

Itoh, Hideshi; Morita, Hodaka

**Working Paper**

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CESifo Working Paper, No. 3533

**Provided in Cooperation with:**

Ifo Institute – Leibniz Institute for Economic Research at the University of Munich

*Suggested Citation:* Itoh, Hideshi; Morita, Hodaka (2011) : Formal contracts, relational contracts, and the threat-point effect, CESifo Working Paper, No. 3533, Center for Economic Studies and Ifo Institute (CESifo), Munich

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# Formal Contracts, Relational Contracts, and the Threat-Point Effect

Hideshi Itoh  
Hodaka Morita

CESIFO WORKING PAPER NO. 3533

CATEGORY 11: INDUSTRIAL ORGANISATION

JULY 2011

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# Formal Contracts, Relational Contracts, and the Threat-Point Effect

## Abstract

Can formal contracts help resolving the holdup problem? We address this important question by studying the holdup problem in repeated transactions between a seller and a buyer in which the seller can make relation-specific investments in each period. In contrast to previous findings, we demonstrate that writing a simple fixed-price contract based on product delivery is of value even when relation-specific investment is purely cooperative. In particular, there is a range of parameter values in which a higher investment can be implemented only if a formal fixed-price contract is written and combined with an informal agreement on additional payments or termination of future trade, contingent upon investments. Furthermore, we show that under an additional natural assumption, focusing our attention on fixed-price contracts as a form of formal contracts is without loss of generality. The key driving force of our result is a possibility that the threat-point effect is negative, i.e., the relation-specific investment decreases the surplus under no trade. This possibility, although very plausible, has been largely ignored in previous theoretical/empirical analyses of the holdup problem.

JEL-Code: D230, D860, L140, L220, L240.

Keywords: holdup problem, formal contract, relational contract, cooperative investment, fixed-price contract, relation-specific investment, repeated transactions, long-term relationships.

*Hideshi Itoh*  
*Graduate School of Commerce and*  
*Management, Hitotsubashi University*  
*Tokyo / Japan*  
*hideshi.itoh@r.hit-u.ac.jp*

*Hodaka Morita*  
*School of Economics / Australian School of*  
*Business / University of New South Wales*  
*Sydney / Australia*  
*H.Morita@unsw.edu.au*

July 2011

This is a substantially revised version of the paper earlier versions of which were circulated under the different title "Formal Contracts, Relational Contracts, and the Holdup Problem." We are grateful to Murali Agastya, Eric Chou, Robert Gibbons, Shingo Ishiguro, Shinsuke Kambe, Michihiro Kandori, Kieron Meagher, Masahiro Okuno-Fujiwara, Heikki Rantakari, Bill Schworm, Kunal Sengupta, Masatoshi Tsumagari, Noriyuki Yanagawa, and the seminar participants at University of Tokyo, University of New South Wales, Chinese University of Hong Kong, Contract Theory Workshop (CTW), Academia Sinica, the 9th World Congress of the Econometric Society held at University College London, University of Bristol, Waseda University, the 2nd Annual Organizational Economics Workshop held at University of Sydney, NBER 2009 Conference on Relational Contracts, and Keio University for helpful comments. Part of this research was conducted while the first author was visiting the Center for Economic Studies (CES), University of Munich, and School of Economics, University of New South Wales. He is grateful to these institutions for their hospitality, as well as to Hitotsubashi University and Japan Society for the Promotion of Science for their financial support. The second author is grateful for financial support from the Australian Research Council.

# 1 Introduction

Relation-specific investments often cause holdup problems when contracting is incomplete. Suppose, as an example, that a seller has an opportunity to make an investment which creates more value inside its relationship to a particular buyer than outside. The relation-specific nature of the investment may result in the buyer's opportunistic behavior. Contracts contingent upon investment-related information could protect the seller, but this is often difficult in reality. So, without adequate contractual protection, the seller's anticipation of the buyer's opportunistic behavior results in a less than socially optimal level of investment.

The holdup problem has played a central role in the economic analysis of organizations and institutions, and many authors have proposed various organizational interventions, such as vertical integration (Klein et al., 1978; Williamson, 1985), as remedies to the problem. In the holdup literature, a fundamental driving force of the inefficiency has been the assumption that contracts contingent upon the nature of relation-specific investments are infeasible, which is a realistic assumption in a wide variety of real-world bilateral trade. At the same time, the courts can often verify delivery of the goods by the seller, and hence formal fixed-price contracts based on product delivery are often feasible. More general formal contracts that may be contingent upon the parties' messages (i.e., reports on the state) should also be considered.

Can formal contracts help resolving or mitigating the holdup problem? Edlin and Reichelstein (1996) and Che and Hausch (1999) have made significant contributions in addressing this important question. Edlin and Reichelstein showed that the answer is yes by demonstrating that a well-designed fixed-price contract can give the seller efficient investment incentives, where they focus on "selfish" investments that benefited the investor (e.g., the seller's investment reduces his/her production costs). Che and Hausch has shown that the answer to this question becomes very different for "cooperative" investments (e.g., the seller's investment improves the buyer's value of the good). They found, among other things, that formal contracts have no values when investments are purely cooperative (that is, the seller's investment benefits the buyer only, and the buyer's investment benefits the seller only). See Section 2 for more details on these papers.

We demonstrate that formal contracts can help mitigating the holdup problem even when relation-specific investment is purely cooperative, by incorporating two key ingredients in our model. The first ingredient concerns the effect of relation-specific investments on the surplus under no trade. We call this effect the "threat-point effect", following Edlin and Hermalin (2000). Most previous contracting models in the holdup literature, including Edlin and Reichelstein (1996) and Che and Hausch (1999), assume that relation-specific

investment has no threat-point effects. In contrast, relation-specific investment does have threat-point effects in most property rights models (Grossman and Hart, 1986; Hart and Moore, 1990; Hart, 1995). In our model, the threat-point effect can be positive or negative. That is, relation-specific investment may reduce the surplus under no trade, as pointed out, for example, by Rajan and Zingales (1998) and Edlin and Hermalin (2000). Concerning physical asset specificity, Rajan and Zingales (1998) pointed out, “The specialization of an asset implies almost by definition a reduction in the outside value of that asset” (p.408). Iyer and Schoar (2008) used custom-printed lots of pens as an example of the negative threat-point effect in their recent field experiment regarding the holdup problem. A specific logo (e.g. company logo) printed on the pens increases the value of the pens for a specific buyer. However, it may decrease the surplus under no trade because, in order to sell the pens to alternative buyers, the seller may have to remove the logo by incurring adjustment costs. A similar idea is incorporated in Hart (1995)’s model, where he argues that, once a seller has made relation-specific investment, the seller will have to make some adjustments to turn its product into a general-purpose one before selling it to alternative buyers (Hart, 1995, p.36).<sup>1</sup> See, for example, Andrabi et al. (2006) and Banerjee and Basu (2009) for recent theoretical analyses that incorporate the idea that relation-specific investment decreases the surplus under no trade.<sup>2</sup>

The second ingredient is repeated transactions between a seller and a buyer. In reality, relation-specific investments are often made under long-term and repeated interaction between parties. Coase (1988) pointed out that A.O. Smith, a large independent manufacturer

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<sup>1</sup>In Hart (1995)’s model, a seller’s relation-specific investment affects the seller’s cost. In particular, the seller’s cost to produce a widget for a specific buyer is  $C(e)$  and the production cost is  $c(e; B)$  for alternative buyers, where  $e$  denotes the level of the seller’s relation-specific investment. Hart assumes that the investment  $e$  is more effective for  $C(e)$  than for  $c(e; B)$  because, “If trade does not occur, [the seller] will sell her widget on the competitive spot market for  $\bar{p}$ , but will have to make some adjustments to turn it into a general-purpose widget. (p.36)” In his model, relation-specific investment increases the surplus under no trade, because it assumes that  $c(e; B)$  is decreasing in  $e$ . This, however, may not necessarily be the case. If adjustment costs are substantially high and increasing in  $e$ ,  $c(e; B)$  can also be increasing in  $e$ , implying that relation-specific investment decreases the surplus under no trade.

<sup>2</sup>Also, even if relation-specific investment itself does not decrease the surplus under no trade, it may still end up decreasing the surplus under no trade if relation-specific investment and general-purpose investments are substitutes for investors. For example, consider a seller who can make two kinds of investments, a relation-specific investment (zero or one unit) that increases the value of its product only for a specific buyer, and a general-purpose investment (zero or one unit) that increases the value of its product for all potential buyers. If the seller can make only one unit of investment because of various resource constraints, then the seller’s relation-specific investment reduces the surplus under no trade by preventing itself from making the general-purpose investment. See Cai (2003) who studies such a multi-dimensional investment model in which increasing relation-specific investment reduces general-purpose investment and hence reduces the outside value. This idea can be applied, among other things, to human asset specificity: If a worker can spend only one hour per day for training, spending the limited time for acquiring skills specific to his employer could reduce the surplus under no trade by preventing him from acquiring general skills that are also applicable to some other potential employers.

of automobile frames, had invested in expensive equipment that was highly specific to its main customer, such as General Motors, for more than 50 years. Also, Coase (2000) found that prior to the acquisition of Fisher Body by General Motors in 1926, Fisher Body had repeatedly made location-specific investments for General Motors. According to Holmström and Roberts (1998, p.83), “Nucor [the most successful steel maker in the United States over the past 20 years] decided to make a single firm, the David J. Joseph Company (DJJ), its sole supplier of scrap. Total dependence on a single supplier would seem to carry significant hold-up risks, but for more than a decade, this relationship has been working smoothly and successfully.”

Despite the important connection between relation-specific investments and long-term relationships, there have been very few theoretical analyses, to the best of our knowledge, that have addressed the holdup problem under infinitely repeated interactions.<sup>3</sup> Edlin and Reichelstein (1996) and Che and Hausch (1999), for example, both focus on spot transactions. This might be because, due to a reasoning based on the Folk Theorem, the holdup problem can obviously be resolved under infinitely repeated interactions if the discount factor is high enough. We show, however, that when the discount factor is not high enough, formal contracts can play a crucial role in resolving the holdup problem under repeated transactions when relation-specific investment is purely cooperative and decreases the surplus under no trade.

Cooperative investments play important roles in reality. Let us consider manufacturer-supplier relationships in the Japanese automobile industry as an example. A common practice in the Japanese automobile industry is that the supplier does the actual design of the customized parts based on functional specifications provided by the manufacturer. These parts are called the “drawing approved” parts by Asanuma (1989, 1992), and the “black-box” parts by Fujimoto (1999). Asanuma (1989, 1992) observed that, in order to complete the design successfully, the supplier should have equipment to design, manufacture, and test trial parts, and the ability to understand and finely adapt to the subtle needs of the manufacturer. Consequently, suppliers need to make a substantial amount of relation-specific investments. Fujimoto (1999) emphasizes the importance of “bundled outsourcing” behind this practice, meaning that a bundle of functionally related tasks like machining and assembly, detailed engineering and manufacturing, or production and inspection, is subcontracted out to one

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<sup>3</sup>Several recent papers introduced dynamic structures into the analysis of the holdup problem. Gul (2001) studied repeated offers, Che and Sákovic (2004) allowed parties to continue to invest until they agree on the terms of trade, and Pitchford and Snyder (2004) also studied the holdup problem involving gradual investment. Watson (2007) examined a contracting model in which verifiable productive actions take place over multiple (finite) periods of time, following unverifiable one-shot investment. See also Watson and Wignall (2009) for a related analysis. None of these papers, however, studied the holdup problem under infinitely repeated interactions.

supplier as a package. He argues that this practice results in asset specificity and facilitates long-term relational transactions. Regarding Japanese automakers' relationships with their partner suppliers, Dyer (1996) found that on average 30.6% of the supplier's total capital equipment investments is valuable only for a particular manufacturer ("not redeployable").<sup>4</sup> Dyer also found that the average number of "guest" engineers sent by partner suppliers to automakers is 7.2, where guest engineers become a part of the design team and are co-located with automaker engineers. These investments in physical asset and human capital are cooperative relation-specific investments because suppliers make these investments in order to improve their abilities to meet functional requirements of the customized parts for particular manufacturers.<sup>5</sup>

According to Asanuma's observations, parts that require a high degree of cooperative relation-specific investments tend to be transacted under repeated bilateral interactions between an automaker and a single supplier, where investments are made on a recurring basis because automakers periodically introduce new models. The supplier's costs for designing customized parts are to be recovered through the sale price of the parts, where prices are not specified in formal contracts but determined through negotiations on a regular basis (Holmstrom and Roberts, 1998). That is, formal price contracts are not utilized to induce relation-specific investments under repeated transactions in the context of manufacturer-supplier relationships in the Japanese automobile industry. The supplier's contributions to improvements in quality, design, and cost reduction affect its ratings by the assembler, and better ratings are rewarded in future transactions through "promotion" to more favorable/challenging assignments.<sup>6</sup> Formal price contracts, however, are often utilized in similar setups in different cases. For example, Crocker and Reynolds (1993) studied several kinds of formal price contracts in defense procurement of the U.S. Air Force, where "By its nature, defense acquisition involves substantial recurring investment in relationship-specific assets by a relatively small group of highly specialized defense contracts."<sup>7</sup>

Under what conditions can formal contracts help to induce relation-specific investments under repeated transactions? Our theoretical analysis identifies the threat-point effect as a critical factor to be considered for cooperative investments, where formal contacts are not

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<sup>4</sup>Dyer studied two Japanese and three U.S. automakers. Each automaker purchasing department manager selected a sample of 50 domestic supplier relationships. In selecting this sample, each automaker was asked to select a sample of 25 "supplier partners with whom you work most closely" and a sample of 25 "most typical arms-length supplier relationships."

<sup>5</sup>See Che and Hausch (1999) for several other real-world examples for cooperative investments.

<sup>6</sup>See Nishiguchi (1994) and Fujimoto (1999) for other evidences and observations.

<sup>7</sup>In his well-known study of coal markets, Joskow (1985, 1987) examined the effects of relation-specific investment on the duration of contracts negotiated between electric utilities and coal suppliers. The focus here is site specific investment, which is not recurring investment by its nature.

useful if the threat-point effect is positive whereas formal contracts can be useful if the effect is negative and the holdup effect is sufficiently large. We therefore suggest that the threat-point effect be incorporated in empirical studies on formal contracts and relation-specific investments under repeated transactions.

