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Intergenerational Redistribution with Asymmetric Information: the Case of Non-Observable Savings

Oliver Lorz

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Abstract: This paper analyzes intergenerational redistribution in a 2-period overlapping-generations model that allows for heterogeneous labor productivities within the working generation. In each period, the government decides about redistributive transfers to maximize the aggregate utility of the members of both generations. There is asymmetric information between the government and the private sector: The government can only observe labor incomes but not individual labor supply or individual productivities. In addition, individual savings and capital incomes are not observable for the government. It is shown that the political economy equilibrium is not Pareto-efficient. Redistribution results in equal consumption levels of individuals with different labor productivities. This destroys incentives for individuals with a high labor productivity to earn a higher labor income than individuals with a low labor productivity.

Keywords: Overlapping generations, redistribution, political economy, asymmetric information.

JEL-Classification: E 62, D 78, D 82.

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1. Introduction⁺

Redistribution between the working generation and the generation of the retired is a considerable part of modern welfare states. Prominent institutions in this respect are unfunded pension systems where the working generation pays contributions to finance benefits for the retired of the same period. Redistribution between generations may be combined with redistribution within generations according to individual characteristics of different members of each generation. For example, contributions to an unfunded pension system may increase with the individual labor income whereas all retired get equal benefits, or redistributive taxes and transfers between individuals of the same generation may supplement the unfunded pension system.

In this paper, a political economy model is set up to analyze redistribution between generations and within generations in a unified framework. In each period, the government is assumed to maximize the aggregate utility of all individuals living in the economy. Unlike existing models on the political economy of intergenerational redistribution, this paper incorporates an endogenous supply of labor and asymmetric information between the government and the private sector.¹ The government can only observe labor incomes but not labor supply or labor productivities.² Redistribution policy then may distort labor supply of the working generation.

This paper is a sequel of the Kiel Working Paper (KWP) 904 (Lorz, 1999). In KWP 904 the government is able to control individual savings and to observe capital incomes. In this situation, the young individuals have no incentives to save voluntarily, because capital income is completely redistributed at the margin by the government of the subsequent period. All savings then have to be done in the form of savings

^{*} I would like to thank Friedrich Breyer for helpful comments and Nicole Petersohn for drawing Figure 1.

¹ For surveys on the political economy of intergenerational redistribution see Breyer (1994) or Verbon (1993). Most closely related to this paper are van de Ven (1996), Meijdam, Verbon (1996) and Verbon et al. (1998). In these models, however, the supply of labor is assumed to be exogenously given.

² This is the same information structure as in the models of optimum income taxation. The seminal paper on optimum income taxation is Mirlees (1971). For optimum income taxation in an overlappinggenerations framework see Stiglitz (1985, 1987).

by the government or with mandatory contributions to a funded pension system. In this paper instead capital incomes are not observable for the government. The government cannot adjust the level of redistributive transfers with respect to observed capital incomes, and incentives for private savings remain intact. Each young individual saves for retirement up to the point where the rate of time preference is equal to the marginal productivity of capital. This implies that the rate of time preference is the same for all individuals.

As this paper shows, the equilibrium consumption levels are equally distributed within the generation of the retired. All retired individuals have the same disposable income after transfers regardless of their labor income in the preceding period. Since the savings equilibrium implies the same rate of time preference for all individuals, not only all retired have the same consumption level but also all members of the young generation in the preceding period. The equilibrium then results in a Pareto-inefficient supply of labor: individuals with a high labor productivity have no incentive to earn a higher labor income than individuals with a low productivity. A Pareto-improvement could be achieved without changing the aggregate supply of labor, if individuals with a high labor productivity worked less than in the political economy equilibrium. This result is derived in a 2-period overlapping generations model with two representative individuals in each generation. The model is presented and the equilibrium is derived in the following section 2. Section 3 shows the Pareto-inefficiency of the political economy equilibrium. Section 4 concludes.

