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Becoming obsolete
Bankruptcy through technological progress

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Abstract

This paper studies product obsolescence, the entry and exit of firms, and the evolution of firm size as foundation of endogenous economic growth. I develop a dynamic general equilibrium model with heterogenous firms to analyze firm behavior in an economic environment that is characterized by a growing knowledge base. It uses a simple mechanism of knowledge diffusion that induces growth of the general knowledge base. The growing knowledge base forces small firms using the least knowledge intensive production processes to leave the economy, since competition from new entrants is tougher, since they developed their products starting from a larger knowledge base. Exiting firms are found to be smaller and less productive than survivors. Another main insight of the model is the evolution of a cohort's firm size distribution.
JEL: O40, L11,

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1. Introduction

This paper studies product obsolescence, the entry and exit of firms, and the evolution of firm size throughout firms' life as foundation of endogenous economic growth. In response to a changing economic environment entry and exit of firms, and even more the appearance and disappearance of products belong to the economy's everyday events. Firms cease operation for different reasons not only in recession but also in phases of boom. Although predominantly young and small firms leave the market, also big firms as well as well-established old ones exit. About 20 out of the 100 biggest firms in the USA cannot be found in the commercial register anymore ten years later. Taking a closer look on the product level brings about, that 54% of the surviving firms change their product

mix every 5 years (Bernard et al. (2010)), and that 22% of all product exits is due to firms that change their product line (Dunne et al. (2005)). Hence, most products are introduced to be dropped out of the market some years later again. Bernard et al. (2010) show, that 89% of added and dropped products are added and dropped by existing firms, which implies only the exchange of products but not the death of the firm.

In economic theory, in contrast, firm exits and product obsolescence is widely unconsidered. Endogenous growth models relying on intentional R&D-investment or accumulation of knowledge build on horizontal or vertical product differentiation or a combination of both. Models building on horizontal product differentiation (see Grossman and Helpman (1991, ch.3), Romer (1990), Jones (1995)) completely lack the possibility of firm exit. The models are driven by ever more products that never disappear. In models with vertical product differentiation (Segerstrom et al. (1990), Grossman and Helpman (1991, ch.4), Aghion and Howitt (1992), or Segerstrom (1998) as an example for non-scale effects) existing products are replaced by new ones of higher quality or, theoretically equivalent, produced using cost-reducing production processes. But the number of product lines and the industrial composition of the economy is not a matter of change in the process of growth. Modeling both dimension of product differentiation (Dinopoulos and Thompson (1998), Young (1998)) does not insert obsolescence of products or product lines, respectively. Once introduced, a product line will be improved but will never disappear. However, obsolescence of products not only in a certain quality specification but the disappearance of an entire product line from a market is an essential part of the product life cycle.

Building on Melitz (2003) I develop a dynamic general equilibrium model with heterogenous firms to analyze firm behavior in an economic environment that is characterized by a growing knowledge base in the development of new products. I focus on the product life within a changing environment. Therefore I assume single product firms that cease to exist at the end of their product's life cycle. Since I am not so much interested in explaining industry dynamics, modeling multi-product firms that allow differentiating the product cycle from the evolution of the firm, complicates the matter without adding to much to my analysis. The model refrains from classifying the very kind of product innovation arising from the obtained production-process and is consistent with models of horizontal as well as vertical differentiation. A new firm might introduce an entirely new product line, add a possibly cheaper variety to an existing one or offer a product improved in quality. The consumer recognizes all of them as new. The model features continued product innovation and endogenous product obsoles-

cence. It uses a simple mechanism of knowledge diffusion that induces growth of the general knowledge base. The growing knowledge base forces small firms which use the least knowledge intensive production processes to leave the economy in every period, because competition from new entrants is tougher since they developed their products starting from a larger knowledge base. The characteristics of exiting firms are in line with the empirical evidence that finds, that exiting firms are smaller and less productive than survivors (see Dunne et al. (2005) and Baldwin (1998)). Clearly, these empirical findings stress differences in firms and their inclination to exit. An appropriate representation in a model requires heterogeneity in firm characteristics.

Firms decide forward-looking whether to enter and when to exit given a stochastic productivity draw at entry that defines the knowledge content of the firm-specific production process. They further take knowledge diffusion into the publicly accessible knowledge base into account. This knowledge diffusion from firms into the overall economy effects the process of Schumpeterian creative destruction and forces firms to exit after some time and to be replaced by new ones with more innovative products that are building on the currently available mass of public knowledge.

The model, that I present in the following draws heavily from Melitz (2003) and adopts the framework of monopolistic competition between heterogenous firms in a general equilibrium setting. Simplifying Hopenhayn (1992) *Melitz* proposed a now widely used representation of firm heterogeneity in average firm's characteristics to describe the aggregate outcome of individual firm's decisions. Hereby, the average that results from firms that are heterogeneous with respect to a given characteristic gives the same aggregate outcome as in a model of representative firms that show this average's value as their common characteristic. Even though in both settings the same aggregate may result, the approaches are not exchangeable with regard to the firm. The use of a representative firm allows to describe the overall dynamics in the economy and the effects of a changing environment on firms in general. This is what happens in models of endogenous growth, like these mentioned above, that assume symmetric firms which all share the same characteristics. But the truly exiting dynamics of the economy arise from the heterogeneity of firms. Using the average of a heterogenous characteristic, where the average arises from previous firm decisions, retains the diversity of market participants. Considering heterogenous firms allows to describe the overall economy but it also makes possible to analyze firm-specific behavior regarding market entry and exit. Moreover, it allows to study interaction between the single firm and the overall economy.

In fact, there is a twofold heterogeneity in the model. On the one hand, firms are characterized by different firm knowledge levels at entry. On the other hand, every firm belongs to a cohort, the group of firms entering at the same point in time. Cohorts entering at different dates in economic history build their product inventions on different levels of the general knowledge stock available in the economy. The general knowledge base of the economy in a particular point in time is a special characteristic of the respective cohort entering at this date and distinguishes cohorts.

The contribution of this model is to endogenously derive product obsolescence which results from knowledge diffusion that enlarges the knowledge base for subsequent product development. Technological progress through purposeful R&D to develop new and to improve existing production processes is commonly recognized as the engine of growth. It is not only the single firm but the whole economy that gains from every new product and its production process, since its development adds to the general knowledge stock due to knowledge spillovers from firms into the economy. Therefore, in the model a cohort of later entering firms enjoys improved starting conditions that give them a relative advantage over incumbent firms. That increases the competitive pressure on the incumbent firms, whose market shares fall, if more efficient firms enter. Knowledge diffusion is thus the force which also drives firms out of the market if their sales can not generate sufficiently high profits to sustain the fixed costs of production. Market exit, which in Melitz (2003) occurs accidently, is a matter of firm decision in my setting. Using market entry and exit decisions the share of firms that leave the market in every period can be derived, which is the endogenous equivalent to *Melitz'* exogenously given probability of market exit.

