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Pareto Efficiency and the Role of  
Border Tax Adjustments

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# Coordinating Climate and Trade Policies: Pareto Efficiency and the Role of Border Tax Adjustments

## Abstract

This paper explores the role of trade instruments in globally efficient climate policies, focusing on the central issue of whether border tax adjustment (BTA) is warranted when carbon prices differ internationally. It shows that tariff policy has a role in easing cross-country distributional concerns that can make non-uniform carbon pricing efficient, and that Pareto-efficiency requires a form of BTA when carbon taxes in some countries are constrained, a special case being identified in which this has the simple structure envisaged in practical policy discussion. It also stresses - a point that has been overlooked in the policy debate - that the case for BTA depends critically on whether climate policies are pursued by carbon taxation or by cap-and-trade.

JEL-Code: H200, F180.

Keywords: environmental taxation, cap-and-trade, international trade, Pareto efficiency, border tax adjustments.

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# I. Introduction

A key concern in countries contemplating reasonably aggressive emissions reduction—which has become still more prominent since the crisis, as they struggle to restore employment—is the fear that their competitive position in world markets jeopardized, by ‘carbon leakage’ as production shifts elsewhere.<sup>1</sup> The likelihood that any mitigation measures will be strongly asymmetric, at least for coming years, amplifies this concern, which is reflected in the inclusion in climate change legislation in both EU<sup>2</sup> and in proposals elsewhere (such as the Waxman-Markey bill in the U.S.) of provisions, for exposed emissions-intensive sectors, for various forms of ‘border tax adjustment’ (BTA)<sup>3</sup>—meaning the levying of some charge on imports, and remission of charge on exports, to the extent that carbon prices are higher domestically than elsewhere. Unsurprisingly, the appropriateness or not of such adjustments has been the focus of heated debate.

The theoretical literature has begun to address the linkages between climate (environment, more generally) and trade policies that are the heart of this question. Much of it has focused on non-cooperative policy formation, commonly characterizing nationally optimal policy, or desirable directions of reform—whether for small or large economies—when one or other instrument, environment or trade, is for some reason constrained away from the optimal: see, among others, Markusen (1975), Baumol and Oates (1988), Copeland (1994), Hoel (1996) and Turunen-Red and Woodland (2004). This is clearly an important perspective, capturing the element of the policy concern that relates directly to national self-interest. But an understanding of the requirements of cooperative policy is also needed: much of the reason why the EU and other advanced economies consider undertaking aggressive mitigation policies, for instance, has less to do with the harm they might themselves suffer from unmitigated climate change than with their concern with (and some historic guilt) for the harm that might be suffered by others. Not least because of the importance of the rhetoric of cooperation in relation to climate policies, the implications of cooperative design provide a central benchmark for policy evaluation.

This collective perspective has received far less attention (an exception being the partial equilibrium treatment in Gros (2009)). The aim in this paper is therefore to explore the interaction between climate and trade policies in that context. It provides a general

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<sup>1</sup>For insightful discussion on these issues see Copeland and Taylor (2004) and Sheldon (2006). Levinson and Taylor (2008) provide empirical evidence that more stringent environmental regulation reduces exports.

<sup>2</sup>Adjustments of this kind, in the context of the EU Emission Trading System, are provided for in Directive 2009/29/EC amending Directive 2003/87/EC. The failed Australian cap-and-trade proposals, in similar spirit, also included provision for allocation of free allowances to exposed sectors.

<sup>3</sup>Also advocated by, for instance, Stiglitz (2006).

treatment, within the standard general equilibrium model of competitive trade, of Pareto-efficient design of climate and tariff policies, unifying and extending results in this area.

The central question of interest here is whether there are circumstances in which some form of BTA is part of a globally efficient response to climate change (or to any other environmental problem with broadly the same border-crossing structure). By ‘border tax adjustment’ we mean, in the broadest sense, differential taxation of tradable commodities that is driven by differences in underlying national carbon prices. Of particular interest is the possibility that this adjustment will take the very simple form commonly envisaged in policy discussions—which is likely the only one conceivably practicable—of setting a charge on imports equal to some notion of carbon tax ‘not paid’ abroad on imports, and remitting tax on exports in similar fashion.

There are of course many other issues raised by the possibility of BTA for carbon prices. These include the questions of whether or not such adjustment is WTO-consistent (see, for instance, Chapter 5 of OECD (2004)),<sup>4</sup> very significant issues of implementability (Moore, 2010); and, not least, the (perhaps limited) empirical significance of the relative producer effects of carbon pricing that might be adjusted for (Houser *et al.* 2008). Nor does the analysis here consider the potential merit of BTA as a credible device by which countries implementing carbon pricing can encourage participation by others.<sup>5</sup> Important though they are, these are not the concern here—which is with the pure efficiency case for climate-motivated border tax adjustment.

The plan of the paper is as follows. Section II sets out the model, which takes carbon taxation to be the instrument of climate policy, and Section III then derives benchmark results for collectively efficient carbon tax and tariff policy when all instruments can be freely set. Section IV establishes the case for some form of border tax adjustment when instruments are constrained in some countries, which Section V then explores in more detail. Section VI considers the case in which at least some countries use not carbon taxation but a cap-and-trade scheme. Section VII concludes.

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<sup>4</sup>There are precedents, notable in the US Superfund tax and, of particular relevance in the climate context, for ozone-depleting chemicals.

<sup>5</sup>Participants themselves presumably gain from the BTAs, and non-participants would then benefit by imposing a carbon price themselves, at least to the extent that since by doing so they would capture revenues otherwise accruing to others (though terms of trade effects would also play a role).

## II. Modeling climate and trade policies

The framework is that of Keen and Wildasin (2004), modified to deal with pollution as a by-product of production. We consider a perfectly competitive general equilibrium model of international trade in which there are  $J$  countries indexed by the superscript  $j$ . In each country there is a representative consumer and a private sector that produces (only)  $N$  tradeable commodities. The  $N$ -vector of international commodity prices is denoted by  $\mathbf{w}$ .<sup>6</sup> (All vectors are column vectors, and a prime indicates transposition). Trade is subject to trade taxes or subsidies, the vector of which is denoted in country  $j$  by  $\boldsymbol{\tau}^j$  consistent with most-favored nation rules, each country is assumed to apply the same tariff rates to all others.<sup>7</sup> The commodity price vector in country  $j$  is then given by the  $N$ -vector  $\mathbf{p}^j = \mathbf{w} + \boldsymbol{\tau}^j$ . Consumption taxes are readily shown to be optimally zero in the present setting, so they are simply excluded.

The production of each commodity generates some pollutant—we have in mind carbon emissions, though there are of course many other possible interpretations<sup>8</sup>—with the  $N$ -vector  $\mathbf{z}^j$  denoting emissions in country  $j$ . Total emissions in country  $j$  are thus given by  $\boldsymbol{\iota}'\mathbf{z}^j$  where  $\boldsymbol{\iota}$  is the  $N$ -vector of 1s; and global emissions, on which—as with the concentration of greenhouse gases in the global atmosphere—damage in each country depends, are

$$k = \sum_{l=1}^J \boldsymbol{\iota}'\mathbf{z}^l. \quad (1)$$

This damage is assumed to arise (only) directly in consumer welfare, not through production; though perhaps not the most realistic assumption in the climate context, this helps relates our results to most familiar approaches in the literature.

The representative consumer of country  $j$  has preferences represented by the expenditure function given by

$$e^j(\mathbf{p}^j, u^j, k) = \min_{\mathbf{x}^j} \{ \mathbf{p}^j' \mathbf{x}^j : U^j(\mathbf{x}^j, k) \geq u^j \}, \quad (2)$$

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<sup>6</sup>Though world prices are something of a fiction, in the sense that no private agent may trade at them, they do matter for the revenues that national governments collect.

<sup>7</sup>As usual, the model is very general in allowing for all types of trade taxes and subsidies. If  $\tau_i^j > 0$  ( $\tau_i^j < 0$ ) and commodity  $i$  is being imported by country  $j$ , then  $\tau_i^j$  is an import tariff (import subsidy); if  $i$  is exported by country  $j$  then  $\tau_i^j$  is an export subsidy (export tax).

<sup>8</sup>And generalizations too. The analysis and main results are readily generalized to allow for  $M$ -types of pollutants. Much the analysis would apply to other pollutants, such as CFCs, whose emissions disperse uniformly in the atmosphere. An alternative specification, capturing varying degrees (and their effects on country  $j$ ), would be to specify emission discharges in country  $j$  as  $k^j = \sum_{l=1}^J \mathbf{b}^j' \mathbf{z}^l$  with  $\mathbf{b}^j = (b^{j1}, \dots, b^{jJ})'$ , where  $b^{ji} \in [0, 1]$  indicates the extent to which the emission discharges of country  $i$  spillovers to country  $j$ . This though does not add significant insights to what follows.

with  $\mathbf{e}_p^j$  the vector of compensated demands and  $e_k^j > 0$ , assumed strictly positive in all countries, the compensation required for a marginal increase in global emissions.