2. The Model

As in KWP 904 an overlapping generations economy is assumed that lasts for two periods (t=1,2). In each period, a young and an old generation are alive. An aggregate consumption good is produced with the factors capital and labor according to the linear-homogenous technology $Y_t = F(K_t, L_t)$. The product markets and the factor markets are competitive. The price of the aggregate good is set equal to one. Each generation consists of two representative individuals (i=1,2). The individuals of the young generation supply labor l_t^i and save s_t^i units of the aggregate good. They differ

with respect to their labor productivity: a physical unit of labor of individual 2 substitutes a units of labor of individual 1 (a<1). This gives an aggregate labor supply of $L_t = l_t^1 + al_t^2$ measured in units of l_t^1 . The aggregate capital stock is given by $K_t = s_{t-1}^1 + s_{t-1}^2 + s_{t-1}^g$; the term s_{t-1}^g denotes public savings determined by the government of t-1.³ Savings of period 0 are exogenously given. The young receive a labor income of $e_{y_t}^1 = l_t^1 F_L(K_t, L_t)$ and $e_{y_t}^2 = al_t^2 F_L(K_t, L_t)$. The old receive a capital income of $e_{a_t}^i = s_{t-1}^i F_K(K_t, L_t)$.

The government of period t may redistribute between the young and the old generation as well as within the generations between the individuals 1 and 2. For this purpose the government raises contributions τ_i^1 and τ_i^2 from the young and pays benefits b_i^1 and b_i^2 to the old.⁴ This gives consumption levels of $c'_{y_i} = e'_{y_i} - \tau'_i$ for the young generation and $c'_{o_i} = e'_{o_i} + b'_i$ for the old generation. In addition to redistributive transfers, the government decides about public savings $s_i^{g,5}$

There is asymmetric information between the government and the private sector: The government can not observe private savings or capital income. The government is therefore not able to pay means-tested benefits to the old or to raise capital income taxes. In addition, the government is not able to observe individual labor productivities or labor supply of the young. It can only observe individual labor incomes, and it may differentiate contributions and benefits according to these labor incomes, i.e. $\tau_t^i = \tau_t^i (e_{y_t}^i)$ and $b_t^i = b_t (e_{y_{t-1}}^i)$.

The utility of a young individual in period 1 is given by the utility function $\tilde{u}_{y_1}^i = u_y(c_{y_1}^i) + \beta u_a(c_{a_2}^i) + v(\bar{l} - l_1^i)$, with \bar{l} denoting individual labor endowment.

³ Private savings and public savings may be positive or negative. Aggregate savings have to be positive.

⁴ These contributions and benefits may be interpreted as net fiscal burdens or benefits from taxes, subsidies and social security payments. Contributions and benefits may be positive or negative.

⁵ Public savings may include savings of a mandatory funded pension system as well as net savings or deficits of the public sector.

The utility of a young individual in period 2 is given by $\tilde{u}_{y_2}' = u_y(c_{y_2}') + v(\bar{l} - l_2')$. The utility of an old individual is given by $\tilde{u}_{o_t}' = u_o(c_{o_t}')$. The government in t is assumed to maximize the unweighted utility of all individuals alive in t. This gives an objective function of the government of $W_t = \tilde{u}_{y_t}^1 + \tilde{u}_{y_t}^2 + \tilde{u}_{o_t}^1 + \tilde{u}_{o_t}^2$.

With respect to the sequence of moves of all participants, the model comprises four stages: In the first stage, the government in period 1 determines the schedules of contributions and benefits $\tau_1(e_{y_1})$ and $b_1(e_{y_0})$ and public savings s_1^g . In the second stage, the young individuals in period 1 decide about their labor supply l_1' and about the level of their private retirement savings s_1' . In the third stage, the government of period 2 decides about $\tau_2(e_{y_2})$ and $b_2(e_{y_1})$. Finally, in the fourth stage, the young in period 2 decide about their labor supply l_2' .