Firm heterogeneity and knowledge diffusion are crucial to the model. While the former makes it possible to distinguish the firms within each cohort and to analyze their behavior, the latter allows to trace the evolution of a cohort itself and relative to other cohorts. Questions of central interest concern the composition of a cohort over time and whether a cohort's behavior depends on the knowledge base at the time of entry. Knowing the distribution within each cohort and a cohort's change over time, allow to describe the firm composition of the overall economy. In the stationary equilibrium the number of firms is constant but the composition of products in the economy is a matter of ongoing change, which is consistent with the empirical evidence reported in Dunne et al. (2005) and Bernard et al. (2010).

A main insight of the model, apart from product obsolescence, is the evolution of a cohort's productivity distribution. In every period some members of

the cohort are forced to leave the market which alters the cohort's appearance according to its changing firm composition. Since always the least productive firms are dropped off and over time those firms which incorporate more knowledge in the production process and are therefore more productive, will be left over. Since more productive firms are bigger the firm size distribution shifts to the right. This finding have to prove their relevance faced with the empirical evidence on firm size distribution as reported by Cabral and Mata (2003), Marsili (2006), and Hutchinson et al. (2010).

The paper is organized as follows.I present the model set-up including consumers' decisions on demand as well as a description of firm characteristics and the firms' pricing behavior in the next section. Market entry and exit of firms is discussed in section 3, and leads into the general equilibrium, which I derive in section 4. In section 5, I analyze the theoretical findings of the equilibrium and relate them to empirical evidence, which in section 6 will be followed by the conclusion.

2. The model

In the following I build up the model starting with the demand-side. Given their preferences, consumers decide on their demand of differentiated goods offered by monopolistically competing single-product firms, whose characteristics and decisions are presented afterwards. To that end, I introduce firm heterogeneity, which is followed by the firms production and pricing decisions. Finally, knowledge diffusion and its impact on firms efficiency is studied.

2.1. Consumers: preferences and demand

Consider a closed economy populated by a fixed number of L infinitely lived, identical consumers. Each of whom inelastically supplies one unit of labor and aims to maximize the present value of lifetime utility from consumption q_τ. Consumption q_τ is an aggregate bundle of $N > 1$ differentiated goods according to the following utility function

$$U = \int_0^\infty e^{-\rho\tau} ln\, q_\tau \, d\tau \qquad \text{with} \quad q_\tau = \left(\sum_{i=1}^{N} q_{i\tau}^{\alpha} \right)^{1/\alpha}, \tag{1}$$

where ρ is the individual discount rate $0 < \rho < 1$ and $\alpha \in (0, 1)$ the differentiation parameter that determines a constant elasticity of substitution $\epsilon = 1/(1-\alpha) > 1$ between any pair of N goods. $q_{i\tau}$ is the individual demand of good i at time τ. As

shown by Dixit and Stiglitz (1977), consumer behavior can be modeled by considering the set of varieties consumed as an aggregate good, q_τ, with aggregate price, P_τ

$$P_\tau^{1-\epsilon} = \left(\sum_i^N p_{i\tau}^{1-\epsilon} \right). \tag{2}$$

as sum over the products' prices, $p_{i\tau}$.
The consumer's maximization problem follows the set-up extensively discussed in (Barro and Sala-i Martin, 2004, ch.2) and (Grossman and Helpman, 1991, ch.3). The consumer maximizes utility in two stages. The chosen time pattern of spending is restricted by the dynamic budget constraint. Since consumers have no other income than wage, the lifetime budget constraint takes the form

$$\int_t^\infty R(\tau)E(\tau)\,d\tau \leq \int_t^\infty R(\tau)w(\tau) \tag{3}$$

with the cumulative interest rate $R(\tau) \equiv e^{-\int_t^\tau r(s)ds}$, expenditure $E(\tau)$, and the wage rate w. This gives the familiar time path of spending

$$\dot{E}/E = r - \rho. \tag{4}$$

The budget in every period, $E(\tau) = \sum_{i=1}^N p_{i\tau}\, q_{i\tau}$, is divided symmetrically to all products taking as given the product prices $p_{i\tau}$ and the time path of expenditure (4) in order to maximize aggregate consumption q_τ. From this static maximization problem the instantaneous overall demand for a single differentiated good by all L consumers in the economy results as

$$Q_{i\tau} = \frac{LE}{p_{i\tau}^\epsilon P_\tau^{1-\epsilon}} \tag{5}$$

which depends negatively on the product price and positively on the aggregate price.

2.2. Firm heterogeneity: knowledge and productivity

There is a large pool of prospective entrants which prior to entry are identical. To enter, firms must invest fixed and sunk entry costs $f^e > 0$ to develop the production technology for a new product. These have to be covered from lifetime profits. The innovation process is not explicitly modeled but summarized in a lottery in which firms get assigned an initial productivity parameter of nominal value φ which is drawn from a common distribution $g(\varphi)$ with positive

support over $(0, \infty)$ and a continuous cumulative distribution function $G(\varphi)$. The productivity parameter represents the firm-specific knowledge stock and relates the stock of firm knowledge k_{i1} of a firm i in its first firm-period $t = 1$ to the generally accessible knowledge $K_\tau^{t=1}$ at the point in time of the economy τ, when the firm is in its first firm period $t = 1$. It takes the form $\varphi_{i1} = k_{i1}/K_\tau^1$ in firm period $t = 1$, when the respective economy existed for τ periods so far.[1] While the firm knowledge stock builds the foundation of firm activities and is specific to the firm, the general accessible knowledge in the economy forms the innovation base of the economy in that period.

Using a stochastic process captures the idea, that even from a product developed by purposeful research its full characteristics, its potential relative to other products, and the full possibilities of the production process are not certainly known to the firm. It realizes the true nature of the product only by production and sale.

The productivity parameter does not say anything about the composition of firm knowledge. It may incorporate only well-known but newly combined knowledge or entirely new one, but it probably consists of a mix of both. The productivity parameter only counts the mass of knowledge and not the distinction from existing knowledge. Even the combination of known process parts may pose a new process and result in an innovative variety. The relative mass of contained knowledge displayed in φ gives the intensity and complexity of the process. It is assumed that each firm's production process does not change over time and therefore the firm-knowledge level $k_{it} = k_i$. Consequently, the value of the productivity parameter relative to the general knowledge base of the entry period (the value of the initial draw) also remains constant in all periods of firm life, $\varphi_{i1} = \varphi_{it} = \varphi_i$.

2.3. Technology and Pricing

The N single product firms use a technology that is described by the production function

$$Q_{it} = L_{it}^Q c(k_i)^{-1} = L_{it}^Q k_i^\kappa \tag{6}$$

that depends on labor used in production by firm i in firm period t, L_{it}^Q, and the efficiency units of knowledge k_i^κ used in the production process of that firm.