We consider a standard setup of the holdup problem in which a buyer purchases 0 or 1 units of a product from a seller. The seller chooses an action  $a$ , that can be interpreted as a level of relation-specific investment, by incurring private costs. The investment increases the product's value for the buyer, but has no effects on the seller's production cost. That is, the investment is purely cooperative. A formal contract can be signed prior to the seller's investment decision.<sup>8</sup> After the investment is made, some state uncertainty, denoted  $\theta$ , is revealed and observed. The buyer and the seller are then free to (re)negotiate the price with exogenously specified bargaining power, whether or not formal contracts have been signed. Let  $\rho(a, \theta)$  denote the negotiation price in the absence of formal contracts. Unlike most previous models considered in the holdup literature, our model allows for the possibility that relation-specific investment decreases the surplus under no trade, and this in turn implies that  $\rho(a, \theta)$  can be decreasing in  $a$ .

We first study the value of formal contracts under spot transaction. We find that formal contracts, even if we allow contracts to be contingent on messages, are of no value in resolving or mitigating the holdup problem under an assumption that  $\rho(a, \theta)$  is either weakly increasing or weakly decreasing in  $a$  for all realizations of uncertainty  $\theta$ . Although we believe that this is a natural assumption, implying that the effects of uncertainty  $\theta$  be not too large to alter the sign of the effects of investment  $a$  on the negotiated price, our contribution actually does not rely on it: If this assumption does not hold, formal contracts can be of value even under spot transaction in our model.<sup>9</sup> In other words, this is a conservative assumption in our attempt to show that formal contracts are of value in resolving the holdup problem even when relation-specific investment is purely cooperative.

Next we consider repeated transactions, and find that formal contracts can help resolving or mitigating the holdup problem even under the conservative assumption mentioned above. Furthermore, we find that it is without loss of generality to confine our attention to formal fixed-price contracts. That is, whenever a simple fixed-price contract can help to resolve or mitigate the holdup problem, the parties cannot get strictly better off by signing a more complex contract that is contingent upon messages. We then show that the qualitative nature of our results remains unchanged under an extension of our model that allows multiple

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<sup>8</sup>The contract can be a simple fixed-price contract based on product delivery or a more complex contract contingent upon the parties' reports on the investment and the state.

<sup>9</sup>Formal *fixed-price* contracts may not be of value without this assumption. However, more complex contracts contingent upon the parties' reports on the state can be of value under spot transaction.

quantities (rather than 0 or 1) to be transacted between the seller and the buyer.

Below we illustrate the logic behind our main findings by using a simpler version of our model in which (i) the level of the seller's investment  $a$  is either  $a = 0$  ("do not invest") or 1 ("invest") and the cost for the investment is  $d > 0$ , (ii) the buyer has the entire bargaining power (that is, the buyer can make a take-it-or-leave-it price offer to the seller), and (iii) there is no uncertainty. We focus on simple fixed-price contracts as the form of formal contracts. Our analysis on more general versions of the model indicates that it is without loss of generality to confine our attention on fixed-price contracts in this simpler version of the model.

Let  $v_a \geq 0$  be the value for the buyer and  $m_a \geq 0$  the alternative-use value under investment level  $a = 0, 1$ , where  $v_1 - v_0 > 0$  holds. As mentioned above, the threat-point effect may be positive (i.e.  $m_1 - m_0 \geq 0$ ) or negative (i.e.  $m_1 - m_0 < 0$ ) in our model. Assume (i)  $v_1 > m_1$ ; (ii)  $v_1 - v_0 > v_1 - m_0 > d$ ; and (iii)  $m_1 - m_0 < d$ . The first two assumptions imply that investment and trade are the efficient outcome while no trade is ex post efficient under no investment ( $m_0 > v_0$ ). The third assumption implies that no investment is efficient if they do not trade, and we will see below that the same assumption will also imply "under-investment" under spot transaction.

Under spot transaction without formal contracts, the buyer's take-it-or-leave-it price offer is  $m_a$  if investment level is  $a = 0, 1$ . Since  $m_0 - (m_1 - d) = -(m_1 - m_0) + d > 0$ , the seller chooses not to invest (under-investment). By our assumption  $m_0 > v_0$ , no trade occurs and the seller obtains the alternative-use value  $m_0$ . The seller would choose to invest if the buyer could commit to cover the investment cost  $d$  by paying  $m_0 + d$  contingent upon investment. Such contingent price contracts, however, are often infeasible in reality, and hence we assume that they are not possible in our model. Simple fixed-price contracts are not helpful under spot transactions. This is because, under non-contingent fixed-price contracts, the buyer virtually commits herself to paying the same price whether or not the investment is made, and hence the seller chooses not to invest.

Now consider repeated transactions. Suppose that the buyer informally promises to pay  $m_0 + d$  contingent upon investment. The seller does invest, if it is anticipated that the buyer keeps the promise. But, once the seller invests, the buyer has a temptation to renege on the implicit promise and purchase the product at the price  $m_1$ . The buyer's renegeing temptation is  $(m_0 + d) - m_1 = d + (m_0 - m_1) > 0$ . Suppose the threat-point effect is negative so that  $m_0 - m_1 > 0$  holds. Then, a simple fixed-price contract eliminates  $m_0 - m_1$  from the renegeing temptation, because, under such a contract, the buyer credibly commits to pay a fixed price regardless of the level of investment. Under the same reasoning, however, the seller would choose not to invest under the fixed-price contract. Therefore, in order to induce the seller to

invest, the buyer needs to combine the fixed-price contract  $p = m_0$  with an implicit promise of covering the investment cost  $d$  as a bonus. The buyer's temptation to renege on this implicit promise is  $d$ , which is less than  $d + (m_0 - m_1)$ .

Hence, a formal fixed-price contract combined with an implicit bonus can help mitigating the holdup problem by reducing the buyer's reneging temptation under repeated transactions. Similar basic logic carries over into more general versions of the model as we show in later sections. In this example, it is optimal for the buyer to promise to pay a bonus since she then need not leave a rent to the seller. However, a formal fixed-price contract along with a termination scheme in which the buyer terminates to trade with the seller if the latter does not invest is equally effective in mitigating the holdup problem. See Section 4 for details.

The rest of the paper is organized as follows: Section 2 relates the present paper to the existing literature. Section 3 presents our base model in which a buyer purchases 0 or 1 units of a product from a seller, and Section 4 analyzes the model. Section 5 explores an extension of the model that allows multiple quantities to be transacted, and Section 6 offers concluding remarks.

## 2 Relationship to the Literature

In this section we discuss the relationship between our paper and the existing theoretical literature, and also our contribution to the empirical literature on the relationship between relational governance and formal contracts. We identify three sets of theoretical literature that are related to our work: (i) literature analyzing whether simple formal contracts resolve or mitigate the holdup problem under spot transactions; (ii) property rights literature that analyzes the threat-point effect under spot transactions; and (iii) literature on relational contracting and its interaction with formal contracts and institutions.

Let us begin with the first set of theoretical literature. Edlin and Reichelstein (1996) considered a bilateral trade relationship in which the seller and the buyer can write a simple contract specifying a fixed trade price and quantity at a future date. The seller then decides how much to invest in a relation-specific asset that lowers the subsequent cost of producing the good. After the investment is made, the buyer and the seller are free to renegotiate the contract with exogenously specified bargaining power. Edlin and Reichelstein found that a well-designed fixed-price contract can give the seller efficient investment incentives.

Che and Hausch (1999) pointed out that these previous studies were limited by their restriction on the nature of the relation-specific investments; that is, these studies focused on "selfish" investments that benefited the investor (e.g., the seller's investment reduces his/her production costs). Che and Hausch convincingly argued through a number of real-world

examples that “cooperative” investments (e.g., the seller’s investment improves the buyer’s value of the good) were equally important, although cooperative investments had received little attention in the literature.

Che and Hausch’s results for cooperative investments, which, as we discussed in Introduction, serve as the starting point of our work, are very different from those of Edlin and Reichelstein for selfish investments although they considered a bilateral trade relationship similar to the one analyzed by Edlin and Reichelstein: Che and Hausch showed that if investments are sufficiently cooperative and a commitment not to renegotiate is impossible for the parties, there exists an intermediate range of bargaining shares for which contracting has no value, i.e., contracting offers the parties no advantages over *ex post* negotiation. In particular, contracting has no value for any parameter range if both investments are purely cooperative (that is, the seller’s investment benefits the buyer only, and the buyer’s investment benefits the seller only).

Several papers argue that the first-best (efficient) purely cooperative investment by the seller can be induced even under spot transactions. MacLeod and Malcomson (1993), the very first paper studying cooperative investment, show the efficiency result when the outside option principle applies to enable the seller to obtain all the returns from investment. Lyon and Rasmusen (2004) point out that if the buyer always has a final opportunity to exercise her option and there is no discounting, then the seller can obtain all the returns from investment by refusing to renegotiate, and show that a buyer-option contract implements the first-best cooperative investment.<sup>10</sup>

Che and Chung (1999) and Schweizer (2006) show that the first-best investment can be induced under particular breach remedies which require that investments be observed by the court. Stremitzer (2010) shows that simple contracts specifying required “quality” (a threshold value to the buyer) can induce the seller’s first-best investment if the court can observe the realized value to the buyer. Note that the efficiency results by Che and Chung (1999), Schweizer (2006), and Stremitzer (2010) require the court to observe more information than that assumed in Che and Hausch (1999) where no investment-related information (including the value to the buyer) is contractible and hence none of the standard breach remedies is available except for specific performance.<sup>11</sup>

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<sup>10</sup>Wickelgren (2006) however argues that this conclusion is sensitive to the renegotiation bargaining structure. He shows that the holdup problem reappears if the parties discount the future and renegotiation continues for sufficiently long periods.

<sup>11</sup>Stremitzer (2010) also shows that the first-best investment can be attained by a contract with lower informational requirements that the court only need observe whether the realized value to the buyer lies above or below the threshold level specified in the contract. Furthermore, he argues that this contract can be implemented in the informational environment of Che and Hausch (1999) because they assume that the court can observe whether or not trade has occurred, which implies that the court can verify whether the

Our paper contributes to this literature by analyzing roles of formal contracts in resolving or mitigating the holdup problem under repeated transactions in the presence of the threat-point effect. In contrast to MacLeod and Malcomson (1993) and Lyon and Rasmusen (2004), we follow Edlin and Reichelstein (1996) and Che and Hausch (1999) by assuming exogenously specified surplus sharing in renegotiation.<sup>12</sup> And to be comparable to Che and Hausch (1999), throughout the paper we adopt the same informational requirements for enforcement as Che and Hausch (1999).<sup>13</sup> Hence in our base model the seller cannot be induced to choose the first-best cooperative investment under spot transactions.

All the papers mentioned above, including Edlin and Reichelstein (1996) and Che and Hausch (1999), assume that the seller's investment has no effect on the surplus under no trade.<sup>14</sup> In contrast to the first set of literature, the second set, which is the literature on the property rights theory originating from Grossman and Hart (1986), Hart and Moore (1990), and Hart (1995), assumes that the parties' investments on an asset affect the asset owner's payoff under no trade. Edlin and Hermalin (2000) call this the threat-point effect. Besides the analysis of the threat-point effect, some of the recent literature following this theory is particularly relevant to our work in that cooperative investments are analyzed (Edlin and Hermalin, 2000; Nöldeke and Schmidt, 1998; Segal and Whinston, 2000) and the possibility of the negative threat-point effect (the seller's investment reduces his payoff at the threat point) is studied (Edlin and Hermalin, 2000; Rajan and Zingales, 1998). However, different from our model, this set of literature adopts the incomplete contract setting where formal contracts contingent on terms of trade, in particular, quantities, are infeasible.<sup>15</sup> Hence, roles of formal contracts are not explored in this literature.

Regarding the third set of theoretical literature, our analysis of informal agreements builds on a general analysis of relational contracts by MacLeod and Malcomson (1989) and Levin (2003). Baker et al. (1994), Schmidt and Schnitzer (1995), Pearce and Stacchetti (1998), and Kvaløy and Olsen (2009) study how formal contracting affects the self-enforceability of informal agreements. These papers do not, however, analyze the holdup problem caused by the relation-specific nature of investments and incompleteness of contracting in the sense that

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good exceeds a certain minimal quality threshold.

<sup>12</sup>Edlin and Reichelstein (1996), in an appendix, present an underlying bargaining game that corresponds to the constant bargaining share.

<sup>13</sup>See Stremitzer (2010) for more on the relationship between the informational environment and cooperative investment.

<sup>14</sup>While the seller's outside option depends on his investment in the general model of MacLeod and Malcomson (1993), it is assumed to be independent when they study one-sided cooperative investment.

<sup>15</sup>Segal and Whinston (2000), who study effects of exclusive contracts on investment incentives, assume in most of their analysis that the only possible contract is the exclusivity provision. In a later section they allow contracts specifying quantities to be feasible in limited situations where the result of Che and Hausch (1999) that contracting has no value is confirmed.

they allow contingent formal contracts while they do not consider *ex post* price negotiation. In our model, on the other hand, formal contracts and *ex post* negotiation play crucial roles. Baker et al. (2002), Halonen (2002), and Morita (2001) analyze the holdup problem in infinitely repeated transactions, but their focus is quite different from ours. Baker et al. (2002) and Halonen (2002) study how asset ownership affects the self-enforceability of relational contracts, and Morita (2001) focuses on the role of partial ownership in resolving the holdup problem under repeated interaction. None of the studies identify the threat-point effect as an important factor in determining the value of formal contracting, nor do they capture the idea that simple formal contracts can play an important role in reducing reneging temptations under repeated transactions.<sup>16</sup> Baker et al. (2001) extend Baker et al. (2002) by allowing formal contracts contingent on contractible characteristics of trade in addition to allocation of asset ownership. Although their model is general, they focus on the difficulty of “bringing the market inside the firm.” Corts (2009) studies effects of repeated interaction on formal contract choice between cost-plus contracts and fixed-price contracts in construction and procurement. Neither the value of formal contracting nor the threat-point effect is analyzed in these papers.

The present paper also sheds new light on recent empirical investigations on the relationship between relational governance and formal contracts. In the empirical literature of transaction cost economics, the majority of previous researchers have studied how several transactional properties (representing asset specificity, uncertainty, and transactional frequency) affect an organizational mode, conceptualized by market, hierarchy, or various hybrid and intermediate modes (see, for example, Shelanski and Klein (1995) and Boerner and Macher (2002) for surveys).

Several researchers have recently made an important contribution to this literature by investigating the relationship between relational governance and formal contracts (see, for example, Banerjee and Duflo (2000); Poppo and Zenger (2002); Corts and Singh (2004); Kalnins and Mayer (2004)). It has often been argued that relational governance and formal contracts are substitutes rather than complements (see Dyer and Singh (1998) and Adler (2001), among others), and that the use of formal contracts may even have undesirable consequences under relational governance (see Macaulay (1963) for an empirical investigation and Bernheim and Whinston (1998) for a theoretical analysis). In contrast, Poppo and Zenger (2002) have recently presented evidence which suggests that relational governance and formal contracts can be complements. In their investigation of informational service

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<sup>16</sup>Although Baker et al. (2002) employ a holdup model different from ours, integration in their model and fixed-price contracts in our model play a similar role of eliminating *ex post* opportunities for price negotiation. See footnotes 18 and 20 for details.

outsourcing they found that, controlling for several transactional properties such as asset specificity, increases in the level of relational governance were associated with greater levels of complexity in formal contracts (see Ryall and Sampson (2009) for a related finding).