To solve the model, the problem of the government finding the optimal contribution schedule is restated as a problem of designing an optimal self-selection mechanism (see Stiglitz, 1982). The government then offers a contribution-income package $(\tau_t^1, e_{\nu_t}^1)$ for individual 1 and $(\tau_t^2, e_{\nu_t}^2)$ for individual 2 – under the self-selection constraints that both individuals have no incentive to choose the wrong package. This is equivalent to finding the optimal combinations (τ_t^1, l_t^1) and (τ_t^2, l_t^2) under the self-selection constraints. The subsequent government then determines the benefits b_{t+1}^1 and b_{t+1}^2 .

This gives the following modified 3-stage game structure: In the first stage, the government of period 1 decides about τ_1^i , b_1^i , l_1^i and s_1^g . In the second stage, the young individuals of period 1 decide about s_1^i . In the third stage, the government of period 2 decides about τ_2^i , b_2^i and l_2^i . The government of period 2 does not know the level of individual savings s_1^i . Figure 1 depicts the modified game in extensive form.

Since the government in 2 does not know s_1^1 and s_1^2 , the second and the third stage of the game may be treated as a simultaneous game between the government in 2 and the

representative individuals in 1. The government chooses the optimal levels of τ'_2 , b'_2 and l'_2 for given levels of s'_1 whereas the individual *i* chooses the optimal level of s'_1 for given τ'_2 , b'_2 and l'_2 .

Figure 1 --- The Modified Redistribution Game



The government in 2 maximizes $W_2 = \tilde{u}_{y_2}^1 + \tilde{u}_{y_2}^2 + \tilde{u}_{o_2}^1 + \tilde{u}_{o_2}^2$, subject to the budget constraint $b_2^1 + b_2^2 = \tau_2^1 + \tau_2^2 + s_1^g F_K(K_2, L_2)$ and the self-selection constraint $u_y(c_{y_2}^1) + v(\bar{l} - l_2^1) \ge u_o(c_{y_2}^2) + v(\bar{l} - al_2^2)$.⁶ According to the self-selection constraint the

⁶ It can be shown that the self-selection constraint for individual 2 is not binding in the equilibrium (see Stiglitz, 1987). Therefore only the self-selection constraint for individual 1 needs to be considered explicitly.

utility of the young individual 1 receiving an income of $e_{y_2}^1$ and paying contributions of τ_2^1 has to be at least as high as the utility of the same individual receiving $e_{y_2}^2$ and paying τ_2^2 . This gives after rearranging the following first order conditions:

$$u'_o(c^1_{\sigma_2}) = u'_o(c^2_{\sigma_2}), \qquad (1)$$

$$u_{y}'(c_{y_{2}}^{1})[1+\sigma_{2}] = u_{a}'(c_{a_{2}}^{1}), \qquad (2)$$

$$u'_{v}(c^{2}_{v_{2}})[1-\sigma_{2}] = u'_{o}(c^{\dagger}_{o_{2}}), \qquad (3)$$

$$\nu' (\bar{l} - l_2^{\dagger}) = u'_{\nu} (c_{\nu_2}^{\dagger}) F_L (K_2, L_2),$$
(4)

$$v'(\bar{l}-l_2^2) - \sigma_2 v'(\bar{l}-al_2^2) a = u'_{\nu}(c_{\nu_2}^2) [1-\sigma_2] a F_L(K_2,L_2),$$
(5)

$$c_{y_2}^{\dagger} + c_{y_2}^2 + c_{o_2}^{\dagger} + c_{o_2}^2 = F(K_2, L_2),$$
 (6)

$$u_{\nu}(c_{\nu_{2}}^{\dagger}) + \nu(\bar{l} - l_{2}^{\dagger}) = u_{\nu}(c_{\nu_{2}}^{2}) + \nu(\bar{l} - al_{2}^{2}).$$
⁽⁷⁾