[1] Note the different period indication. Lifetime periods of infinitely lived consumers that can also be understood as a point in time in an economy's history are signed by τ. Differently, periods lived by firms are indicated with t. The index always refers to the period count of the entity that the variable characterizes. If the variable is bound to firms, the index is t. If it is a characteristic of the overall economy this index will be τ. The superscript always refers to the alternative level, overall economy or firm level, respectively.

The cost-function $c(k_i) = k_i^{-\kappa}$ depicts a constant elasticity of cost-reduction from rising knowledge with $0 < \kappa < 1$. A firm that incorporates a higher knowledge level than competitors produces the same amount with less labor.

All firms face the same demand schedule and follow the same production technology. They maximize their firm value

$$V_i = \int_1^{T_i+1} \pi_{it}\, dt \tag{7}$$

as the sum of instantaneous profits π_{it} over all T_i periods of firm life of firm i, where the age at death of that firm T_i depends on the initial productivity draw, $T_i = T(\varphi_i)$. The upper bound of the integral is $T_i + 1$, since a firm lives from the beginning of the first period until the end of period T_i which means to sum up until the beginning of the following period, $T_i + 1$. The period profits π_{it} are given by

$$\pi_{it} = [p_{it} - c(k_i)]Q_{it} - f^Q \tag{8}$$

where f^Q denotes fixed costs of production. There is no time-discounting in this model. The optimization of the firm value reduces to the maximization of period profits, (8). The optimal pricing strategy is given by

$$p_{it} = \frac{\alpha S_{it} - 1}{\alpha S_{it} - \alpha}\, c(k_i) \tag{9}$$

where S_{it} denotes the market share. It is reasonable to state, that in the overall economy there are $N \to \infty$ firms. With an infinite number of firms strategic interaction in price setting ceases to exist. This is also visible in the market share $S_{it} = (p_{it}/P_\tau^t)^{1-\epsilon}$, defined as the value of a firm's output relative to the value of the economy's output, which tends to be zero. Hence, pricing reduces to

$$p_{it} = \frac{1}{\alpha k_i^\kappa}. \tag{10}$$

All firms charge the usual mark-up from product differentiation. Depending on their constant firm-specific knowledge level, $k_i = \varphi_{i1} K_\tau^{i1}$, they set different but throughout time constant prices, $p_{it} = p_i(k_i)$. In fact, efficiency units of knowledge k_i^κ govern price setting. With different prices at the same price level P_τ firms face a firm-specific demand. Substituting $p_i(k_i)$ in (5) the demand of firm i in firm period t at price level P_τ is

$$Q_{it} = \alpha^\epsilon (k_i)^{\kappa\epsilon} \frac{LE}{P_\tau^{1-\epsilon}}. \tag{11}$$

The demand is the higher the more knowledge is incorporated in the production process, since the price depends negatively on the firm knowledge level. The higher the firm-specific knowledge stock is, the lower will be the price the firm charges, which leads to a higher demand by consumers.

2.4. Knowledge diffusion and real productivity

I assume, that by production and sale of a good does not only the producing firm learns about the market value of the product but also the public learns about the product and the underlying production process. Further, I assume that not all firm knowledge about the production process can be read from the product, but some share $0 < \zeta < 1$ of a firm's knowledge diffuses into the economy and adds to the general stock of knowledge. Knowledge spillovers thus enlarge the general knowledge base $K_{\tau+1} > K_\tau$ which spurs the invention of subsequent products. However, firms are not able to absorb knowledge, that diffuses from other firms into their production process. An increase of the general knowledge base is similar to depreciation of the firm specific knowledge stock. Firm knowledge does not disappear or becomes useless in the production process. Thus, in this model depreciation is not a physical but an economic concept. The position of incumbents relative to entrants worsens. Starting from a larger general knowledge base an entering firm with the same value of the initial productivity draw φ combines this with a higher knowledge base. This creates a higher nominal knowledge content, which leads to a lower product price (10). Nevertheless, in every entry period there will be a last entering firm j, that charges the same price, because it reaches the same nominal amount of firm-specific knowledge as incumbent firm i,

$$k_i = \varphi_{i1} K_\tau^{i1} = \varphi_{j1} K_{\tau+\Delta}^{j1} = k_j. \tag{12}$$

The initial productivity draw φ_{j1} of firm j, that leads to the same knowledge level given the current knowledge base $K_{\tau+\Delta}^{j1}$, represents the benchmark for firm i in its firm period t_i. Δ counts the number of periods passed between the entry of firm i and j or simply since the entry of firm i. Be aware, that between the measurement of the general knowledge K_τ and $K_{\tau+\Delta}$ lie $t_i - 1$ periods. Firm i entered $t_i - 1$ periods before firm j. This defines the time interval $\Delta = t_i - 1$ between firm i and a later entering one. Firm j does not indicate a special firm that is traced throughout firm life in comparison to firm i. Instead, it indicates always that firm of the last entering cohort, which shows the same firm-specific knowledge as firm i. $K_{\tau+\Delta}^{j1}$ gives the general knowledge base valid for this firm j in its entry period, where the subindex relates it to the knowledge base that formed the

foundation Δ periods ago. Because of knowledge diffusion the general base is growing throughout time. Therefore, to reach the same nominal level of firm-specific knowledge firm j has to draw a lower productivity value than firm i Δ periods ago. The initial productivity draw of firm j states the "true" value of firm knowledge k_i in its firm period t_i which I will call real productivity value, φ_{it}^{re}, hereafter. Since the nominal value of the firm-specific knowledge remains unchanged throughout firm life it has to be true that $\varphi_{it}^{re} K_{\tau+\Delta}^{it} = \varphi_{i1} K_{\tau}^{i1}$.

To analyze the economy at a given point in time τ it is convenient to express all firm-specific knowledge values in terms of the general knowledge base valid at that time using real productivity values, i.e. $k_i = \varphi_{it}^{re} K_{\tau}^{it}$. The evolution of the real productivity is driven by the change in the knowledge base, which will be derived next.

To account for the assumptions that only some part of a firm's process may be really new to the economy and that production processes need not to be entirely different from each other, only the average of diffusing firm-specific knowledge shows up in the knowledge base of the next period. Average diffusion, K_{τ}^{diff}, can be derived by summing up the weighted firm-specific knowledge diffusion. I think, it is reasonable to assume, that firms add to overall knowledge to the same extent as they contribute to the economy's output. Alternatively, K_{τ}^{diff} can be obtained as the knowledge diffusion $\zeta \bar{k}$ from the average firm whose knowledge content is expressed by $\bar{k}$. Because at a given general knowledge level every firm knowledge is represented in a real productivity value, it is sufficient to determine the average real productivity, $\bar{\varphi}^{re}$. The average real productivity in the economy results from its distribution $x(\varphi^{re})$ that has positive support over $(0, \infty)$ and the cumulative continuous distribution $X(\varphi^{re})$. The real productivity distribution is based on the distribution of the initial productivity, $g(\varphi)$, but is not identical with this distribution. While $x(\varphi^{re})$ refers to all firms in the economy, $g(\varphi)$ is the distribution of the productivity of a cohort in the first period. In that period nominal and real productivity values coincide. In equilibrium the real productivity distribution $x(\varphi^{re})$ and its support will prove to remain unchanged. This implies a constant value of the average real productivity. For this reason I delete the time-index τ in all expressions that depend on the average real productivity value.

