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## Poverty and life cycle effects: A nonparametric analysis for Germany

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# Poverty and life cycle effects.

A nonparametric analysis for Germany

**preliminary version**

Andreas Stich\*

## **Abstract**

Most empirical studies on poverty consider the extent of poverty either for the entire society or for separate groups like elderly people. However, these papers do not show what the situation looks like for persons of a certain age. In this paper poverty measures depending on age are derived using the joint density of income and age. The density is nonparametrically estimated by weighted Gaussian kernel density estimation. Applying the conditional density of income to several poverty measures provides an insight into the extent of poverty over the life cycle. Furthermore confidence bands for the poverty age profiles were computed.

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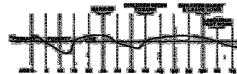
## 1 Introduction

One of the most discussed problems in the world today is poverty. Not only in the developing countries but also in industrial societies like the U.S.A. or Germany poverty endangers the well-being of people. It is an important duty of the government to fight against poverty. To examine the efficiency of actions against poverty, indices are needed which measure the extent of poverty.

In the recent literature many proposals for poverty measures can be found. For an overview see, e.g., Seidl (1988) or Scheurle (1991), but most of the papers only take into account the whole society or particular groups. An interesting point, often missing in empirical studies, is the dependence of poverty on age, i.e., poverty in the life cycle.

Alcock (1993) gives an exhaustive overview of determinants which influence poverty one of which is age. In chapter 10 he describes several causes why the risk of poverty for old people is higher than for young peoples. Rowntree (1901) found that poverty fluctuates over the life course. He noticed that labourers are particularly prone to poverty during childhood, in the early middle life and in old age (see Rowntree (1901), p. 137) and illustrates his result with figure 1 (see below). About a century later Hedstrom and Ringen (1990) confirmed the results of Rowntree for seven countries and found two areas of poverty: childhood and old age. For their research they used seven age groups with bounds (0-24, 25-34, 35-44, 45-54, 55-65, 65-74, 75+). They found that in four out of seven countries (including West Germany) poverty rates follow a Rowntree type of cycle, i.e., the risk of poverty is highest in the youngest and oldest age groups. Osberg (1996) studies poverty in Canada in the years 1975 – 1994 for five age groups. The drawback of this classification is that changes in the bounds of the periods of poverty over time cannot be observed.

Figure 1: Poverty in the life cycle



Source: Rowntree, B.S. (1901): Poverty. A study of town life, p. 137

Changes of the periods of poverty can only be observed if age dependent poverty measures are constructed. This will be done in this paper. It follows a line of research recently used in the measurement of inequality and mobility (Trede (1995), Gerfin (1994) and Pudney (1993)). The poverty age profiles of Germany for the years 1984-1995 are computed. The

joint density of income and age is estimated via Gaussian kernel density estimation and age dependent poverty measures are created using the conditional density function; an application of kernel density estimation for income distributions is given in Cowell et al. (1996). This paper evaluates point estimators and confidence bands following a procedure introduced by Härdle (1991).

The next section derives the kernel estimator and presents the poverty measures. Section 3 describes the data and section 4 deals with the results. The last section concludes.

## 2 The Tools

In this paper two areas of statistics are needed: First, the estimation of a bivariate density and, second, the measurement of poverty.

Before computing age-dependent poverty measures, the joint density of the variables age and income has to be estimated. Estimating the income distribution with parametric methods concerns many technical problems (see, e.g., Brachmann et al. (1996)) even in the one dimensional case. Besides, the recent literature claims that conventional parametric models of income distributions fail to reproduce some important features of the true distribution (see, e.g., Cowell et al. (1996)). Therefore, a nonparametric method is needed: multidimensional kernel density estimation. Let  $f_{X,A}$  be the joint density function of income and age. The Gaussian kernel estimator is (see Scott (1992))

$$\hat{f}(x, a) = \frac{1}{nh_X h_A} \sum_{i=1}^n \phi\left(\frac{x - x_i}{h_X}\right) \phi\left(\frac{a - a_i}{h_A}\right) \quad (2.1)$$

where  $\phi(\cdot)$  is the pdf of the standard normal distribution,  $x_i$  is income belonging to the  $i$ -th observation and  $a_i$  the respective age.  $h_X$  and  $h_A$  are the bandwidths. Scott (1992) gives the following rule of thumb for bandwidth choice in this case:

$$h_k = \sigma_k \cdot n^{-1/6} \quad k = A, X \quad (2.2)$$

where  $\sigma_k^2$  is the variance of  $k = A, X$ .

Note that the kernel estimator for the marginal density of age is

$$\hat{f}(a) = \frac{1}{nh_A} \sum_{i=1}^n \phi\left(\frac{a - a_i}{h_A}\right). \quad (2.3)$$

So the estimator of the conditional density function of income is

$$\hat{f}(x|a) = \frac{\hat{f}(x, a)}{\hat{f}(a)} = \frac{\sum_{i=1}^n \phi\left(\frac{x - x_i}{h_X}\right) \phi\left(\frac{a - a_i}{h_A}\right)}{h_X \sum_{i=1}^n \phi\left(\frac{a - a_i}{h_A}\right)}. \quad (2.4)$$

A nonparametric estimator of the regression curve  $m(a) = E(X|A = a)$  is given by the Nadaraya–Watson–estimator

$$\hat{m}(a) = \frac{\sum_{i=1}^n K\left(\frac{a - a_i}{h_A}\right) x_i}{\sum_{i=1}^n K\left(\frac{a - a_i}{h_A}\right)} \quad (2.5)$$

with a kernel function  $K$ .

Measuring poverty in a society can be divided into two steps. First it has to be decided whether a person is poor or not. A simple way to do this is fixing a threshold value, i.e., the poverty line (say  $p$ ). Everyone with income lower than, or equal to, the poverty line is poor, otherwise he or she will be called rich. This threshold can be an absolute value, e.g., the minimum amount of money which someone needs for living, or a relative value, e.g., 40%, 50% or 60% of the mean or median. For an overview see Seidl (1988). In this paper 40%, 50% and 60% of the median is used as poverty line.

Next an index has to be created which measures poverty. Since the seminal article of Sen (1976) three components of poverty are widely accepted (see Schaich (1995)):

- (i) the fraction of the poor
- (ii) the amount which is needed to lift the income of all poor to the poverty line, i.e., the income gap
- (iii) the disparity within the poor.

Furthermore, Sen (1976) demands the independence of the poverty indices from the income of the rich.

Schaich (1995) divides poverty measures into simple and complex poverty indices. Simple measures fulfill only one of the above mentioned properties. There are, among others, the Head Count Ratio, i.e., the share of the poor in the society

$$HCR = F(p) = \int_0^p f(x) dx \quad (2.6)$$

where  $F(\cdot)$  is the distribution function and  $f(\cdot)$  the density of income. The HCR only fulfills (i). An index fulfilling (ii) is the Income Gap Ratio. This function is defined as the amount of money which is needed to lift the income of all poor to the poverty line divided by the maximum income gap.

$$IGR = \int_0^p \frac{p-x}{p} dF^* = 1 - \frac{\int_0^p x dF^*}{p} = 1 - \frac{\mu^*}{p} \quad (2.7)$$

where  $F^*$  denotes the cumulative distribution function and  $\mu^*$  denotes the mean of the income of the poor.

A complex index can be created as a function of the  $HCR$ , the  $IGR$  and a function measuring disparity, i.e., an inequality index, for the poor. A widely used function is

$$P_I(F) = HCR(IGR + (1 - IGR)I_{po}) \quad (2.8)$$

where  $I_{po}$  is an inequality measure for the poor. Sen (1976) used (2.8) with the Gini-index as inequality index. Other inequality indices could be the Atkinson measures

$$A^\varepsilon = 1 - \left[ \int_0^\infty \left( \frac{x}{\mu} \right)^{1-\varepsilon} dF \right]^{\frac{1}{1-\varepsilon}} \quad \varepsilon \in \mathbb{R}, \varepsilon \neq 1 \quad (2.9)$$

or the generalized Theil measures

$$T^\lambda = \frac{1}{\lambda(\lambda - 1)} \left[ \int_0^\infty \left( \frac{x}{\mu} \right)^\lambda dF - 1 \right] \quad \lambda \in \mathbb{R}, \lambda \neq 0, 1 \quad (2.10)$$

(see, e.g., Blackorby and Donaldson (1978, 1980, 1984)).

With

$$I_{po} = \frac{Var(F^*)}{\mu^{*2}} = CV_{po}^2, \quad (2.11)$$

the coefficient of variation, a slight variation of (2.7) is a member of the class of Foster et al. (1984)

$$P_{CV}(F) = HCR(IGR^2 + (1 - IGR)^2 CV_{po}^2). \quad (2.12)$$

Blackburn (1989) also uses the Theil measure. But he defines his poverty measure as

$$P_{BL}(F) = HCR(IGR - \ln(1 - T_{po}^\lambda)). \quad (2.13)$$

Estimators can be created by using the kernel estimator of the conditional pdf. Note that using the marginal density produces the usual, unconditional, poverty estimator.

Usual estimators are constructed for simple random sampling. However, since most panel surveys use a more complicated sampling design, other (weighted) estimators are required. If the inclusion probabilities are known, i.e., the probability that an element of the population is at least once in the sample a linear unbiased estimator can be created, which takes the sampling design into account. This is the well known Horvitz–Thompson–estimator (see, e.g., Särndal et al. (1992)). The estimator of the population total is:

$$T_{HT}(x) = \sum_{i \in \omega} x_i \pi_i^{-1}$$

where  $\omega$  is the sample and  $\pi_i$  the inclusion probability of  $x_i$ . The distribution function can be estimated by

$$F_{HT}(x) = \frac{1}{\hat{N}} \sum_{i \in \omega} \mathbf{1}_{x_i \leq x} \pi_i^{-1}$$

with  $\mathbf{1}$  the indicator function and

$$\hat{N} = \sum_{i \in \omega} \pi_i^{-1}$$

the estimated population size. Let  $x_{(1)} \leq \dots \leq x_{(n)}$  be the ordered values of the sample. A HT–estimator for the median is given by (see Särndal et al. (1992), ch. 5.7)

$$\tilde{x}_{HT} = \begin{cases} x_{(l)} & \text{if } B_{l-1} < 0.5\hat{N} < B_l \\ \frac{1}{2}(x_{(l)} - x_{(l+1)}) & \text{if } B_l = 0.5\hat{N} \end{cases}$$

with  $B_l = \sum_{i=1}^l \pi_{[i]}^{-1}$  and  $\pi_{[i]}$  the inclusion probability of  $x_{(i)}$ .

The usual kernel density estimator can be interpreted as an average of the kernel function at points  $a_i$ ,  $i = 1, \dots, n$  using the weight  $1/n$  for every observation. In the framework of HT-estimators, a generalized kernel estimator would be

$$\hat{f}_{HT}(a) = \frac{1}{\hat{N}h_A} \sum_{i=1}^n \pi_i^{-1} \phi\left(\frac{a - a_i}{h_A}\right), \quad (2.14)$$

i.e., a weighted average of the kernel function. The inverse inclusion probability can be interpreted as apparent frequency (see Stuart (1984), p. 81), i.e., the element with inclusion probability  $\pi_i$  represents  $\pi_i^{-1}$  elements in the population. Hence the HT-kernel density estimator can be seen as the usual kernel density estimator of the (estimated) population.

Note that for simple random sampling without replacement  $\pi_i = n/N$  and  $\hat{N} = N$  and thus  $\hat{f}_{HT} \equiv \hat{f}$ .

In the two-dimensional case the HT-kernel density estimator is

$$\hat{f}_{HT}(x, a) = \frac{1}{\hat{N}h_X h_A} \sum_{i=1}^n \pi_i^{-1} \phi\left(\frac{x - x_i}{h_X}\right) \phi\left(\frac{a - a_i}{h_A}\right) \quad (2.15)$$

and the Nadaraya-Watson-estimator can be written as

$$\hat{m}_{HT}(a) = \frac{\sum_{i=1}^n \pi_i^{-1} K\left(\frac{a - a_i}{h_A}\right) x_i}{\sum_{i=1}^n \pi_i^{-1} K\left(\frac{a - a_i}{h_A}\right)} \quad (2.16)$$

Confidence intervals are computed via the bootstrap-method. Let  $x^*$  be a random sample uniformly drawn from  $x$ , with replacement and  $PI$  the poverty measure of interest. Estimating the poverty index for  $x^*$  yields  $\widehat{PI}^*$ . This is repeated a large number of times,  $B = 1000$ , say, giving  $B$  poverty indices  $\widehat{PI}_b^*$ ,  $b = 1, \dots, B$ . Let  $\widehat{PI}_{(1)}^* \leq \dots \leq \widehat{PI}_{(B)}^*$  be the order statistic of the resampled poverty measure. A confidence interval is given by (see Hall (1992), Chapter 1.3).

$$[2\widehat{PI} - \widehat{PI}_{((1-\alpha/2)B)}^*, 2\widehat{PI} - \widehat{PI}_{((\alpha/2)B)}^*].$$

An overview on further (more complicated) bootstrap confidence intervals can be found in DiCiccio and Efron (1996).

The confidence bands are computed by a procedure which Trede (1995) used to evaluate confidence bands for mobility age profiles. It goes back to a procedure derived by Härdle (1991, ch. 7.2).

Let  $y_i = (x_i, a_i)$ ,  $i = 1, \dots, n$ , be randomly drawn independent observations. Given the  $y_i$  one can estimate the poverty age profiles for the different indices  $\widehat{PI}(a)$  with  $PI(a) = HCR(F(x|a))$ ,  $IGR(F(x|a))$  or  $P_I(F(x|a))$  respectively using (2.4) and evaluate it at a number of certain ages  $a_0 = (a_{01}, \dots, a_{0m})$ . Since  $\widehat{PI}(a)$  is a random variable a confidence band of the entire function will give further information on the change of poverty during the life cycle. To achieve this bootstrap procedures rely on resampling. Let  $y_i^*$ ,  $i = 1, \dots, n$ ,

be a random sample uniformly drawn from  $y_i$ , with replacement. Estimating the poverty age profile for the  $y_i^*$  yields  $\widehat{PI}^*(a)$ . This is repeated a large number of times,  $B = 1000$ , say, giving  $B$  poverty age profiles  $\widehat{PI}_b^*$ ,  $b = 1, \dots, B$ , each evaluated at ages  $a_0$ . Let  $\widehat{PI}_{(1)}^*(a_{0j}) \leq \dots \leq \widehat{PI}_{(B)}^*(a_{0j})$ ,  $j = 1, \dots, m$  be the order statistics of the resampled poverty age profiles at age  $a_{0j}$ .

