(\delta_3 V^L\left(x_n^{H-}, \sigma_0\right) + (1-\delta_3)\bar{V}^L(\sigma_0)\right).$$

In contrast, the proposer's payoff from the offer x_n^{H+} is bounded below by

$$p^H(\sigma_0)\left((1-\delta_1)\,V^H\left(x_n^{H+},\sigma_0\right)+\delta_1\bar{V}^H\left(\sigma_0\right)\right)+p^L(\sigma_0)\bar{V}^L\left(\sigma_0\right).$$

By (A-12) the latter is strictly greater, contradicting the optimality of the offer x_n.

Part 2: The proof of Part 2 is a simpler version of that of Part 1a. In the Preliminaries, omit (A-8), (A-9), and the sequence x_n^{L-}. Instead, choose $N(\varepsilon,\delta)$ such that for $n \geq N(\varepsilon,\delta)$, $P_n^H\left(x_n^{H+},\underline{b}\right) \geq 1-\delta_1$, $P_n^L\left(x_n^{H+},\bar{b}\right) \leq \delta_1$, $P_n^\omega\left(x_n^{H-},\bar{b}\right) \leq \delta_2$ for $\omega = L,H$, along with $P_n^L(1,\underline{b}) \leq \delta_3$. (The last inequality is a consequence of Lemma 7.) The conclusion of Part A remains the same, but needs no proof. Parts B and C are unchanged. ∎

Proof of Proposition 4

We prove the result in reverse order, starting with the easier half:

Part (II): Since W is negative, it must be the case that $x_L(\lambda;\alpha) \neq \infty$ for λ sufficiently small, and $x_U(\underline{b},\lambda) < x_L(\lambda;\alpha)$. From Proposition 3, as $n \to \infty$ voters' expected payoff under majority converges to

$$E_{\sigma_i,\omega}\left[\bar{U}^\omega\left(\sigma_i,\lambda\right)\right]+E_{\sigma_i,\omega}\left[\Delta^\omega\left(x_L\left(\lambda;\alpha\right),\sigma_i,\lambda\right)\right]. \tag{A-16}$$

By Proposition 2, the proposer's offer against unanimity is weakly below $x_U(\underline{b},\lambda)$. From Lemma 5, voters accept this offer with probability 1. Since Δ^ω is increasing in the offer, Lemma 4 implies that (for all n) voters' equilibrium payoff is weakly less than

$$E_{\sigma_i,\omega}\left[\bar{U}^\omega\left(\sigma_i,\lambda\right)\right]+E_{\sigma_i,\omega}\left[\Delta^\omega\left(x_U\left(\underline{b},\lambda\right),\sigma_i,\lambda\right)\right]. \tag{A-17}$$

Since Δ^ω is increasing, this is strictly less than (A-16), the voter payoff under majority as $n \to \infty$. The result follows.

Part (I): From Proposition 3, if W is everywhere positive the voters' payoff under majority converges to

$$E_{\sigma_i,\omega}\left[\bar{U}^\omega\left(\sigma_i\right)\right]+p^H E_{\sigma_i}\left[\Delta^H\left(x_H\left(\lambda\right),\sigma_i,\lambda\right)|H\right], \tag{A-18}$$

which at $\lambda = 0$ is simply $E_{\sigma_i,\omega}\left[\bar{U}^{\omega}(\sigma_i)\right]$. We claim that there exist $\delta > 0$, $\tilde{\lambda} > 0$ and $\tilde{N}$ such that the voters' equilibrium payoff under unanimity rule exceeds $E_{\sigma_i,\omega}\left[\bar{U}^{\omega}(\sigma_i)\right]+\delta/2$ for all $\lambda \leq \tilde{\lambda}$ and $n \geq \tilde{N}$. Since we can choose $\hat{\lambda} \leq \tilde{\lambda}$ such $p^H E_{\sigma_i}\left[\Delta^H(x_H(\lambda), \sigma_i, \lambda)\,|H\right] \leq \delta/4$ for all $\lambda \leq \hat{\lambda}$, it follows from (A-18) and the claim that the voters' payoff is greater under unanimity when n is large enough.

To prove the claim, let $\check{\lambda}$, κ and N be as given in Proposition 2. For a given offer x and belief b, since $\sigma_n^* \to \underline{\sigma}$ as $n \to \infty$ (see the proof of Lemma 5) the limit of voters' expected payoffs under unanimity is

$$\begin{aligned} u(x,b,\lambda) \;\equiv\; & E_{\sigma_i,\omega}\left[\bar{U}^{\omega}(\sigma_i)\right] \\ & + bP^H(x,b,\lambda)\,E_{\sigma_i}\left[\Delta^H(x,\sigma_i,\lambda)\,|H\right] + (1-b)\,P^L(x,b,\lambda)\,E_{\sigma_i}\left[\Delta^L(x,\sigma_i,\lambda)\,|L\right]. \end{aligned}$$

From Lemma 5, u is continuous, and differentiable except at $x_H(\lambda)$ and $x_U(\lambda, b)$. We next show that, similar to Lemma 4,[45]

$$\frac{\partial}{\partial x}u(x,b,\lambda) \geq bP^H(x,b,\lambda)\,\frac{\partial}{\partial x}E_{\sigma_i}\left[\Delta^H(x,\sigma_i,\lambda)\,|H\right] + (1-b)\,P^L(x,b,\lambda)\,\frac{\partial}{\partial x}E_{\sigma_i}\left[\Delta^L(x,\sigma_i,\lambda)\,|L\right]. \tag{A-19}$$