Parallel to Melitz (2003) the average real productivity, $\bar{\varphi}^{re}$, has to be described as weighted harmonic mean of all productivity values with the relative

output shares as weights and is given by [2]

$$\bar{\varphi}^{re} = \left(\int_0^{\infty} (\varphi^{re})^{\Omega} x(\varphi^{re})\, d\varphi^{re} \right)^{\frac{1}{\Omega}} \tag{13}$$

with $\Omega = \kappa(\epsilon - 1)$. Applying this kind of average directly to knowledge diffusion gives

$$\begin{aligned} K_{\tau}^{diff} &= \zeta \left(\int_0^{\infty} \frac{1}{k_i^{\kappa}} \left(\frac{\varphi^{re}}{\bar{\varphi}} \right)^{\kappa\epsilon} x(\varphi^{re}) d\varphi^{re} \right)^{-1/\kappa} \\ &= \zeta K_{\tau} \bar{\varphi}^{re}. \end{aligned} \tag{14}$$

Equation (14) states, that the economy is characterized by a constant growth rate of the economy's knowledge base $\delta = \zeta \bar{\varphi}^{re}$, if the average productivity measure $\bar{\varphi}^{re}$ does not depends on the point in time of economic history, which will result with a constant lower productivity bound. Hence, general knowledge increases exponentially. The knowledge base of the economy in firm period t_i, $\Delta = t_i - 1$ period after entry, results as

$$K_{\tau+\Delta}^{t_i} = K_{\tau}^{1}(1 + \delta)^{\Delta}. \tag{15}$$

This relation gives further insight into the price index (17) and the real value of a firm's productivity which both depend negatively on the knowledge base of the economy. Inserting (15) into (12) and solving for the real productivity gives

$$\varphi_{it}^{re} = \frac{\varphi_i}{(1 + \delta)^{(t-1)}}. \tag{16}$$

To see the dependency of the aggregate price on the knowledge base, the price index given in (2) has to be rewritten as continuous weighted sum of all prices, since these depend on continuous real productivities following the distribution $x(\varphi^{re})$,

$$P_{\tau} = \left[\int_0^{\infty} N\, p(\varphi^{re})^{1-\epsilon} x(\varphi^{re}) d\varphi^{re} \right]^{\frac{1}{1-\epsilon}}. \tag{17}$$

Inserting the firms' price (10) in terms of real productivity $k_i^{\tau} = \varphi_{it}^{re} K_{\tau}$, yields the average productivity expression given in (13). The price index simplifies to

$$P_{\tau} = N^{\frac{1}{1-\epsilon}} \alpha^{-1} (\bar{\varphi}^{re}\, K_{\tau})^{-\kappa} \tag{18}$$

[2] $\bar{\varphi}^{re}$ results from $(\bar{\varphi}^{re})^{-\kappa} = \int_0^{\infty} \varphi^{re\,-\kappa} [q(\varphi_i^{re})/q(\bar{\varphi}^{re})]^{\kappa\epsilon}\, x(\varphi^{re})\, d\varphi^{re}$, which is the weighted harmonic mean with the output shares of any two firms as ratio of these firms' real productivities in efficiency units $q(\varphi^{re})/q(\bar{\varphi}^{re}) = (\varphi^{re}/\bar{\varphi}^{re})^{\kappa\epsilon}$.

which ceteris paribus is declining throughout time because of the rising knowledge base.

The average productivity derived in (13) is the central variable in the model. It is not only the weighted average of firm productivities but indicates the average firm that represents the aggregate. Through the distribution $x(\varphi^{re})$ it contains all information on firms. Taking the number of firms in the economy into account it is one determinant in the value of all aggregates. The overall revenues are simply $Rev = PQ = Nrev(\bar{\varphi}^{re})$ and the aggregate profit $\Pi = N\pi(\bar{\varphi}^{re})$. Dividing an aggregate variable by the number of firms gives exactly this variable's value for the average firm.

3. Entry and exit

There are two essential questions a firm has to decide on, whether to enter into the market and when to exit. Despite the natural order of entry and exit in time, I start with the analysis of the exit decision.

3.1. Market exit

Consider a firm that has successfully entered into the market and charges the price derived in (10) all throughout firm life. As the aggregate price is ever declining, the demand Q_{it} of firm i's product is decreasing and the firm's market position is worsening from period to period. Eventually, revenues will no longer cover variable and fixed costs of production. This development is easy to see if the per-period profit (8) is written in terms of real productivity

$$\pi_{it} = (1-\alpha)\frac{LE}{N}\left(\frac{\varphi_{it}^{re}}{\bar{\varphi}^{re}}\right)^{\Omega} - f^{Q}. \tag{19}$$

There will be a value of $\varphi^{re} = \varphi^{*}$, at which firms will realize zero profits $\pi_{it} = 0$. With further growth of the knowledge base a firm will thus have to exit the market in the subsequent period. This will be referred to as the market exit condition (MEC) characterized by the critical exit productivity φ^{*}. Eventually, because of ongoing knowledge diffusion this real productivity is reached by all firms of a cohort with certainty. A firm getting assigned a productivity $\varphi < \varphi^{*}$ at entry will decide to immediately exit and never produce, while all firms with $\varphi \geq \varphi^{*}$ will reach death ages $T \in (1, \infty)$. The lower bound of real productivities, φ^{*}, whose value equals the value of the lower bound of nominal productivity, determines the equilibrium distributions of the nominal productivity of a cohort at entry, $\mu(\varphi)$, as

well as of the real productivity in the overall economy $\lambda(\varphi^{re})$. Both distributions depend on market participation. For the cohort's first period, market participation means successful entry that happens with ex-ante probability $1 - G(\varphi^*)$ and the equilibrium distribution is based on the ex-ante distribution of productivity $g(\varphi)$,

$$\mu(\varphi) = \begin{cases} \frac{g(\varphi)}{1-G(\varphi^*)} & \text{if } \varphi > \varphi^* \\ 0 & \text{otherwise.} \end{cases} \tag{20}$$

The corresponding equilibrium distribution of real productivity conditional on market participation looks similar but depends on the distribution of the real productivity of all firms $h(\varphi^{re})$, which comprises not just a cohort in its first year but all firms at a given point in time. It reads

$$\lambda(\varphi^{re}) = \begin{cases} \frac{x(\varphi^{re})}{1-X(\varphi^*)} & \text{if } \varphi^{re} > \varphi^* \\ 0 & \text{otherwise,} \end{cases} \tag{21}$$

with $1-X(\varphi^*)$ as the probability of market participation.