Emissions  $\mathbf{z}^j$  are subject to pollution taxes, given by the  $N$ -vector  $\mathbf{s}^j$ ; these note, are in general sector-specific.<sup>9</sup> The production sector in country  $j$  is competitive and characterized by a revenue function

$$r^j(\mathbf{p}^j, \mathbf{s}^j, \mathbf{v}^j) = \max_{\mathbf{y}^j, \mathbf{z}^j} \{ \mathbf{p}^{j'} \mathbf{y}^j - \mathbf{s}^{j'} \mathbf{z}^j : (\mathbf{y}^j, \mathbf{z}^j) \in T^j(\mathbf{v}^j) \}, \quad (3)$$

where  $T^j(\mathbf{v}^j)$  is the technology set,  $\mathbf{v}^j$  being the vector of factor endowments, and  $\mathbf{y}^j$  is the (net) output of tradeable goods. The revenue function in (3) is convex, linearly homogeneous function of prices, and assumed to be twice continuously differentiable.<sup>10</sup> (The fossil fuels from whose use carbon emissions arise are not explicitly identified, though they can be thought of as being amongst the  $N$  commodities, since our interest here is not in their pricing). Hotelling's lemma implies that  $\mathbf{r}_p^j$  is the vector of net supply functions for tradeable commodities; it also follows from (3) that  $\mathbf{r}_s^j(\mathbf{p}^j, \mathbf{s}^j) = -\mathbf{z}^j$ : emissions are given by (minus) the derivative of the revenue function with respect to the sectoral carbon tax rates.

Tax revenues from all sources are assumed to be returned to the consumers in a lump sum fashion. At some points, unrequited commodity transfers between countries will be allowed; denoting by the  $N$ -vector  $\boldsymbol{\alpha}^j$  that received by  $j$ , these must satisfy

$$\sum_{j=1}^J \boldsymbol{\alpha}^j = \mathbf{0}_{N \times 1}. \quad (4)$$

The consumer budget constraint in country  $j$  is, therefore,

$$e^j(u^j, \mathbf{p}^j, k^j) = r^j(\mathbf{p}^j, \mathbf{s}^j) - \mathbf{s}^{j'} \mathbf{r}_s^j(\mathbf{p}^j, \mathbf{s}^j) + \boldsymbol{\tau}^{j'} (\mathbf{e}_p^j(u^j, \mathbf{p}^j, k) - \mathbf{r}_p^j(\mathbf{p}^j, \mathbf{s}^j)) + \mathbf{w}' \boldsymbol{\alpha}^j. \quad (5)$$

This simply says that expenditure  $e^j(u^j, \mathbf{p}^j, k^j)$  must equal GDP, given by  $r^j(\mathbf{p}^j, \mathbf{s}^j)$ , plus any pollution tax revenues,  $\mathbf{s}^{j'} \mathbf{z}^j$ , tariff revenue, given by  $\boldsymbol{\tau}^{j'} (\mathbf{e}_p^j(u^j, \mathbf{p}^j, k) - \mathbf{r}_p^j(\mathbf{p}^j, \mathbf{s}^j))$ , and transfers received by country  $j$ , the value of which is  $\mathbf{w}' \boldsymbol{\alpha}^j$ .

Market clearing requires that

$$\sum_{j=1}^J \{ \mathbf{e}_p^j(u^j, \mathbf{p}^j, k) - \mathbf{r}_p^j(\mathbf{p}^j, \mathbf{s}^j) \} = \mathbf{0}_{(N-1) \times 1}, \quad (6)$$

where, by Walras' Law, the market-clearing equation for the first commodity is dropped.

<sup>9</sup>As in, among others, Copeland (1994), Hoel (1996) and Turunen-Red and Woodland (2004).

<sup>10</sup>For the properties of the revenue function see Dixit and Normal (1980) and Woodland (1982).

The same commodity is taken as numeraire, and without loss of generality, to be untaxed in all countries: so  $\tau_1^j = 0$  and  $p_1^j = 1$ , for  $j = 1, \dots, J$ .

Given tariffs  $\boldsymbol{\tau}^j$  and carbon tax vectors  $\mathbf{s}^j$ , for  $j = 1, \dots, J$ , a vector of international transfers satisfying (4), the market equilibrium conditions (6), and the national budget constraints (5), the system may be solved for the equilibrium world price vector  $\mathbf{w}$  and the vector of national utilities  $\mathbf{u} = (u^1, \dots, u^J)'$ .<sup>11</sup>

The analysis that follows uses Tucker's Theorem of the Alternative to characterize Pareto-efficient environmental and tariffs structures. The necessary conditions for this are derived in Appendix A. They involve variables  $\sigma^j$  that can be interpreted as the (negative) of the implicit social marginal value—evaluated at the Pareto-efficient allocation being characterized—of the utility of country  $j$ .<sup>12</sup> If country  $i$ , say, is more 'income-needy' than country  $j$ , then  $\sigma^i < \sigma^j$ .

### III. Unconstrained carbon tax and tariff policies

To fix ideas, this section considers the relatively straightforward case in which there are no constraints on the carbon taxes and tariffs that can be set in any country. Then:<sup>13</sup>

**Proposition 1** *At any Pareto-efficient allocation, in every country  $j$ :*

(a) *The vector of carbon taxes in country  $j$  is given by*

$$\sigma^j \mathbf{s}^j = \sum_{i=1}^J \sigma^i e_k^i \boldsymbol{\nu}' \gg \mathbf{0}'_{1 \times N}, \quad (7)$$

*so that for any countries  $j$  and  $i$ ,  $\mathbf{s}^j = \theta^{ij} \mathbf{s}^i \gg \mathbf{0}'_{1 \times N}$ , where  $\theta^{ij} = \sigma^i / \sigma^j$ .*

(b) *The tariff vectors of any pair of countries  $j$  and  $i$  are collinear:*

$$\boldsymbol{\tau}^j = \theta^{ij} \boldsymbol{\tau}^i. \quad (8)$$

**Proof:** See Appendix B.

The interpretation of part (a) is straightforward. Pareto efficiency requires that each country set its carbon tax in each sector  $n$  to equate the value of the income loss that a

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<sup>11</sup>Differentiability of all functions at the initial equilibrium is assumed. Standard assumptions hold so an equilibrium exists.

<sup>12</sup>If policy were evaluated by an explicit social welfare function  $\omega(V^1, \dots, V^J)$ ,  $V^l$  being indirect utility in country  $l$ ,  $\sigma^l$  would correspond to  $-(\partial\omega(V^1, \dots, V^J)/\partial V^l) (e_u^l / (1 - \boldsymbol{\tau}^l \boldsymbol{\eta}^l))$ , where  $\partial\omega(V^1, \dots, V^J)/\partial V^l > 0$ , and  $-e_u^l / (1 - \boldsymbol{\tau}^l \boldsymbol{\eta}^l) < 0$  with  $\boldsymbol{\eta}^l = \mathbf{e}_{\mathbf{p}u}^l / e_u^l$ ,  $e_u^l > 0$ .  $-e_u^l / (1 - \boldsymbol{\tau}^l \boldsymbol{\eta}^l) < 0$  relates to the Hatta normality condition: see Turunen-Red and Woodland (1988).

<sup>13</sup>The notation  $\mathbf{q} \gg \mathbf{0}$  means that all elements of the vector  $\mathbf{q}$  are strictly positive.



small increase in its carbon tax causes itself,  $\sigma^j s_n^j$ , to the sum of the marginal benefits conveyed to all countries by a marginal cut in emissions,  $\sum_{i=1}^J \sigma^i e_k^i$ . An immediate implication, since the marginal damage from emissions is the same whichever sector they originate in, is that each country should apply the same carbon tax to all activities: within each country, carbon taxes are optimally uniform across sectors. But while each country sets a single carbon tax rate, part (a) also shows that the level of that tax generally differs across countries. Recalling the interpretation of  $\sigma^j$ , Pareto efficiency requires that more ‘income-needy’ countries impose lower carbon taxes.<sup>14</sup> This is intuitively natural, and to the same effect consistent with the results of Chichilnisky and Heal (1994) and Sandmo (2005, 2006)—and indeed with much of the policy debate, which has emphasized the lesser ability of lower income countries to cope with aggressive carbon pricing. There is though one subtle difference between this and previous results: here the simple equity-based modification of the Pigovian rule applies even though distorting taxes—tariffs—may also be deployed.