We contribute to this line of investigation by exploring the relationship between relational governance and formal contracts in the presence of the holdup problem. Our analysis identifies whether the threat-point effect is positive or negative in investment as an important factor in determining the value of formal contracts. We find that relational governance and formal contracts can be complements or substitutes, and that the use of formal contracts may even have undesirable consequences. Our analysis indicates that they are complements when the holdup problem is so serious that the relation-specific investment reduces the negotiated price, and a necessary condition for this is that the investment reduces the surplus under no trade (negative threat-point effect). We therefore suggest that some proxies for the threat-point effect be incorporated in future empirical studies.

### 3 Model

We consider repeated transactions between an upstream party (seller) and a downstream party (buyer).<sup>17</sup> In each period, the seller chooses an action (e.g., investment level)  $a \in A$  by incurring private cost  $d(a)$ . Although  $a$  can be fairly general (e.g., multi-dimensional), to simplify exposition we assume  $a$  is real-valued and is measured in terms of the costs of action, and hence  $d(a) = a$ . The set of feasible actions  $A \subset \mathbb{R}$  can be finite with more than one elements, countably infinite, or continuous. We assume there exists the least costly action in  $A$ , denoted by  $\underline{a} \geq 0$ .

The seller's action affects (i) the value of the seller's product for the buyer and (ii) the alternative-use value of the product. Let  $v(a, \theta)$  be the value for the buyer, where  $\theta$  is the state of nature drawn from support  $\Theta = [\underline{\theta}, \bar{\theta}]$  by a cumulative distribution function  $F(\cdot)$ , and let  $m(a, \theta)$  be the alternative-use value, which we assume accrues to the seller, and the payoff to an alternative user is zero. The buyer's outside payoff is independent of the seller's action and normalized to zero, and hence the total no-trade surplus is equal to  $m(a, \theta)$ . To highlight the value of formal contracting in repeated transaction, we assume that at most one unit of an indivisible product is traded, and the seller's production cost is normalized to zero. Hence the seller's investment is purely "cooperative." Later in Section 5 we will extend our main results to the case in which different levels of quantity are traded.<sup>18</sup>

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<sup>17</sup>Throughout the paper we suppose the seller is a male and the buyer is a female for convenience.

<sup>18</sup>Although we do not study allocation of asset ownership but focus on roles of formal contracting, our model can be interpreted in terms of asset ownership as follows: There is a physical asset that is owned by the seller. The seller invests in the asset and uses it to generate value  $v(a, \theta)$  for the buyer when they trade. If

We assume  $v(a, \theta)$  is strictly increasing in  $a$  and  $v(a, \theta) > a$  for all  $a$  and  $\theta$ . The alternative-use value  $m(a, \theta)$  may be increasing or decreasing in  $a$  as discussed in the previous sections. We, however, follow the holdup literature by assuming that investment affects  $v(a, \theta)$  at least as much as  $m(a, \theta)$  at margins:<sup>19</sup>

$$v(a, \theta) - v(a', \theta) \geq m(a, \theta) - m(a', \theta) \quad \text{for all } \theta \text{ and } a > a'. \quad (1)$$

Denote the efficient action by  $a^*$ :  $a^* = \arg \max_a (E[v^+(a, \theta)] - a)$  where  $v^+(a, \theta) = \max\{v(a, \theta), m(a, \theta)\}$  and  $E[x(\theta)] = \int x(\theta) dF(\theta)$ . We assume  $a^*$  is unique and  $a^* > \underline{a}$ . For simplicity, we assume  $v^+(a^*, \theta) = v(a^*, \theta)$  for all  $\theta$ : under the first-best investment, trade is efficient in all states.

We assume that  $a$ ,  $\theta$ ,  $v$ , and  $m$  are observable to both parties but unverifiable, while delivery of the product and transfer payments are verifiable. Consistent with Che and Hausch (1999), we assume that the court can observe no investment-related information (including the value for the buyer) and hence none of the standard breach remedies is available except for specific performance.

Suppose that at the beginning of each period the seller and the buyer can agree on a compensation plan, with the seller promising a particular action. The compensation plan consists of  $\{w, \bar{p}, (b(a, \theta))_{a \in A, \theta \in \Theta}\}$ , where  $w$  is paid from the buyer to the seller at the beginning of each period and serves the distribution purpose only,  $\bar{p}$  is a formal fixed-price contract contingent on the delivery of the product, and  $b(a, \theta)$  is an additional informal payment made by the buyer when the seller's action is  $a$  and state  $\theta$  realizes (negative payments mean transfers from the seller to the buyer). Since delivery of the product is verifiable, we assume that  $\bar{p}$  is enforced with a specific performance damage clause. On the other hand,  $b(a, \theta)$  is not enforceable because neither  $a$  nor  $\theta$  is verifiable. In order to investigate the value of the formal fixed-price contract in resolution of the holdup problem, we compare it with the case of no formal contract, in which  $\bar{p}$  is not specified in a compensation plan. When  $\bar{p}$  is not specified,  $b(a, \theta)$  is the (informal) price paid by the buyer contingent on the level of

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they do not trade, the seller uses the asset in trade with third parties to obtain  $m(a, \theta)$ . This payoff structure is essentially identical to nonintegration (where the asset is owned by the seller) in Baker et al. (2002). They compare this arrangement with integration (buyer ownership). In Edlin and Hermalin (2000) and Nöldeke and Schmidt (1998), the final owner of the asset enjoys return  $r(a, e)$ . The seller chooses to invest  $a \geq 0$  first and then the buyer chooses to invest  $e \geq 0$  after observing  $a$ . They consider the buyer option contract in which the buyer has the option to buy back the asset at the prespecified price after the seller invests. If the buyer does not exercise the option, her optimal choice is  $e = 0$ , and hence their models translate into ours by  $v(a) = \max_e r(a, e) - e$  and  $m(a) = r(a, 0)$  (assuming there is no uncertainty, for simplicity).

<sup>19</sup>This assumption holds in the models of Edlin and Hermalin (2000) and Nöldeke and Schmidt (1998) if the seller's investment  $a$  and the buyer's investment  $e$  are complements at the margin. Edlin and Hermalin (2000) show that if (1) holds with strict inequalities and the buyer decides to exercise the option before she chooses her investment, the seller cannot be induced to choose the first-best investment.

the seller's action, the state of nature, and the delivery of the product.

Our focus on fixed-price contracts as a form of formal contracts can be justified by our objective to show that even writing a simple fixed-price formal contract can help mitigate the holdup problem under repeated transactions but it is not the case under spot transactions (Proposition 4 (a)). Furthermore, in subsection 4.5 we identify a natural condition under which more general formal contracts that may be contingent on messages are of no value, and hence confining our attention to fixed-price contracts as a form of formal contracts is without loss of generality.

In each period, the timing is as follows. First, the seller and the buyer can agree on a compensation plan. If  $\bar{p}$  is specified in the plan, the agreement includes signatures by the seller and the buyer on the formal fixed-price contract. Second, the seller chooses an action. Third, the state of nature realizes. Fourth, after observing the seller's action and the state of nature, the buyer and the seller negotiate to an ex post efficient outcome if there is inefficiency. This applies either when trade is inefficient under a fixed-price contract, or when trade is efficient under no formal contract. In these cases, we assume that the transfer is determined by the generalized Nash bargaining solution. Let  $\alpha \in [0, 1)$  be the seller's share of the surplus, and hence the buyer's share is  $1 - \alpha$ . Finally, the seller produces and sells the product to the buyer according to the compensation plan or at the negotiated price, or in the outside market.

## 4 Analysis

This section explores the value of formal contracts in resolving or mitigating the holdup problem under both spot and repeated transactions. In subsections 4.1–4.4, we focus on fixed-price contracts as a form of formal contracts. In subsection 4.5, we show that this focus is without loss of generality when  $\rho(a, \theta)$  is either weakly increasing or weakly decreasing in  $a$  for all realizations of uncertainty  $\theta$ , where  $\rho(a, \theta)$  denotes the negotiation price in the absence of formal contracts (see subsection 4.1 for the definition of  $\rho(a, \theta)$ ).

### 4.1 Spot Transactions

When the seller and the buyer meet only once, or they do not use history dependent strategies, a standard holdup problem can arise. Since  $b(a, \theta)$  does not play any role in spot transactions, we simply set  $b(a, \theta) \equiv 0$  in this subsection.

Suppose that no formal fixed-price contract is written at the beginning. If state  $\theta$  satisfying  $v(a, \theta) \geq m(a, \theta)$  realizes, the seller and the buyer negotiate trade and a price. The

negotiated price, denoted by  $\rho(a, \theta)$ , is given by

$$\rho(a, \theta) = m(a, \theta) + \alpha(v(a, \theta) - m(a, \theta)) = \alpha v(a, \theta) + (1 - \alpha)m(a, \theta).$$

The seller's payoff is then  $\rho(a, \theta) - a$ . On the other hand, if  $v(a, \theta) < m(a, \theta)$  holds in the realized state, there is no negotiation and trade does not occur. The seller's payoff is  $m(a, \theta) - a$ .

Define  $\rho^+(a, \theta)$  by

$$\rho^+(a, \theta) = \max\{\rho(a, \theta), m(a, \theta)\} = m(a, \theta) + \alpha \max\{v(a, \theta) - m(a, \theta), 0\}.$$

Then the seller chooses action  $a$  that maximizes  $E[\rho^+(a, \theta)] - a$ . Denote the optimal action under no contract by  $a^o$ :

$$a^o \in \arg \max_a (E[\rho^+(a, \theta)] - a) \quad (2)$$

In this setup it is easy to show that the seller does not overinvest.

**Proposition 1** If no formal fixed-price contract is written at the beginning, the seller does not overinvest under spot transaction:  $a^* \geq a^o$ .

**Proof** Suppose instead  $a^* < a^o$ . Since  $a^*$  is uniquely efficient,  $E[v(a^*, \theta)] - a^* > E[v^+(a^o, \theta)] - a^o$ , or

$$a^o - a^* > E[v^+(a^o, \theta)] - E[v(a^*, \theta)]$$

holds. On the other hand, since  $a^o$  is optimal under spot transaction,

$$a^o - a^* \leq E[\rho^+(a^o, \theta)] - E[\rho^+(a^*, \theta)]$$

By  $\alpha < 1$ ,  $a^o > a^*$ , and (1),

$$\rho^+(a^o, \theta) - \rho^+(a^*, \theta) \leq v(a^o, \theta) - v(a^*, \theta) \leq v^+(a^o, \theta) - v(a^*, \theta)$$

holds for all  $\theta$ . Therefore

$$a^o - a^* \leq E[v^+(a^o, \theta)] - E[v(a^*, \theta)]$$

must hold, which is a contradiction. **Q.E.D.**

Although the seller's action increases  $v(a, \theta)$ , he cannot capture the full marginal contribution ( $\alpha < 1$ ). This "holdup effect" may be partially offset by the threat-point effect

that the seller's action improves his bargaining position via  $m(a, \theta)$  if it is increasing in  $a$ . However, this effect at most compensates for the return lost by the holdup effect, due to assumption (1). To make the analysis interesting, we hereafter assume  $a^* > a^o$ : There exists  $a < a^*$  such that the following inequality holds:

$$a - a^* < E[\rho^+(a, \theta)] - E[\rho^+(a^*, \theta)].$$

We denote the joint surplus from trade at action  $a$  by  $\pi(a) \equiv E[v^+(a, \theta)] - a$ , and the joint surplus at no trade by  $\bar{\pi} \equiv \max_a(E[m(a, \theta)] - a)$ . To simplify analysis (particularly under repeated transaction), we assume  $v(a^o, \theta) \leq m(a^o, \theta)$  for all  $\theta$ : for all states no trade is efficient under action  $a^o$ . This implies  $v^+(a^o, \theta) = m(a^o, \theta)$  for all  $\theta$ , and hence  $\bar{\pi} = \pi(a^o)$ : the equilibrium outcome under spot transaction without formal contracting is that, in neither state the seller and the buyer trade.

Next, suppose that the buyer and the seller sign a formal fixed-price contract  $\bar{p}$  at the beginning. If  $v(a, \theta) \geq m(a, \theta)$ , there is no room for negotiation and the parties trade with price  $\bar{p}$ . The seller's payoff is  $\bar{p} - a$ . If  $v(a, \theta) < m(a, \theta)$ , however, they negotiate to cancel the contract and break off trade. The seller's payoff is then  $\bar{p} + \alpha(m(a, \theta) - v(a, \theta)) - a$ . While there is renegotiation if  $v(a, \theta) < m(a, \theta)$ , the payment to the seller is decreasing in  $a$  by assumption (1).

Define  $\rho^-(a, \theta)$  by

$$\rho^-(a, \theta) = \alpha \max\{m(a, \theta) - v(a, \theta), 0\} = \rho^+(a, \theta) - \rho(a, \theta).$$

The seller chooses  $a$  to maximize  $\bar{p} + E[\rho^-(a, \theta)] - a$ , the solution of which is obviously  $a = \underline{a}$ . The outcome is no better than the case with no formal fixed-price contract, where although the seller underinvests, he may choose action higher than  $\underline{a}$ . Fixed-price contracts are of no value under spot transactions, irrespective of the sign of the threat-point effect.<sup>20</sup>

## 4.2 Relational Contract without Formal Fixed-Price Contract

We now consider the case in which the seller and the buyer engage in infinitely repeated transactions, with the common discount factor  $\delta \in (0, 1)$ . Since they have symmetric information in our model, we follow MacLeod and Malcomson (1989) and Levin (2003) that

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<sup>20</sup>Baker et al. (2002) assume that under integration (buyer ownership) the buyer can simply take the product without paying the seller. Their integration is hence equivalent to the fixed-price contract with  $\bar{p} = 0$  for the case of  $v(a, \theta) \geq m(a, \theta)$ . In contrast to our model, however, Baker et al. (2002) assume that  $v(a, \theta) < m(a, \theta)$  never happens. If  $m(a, \theta)$  were tied to the seller even under buyer ownership, their integration and our fixed-price contracts would be identical for the case of  $v(a, \theta) < m(a, \theta)$  as well. However, if  $m(a, \theta)$  were tied to the asset, then the buyer owning the asset and fixed-price contracts would no longer be equivalent.

provide the definite treatment of models of ongoing contractual relationships. A *relational contract* is a complete plan for the relationship describing the compensation plan and the seller's action for every period and history. Since a relational contract is in general contingent on the seller's action which is observable but unverifiable, it must satisfy conditions under which it is neither party's interest to renege on the contract: it must be *self-enforcing*, i.e., a subgame perfect equilibrium of the repeated game.