In (13) I derived the unconditional average real productivity. Applying this equation, the average real productivity at entry conditional on market participation, $\bar{\varphi}^{re}$, is obtained as weighted sum starting at the lower bound φ^* with the conditional distributions of the real productivity, $\lambda(\varphi^{re})$, as weights.

$$\bar{\varphi}^{re}(\varphi^*) = \frac{1}{1 - X(\varphi^*)} \left(\int_{\varphi^*}^{\infty} (\varphi^{re})^{\Omega} x(\varphi^{re})\, d\varphi^{re} \right)^{\frac{1}{\Omega}} \tag{22}$$

In the same way also the average nominal productivity at entry conditional on market participation, $\tilde{\varphi}$, is derived as weighted sum with the conditional distribution of the nominal productivity, $\mu(\varphi)$, as weights.[3]

$$\tilde{\varphi}(\varphi^*) = \frac{1}{1 - G(\varphi^*)} \left(\int_{\varphi^*}^{\infty} \varphi^{\Omega} g(\varphi)\, d\varphi \right)^{\frac{1}{\Omega}}. \tag{23}$$

To see how the productivity distribution in the economy and in a cohort's first period of life are related to each other, consider a cohort of firms entering successfully with productivities distributed according to $\mu(\varphi)$. In every period of a cohort's life, t_C, where the index $C = 1, 2, 3...$ indicates the cohort, some cohort members leave the economy. The lowest nominal productivity surviving within

[3]Hereafter, to keep equations clearly arranged $\bar{\varphi}^{re}$ and $\tilde{\varphi}$ are written without indicating the dependency on φ^* again, $\bar{\varphi}^{re} = \bar{\varphi}^{re}(\varphi^*)$ and $\tilde{\varphi} = \tilde{\varphi}(\varphi^*)$.

the cohort increases from period to period. Hence, the distribution of nominal productivities, φ, of the remaining firms changes and moves towards those endowed with higher productivity. Tracing a cohort throughout time, where in every point in time always the least productive firms leave the market, it is possible to give the distribution of the nominal productivity, φ, of the remaining firms. The average productivity of a cohort in nominal terms then still has the structure of (23). Differently to the conditional distribution of the entry period, the lower productivity bound increases, while the initial draw, $g(\varphi)$ is still valid. It is possible to derive the conditional distribution of the nominal productivity value of a cohort for every period until the last member leaves the market.

Tracing the cohort in real productivity values gives a different picture. Since the real value of the firm knowledge level declines from period to period as the general knowledge base increases, the real productivities of the cohort become more and more concentrated in lower values of the distribution. The lower bound remains unchanged but the highest values disappear from period to period until only one firm remains that just reaches the lower productivity bound, φ^*. All distributions are independent of the number of firms in the economy or the cohort as (22) and (23) show.

If the critical exit productivity exists and is unique, a stationary equilibrium with unchanged distributions of initial and real productivity values of the cohort and the overall economy, respectively, results This means, that it is sufficient to describe one cohort throughout all firms' life to display all subsequently entering cohorts, that will see the same development. Furthermore, the cohort can be used to describe the economy in a given moment. In equilibrium, a certain number of cohorts of different ages $t \in (1, \infty)$ exists. There is one cohort that just entered the market, in another one is exactly the last firm left and going to see its last period, while most of the cohorts have already been in the market for some periods and still have some periods to go.

Every of these cohorts attributes to the distribution of nominal productivity that every single cohort exhibits at the same age once in its lifetime. Hence, summing up the productivity distributions of a single cohort gives the aggregate of all cohorts in equilibrium. The economy's overall distribution of productivity in initial as well as in real values is equivalent to the sum of all the distributions that a single cohort displays from $t = (1, \infty)$ until the last member firm leaves the economy. Hence, the distribution of real productivity of the overall economy, $x(\varphi^{re})$, and of the nominal productivity of the cohort at entry, $g(\varphi)$, as well as the equilibrium distributions, $\lambda(\varphi^{re})$ and $\mu(\varphi)$, respectively, are different but not independent from each other and $\bar{\varphi}^{re} < \tilde{\varphi}$.

In equilibrium exiting firms are replaced by new ones entering with productivities that follow the same distribution of initial productivities, $\mu(\varphi)$. Although the age of leaving firms $T \in (1, \infty)$ has the same distribution as the initial productivities conditional on successful entry, the overall distribution $x(\varphi^{re})$ remains unchanged. To clarify the equality of distributions of entering and exiting firms all productivity values are written in terms of the initial draw. Applying (16) to the exit productivity gives the age at market exit

$$T_i = \frac{ln\varphi_{i1} - ln\varphi^*}{ln(1+\delta)} + 1 \tag{24}$$

and shows the direct correspondence of the age at exit, T_i, to the initial productivity, φ_{i1}. Since the distribution $g(\varphi)$ is exogenously given and the exit productivity is endogenously determined and stationary, the distribution conditional on successful entry, $\mu(\varphi)$, is constant. All firms entering in different point in time in the past were drawing from the same distribution. Being characterized at exit by a certain age is described by the "probability" of having drawn the corresponding initial productivity T_i periods ago. Thus, the productivity distribution of exiting firms equals the productivity distribution of entrants.

Since the profit of the average firm characterizes the economy, it is worth to formulate it again depending on the exit productivity. From the MEC (19) results the revenue at market exit productivity

$$rev(\varphi^*) = rev(\bar{\varphi}^{re}) \left(\frac{\varphi^*}{\bar{\varphi}^{re}} \right)^{\Omega} = \epsilon f^Q \tag{25}$$

with the average revenue $rev(\bar{\varphi}^{re}) = \frac{LE}{N}$. This gives the average profit in the economy as a function of the exit productivity φ^*

$$\pi(\bar{\varphi}^{re}) = f^Q \left[\left(\frac{\bar{\varphi}^{re}}{\varphi^*} \right)^{\Omega} - 1 \right]. \tag{26}$$

3.2. Free entry

Starting up a new firm is costly since the production process of a new product has to be developed and a new organization has to be installed. The entrant devotes an amount of f^e units of labor, which is thereafter sunk, to obtain its firm endowment. In an equilibrium with free entry, firms only consider to spend the investment cost, if the expected firm value $E(V)$, covers at least the market entry costs,

$$EV = (1 - G(\varphi^*))\, \tilde{V} \geq f^e. \tag{27}$$

The firm value $\tilde{V}$ of a firm that shows the average productivity of the entering cohort, $\tilde{V} = V(\tilde{\varphi})$, conditional on successful entry is the expected firm value. The firm with initial draw $\tilde{\varphi}$ represents the entire cohort in the period of entry and its firm life is representative for the life of all members of the cohort in average. To distinguish it from the average of all firms active in the economy indicated by $\bar{\varphi}^{re}$, it is called the representative firm displaying the representative productivity $\tilde{\varphi}$. In fact, with free entry there is an unbounded mass of prospective entrants and hence, the value of EV net of market entry costs could not be positive.