The term $\sigma_2 > 0$ denotes the shadow price of the self-selection constraint in period 2. Equations (1)-(3) determine the equilibrium distribution of disposable incomes within and between generations in period 2. According to (1) the government in period 2 redistributes within the old generation up to the point where the consumption levels of all individuals in this generation are equalized. This result holds despite the fact that the government cannot observe capital incomes of the old. The self-selection constraint prohibits such an extreme income redistribution within the young generation in period 2. Instead, as equations (2) and (3) show, the more productive individual 1 receives a higher disposable income than the less productive individual 2. With respect to redistribution between generations, equations (2) and (3) imply $u'_{y}(c'_{y_2}) < u'_{y}(c'_{y_2}) < u'_{y}(c^2_{y_2})$. The marginal utility of the members of the old generation is in between the marginal utilities of the members of the young generation.

The conditions for equilibrium labor supply are given by (4) and (5). These are standard conditions for optimal labor supply in an optimum income tax framework (see Stiglitz, 1982). Labor supply of the high-productivity individual is not distorted, whereas a positive tax is raised at the margin on labor supply of the low-productivity individual. Together with (2) and (3) the self-selection constraint (7) implies $l_2^1 > a l_2^2$. The high-productivity individual receives a higher labor income in period 2.

The following equation denotes the first order condition for optimum individual savings:

$$u_{y}'(c_{y_{1}}^{i}) = \beta u_{o}'(c_{o2}^{i}) F_{\kappa}(K_{2}, L_{2}).$$
(8)

For given expected retirement benefits, both individuals save to equalize the marginal rate of intertemporal substitution with the marginal productivity of capital. The rate of time preference is then equal to the interest rate. For $c_{02}^1 = c_{02}^2$ equation (8) implies $c_{y_1}^1 = c_{y_1}^2$. With their savings decision the young individuals transmit the effects of redistribution in the subsequent period to the current period. Equal consumption levels of all members of the old generation in period 2 then imply equal consumption levels of all members of the young generation in period 1. The distribution of consumption within the young generation is predetermined for the government in period 1.

Equations (1)–(8) determine c'_{y_1} , c'_{y_2} , c'_{o_2} and l'_2 as functions of l'_1 , τ'_1 and s''_1 .⁷ To interpret the properties of these terms equation (8) is rewritten as follows:

$$u'_{y} \left(e^{1}_{y_{1}} - \tau^{1}_{1} - s^{1}_{1} \right) = u'_{o} \left(c^{1}_{o_{2}} \right) \beta F_{K} \left(K_{2}, L_{2} \right),$$

$$u'_{y} \left(e^{2}_{y_{1}} - \tau^{2}_{1} - s^{2}_{1} \right) = u'_{o} \left(c^{1}_{o_{2}} \right) \beta F_{K} \left(K_{2}, L_{2} \right),$$
(9.a)
$$(9.b)$$

with
$$c_{o_2}^1 = c_{o_2}^1(K_2)$$
, $L_2 = L_2(K_2)$ and $K_2 = s_1^1 + s_1^2 + s_1^8$.

The terms $c_{o2}^{1}(K_{2})$ and $L_{2}(K_{2})$ are determined by (2)-(7). From (9.a) and (9.b) the following relationships can be derived:

$$\frac{dK_2}{d\tau_1'} = -\frac{dK_2}{ds_1^8},\tag{10}$$

 $^{^{7}}$ It is assumed that a unique interior solution for the system (1)-(8) exists and that the resulting endogenous variables are differentiable with respect to the policy variables of period 1.

$$\frac{dc_{y_1}^1}{d\tau_1'} = \frac{dc_{y_1}^2}{d\tau_1'} = -\frac{dc_{y_1}^1}{ds_1^g} = -\frac{dc_{y_1}^2}{ds_1^g}.$$
(11)

Equations (10) and (11) show that the contributions of individual 1 have the same marginal effects on consumption and the capital stock as the contributions of individual 2. Suppose, for example, that the contributions of individual 1 are reduced and the contributions of individual 2 are increased by the same amount. Both individuals then adjust their voluntary savings to keep consumption during their working life unchanged. Aggregate savings are unaffected. The government in period 2 lowers the benefits for individual 2 and increases the benefits for individual 1 to keep consumption of both individuals during retirement constant.