The *optimal contract* is a self-enforcing relational contract that maximizes the joint surplus. Without loss of generality we can focus on *stationary contracts* under which in every period the parties agree on the same compensation plan and the seller chooses the same action on the equilibrium path. Furthermore, if either party reneges on the payment or action, they negotiate to determine the trade in the current period, and, from the next period on, they revert to no trade, which is the worst equilibrium outcome. Furthermore, for any optimal stationary contract, there is an optimal stationary contract with the same equilibrium behavior and the property that even off the equilibrium path the seller and the buyer cannot jointly benefit from renegotiating to a new self-enforcing contract.<sup>21</sup> This implies that, for any optimal stationary contract, there is an optimal renegotiation-proof stationary contract with the same equilibrium behavior.<sup>22</sup> That is, although our focus on stationary contracts means that we are assuming that the parties cannot commit not to renegotiate the relational contract, this assumption is for simplicity and not critical for our results.

In this subsection, we assume no formal fixed-price contract is written. The effects of writing a formal fixed-price contract are analyzed in the next subsection. We obtain conditions under which there exists a self-enforcing (stationary) relational contract that implements a given action  $\hat{a} > a^o$  satisfying  $\pi(\hat{a}) > \bar{\pi}$ .

The relational contract in this subsection includes the following efficient trade decision, denoted by  $e(a, \theta) \in \{0, 1\}$ , and compensation plan  $b(a, \theta)$ :

- If  $v(a, \theta) \geq m(a, \theta)$ , then trade ( $e(a, \theta) = 1$ ) and the buyer pays  $b(a, \theta)$ .
- If  $v(a, \theta) < m(a, \theta)$ , then no trade ( $e(a, \theta) = 0$ ) and the buyer pays  $b(a, \theta)$ .

If the parties follow the trade decision and compensation plan as above, the seller's incentive compatibility constraints are given as follows.

$$\underline{E[b(\hat{a}, \theta) + (1 - e(\hat{a}, \theta))m(\hat{a}, \theta)] - \hat{a} \geq E[b(a, \theta) + (1 - e(a, \theta))m(a, \theta)] - a \quad \text{for all } a \quad (3)}$$

<sup>21</sup>Although Levin (2003) does not analyze a case where the parties engage in *ex post* price negotiation in each period, it is straightforward to generalize his results to such a situation.

<sup>22</sup>A result in Levin (2003) implies that, for any optimal stationary contract, there is an optimal stationary contract in which from the next period on after a deviation, the term of transaction is altered to hold the deviating party to the same payoff as the one in the spot transaction outcome but still keep them on the *ex post* efficiency frontier. Such an optimal stationary contract is renegotiation proof.

Note that future payoffs do not appear in the constraints. A logic similar to the existence of optimal stationary contracts can be applied to show that we can further restrict our attention to contracts that provide the seller's investment incentives with discretionary payments alone. However, this does not imply that the optimal stationary contract specifies discretionary payments: one can construct an equivalent optimal stationary contract in which there is no discretionary payment and the relationship will be terminated after the seller chooses an action different from the one specified in the contract.

We next derive the buyer's self-enforcing condition. If  $v(a, \theta) \geq m(a, \theta)$  holds, the buyer's short-term gain from not paying  $b(a, \theta)$  and negotiating to trade by price  $\rho(a, \theta)$  instead is  $b(a, \theta) - \rho(a, \theta)$ . If  $v(a, \theta) < m(a, \theta)$  holds, then the buyer's gain is  $b(a, \theta)$  since trade is inefficient. The buyer's reneging temptation is hence written as  $\max_{a, \theta} \{b(a, \theta) - e(a, \theta)\rho(a, \theta)\}$ . The buyer will then lose her future per period gain  $E[e(\hat{a}, \theta)v(\hat{a}, \theta)] - w - E[b(\hat{a}, \theta)]$ . The buyer therefore honors the agreement if and only if

$$\max_{a, \theta} \{b(a, \theta) - e(a, \theta)\rho(a, \theta)\} \leq \frac{\delta}{1 - \delta} (E[e(\hat{a}, \theta)v(\hat{a}, \theta)] - w - E[b(\hat{a}, \theta)]) \quad (4)$$

The seller's self-enforcing condition is obtained in a similar fashion. The seller honors the agreement if and only if

$$-\min_{a, \theta} \{b(a, \theta) - e(a, \theta)\rho(a, \theta)\} \leq \frac{\delta}{1 - \delta} (w + E[b(\hat{a}, \theta)] + (1 - e(\hat{a}, \theta))m(\hat{a}, \theta) - \hat{a} - \bar{\pi}). \quad (5)$$

Combining (4) and (5) yields a single necessary condition:

$$\max_{a, \theta} \{b(a, \theta) - e(a, \theta)\rho(a, \theta)\} - \min_{a, \theta} \{b(a, \theta) - e(a, \theta)\rho(a, \theta)\} \leq \frac{\delta}{1 - \delta} (\pi(\hat{a}) - \bar{\pi}). \quad (6)$$

And (3) and (6) are also sufficient for investment  $\hat{a}$  to be implemented: one can find an appropriate  $w$  such that (4), (5), and the parties' participation constraints are satisfied.

Note that the left-hand side of (6), as well as Baker et al. (2002)'s corresponding condition,<sup>23</sup> still depends on compensation plan  $b(a, \theta)$ . In the next proposition, we go further to obtain the necessary and sufficient condition for compensation plan implementing  $\hat{a}$  to exist. Define the change in expected payments by  $\Delta^+(a, a') \equiv E[\rho^+(a, \theta) - \rho^+(a', \theta)]$ .

**Proposition 2** Suppose no formal fixed-price contract is written. The seller's action  $\hat{a}$  satisfying  $\pi(\hat{a}) > \bar{\pi}$  can be implemented by a relational contract if and only if (DE-NC)

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<sup>23</sup>Condition (6) roughly corresponds to the enforcement condition under nonintegration in Baker et al. (2002, condition in p.55) where trade is assumed to be always efficient and hence  $e(a, \theta) \equiv 1$ .

holds.

$$\hat{a} - a^o - \Delta^+(\hat{a}, a^o) \leq \frac{\delta}{1-\delta} (\pi(\hat{a}) - \bar{\pi}) \quad (\text{DE-NC})$$

**Proof** See Appendix.

The necessary and sufficient condition (DE-NC) depends only on the parameters under the action to be implemented ( $\hat{a}$ ) and the action most preferred by the seller under spot transactions ( $a^o$ ). Intuitively, the seller's incentive compatibility constraints are binding at  $a = a^o$ , and the buyer must pay the seller sufficiently higher ( $\hat{a} - a^o$ ) for  $\hat{a}$  than for  $a^o$ . However, the higher pay for  $\hat{a}$  results in renegeing temptations for both parties. The buyer faces the temptation not to pay the informal price  $b(\hat{a}, \theta)$  but to pay the negotiated price  $\rho^+(\hat{a}, \theta)$ . The seller faces the temptation to choose  $a^o$ , and not to pay a penalty  $-b(a^o, \theta)$  but to receive  $\rho^+(a^o, \theta)$ . The total renegeing temptation is thus equal to the left-hand side of (DE-NC), which is nonnegative because the optimality of  $a^o$  under spot transactions without formal contracts (2) implies

$$\Delta^+(a^o, a) \geq a^o - a \quad \text{for all } a. \quad (7)$$

This renegeing temptation must be at most as large as the total future loss.

Note that the right-hand side of (6) or (DE-NC) does not depend on the compensation plan. There is hence no compensation plan that makes the total renegeing temptation given in the left-hand side of (6) smaller than the left-hand side of (DE-NC). Therefore, the compensation plan that is constructed by

$$\begin{aligned} b(\hat{a}, \theta) &= e(\hat{a}, \theta)\rho(\hat{a}, \theta) - \Delta^+(\hat{a}, a^o) + \hat{a} - a^o \\ b(a, \theta) &= e(a, \theta)\rho(a, \theta), \quad \text{for all } a \neq \hat{a} \end{aligned}$$

in the proof of the proposition minimizes the left-hand side of (6), and in this sense, it is an optimal contract implementing a given action  $\hat{a}$ .

### 4.3 Relational Contract with Formal Fixed-Price Contract

As discussed in Subsection 4.1, formal fixed-price contracts play no role in resolving the hold-up problem under spot transactions. The story is, however, different for repeated transactions.

In this subsection, we continue considering the case in which the seller and the buyer engage in infinitely repeated transactions, focusing on stationary contracts as in the previous subsection. Unlike in the previous subsection, however, we consider the case in which the

buyer and the seller sign a formal fixed-price contract  $\bar{p}$  at the beginning of each period on the equilibrium path. Note that if either party reneges on additional payments  $b(a, \theta)$  when trade is actually efficient, no price negotiation arises because the formal fixed-price contract  $\bar{p}$  is enforced. From the next period on, the parties revert to no trade. We derive conditions for a self-enforcing (stationary) relational contract implementing a given action  $\hat{a}$  to exist.

The relational contract along with a formal fixed-price contract  $\bar{p}$  includes the following efficient trade decision and compensation plan:

- If  $v(a, \theta) \geq m(a, \theta)$ , then trade following the formal fixed-price contract ( $e(a, \theta) = 1$ ) and the buyer pays  $b(a, \theta)$  in addition to  $\bar{p}$ .
- If  $v(a, \theta) < m(a, \theta)$ , then cancel the formal contract to no trade ( $e(a, \theta) = 0$ ) and the buyer pays  $b(a, \theta)$ .

As in the no formal contract case, without loss of generality we assume that incentives are provided via discretionary payments alone. The seller's incentive compatibility constraints are then given as follows.

$$\begin{aligned} E[b(\hat{a}, \theta) + e(\hat{a}, \theta)\bar{p} + (1 - e(\hat{a}, \theta))m(\hat{a}, \theta)] - \hat{a} \\ \geq E[b(a, \theta) + e(a, \theta)\bar{p} + (1 - e(a, \theta))m(a, \theta)] - a \quad \text{for all } a \end{aligned} \quad (8)$$

The buyer's reneging temptation is derived as follows. First, when  $v(a, \theta) \geq m(a, \theta)$ , the buyer can refuse to pay  $b(a, \theta)$  though she has to follow the formal contract and pay  $\bar{p}$ . Her short-term gain is  $b(a, \theta)$ . Next when  $v(a, \theta) < m(a, \theta)$ , the buyer can refuse to cancel the formal contract and to pay  $b(a, \theta)$ , and instead negotiate to obtain

$$v(a, \theta) - \bar{p} + (1 - \alpha)(m(a, \theta) - v(a, \theta)) = \rho(a, \theta) - \bar{p}.$$

The buyer's reneging temptation is thus written as  $\max_{a, \theta} \{b(a, \theta) + (1 - e(a, \theta))(\rho(a, \theta) - \bar{p})\}$ . The buyer honors the agreement if and only if

$$\max_{a, \theta} \{b(a, \theta) + (1 - e(a, \theta))(\rho(a, \theta) - \bar{p})\} \leq \frac{\delta}{1 - \delta} (E[e(\hat{a}, \theta)v(\hat{a}, \theta)] - w - \bar{p} - E[b(\hat{a}, \theta)]). \quad (9)$$

Similarly, the seller honors the agreement if and only if

$$-\min_{a, \theta} \{b(a, \theta) + (1 - e(a, \theta))(\rho(a, \theta) - \bar{p})\} \leq \frac{\delta}{1 - \delta} (w + \bar{p} + E[b(\hat{a}, \theta) + (1 - e(\hat{a}, \theta))m(\hat{a}, \theta)] - \hat{a} - \bar{\pi}). \quad (10)$$

Combining these conditions yields<sup>24</sup>

$$\begin{aligned} & \max_{a,\theta} \{b(a,\theta) + (1 - e(a,\theta))(\rho(a,\theta) - \bar{p})\} - \min_{a,\theta} \{b(a,\theta) + (1 - e(a,\theta))(\rho(a,\theta) - \bar{p})\} \\ & \leq \frac{\delta}{1 - \delta} (\pi(\hat{a}) - \bar{\pi}) \end{aligned} \quad (11)$$

Define the change in expected payments by

$$\begin{aligned} \Delta^-(a, a') &= E[\rho^-(a, \theta) - \rho^-(a', \theta)] \\ &= \alpha E[\max\{m(a, \theta) - v(a, \theta), 0\} - \max\{m(a', \theta) - v(a', \theta), 0\}], \end{aligned}$$

which is nonpositive if  $a > a'$ .

**Proposition 3** The seller's action  $\hat{a}$  satisfying  $\pi(\hat{a}) > \bar{\pi}$  can be implemented by combining a formal fixed-price contract and a relational contract if and only if (DE-FP) holds.

$$\hat{a} - \underline{a} - \Delta^-(\hat{a}, \underline{a}) \leq \frac{\delta}{1 - \delta} (\pi(\hat{a}) - \bar{\pi}) \quad (\text{DE-FP})$$

**Proof** See Appendix.

The necessary and sufficient condition (DE-FP) depends only on two actions  $\hat{a}$  and  $\underline{a}$  the latter of which is the one most preferred by the seller under a spot transaction with a fixed-price contract. Intuitively, the buyer faces the temptation not to pay an informal price  $b(\hat{a}, \theta)$  but to pay  $\alpha \max\{m(\hat{a}, \theta) - v(\hat{a}, \theta), 0\}$  in addition to  $\bar{p}$ . The seller faces the temptation to choose  $\underline{a}$ , and not to pay a penalty  $-b(\underline{a}, \theta)$  but to receive  $\alpha \max\{m(\underline{a}, \theta) - v(\underline{a}, \theta), 0\}$  in addition to  $\bar{p}$ . The total renegeing temptation is thus equal to the left-hand side of (DE-FP).

#### 4.4 Comparison

We can analyze the value of writing a formal fixed-price contract in repeated transactions by comparing two conditions, (DE-NC) for the case of no formal fixed-price contract, and (DE-FP) for the case of writing a formal fixed-price contract.