Depending on the initial draw every firm follows its deterministic time-path of profits which is governed by the firm specific path of real productivity. For the representative firm the firm value is given by the sum of period profits (see eq. 7) until period $\tilde{T}$, the age of the firm at death. The period profits are given by (19). The value of the representative firm reads

$$V(\tilde{\varphi}) = \int_1^{\tilde{T}+1} \left((1-\alpha)\frac{LE}{N} \left(\frac{\tilde{\varphi}(1+\delta)^{1-t}}{\bar{\varphi}^{re}} \right)^{\Omega} - f^{Q} \right) dt. \tag{28}$$

Although the average productivity at entry, which is the productivity draw of the representative firm, is based on the distribution of the initial draw, the generated revenues throughout firm life depend on the distribution of real productivity which governs the price index in the demand function in (17). In equilibrium, the only time-dependent variable of the above firm value $V(\tilde{\varphi})$ is the productivity discounting $(1+\delta)$. Solving the integral in (17) and using (27) gives the representative firm's value. Equation (28) is the free entry condition (FEC),

$$\tilde{V} = \frac{f^{e}}{1-G(\varphi^{*})} = \frac{(1-\alpha)^{2}}{\kappa\alpha}\frac{LE}{N}\left(\frac{\tilde{\varphi}}{\bar{\varphi}^{re}}\right)^{\Omega}\left(1-(1+\delta)^{-\tilde{T}\Omega}\right) - \tilde{T}f^{Q} \tag{29}$$

which is a function of the exit productivity, φ^{*}. Note, that equilibrium condition (29) includes three endogenous variables. The endogenous age in the exit period, $\tilde{T}$, given in (24) displays φ^{*} in the nominator and in the denominator since $\delta = \zeta\bar{\varphi}(\varphi^{*})$. Furthermore, average and representative productivity depend on the same lower bound of their distributions, $\lambda(\varphi^{re})$ and $\mu(\varphi)$, respectively.

4. General equilibrium

In this section the general equilibrium of the economy will be solved. The factor markets' equilibria, i.e. the equilibrium in the goods market results from the conditions of market entry and exit. The equilibrium in the capital and labor market will be added to this.

The market exit and free entry conditions give two different relationships linking average revenues $rev(\bar{\varphi}^{re}) = LE/N$ to the exit productivity φ^*. Rearranging (19) at $\varphi^{re} = \varphi^*$ and using (29) yields

$$rev(\bar{\varphi}^{re}) = \frac{LE}{N} = \epsilon f^{Q} \left(\frac{\bar{\varphi}^{re}}{\varphi^*}\right)^{\Omega} \tag{30}$$

$$rev(\bar{\varphi}^{re}) = \left(1 - (1+\delta)^{-\tilde{T}\Omega}\right)^{-1} \frac{\kappa\alpha}{(1-\alpha)^2} \left(\frac{\bar{\varphi}^{re}}{\tilde{\varphi}}\right)\left(\frac{f^{e}}{1 - G(\varphi^*)} + \tilde{T} f^{Q}\right). \tag{31}$$

Together with the age at death $\tilde{T}$ of the representative firm (see eq. (24) at $\varphi_i = \tilde{\varphi}$) these equations describe not only product market clearing but define the value of φ^* and the average revenues, $rev(\bar{\varphi}^{re})$, in the stationary equilibrium.

From the endogenously determined constant level of φ^*, that is bounding the exogenously given productivity distribution $g(\varphi)$ and the derived distribution of real productivity $h(\varphi^{re})$, follow constant values of the means of the equilibrium distributions and the variables, that are based on the latter. Most importantly, the number of firms in the economy as well as the number of exiting and entering firms will be stationary.

While the size of a cohort is a matter of capital market clearing, the size of the economy results from labor market clearing, that I want to go on with. Aggregate labor $L = L^{e} + L^{P}$ reflects labor units employed in entry L^{e} and labor used for production L^{P}=$L^{Q} + F^{Q}$, that divides in employment in manufacturing, L^{Q}, and labor covering aggregate fixed costs of production, F^{Q}=Nf^{Q}. In this model consumers have no other sources of income than wage and all revenues are either paid to labor as reward for work in production or account for fixed costs of production and entry. Thus, aggregate revenue $Rev = Nrev(\bar{\varphi}^{re})$ is fixed by the exogenously given constant population valuated at the numeraire wage, $Rev = L$. The number of firms in equilibrium is

$$N = \frac{Rev}{rev(\bar{\varphi}^{re})} = \frac{L}{\epsilon f^{Q}} \left(\frac{\varphi^*}{\bar{\varphi}^{re}}\right)^{\Omega}. \tag{32}$$

Inserting this result into (18) determines the price index in equilibrium,

$$P_{\tau} = \frac{\alpha^{-1}}{(\varphi^* K_{\tau})^{\kappa}} \left(\frac{f^{Q}}{(1-\alpha)L}\right)^{\frac{1-\alpha}{\alpha}} \tag{33}$$

which depends on the critical exit productivity and the general knowledge base in a given moment.

To close the model, I turn to the capital market. In fact, the capital market is only an implicit one in this model. There is no interest rate. Hence, there is no riskless loan or bond in addition to or as alternative to firm ownership by consumers. But even without a capital market the aggregated profits in the economy have to cover the investment by all new entrants. Hence, expected profit flows over firm life paid to firm owners equal the investment costs for market entry. In every point in time a mass of exiting firms, N^{exit}, of all possible firm ages drop out of the market. These firm are replaced by successful entrants, i.e. $N^{exit} = N^{entry} = (1 - G(\varphi^*))N^e$. Entrants and dropouts are characterized by the same distribution of initial productivity. N^e denotes the number of firms attempting market entry. Investment by all firms trying to enter requires $L^e = N^e f^e = F^e$ units of labor. The incurred costs are covered by aggregated profits, $F^e = N\pi(\bar{\varphi}^{re})$. The number of entrants is then given by

$$N^{entry} = \frac{(1 - G(\varphi^*))\, N\, \pi(\bar{\varphi})}{f^e}. \tag{34}$$

Recall that market entry costs conditional on successful entry are the sum of lifetime profits of the representative firm, $f^e/(1 - G(\varphi^*)) = \tilde{V}$. Therefore it is the ratio between the average firm profit and the firm value of the representative firm that influences the number of firms leaving and entering the economy in the stationary equilibrium. Inserting the average profit (26) and substituting the number of firms by (32) gives the number of successful entrants

$$N^{entry} = \frac{(1 - G(\varphi^*))}{\epsilon f^e} L \left(1 - \left(\frac{\varphi^*}{\bar{\varphi}^{re}}\right)^{\Omega}\right). \tag{35}$$

This completes the characterization of the stationary equilibrium, which I will further analyze in the following. Note, that apart from the endogenous variables derived in this section (exit productivity, aggregate price, number of entrants, overall revenues) the vector that defines the equilibrium includes also the wage (numeraire), the age of the representative firm at market exit as given in (24) and the firm price derived in (10).