In addition, according to (10) and (11), an increase of the contributions of the young has the same effects on consumption and the capital stock as a reduction of government savings. A mandatory funded pension system with public savings financed by contributions of the young then is completely neutralized by private savings and redistribution policy of the subsequent government. Without loss of generality s_i^g can be set equal to zero.

The government in period 1 maximizes $W_1 = \tilde{u}_{y_1}^1 + \tilde{u}_{y_1}^2 + \tilde{u}_{o_1}^1 + \tilde{u}_{o_1}^2$ subject to the budget constraint $b_1^1 + b_1^2 + s_1^g = \tau_1^1 + \tau_1^2 + s_0^g F_K(K_1, L_1)$, the self-selection constraint $u_y(c_{y_1}^1) + \beta u_o(c_{o_2}^1) + v(\bar{l} - l_1^1) \ge u_o(c_{y_1}^2) + \beta u_o(c_{o_2}^2) + v(\bar{l} - al_1^2)$ and subject to the solution of the second and the third stage of the game. Since $c_{y_1}^1 = c_{y_1}^2$ and $c_{o_2}^1 = c_{o_2}^2$, the self-selection constraint in period 1 becomes $l_1^1 \le al_1^2$. In an interior equilibrium, the following first order conditions have to be satisfied (see appendix):

$$u'_{o}(c^{1}_{o1}) = u'_{o}(c^{2}_{o1}), \qquad (12)$$

$$v'(l-l_1^1) + \sigma_1 = u'_o(c_{o_1}^1)F_{L_1}, \qquad (13)$$

$$v'(l-l_1^2) - a\sigma_1 = u'_o(c_{o1}^1)aF_{L_1}, \qquad (14)$$

$$l_1^1 = a l_1^2 \,. \tag{15}$$

According to (15) the representative individual with the high labor productivity receives the same labor income as the individual with the low labor productivity. Redistribution policy in period 2 and private savings imply that both individuals have the same consumption level in both periods. Individual 1 then has no incentive to earn a higher labor income than individual 2. The following section 3 shows that this outcome is Pareto-inefficient.

3. Pareto-Inefficiency of the Equilibrium

The following policy experiment shows the potential for a Pareto-improvement starting from the equilibrium: Assume that l_1^1 is increased by Δl_1^1 ($\Delta l_1^1 \rightarrow 0$) and l_1^2 is reduced by Δl_1^2 such that $\Delta L_1 = 0$ ($a\Delta l_1^2 = -\Delta l_1^1$). In addition, individual taxes and transfers are adjusted to keep the budget constraints and the self-selection constraints satisfied as equalities and to keep individual savings constant. The self-selection constraint of period 1 implies the following:⁸

$$-u_{\gamma}'(c_{\gamma_{1}}^{1})\Delta\tau_{1}^{1} + \beta u_{\sigma}'(c_{\sigma_{2}}^{1})\Delta b_{2}^{1} - \nu'(\bar{l} - l_{1}^{1})\Delta l_{1}^{1} = -u_{\gamma}'(c_{\gamma_{1}}^{2})\Delta\tau_{1}^{2} + \beta u_{\sigma}'(c_{\sigma_{2}}^{2})\Delta b_{2}^{2} - \nu'(\bar{l} - al_{1}^{2})\Delta l_{1}^{2}$$
(16)

The budget constraint of period 1 implies $\Delta \tau_1^2 = -\Delta \tau_1^1$, the budget constraint of period 2 gives $\Delta b_2^2 = -\Delta b_2^1$. The first order condition $u'_{\gamma}(c'_{\gamma_1}) = \beta F_{\kappa_2} u'_{\sigma}(c'_{\sigma_2})$ for individual savings implies:

$$\Delta b_{2}' = -\frac{u_{y}''(c_{y_{1}}')}{\beta F_{\kappa_{2}} u_{o}''(c_{o_{2}}')} \Delta \tau_{1}'.$$
(17)