The confidence bands of the entire profiles can be found by the following procedure (see also Trede (1995) and Härdle (1991), ch. 7.2). Define the lower boundary

$$\underline{PI}_k(a_0) = (\widehat{PI}_{(k)}^*(a_{01}), \dots, \widehat{PI}_{(k)}^*(a_{0m}))$$

and the upper boundary

$$\overline{PI}_k(a_0) = (\widehat{PI}_{(B-k+1)}^*(a_{01}), \dots, \widehat{PI}_{(B-k+1)}^*(a_{0m})).$$

Obviously, no resampled poverty age profile leaves the area enclosed by  $\underline{PI}_1(a_0)$  and  $\overline{PI}_1(a_0)$ . Let  $k^*$  be the largest integer  $k$  such that the proportion of resampled poverty age profiles lying completely inside the boundaries is greater or equal to  $(1 - \alpha)$ . The lower and upper boundaries  $\underline{PI}_{k^*}(a_0)$  and  $\overline{PI}_{k^*}(a_0)$  constitute a  $(1 - \alpha)$  confidence band.

To compute the confidence bands the WARP algorithm of Härdle (1991) is used. Trede (1995) describes the procedure for the two-dimensional case. This method is adapted to the HT-design. Let  $y_i = (a_i, x_i)$  be the observations. The usual kernel density estimate is computationally intensive, depending heavily on the number of observations. The idea of WARP (Weighted Average of Rounded Points) is to reduce the required number of computer operations by grouping several neighboring observations together into a single observation. To account for the different numbers of observations in each group the groups are weighted: the more observations fall into a certain group the higher the weight.

Let  $h_A, h_X$  be the bandwidths for the two variables  $a, x$ . Define rectangles  $B(i, j)$ , called bins in the sequel, of equal height  $\delta_A = h_A/M_A$  and equal width  $\delta_X = h_X/M_X$  where  $M_A, M_X$  are the parameters responsible for the trade-off between speed (small  $M_k$ ) and accuracy (large  $M_k$ ). The bins  $B(i, j)$  must pave the entire plane  $(\min(a_i), \max(a_i)) \times (\min(x_i), \max(x_i))$  plus a sufficiently wide margin which depends on the type of kernel and the bandwidths. For a Gaussian kernel the margin should not fall short of  $8h_A$  at the top and the bottom and  $8h_X$  at the left and the right.

Next, the estimated number of observations within each bin,  $c(i, j)$ , is determined.

$$c(i, j) = \sum_{k \in B(i, j)} \pi_k^{-1}. \quad (2.17)$$

The bins at the margin are empty by construction; they are but a computational convenience as will be seen below. The density approximation is constant within each bin; for simplicity the estimation is only done for the bins' midpoints.

A grid of weights having height  $8M_A + 1$  and width  $8M_X + 1$  is defined by

$$w(i, j) = \frac{\phi(i/M_A)}{\sum_{k=1}^n \phi(k/M_A)} M_A \cdot \frac{\phi(j/M_X)}{\sum_{k=1}^n \phi(k/M_X)} M_X \quad (2.18)$$

for  $i = -4M_A, \dots, 4M_A$  and  $j = -4M_X, \dots, 4M_X$ . Summing up the weights yields  $\sum \sum w(i, j) = M_A M_X$ . The density estimate at the midpoint of bin  $B(k, l)$  is

$$\hat{f}(k, l) = \frac{1}{\hat{N} h_A h_X} \sum_{i=-4M_A}^{4M_A} \sum_{j=-4M_X}^{4M_X} c(k + i, l + j) w(i, j) \quad (2.19)$$

for  $k = 4M_A + 1, \dots, I - 4M_A$  and  $l = 4M_X + 1, \dots, J - 4M_X$  with  $I = ((\max(a_i) - \min(a_i)) + 1)/\delta_A + 16M_A$  and  $J = ((\max(x_j) - \min(x_j)) + 1)/\delta_X + 16M_X$ . The function  $\hat{f}(\cdot, \cdot)$  is a valid density estimate since  $\sum \sum f(k, l) = 1/(\delta_A \delta_X)$ . Remember that the density is constant within each bin; as the bins have height  $\delta_A$  and width  $\delta_X$  the total probability mass under  $\hat{f}$  equals unity.

### 3 The Data

This section examines the dependence of poverty on age in Germany in 1984 – 1995 using the German Socio–Economic Panel (SOEP) data. The panel participants are interviewed annually; the data are recorded on the household level as well as on the individual level.

The panel includes a large section on income and earnings in the preceding year. The main respondent of a household is asked in each wave of the SOEP: "If you add all: What is the total monthly amount of the household net income of all household members today? Please report the monthly net amount, i.e. the amount after tax and social insurance. Regular payments like aid for dwelling and children, student loans, alimony payments, etc. add to the amount! In the case of "I don't know", please estimate the amount".

There are of course several limitations regarding this information. First, about 50% underestimate their true net income. (see Rendtel et al. (1992)). Second, the values are rounded. While Rendtel et al. (1992) show that the first point is irrelevant when the poverty line depends on an income distribution parameter, there is no solution for the second aspect. This has to be taken into consideration when the results are interpreted.

The East German observations are not taken into account until 1991. Before this year only the sample containing the West Germans and the sample containing foreigners are used. All observations with income less than 0 or missing age data are deleted. The number of the remaining observations can be found in table 1.

Table 1: Number of valid observations

year	1984	1985	1986	1987	1988	1989
$n$	15319	13761	13159	13003	12353	11967
year	1990	1991	1992	1993	1994	1995
$n$	11741	17246	16856	16453	16130	16568

The aim of this article is to estimate the dependence of poverty on age in the German society. For this purpose ideally the welfare level of every person ought to be calculated. However, the variable used is net income of the household. So the household net income

has to be transformed to a personal income which contains the same level of welfare for every member of the household. The economies of scale inside the household should also be considered. To perform this conversion equivalence scales are developed. In this paper the equivalence scale of Buhmann et al. (1988) is used, which has a single parameter ( $\theta$ ), the elasticity of the scale rate with respect to household size. This equivalence scale is characterized by the following equation:

$$X = \frac{HI}{S^\theta} \quad (3.1)$$

where equalized income ( $X$ ) equals net household income ( $HI$ ) divided by household size ( $S$ ) raised to the power of  $\theta$ . Economies of scale can be thought of as a function of  $\theta$ . At the one end, where  $\theta = 0$ , scale economies are perfect so that a family of any number of members can live exactly as well as a one person household with the same amount of disposable income. At the other end, where  $\theta = 1$ , no economies of scale exist and a household of two requires twice as much disposable income as a household of one to reach the same level of welfare. The equivalence income is the income which every person would have if he or she lived in a one person household with the same welfare level as his or her present family.

Burkhauser et al. (1996) computed  $\theta$  for different equivalence scales used for Germany and the U.S.A. The "official" scale of Germany embedded in German public welfare law (BSHG) since 1991 implies roughly  $\theta = 0.81$ . This scale has weight 1 for the family head, 0.8 for every further adult person, 0.9 for children between 14 and 17 years, 0.65 for children between 7 and 13 years and 0.5 for children younger than 7 years.

The ELES scale, developed by Merz et al. (1993), implies a value of  $\theta = 0.38$  for Germany. Between these bounds the value of the international experts scale can be found. This scale was developed by several analysts and used in various studies undertaken on behalf of the Office of Economic Cooperation and Development (Förster (1994), Atkinson et al. (1994)), as well as the Statistical Office of the European Community (Hagenaars et al. (1994)) and by Ruggles (1990). The elasticity of this scale is  $\theta = 0.50$ .

Although Burkhauser et al. (1996) found that overall inequality and poverty measures are not sensitive with respect to the equivalence scale used, the scales yield different results when subgroups of the population (e.g. people aged 65+) are considered. Therefore, this study computes the poverty indices for  $\theta = 0.81$ ,  $\theta = 0.5$  and  $\theta = 0.38$ .

Osberg (1996) remarks that the choice of the poverty line has great influence on estimated poverty. He shows that 27.6% of the senior citizens of Canada were poor according to the low income cut-off poverty line of Statistics Canada (further information can be found in Osberg (1996), p. 9) in 1994 while using the one-half of the median poverty line leads to a rate of poverty of just 1.5% among the senior citizens. Because of this, three poverty lines are considered in this paper, namely 40%, 50% and 60% of the median of the equivalence income.

## 4 The Results

At first the joint density of age and (equivalence) income has to be estimated. The bandwidths for the kernel density estimation are computed according to (2.2). They can

be found in table 2. These numbers show that the bandwidth for age is almost always 4 years or so whereas the bandwidth for income depends on the year and  $\theta$ . The income data in the SOEP are heavily rounded so that the optimal bandwidths have to be handled with caution. In the kernel density estimation the bandwidth for age is set to  $h_A = 4$  and the bandwidth for income is set as in table 2.

Table 2: Bandwidths for different equivalence incomes 1984 – 1995

year	$\theta = 0.38$		$\theta = 0.50$		$\theta = 0.81$	
	$h_A$	$h_X$	$h_A$	$h_X$	$h_A$	$h_X$
1984	4.13	281.01	4.13	252.26	4.13	198.61
1985	4.19	247.31	4.19	211.45	4.19	154.63
1986	4.22	335.80	4.22	293.37	4.22	214.58
1987	4.24	280.64	4.24	249.13	4.24	192.28
1988	4.30	275.59	4.30	241.04	4.30	181.06
1989	4.32	260.88	4.32	233.45	4.32	186.55
1990	4.38	316.34	4.38	270.66	4.38	195.77
1991	4.09	212.43	4.09	188.13	4.09	149.41
1992	4.11	216.25	4.11	192.26	4.11	155.31
1993	4.13	241.28	4.13	214.57	4.13	171.94
1994	4.16	243.05	4.16	216.10	4.16	173.47
1995	4.12	270.53	4.12	242.52	4.12	196.51

Figure 2: Surface plot of the joint density for income and age with  $\theta = 0.38$  for 1995

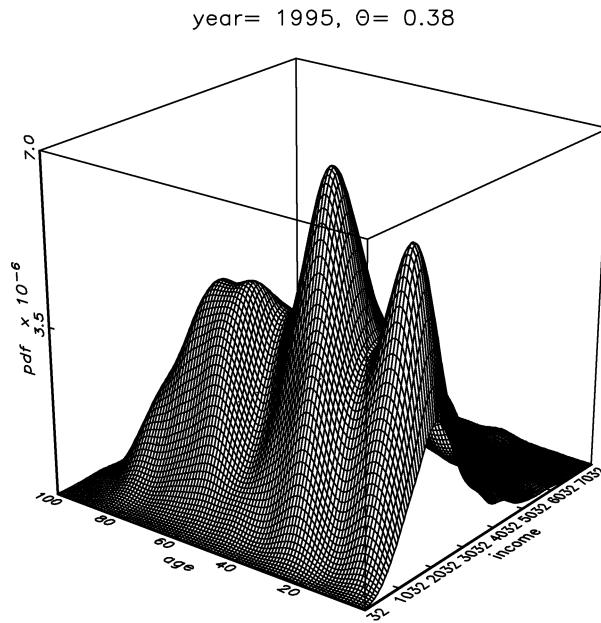


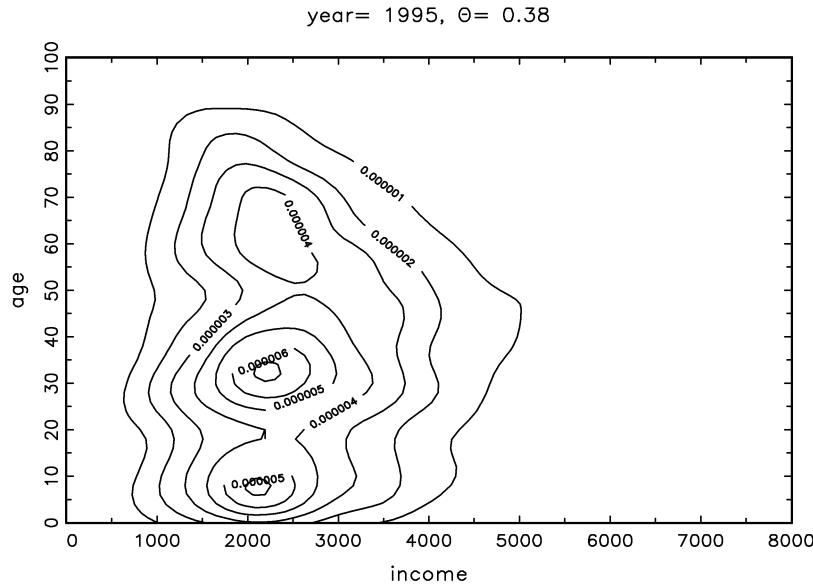
Figure 3: Contour plot of the joint density for income and age with  $\theta = 0.38$  for 1995

Figure 4: Nonparametric regression of income on age for 1995

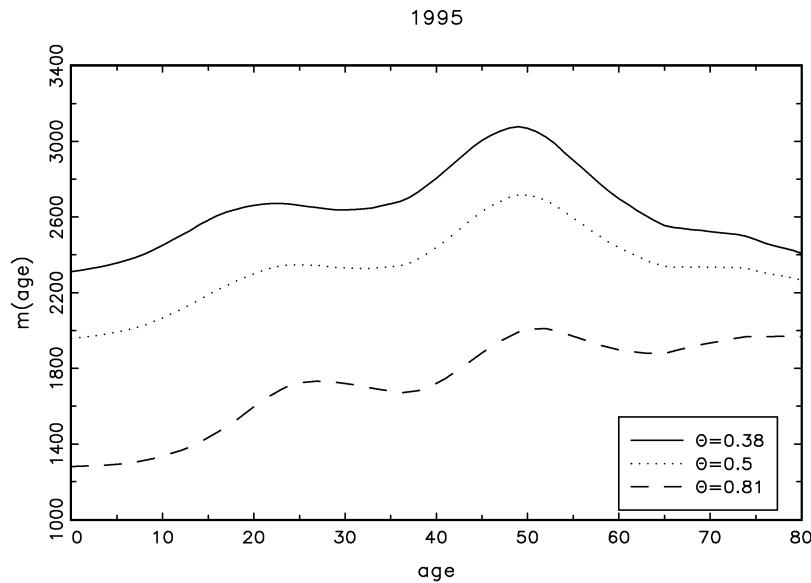


Table 3: Median-estimates for different equivalence-scales 1984 – 1995

year	$\theta = 0.38$	$\theta = 0.50$	$\theta = 0.81$
1984	1735.97	1500.00	1106.14
1985	1771.49	1562.00	1149.98

year	$\theta = 0.38$	$\theta = 0.50$	$\theta = 0.81$
1986	1877.32	1674.32	1200.00
1987	1921.09	1732.05	1250.00
1988	1976.13	1750.00	1281.67
1989	2024.71	1800.00	1333.88
1990	2173.74	1950.00	1430.00
1991	2007.69	1788.85	1301.34
1992	2184.84	1979.90	1425.95
1993	2305.48	2061.22	1507.29
1994	2371.35	2121.32	1592.97
1995	2437.23	2163.72	1626.68