This brings us to part (b) of Proposition 1, which is more striking. To see why Pareto efficiency requires collinear tariff vectors, consider some change in world prices that increases country  $j$ ’s import of good  $n$  by one unit, and increases  $i$ ’s exports by one unit. With carbon taxes optimally set, this increases the shadow value of  $j$ ’s real income by  $\sigma^j \tau_n^j$  and reduces that of  $i$  by  $\sigma^i \tau_n^i$ ; Pareto efficiency then requires that  $\tau_n^j = (\sigma^i / \sigma^j) \tau_n^i$ ; and this can hold for all  $n$  only if the tariff vectors are collinear. The importance of this is in emphasizing that production inefficiency is generally part of a Pareto efficient allocation. To see this, recall that producer prices in country  $j$  are  $\mathbf{p}^j = \mathbf{w} + \boldsymbol{\tau}^j$ ; this means that global production will be efficient—in the narrow sense that it is impossible to increase global output of any good without either reducing the global output of some other or increasing aggregate emissions—only if the tariff vectors  $\boldsymbol{\tau}^j$  are the same for all countries. But there is no reason to suppose that  $\boldsymbol{\tau}^j = \mathbf{0}_{(N-1) \times 1}$  for all countries, nor that  $\theta^{ij} = \sigma^i / \sigma^j$  takes the same value for all  $j$ .

There is generally production inefficiency in allocations characterized by Proposition 1 in a broader sense too, reflecting also environmental concerns. Maximizing the net output of some good without either reducing the net output of any other or increasing global emissions requires that both producer prices  $\mathbf{p}$  and carbon taxes  $\mathbf{s}$  be equalized across countries. Proposition 1 points to violations on both of these margins (or neither), driven by distributional concerns: in each case, relative welfare weights shape the proportionality factor between the (sectorally uniform) carbon taxes and tariffs applied by each country.

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<sup>14</sup>Notice that part (a) of Proposition 1 relates to the analysis in Chichilnisky and Heal (1994), but the analysis here is casted in terms of explicit fiscal instruments.

Proposition 1 applies whether or not international transfers between countries can be deployed. If they can be then, of course, Pareto-efficiency requires equalizing the  $\sigma^j$  across countries. Part (a) of Proposition 1 then implies that Pareto efficiency requires the same level of carbon taxes in every country, and part (b) that  $\tau^j = \tau^i$ . All Pareto-efficient allocations are thus characterized by production efficiency. The same may be true, however, even without international transfers. The reason is as in Keen and Wildasin (2004), and Turunen-Red and Woodland (2004): if there are more goods on which the tariff rates may be varied than there are countries (and sufficient rank in the corresponding matrix of net exports), offsetting tariffs can be designed so as to achieve any desired reallocation of tariff revenue between countries. Explicit transfers are then redundant. Hence:

**Proposition 2** *If there are no constraints on lump transfers between countries, or there are at least as many goods as countries (and an appropriate rank condition is satisfied), then, at any Pareto-efficient allocation, for all countries  $i, j$ :*

(a) *Carbon taxes for any countries  $j$  and  $i$  satisfy*

$$\mathbf{s}^{j'} = \sum_{i=1}^J e_k^i \boldsymbol{\nu}^i = \mathbf{s}^{i'} \gg \mathbf{0}'_{1 \times N}, \quad (9)$$

and

(b) *tariff vectors of any pair of countries  $j$  and  $i$  satisfy*

$$\boldsymbol{\tau}^j = \boldsymbol{\tau}^i. \quad (10)$$

**Proof:** See Appendix C.

Carbon taxes are thus set at first best Pigovian levels, and tariff policy has no substantive role (but could be normalized away, for instance, by inclusion in the common vector of world prices  $\mathbf{w}$ ).

In the relatively unconstrained world of Propositions 1 and 2, the alignment of climate and trade policies is, thus, fairly straightforward. Importantly for present purposes, while tariff policy is generally not redundant there is nothing in Propositions 1 and 2 that is in the nature of a border tax adjustment. A case for BTA can thus arise only in more constrained circumstances, and it is this possibility that we now turn.

## IV. Pareto efficiency and the role of border tax adjustments

Imagine then that for some reason—perhaps unmodeled political constraints—not all carbon taxes and tariffs are freely variable. Specifically, suppose—going to something of the opposite extreme to the circumstances of the previous section—that they can be freely set in country  $h$  but everywhere else are fixed at arbitrary levels. We refer to these countries as ‘unconstrained’ and ‘constrained’ respectively (and occasionally to  $h$  as ‘home’), and will have in mind in the informal discussion that carbon taxes in the latter—which may be sector-specific—are ‘too low’ (relative to the first-best Pigovian carbon-tax). The global economy is thus constrained<sup>15</sup> inside the global utility possibility frontier, and the question is: How should carbon taxes and tariffs then be set in country  $h$ ?

The following result establishes the two key features of any constrained Pareto-efficient allocation in these circumstances (now reverting to a world in which it may not be possible to fully address all equity concerns):

**Proposition 3** *Suppose that carbon taxes and tariffs are fixed at arbitrary levels in all countries except  $h$ . Then constrained Pareto efficiency requires that:*

(a) *Carbon taxes are given by*

$$\sigma^h \mathbf{s}^{h'} = \left\{ \sum_{j=1}^J \sigma^j e_k^j + \sum_{j \neq h}^J (\sigma^h \boldsymbol{\tau}^h - \sigma^j \boldsymbol{\tau}^j)' \mathbf{e}_{\mathbf{p}^k}^j \right\} \boldsymbol{\nu}', \quad (11)$$

and

(b) *tariffs be set so that*

$$\begin{aligned} \sigma^h \boldsymbol{\tau}^{h'} = & \left\{ - \sum_{j=1}^J \sigma^j \mathbf{m}^{j'} + \sum_{j \neq h}^J (\sigma^h \mathbf{s}^h - \sigma^j \mathbf{s}^j)' \mathbf{r}_{\mathbf{sp}}^j \right. \\ & \left. + \sum_{j \neq h}^J (\sigma^h \boldsymbol{\tau}^h - \sigma^j \boldsymbol{\tau}^j)' \mathbf{e}_{\mathbf{p}^k}^j \sum_{l \neq h}^J (-\boldsymbol{\nu}' \mathbf{r}_{\mathbf{sp}}^l) + \sum_{j \neq h}^J \sigma^j \boldsymbol{\tau}^{j'} \hat{\boldsymbol{\pi}}_{\mathbf{pp}}^j \right\} \hat{\boldsymbol{\pi}}_{\mathbf{pp}}^{-1}, \end{aligned} \quad (12)$$

where  $\hat{\boldsymbol{\pi}}_{\mathbf{pp}}^j \equiv \mathbf{e}_{\mathbf{pp}}^j - \mathbf{r}_{\mathbf{pp}}^j + \mathbf{e}_{\mathbf{p}^k}^j \sum_{l \neq h}^J (-\boldsymbol{\nu}' \mathbf{r}_{\mathbf{sp}}^l)$  and  $\hat{\boldsymbol{\pi}}_{\mathbf{pp}}^{-1} \equiv \left( \sum_{j \neq h}^J \hat{\boldsymbol{\pi}}_{\mathbf{pp}}^j \right)^{-1}$ .

**Proof:** See Appendix D.

Part (a) of Proposition 3 shows that that the unconstrained carbon tax in country  $h$  is not set equal to the welfare-weighted marginal damage from emissions. This is because the

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<sup>15</sup>Leaving aside the case in which the arbitrary rates in all unconstrained countries happen to coincide with those of some Pareto efficient allocation.

carbon tax set in  $h$ , by affecting emissions and hence demand structures in the constrained countries, impacts distortions associated with the tariffs set there. To the extent, for instance, that the fall in emissions implied by increasing the carbon tax in  $h$  increases demand in some constrained country  $j$  of goods that tariff distortions imply are under-imported there (the tariff imposed by country  $j$  being greater than any export subsidy imposed by country  $h$ ), so that  $(\sigma^h \boldsymbol{\tau}^h - \sigma^j \boldsymbol{\tau}^j)' \mathbf{e}_{pk}^j \boldsymbol{\iota}' < \mathbf{0}'_{1 \times N}$ , this calls for  $\mathbf{s}^h$  to be set higher than would otherwise be the case. In this way, the unconstrained carbon tax is used to reduce the distortions associated with imperfections of collective tariff policies. If, for example, a warming in climate leads in country  $j$  to reduced demand for heating equipment that is subject to a large import tariff, this becomes an argument for a higher carbon tax in country  $h$ .