Lifetime utility of both individuals is influenced by the policy experiment as follows:

$$\Delta \tilde{u}_{y_1}^{l} = -u_y' (c_{y_1}^{l}) \Delta \tau_1^{l} + \beta u_o' (c_{o_2}^{l}) \Delta b_2^{l} - v' (\bar{l} - l_1^{l}) \Delta l_1^{l}, \qquad (18)$$

$$\Delta \tilde{u}_{y_{1}}^{2} = -u_{y}' \Big(c_{y_{1}}^{2} \Big) \Delta \tau_{1}^{2} + \beta u_{o}' \Big(c_{o2}^{2} \Big) \Delta b_{2}^{2} - v' \Big(\bar{l} - l_{1}^{2} \Big) \Delta l_{1}^{2} \,. \tag{19}$$

⁸ The policy experiment has no implications for the self-selection constraint of period 2.

This gives after inserting:

$$\Delta \tilde{u}_{y_1}^{1} = \nu' (\bar{l} - l_1^{1}) \frac{1-a}{2a} \Delta l_1^{1} > 0, \qquad (20)$$

$$\Delta \tilde{u}_{y_{1}}^{2} = \left[\nu' \left(\bar{l} - l_{1}^{2} \right) \frac{1}{a} - \nu' \left(\bar{l} - a l_{1}^{2} \right) \frac{1+a}{2a} \right] \Delta l_{1}^{1} > 0 \,.$$
⁽²¹⁾

The utility of both young individuals in period 1 increases. The equilibrium is not Pareto-efficient.

4. Conclusion

In this paper, the equilibrium allocation has been derived for an overlappinggenerations economy where the government decides about redistributive transfers and the private sector determines labor supply and savings. The results of this paper complement the results of KWP 904 where savings are determined by the government. The equilibrium in both papers is Pareto-inefficient. The inefficiencies result from the inability of the government to determine redistribution policy of the subsequent government. In a Pareto-efficient equilibrium all individuals must have the same rate of time preference. In addition, individuals with a high labor productivity must receive a higher labor income than individuals with a low labor productivity (see Stiglitz, 1985, 1987).⁹ With an equal distribution within the old generation, at least one of these conditions is not satisfied. Either the rates of time preference differ as in KWP 904 or all individuals receive the same labor income as in this paper.¹⁰

⁹ The second condition must hold as long as the self-selection constraint for the high-productivity individuals is binding.

¹⁰ With savings controlled by the government, an additional inefficiency may result from inefficiently low aggregate savings (see Lang, 1996, Hori, 1997, and Lorz, 1999).

Appendix: Solution of the First Stage of the Game

With $c_{o_2}^1 = c_{o_2}^2$ and $c_{y_1}^1 = c_{y_1}^2$ the government in period 1 maximizes the following Lagrange-function ($s_1^s = 0$):

$$L = 2u_{y}(c_{y_{1}}^{1}) + 2\beta u_{o}(c_{o_{2}}^{1}) + v(l - l_{1}^{1}) + v(l - l_{1}^{2}) + u_{o}(c_{o_{1}}^{1}) + u_{o}(c_{o_{1}}^{1}) + \lambda_{1}[\tau_{1}^{1} + \tau_{1}^{2} + s_{0}^{g}F_{K}(K_{1}, L_{1}) - b_{1}^{1} - b_{1}^{2}] - \sigma_{1}[l_{1}^{1} - al_{1}^{2}]$$

with $c_{y_{1}}^{1} = c_{y_{1}}^{1}(\tau_{1}^{\prime}, l_{1}^{\prime}, l_{1}^{2}), c_{o_{2}}^{1} = c_{o_{2}}^{1}(\tau_{1}^{\prime}, l_{1}^{1}, l_{1}^{2})$ and $c_{o_{1}}^{\prime} = s_{0}^{\prime}F_{K}(K_{1}, L_{1}) + b_{1}^{\prime}$
(A.1)