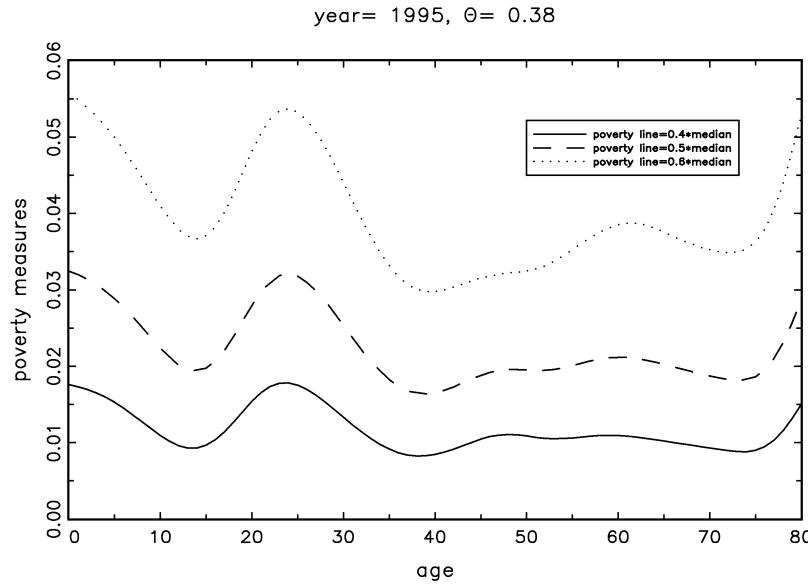
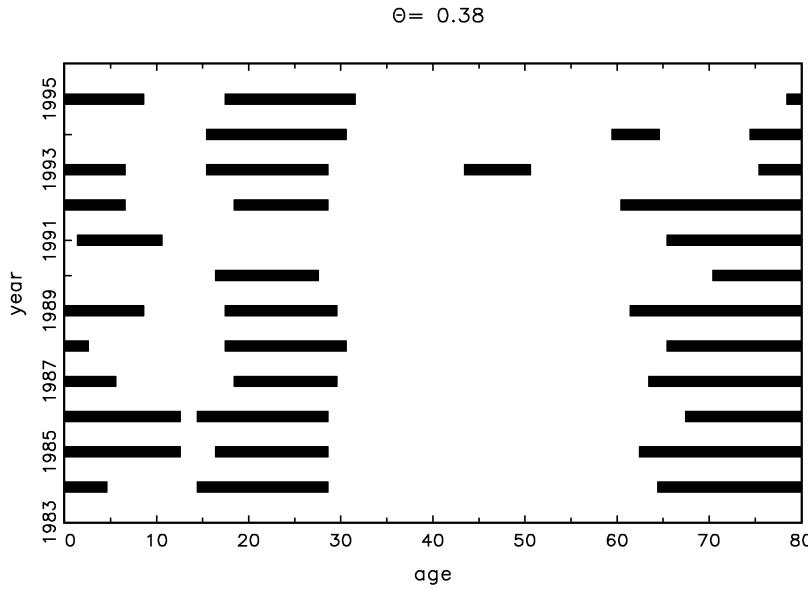
Figure 5: Different poverty measures for  $\theta = 0.38$  and  $p = 0.4 \cdot \bar{x}_{HT}$  for 1995



Next the parameters  $M_A$  and  $M_X$  for the WARP-algorithm have to be chosen. After careful consideration they are set to  $M_X = 5$  and  $M_A = 4$ . Meaning that the height of the grids is one year and the width is around 50 DM.

In figures 2 and 3 the joint density is plotted for 1995 and  $\theta = 0.38$ . The density has 3 peaks which are at an equivalence income of 2000 DM. Ordered by height these peaks are located at age 30, 9 and 65 years, respectively.

The nonparametric regression (see figure 4) shows for  $\theta = 0.38$  and  $\theta = 0.5$  a roughly concave form. This is in coincidence with the expected form, because income increases in younger years until it reaches its maximum and decreases in old age. For  $\theta = 0.81$  this observation cannot be done. Though there is a slight bend at age 60, the age dependent income increases in older age. This can be explained by the high weights of the BSHG-scale. In old age there are many households with one or two persons, while the average number of persons in households in younger years is usually higher. These families are

Figure 6: Atkinson poverty measures for different poverty lines and  $\theta = 0.38$  for 1995Figure 7: Areas of high poverty risk 1984 – 1995 estimated with the Atkinson poverty measure and  $\theta = 0.38$ 

discriminated by the BSHG-scale because of the high weights and thus the equivalence income in these households decreases dramatically. This can be seen by the interpretation of the poverty age curves, too. This effect can be explained as follows: The higher  $\theta$  the less economies of scale are given, the more the equivalence income decreases (see, e.g.

Figure 8: Areas of high poverty risk 1984 – 1995 estimated with the Atkinson poverty measure and  $\theta = 0.5$

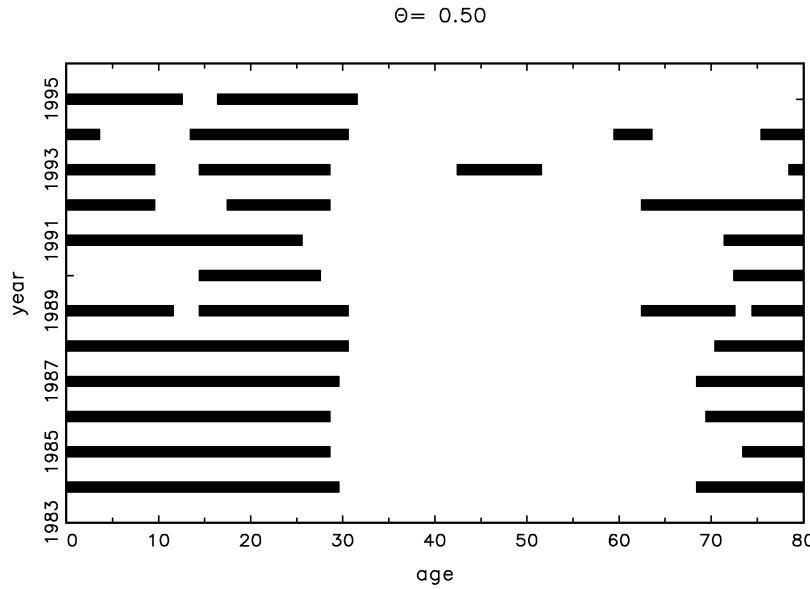


Figure 9: Areas of high poverty risk 1984 – 1995 estimated with the Atkinson poverty measure and  $\theta = 0.81$

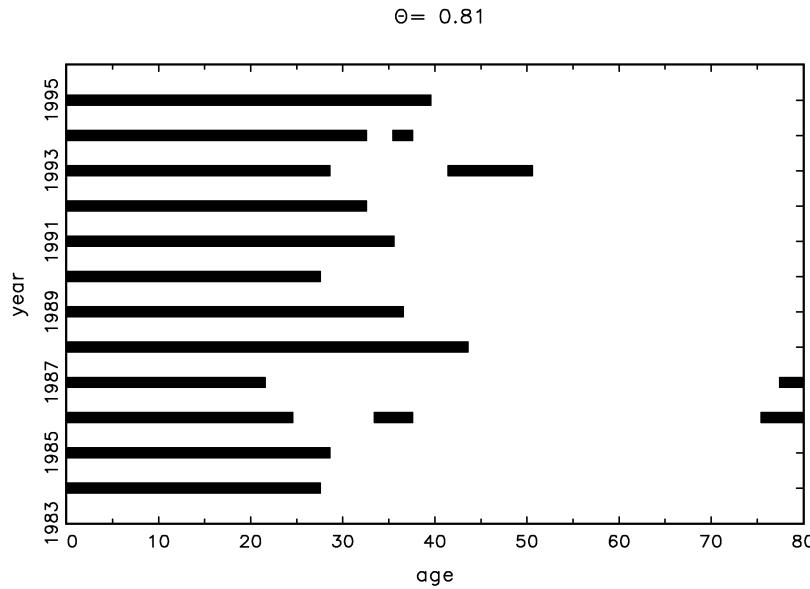
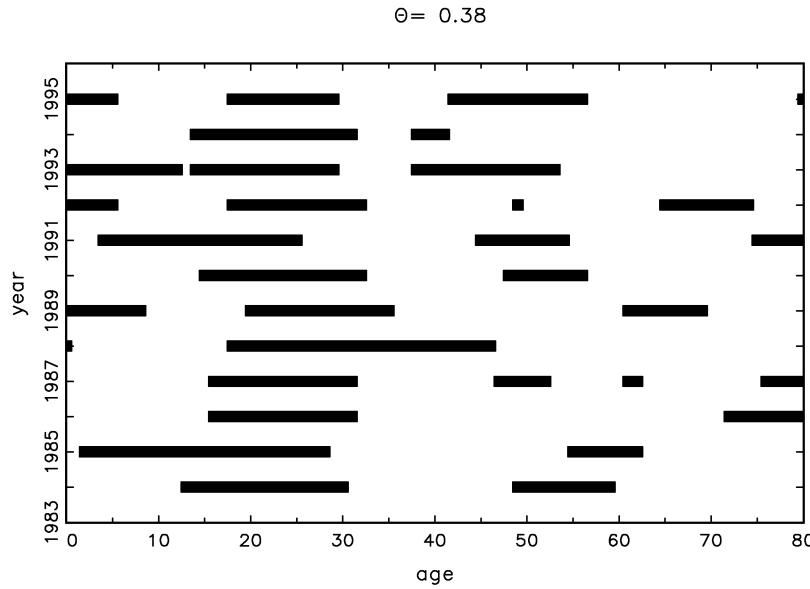


figure 4). This can be seen in table 3 which gives the median estimates for the different choices of the economies of scale parameter, too.

Both, the median estimates and the bandwidths, show a structural break in 1991; this is due to the inclusion of the East German sample which is taken into account since 1991.

Figure 10: Areas of high poverty risk 1984 – 1995 estimated with the Income Gap Ratio and  $\theta = 0.38$



Now the question arises which poverty measures should be computed. Not only the simple measures  $HCR$  and  $IGR$  but also some complex measures should be computed. Taking the behaviour of the disparity measures into account there should be one with high sensitivity in the low income area and one with high sensitivity in the high income area.

In this paper the Blackburn poverty measure with  $\lambda = 2$  is calculated. The Theil measure is very sensitive in the upper area when its parameter is high (see Schmid (1991)). Calculating (2.7) with the Theil measure and  $\lambda = 2$  permits to investigate the influence of the different functional forms of the poverty measures. To make available an inequality measure with high sensitivity in the lower area, the Atkinson measure with  $\varepsilon = 2$  (see Schmid (1991)) is used in (2.7). Furthermore, the poverty measure based on the coefficient of variation as a member of the class of Forster is calculated. Notice that the coefficient of variation is highly sensitive in the upper tail (see Schmid (1991)).

With these indices the age dependent poverty levels as well as the overall poverty levels are calculated. Confidence intervals for the overall indices can be found in tables 10 – 27 (see appendix). They are computed using the marginal density of income in the formulae of the poverty measures.

Accurate observation gives the equal reaction of all poverty measures except the  $IGR$  (see figure 5). The only difference is the niveau of the different measures. Because of this only the Atkinson poverty measure and the  $IGR$  are taken into consideration.

Furthermore, the choice of the poverty line has, except of the level, no influence of the poverty age curve, as can be seen in figure 6. Therefore only the 40% of the median will be used as poverty line.

The age dependent poverty levels and their corresponding 95% confidence bands can be found in figures 14 and 16 – 18 for the Atkinson poverty measures and in figures 15 and

19 – 21 for the *ICR* in the appendix. They are computed up to an age of 80 years since above this age the number of observations is too small (between 164 and 289) to give exact estimates for the poverty measures. The constant lines in the figures are the values of the overall indices and the 95% confidence intervals. So it is possible to compare these indices with the age dependent values.

Because of the difficult interpretation of these figures, the parts of lifetime are indicated during which the risk of poverty is above average in figures 7 – 10 and tables 4 – 9. In the other parts the risk of poverty is below average. In figures 11 – 13 the periods of significantly high and low poverty are given for the Atkinson poverty measure.

As can be seen in the different figures (e.g. figure 7) for  $\theta = 0.38$  and  $\theta = 0.5$  only a few periods with significant deviations can be found. For  $\theta = 0.38$  in single years short periods of significantly low poverty can be located between the age of 40 and 60 and in 4 out of 12 years a significantly high risk of poverty can be identified above an age of 75. The result for  $\theta = 0.5$  is similar, but the times of significant high poverty are not worth mentioning (1995 at an age of 0 – 5 and 1991 at an age of 79 – 80). The spells of significantly low poverty are spread irregularly between the age of 37 and 66 in 5 out of 12 years (see figure 8). Using the BSHG-scale gives a clearer picture. In almost all years (except 1990 and 1994) a significantly high risk of poverty can be found in the first two decades of lifetime. Above an age of 50 there are long spells of low poverty (see figure 9).

Measuring poverty with the *IGR* shows no significant deviations of the poverty age curves from the poverty level of the whole population.

Looking at figures 7 – 10 gives further information on the poverty age curves. The Atkinson curves show for  $\theta = 0.38$  the typical Rowntree type of cycle in almost all years (except 1991 and 1990). There are only small changes in the first periods of over average poverty (1st spell: 0 – 5 in 1984 to 0 – 9 in 1995; 2nd spell: 15 – 29 in 1984 to 18 – 32 in 1995). But the bounds of the third spell change. Until 1992 they remain the same (see table 4) at age 65 and 80 then the spell is shortened in the next 3 years to 79 – 80. A similar behaviour can be found for  $\theta = 0.5$ . But here the first two periods can not be divided until 1989. The third spell is established at older age than for  $\theta = 0.38$ ; it begins at an age of 70 (1989 and 1992 at 63), decreases until 1993 and vanishes 1995. For  $\theta = 0.81$  there is only one spell of outstanding poverty growing from 0 – 28 in 1984 to 0 – 40 in 1995. In old age only in 1986 (age 76 – 80) and 1987 (age 78 – 80) spells of over average poverty risk can be found. Worth mentioning is the period of high poverty at the age spell 43 – 52 in the year 1993. But this deviation is not significant so it will not be considered further on.