One other aspect of part (a) bears emphasis: since  $\mathbf{s}^h$  is collinear with  $\boldsymbol{\iota}$ , the carbon tax in the unconstrained country  $h$  should be uniform across sectors, whether or not it is uniform in the constrained countries. The best way to respond, if need be, to sectoral differentiation abroad, is through the tariff structure. The proper task of the carbon tax is to address inefficiencies in the level of emissions.

Part (b), characterizing Pareto efficient tariff design in country  $j$ , is still more complex, with four effects at work;

i) A term  $\sum_{j \neq h}^J (\sigma^h \mathbf{s}^h - \sigma^j \mathbf{s}^j)' \mathbf{r}_{\mathbf{sp}}^j \hat{\boldsymbol{\pi}}_{\mathbf{pp}}^{-1}$  that reflects differences in carbon taxes between the unconstrained and all other countries, adjusted for equity considerations and reflecting too the extent to which production in the constrained countries is affected by the carbon taxes applied there. This then is a BTA in the broad sense defined in the introduction, with a pivotal role played by the responsiveness of net outputs to the local carbon taxes, as given by the matrix<sup>16</sup>  $\mathbf{r}_{\mathbf{sp}}^j$ . If  $\mathbf{r}_{\mathbf{sp}}^j = \mathbf{0}_{N \times (N-1)}$ , so that carbon taxation has no impact on production in the constrained country, this BTA-type term vanishes.<sup>17</sup>

ii) An aggregate of the terms-of-trade-effects arising in each country from changes in  $h$ 's tariff policy, each weighted by equity concerns,  $\sum_{j=1}^J \sigma^j \mathbf{m}^j \hat{\boldsymbol{\pi}}_{\mathbf{pp}}^{-1}$ , giving the welfare-weighted sum of the changes in real income generated in each country by increasing tariffs in  $h$ . This vanishes if international transfers (explicit or implicit) are optimally deployed, as discussed above.

iii) A term  $\sum_{j \neq h}^J (\sigma^h \boldsymbol{\tau}^h - \sigma^j \boldsymbol{\tau}^j)' \mathbf{e}_{pk}^j \sum_{l \neq h}^J (-\boldsymbol{\iota}' \mathbf{r}_{\mathbf{sp}}^l) \hat{\boldsymbol{\pi}}_{\mathbf{pp}}^{-1}$ , similar to that in part (a) of the

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<sup>16</sup>This is Copeland's (1994) indicator of sectoral pollution intensity.

<sup>17</sup>This is closely related to the observation of Lockwood and Whalley (2010) that a case for BTA can arise only when differential carbon taxes affect relative producer prices: otherwise the exchange rate (or domestic price level) will accommodate such differences automatically.

Proposition, but now reflecting the impact on tariff distortions of the change in emissions induced by changing tariffs in country  $h$  rather than those from changing carbon taxes.

iv) The change in welfare-weighted tariff revenues  $\sum_{j \neq h}^J \sigma^j \tau^{j'} \hat{\pi}_{\text{pp}}^j \hat{\pi}_{\text{pp}}^{-1}$  (in all constrained countries) brought about by increasing tariffs in  $h$ .

The fairly general case of Proposition 3 is clearly also fairly complex and opaque. For further insight, the next section turns to various special cases.

## V. Border tax adjustment with inefficient carbon pricing: Further analysis

A first simplification is to suppose that there are only two countries, that international transfers can be freely deployed (so that  $\sigma^j$  is equated across countries), and that compensated demands for the non-numeraire commodities are independent of emissions,  $\mathbf{e}_{\text{pk}}^2 = \mathbf{0}_{(N-1) \times 1}$ . Then Proposition 3 gives:

**Corollary 1** *Suppose there are only two countries, 1 and 2, with carbon taxes and tariffs in country 2 fixed at arbitrary levels, that lump sum transfers between the two countries are unconstrained and that compensated demands for the non-numeraire commodities are independent of emissions. Then Pareto efficiency requires that  $\mathbf{s}^1$  and  $\boldsymbol{\tau}^1$  be set so that:*

$$(a) \quad \mathbf{s}^{1'} = (e_k^1 + e_k^2) \boldsymbol{\iota}', \quad (13)$$

and

$$(b) \quad \boldsymbol{\tau}^{1'} = \boldsymbol{\tau}^{2'} + (\mathbf{s}^1 - \mathbf{s}^2)' \mathbf{r}_{\text{sp}}^2 (\mathbf{e}_{\text{pp}}^2 - \mathbf{r}_{\text{pp}}^2)^{-1}. \quad (14)$$

Now the unconstrained Pigovian tax is set at its first-best Pigovian level, for reasons clear from the discussion of Proposition 3(a). And the unconstrained tariff—more precisely, the difference between the unconstrained and constrained tariff—differs from zero only to the extent that the carbon tax abroad is not set at its first best level. In this case, the sole purpose of tariff policy is thus to provide a border tax adjustment.

The nature of the BTA called for is though somewhat complex, reflecting the responsiveness of net import demand, and the impact of carbon pricing on emissions, in the constrained country. (This, incidentally, provides an answer to one question that has lingered in the literature: whether the border tax adjustment should reflect technology in the home or in the foreign country: Proposition 3 shows that constrained Pareto efficiency

requires that adjustment (both the tariff on imports and the refund on exports) be by the latter).

To see the intuition underlying the form of BTA called for in part(b) of Corollary 1, suppose for simplicity that (in addition to the assumptions of the corollary) all carbon taxes and tariffs are zero in the constrained country, 2. Recalling that optimality requires that any conceivable marginal change in policy have zero impact (given the availability of international transfers) on the sum of utilities, consider the particular policy of combining a change in world prices, and hence of producer prices in the constrained country, of  $\mathbf{dw} = \mathbf{dp}^2$ , with an offsetting change in the unconstrained tariff,  $\mathbf{d}\tau^1 = -\mathbf{dw}$ . It can then be shown, since producer prices (and the carbon tax) in the unconstrained country are unchanged, that the consequent change in global welfare is<sup>18</sup>

$$e_u^1 du^1 + e_u^2 du^2 = [(e_k^1 + e_k^2)\boldsymbol{\iota}'\mathbf{r}_{\mathbf{sp}}^2 - \boldsymbol{\tau}^{1'}(\mathbf{e}_{\mathbf{pp}}^2 - \mathbf{r}_{\mathbf{pp}}^2)] \mathbf{dp}^2. \quad (15)$$

Recalling that  $\mathbf{r}_{\mathbf{s}}^2 = -\mathbf{z}^2$ , the first effect on the right of (15) is  $-(e_k^1 + e_k^2)dk^2$ , where  $k^2 = \boldsymbol{\iota}'\mathbf{z}^2$  denotes aggregate emissions in country 2; this term thus captures the global social benefit of any reduction in country 2's emissions induced by the change in producer prices there. The second term is  $-\boldsymbol{\tau}^{1'}\mathbf{dm}^2 = -\boldsymbol{\tau}^{1'}(\mathbf{e}_{\mathbf{pp}}^2 - \mathbf{r}_{\mathbf{pp}}^2)$ , which in turn is equal in equilibrium to  $\boldsymbol{\tau}^{1'}\mathbf{dm}^1$ ; this effect, reflecting the impact of the reform on the distortion of trade implied by the initial tariff structure is thus harmful to the extent that it decreases 1's imports of goods that are subject to a positive tariff. Optimal policy implies balancing these two effects, so that

$$\boldsymbol{\tau}^{1'} = (e_k^1 + e_k^2)\boldsymbol{\iota}'\mathbf{r}_{\mathbf{sp}}^2 (\mathbf{e}_{\mathbf{pp}}^2 - \mathbf{r}_{\mathbf{pp}}^2)^{-1}, \quad (16)$$

which is precisely as the two parts of Corollary 1 imply in this case. The kind of policy this implies is a reduction in the producer price of 'dirty' goods in the constrained country, to discourage their production there, combined with—indeed induced by—a tariff that offsets the tendency for the unconstrained country to consequently import more of those dirty goods.