5. Analysis of the equilibrium

To analyze the equilibrium I start by considering the importance of knowledge diffusion to the overall dynamics of the economy. The growth rate of consumption and the share of exiting firms in every period are paid particular attention. First, I will turn to the impact of the country size on the outcome of

firm-level variables and the welfare in the overall economy. This is followed in the second sub-section by an analysis of the resulting distributions with market entry and exit and serves to evaluate the model in the light of empirical findings on firm entry and exit.

5.1. Characteristics of the equilibrium

A main feature of this model is the growing knowledge base caused by knowledge diffusion from firms as given in (15). This allows subsequent firms to enter with ever more sophisticated products, that contain more knowledge and feature richer functionality and quality than older products. Equivalently to the firm, which is called more productive if it shows at a given knowledge base a higher firm-specific knowledge stock than its competitors, the general knowledge base is to be interpreted as the economy's overall efficiency or productivity. A rising overall efficiency results in lower product prices, which lead to a falling aggregate price and cause the aggregate demand to increase. The aggregated demand in the economy Q_τ is proportional to single consumer's demand for all products which are given by the CES-function in (1),

$$Q_\tau = L\bar{q}_\tau N^{(1-\alpha)/\alpha} = (\bar{\varphi}^{re} K_\tau)^\kappa \bar{L}_i^Q N^{(1-\alpha)/\alpha}, \tag{36}$$

where $\bar{q}_\tau$ is understood as average demand which equals the single demand of the average product characterized by $\bar{\varphi}^{re}$. Alternatively, aggregated demand can be expressed as the N-fold overall demand for the average product taking into account the CES-specification of utility. Producing the demand of the economy involves in average $\bar{L}_i^Q = (\epsilon - 1) f^Q (\bar{\varphi}^{re})^\Omega (\varphi^*)^{-\Omega}$ workers in production.

Taking logs and time derivatives of Q_τ gives the growth rate of the economy

$$\begin{aligned} \frac{\dot{Q}}{Q} &= \left(\frac{1}{\alpha} - 1\right)\frac{\dot{N}}{N} + \frac{\dot{\bar{L}}_i^Q}{\bar{L}_i^Q} + \kappa\frac{\dot{\bar{\varphi}}^{re}}{\bar{\varphi}^{re}} + \kappa\frac{\dot{K}_\tau}{K_\tau} \\ &= \kappa\zeta\bar{\varphi}^{re} \end{aligned} \tag{37}$$

which reduces to the constant growth rate of the knowledge base (14) evaluated at efficiency units. The equilibrium property of stationarity means essentially that $\dot{\bar{\varphi}}^{re}/\bar{\varphi}^{re} = 0$. Since the number of firms (32) depends on exogenously given or stationary variables only, the number of firms remains constant, $\dot{N}/N = 0$. Producing the demand of the economy involves on average $\bar{L}_i^Q$ workers in production, $\bar{L}_i^Q = (\epsilon - 1) f^Q (\bar{\varphi}^{re})^\Omega (\varphi^*)^{-\Omega}$. This is derived applying product market clearing which requires that the demand for the average product (5) equals the production technology (6). Substituting the equilibrium values of the product

price (10) and the price index (33) into (5) and rearranging gives the number of production workers employed by the average firm. This number is independent from the only time-variant variable of the model - the general knowledge level. Therefore it does not change in time, $\dot{\bar{L}}_i^Q / \bar{L}_i^Q = 0$. Hence, the economy grows at a constant rate, which is caused by knowledge diffusion but endogenously determined through the individual decision of the heterogenous firms.

Knowledge diffusion drives the economy and for the same reason firms also lose their former market position and eventually have to leave the market. At first the product ceases to be new and after a while it becomes obsolete and disappears from the market, since it cannot be sold covering production costs. Eq. (35) gives the number of exiting firms. Dividing this expression by the number of firms in the economy (32) gives the share of firms that exit in every period.

$$\frac{N^{entry}}{N} = (1 - G(\varphi^*)) \frac{f^Q}{f^e} \left(\left(\frac{\bar{\varphi}^{re}}{\varphi^*} \right)^{\Omega} - 1 \right). \tag{38}$$

This ratio can be understood as endogenous equivalent to the probability of market exit in Melitz (2003), which is exogenously given. *Melitz* assumed, that firms are forced to exit because of a bad shock that occurs in every period with an exogenously given probability. However, in this model firm exit is caused by changing market conditions and the share of exiting firms arises endogenously from firm decision on market entry and exit. Both decision are reflected in the fixed costs of production (f^Q) and of market entry (f^e), respectively. In every period all firms decide whether to leave the market given the fixed production costs that have to be covered from revenues. Therefore, the number of firms in the economy (32) adds the fixed costs of production to (38). The value of the representative firm displaying the market entry decision adds entry costs. Since both decisions together determine the exit productivity and thereby the ex-post productivity distributions, firm heterogeneity is captured as well. Last but not least, consumer and production are included as well. The auxiliary parameter $\Omega = \kappa\alpha/(1-\alpha)$ contains the differentiation parameter and the parameter of cost-reduction from knowledge in the production process. Hence, the endogenously resulting share of exiting firms summarizes all crucial parameters of the model.

Taking a closer look on the firm-specific variables it turns out that the exit productivity and the derived averages $\bar{\varphi}^{re}$ and $\tilde{\varphi}^*$, the average revenue $rev(\bar{\varphi}^{re})$, average firm profit $\pi(\bar{\varphi}^{re})$ and the representative firm value are independent of the country's size, L. However, the variables describing the aggregate, the overall revenue, Rev, the number of firms N and hence the number of entrants, N^{entry},

increase proportionally with the country size. This dependency on the scale of the economy is different from the scale effects to be found in the earlier models of endogenous growth (see Romer (1990), Grossman and Helpman (1991), Aghion and Howitt (1992)) where an increase in the labor endowment of the economy leads to a higher growth rate of productivity. As following endogenous growth models by Peretto (1998), Young (1998) and Peretto and Smulders (2002) depend on the average R&D-investment, the economy of the proposed model is driven by the average knowledge diffusion. A larger economy is characterized by higher aggregate values. But these are divided on more firms. Because of the stationarity of the equilibrium the average firm's characteristics including knowledge diffusion remain constant. Although the growth rate is independent of scale effects, a larger economy with a higher number of products (firms) is characterized by a higher welfare level than an economy with less workers with the same general knowledge base.

$$W = P^{-1} = \alpha(\varphi^* K_\tau)^\kappa \left(\frac{(1-\alpha)L}{f^Q}\right)^{\frac{1-\alpha}{\alpha}} . \tag{39}$$

This results from utility that follows a CES-function, which features love of variety. Hence, consumers benefit from the higher number of products. More products in the economy lead to a lower aggregate price meaning a higher welfare level. It can be summarized, that economies differing only in the number of workers will be equal in all the firm-level characteristics including the growth rate of the economy but they reach different welfare levels, which are growing with the overall knowledge base.