Choosing the *IGR* to calculate the poverty age curves changes the picture. Now there can hardly be found a difference in the curves using different values for  $\theta$  (see tables 7 – 9). On the other hand the spells of high poverty change irregularly over time. Only in the third decade of lifetime a period of high poverty risk can be found in all years. But almost all of these deviations are not significant.

It can be summarized that the poverty age curves for Germany follow the Rowntree type of cycle and there are no significant changes in the reported 12 years. The change of the periods of high poverty risk can be explained by the dependence of the economies of scale on  $\theta$ , because of different household sizes at different age levels. Only the BSHG-scale

gives a different result; The risk of poverty decreases with increasing age. This behaviour can be explained by the weighting of the official German equivalence scale. This scale treats one-person households basically different with respect to other equivalence scales (see Burkhauser et al. (1996)). The decreasing poverty in old age is confirmed by the research of Burkhauser et al. (1996), which find lower poverty for older age groups with respect to other groups for  $\theta = 0.81$ , while this relation is reversed for other (lower) values of  $\theta$ . They state:

*Our research suggests that the official German scale is out of line with other measures of economies of scale for Germany or other countries.*

This result can be confirmed by this study.

An interesting aspect is whether the unification of Germany had changed the poverty age curves or not. From table 3 it can be seen that the equivalence income is decreased and from tables 13 – 15 follows that the proportion of the poor is increased, but all other poverty measures show that the poverty is decreased. The form of the poverty age curves is not lasting influenced by the unification, too.

Furthermore the poverty in Germany, measured with the here used indices, has slightly decreased in the observed 12 years. (see tables 10 – 27). Almost all of these changes are significant but the tendency is not unique in the 12 years.

## 5 Conclusion

Calculating age dependent poverty measures for Germany reveals that poverty decreases with increasing age until an age of around 50 years and then increases. This is in coincidence with a priori knowledge about the development of poverty over the life cycle. The different poverty measures behave *all* in the same way. Their poverty age profiles lie in the same ranges above or below the overall measures. Only the degree of poverty is different. This can be seen for all years from 1984 to 1995. This conclusion cannot be drawn for the different equivalence scales. Although all measures react equally for a fixed  $\theta$  the single measures show different behaviour for different  $\theta$ . Because of this the equivalence scale should be chosen carefully, whereas the computation of one complex poverty measure is sufficient to estimate the life cycle effect of poverty.

Further research has to be done with more sophisticated bandwidth selection methods, because in this paper only the rule of thumb is used to compute the bandwidths.

Astonishingly, about a century after the research of Rowntree (1901) the same cycle of poverty can be found. This means that the industrialization, development, social policy and unification which took place during the last 90 years did not essentially change the age profile of poverty.

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## Appendix

Table 4: Areas with high risk of poverty  $\theta = 0.38$  for the years 1984 – 1995 estimated with the Atkinson poverty measure

year	1. area	2. area	3. area	4. area
1984	0 – 5	15 – 29		65 – 80
1985	0 – 13	17 – 29		63 – 80
1986	0 – 13	15 – 29		68 – 80
1987	0 – 6	19 – 30		64 – 80
1988	0 – 3	18 – 31		66 – 80
1989	0 – 9	18 – 30		62 – 80
1990		17 – 28		71 – 80
1991	2 – 11			66 – 80
1992	0 – 7	19 – 29		61 – 80
1993	0 – 7	16 – 29	44 – 51	76 – 80
1994		16 – 31	60 – 65	75 – 80
1995	0 – 9	18 – 32		79 – 80

Table 5: Areas with high risk of poverty  $\theta = 0.5$  for the years 1984 – 1995 estimated with the Atkinson poverty measure

year	1. area	2. area	3. area	4. area
1984	0 – 30		69 – 80	
1985	0 – 29		74 – 80	
1986	0 – 29		70 – 80	
1987	0 – 30		69 – 80	
1988	0 – 31		71 – 80	
1989	0 – 12	15 – 31	63 – 73	75 – 80
1990		15 – 28		73 – 80
1991	0 – 26			72 – 80
1992	0 – 10	18 – 29	63 – 80	
1993	0 – 10	15 – 29	43 – 52	79 – 80
1994	0 – 4	14 – 31	60 – 64	76 – 80
1995	0 – 13	17 – 32		

Table 6: Areas with high risk of poverty  $\theta = 0.81$  for the years 1984 – 1995 estimated with the Atkinson poverty measure

year	1. area	2. area	3. area
1984	0 – 28		
1985	0 – 29		
1986	0 – 25	34 – 38	76 – 80
1987	0 – 22		78 – 80
1988	0 – 44		
1989	0 – 37		

year	1. area	2. area	3. area
1990	0 – 28		
1991	0 – 36		
1992	0 – 33		
1993	0 – 29	42 – 51	
1994	0 – 33	36 – 38	
1995	0 – 40		

Table 7: Areas with high risk of poverty  $\theta = 0.38$  for the years 1984 – 1995 estimated with the Income Gap Ratio

year	1. area	2. area	3. area	4. area
1984	13 – 31	49 – 60		
1985	2 – 29	55 – 63		
1986	16 – 32	72 – 87		
1987	16 – 32	47 – 53	61 – 63	76 – 80
1988	0 – 1	18 – 47		
1989	0 – 9	20 – 36	61 – 70	
1990	15 – 33	48 – 57		
1991	4 – 26	45 – 55	75 – 80	
1992	0 – 6	18 – 33	49 – 50	65 – 75
1993	0 – 13	14 – 30	38 – 54	
1994	14 – 32	38 – 42		
1995	0 – 6	18 – 30	42 – 57	

Table 8: Areas with high risk of poverty  $\theta = 0.5$  for the years 1984 – 1995 estimated with the Income Gap Ratio

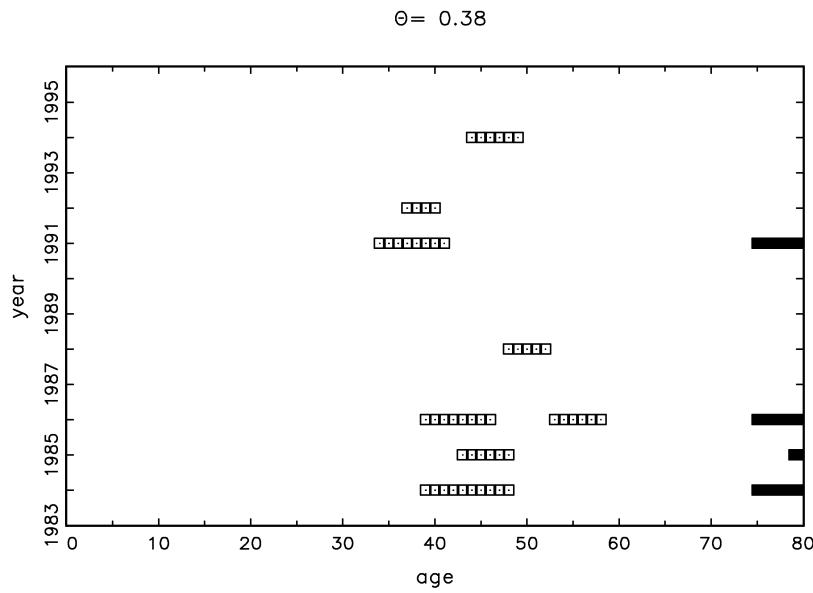
year	1. area	2. area	3. area	4. area
1984	14 – 31	49 – 60		
1985	0 – 28	53 – 64		
1986	16 – 31	46 – 50	72 – 80	
1987	17 – 30	47 – 51	61 – 65	74 – 80
1988	0 – 2	18 – 47		
1989	0 – 8	22 – 35	62 – 71	
1990	16 – 32	47 – 58	79 – 80	
1991	4 – 26	43 – 57	79 – 80	
1992	0 – 7	19 – 34	48 – 51	65 – 75
1993	0 – 11	16 – 29	38 – 54	
1994	15 – 32	37 – 43	60 – 62	77 – 80
1995	0 – 7	19 – 30	42 – 53	79 – 80

Table 9: Areas with high risk of poverty  $\theta = 0.81$  for the years 1984 – 1995 estimated with the Income Gap Ratio

year	1. area	2. area	3. area	4. area

year	1. area	2. area	3. area	4. area
1984	13 – 29	49 – 59	70 – 80	
1985	0 – 27	57 – 66		
1986	0 – 11	18 – 26	70 – 80	
1987	14 – 18	61 – 94		
1988	0 – 2	19 – 32	36 – 47	65 – 75
1989	0 – 8	22 – 34	62 – 74	78 – 80
1990	18 – 30	45 – 59	73 – 80	
1991	7 – 27	43 – 58		
1992	0 – 8	21 – 34	49 – 59	63 – 76
1993	0 – 7	16 – 28	39 – 55	
1994	17 – 34	37 – 42	57 – 63	74 – 80
1995	0 – 9	20 – 32	35 – 47	

Figure 11: Areas of significant high or low poverty risk estimated with the Atkinson poverty measure and  $\theta = 0.38$  1984 – 1995



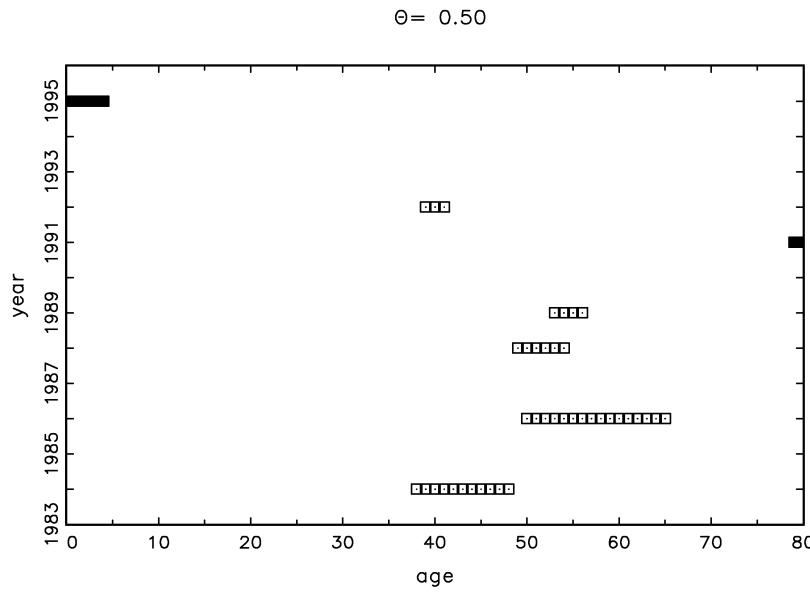
$\square \equiv$  significant low poverty risk at  $\alpha = 0.05$

$\blacksquare \equiv$  significant high poverty risk at  $\alpha = 0.05$ .

Table 10: Confidence intervals for the Atkinson poverty measures for  $\theta = 0.38$  for the years 1984 – 1995

year	$p = 0.4 \cdot \tilde{x}_{HT}$	$p = 0.5 \cdot \tilde{x}_{HT}$	$p = 0.6 \cdot \tilde{x}_{HT}$
1984	[ 0.0232, 0.0288]	[ 0.0386, 0.0464]	[ 0.0590, 0.0684]
1985	[ 0.0184, 0.0227]	[ 0.0307, 0.0365]	[ 0.0516, 0.0597]
1986	[ 0.0235, 0.0279]	[ 0.0362, 0.0421]	[ 0.0580, 0.0658]
1987	[ 0.0162, 0.0204]	[ 0.0305, 0.0371]	[ 0.0477, 0.0561]

Figure 12: Areas of significant high or low poverty risk estimated with the Atkinson poverty measure and  $\theta = 0.5$  1984 – 1995

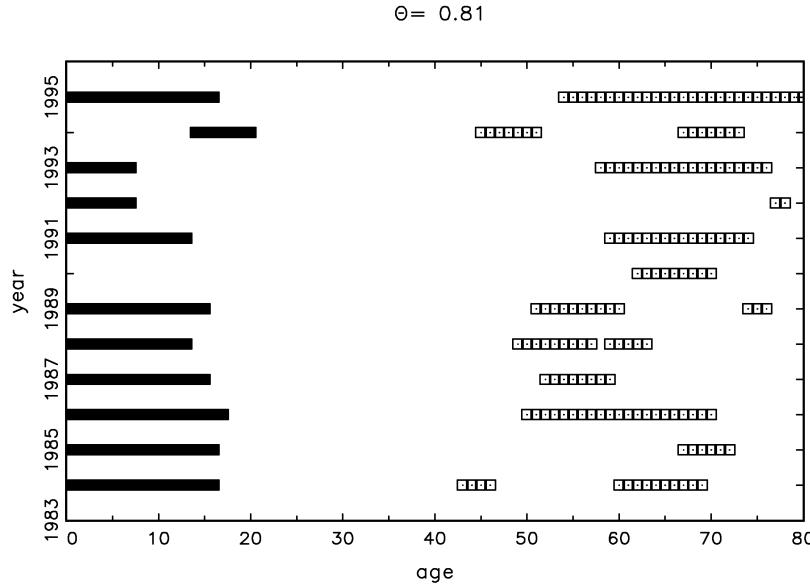


year	$p = 0.4 \cdot \tilde{x}_{HT}$	$p = 0.5 \cdot \tilde{x}_{HT}$	$p = 0.6 \cdot \tilde{x}_{HT}$
1988	[ 0.0162, 0.0219]	[ 0.0276, 0.0340]	[ 0.0471, 0.0573]
1989	[ 0.0137, 0.0188]	[ 0.0256, 0.0329]	[ 0.0426, 0.0526]
1990	[ 0.0163, 0.0222]	[ 0.0304, 0.0391]	[ 0.0470, 0.0579]
1991	[ 0.0141, 0.0176]	[ 0.0273, 0.0325]	[ 0.0468, 0.0541]
1992	[ 0.0128, 0.0175]	[ 0.0245, 0.0307]	[ 0.0420, 0.0500]
1993	[ 0.0133, 0.0201]	[ 0.0240, 0.0326]	[ 0.0412, 0.0530]
1994	[ 0.0124, 0.0176]	[ 0.0237, 0.0312]	[ 0.0405, 0.0508]
1995	[ 0.0105, 0.0136]	[ 0.0203, 0.0253]	[ 0.0374, 0.0442]