This gives the following first order conditions:

$$u'_o(c'_{o1}) = \lambda_1, \tag{A.2}$$

$$2u'_{y}(c_{y_{1}}^{i})\frac{dc_{y_{1}}^{i}}{d\tau_{1}^{i}}+2\beta u'_{o}(c_{o_{2}}^{i})\frac{dc_{o_{2}}^{i}}{d\tau_{1}^{i}}=-\lambda_{1}, \qquad (A.3)$$

$$2u'_{y}\left(c_{y_{1}}^{l}\right)\frac{dc_{y_{1}}^{l}}{dl_{1}^{l}}+2\beta u'_{o}\left(c_{o2}^{l}\right)\frac{dc_{o2}^{l}}{dl_{1}^{l}}=v'\left(l-l_{1}^{l}\right)+\sigma_{1}-\lambda_{1}F_{\kappa l}K_{1}, \qquad (A.4)$$

$$2u'_{y}\left(c_{y_{1}}^{1}\right)\frac{dc_{y_{1}}^{1}}{dl_{1}^{2}}+2\beta u'_{o}\left(c_{o2}^{1}\right)\frac{dc_{o2}^{1}}{dl_{1}^{2}}=v'\left(l-l_{1}^{2}\right)-a\sigma_{1}-\lambda_{1}aF_{\kappa l_{1}}K_{1}.$$
 (A.5)

From (9.a) and (9.b) the following equations can be derived:

$$\frac{dc_{y_1}^{i}}{dl_1^{i}} = -\frac{dc_{y_1}^{i}}{d\tau_1^{i}} \left[\frac{de_{y_1}^{i}}{dl_1^{i}} + \frac{de_{y_1}^{2}}{dl_1^{i}} \right],$$
(A.6)
$$\frac{dc_{o2}^{i}}{dl_1^{i}} = -\frac{dc_{o2}^{i}}{d\tau_1^{i}} \left[\frac{de_{y_1}^{i}}{dl_1^{i}} + \frac{de_{y_1}^{2}}{dl_1^{i}} \right].$$
(A.6)

The definitions $e_{y_1}^1 = F_{L_1} l_1^1$ and $e_{y_1}^2 = a F_{L_1} l_1^2$ imply:

$$\frac{de_{y_1}^1}{dl_1^1} + \frac{de_{y_1}^2}{dl_1^1} = F_{L_1} + L_1 F_{L_1}, \tag{A.8}$$

$$\frac{de_{y_1}^1}{dl_1^2} + \frac{de_{y_1}^2}{dl_1^2} = a \Big[F_{L_1} + L_1 F_{LL_1} \Big].$$
(A.9)

Inserting (A.6)-(A.9) into (A.4) and (A.5) gives:

$$\lambda_{1}[F_{L1} + L_{1}F_{LL1}] = \nu'(l - l_{1}^{1}) + \sigma_{1} - \lambda_{1}F_{KL1}K_{1}, \qquad (A.10)$$
$$a\lambda_{1}[F_{L1} + L_{1}F_{LL1}] = \nu'(l - l_{1}^{2}) - a\sigma_{1} - \lambda_{1}aF_{KL1}K_{1}. \quad (A.11)$$

With $K_1 F_{KL_1} + L_1 F_{LL_1} = 0$ this gives:

$$v'(l-l_1^{l}) + \sigma_1 = \lambda_1 F_{L_1}, \tag{A.12}$$

$$\nu'(l-l_1^2) - a\sigma_1 = a\lambda_1 F_{L_1}.$$
 (A.13)

Equations (A.12) and (A.13) imply that $\sigma_1 > 0$. The self-selection constraint then has to be satisfied as an equality $l_1 = al_2$.

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