Inequality (A-19) is equivalent to

$$b\frac{\partial}{\partial x}P^H(x,b,\lambda)\,E_{\sigma_i}\left[\Delta^H(x,\sigma_i,\lambda)\,|H\right] + (1-b)\,\frac{\partial}{\partial x}P^L(x,b,\lambda)\,E_{\sigma_i}\left[\Delta^L(x,\sigma_i,\lambda)\,|L\right] \geq 0, \tag{A-20}$$

which holds trivially for $x < x_H(\lambda)$ and $x > x_U(\lambda, b)$, since in these ranges the limiting acceptance probabilities are 0 and 1 respectively. For x between $x_H(\lambda)$ and $x_U(\lambda, b)$, Lemma 5 implies

$$\frac{\frac{\partial}{\partial x}P^L(x,b,\lambda)}{\frac{\partial}{\partial x}P^H(x,b,\lambda)} = \left(-\frac{\Delta^H(x,\underline{\sigma},\lambda)}{\Delta^L(x,\underline{\sigma},\lambda)}\frac{b}{1-b}\ell(\underline{\sigma})\right)\frac{\frac{\ell(\underline{\sigma})}{1-\ell(\underline{\sigma})}+1}{\frac{\ell(\underline{\sigma})}{1-\ell(\underline{\sigma})}} = -\frac{\Delta^H(x,\underline{\sigma},\lambda)}{\Delta^L(x,\underline{\sigma},\lambda)}\frac{b}{1-b}.$$

So (A-19) is equivalent to

$$bE_{\sigma_i}\left[\Delta^H(x,\sigma_i,\lambda)\,|H\right] + (1-b)\,E_{\sigma_i}\left[\Delta^L(x,\sigma_i,\lambda)\,|L\right]\left(-\frac{\Delta^H(x,\underline{\sigma},\lambda)}{\Delta^L(x,\underline{\sigma},\lambda)}\frac{b}{1-b}\right) \geq 0.$$

[45] Lemma 4 cannot be applied directly because it deals with an arbitrary finite number of voters, not the limit as $n \to \infty$.

Recalling that $\Delta^L(x, \underline{\sigma}, \lambda) < 0$ for the offers under consideration, (A-19) is in turn equivalent to

$$-\Delta^L(x, \underline{\sigma}, \lambda) E_{\sigma_i}\left[\Delta^H(x, \sigma_i, \lambda) | H\right] \geq -E_{\sigma_i}\left[\Delta^L(x, \sigma_i, \lambda) | L\right] \Delta^H(x, \underline{\sigma}, \lambda).$$

This holds since by Assumption 2, $E_{\sigma_i}\left[\Delta^\omega(x, \sigma_i)\right] \geq \Delta^\omega(x, \hat{\sigma})$, and so (A-19) holds.

As x approaches x_H from above, $P^L(x, b, \lambda)/P^H(x, b, \lambda) \to 0$. By Assumption 4 $\Delta^H(x, \sigma, 0)$ is strictly positive over $(x_H(0), 1)$. So $\frac{\partial}{\partial x} u(x, b, 0) > 0$ for all x close enough to $x_H(0)$, and hence $u(x, b, 0) > E_{\sigma_i,\omega}\left[\bar{U}^\omega(\sigma_i)\right]$ for x close enough to $x_H(0)$. Next, note that there is no $x \in (x_H(0), 1)$ such that $u(x, b, 0) = E_{\sigma_i,\omega}\left[\bar{U}^\omega(\sigma_i)\right]$: for if such an offer x existed, by continuity there exists some $\tilde{x} \in (0, 1)$ such that $u(\tilde{x}, b, 0) = E_{\sigma_i,\omega}\left[\bar{U}^\omega(\sigma_i)\right]$ and $u'(\tilde{x}, b, 0) \leq 0$. But then (by (A-19))

$$\begin{aligned} bP^H(\tilde{x}, b, 0)\Delta^H(\tilde{x}, \sigma_i, 0) + (1-b)P^L(\tilde{x}, b, 0)\Delta^L(\tilde{x}, \sigma_i, 0) &= 0 \\ bP^H(\tilde{x}, b, 0)\frac{\partial}{\partial x}\Delta^H(\tilde{x}, \sigma_i, 0) + (1-b)P^L(\tilde{x}, b, 0)\frac{\partial}{\partial x}\Delta^L(\tilde{x}, \sigma_i, 0) &\leq 0, \end{aligned}$$

contradicting Assumption 4. So $u(x, b, 0)$ is strictly greater than $E_{\sigma_i,\omega}\left[\bar{U}^\omega(\sigma_i)\right]$ over $(x_H(0), 1)$. By continuity there exists $\delta > 0$ and $\tilde{\lambda} \leq \check{\lambda}$ such that $\min_{x \in [x_H(\lambda)+\kappa, 1-\kappa]} u(x, b, \lambda) > E_{\sigma_i,\omega}\left[\bar{U}^\omega(\sigma_i)\right] + \delta$ for all $\lambda \leq \tilde{\lambda}$. By the uniform convergence of acceptance probabilities with respect to the parameter λ, the claim follows. ∎