Table 11: Confidence intervals for the Atkinson poverty measures for  $\theta = 0.50$  for the years 1984 – 1995

year	$p = 0.4 \cdot \tilde{x}_{HT}$	$p = 0.5 \cdot \tilde{x}_{HT}$	$p = 0.6 \cdot \tilde{x}_{HT}$
1984	[ 0.0197, 0.0246]	[ 0.0339, 0.0408]	[ 0.0540, 0.0631]
1985	[ 0.0153, 0.0192]	[ 0.0287, 0.0347]	[ 0.0478, 0.0561]
1986	[ 0.0218, 0.0261]	[ 0.0376, 0.0438]	[ 0.0594, 0.0677]
1987	[ 0.0147, 0.0188]	[ 0.0285, 0.0348]	[ 0.0455, 0.0535]
1988	[ 0.0138, 0.0183]	[ 0.0267, 0.0337]	[ 0.0432, 0.0514]
1989	[ 0.0129, 0.0177]	[ 0.0239, 0.0310]	[ 0.0406, 0.0502]
1990	[ 0.0154, 0.0212]	[ 0.0280, 0.0368]	[ 0.0439, 0.0546]
1991	[ 0.0127, 0.0159]	[ 0.0253, 0.0302]	[ 0.0446, 0.0513]
1992	[ 0.0114, 0.0154]	[ 0.0225, 0.0281]	[ 0.0397, 0.0470]
1993	[ 0.0120, 0.0179]	[ 0.0216, 0.0290]	[ 0.0383, 0.0484]

Figure 13: Areas of significant high or low poverty risk estimated with the Atkinson poverty measure and  $\theta = 0.81$  1984 – 1995



year	$p = 0.4 \cdot \tilde{x}_{HT}$	$p = 0.5 \cdot \tilde{x}_{HT}$	$p = 0.6 \cdot \tilde{x}_{HT}$
1994	[ 0.0110, 0.0152]	[ 0.0203, 0.0258]	[ 0.0361, 0.0436]
1995	[ 0.0088, 0.0118]	[ 0.0198, 0.0242]	[ 0.0345, 0.0408]

Table 12: Confidence intervals for the Atkinson poverty measures for  $\theta = 0.81$  for the years 1984 – 1995

year	$p = 0.4 \cdot \tilde{x}_{HT}$	$p = 0.5 \cdot \tilde{x}_{HT}$	$p = 0.6 \cdot \tilde{x}_{HT}$
1984	[ 0.0248, 0.0297]	[ 0.0380, 0.0443]	[ 0.0605, 0.0690]
1985	[ 0.0165, 0.0202]	[ 0.0309, 0.0363]	[ 0.0523, 0.0597]
1986	[ 0.0230, 0.0269]	[ 0.0358, 0.0409]	[ 0.0574, 0.0646]
1987	[ 0.0182, 0.0223]	[ 0.0315, 0.0374]	[ 0.0498, 0.0577]
1988	[ 0.0154, 0.0203]	[ 0.0274, 0.0337]	[ 0.0476, 0.0565]
1989	[ 0.0149, 0.0191]	[ 0.0260, 0.0317]	[ 0.0457, 0.0538]
1990	[ 0.0141, 0.0200]	[ 0.0272, 0.0363]	[ 0.0435, 0.0547]
1991	[ 0.0114, 0.0148]	[ 0.0242, 0.0291]	[ 0.0426, 0.0493]
1992	[ 0.0120, 0.0158]	[ 0.0224, 0.0269]	[ 0.0403, 0.0468]
1993	[ 0.0116, 0.0162]	[ 0.0221, 0.0287]	[ 0.0379, 0.0463]
1994	[ 0.0116, 0.0149]	[ 0.0235, 0.0284]	[ 0.0409, 0.0475]
1995	[ 0.0126, 0.0153]	[ 0.0268, 0.0312]	[ 0.0461, 0.0521]

Table 13: Confidence intervals for the Head Count Ratios for  $\theta = 0.38$  for the years 1984 – 1995

year	$p = 0.4 \cdot \tilde{x}_{HT}$	$p = 0.5 \cdot \tilde{x}_{HT}$	$p = 0.6 \cdot \tilde{x}_{HT}$
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year	$p = 0.4 \cdot \bar{x}_{HT}$	$p = 0.5 \cdot \bar{x}_{HT}$	$p = 0.6 \cdot \bar{x}_{HT}$
1984	[ 0.0467, 0.0540]	[ 0.0830, 0.0932]	[ 0.1352, 0.1486]
1985	[ 0.0425, 0.0492]	[ 0.0729, 0.0819]	[ 0.1325, 0.1448]
1986	[ 0.0515, 0.0581]	[ 0.0791, 0.0875]	[ 0.1359, 0.1475]
1987	[ 0.0369, 0.0432]	[ 0.0791, 0.0890]	[ 0.1260, 0.1389]
1988	[ 0.0406, 0.0489]	[ 0.0679, 0.0815]	[ 0.1312, 0.1463]
1989	[ 0.0334, 0.0410]	[ 0.0665, 0.0771]	[ 0.1184, 0.1326]
1990	[ 0.0353, 0.0433]	[ 0.0737, 0.0857]	[ 0.1175, 0.1322]
1991	[ 0.0391, 0.0463]	[ 0.0807, 0.0908]	[ 0.1411, 0.1537]
1992	[ 0.0383, 0.0484]	[ 0.0733, 0.0841]	[ 0.1265, 0.1402]
1993	[ 0.0365, 0.0473]	[ 0.0640, 0.0781]	[ 0.1181, 0.1350]
1994	[ 0.0337, 0.0421]	[ 0.0683, 0.0819]	[ 0.1221, 0.1413]
1995	[ 0.0334, 0.0425]	[ 0.0627, 0.0741]	[ 0.1210, 0.1381]

Table 14: Confidence intervals for the Head Count Ratios for  $\theta = 0.50$  for the years 1984 – 1995

year	$p = 0.4 \cdot \bar{x}_{HT}$	$p = 0.5 \cdot \bar{x}_{HT}$	$p = 0.6 \cdot \bar{x}_{HT}$
1984	[ 0.0387, 0.0447]	[ 0.0727, 0.0809]	[ 0.1234, 0.1343]
1985	[ 0.0352, 0.0414]	[ 0.0734, 0.0826]	[ 0.1330, 0.1449]
1986	[ 0.0472, 0.0533]	[ 0.0889, 0.0978]	[ 0.1494, 0.1611]
1987	[ 0.0341, 0.0400]	[ 0.0750, 0.0844]	[ 0.1219, 0.1338]
1988	[ 0.0361, 0.0436]	[ 0.0767, 0.0889]	[ 0.1198, 0.1358]
1989	[ 0.0314, 0.0386]	[ 0.0648, 0.0755]	[ 0.1197, 0.1336]
1990	[ 0.0346, 0.0420]	[ 0.0723, 0.0833]	[ 0.1163, 0.1299]
1991	[ 0.0353, 0.0417]	[ 0.0762, 0.0853]	[ 0.1379, 0.1493]
1992	[ 0.0326, 0.0411]	[ 0.0652, 0.0779]	[ 0.1174, 0.1338]
1993	[ 0.0342, 0.0460]	[ 0.0603, 0.0732]	[ 0.1142, 0.1304]
1994	[ 0.0335, 0.0425]	[ 0.0574, 0.0693]	[ 0.1074, 0.1240]
1995	[ 0.0255, 0.0359]	[ 0.0690, 0.0802]	[ 0.1056, 0.1257]

Table 15: Confidence intervals for the Head Count Ratios for  $\theta = 0.81$  for the years 1984 – 1995

year	$p = 0.4 \cdot \bar{x}_{HT}$	$p = 0.5 \cdot \bar{x}_{HT}$	$p = 0.6 \cdot \bar{x}_{HT}$
1984	[ 0.0528, 0.0594]	[ 0.0805, 0.0893]	[ 0.1380, 0.1499]
1985	[ 0.0390, 0.0448]	[ 0.0818, 0.0907]	[ 0.1451, 0.1573]
1986	[ 0.0515, 0.0575]	[ 0.0793, 0.0870]	[ 0.1359, 0.1469]
1987	[ 0.0457, 0.0521]	[ 0.0826, 0.0916]	[ 0.1343, 0.1467]
1988	[ 0.0426, 0.0498]	[ 0.0760, 0.0862]	[ 0.1405, 0.1544]
1989	[ 0.0377, 0.0442]	[ 0.0687, 0.0779]	[ 0.1317, 0.1442]
1990	[ 0.0342, 0.0412]	[ 0.0743, 0.0846]	[ 0.1193, 0.1323]
1991	[ 0.0355, 0.0414]	[ 0.0834, 0.0926]	[ 0.1416, 0.1532]
1992	[ 0.0349, 0.0433]	[ 0.0613, 0.0732]	[ 0.1203, 0.1382]
1993	[ 0.0346, 0.0440]	[ 0.0674, 0.0803]	[ 0.1127, 0.1311]

year	$p = 0.4 \cdot \tilde{x}_{HT}$	$p = 0.5 \cdot \tilde{x}_{HT}$	$p = 0.6 \cdot \tilde{x}_{HT}$
1994	[ 0.0360, 0.0434]	[ 0.0789, 0.0905]	[ 0.1321, 0.1475]
1995	[ 0.0318, 0.0439]	[ 0.0886, 0.0989]	[ 0.1477, 0.1613]

Table 16: Confidence intervals for the Income Gap Ratios for  $\theta = 0.38$  for the years 1984 – 1995

year	$p = 0.4 \cdot \tilde{x}_{HT}$	$p = 0.5 \cdot \tilde{x}_{HT}$	$p = 0.6 \cdot \tilde{x}_{HT}$
1984	[ 0.2585, 0.2904]	[ 0.2526, 0.2785]	[ 0.2517, 0.2722]
1985	[ 0.2375, 0.2570]	[ 0.2573, 0.2736]	[ 0.2375, 0.2521]
1986	[ 0.2373, 0.2544]	[ 0.2772, 0.2912]	[ 0.2608, 0.2727]
1987	[ 0.2545, 0.2767]	[ 0.2170, 0.2352]	[ 0.2354, 0.2501]
1988	[ 0.2218, 0.2451]	[ 0.2484, 0.2916]	[ 0.2296, 0.2481]
1989	[ 0.2403, 0.2677]	[ 0.2275, 0.2523]	[ 0.2214, 0.2420]
1990	[ 0.2671, 0.2976]	[ 0.2281, 0.2526]	[ 0.2426, 0.2631]
1991	[ 0.2373, 0.2546]	[ 0.2289, 0.2461]	[ 0.2280, 0.2437]
1992	[ 0.1975, 0.2390]	[ 0.2178, 0.2465]	[ 0.2228, 0.2479]
1993	[ 0.2127, 0.2498]	[ 0.2391, 0.2822]	[ 0.2313, 0.2590]
1994	[ 0.2157, 0.2574]	[ 0.2104, 0.2442]	[ 0.2088, 0.2362]
1995	[ 0.1951, 0.2350]	[ 0.2317, 0.2616]	[ 0.2127, 0.2419]

Table 17: Confidence intervals for the Income Gap Ratios for  $\theta = 0.50$  for the years 1984 – 1995

year	$p = 0.4 \cdot \tilde{x}_{HT}$	$p = 0.5 \cdot \tilde{x}_{HT}$	$p = 0.6 \cdot \tilde{x}_{HT}$
1984	[ 0.2905, 0.3205]	[ 0.2697, 0.2941]	[ 0.2594, 0.2792]
1985	[ 0.2554, 0.2751]	[ 0.2301, 0.2474]	[ 0.2186, 0.2334]
1986	[ 0.2489, 0.2662]	[ 0.2331, 0.2474]	[ 0.2275, 0.2397]
1987	[ 0.2585, 0.2810]	[ 0.2214, 0.2394]	[ 0.2398, 0.2540]
1988	[ 0.2262, 0.2483]	[ 0.2053, 0.2253]	[ 0.2341, 0.2641]
1989	[ 0.2304, 0.2579]	[ 0.2152, 0.2382]	[ 0.2068, 0.2268]
1990	[ 0.2392, 0.2728]	[ 0.2098, 0.2360]	[ 0.2310, 0.2516]
1991	[ 0.2354, 0.2554]	[ 0.2255, 0.2430]	[ 0.2241, 0.2389]
1992	[ 0.2235, 0.2623]	[ 0.2292, 0.2607]	[ 0.2322, 0.2577]
1993	[ 0.1924, 0.2402]	[ 0.2351, 0.2762]	[ 0.2250, 0.2549]
1994	[ 0.1823, 0.2341]	[ 0.2329, 0.2630]	[ 0.2287, 0.2506]
1995	[ 0.2258, 0.2744]	[ 0.2018, 0.2251]	[ 0.2307, 0.2665]

Table 18: Confidence intervals for the Income Gap Ratios for  $\theta = 0.81$  for the years 1984 – 1995

year	$p = 0.4 \cdot \tilde{x}_{HT}$	$p = 0.5 \cdot \tilde{x}_{HT}$	$p = 0.6 \cdot \tilde{x}_{HT}$
1984	[ 0.2372, 0.2605]	[ 0.2770, 0.2953]	[ 0.2608, 0.2755]
1985	[ 0.2533, 0.2704]	[ 0.2327, 0.2469]	[ 0.2261, 0.2387]
1986	[ 0.2333, 0.2481]	[ 0.2743, 0.2865]	[ 0.2598, 0.2702]

year	$p = 0.4 \cdot \tilde{x}_{HT}$	$p = 0.5 \cdot \tilde{x}_{HT}$	$p = 0.6 \cdot \tilde{x}_{HT}$
1987	[ 0.2089, 0.2261]	[ 0.2210, 0.2356]	[ 0.2334, 0.2462]
1988	[ 0.2012, 0.2197]	[ 0.2283, 0.2443]	[ 0.2160, 0.2310]
1989	[ 0.2151, 0.2377]	[ 0.2358, 0.2544]	[ 0.2187, 0.2341]
1990	[ 0.2371, 0.2712]	[ 0.2106, 0.2346]	[ 0.2363, 0.2546]
1991	[ 0.2195, 0.2427]	[ 0.1929, 0.2110]	[ 0.2129, 0.2272]
1992	[ 0.1940, 0.2334]	[ 0.2324, 0.2677]	[ 0.2163, 0.2440]
1993	[ 0.1973, 0.2304]	[ 0.2084, 0.2399]	[ 0.2251, 0.2580]
1994	[ 0.2103, 0.2349]	[ 0.1963, 0.2164]	[ 0.2175, 0.2346]
1995	[ 0.2396, 0.2921]	[ 0.2058, 0.2192]	[ 0.2197, 0.2331]