To close the analysis of the equilibrium I turn to the productivity distributions in the economy. The ex-ante distribution of initial productivity $g(\varphi)$ is exogenously given and serves as base for the endogenously derived ex-post distribution of the initial draw $\mu(\varphi)$ and of the real productivity distribution $\lambda(\varphi^{re})$, which describes the overall economy. As explained above, the productivity distribution of exiting firms equals the productivity distribution of entrants if this main firm characteristic is expressed in nominal productivity values of the first period's initial draw. Firm exit brings about an increase of the currently lowest productivity value of the remaining firms of the cohort. This causes the productivity distribution of a cohort in nominal terms to move to the right. This means, that firms using a production process with a higher knowledge content and producing a more complex good live longer than firms selling a more simple product. Less productive firms will thus be replaced earlier by new entrants,

that produce a similar good at lower costs using a production process founded on the latest knowledge contributions. The moment of exit depends on the firm characteristic at entry. Thus firms select in the decision to exit or to continue. As described in section 3.1 the real productivity distribution and age composition of the economy arises from the aggregate of a cohort's life. Since firms with a low productivity draw exit the market while still young, the economy is dominated by older firms with higher nominal productivity values. There is a disproportionate number of highly productive firms in the economy.

5.2. Empirical evidence

The findings on the productivity distribution suit remarkably well the empirical findings of Cabral and Mata (2003) on the firm size distribution. They analyzed Portuguese manufacturing firm data to extract and theoretically explain stylized facts about the firm size distribution and its evolution. They use employment as the measure of firm size. In this model, firm size depends on productivity. However, the number of workers used in production, which is given for the average firm above, can be easily reformulated as a function of productivity. Thus, this model's productivity measure corresponds to the firm size investigated by Cabral and Mata (2003). They find that the firm size distribution of a given cohort at the time of market entry is very skewed to the right and more skewed than the overall firm size distribution. At birth, the share of small firms in a cohort is higher than in the overall economy. While the size distribution of the cohort gradually evolves towards a more symmetric distribution, total firm size distribution proves to be fairly stable over time and is somewhat skewed to the right. These findings suit well the results of the proposed model. Since the first exiting firms are the least productive ones of a cohort. They are smaller in terms of employment and demand and still young at firm death. The surviving firms show a higher productivity and the productivity distribution of the cohort in terms of the initial draw is moving to the right. While the cohort's productivity distribution is a matter of change, the distribution of productivity in the overall economy remains stable throughout time, which results from the stationarity of the equilibrium. Since the economy is dominated by more productive firms, the distribution of the overall economy is also right-skewed.

These findings contrast the results of earlier works of the 1950ies and 1960ies (Hart and Prais (1956), Simon and Bonnini (1958), Mansfield (1962), Ijiri and Simon (1964)) which found that firm size distribution is stable and approximately lognormal. Doubts in these findings have arisen with the studies of Evans (1987) and Hall (1987) suggesting a size distribution evolving over time and

differing from a lognormal distribution. Right skewness of a cohort's size distribution at entry is confirmed by Van Ark and Monnikhof (1996). Their data set shows firm size patterns for five OECD-countries similar to the Portuguese, which more recently were also found by Hutchinson et al. (2010) for the UK and Belgium.

But Cabral and Mata (2003) do not only describe firm size distribution. They also aim at identifying the differences between the survivors and the overall cohort at entry to explain the observed changes in the distribution. They show that the size distribution that characterizes the survivors of a cohort at birth differs only slightly from that of the whole cohort at birth. If after some years the distribution of the cohort diminished by the exiting firm has moved to the right, the main characteristic of the surviving firms must have changed over time. Precisely, in average surviving firms see firm size growth, which *Cabral and Mata* recognize as the main characteristic of firm survival and equate it to aging. In their opinion, surviving firms mature because they are able to overcome beginning financial restrictions. Whether the firm is large at entry is of minor importance for the generation of financial support, since size is not an indicator of productivity in their model. However, they do not exclude selection as a reason for firm survival, but see aging as the dominating effect. The conventional wisdom that the exit rate is higher among young and small firms applies to their data as well. To the contrary, my model suggests selection as the only reason for the evolution of the surviving firms' size distribution. Less productive firms die first, because they are less productive.

6. Conclusion

In this paper, I developed a dynamic general equilibrium model of heterogenous firms and endogenous growth without scale effects that allows to analyze market entry of firms and their endogenous exit. Products become obsolete. Knowledge diffusion from firms induces growth of the general knowledge base, which improves the starting condition of later entrants relative to incumbents whose market position is worsening. Knowledge diffusion works in the same way as depreciation of the firm specific knowledge stock. Since new entering firms offer on average products of higher knowledge content, in every period firms (and their product) characterized by the currently least knowledge intensive production processes are pushed out of the market. This market exit occurs not accidently caused by a negative shock as in Melitz (2003), but is driven by the economy's constant knowledge growth, which induces existing products to become obsolete. The endogenously determined share of exiting firms resulting

from the firm decisions on entry and exit plays a key role in the evolution of the economy.

Although the firm composition is changing from period to period, the economy itself is stable in the sense, that the number of firms is constant and the values of firm-level characteristics are stationary. Ongoing knowledge growth shows up in a decreasing price level and leads to an ever increasing welfare level. The welfare level is the only distinct difference between two economies, that differ only in the number of firms. A larger economy allows for more products which drives down the aggregate price.

In this model, considering heterogenous firms allows to describe the interaction between an individual firm and the overall economy. Knowledge diffusion makes it possible to distinguish cohorts of entering firms and trace their evolution. It turns out, that the distribution of firm productivity of a cohort moves towards higher productivity values in terms of the initial nominal productivity draw. Surviving firms' production processes rely on more knowledge relative to that of members of the cohort that dropped out early. The model explains the empirically observed exit of predominantly young and small firms and the observed fact that new entrants are characterized on average by a higher firm specific knowledge level.

Yet, the model neglects the accumulation of further firm-specific knowledge to counteract knowledge diffusion. This gives the direction for further research. Investment of firms in research and development aims at innovations (new products) as well as at improvements, which concerns the quality of products and the costs of production. This observation is a natural starting point for further exploration of the model, e.g. introduction of in-house R&D. Preserving product obsolescence through knowledge diffusion, the question remains, whether it is rational to invest in the improvement of existing products or whether a natural reaction would be to develop a new product or start a multi-product firm to escape from firm death.

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