A more direct piece of intuition may also be helpful. Imagine that both countries initially set their carbon taxes at Pigovian levels (so that  $\mathbf{s}^1 = \mathbf{s}^2 \equiv (e_k^1 + e_k^2)\boldsymbol{\iota}$ ), and deploy no tariffs. Now suppose, however, the carbon tax is removed in country 2, and the tariff in country 1 changed in response as Corollary 1 requires. To a linear approximation, the

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<sup>18</sup>Since  $e_u^j > 0$ ,  $j = 1, 2$ , is the reciprocal of the marginal utility of income the left hand side of (15) represents the change in global utility in terms of the numeraire good.

change in country 1's imports (maintaining the simplifying assumptions above) is

$$\mathbf{d}\mathbf{m}^{1'} = \boldsymbol{\tau}^{1'} (\mathbf{e}_{\mathbf{pp}}^2 - \mathbf{r}_{\mathbf{pp}}^2) - (e_k^1 + e_k^2) \boldsymbol{\iota}' \mathbf{r}_{\mathbf{sp}}^2, \quad (17)$$

But (16) implies that  $\mathbf{d}\mathbf{m}^1 = \mathbf{0}_{(N-1) \times 1}$ . That is, optimal tariff policy in the unconstrained country undoes the trade impact of suboptimal carbon pricing in the other country, as to re-establish (approximately) the same pattern of net trade as prevails when both deploy first-best Pigovian taxes.

These results clearly in general imply quite complex structures for the optimal tariff rates themselves. In the special case in which there are only two tradeable goods, one of which (the numeraire) is 'clean', in the sense that its production generates no pollution the interpretation of optimal tariff policy is especially clear. In obvious notation, (16) becomes

$$\tau^1 = (e_k^1 + e_k^2) \frac{dk^2}{dp^2} \frac{dp^2}{dm^2}. \quad (18)$$

The optimal tariff can thus be thought of attaching a shadow price to imports in the unconstrained country that reflects their contribution to emissions abroad.

But even the simple form of the optimal tariff in (18) is quite different from the more mechanical calculation commonly considered in the policy-oriented literature (and, likely, the only type that could conceivably have sufficient verifiability for practical application). This typically envisages charging on imports (and refunding on exports) an amount equal to the shortfall of the carbon tax actually paid abroad, directly and indirectly, relative to that which would have been paid had the home country carbon tax applied. This is the amount

$$(s^{1*} \boldsymbol{\iota} - \mathbf{s}^2)' \boldsymbol{\phi}^2 (\mathbf{I}_N - \mathbf{A}^2)^{-1}, \quad (19)$$

where the  $N$ -vector  $\boldsymbol{\phi}^2$  gives emissions per unit of output, the typical element of the matrix  $\mathbf{A}^2$  is the use of good  $j$  per unit of gross output of  $n$ , and  $s^{1*} \equiv e_k^1 + e_k^2$  is the uniform Pigovian tax in the unconstrained country. To express the efficient border tax adjustment in these terms, it is shown in Appendix E, that if emissions per unit of output (described by the vector  $\boldsymbol{\phi}^2$ ) are constant in each sector and there are no substitution effects in demand between non-numeraire commodities, then part (b) of Corollary 1 gives

$$\boldsymbol{\tau}^{1'} = \boldsymbol{\tau}^{2'} - (s^{1*} \boldsymbol{\iota} - \mathbf{s}^2)' \boldsymbol{\phi}^2 \left\{ (\mathbf{r}_{\mathbf{p}}^{2'} \otimes \mathbf{I}_N) \frac{\partial \left( [\mathbf{I}_N - \mathbf{A}^2(\mathbf{p}^2)]^{-1} \right)}{\partial \mathbf{p}^2} (\mathbf{r}_{\mathbf{pp}}^2)^{-1} + [\mathbf{I}_N - \mathbf{A}^2(\mathbf{p}^2)]^{-1} \right\}. \quad (20)$$

And hence:

**Proposition 4** *In the circumstances of Corollary 1, suppose further that emissions per unit of output  $\phi^2$  are constant, there are no substitution effects in demand between non-numeraire commodities ( $\mathbf{e}_{\mathbf{pp}}^2 = \mathbf{0}_{(N-1) \times (N-1)}$ ), and that substitution between produced inputs is negligible (in the sense that the matrix  $\mathbf{A}^2$  is independent of prices). Then, with  $s^{1*} = e_k^1 + e_k^2$ , Pareto efficiency requires that:*

$$\boldsymbol{\tau}^{1'} = \boldsymbol{\tau}^{2'} - (s^{1*} \boldsymbol{\iota} - \mathbf{s}^2)' \phi^2 (\mathbf{I}_N - \mathbf{A}^2)^{-1} . \quad (21)$$

Here then is a case in which collectively efficient policy has a remarkably simple form. The unconstrained carbon tax should be set at the first-best Pigovian level, and border tax adjustment should take the form of a countervailing charge on imports (and refund on exports) corresponding mechanically to the tax ‘under-paid’ in the foreign country. One important difference from common proposals, however, is that, to the extent that technologies differ between the two countries, the rebate on exports will generally not equal the carbon tax paid at home.

The assumptions needed to arrive at Proposition 4 are, of course, extremely strong. It does suggest, nevertheless, that—conceptually at least—proposals for border adjusting carbon taxes commonly encountered are not wholly misplaced, even from the perspective of global rather than national welfare.

## VI. Border tax adjustment and cap-and-trade

The analysis so far has assumed that the climate instruments deployed, if any, are in the form of carbon taxes. An alternative, however, is cap-and trade: not levying a charge directly on emissions, but instead issuing a fixed number of tradable emission rights. This alternative is of considerable practical importance, perhaps even more so than carbon taxation: as noted at the outset, it is schemes of this kind that have been adopted by the EU and which have made most headway in the U.S. The question then is whether the conclusions above continue to apply when the instrument of climate policy is not tax, but national-level cap-and-trade.

The essence of the results in Section III—when instrument choice is unconstrained—clearly apply essentially unchanged. This is a simple consequence of the familiar equivalence, under perfect certainty (as assumed here) of carbon taxation and cap-and-trade,<sup>19</sup> and of the result above that sectoral differentiation of carbon taxation (which could not be replicated by permits tradable between sectors) cannot be part of a Pareto-efficient

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<sup>19</sup>There is large literature on the choice between taxation and cap-and-trade under uncertainty: see, for instance, Pizer (2002).



allocation: analogues of Propositions 1 and 2 thus hold with the characterizations of carbon taxes reinterpreted as characterizing emissions caps in terms of the associated shadow value of emissions. (Whether the pollution permits are auctioned or allocated free of charge, critical in practice, is immaterial here, given the lump sum return of any revenues raised.)

What though if, as in Sections IV and V, the instrument choice is constrained in some country? (For brevity, we here assume just two-countries, with lump-sum transfers between them available; we also omit proofs<sup>20</sup>).

Suppose first that carbon taxation is used in the constrained country. Matters are then straightforward, since any allocation that can be achieved when the unconstrained country uses carbon taxation—as above—can be replicated by instead fixing there an appropriate emissions level; and vice versa. So:

**Proposition 5** *Suppose carbon taxes are used in the constrained country. Then Propositions 3 and 4 and Corollary 1 continue to apply, appropriately reinterpreted, when the unconstrained country uses cap-and-trade.*

Things are very different, however, if the constrained country uses cap-and-trade:

**Proposition 6** *Suppose the constrained country uses cap-and-trade. Then:*

(a) *At any Pareto-efficient allocation at which that cap binds, no tariffs are levied by the unconstrained country—so there is no border tax adjustment;*

(b) *At any Pareto efficient allocation at which the cap does not bind, tariffs in the unconstrained country embody border tax adjustments of the kind characterized in Propositions 3 and 4 and Corollary 1 for the case in which the carbon tax in the constrained country/ies is zero.*

The intuition is straightforward. Policies adopted in the unconstrained country can have no impact on emissions in the constrained country so long as the emission cap in the latter is binding. So if it is Pareto efficient for that cap to bind, deploying tariffs in the unconstrained country can serve no useful purpose. So—following part (a) of the proposition—there is no case for any BTA. It could be, however, that in some efficient allocations the unconstrained country sets its tariff so as to drive emissions abroad below the cap. In that case, the situation in the constrained country is the same, at the margin, as if it set a carbon tax of zero; and so—following part (b) of the proposition—the earlier results for that case apply.

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<sup>20</sup>These are straightforward once the structure of Section II is reformulated in terms of emission levels rather than carbon taxes.

It is part (a) of Proposition 6 that is, however, most striking. The point it makes appears to have been largely unnoticed in the policy debate: while there may be a case in terms of the collective good for BTAs when carbon taxes abroad are constrained at inefficiently low levels, there is no such case if it is instead emission targets abroad that are set at an inefficiently low (but binding) level.