Table 19: Confidence intervals for the Blackburn poverty measures for  $\theta = 0.38$  for the years 1984 – 1995

year	$p = 0.4 \cdot \tilde{x}_{HT}$	$p = 0.5 \cdot \tilde{x}_{HT}$	$p = 0.6 \cdot \tilde{x}_{HT}$
1984	[ 0.0171, 0.0212]	[ 0.0291, 0.0344]	[ 0.0461, 0.0526]
1985	[ 0.0136, 0.0169]	[ 0.0249, 0.0295]	[ 0.0415, 0.0476]
1986	[ 0.0168, 0.0201]	[ 0.0295, 0.0341]	[ 0.0471, 0.0531]
1987	[ 0.0126, 0.0159]	[ 0.0227, 0.0273]	[ 0.0388, 0.0449]
1988	[ 0.0121, 0.0154]	[ 0.0231, 0.0281]	[ 0.0394, 0.0462]
1989	[ 0.0106, 0.0145]	[ 0.0199, 0.0254]	[ 0.0342, 0.0412]
1990	[ 0.0128, 0.0173]	[ 0.0226, 0.0285]	[ 0.0376, 0.0453]
1991	[ 0.0120, 0.0151]	[ 0.0238, 0.0285]	[ 0.0415, 0.0479]
1992	[ 0.0106, 0.0138]	[ 0.0211, 0.0258]	[ 0.0371, 0.0434]
1993	[ 0.0101, 0.0151]	[ 0.0201, 0.0274]	[ 0.0354, 0.0447]
1994	[ 0.0099, 0.0135]	[ 0.0195, 0.0245]	[ 0.0342, 0.0410]
1995	[ 0.0092, 0.0120]	[ 0.0190, 0.0236]	[ 0.0344, 0.0407]

Table 20: Confidence intervals for the Blackburn poverty measures for  $\theta = 0.50$  for the years 1984 – 1995

year	$p = 0.4 \cdot \tilde{x}_{HT}$	$p = 0.5 \cdot \tilde{x}_{HT}$	$p = 0.6 \cdot \tilde{x}_{HT}$
1984	[ 0.0157, 0.0194]	[ 0.0266, 0.0317]	[ 0.0428, 0.0491]
1985	[ 0.0120, 0.0151]	[ 0.0223, 0.0267]	[ 0.0378, 0.0437]
1986	[ 0.0161, 0.0193]	[ 0.0279, 0.0322]	[ 0.0451, 0.0509]
1987	[ 0.0117, 0.0149]	[ 0.0218, 0.0262]	[ 0.0378, 0.0438]
1988	[ 0.0108, 0.0139]	[ 0.0208, 0.0254]	[ 0.0369, 0.0434]
1989	[ 0.0096, 0.0132]	[ 0.0182, 0.0232]	[ 0.0320, 0.0385]
1990	[ 0.0112, 0.0154]	[ 0.0201, 0.0257]	[ 0.0351, 0.0421]
1991	[ 0.0108, 0.0136]	[ 0.0220, 0.0263]	[ 0.0395, 0.0452]
1992	[ 0.0100, 0.0130]	[ 0.0202, 0.0248]	[ 0.0363, 0.0425]
1993	[ 0.0092, 0.0136]	[ 0.0185, 0.0251]	[ 0.0333, 0.0417]
1994	[ 0.0088, 0.0121]	[ 0.0180, 0.0227]	[ 0.0325, 0.0387]
1995	[ 0.0085, 0.0111]	[ 0.0180, 0.0221]	[ 0.0334, 0.0395]

Table 21: Confidence intervals for the Blackburn poverty measures for  $\theta = 0.81$  for the years 1984 – 1995

year	$p = 0.4 \cdot \tilde{x}_{HT}$	$p = 0.5 \cdot \tilde{x}_{HT}$	$p = 0.6 \cdot \tilde{x}_{HT}$
1984	[ 0.0176, 0.0210]	[ 0.0306, 0.0350]	[ 0.0485, 0.0542]
1985	[ 0.0131, 0.0159]	[ 0.0248, 0.0288]	[ 0.0426, 0.0480]
1986	[ 0.0164, 0.0193]	[ 0.0292, 0.0333]	[ 0.0469, 0.0524]
1987	[ 0.0128, 0.0157]	[ 0.0239, 0.0282]	[ 0.0408, 0.0466]
1988	[ 0.0114, 0.0141]	[ 0.0227, 0.0269]	[ 0.0395, 0.0455]
1989	[ 0.0108, 0.0138]	[ 0.0211, 0.0255]	[ 0.0371, 0.0430]
1990	[ 0.0106, 0.0147]	[ 0.0204, 0.0258]	[ 0.0363, 0.0433]
1991	[ 0.0099, 0.0128]	[ 0.0205, 0.0247]	[ 0.0382, 0.0438]
1992	[ 0.0096, 0.0121]	[ 0.0197, 0.0234]	[ 0.0355, 0.0404]
1993	[ 0.0092, 0.0129]	[ 0.0184, 0.0241]	[ 0.0335, 0.0412]
1994	[ 0.0100, 0.0128]	[ 0.0203, 0.0243]	[ 0.0371, 0.0429]
1995	[ 0.0118, 0.0144]	[ 0.0234, 0.0275]	[ 0.0418, 0.0475]

Table 22: Confidence intervals for the Theil poverty measures for  $\theta = 0.38$  for the years 1984 – 1995

year	$p = 0.4 \cdot \tilde{x}_{HT}$	$p = 0.5 \cdot \tilde{x}_{HT}$	$p = 0.6 \cdot \tilde{x}_{HT}$
1984	[ 0.0144, 0.0176]	[ 0.0246, 0.0288]	[ 0.0392, 0.0443]
1985	[ 0.0117, 0.0143]	[ 0.0212, 0.0249]	[ 0.0358, 0.0407]
1986	[ 0.0144, 0.0171]	[ 0.0247, 0.0283]	[ 0.0399, 0.0447]
1987	[ 0.0107, 0.0133]	[ 0.0199, 0.0235]	[ 0.0335, 0.0384]
1988	[ 0.0105, 0.0132]	[ 0.0198, 0.0238]	[ 0.0341, 0.0396]
1989	[ 0.0091, 0.0122]	[ 0.0173, 0.0217]	[ 0.0298, 0.0354]
1990	[ 0.0108, 0.0143]	[ 0.0196, 0.0243]	[ 0.0323, 0.0384]
1991	[ 0.0104, 0.0129]	[ 0.0206, 0.0245]	[ 0.0361, 0.0413]
1992	[ 0.0093, 0.0120]	[ 0.0184, 0.0222]	[ 0.0322, 0.0373]
1993	[ 0.0089, 0.0130]	[ 0.0173, 0.0231]	[ 0.0307, 0.0381]
1994	[ 0.0087, 0.0116]	[ 0.0170, 0.0211]	[ 0.0300, 0.0354]
1995	[ 0.0080, 0.0104]	[ 0.0165, 0.0203]	[ 0.0300, 0.0352]

Table 23: Confidence intervals for the Theil poverty measures for  $\theta = 0.50$  for the years 1984 – 1995

year	$p = 0.4 \cdot \tilde{x}_{HT}$	$p = 0.5 \cdot \tilde{x}_{HT}$	$p = 0.6 \cdot \tilde{x}_{HT}$
1984	[ 0.0129, 0.0158]	[ 0.0224, 0.0263]	[ 0.0364, 0.0412]
1985	[ 0.0102, 0.0127]	[ 0.0193, 0.0228]	[ 0.0330, 0.0379]
1986	[ 0.0137, 0.0163]	[ 0.0241, 0.0276]	[ 0.0391, 0.0437]
1987	[ 0.0100, 0.0125]	[ 0.0190, 0.0225]	[ 0.0326, 0.0374]
1988	[ 0.0094, 0.0119]	[ 0.0183, 0.0221]	[ 0.0319, 0.0372]
1989	[ 0.0083, 0.0112]	[ 0.0159, 0.0200]	[ 0.0282, 0.0335]
1990	[ 0.0096, 0.0129]	[ 0.0177, 0.0222]	[ 0.0304, 0.0360]
1991	[ 0.0094, 0.0116]	[ 0.0192, 0.0226]	[ 0.0345, 0.0390]
1992	[ 0.0086, 0.0111]	[ 0.0175, 0.0212]	[ 0.0314, 0.0364]
1993	[ 0.0082, 0.0118]	[ 0.0160, 0.0213]	[ 0.0290, 0.0357]
1994	[ 0.0078, 0.0105]	[ 0.0156, 0.0194]	[ 0.0282, 0.0332]

year	$p = 0.4 \cdot \tilde{x}_{HT}$	$p = 0.5 \cdot \tilde{x}_{HT}$	$p = 0.6 \cdot \tilde{x}_{HT}$
1995	[ 0.0074, 0.0095]	[ 0.0159, 0.0193]	[ 0.0289, 0.0338]

Table 24: Confidence intervals for the Theil poverty measures for  $\theta = 0.81$  for the years 1984 – 1995

year	$p = 0.4 \cdot \tilde{x}_{HT}$	$p = 0.5 \cdot \tilde{x}_{HT}$	$p = 0.6 \cdot \tilde{x}_{HT}$
1984	[ 0.0150, 0.0177]	[ 0.0255, 0.0290]	[ 0.0410, 0.0456]
1985	[ 0.0112, 0.0134]	[ 0.0214, 0.0247]	[ 0.0370, 0.0414]
1986	[ 0.0141, 0.0165]	[ 0.0246, 0.0277]	[ 0.0398, 0.0442]
1987	[ 0.0112, 0.0136]	[ 0.0208, 0.0244]	[ 0.0353, 0.0400]
1988	[ 0.0101, 0.0123]	[ 0.0197, 0.0231]	[ 0.0345, 0.0394]
1989	[ 0.0095, 0.0119]	[ 0.0182, 0.0217]	[ 0.0324, 0.0372]
1990	[ 0.0092, 0.0124]	[ 0.0179, 0.0223]	[ 0.0313, 0.0370]
1991	[ 0.0087, 0.0111]	[ 0.0182, 0.0217]	[ 0.0335, 0.0381]
1992	[ 0.0085, 0.0106]	[ 0.0171, 0.0200]	[ 0.0309, 0.0351]
1993	[ 0.0081, 0.0112]	[ 0.0162, 0.0209]	[ 0.0292, 0.0354]
1994	[ 0.0088, 0.0111]	[ 0.0179, 0.0214]	[ 0.0325, 0.0373]
1995	[ 0.0101, 0.0122]	[ 0.0207, 0.0240]	[ 0.0365, 0.0411]

Table 25: Confidence intervals for the Coefficient of Variation poverty measures for  $\theta = 0.38$  for the years 1984 – 1995

year	$p = 0.4 \cdot \tilde{x}_{HT}$	$p = 0.5 \cdot \tilde{x}_{HT}$	$p = 0.6 \cdot \tilde{x}_{HT}$
1984	[ 0.0061, 0.0078]	[ 0.0101, 0.0122]	[ 0.0153, 0.0180]
1985	[ 0.0047, 0.0060]	[ 0.0081, 0.0099]	[ 0.0135, 0.0158]
1986	[ 0.0061, 0.0074]	[ 0.0099, 0.0117]	[ 0.0156, 0.0179]
1987	[ 0.0042, 0.0055]	[ 0.0076, 0.0094]	[ 0.0124, 0.0147]
1988	[ 0.0041, 0.0053]	[ 0.0074, 0.0092]	[ 0.0125, 0.0150]
1989	[ 0.0035, 0.0050]	[ 0.0065, 0.0086]	[ 0.0109, 0.0137]
1990	[ 0.0043, 0.0061]	[ 0.0076, 0.0101]	[ 0.0122, 0.0152]
1991	[ 0.0038, 0.0049]	[ 0.0074, 0.0091]	[ 0.0129, 0.0153]
1992	[ 0.0033, 0.0046]	[ 0.0066, 0.0084]	[ 0.0115, 0.0139]
1993	[ 0.0033, 0.0052]	[ 0.0063, 0.0090]	[ 0.0109, 0.0146]
1994	[ 0.0032, 0.0046]	[ 0.0062, 0.0081]	[ 0.0107, 0.0133]
1995	[ 0.0028, 0.0038]	[ 0.0057, 0.0073]	[ 0.0104, 0.0127]

Table 26: Confidence intervals for the Coefficient of Variation poverty measures for  $\theta = 0.50$  for the years 1984 – 1995

year	$p = 0.4 \cdot \tilde{x}_{HT}$	$p = 0.5 \cdot \tilde{x}_{HT}$	$p = 0.6 \cdot \tilde{x}_{HT}$
1984	[ 0.0054, 0.0069]	[ 0.0090, 0.0110]	[ 0.0142, 0.0167]
1985	[ 0.0040, 0.0052]	[ 0.0073, 0.0089]	[ 0.0122, 0.0144]
1986	[ 0.0057, 0.0070]	[ 0.0096, 0.0113]	[ 0.0152, 0.0174]
1987	[ 0.0039, 0.0051]	[ 0.0071, 0.0088]	[ 0.0119, 0.0141]

year	$p = 0.4 \cdot \tilde{x}_{HT}$	$p = 0.5 \cdot \tilde{x}_{HT}$	$p = 0.6 \cdot \tilde{x}_{HT}$
1988	[ 0.0035, 0.0046]	[ 0.0067, 0.0084]	[ 0.0115, 0.0138]
1989	[ 0.0032, 0.0046]	[ 0.0059, 0.0078]	[ 0.0101, 0.0126]
1990	[ 0.0038, 0.0056]	[ 0.0068, 0.0091]	[ 0.0111, 0.0139]
1991	[ 0.0034, 0.0044]	[ 0.0067, 0.0083]	[ 0.0121, 0.0142]
1992	[ 0.0030, 0.0042]	[ 0.0062, 0.0078]	[ 0.0110, 0.0133]
1993	[ 0.0030, 0.0047]	[ 0.0058, 0.0082]	[ 0.0101, 0.0134]
1994	[ 0.0029, 0.0042]	[ 0.0055, 0.0073]	[ 0.0098, 0.0122]
1995	[ 0.0025, 0.0034]	[ 0.0054, 0.0068]	[ 0.0099, 0.0121]