The conditions differ only in terms of the renegeing temptations given on the left-hand sides, and the renegeing temptations are different in two respects. One difference is captured by the term  $\Delta^+(\hat{a}, a^o)$  in (DE-NC) and  $\Delta^-(\hat{a}, \underline{a})$  in (DE-FP).  $\Delta^+(\hat{a}, a^o)$  is nonzero when trade is efficient under either  $\hat{a}$  or  $a^o$  so that after renegeing, the seller and the buyer negotiate the

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<sup>24</sup>Condition (11) roughly corresponds to the enforcement condition under integration in Baker et al. (2002, condition (10) in p.52). Since trade is always efficient in Baker et al. (2002), the left-hand side is simplified to  $\max_{a,\theta} b(a, \theta) - \min_{a,\theta} b(a, \theta)$ .

price to trade the product under no formal contract. On the other hand,  $\Delta^-(\hat{a}, \underline{a})$  is nonzero when trade is inefficient under either  $\hat{a}$  or  $\underline{a}$  so that after reneging, they negotiate the transfer to cancel trade under a formal fixed-price contract.

The other difference, captured by the term  $(\hat{a} - a^o)$  in (DE-NC) and  $(\hat{a} - \underline{a})$  in (DE-FP), arises because the seller's optimal action under a spot transaction may be different. It is  $\underline{a}$  under a formal fixed-price contract, while  $a^o$ , the optimal action under no formal contract, may be higher than  $\underline{a}$ . Without formal contracting, the seller may choose an action higher than the least costly level because his action has positive effects on the price determined by negotiation, which depends on the seller's share ( $\alpha$ ) and the marginal values under trade and no trade. Since the value under trade  $v(a, \theta)$  is increasing in action, it provides the seller with an incentive to choose a higher action if the seller's share is positive. Furthermore, if the alternative-use value  $m(a, \theta)$  increases with action so that the threat-point effect is positive, it provides an additional incentive to increase action, although the effect is not as large as that of the value for the buyer because of (1). And even if the alternative-use value is decreasing so that the threat-point effect is negative, the marginal benefit of action for the buyer captured by the seller may be so large that the seller is induced to choose  $a^o > \underline{a}$ . On the other hand, the incentive via the price determined by negotiation is never positive under a fixed-price contract. If trade is always efficient, there will be no negotiation and hence the seller is paid a constant amount equal to the fixed price. If trade is inefficient, higher action reduces the total surplus and hence the negotiated price, whether the threat-point effect is positive or negative. The seller hence chooses the least costly action under formal fixed-price contracts.

The following comparative result is now immediate.

**Proposition 4** Consider the implementation of  $\hat{a}$  satisfying  $\pi(\hat{a}) > \bar{\pi}$ .

- (a) Suppose  $(a^o - \underline{a}) + \Delta^+(\hat{a}, a^o) - \Delta^-(\hat{a}, \underline{a}) < 0$  holds. If  $\hat{a}$  can be implemented under repeated transactions without any formal contract, the same action can be implemented under repeated transactions with an appropriate fixed-price contract. And there is a range of parameter values in which  $\hat{a}$  can be implemented only if a formal fixed-price contract is written.
- (b) Suppose  $(a^o - \underline{a}) + \Delta^+(\hat{a}, a^o) - \Delta^-(\hat{a}, \underline{a}) > 0$  holds. If  $\hat{a}$  can be implemented under repeated transactions with a formal fixed-price contract, the same action can be implemented under repeated transactions without any formal contract. And there is a range of parameter values in which  $\hat{a}$  can be implemented only if no formal fixed-price contract is written.

Proposition 4 (a) shows that in contrast to a well-known result in the case of spot transactions that “formal contracting has no value,” a simple formal fixed-price contract, combined with informal discretionary payments or termination of trade, can help mitigate the holdup problem under repeated transactions. The condition reflects two sources of differences in the renegeing temptation explained above.

To better understand the condition and develop intuition behind the result, we first suppose  $a^o = \underline{a}$ : under spot transactions, the seller faces no incentive to choose action higher than the least costly level. This is the case in our two-investment example in Introduction. Then the condition in (a) is equivalent to

$$\Delta^+(\hat{a}, \underline{a}) - \Delta^-(\hat{a}, \underline{a}) = E[\rho(\hat{a}, \theta) - \rho(\underline{a}, \theta)] < 0, \quad (12)$$

that is, the total negotiated price is lower for the higher action  $\hat{a}$  than for  $\underline{a}$ .

Suppose further that  $\Delta^-(\hat{a}, \underline{a}) = 0$ , that is, there is no price negotiation under fixed-price contracts. This condition holds when, for example,  $v(\hat{a}, \theta) \geq m(\hat{a}, \theta)$  and  $v(\underline{a}, \theta) = m(\underline{a}, \theta)$  for all  $\theta$ .<sup>25</sup> Condition (12) then becomes  $\Delta^+(\hat{a}, \underline{a}) < 0$ , implying that price negotiation brings negative incentive under no formal contract. In this case, by eliminating the effect of the price negotiation on the renegeing temptation, a well-designed formal fixed-price contract can reduce the renegeing temptation from  $(\hat{a} - \underline{a}) - E[\rho(\hat{a}, \theta) - \rho(\underline{a}, \theta)]$  to  $(\hat{a} - \underline{a})$ . Therefore, there is a range of parameter values in which (DE-FP) holds while (DE-NC) does not.

More generally without assuming  $\Delta^-(\hat{a}, \underline{a}) = 0$ , price negotiation can happen under a fixed-price contract when trade is inefficient, and under no formal contract when trade is efficient. The total difference of renegeing temptation is thus always summarized by (12), and this being negative implies the renegeing temptation is smaller under a fixed-price contract.

Since the post-renegeing price is determined by the generalized Nash bargaining solution,  $E[\rho(\hat{a}, \theta) - \rho(\underline{a}, \theta)]$  is rewritten as

$$E[\rho(\hat{a}, \theta) - \rho(\underline{a}, \theta)] = E[\alpha (v(\hat{a}, \theta) - v(\underline{a}, \theta)) + (1 - \alpha) (m(\hat{a}, \theta) - m(\underline{a}, \theta))].$$

A necessary condition for (12) is thus  $E[m(\hat{a}, \theta)] < E[m(\underline{a}, \theta)]$ : Increasing action from  $\underline{a}$  to  $\hat{a}$  weakens the seller’s bargaining position by reducing his expected payoff at the threat point. We have already argued in Section 1 that this is plausible under some settings. Under repeated transactions, this negative threat-point effect brings a new negative effect of raising the total renegeing temptation without a formal fixed-price contract. The formal fixed-price contract can eliminate the negative “market incentive” and hence can be valuable

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<sup>25</sup>More generally, this condition holds when trade is always efficient, that is,  $v(a, \theta) \geq m(a, \theta)$  for all  $a$  and  $\theta$ .

if in addition the holdup effect is sufficiently large ( $\alpha$  is sufficiently small).

On the other hand, Proposition 4 (b) implies that if the threat-point effect is zero (as in Che and Hausch (1999)) or positive, the formal fixed-price contract has no value even under repeated interactions. Furthermore, reducing such a positive “market incentive” by writing a formal fixed-price contract may decrease the total surplus under repeated transactions. Note that the result follows even though we are assuming  $a^o = \underline{a}$ , that is, the threat-point effect is not large enough to increase the seller’s action from the least costly level under spot transactions. The formal fixed-price contract has a negative value because of the increasing renegeing temptation under repeated transactions.<sup>26</sup>

The two-investment example in Section 1 corresponds to  $\alpha = 0$  (the buyer’s take-it-or-leave-it offer), and hence the sign of  $\Delta_m = m_1 - m_0$  is the same as that of  $E[\rho(\hat{a}, \theta)] - E[\rho(\underline{a}, \theta)]$ : the sign of the threat-point effect fully determines the value of writing a formal fixed-price contract. In more general settings analyzed in this section, the holdup effect must be sufficiently large ( $\alpha$  sufficiently small) and the threat-point effect must intensify this by being sufficiently negative.

We have so far developed intuition under assumption  $a^o = \underline{a}$ , in order to clarify how crucial is the marginal effect of action on the post-renegeing price, and in particular the threat-point effect, for the value of writing a formal fixed-price contract. Now consider a more general case of  $a^o \geq \underline{a}$ . Suppose the seller’s effort incentive through price negotiation is so strong that the seller is induced to choose an action higher than the least costly level even under a spot transaction ( $a^o > \underline{a}$ ). This advantage of not writing a formal fixed-price contract under a spot transaction plays an additional beneficial role of reducing the renegeing temptation under repeated transactions, because the incentive necessary to induce the seller to choose  $\hat{a}$  decreases from  $\hat{a} - \underline{a}$  to  $\hat{a} - a^o$ . The condition for writing a formal fixed-price contract to be valuable is now  $(a^o - \underline{a}) + \Delta^+(\hat{a}, a^o) - \Delta^-(\hat{a}, \underline{a}) < 0$ : the value of writing a formal fixed-price contract thus may not be positive even if the expected post-renegeing price is decreasing.

However, writing a formal fixed-price contract can still be beneficial if  $\Delta^+(\hat{a}, a^o) - \Delta^-(\hat{a}, \underline{a})$  is sufficiently negative and dominates the effect of increasing the renegeing temptation by  $a^o - \underline{a}$ . It is easy to construct an example in which despite  $a^o > \underline{a}$ , writing a formal fixed-price contract is of value under repeated transactions.

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<sup>26</sup>This result has a flavor of an endogenous incomplete contract. Bernheim and Whinston (1998) show that parties may optimally leave some verifiable aspects of performance unspecified (“strategic ambiguity”) in order to alter the set of feasible self-enforcing informal agreements. Not writing a formal fixed-price contract in our model may be classified as one form of strategic ambiguity, although the underlying models and logics are different. While we model the dynamic contracting problem in the context of infinitely repeated interaction and emphasize the effect on the alternative-use values, they consider two-period dynamic models with or without intertemporal payoff linkages.

## 4.5 General Formal Contracts

We have so far restricted our analysis to two polar cases of formal contracting, either no formal contract or a fixed-price contract. In this subsection, we consider general formal contracts that may be contingent on the messages sent by the buyer and the seller. We first extend the well-known result of Che and Hausch (1999) concerning the foundation of incomplete contracting to our setup in which the seller's action affects the surplus under no trade, and show that formal contracts are of no value under spot transactions when  $\rho(a, \theta)$  is either weakly increasing or weakly decreasing in  $a$  for all  $\theta$ . We then consider general formal contracts under repeated transactions and show that, under the same condition, it is without loss of generality to confine our attention to two polar cases: either not writing a formal contract or writing a fixed-price contract attains the highest investment incentives.

### Spot Transaction

A general formal contract is written as  $\{p(\eta_b, \eta_s), q(\eta_b, \eta_s)\}$ , where  $\eta_b = (a_b, \theta_b)$  is the buyer's report,  $\eta_s = (a_s, \theta_s)$  is the seller's report. For all reports  $(\eta_b, \eta_s)$ , the contract specifies trade decision  $q(\eta_b, \eta_s) \in \{0, 1\}$  and payment from the buyer to the seller  $p(\eta_b, \eta_s)$ . If the trade decision is inefficient for action and state  $\eta = (a, \theta)$ , the parties renegotiate the contract to the efficient trade decision  $e(\eta)$ , where  $e(\eta) = 1$  if  $v(\eta) - m(\eta) \geq 0$  and  $e(\eta) = 0$  if  $v(\eta) - m(\eta) < 0$ . The ex post payoffs in  $\eta$ , resulting from the contract and renegotiation, are as follows:

$$\begin{aligned} u_B(\eta_b, \eta_s \mid \eta) &= v(\eta)q(\eta_b, \eta_s) - p(\eta_b, \eta_s) + (1 - \alpha)e(\eta)(v(\eta) - m(\eta))(1 - q(\eta_b, \eta_s)) \\ &\quad + (1 - \alpha)(1 - e(\eta))(m(\eta) - v(\eta))q(\eta_b, \eta_s) \\ u_S(\eta_b, \eta_s \mid \eta) &= p(\eta_b, \eta_s) + m(\eta)(1 - q(\eta_b, \eta_s)) + \alpha e(\eta)(v(\eta) - m(\eta))(1 - q(\eta_b, \eta_s)) \\ &\quad + \alpha(1 - e(\eta))(m(\eta) - v(\eta))q(\eta_b, \eta_s) \end{aligned}$$

Note  $u_B(\eta_b, \eta_s \mid \eta) + u_S(\eta_b, \eta_s \mid \eta) = \max\{v(\eta), m(\eta)\}$  holds for all  $\eta$ .

For each  $\eta = (a, \theta)$ , truth telling must form a Nash equilibrium:

$$\begin{aligned} u_S(\eta) &\equiv u_S(\eta, \eta \mid \eta) \geq u_S(\eta, \hat{\eta} \mid \eta), \quad \forall \hat{\eta} \\ u_B(\eta) &\equiv u_B(\eta, \eta \mid \eta) \geq u_B(\hat{\eta}, \eta \mid \eta), \quad \forall \hat{\eta} \end{aligned}$$

Using the zero-sum feature of the payoffs yields  $u_B(\hat{\eta}) \geq u_B(\eta, \hat{\eta} \mid \hat{\eta})$  if and only if  $u_S(\hat{\eta}) \leq$

$u_S(\eta, \hat{\eta} \mid \hat{\eta})$ . Thus we must have

$$\begin{aligned} u_S(\hat{\eta}) - u_S(\eta) &\leq u_S(\eta, \hat{\eta} \mid \hat{\eta}) - u_S(\eta, \hat{\eta} \mid \eta) \\ &= (1 - q(\eta, \hat{\eta})) (\rho^+(\hat{\eta}) - \rho^+(\eta)) + q(\eta, \hat{\eta}) (\rho^-(\hat{\eta}) - \rho^-(\eta)) \end{aligned} \quad (13)$$

**Proposition 5** Suppose  $\rho(a, \theta)$  is either weakly increasing in  $a$  for all  $\theta$ , or weakly decreasing in  $a$  for all  $\theta$ . Then formal contracts are of no value under spot transactions.