## VII. Concluding remarks

This paper has explored the interplay between climate- and trade-related instruments in forming globally efficient responses to climate concerns. One role that emerges for tariff policies is in easing the constraints stemming from cross-country distributional concerns that can make non-uniform carbon pricing efficient. The other potential role, on which most of the analysis has focused, is in mitigating distortions that arise from cross-country differences in carbon prices. The paper has identified circumstances in which global efficiency does indeed require some form of BTA (and others in which it does not), and has fully characterized the form of adjustment needed.

The first role emerges most clearly when there are no constraints on the rates at which carbon taxes (or emission levels under cap-and-trade schemes) and tariffs can be set. The implications of Pareto-efficiency are then straightforward: carbon prices should be uniform across sectors within countries (or permits tradable across them), but equity considerations may call for them to be lower in countries judged less needy. The only possible role for tariffs is as an indirect way to alleviate the underlying cross-country equity concerns that can warrant different carbon prices, a task quite different from that of responding to distortions arising from the differences in carbon prices.

It has also been seen, however, that global efficiency requires a more purposive use of tariff policy in recognition of climate concerns—a form of BTA—if climate policies are constrained in countries that deploy carbon taxes. It remains optimal to set those carbon prices that can be set freely—whether explicitly by taxation or implicitly by cap-and-trade—in line with (a simple modification of) the Pigou rule (and not to differentiate them across sectors). But tariffs should now be set so as to recognize the impact on emissions of sourcing domestic demand from countries with carbon taxes that are inappropriate from the collective perspective. The results here fully characterize the BTA required, and show that in a special but instructive case, it takes the simple form—as envisaged in practical policy debate and proposals—of a charge on imports (and rebate on export) equal to the carbon tax ‘not paid’ abroad.

Importantly, this case for BTA does not apply if it is cap-and-trade policies, not carbon

taxation, that is the constrained instrument. This is because emissions cannot then be affected by policies elsewhere. While there has been some discussion of the practical differences between implementing BTAs under carbon taxes and cap-and-trade, the wider point that the underlying economic case is entirely different in the two cases—and much weaker under cap-and-trade—seems not to have been recognized. There may be a case for BTA in terms of national self-interest; but in terms of collective efficiency there is not.

The analysis here is of course severely limited in several respects. Factors have been assumed immobile, for example, precluding the possibility of carbon leakage through location choices that is a major concern in policy debates. And implementation of any form of BTA in any event raises a host of legal and practical issues. What the analysis here does establish, however, is that while practical proposals are naturally driven by national (or sectoral) self-interest, a strong conceptual case can be made for the use of BTAs along broadly the lines often proposed—in relation to carbon taxes, but not cap-and-trade—in the more appealing terms of global efficiency.

# Appendices

## Appendix A: Necessary conditions for Pareto efficiency

Perturbing (5) for country  $j$ , using (1),  $\mathbf{p}^j = \mathbf{w} + \boldsymbol{\tau}^j$  and recalling that  $\mathbf{r}_s^j = -\mathbf{z}^j$ , one obtains

$$\lambda_u^j du^j = \lambda_w^{j'} d\mathbf{w} + \lambda_\tau^{j'} d\boldsymbol{\tau}^j + \sum_{i \neq j}^J \lambda_\tau^{j/i'} d\boldsymbol{\tau}^i + \lambda_s^{j'} d\mathbf{s}^j + \sum_{i \neq j}^J \lambda_s^{j/i'} d\mathbf{s}^i, \quad (\text{A.1})$$

where

$$\lambda_u^j \equiv e_u^j - \boldsymbol{\tau}^{j'} \mathbf{e}_{\mathbf{p}u}^j, \quad (\text{A.2})$$

$$-\lambda_w^{j'} \equiv \mathbf{m}^{j'} + \lambda_k^j \sum_{l=1}^J (-\boldsymbol{\iota}' \mathbf{r}_{\mathbf{sp}}^l) + \mathbf{s}^{j'} \mathbf{r}_{\mathbf{sp}}^j - \boldsymbol{\tau}^{j'} (\mathbf{e}_{\mathbf{pp}}^j - \mathbf{r}_{\mathbf{pp}}^j), \quad (\text{A.3})$$

$$-\lambda_\tau^{j'} \equiv \lambda_k^j (-\boldsymbol{\iota}' \mathbf{r}_{\mathbf{sp}}^j) + \mathbf{s}^{j'} \mathbf{r}_{\mathbf{sp}}^j - \boldsymbol{\tau}^{j'} (\mathbf{e}_{\mathbf{pp}}^j - \mathbf{r}_{\mathbf{pp}}^j), \quad (\text{A.4})$$

$$-\lambda_\tau^{j/i'} \equiv \lambda_k^j (-\boldsymbol{\iota}' \mathbf{r}_{\mathbf{sp}}^i), \quad (\text{A.5})$$

$$-\lambda_s^{j'} \equiv \lambda_k^j (-\boldsymbol{\iota}' \mathbf{r}_{\mathbf{ss}}^j) + \mathbf{s}^{j'} \mathbf{r}_{\mathbf{ss}}^j + \boldsymbol{\tau}^{j'} \mathbf{r}_{\mathbf{ps}}^j, \quad (\text{A.6})$$

$$-\lambda_s^{j/i'} \equiv \lambda_k^j (-\boldsymbol{\iota}' \mathbf{r}_{\mathbf{ss}}^i), \quad (\text{A.7})$$

$$\lambda_k^j = e_k^j - \boldsymbol{\tau}^{j'} \mathbf{e}_{\mathbf{p}k}^j, \quad (\text{A.8})$$

with  $\mathbf{m}^{j'} \equiv (\mathbf{e}_{\mathbf{p}}^j - \mathbf{r}_{\mathbf{p}}^j)'$  denoting the  $(N-1)$ -vector of imports of country  $j$ . Notice that (A.5) and (A.7) refer to the effects on country  $j$  from changes in carbon taxes and tariffs in all other countries.

Perturbing now equations (6), one obtains

$$\sum_{j=1}^J \mathbf{e}_{\mathbf{p}u}^j du^j = \boldsymbol{\pi}_{\mathbf{pp}} d\mathbf{w} + \sum_{j=1}^J \boldsymbol{\pi}_{\mathbf{pp}}^j d\boldsymbol{\tau}^j + \sum_{j=1}^J \boldsymbol{\pi}_{\mathbf{ps}}^j d\mathbf{s}^j, \quad (\text{A.9})$$

where

$$-\boldsymbol{\pi}_{\mathbf{pp}} \equiv \sum_{j=1}^J \left\{ \mathbf{e}_{\mathbf{pp}}^j - \mathbf{r}_{\mathbf{pp}}^j + \mathbf{e}_{\mathbf{p}k}^j \sum_{l=1}^J (-\boldsymbol{\iota}' \mathbf{r}_{\mathbf{sp}}^l) \right\}, \quad (\text{A.10})$$

$$-\boldsymbol{\pi}_{\mathbf{pp}}^j \equiv \mathbf{e}_{\mathbf{pp}}^j - \mathbf{r}_{\mathbf{pp}}^j + \sum_{l=1}^J \mathbf{e}_{\mathbf{p}k}^l (-\boldsymbol{\iota}' \mathbf{r}_{\mathbf{sp}}^j), \quad (\text{A.11})$$

$$-\boldsymbol{\pi}_{\mathbf{ps}}^j \equiv \mathbf{r}_{\mathbf{ps}}^j + \sum_{l=1}^J \mathbf{e}_{\mathbf{p}k}^l (-\boldsymbol{\iota}' \mathbf{r}_{\mathbf{ss}}^j). \quad (\text{A.12})$$