Table 27: Confidence intervals for the Coefficient of Variation poverty measures for  $\theta = 0.81$  for the years 1984 – 1995

year	$p = 0.4 \cdot \tilde{x}_{HT}$	$p = 0.5 \cdot \tilde{x}_{HT}$	$p = 0.6 \cdot \tilde{x}_{HT}$
1984	[ 0.0064, 0.0078]	[ 0.0103, 0.0121]	[ 0.0162, 0.0184]
1985	[ 0.0043, 0.0053]	[ 0.0079, 0.0094]	[ 0.0135, 0.0156]
1986	[ 0.0059, 0.0071]	[ 0.0098, 0.0113]	[ 0.0155, 0.0176]
1987	[ 0.0045, 0.0057]	[ 0.0079, 0.0096]	[ 0.0131, 0.0153]
1988	[ 0.0038, 0.0048]	[ 0.0071, 0.0087]	[ 0.0124, 0.0146]
1989	[ 0.0036, 0.0048]	[ 0.0066, 0.0083]	[ 0.0116, 0.0138]
1990	[ 0.0035, 0.0051]	[ 0.0066, 0.0088]	[ 0.0112, 0.0140]
1991	[ 0.0031, 0.0042]	[ 0.0063, 0.0079]	[ 0.0115, 0.0136]
1992	[ 0.0031, 0.0040]	[ 0.0060, 0.0074]	[ 0.0108, 0.0126]
1993	[ 0.0030, 0.0044]	[ 0.0058, 0.0079]	[ 0.0101, 0.0130]
1994	[ 0.0032, 0.0042]	[ 0.0063, 0.0078]	[ 0.0113, 0.0134]
1995	[ 0.0036, 0.0045]	[ 0.0073, 0.0087]	[ 0.0129, 0.0150]

Figure 14: Atkinson poverty measures for 1984, 1994 and 1995

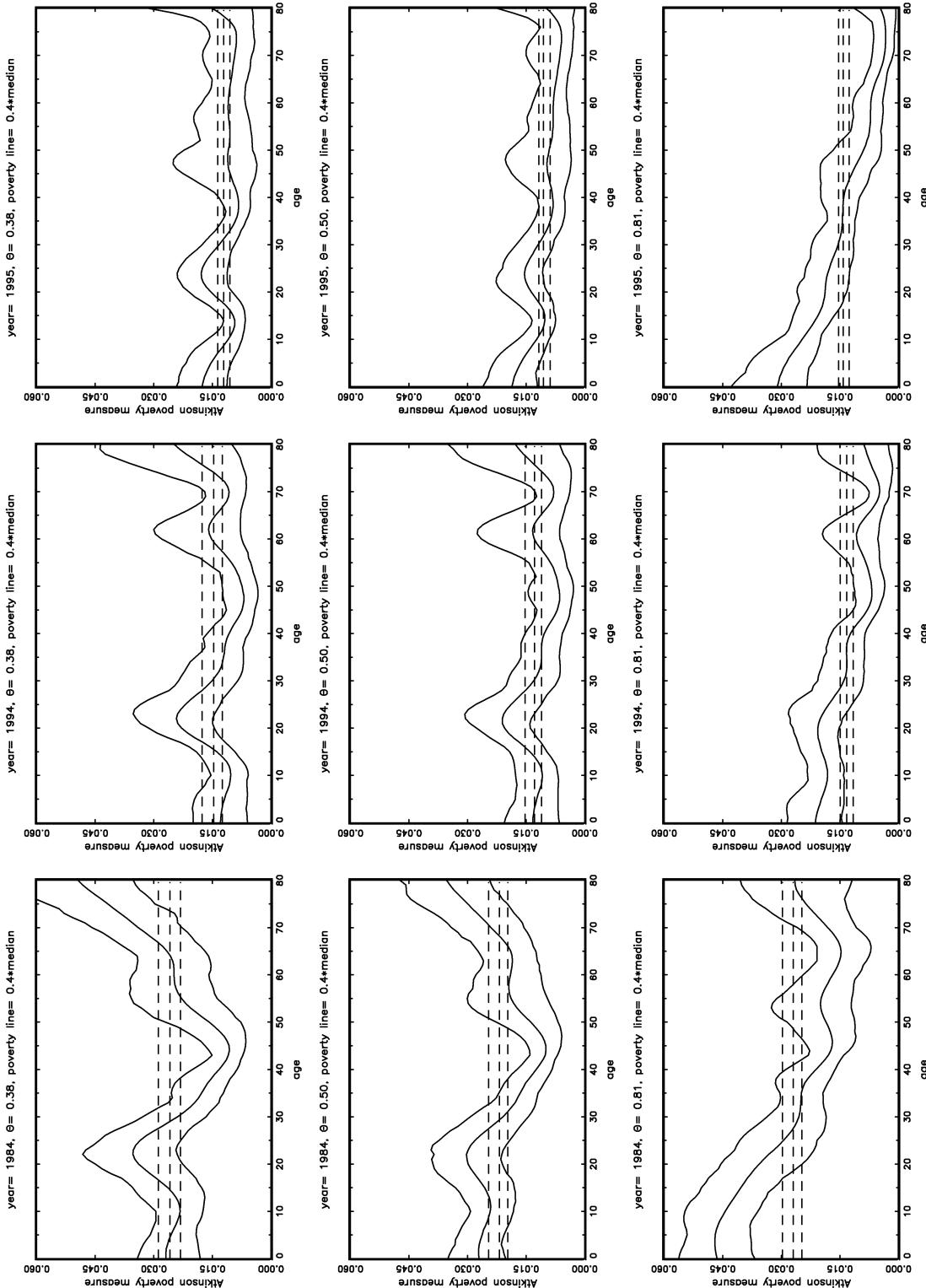


Figure 15: Income Gap Ratios for 1984, 1994 and 1995

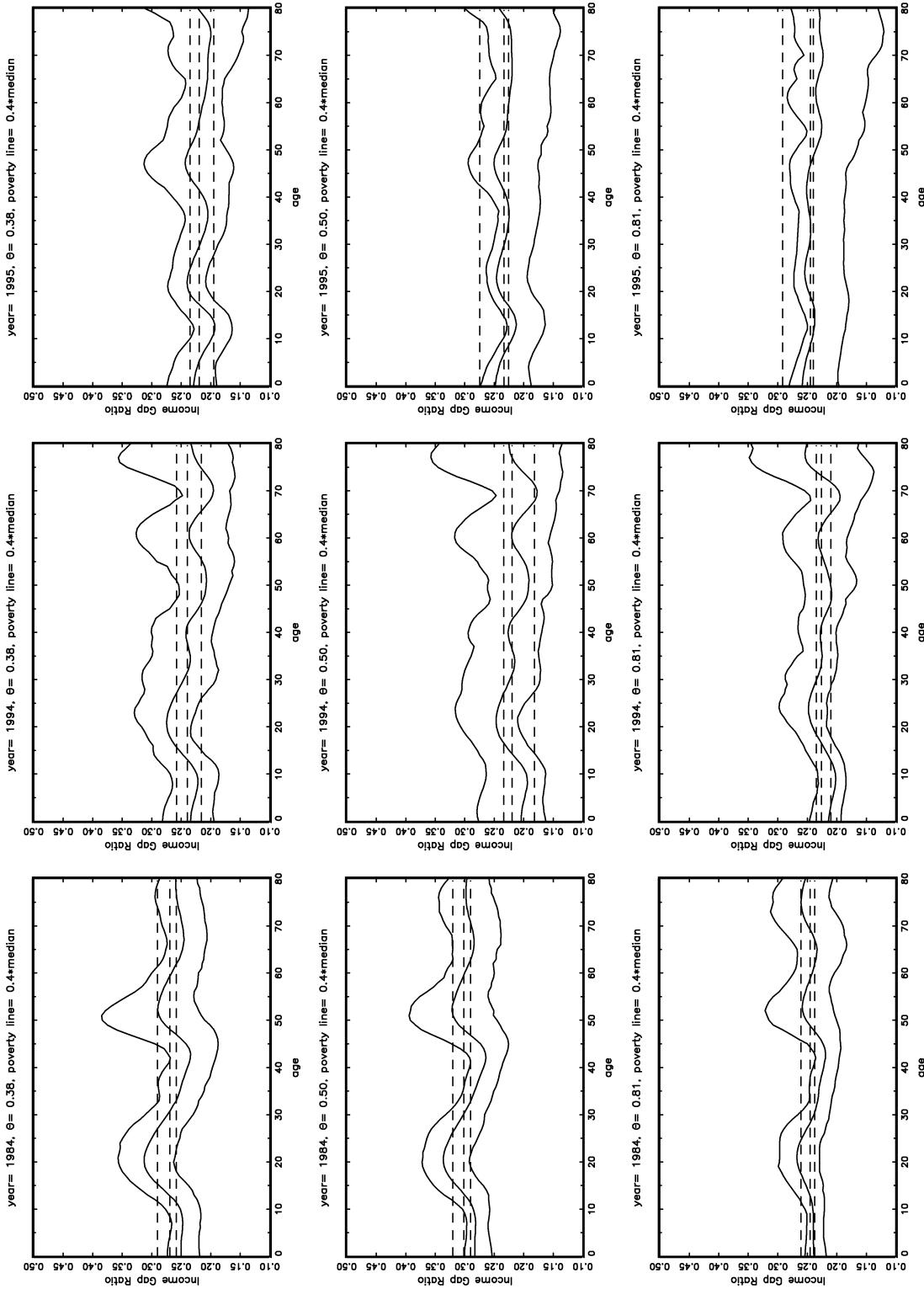


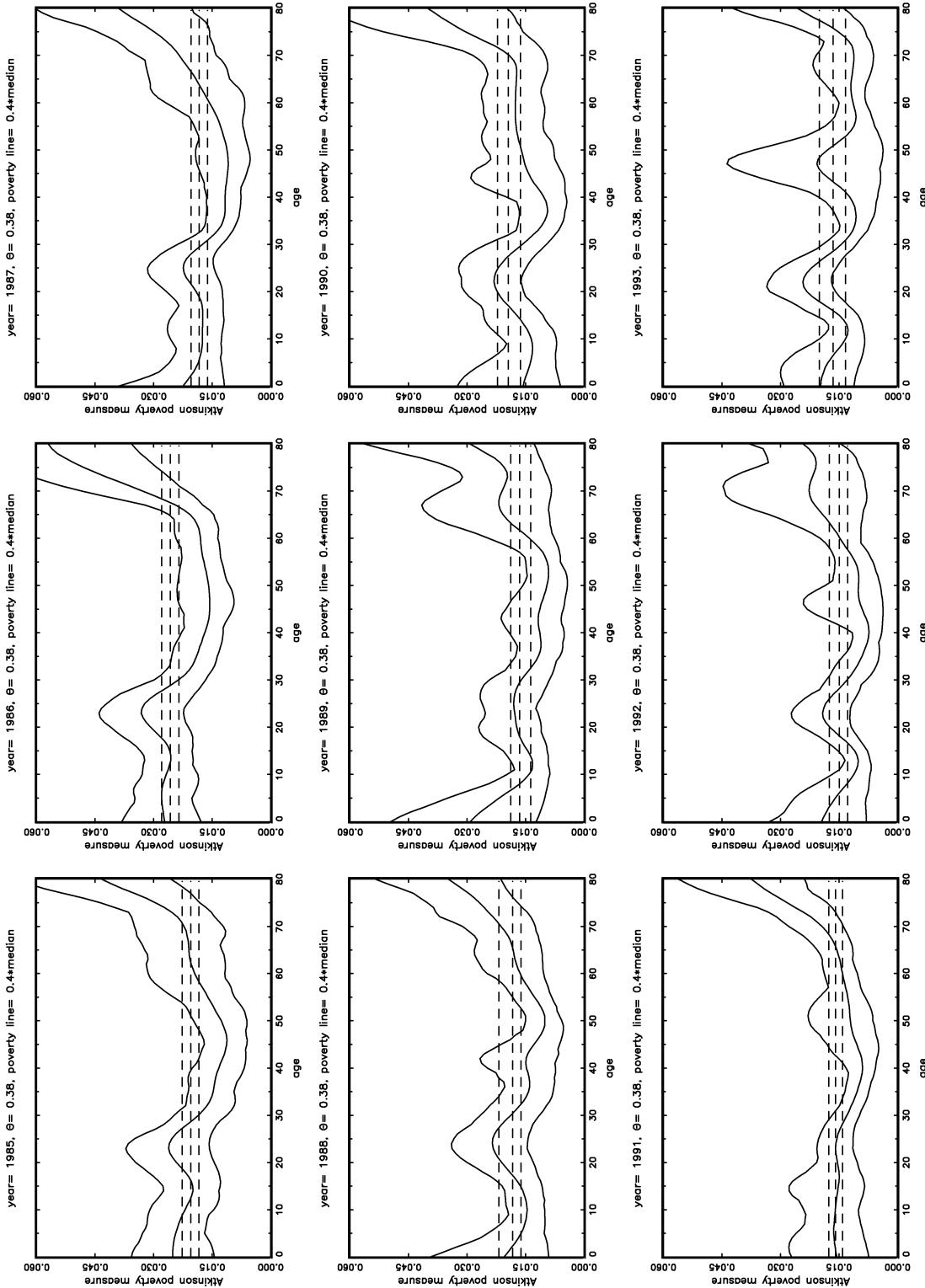
Figure 16: Atkinson poverty measures for  $\theta = 0.38$ ,  $p = 0.4 \cdot \tilde{x}_{HT}$  for 1985 – 1993