**Proof** Suppose instead there is a contract under which the seller's optimal choice is  $\hat{a} > a^o$  satisfying  $\pi(\hat{a}) > \bar{\pi} = \pi(a^o)$ . Then by the seller's incentive compatibility constraints the following inequality must hold.

$$E[u_S(\hat{a}, \theta)] - E[u_S(a^o, \theta)] \geq \hat{a} - a^o$$

By specificity (1) and  $\hat{a} > a^o$ ,  $\rho^-(\hat{\eta}) - \rho^-(\eta) \leq 0$  holds for all  $\theta$  where  $\hat{\eta} = (\hat{a}, \theta)$  and  $\eta = (a^o, \theta)$ . Hence by (13)  $u_S(\hat{\eta}) - u_S(\eta) \leq (1 - q(\eta, \hat{\eta})) (\rho^+(\hat{\eta}) - \rho^+(\eta))$  for all  $\theta$ . We thus obtain

$$E[u_S(\hat{a}, \theta) - u_S(a^o, \theta)] \leq E[(1 - q((a^o, \theta), (\hat{a}, \theta))) (\rho^+(\hat{a}, \theta) - \rho^+(a^o, \theta))]$$

Now suppose first  $\rho(\hat{a}, \theta) \geq \rho(a^o, \theta)$  for all  $\theta$ . Then

$$\hat{a} - a^o \leq E[(1 - q((a^o, \theta), (\hat{a}, \theta))) (\rho^+(\hat{a}, \theta) - \rho^+(a^o, \theta))] \leq \Delta^+(\hat{a}, a^o)$$

which contradicts  $\hat{a} \neq \arg \max_a E[\rho^+(a, \theta)] - a$

Next suppose  $\rho(\hat{a}, \theta) \leq \rho(a^o, \theta)$  for all  $\theta$ . Then

$$\hat{a} - a^o \leq E[(1 - q((a^o, \theta), (\hat{a}, \theta))) (\rho^+(\hat{a}, \theta) - \rho^+(a^o, \theta))] \leq 0$$

which contradicts  $\hat{a} > a^o$ .

**Q.E.D.**

Proposition 5 identifies a sufficient condition for the well-known result of Che and Hausch (1999) to hold when alternative-use value depends on  $a$ . Formal contracts cannot improve the seller's effort incentives from the no contract case if the effects of uncertainty  $\theta$  is not too large to alter the sign of the effects of investment  $a$  on the negotiated price. For example, this condition holds if both  $v(a, \theta)$  and  $m(a, \theta)$  are additively separable in terms of  $a$  and  $\theta$ :  $v(a, \theta) = v(a) + x(\theta)$  and  $m(a, \theta) = m(a) + y(\theta)$ , with  $\alpha v(a) + (1 - \alpha)m(a)$  being monotone in  $a$ . It also holds if  $\theta$  does not affect the negotiated price because, either there is no uncertainty in the value for the buyer and the alternative-use value, or, as in Edlin and Hermalin (2000),

the parties can renegotiate only *before* uncertainty resolves.<sup>27</sup> Finally, the condition holds if, as in Che and Hausch (1999), the alternative-use value  $m(a, \theta)$  does not depend on  $a$ .

If the condition is violated, formal contracts contingent on messages may be of value, although fixed-price contracts are not, as the following example shows.

### Example 1

Let  $\theta \in \{\theta_L, \theta_H\}$  and  $\beta = \Pr\{\theta = \theta_H\}$ . The agent's action is either  $a = 0$  (no investment) or  $a = 1$  (investment), the latter of which costs him  $d > 0$ . Denote  $v_{at} = v(a, \theta_t)$ ,  $m_{at} = m(a, \theta_t)$ ,  $\rho_{at} = \rho(a, \theta)$ , and so on, for  $a = 0, 1$  and  $t = L, H$ . The marginal effects of the seller's action on these values are denoted by  $\Delta_{vt} = v_{1t} - v_{0t}$  and  $\Delta_{mt} = m_{1t} - m_{0t}$ .

The key feature of the example is that the negotiated price is increasing in investment in state  $\theta_H$  while it is decreasing in state  $\theta_L$ . We assume  $\alpha = 0$  so that the negotiated price is  $\rho_{at} = m_{at}$ . And we assume no-trade surplus satisfies the following conditions:  $m_{0L} = m_{0H} = m_0$  and  $m_{1H} > m_0 > m_{1L}$ . The first assumption is for simplicity: state does not affect no-trade surplus under no investment. The second assumption implies  $\Delta_{mH} > 0$  and  $\Delta_{mL} < 0$ : the threat-point effect is positive in state  $\theta_H$ , but negative in state  $\theta_L$ .

As for  $v_{at}$ , we assume  $v_{1H} > m_{1H}$ ,  $m_0 \geq v_{0H}$ , and  $v_{1L} > m_0 \geq v_{0L}$ . These assumptions imply  $\Delta_{vt} > 0$  and  $\Delta_{vt} > \Delta_{mt}$ . And for all states trade is efficient when the seller invests while it is inefficient when he does not.

We assume  $\beta\Delta_{mH} + (1 - \beta)\Delta_{mL} < d < \beta\Delta_{mH}$ . The first inequality implies underinvestment occurs if no contract is written. And the seller obviously chooses  $a = 0$  under fixed-price contract.

Now consider the following form of formal contracts:  $\{p_j^i, q_j^i\}$  where  $i \in \{0, 1\}$  is the buyer's report concerning the seller's action,  $j \in \{L, H\}$  is the seller's report concerning state,  $p_j^i$  is the payment by the buyer to the seller, and  $q_j^i \in \{0, 1\}$  is the decision of trade (1) or no trade (0). Note fixed-price contract corresponds to  $p_j^i \equiv \bar{p}$  and  $q_j^i \equiv 1$ . The trade decision and the payment are specified as follows.  $q_L^0 = q_L^1 = 1$ ,  $p_L^0 = p_L^1 = m_0$ ,  $q_H^0 = q_H^1 = 0$ , and  $p_H^0 = p_H^1 = 0$ . The idea is to utilize positive "market incentive" by not specifying trade when state is  $\theta_H$ , while the formal contract specifying trade in state  $\theta_L$  prevents the negotiated price from affecting the seller's incentive negatively.

We first show that this contract induces investment by the seller, provided that both the buyer and the seller report truthfully. If the seller invests, his expected payoff is  $\beta m_{1H} +$

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<sup>27</sup>Under the assumption of this timing, Edlin and Hermalin (2000) show a result similar to ours that an optimal second-best arrangement among general mechanisms where both parties send messages about the seller's action is for the buyer to transfer ownership to the seller ex ante and simply bargain for the asset after the seller has invested, which arrangement corresponds to our no contract case.

$(1 - \beta)m_0 - a$ . If the seller does not invest, his expected payoff is  $\beta m_0 + (1 - \beta)m_0 = m_0$ . It is optimal for the seller to invest because of assumption  $\beta\Delta_{mH} > d$ .

We next show that the buyer reports truthfully. Suppose the seller's investment is  $a$ , the true state is  $\theta_t$ , and the seller reports the state truthfully. The buyer's payoff is  $v_{at}q_t^i - p_t^i + (1 - q_t^i)(v_{at} - m_{at})$ , which does not depend on the buyer's report  $i$  by the construction of the formal contract. Hence she has no incentive to misreport.

The remaining task is to show that the seller reports truthfully. Suppose the seller's investment is  $a$ , the true state is  $\theta_t$ , and the buyer reports  $a$  truthfully. If the seller reports  $\theta_H$ , his payoff is  $m_{at} - da$ . If the seller reports  $\theta_L$ , his payoff is  $m_0 - da$ . When the true state is  $\theta_H$ ,  $m_{aH} - da \geq m_0 - da$  must hold, which conditions are satisfied for  $a = 0, 1$  because  $m_{1H} > m_{0H} = m_0$ . Finally, when the true state is  $\theta_L$ ,  $m_{aL} - da \leq m_0 - da$  must hold, which inequalities are satisfied because  $m_{1L} < m_{0L} = m_0$ .

## Repeated Transactions

We next extend the analysis of general formal contracts to repeated transactions. Consider formal (short-term) contracts  $\{p(\eta_b, \eta_s), q(\eta_b, \eta_s)\}$  along with the relational contract that consists of the following promises:

- The seller chooses  $\hat{a}$  and both the buyer and the seller report truthfully.
- If  $v(\eta) \geq m(\eta)$  and  $q(\eta, \eta) = 1$ , then trade following the formal contract ( $e(\eta) = 1$ ) and the buyer pays  $b(\eta)$  in addition to  $p(\eta, \eta)$
- If  $v(\eta) \geq m(\eta)$  and  $q(\eta, \eta) = 0$ , then cancel the formal contract to trade ( $e(\eta) = 1$ ) and the buyer pays  $b(\eta)$
- If  $v(\eta) < m(\eta)$  and  $q(\eta, \eta) = 1$ , then cancel the formal contract to no trade ( $e(\eta) = 0$ ) and the buyer pays  $b(\eta)$
- If  $v(\eta) < m(\eta)$  and  $q(\eta, \eta) = 0$ , then no trade following the formal contract ( $e(\eta) = 0$ ) and the buyer pays  $b(\eta)$  in addition to  $p(\eta, \eta)$

The ex post payoffs in state  $\eta$  when both report truthfully are given as follows:

$$\begin{aligned} u_B(\eta) &= e(\eta)v(\eta) - b(\eta) - p(\eta, \eta)R(\eta) \\ u_S(\eta) &= (1 - e(\eta))m(\eta) + b(\eta) + p(\eta, \eta)R(\eta) \end{aligned}$$

where  $R(\eta) = e(\eta)q(\eta, \eta) + (1 - e(\eta))(1 - q(\eta, \eta))$ . Note that  $u_B(\eta) + u_S(\eta) = e(\eta)v(\eta) + (1 - e(\eta))m(\eta) = \max\{v(\eta), m(\eta)\}$  holds for all  $\eta$ .

If the parties follow the informal promises, the seller's incentive compatibility constraints are given as follows.

$$\begin{aligned} & E[(1 - e(\hat{a}, \theta))m(\hat{a}, \theta) + b(\hat{a}, \theta) + p((\hat{a}, \theta), (\hat{a}, \theta))R(\hat{a}, \theta)] - \hat{a} \\ & \geq E[(1 - e(a, \theta))m(a, \theta) + b(a, \theta) + p((a, \theta), (a, \theta))R(a, \theta)] - a \quad \text{for all } a \end{aligned} \quad (14)$$

The buyer's payoff from deviating in state  $\hat{\eta}$  by reporting  $\eta$  is written as

$$\begin{aligned} u_B(\eta, \hat{\eta} \mid \hat{\eta}) &= e(\hat{\eta})v(\hat{\eta}) + (1 - e(\hat{\eta}))q(\eta, \hat{\eta})m(\hat{\eta}) - p(\eta, \hat{\eta}) \\ & \quad - (1 - q(\eta, \hat{\eta}))\rho^+(\hat{\eta}) - q(\eta, \hat{\eta})\rho^-(\hat{\eta}) \end{aligned}$$

For example, suppose  $e(\hat{\eta}) = 1$  and  $q(\eta, \hat{\eta}) = 1$ . The buyer deviates by reporting  $\eta$  and not paying  $b(\hat{\eta})$ , and hence her payoff is  $v(\hat{\eta}) - p(\eta, \hat{\eta})$ . Note there is no negotiation after renegeing in this case. As another case, suppose  $e(\hat{\eta}) = 0$  and  $q(\eta, \hat{\eta}) = 1$ . In this case the buyer reports  $\eta$ , does not pay  $b(\hat{\eta})$  but negotiate to obtain  $v(\hat{\eta}) - p(\eta, \hat{\eta}) + (1 - \alpha)(m(\hat{\eta}) - v(\hat{\eta}))$ . Her payoff is thus  $m(\hat{\eta}) - p(\eta, \hat{\eta}) - \alpha(m(\hat{\eta}) - v(\hat{\eta})) = m(\hat{\eta}) - p(\eta, \hat{\eta}) - \rho^-(\hat{\eta})$ . One can check the other two cases similarly to obtain the buyer's payoff as above. The buyer's renegeing temptation is thus  $\max_{\eta, \eta'} [u_B(\eta', \eta \mid \eta) - u_B(\eta)]$ .

Similarly, the seller's payoff from deviating in state  $\eta$  by reporting  $\hat{\eta}$  is written as

$$\begin{aligned} u_S(\eta, \hat{\eta} \mid \eta) &= (1 - e(\eta))(1 - q(\eta, \hat{\eta}))m(\eta) + p(\eta, \hat{\eta}) \\ & \quad + (1 - q(\eta, \hat{\eta}))\rho^+(\eta) + q(\eta, \hat{\eta})\rho^-(\eta) \end{aligned}$$

The seller's renegeing temptation is hence  $-\min_{\eta, \eta'} [u_S(\eta', \eta \mid \eta') - u_S(\eta')]$ .

The sum of these renegeing temptations are rewritten as follows:

$$\begin{aligned} & \max_{\eta, \eta'} [u_B(\eta', \eta \mid \eta) - u_B(\eta)] - \min_{\eta, \eta'} [u_S(\eta', \eta \mid \eta') - u_S(\eta')] \\ & \geq \max_{\theta} [u_B((a, \theta), (\hat{a}, \theta) \mid (\hat{a}, \theta)) - u_B(\hat{a}, \theta)] - \min_{\theta} [u_S((a, \theta), (\hat{a}, \theta) \mid (a, \theta)) - u_S(a, \theta)] \\ & \geq E[u_B((a, \theta), (\hat{a}, \theta) \mid (\hat{a}, \theta)) - u_B(\hat{a}, \theta)] - E[u_S((a, \theta), (\hat{a}, \theta) \mid (a, \theta)) - u_S(a, \theta)] \\ & \geq \hat{a} - a - E[(1 - q((a, \theta), (\hat{a}, \theta))) (\rho^+(\hat{a}, \theta) - \rho^+(a, \theta)) + q((a, \theta), (\hat{a}, \theta)) (\rho^-(\hat{a}, \theta) - \rho^-(a, \theta))] \\ & = \hat{a} - a - E[(\rho^+(\hat{a}, \theta) - \rho^+(a, \theta)) - q((a, \theta), (\hat{a}, \theta)) (\rho(\hat{a}, \theta) - \rho(a, \theta))] \end{aligned}$$

Now consider the implementation of action  $\hat{a} > a^o$  and suppose  $\rho(a, \theta)$  is increasing in  $a$  for all  $\theta$ . By setting  $a = a^o$ , we obtain

$$\begin{aligned} & \max_{\eta, \eta'} [u_B(\eta', \eta) - u_B(\eta)] - \min_{\eta, \eta'} [u_S(\eta', \eta) - u_S(\eta)] \\ & \geq \hat{a} - a^o - \Delta^+(\hat{a}, a^o) \end{aligned}$$

The right-hand side is attained by no contract. Hence the reneging temptation is minimized by not writing a formal contract.

Next suppose  $\rho(a, \theta)$  is decreasing in  $a$  for all  $\theta$ . By setting  $a = \underline{a}$ , we obtain

$$\begin{aligned} & \max_{\eta, \eta'} [u_B(\eta', \eta) - u_B(\eta)] - \min_{\eta, \eta'} [u_S(\eta', \eta) - u_S(\eta)] \\ & \geq \hat{a} - \underline{a} - \Delta^-(\hat{a}, \underline{a}) \end{aligned}$$

The right-hand side is attained by a fixed-price contract. Hence the reneging temptation is minimized by writing a fixed-price contract. We have hence shown the following result.