Stacking now (A.1) for all countries  $j$  and (A.9) gives

$$\boldsymbol{\Lambda}_u d\mathbf{u} = \boldsymbol{\Lambda}_w d\mathbf{w} + \boldsymbol{\Lambda}_\tau d\boldsymbol{\tau} + \boldsymbol{\Lambda}_s d\mathbf{s}, \quad (\text{A.13})$$

where the matrices  $\Lambda_{\mathbf{u}}, \Lambda_{\mathbf{w}}, \Lambda_{\boldsymbol{\tau}}, \Lambda_{\mathbf{s}}$  are given by

$$\begin{aligned}
\Lambda_{\mathbf{u}} &= \begin{bmatrix} \lambda_u^1 & 0 & \cdots & 0 \\ 0 & \lambda_u^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_u^J \\ \mathbf{e}_{\mathbf{p}u}^1 & \mathbf{e}_{\mathbf{p}u}^2 & \cdots & \mathbf{e}_{\mathbf{p}u}^J \end{bmatrix} \mathbf{d}\mathbf{u} = \begin{bmatrix} du^1 \\ du^2 \\ \vdots \\ du^J \end{bmatrix} \\
\Lambda_{\mathbf{w}} &\equiv \begin{bmatrix} \lambda_{\mathbf{w}}^{1'} \\ \vdots \\ \lambda_{\mathbf{w}}^{J'} \\ \pi_{\mathbf{p}\mathbf{p}} \end{bmatrix} \mathbf{d}\mathbf{w} \equiv \begin{bmatrix} dw_2 \\ dw_3 \\ \vdots \\ dw_N \end{bmatrix} \\
\Lambda_{\boldsymbol{\tau}} &\equiv \begin{bmatrix} \lambda_{\boldsymbol{\tau}}^{1'} & \lambda_{\boldsymbol{\tau}}^{1/2'} & \cdots & \lambda_{\boldsymbol{\tau}}^{1/J'} \\ \lambda_{\boldsymbol{\tau}}^{2/1'} & \lambda_{\boldsymbol{\tau}}^{2'} & \cdots & \lambda_{\boldsymbol{\tau}}^{2/J'} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{\boldsymbol{\tau}}^{J/1'} & \lambda_{\boldsymbol{\tau}}^{J/2'} & \cdots & \lambda_{\boldsymbol{\tau}}^{J'} \\ \pi_{\mathbf{p}\mathbf{p}}^1 & \pi_{\mathbf{p}\mathbf{p}}^2 & \cdots & \pi_{\mathbf{p}\mathbf{p}}^J \end{bmatrix} \mathbf{d}\boldsymbol{\tau} = \begin{bmatrix} d\boldsymbol{\tau}^1 \\ d\boldsymbol{\tau}^2 \\ \vdots \\ d\boldsymbol{\tau}^J \end{bmatrix} \\
\Lambda_{\mathbf{s}} &\equiv \begin{bmatrix} \lambda_{\mathbf{s}}^{1'} & \lambda_{\mathbf{s}}^{1/2'} & \cdots & \lambda_{\mathbf{s}}^{1/J'} \\ \lambda_{\mathbf{s}}^{2/1'} & \lambda_{\mathbf{s}}^{2'} & \cdots & \lambda_{\mathbf{s}}^{2/J'} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{\mathbf{s}}^{J/1'} & \lambda_{\mathbf{s}}^{J/2'} & \cdots & \lambda_{\mathbf{s}}^{J'} \\ \pi_{\mathbf{p}\mathbf{s}}^1 & \pi_{\mathbf{p}\mathbf{s}}^2 & \cdots & \pi_{\mathbf{p}\mathbf{s}}^J \end{bmatrix} \mathbf{d}\mathbf{s} \equiv \begin{bmatrix} ds^1 \\ ds^2 \\ \vdots \\ ds^J \end{bmatrix}. \tag{A.14}
\end{aligned}$$

Notice that  $\Lambda_{\mathbf{u}}$  is of dimension  $(J + N - 1) \times J$ ,  $\Lambda_{\mathbf{w}}$  of dimension  $(J + N - 1) \times (N - 1)$ ,  $\Lambda_{\boldsymbol{\tau}}$  of  $(J + N - 1) \times J(N - 1)$ , and  $\Lambda_{\mathbf{s}}$  of dimension  $(J + N - 1) \times JN$ .

By Tucker's theorem of the alternative,<sup>21</sup> either the system in (A.13) has a solution with  $\mathbf{d}\mathbf{u} > \mathbf{0}_{J \times 1}$  for some perturbation  $(\mathbf{d}\mathbf{w}, \mathbf{d}\boldsymbol{\tau}, \mathbf{d}\mathbf{s})$  so that the initial equilibrium is Pareto efficient or there exists some  $(J + N - 1)$ -vector  $\mathbf{y}$  such that

$$\mathbf{y}' \Lambda_{\mathbf{u}} \ll \mathbf{0}'_{1 \times J}, \tag{A.15}$$

$$\mathbf{y}' \Lambda_{\mathbf{l}} = \mathbf{0}', \mathbf{l} = \mathbf{w}, \boldsymbol{\tau}, \mathbf{s}. \tag{A.16}$$

It proves helpful, for later use, to partition the vector  $\mathbf{y} = (\boldsymbol{\sigma}, \mathbf{v})'$ , where  $\boldsymbol{\sigma} = (\sigma^1, \dots, \sigma^J)'$  and  $\mathbf{v} = (v_2, \dots, v_N)'$ .  $\square$

## Appendix B: Proof of Proposition 1

For the initial equilibrium to be Pareto-efficient (and with explicit transfers unavailable), it must be the case that (A.15) and (A.16) hold. It is straightforward to show—following the same steps as in the proof of Proposition 3 below—that  $\sigma^j \boldsymbol{\tau}^j = \mathbf{v}$  for every country  $j$  and so for any pair of countries  $j$  and  $i$  it is the case that  $\boldsymbol{\tau}^j = \theta^{ij} \boldsymbol{\tau}^i$ , where  $\theta^{ij} = \sigma^i / \sigma^j$ ,

<sup>21</sup>See Mangasarian (1969, p.24) for a statement.

as required by part (b) of the proposition. Part (a) follows from making use of  $\sigma^j \boldsymbol{\tau}^j = \mathbf{v}$  into, following (A.16),  $\mathbf{y}' \boldsymbol{\Lambda}_s = \mathbf{0}'_{1 \times N}$ .  $\square$

## Appendix C: Proof of Proposition 2

With explicit lump-sum transfers the alternative also requires (with  $\mathbf{y} = (\boldsymbol{\sigma}, \mathbf{v}, \varpi)'$ , now being a  $J + N$ -vector)

$$\mathbf{y}' \boldsymbol{\Lambda}_\alpha = \mathbf{0}'_{1 \times J(N-1)}, \quad (\text{C.1})$$

where

$$\boldsymbol{\Lambda}_\alpha \equiv \begin{bmatrix} -\mathbf{w}' & \mathbf{0}' & \cdots & \mathbf{0}' \\ \mathbf{0}' & -\mathbf{w}' & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0}' & \mathbf{0}' & \cdots & -\mathbf{w}' \\ \bar{\mathbf{0}} & \bar{\mathbf{0}} & \bar{\mathbf{0}} & \bar{\mathbf{0}} \\ \mathbf{w}' & \mathbf{w}' & \cdots & \mathbf{w}' \end{bmatrix} \mathbf{d}\boldsymbol{\alpha} \equiv \begin{bmatrix} \mathbf{d}\alpha^1 \\ \mathbf{d}\alpha^2 \\ \vdots \\ \mathbf{d}\alpha^J \end{bmatrix}, \quad (\text{C.2})$$

where  $\bar{\mathbf{0}}$  is a matrix of dimension  $(N-1) \times JN$  and  $\boldsymbol{\Lambda}_\alpha$  is of dimension  $(J+N) \times J(N-1)$ . The above implies that in every country  $j$

$$\sigma^j \mathbf{w}' - \varpi \mathbf{w}' = \mathbf{0}'_{1 \times (N-1)}, \quad (\text{C.3})$$

and so

$$\sigma^j = \sigma, \quad (\text{C.4})$$

for all countries  $j = 1, \dots, J$ .

With lump-sum transfers being unavailable, it can be easily verified—following from (A.16) and  $\mathbf{y}' \boldsymbol{\Lambda}_w = \mathbf{0}'_{1 \times (N-1)}$ —that

$$\sum_{j=1}^J \sigma^j \left\{ \mathbf{m}^{j'} + e_k^j \sum_{l=1}^J (-\boldsymbol{\iota}' \mathbf{r}_{\text{sp}}^l) + \mathbf{s}^{j'} \mathbf{r}_{\text{sp}}^j \right\} = \mathbf{0}'_{1 \times (N-1)}, \quad (\text{C.5})$$

which upon using part (a), for every country  $j = 1, \dots, J$  reduces to

$$\hat{\mathbf{M}}' \boldsymbol{\sigma} = \mathbf{0}_{(N-1) \times 1}, \quad (\text{C.6})$$

where

$$\hat{\mathbf{M}}' = \begin{bmatrix} m_2^1 & m_2^2 & \cdots & m_2^J \\ m_3^1 & m_3^2 & \cdots & m_3^J \\ \vdots & \vdots & \ddots & \vdots \\ m_N^1 & m_N^2 & \cdots & m_N^J \end{bmatrix}_{(N-1) \times J}, \quad (\text{C.7})$$

is the global import matrix with the element corresponding to the numeraire having been removed from the matrix. This is of dimension  $(N-1) \times J$  but with at most  $J-1$  independent columns. To solve for the vector  $\boldsymbol{\sigma}$  one requires that the number of goods is at least as large as the number of countries that is,  $N-1 \geq J-1$ . If this is the case then  $\sigma^j = \sigma$ , for  $j = 1, \dots, J$ .  $\square$