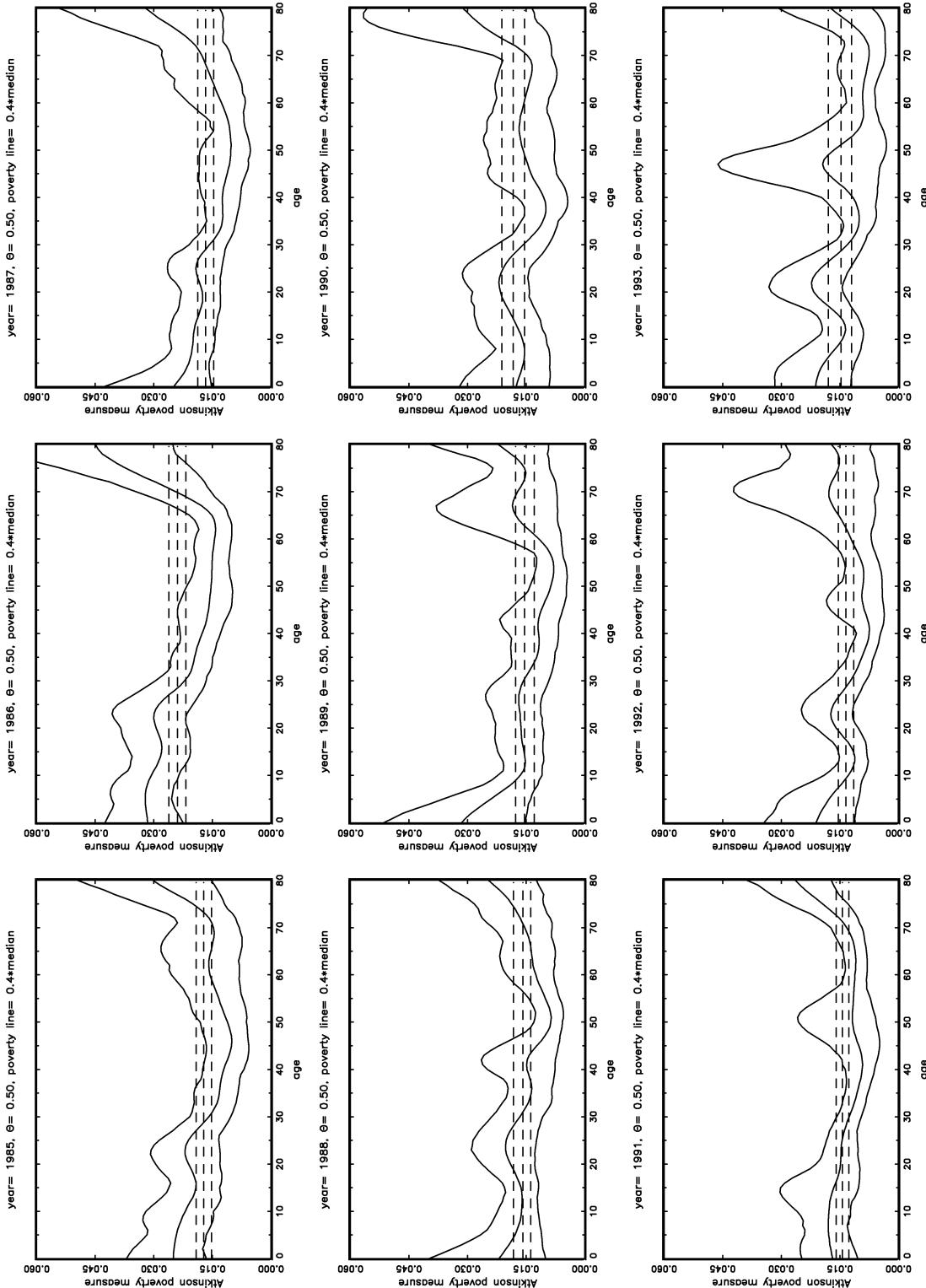
Figure 17: Atkinson poverty measures for  $\theta = 0.50$ ,  $p = 0.4 \cdot \tilde{x}_{HT}$  for 1985 – 1993

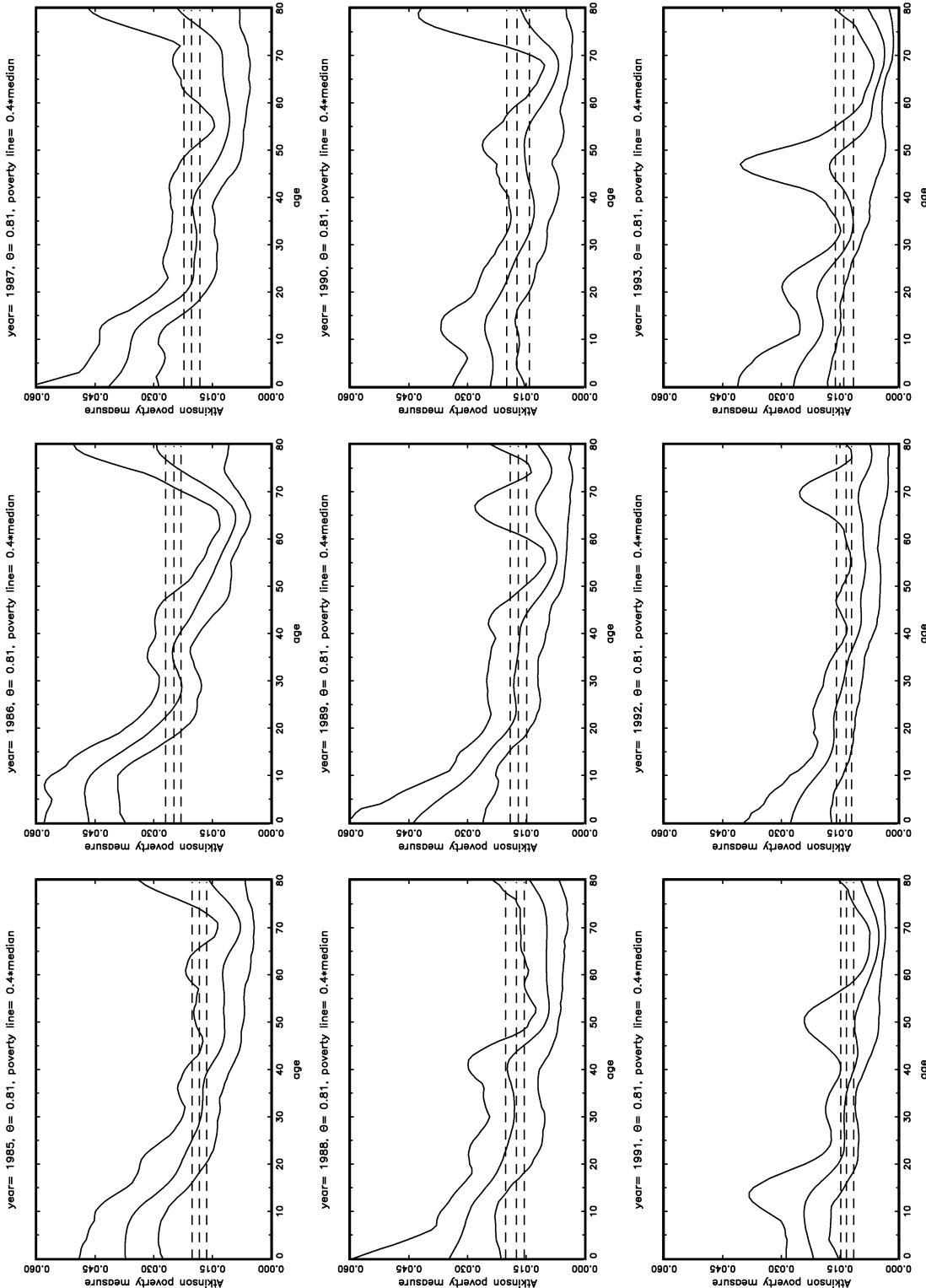
Figure 18: Atkinson poverty measures for  $\theta = 0.81$ ,  $p = 0.4 \cdot \tilde{x}_{HT}$  for 1985 – 1993

Figure 19: Income Gap Ratios poverty measures for  $\theta = 0.38$ ,  $p = 0.4 \cdot \tilde{x}_{HT}$  for 1985 – 1993

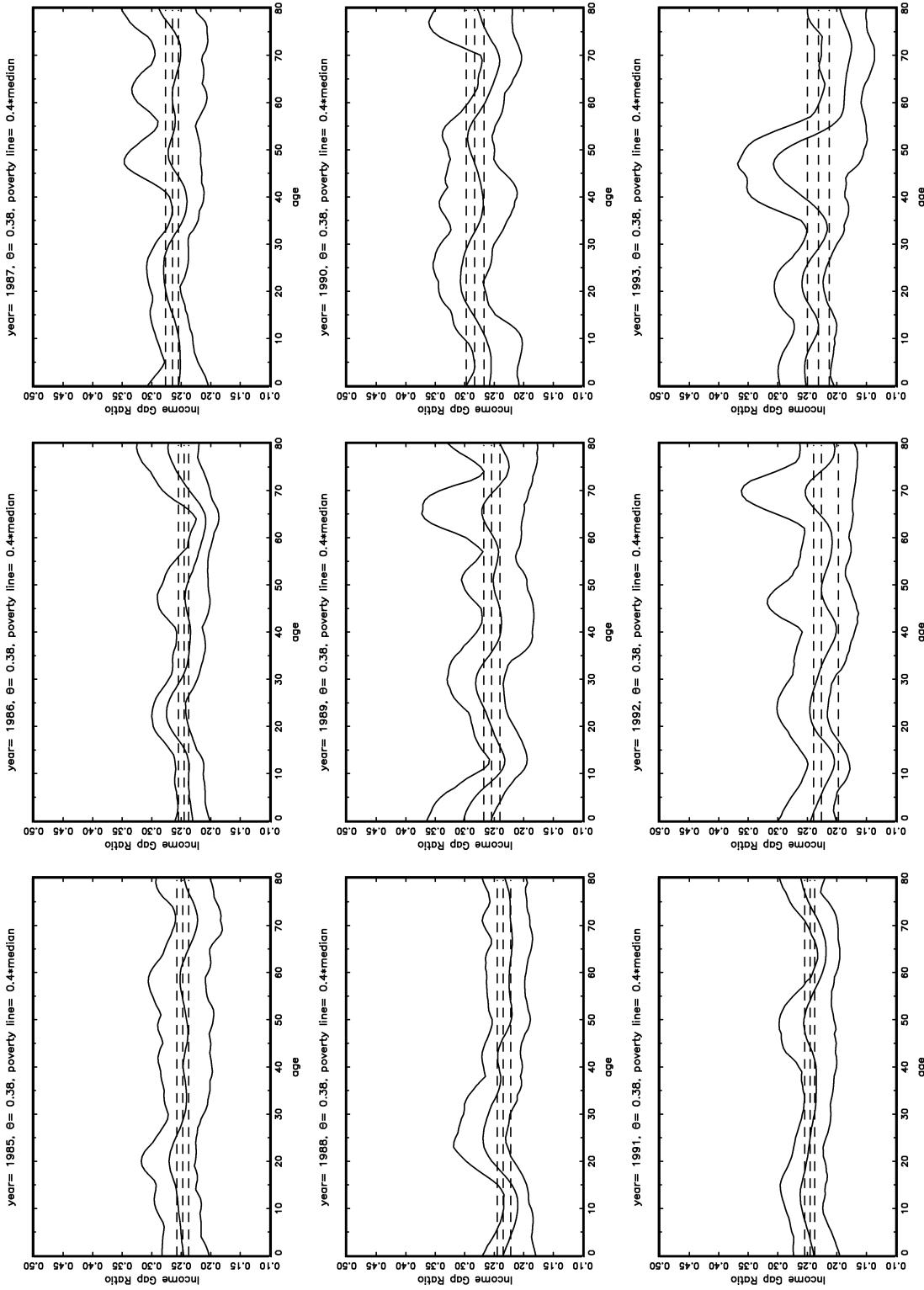


Figure 20: Income Gap Ratios poverty measures for  $\theta = 0.50$ ,  $p = 0.4 \cdot \tilde{x}_{HT}$  for 1985 – 1993

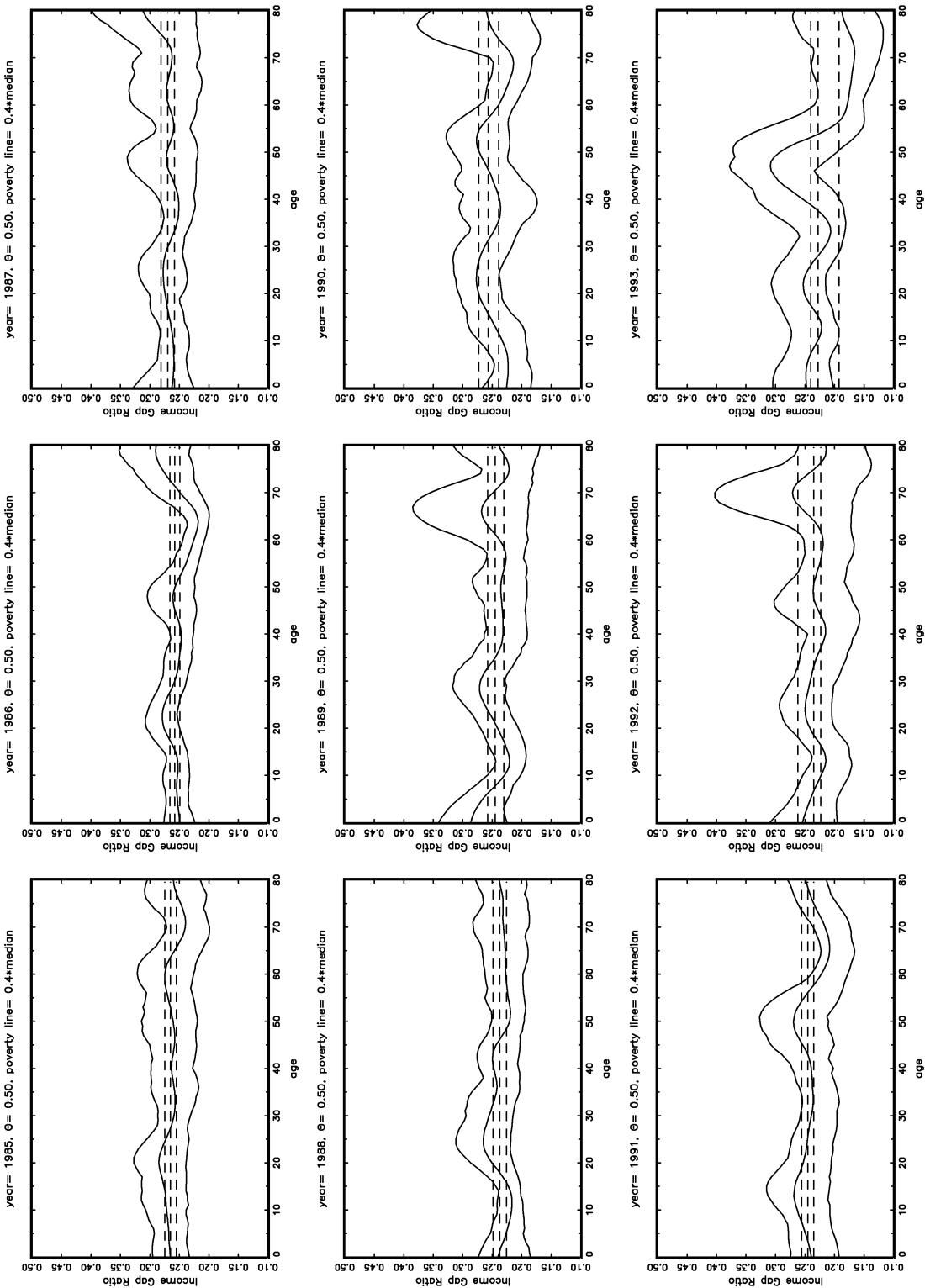


Figure 21: Income Gap Ratios poverty measures for  $\theta = 0.81$ ,  $p = 0.4 \cdot \tilde{x}_{HT}$  for 1985 – 1993