**Proposition 6** Suppose  $\rho(a, \theta)$  is either weakly increasing in  $a$  for all  $\theta$ , or weakly decreasing in  $a$  for all  $\theta$ . Then it is without loss of generality to confine attention to no contract or fixed-price contracts under repeated transactions.

## 5 Multiple Quantities

In the main model analyzed in the previous section, we have made an assumption that at most one unit of the product is traded, which has helped us understand the driving forces of our main results as clearly as possible. In this section, we introduce the possibility of trading more than one unit into our model. As in the previous section, it can be shown that confining our attention to fixed-price contracts as a form of formal contracts is without loss of generality under a natural condition that is analogous to the one identified in the previous section. Given this, we focus on formal fixed-price contracts in this section. We first derive a sufficient condition for writing a fixed-price contract to be of no value under a spot transaction, and illustrate through an example that if the condition does not hold, formal fixed-price contracts can be valuable even under spot transaction, because of the negative threat-point effect. We then show, using the same example, the following two results: (i) When writing a fixed-price contract is of no value under a spot transaction, it can be valuable under a range of parameterizations if the transaction is repeated. (ii) When writing a fixed-price contract is of value under a spot transaction, it can be valuable under a broader range of parameterizations if the transaction is repeated. The driving force of both (i) and (ii) is

the role that formal contracts can play under repeated transactions in mitigating the parties' reneging temptation.

Let  $Q$  be the set of feasible quantity levels. We assume  $Q$  is countable (either finite or infinite):  $Q = \{0, q_1, q_2, \dots\}$  with  $0 < q_1 < q_2 < \dots$ , where 0 corresponds to no trade. The value to the buyer is written as  $v(q, a, \theta)$ . We assume  $v(0, a, \theta) = 0$ ,  $v(q, a, \theta)$  is strictly increasing in  $q$  for all  $(a, \theta)$ , and  $v(q, a, \theta)$  is strictly increasing in  $a$  for all  $q > 0$  and  $\theta$ . Since there are multiple quantities to be traded, we introduce the seller's production cost  $c(q, \theta)$ . We assume  $c(0, \theta) = 0$ , and  $c(q, \theta)$  is strictly increasing in  $q$  for all  $\theta$ .

Define  $\phi(q, a, \theta) = v(q, a, \theta) - c(q, \theta)$ . We assume for each  $(a, \theta)$  there exists a unique efficient quantity  $q^*(a, \theta) = \arg \max_{q \in Q} \phi(q, a, \theta)$ , and  $0 < q^*(a, \theta) < +\infty$  for all  $(a, \theta)$ . We assume quantity and action are complementary:

$$\phi(q, a, \theta) - \phi(q, a', \theta) \geq \phi(q', a, \theta) - \phi(q', a', \theta) \quad \text{for all } q > q', a > a', \text{ and } \theta. \quad (15)$$

This assumption is sufficient for  $q^*(a, \theta)$  to be increasing in  $a$ .

We write  $\phi(a, \theta) = \phi(q^*(a, \theta), a, \theta)$ ,  $v(a, \theta) = v(q^*(a, \theta), a, \theta)$ , and  $c(a, \theta) = c(q^*(a, \theta), \theta)$ . The assumption of "specificity," corresponding to (1), is written as

$$\phi(a, \theta) - \phi(a', \theta) \geq m(a, \theta) - m(a', \theta) \quad \text{for all } \theta \text{ and } a > a'. \quad (16)$$

The first-best investment  $a^*$  is assumed to be unique, and is defined similarly by  $a^* = \arg \max_a E[\phi^+(a, \theta)] - a$  where  $\phi^+(a, \theta) = \max\{\phi(a, \theta), m(a, \theta)\}$ . As before, we assume  $\phi^+(a^*, \theta) = \phi(a^*, \theta)$  for all  $\theta$ .

We obtain a sufficient condition for fixed-price contracts to be of no value under spot transactions. First we analyze the case in which no formal contract is written. As before, let  $e(a, \theta) \in \{0, 1\}$  indicates the efficiency of trade:

$$e(a, \theta) = \begin{cases} 1 & \text{if } \phi(a, \theta) \geq m(a, \theta) \\ 0 & \text{if } \phi(a, \theta) < m(a, \theta) \end{cases}$$

Then if  $e(a, \theta) = 1$ , the buyer and the seller negotiate transfer to trade  $q^*(a, \theta)$ . The buyer's payoff is then  $(1 - \alpha)(\phi(a, \theta) - m(a, \theta))$ , and the seller's payoff is  $\rho(a, \theta) - a = m(a, \theta) + \alpha(\phi(a, \theta) - m(a, \theta)) - a$ . On the other hand, if  $e(a, \theta) = 0$ , there is no negotiation and no trade occurs. The buyer's payoff is 0 and the seller's payoff is  $m(a, \theta) - a$ .

The seller's decision is written as follows:

$$\max_a E[\rho^+(a, \theta)] - a = \max_a E[m(a, \theta) + \alpha \max\{\phi(a, \theta) - m(a, \theta), 0\}] - a. \quad (17)$$

Denote the optimal investment by  $a^o$ . Proposition 1 applies here, with minor modification, to show  $a^o \leq a^*$ . We hereafter assume  $a^o < a^*$ .

Next consider a formal contract  $(\bar{p}, \bar{q})$  meaning the parties trade quantity  $\bar{q}$  and the buyer pays  $\bar{p}$  to the seller. Under this contract, if  $e(a, \theta) = 1$ , they negotiate to trade  $q^*(a, \theta)$ . The buyer's payoff and the seller's payoff are, respectively, obtained as follows:

$$\text{The buyer's payoff: } v(\bar{q}, a, \theta) - \bar{p} + (1 - \alpha)(\phi(a, \theta) - \phi(\bar{q}, a, \theta));$$

$$\text{The seller's payoff: } \bar{p} - c(\bar{q}, \theta) + \alpha(\phi(a, \theta) - \phi(\bar{q}, a, \theta)) - a.$$

Note that the special case of  $\bar{q} = q^*(a, \theta)$  is covered here as well.

On the other hand, if  $e(a, \theta) = 0$ , the parties cancel the contract and negotiate to no trade. The payoffs are then as follows:

$$\text{The buyer's payoff: } \phi(a, \theta) - \bar{p} + (1 - \alpha)(m(a, \theta) - \phi(\bar{q}, a, \theta)) ;$$

$$\text{The seller's payoff: } \bar{p} - c(\bar{q}, \theta) + \alpha(m(a, \theta) - \phi(\bar{q}, a, \theta)) - a.$$

Using  $\rho^-(a, \theta) = \alpha(1 - e(a, \theta))(m(a, \theta) - \phi(a, \theta))$ , we can write the seller's investment decision as follows:

$$\max_a \alpha E[\phi(a, \theta) - \phi(\bar{q}, a, \theta)] + E[\rho^-(a, \theta)] - a \quad (18)$$

Denote the optimal action under fixed-price contract  $(\bar{p}, \bar{q})$  by  $a^1(\bar{q})$ . The optimal fixed-price contract is the one that maximizes  $a^1(\bar{q})$ . (18) implies the optimal contract minimizes the marginal effect of  $E[\phi(\bar{q}, a, \theta)]$ , which is attained at  $\bar{q} = q_1$  under assumption (15): Specifying the minimum positive quantity in the contract provides the strongest incentive. Define  $a^1 = a^1(q_1)$ .

**Proposition 7** Define  $\rho(q, a, \theta) = m(a, \theta) + \alpha(\phi(q, a, \theta) - m(a, \theta))$

(a)  $a^o \geq a^1$  if  $E[\rho(q_1, a, \theta)]$  is increasing in  $a$ .

(b)  $a^o \leq a^1$  if  $E[\rho(q_1, a, \theta)]$  is decreasing in  $a$

**Proof** Rewrite the objective function in (18) as follows:

$$\begin{aligned} & \alpha E[\phi(a, \theta) - \phi(q_1, a, \theta)] + E[\rho^-(a, \theta)] - a \\ &= E[\alpha(\phi(a, \theta) - \phi(q_1, a, \theta)) + \alpha(1 - e(a, \theta))(m(a, \theta) - \phi(a, \theta))] - a \\ &= E[m(a, \theta) + \alpha(\phi(a, \theta) - e(a, \theta)m(a, \theta)) - m(a, \theta) - \alpha(\phi(q_1, a, \theta) - m(a, \theta))] - a \\ &= E[\rho^+(a, \theta) - \rho(q_1, a, \theta)] - a \end{aligned} \quad (19)$$

Now the conclusion follows from (17) and (19).

**Q.E.D.**

Applying Proposition 7 to our model in Section 3 where  $Q = \{0, 1\}$  and  $q_1 = 1 = q^*(a, \theta)$  yields the following. There the condition that  $E[\rho(q_1, a, \theta)]$  is increasing in  $a$  is equivalent to  $E[\rho(a, \theta)]$  being increasing in  $a$ , and hence  $a^o \geq \underline{a} = a^1$ . On the other hand, the condition that  $E[\rho(q_1, a, \theta)]$  is decreasing in  $a$  is equivalent to  $E[\rho(a, \theta)]$  being decreasing in  $a$ , and hence  $a^o = \underline{a} = a^1$ : the least costly action is chosen either under no formal contracting or under a fixed-price contract. Fixed-price contracts hence cannot improve investment incentives from the case of no formal contracts under spot transactions.

However, if  $|Q| \geq 3$ ,  $a^1 > a^o$  is possible. In the following example we first illustrate this possibility. More importantly, we then use the same example to illustrate that under repeated transactions, formal fixed-price contracts can be valuable under a broader range of parameter values when  $a^1 > a^o$ , and there appears a range of parameter values in which fixed-price contracts are of value even though writing a fixed-price contract is not valuable under a spot transaction ( $a^1 \leq a^o$ ).

## Example 2

Let  $Q = \{0, q_1, q_2\}$  and  $A = \{0, 1\}$ . There is no uncertainty and hence we drop  $\theta$ . We also use notations  $\phi_a(q) = \phi(q, a)$ ,  $\phi_a = \phi(q^*(a), a)$ ,  $m_a = m(a)$ , and so on, for action  $a = 0, 1$ . Assume  $\phi_1 = \phi_1(q_2) > \phi_1(q_1) > m_1$ , and  $m_0 \geq \phi_0 = \phi_0(q_2) > \phi_0(q_1)$ . Hence  $q^*(1) = q^*(0) = q_2$  holds, and trading  $q_2$  is efficient under  $a = 1$  while no trade is efficient under action 0. We can choose these values so as to satisfy the other assumptions (15) and (16). The efficient action is assumed to be  $a^* = 1$ :

$$\phi_1 - m_0 > d$$

where  $d > 0$  is the cost of investment ( $a = 1$ ). Define  $\rho_a(q) = m_a + \alpha(\phi_a(q) - m_a)$  and  $\rho_a^+ = m_a + \alpha \max\{\phi_a - m_a, 0\}$ . Suppose

$$\Delta_m + \alpha(\phi_1 - m_1) < d \tag{20}$$

where  $\Delta_m = m_1 - m_0$  represents the threat-point effect. This assumption implies  $\rho_1^+ - \rho_0^+ < d$ , that is, the seller chooses not to invest ( $a = 0$ ) under no contract.

There are two kinds of fixed-price contracts. First, consider a fixed-price contract specifying quantity  $q_2$ :  $(\bar{p}, \bar{q} = q_2)$ . Since  $\bar{q} = q_2$  is the efficient quantity, there is no negotiation under investment, and hence the seller chooses no investment.

Next consider a fixed-price contract specifying quantity  $q_1$ :  $(\bar{p}, \bar{q} = q_1)$ . Under this

contract, if the seller invests, the parties negotiate to  $q_2$ , and hence the seller's payoff is  $\bar{p} - c(q_1) + \alpha(\phi_1(q_2) - \phi_1(q_1)) - d$ . If the seller does not invest, they cancel the contract and the seller's payoff is  $\bar{p} - c(q_1) + \alpha(m_0 - \phi_0(q_1))$ . The seller chooses to invest if

$$\alpha(\phi_1 - m_0) - \alpha(\phi_1(q_1) - \phi_0(q_1)) \geq d \quad (21)$$

which is rewritten as

$$(\rho_1^+ - \rho_0^+) - (\rho_1(q_1) - \rho_0(q_1)) \geq d. \quad (22)$$

By (20) and (22), fixed-price contracts cannot improve investment incentives if  $\rho_1(q_1) \geq \rho_0(q_1)$  or  $\alpha(\phi_1(q_1) - \phi_0(q_1)) \geq -(1 - \alpha)\Delta_m$ . However, if

$$\alpha(\phi_1(q_1) - \phi_0(q_1)) < -(1 - \alpha)\Delta_m \quad (23)$$

holds, fixed-price contracts may be of value. For example, suppose that the threat-point effect is negative ( $\Delta_m < 0$ ) and  $\alpha(\phi_1 - m_0)$  satisfies  $d - \Delta_m > \alpha(\phi_1 - m_0) > d$ . Then if  $\phi_1(q_1) - \phi_0(q_1) > 0$  is sufficiently close to zero, (21) holds, and there is a fixed-price contract implementing investment.

Now consider repeated transactions. The renegeing temptation under no formal contract is

$$d - (\rho_1^+ - \rho_0^+) = d - \Delta_m - \alpha(\phi_1 - m_1), \quad (24)$$

similar to the one in the previous model. The renegeing temptation under a fixed-price contract ( $\bar{p}, \bar{q} = q_2$ ) is also similar and equal to

$$d + \alpha(m_0 - \phi_0). \quad (25)$$

The renegeing temptation under a fixed-price contract ( $\bar{p}, \bar{q} = q_1$ ) is derived as follows. The buyer and the seller agree to cancel the contract and trade  $q_2$  with the buyer's payment  $b_1$  if the seller invests, and not to trade with the buyer's payment  $b_0$  if the seller does not invest. The payoffs are  $v_1(q_2) - b_1$  and  $b_1 - c(q_2)$  for the buyer and the seller, respectively, if  $a = 1$ , and  $-b_0$  and  $b_0 + m_0$  if  $a = 0$ . The seller's incentive compatibility constraint is hence  $b_1 - c(q_2) - b_0 - m_0 \geq d$ .