## Appendix D: Proof of Proposition 3

This proceeds by supposing that all countries, except country  $h$ , are constrained in the use of climate and trade policies. For arbitrary  $\tau^j, \mathbf{s}^j$ , (A.15) requires, for every country  $i$ , that

$$\sigma^i (e_u^i - \tau^{iu} \mathbf{e}_{pu}^i) + \mathbf{v}' \mathbf{e}_{pu}^i < 0, \quad (\text{D.1})$$

following  $\mathbf{y}' \Lambda_1 = \mathbf{0}'$ ,  $\mathbf{l} = \boldsymbol{\tau}^h, \mathbf{s}^h$ , requires, respectively, that

$$\begin{aligned} & \sigma^h \{ \lambda_k^h (-\boldsymbol{\iota}' \mathbf{r}_{sp}^h) + \mathbf{s}^{h'} \mathbf{r}_{sp}^h - \boldsymbol{\tau}^{h'} \mathbf{m}_{pp}^h \} + \sum_{l \neq h}^J \sigma^l \lambda_k^l (-\boldsymbol{\iota}' \mathbf{r}_{sp}^h) + \mathbf{v}' \left\{ \mathbf{m}_{pp}^h + \sum_{l=1}^J \mathbf{e}_{pk}^l (-\boldsymbol{\iota}' \mathbf{r}_{sp}^h) \right\} \\ & = \mathbf{0}'_{1 \times (N-1)}, \end{aligned} \quad (\text{D.2})$$

and

$$\begin{aligned} & \sigma^h \{ \lambda_k^h (-\boldsymbol{\iota}' \mathbf{r}_{ss}^h) + \mathbf{s}^{h'} \mathbf{r}_{ss}^h + \boldsymbol{\tau}^{h'} \mathbf{r}_{ps}^h \} + \sum_{l \neq h}^J \sigma^l \lambda_k^l (-\boldsymbol{\iota}' \mathbf{r}_{ss}^h) + \mathbf{v}' \left\{ \mathbf{r}_{ps}^h - \sum_{l=1}^J \mathbf{e}_{pk}^l (-\boldsymbol{\iota}' \mathbf{r}_{ss}^h) \right\} \\ & = \mathbf{0}'_{1 \times N}, \end{aligned} \quad (\text{D.3})$$

where

$$\mathbf{m}_{pp}^h \equiv \mathbf{e}_{pp}^h - \mathbf{r}_{pp}^h. \quad (\text{D.4})$$

Write now (D.2) as

$$\sigma^h \{ \lambda_k^h (-\boldsymbol{\iota}' \mathbf{r}_{sp}^h) + \mathbf{s}^{h'} \mathbf{r}_{sp}^h \} + \sum_{l \neq h}^J \sigma^l \lambda_k^l (-\boldsymbol{\iota}' \mathbf{r}_{sp}^h) + \mathbf{v}' \sum_{l=1}^J \mathbf{e}_{pk}^l (-\boldsymbol{\iota}' \mathbf{r}_{sp}^h) = (\sigma^h \boldsymbol{\tau}^h - \mathbf{v})' \mathbf{m}_{pp}^h, \quad (\text{D.5})$$

and (D.3) as

$$\sigma^h \{ \lambda_k^h (-\boldsymbol{\iota}' \mathbf{r}_{ss}^h) + \mathbf{s}^{h'} \mathbf{r}_{ss}^h \} + \sum_{l \neq h}^J \sigma^l \lambda_k^l (-\boldsymbol{\iota}' \mathbf{r}_{ss}^h) + \mathbf{v}' \sum_{l=1}^J \mathbf{e}_{pk}^l (-\boldsymbol{\iota}' \mathbf{r}_{ss}^h) = -(\sigma^h \boldsymbol{\tau}^h - \mathbf{v})' \mathbf{r}_{ps}^h. \quad (\text{D.6})$$

Post-multiplying (D.6) by, assuming it exists, the inverse of  $\mathbf{r}_{ss}^h$  and substituting this into the left-hand-side of (D.5) gives (after making use of the inverse of  $\mathbf{m}_{pp}^h + \mathbf{r}_{ps}^h (\mathbf{r}_{ss}^h)^{-1} \mathbf{r}_{sp}^h$  and rearranging)

$$\sigma^h \boldsymbol{\tau}^h = \mathbf{v}. \quad (\text{D.7})$$

Substituting now (D.7) into (D.6), and simplifying, gives (11). Country  $h$ 's tariff vector in (12) is now obtained by substituting (D.7) into, following from (A.16),  $\mathbf{y}' \Lambda_w = \mathbf{0}'_{1 \times (N-1)}$ , and simplifying by making use also of (11) and (D.4).  $\square$

## Appendix E: Derivation of equation 20

Recall that part (a) of Corollary 1 implies that carbon taxes are uniform—in the sense that  $s^{1*} = e_k^1 + e_k^2$ —whereas part (b) implies, if there are no substitution effects in demand ( $\mathbf{e}_{pp}^2 = \mathbf{0}_{(N-1) \times (N-1)}$ ), that

$$\boldsymbol{\tau}^{1'} = \boldsymbol{\tau}^{2'} - (s^{1*} \boldsymbol{\iota} - \mathbf{s}^2)' \mathbf{r}_{sp}^2 (\mathbf{r}_{pp}^2)^{-1}. \quad (\text{E.1})$$

The emissions vector in country 2 is given by

$$\mathbf{z}^2 = \Phi^2 \mathbf{g}^2, \quad (\text{E.2})$$

where  $\mathbf{g}^2$  is gross output and  $\Phi^2$  is an  $N \times N$  diagonal matrix, with the element  $\phi_n^2$  giving the level of pollutant associated with the production of the  $n$ th good. Making use of the standard input-output relation it is the case that

$$\mathbf{g}^2 = [\mathbf{I}_N - \mathbf{A}^2(\mathbf{p}^2)]^{-1} \mathbf{y}^2, \quad (\text{E.3})$$

where  $\mathbf{A}^2$  is an  $N \times N$  matrix with the typical element  $a_{nj}^2$  giving the use of good  $j$  per unit of gross output of  $n$ . Substituting (E.3) in (E.2), and making use of the fact that  $\mathbf{z}^2 = -\mathbf{r}_s^2$  and  $\mathbf{y}^2 = \mathbf{r}_p^2(\mathbf{p}^2, \mathbf{s}^2)$  it is the case that

$$-\mathbf{r}_s^2(\mathbf{p}^2, \mathbf{s}^2) = \Phi^2 [\mathbf{I}_N - \mathbf{A}^2(\mathbf{p}^2)]^{-1} \mathbf{r}_p^2(\mathbf{p}^2, \mathbf{s}^2). \quad (\text{E.4})$$

Differentiating with respect to  $\mathbf{p}^2$ —and with  $\Phi^2$  being independent of  $\mathbf{p}^2$ —one, following from Proposition 93 in Dhyrnes (1978), arrives at

$$-\mathbf{r}_{sp}^2(\mathbf{p}^2, \mathbf{s}^2) = \Phi^2 \left\{ (\mathbf{r}_p^{2'} \otimes \mathbf{I}_N) \frac{\partial ([\mathbf{I}_N - \mathbf{A}^2(\mathbf{p}^2)]^{-1})}{\partial \mathbf{p}^2} + [\mathbf{I}_N - \mathbf{A}^2(\mathbf{p}^2)]^{-1} \mathbf{r}_{pp}^2 \right\}, \quad (\text{E.5})$$

where, following from Proposition 105 in Dhyrnes (1978), it is the case that

$$\frac{\partial ([\mathbf{I}_N - \mathbf{A}^2(\mathbf{p}^2)]^{-1})}{\partial \mathbf{p}^2} = - \left( [\mathbf{I}_N - \mathbf{A}^{2'}(\mathbf{p}^2)]^{-1} \otimes [\mathbf{I}_N - \mathbf{A}^2(\mathbf{p}^2)]^{-1} \right) \frac{\partial \text{vec}([\mathbf{I}_N - \mathbf{A}^2(\mathbf{p}^2)])}{\partial \mathbf{p}^2}, \quad (\text{E.6})$$

where  $\text{vec}([\mathbf{I}_N - \mathbf{A}^2(\mathbf{p}^2)])$  denotes the  $N^2$ -element column vector. Substituting (E.6) into (E.5)—and appropriately replacing  $\Phi^2$  with the vector  $\phi^2$ —and that into (E.1) one arrives—abusing notation somewhat—at (20).  $\square$



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