When the seller chooses  $a = 1$ , the buyer can deviate by not paying  $b_1$  but negotiating transfer. The gain in her payoff is then

$$\begin{aligned} v_1(q_1) - \bar{p} + (1 - \alpha)(\phi_1 - \phi_1(q_1)) - (v_1(q_2) - b_1) \\ = b_1 - \bar{p} + (1 - \alpha)(\phi_1 - \phi_1(q_1)). \end{aligned}$$

When the seller chooses  $a = 0$ , he can refuse to cancel the contract and instead negotiate a transfer. The gain in his payoff is then

$$\begin{aligned} & \bar{p} - c(q_1) + \alpha(m_0 - \phi_0(q_1)) - (b_0 + m_0) \\ & = -b_0 + \bar{p} - c(q_1) + \alpha(m_0 - \phi_0(q_1)). \end{aligned}$$

Summing up these gains and using the seller's incentive compatibility constraint yields the renegeing temptation as follows.

$$d - \alpha(\phi_1 - m_0) + \alpha(\phi_1(q_1) - \phi_0(q_1)). \quad (26)$$

By comparing (25) and (26) we find that the renegeing temptation is at least as high under a fixed-price contract  $(\bar{p}, \bar{q} = q_2)$  than under  $(\bar{p}, \bar{q} = q_1)$ , using (15). A fixed-price contract is thus of value if (26) is smaller than (24), which condition is written as follows:

$$\alpha(\phi_1 - m_0) - \alpha(\phi_1(q_1) - \phi_0(q_1)) \geq \rho_1 - m_0 = \rho_1^+ - \rho_0^+. \quad (27)$$

Now first suppose  $\rho_1(q_1) \geq \rho_0(q_1)$  or equivalently  $\alpha(\phi_1(q_1) - \phi_0(q_1)) \geq -(1 - \alpha)\Delta_m$ . Fixed-price contracts are then of no value under spot transactions. However, since  $d > \rho_1^+ - \rho_0^+$  holds and the left-hand side of (27) is smaller than  $d$ , there is a range of parameter values in which (27) is satisfied, and hence a fixed-price contract is valuable under repeated transactions. Next suppose  $\rho_1(q_1) < \rho_0(q_1)$ . Fixed-price contracts are valuable even under spot transactions if (21) holds. However, since the right-hand side of (27) is smaller than that of (21), a fixed-price contract is valuable in a broader range of parameter values under repeated transactions than spot transactions.

In summary, our main assertion that repeated transactions, along with the negative threat-point effect, contributes crucially to the value of formal contracting turns out to be valid for the case of multiple quantities.

## 6 Concluding Remarks

This paper has offered a new perspective on the role that formal contracts can play in resolving or mitigating the holdup problem. In situations where no formal contract is valuable under spot transactions due to the purely cooperative nature of the relation-specific investment, we have shown that writing a simple fixed-price contract can be valuable under repeated transactions. In our model, there is a range of parameter values in which a formal fixed-price contract combined with an informal agreement of either discretionary payments

or termination of trade can help resolve or mitigate the holdup problem, whereas there is another range of parameter values in which not writing a formal fixed-price contract but entirely relying on an informal agreement increases the total surplus of the buyer and the seller.

Furthermore, we have shown that under an additional natural assumption, more complex formal contracts contingent upon messages cannot provide higher investment incentives than an appropriate simple fixed-price contract and hence confining our attention to fixed-price contracts as a form of formal contracts is without loss of generality. The key driving force of our result is a possibility that relation-specific investment decreases the surplus under no trade, that is, the threat-point effect is negative. This possibility, although very plausible, has been largely ignored in previous theoretical/empirical analyses of the holdup problem under repeated transactions. Our findings suggest that independent variables capturing the threat-point effect be incorporated in future empirical studies on formal contracts and the holdup problem.

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## Appendix

### Proof of Proposition 2

In the main text we have shown that the seller's incentive compatibility constraints (3) and the self-enforcing condition (6) are necessary and sufficient for  $\hat{a}$  to be implemented:

$$E[b(\hat{a}, \theta) + (1 - e(\hat{a}, \theta))m(\hat{a}, \theta)] - \hat{a} \geq E[b(a, \theta) + (1 - e(a, \theta))m(a, \theta)] - a \quad \text{for all } a \quad (3)$$

$$\max_{a, \theta} \{b(a, \theta) - e(a, \theta)\rho(a, \theta)\} - \min_{a, \theta} \{b(a, \theta) - e(a, \theta)\rho(a, \theta)\} \leq \frac{\delta}{1 - \delta} (\pi(\hat{a}) - \bar{\pi}) \quad (6)$$

**Necessity** Suppose  $\hat{a} > a^o$  can be implemented: There exists a compensation plan  $(b(a, \theta))_{a \in A, \theta \in \Theta}$  satisfying (3) and (6). The left-hand side of (6) is rewritten as follows:

$$\begin{aligned} & \max_{a, \theta} \{b(a, \theta) - e(a, \theta)\rho(a, \theta)\} - \min_{a, \theta} \{b(a, \theta) - e(a, \theta)\rho(a, \theta)\} \\ & \geq \max_{\theta} \{b(\hat{a}, \theta) - e(\hat{a}, \theta)\rho(\hat{a}, \theta)\} - \min_{\theta} \{b(a^o, \theta) - e(a^o, \theta)\rho(a^o, \theta)\} \\ & \geq E[b(\hat{a}, \theta) - e(\hat{a}, \theta)\rho(\hat{a}, \theta)] - E[b(a^o, \theta) - e(a^o, \theta)\rho(a^o, \theta)] \\ & \geq \hat{a} - a^o - E[(1 - e(\hat{a}, \theta))m(\hat{a}, \theta)] + E[(1 - e(a^o, \theta))m(a^o, \theta)] \\ & \quad - E[e(\hat{a}, \theta)\rho(\hat{a}, \theta)] + E[e(a^o, \theta)\rho(a^o, \theta)] \\ & = \hat{a} - a^o - E[\rho^+(\hat{a}, \theta) - \rho^+(a^o, \theta)] \end{aligned}$$

This is the left-hand side of (DE-NC), by  $\Delta^+(\hat{a}, a^o) \equiv E[\rho^+(\hat{a}, \theta) - \rho^+(a^o, \theta)]$ .

**Sufficiency** Supposing (DE-NC), we construct a compensation plan that satisfies (3) and (6). Define  $b(a, \theta)$  as follows:<sup>28</sup>

$$\begin{aligned} b(\hat{a}, \theta) &= e(\hat{a}, \theta)\rho(\hat{a}, \theta) - \Delta^+(\hat{a}, a^o) + \hat{a} - a^o \\ b(a, \theta) &= e(a, \theta)\rho(a, \theta), \quad \text{for all } a \neq \hat{a} \end{aligned} \quad (\text{A1})$$

(3) is satisfied for  $a = a^o$ :

$$\begin{aligned} & E[b(\hat{a}, \theta)] + E[(1 - e(\hat{a}, \theta))m(\hat{a}, \theta)] - E[b(a^o, \theta)] - E[(1 - e(a^o, \theta))m(a^o, \theta)] \\ & = E[\rho^+(\hat{a}, \theta)] - \Delta^+(\hat{a}, a^o) + \hat{a} - a^o - E[\rho^+(a^o, \theta)] \\ & = \hat{a} - a^o \end{aligned}$$

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<sup>28</sup>The fixed payment  $w$  is only used to guarantee that (4), (5), and the participation constraints are satisfied.

For  $a \neq a^\circ$ , (3) holds because

$$\begin{aligned}
& E[b(\hat{a}, \theta)] + E[(1 - e(\hat{a}, \theta))m(\hat{a}, \theta)] - E[b(a, \theta)] - E[(1 - e(a, \theta))m(a, \theta)] \\
&= \hat{a} - a^\circ + E[b(a^\circ, \theta)] + E[(1 - e(a^\circ, \theta))m(a^\circ, \theta)] - E[b(a, \theta)] - E[(1 - e(a, \theta))m(a, \theta)] \\
&= \hat{a} - a^\circ + \Delta^+(a^\circ, a) \\
&\geq \hat{a} - a^\circ + a^\circ - a = \hat{a} - a
\end{aligned}$$

where the last inequality follows from (7).

We next show

$$\max_{a, \theta} \{b(a, \theta) - e(a, \theta)\rho(a, \theta)\} = \max_{\theta} \{b(\hat{a}, \theta) - e(\hat{a}, \theta)\rho(\hat{a}, \theta)\}. \quad (\text{A2})$$

First,  $b(\hat{a}, \theta) - e(\hat{a}, \theta)\rho(\hat{a}, \theta) = \Delta^+(a^\circ, \hat{a}) + \hat{a} - a^\circ \geq 0$  holds by (7). And for  $a \neq \hat{a}$ ,  $b(a, \theta) - e(a, \theta)\rho(a, \theta) = 0$ , and hence we obtain (A2). Similarly, we can show

$$\min_{a, \theta} \{b(a, \theta) - e(a, \theta)\rho(a, \theta)\} = \min_{\theta} \{b(a^\circ, \theta) - e(a^\circ, \theta)\rho(a^\circ, \theta)\}.$$

Therefore

$$\begin{aligned}
& \max_{a, \theta} \{b(a, \theta) - e(a, \theta)\rho(a, \theta)\} - \min_{a, \theta} \{b(a, \theta) - e(a, \theta)\rho(a, \theta)\} \\
&= \hat{a} - a^\circ - \Delta^+(\hat{a}, a^\circ)
\end{aligned}$$

which completes the proof.

### Proof of Proposition 3

As shown in the main text, the seller's incentive compatibility constraints (8) and the self-enforcing condition (11) are necessary and sufficient for  $\hat{a}$  to be implemented:

$$\begin{aligned}
& E[b(\hat{a}, \theta) + e(\hat{a}, \theta)\bar{p} + (1 - e(\hat{a}, \theta))m(\hat{a}, \theta)] - \hat{a} \\
&\geq E[b(a, \theta) + e(a, \theta)\bar{p} + (1 - e(a, \theta))m(a, \theta)] - a \quad \text{for all } a
\end{aligned} \quad (8)$$

$$\begin{aligned}
& \max_{a, \theta} \{b(a, \theta) + (1 - e(a, \theta))(\rho(a, \theta) - \bar{p})\} - \min_{a, \theta} \{b(a, \theta) + (1 - e(a, \theta))(\rho(a, \theta) - \bar{p})\} \\
&\leq \frac{\delta}{1 - \delta} (\pi(\hat{a}) - \bar{\pi})
\end{aligned} \quad (11)$$

**Necessity** The left-hand side of (11) is rewritten as follows:

$$\begin{aligned}
& \max_{a,\theta} \{b(a, \theta) + (1 - e(a, \theta))(\rho(a, \theta) - \bar{p})\} - \min_{a,\theta} \{b(a, \theta) + (1 - e(a, \theta))(\rho(a, \theta) - \bar{p})\} \\
& \geq \max_{\theta} \{b(\hat{a}, \theta) + (1 - e(\hat{a}, \theta))(\rho(\hat{a}, \theta) - \bar{p})\} \\
& \quad - \min_{\theta} \{b(\underline{a}, \theta) + (1 - e(\underline{a}, \theta))(\rho(\underline{a}, \theta) - \bar{p})\} \\
& \geq E[b(\hat{a}, \theta) + (1 - e(\hat{a}, \theta))(\rho(\hat{a}, \theta) - \bar{p})] - E[b(\underline{a}, \theta) + (1 - e(\underline{a}, \theta))(\rho(\underline{a}, \theta) - \bar{p})] \\
& \geq \hat{a} - \underline{a} - E[e(\hat{a}, \theta)\bar{p} + (1 - e(\hat{a}, \theta))(m(\hat{a}, \theta) + \bar{p} - \rho(\hat{a}, \theta))] \\
& \quad + E[e(\underline{a}, \theta)\bar{p} + (1 - e(\underline{a}, \theta))(m(\underline{a}, \theta) + \bar{p} - \rho(\underline{a}, \theta))] \\
& = \hat{a} - \underline{a} - E[\rho^-(\hat{a}, \theta) - \rho^-(\underline{a}, \theta)].
\end{aligned}$$

This is the left-hand side of (DE-FP) by  $\Delta^-(\hat{a}, \underline{a}) = E[\rho^-(\hat{a}, \theta) - \rho^-(\underline{a}, \theta)]$ .

**Sufficiency** To show the sufficiency part, define  $b(a, \theta)$  as follows:

$$\begin{aligned}
b(\hat{a}, \theta) &= \hat{a} - \underline{a} - (1 - e(\hat{a}, \theta))(\rho(\hat{a}, \theta) - \bar{p}) - E[\rho^-(\hat{a}, \theta)] \\
b(a, \theta) &= -(1 - e(a, \theta))(\rho(a, \theta) - \bar{p}) - E[\rho^-(a, \theta)] \quad \text{for all } a \neq \hat{a}
\end{aligned}$$

Then the incentive compatibility constraints (8) are satisfied:

$$\begin{aligned}
& E[b(\hat{a}, \theta)] + E[e(\hat{a}, \theta)\bar{p}] + E[(1 - e(\hat{a}, \theta))m(\hat{a}, \theta)] \\
& \quad - E[b(a, \theta)] - E[e(a, \theta)\bar{p}] - E[(1 - e(a, \theta))m(a, \theta)] \\
& = \hat{a} - \underline{a} \geq \hat{a} - a \quad \text{for all } a \neq \hat{a}
\end{aligned}$$

We next show the following:

$$\begin{aligned}
\max_{a,\theta} \{b(a, \theta) + (1 - e(a, \theta))(\rho(a, \theta) - \bar{p})\} &= \hat{a} - \underline{a} - E[\rho^-(\hat{a}, \theta)] \\
\min_{a,\theta} \{b(a, \theta) + (1 - e(a, \theta))(\rho(a, \theta) - \bar{p})\} &= -E[\rho^-(\underline{a}, \theta)]
\end{aligned}$$

First, for  $a = \hat{a}$ ,

$$b(\hat{a}, \theta) + (1 - e(\hat{a}, \theta))(\rho(\hat{a}, \theta) - \bar{p}) = \hat{a} - \underline{a} - E[\rho^-(\hat{a}, \theta)]$$

holds for all  $\theta$ . Second, for  $a \neq \hat{a}$ ,

$$b(a, \theta) + (1 - e(a, \theta))(\rho(a, \theta) - \bar{p}) = -E[\rho^-(a, \theta)] \geq -E[\rho^-(\underline{a}, \theta)]$$

holds for all  $\theta$  since  $\underline{a}$  is the minimum level of investment and  $\rho^-(a, \theta)$  is decreasing in  $a$ . Finally,  $\hat{a} - \underline{a} - E[\rho^-(\hat{a}, \theta)] + E[\rho^-(\underline{a}, \theta)] > 0$  is satisfied. Therefore,

$$\begin{aligned} & \max_{a, \theta} \{b(a, \theta) + (1 - e(a, \theta))(\rho(a, \theta) - \bar{p})\} - \min_{a, \theta} \{b(a, \theta) + (1 - e(a, \theta))(\rho(a, \theta) - \bar{p})\} \\ &= \hat{a} - \underline{a} - E[\rho^-(\hat{a}, \theta) - \rho^-(\underline{a}, \theta)] \end{aligned}$$

holds, which completes the proof.