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**Licences, "Use or Lose"
Provisions and the
Time of Investment**

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Summary

Exclusive rights, like mineral leases and radio spectrum licences, often hold option-like features. This occurs when licencees do not face the obligation to develop the lease or to undertake the investment required to use the assigned spectrum. However, to avoid licences being unused for lengthy periods, regulators sometimes set time limits, after which the exclusive right of exercise may be revoked, prior to its term, because of inaction. This paper looks at the potential impact of "use or lose" provisions upon the private time of investment. We find that these provisions may either increase or reduce the probability of early investment, depending on the risk of losing the licence and the expectations about ongoing deployment costs.

Keywords: Licences, Real Options, Use or Lose Provisions, Time of Investment

JEL Classification: L51, D44, D92

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Licences, Use or Lose Provisions and the Time of Investment

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Abstract

Exclusive rights, like mineral leases and radio spectrum licences, often hold option-like features. This occurs when licencees do not face the obligation to develop the lease or to undertake the investment required to use the assigned spectrum. However, to avoid licences being unused for lengthy periods, regulators sometimes set time limits, after which the exclusive right of exercise may be revoked, prior to its term, because of inaction. This paper looks at the potential impact of "use or lose" provisions upon the private time of investment. We find that these provisions may either increase or reduce the probability of early investment, depending on the risk of losing the licence and the expectations about on-going deployment costs.

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1 Introduction

Rights, but not obligations, to invest capital in productive assets, can be classified by whether they are shared or proprietary options (Kelster, 1984).

The former are collective opportunities for a number of competitive firms or of a whole industry, while the latter provide exclusive rights of exercise resulting from early investment (*e.g.* real estate), patents, copyrights, trademarks, or from a firm's managerial resources, technological knowledge or reputation which competitors cannot duplicate (Dixit and Pindyck, 1994).

Proprietary options can also be embedded in other exclusive rights of exercise, granted by public authorities, such as concessions to supply utility markets, mineral leases or spectrum licences.

This occurs when licensors do not impose the obligation to develop the lease or to supply the market by using the assigned radio spectrum.

Whether or not licencees should be granted with such flexibility, remains an open and controversial political issue.¹

As for the electromagnetic spectrum, advocates of roll-out obligations maintain that regulators should deter speculative spectrum warehousing with no specific use intended. For instance, various studies have showed that the assigned spectrum often remains idle (FCC, 2002), and band idleness is counted by the supporters of a "spectrum commons" among the arguments against the "proprietary rights" approach (Freyens, 2009).

On the other hand, critics of deployment requirements maintain that, especially in industries which are experiencing rapid developments on both the supply and demand side, the ability to wait and see, before committing a capital outlay, allows firms to avoid costly errors. This option value, arising from the asymmetry between the right and the obligation, is clearest in new investment-intensive ICT services, for which it may turn out there is insufficient demand.

Even regulators who have somehow recognized the merits of not imposing roll-out requirements have expressed concern about allowing licencees to sit on the spectrum for an indefinite period, and have called for reforms allowing to revoke licences, prior to their term, in case of inaction.

¹For example, on June 2008 the US House of Representative voted on the Responsible Federal Oil and Gas Lease Act, aimed at prohibiting the Secretary of Interior from issuing new Federal oil and gas leases to holders of existing leases which do not develop their leases or relinquish such leases. However, the bill failed to get the two-thirds support necessary for the passage.

For example, in a Discussion Paper on Radio Spectrum Policy and Planning released in 2007, New Zealand's Ministry of Economic Development stated that "use or lose provisions should apply to acquired spectrum [;] the purpose of a use or lose provision is to spur investment at an early date and avoid spectrum being unused for lengthy periods" (Ministry of Economic Development, 2007, § 6.4).

Using the analogy with financial options, adding use or lose (hereafter "UOL") clauses is equivalent to shorten the time to maturity of the real option embedded in a contract which gives the right, but does not impose the obligation, to buy an asset, namely the entitlement to the stream of profits stemming from using the licence, by affording a sunk capital cost (the exercise price).

However, contrary to standard financial contracts, a quick read through licencing policies and regulation reveal that UOL provisions, as currently applied, often involve an uncertain time to maturity.

This may occur either because, when setting in advance a specific deadline, licensors may then decide not to avail themselves of the revocation clause, or because they simply retain the right to discretionary revoke the licence or issue thinly-veiled warnings.

For example, Mexican Regulation of Satellite Communications allowed the Secreteria de Comunicaciones y Transportes to revoke concessions to broadcast DTH satellite services, prior to their term, if the concessionaire failed to use it within 180 days after it was granted. In Norway, pursuant to the Electronic Communications Act, the Ministry of Transport and Communications may revoke spectrum licences in the event of low utilization be the result of the market power of the licensee. On February 2009, in the Philippines, the National Telecommunications Commission warned telecoms companies that was considering plans to recall frequencies that were currently not being used.² Similarly, on July 2009, in Malaysia, the Information, Communication and Cultural Minister stated that the Government could revoke the licences of the companies that were awarded the 2.3 Ghz spectrum and that had yet to provide the required wireless broadband services using WiMAX technology³.

This paper tries to shed lights on the potential impact of UOL provisions

²"NTC's 'use or lose it' warning to operators". *TeleGeography CommsUpdate*. www.teleogeography.com [February 2, 2009].

³"WiMax licence holders warned to use or lose it". *TheStar Online*. <http://thestar.com.my> [July 30, 2009].

upon the private time of investment, and it is motivated by two specific questions. First, does the risk of losing the licence spur deployment at an early date? Second, is the effectiveness of UOL provisions affected by expectations about on-going deployment costs?

To answer these questions, we develop a model where the holder of a simple⁴ proprietary option ("the licensee") is assumed to face uncertainty about the returns of the irreversible investment required to exploit his/her exclusive right of exercise, and licencing conditions allow the concedent authority ("the regulator") to discretionarily revoke the licence because of inaction.

A main result of the paper is that, contrary to the conventional wisdom, reducing the expected time to maturity does not necessarily spur investment at an early date. For instance, in industries which are likely to experience declining deployment costs, "stringent" UOL provisions may even involve a perverse effect, by reducing, rather than increasing, the probability of early investment.

The rest of the paper is organised as follows. Section 2 outlines the model. Section 3 derives the value of a licence with uncertain time to maturity, and illustrates the relationship between the optimal private trigger value and the probability of losing the proprietary option. Section 4 evaluates the expected time of investment, with and without uncertain time to maturity. Section 5 concludes, and the Appendix contains the proofs omitted in the text.

2 The Model

Suppose a risk-neutral firm⁵ has acquired, say by an auction, an exclusive and discretionary opportunity to undertake a development project yielding a per period cashflow x_t .

The required instantaneous investment (K) is sunk, and the project can neither be changed, nor temporarily stopped, nor shut down. Operating and maintenance costs are comparatively small and set to zero.

⁴Both shared and proprietary options may be further distinguished by whether they are simple or compound. The former include "commercial one-stage projects that derive their value from expected cash flows", while the latter are projects which "do not derive their value primarily from cash inflows, but from strategic value" (Smit and Trigeorgis, 2004, p.22-23).

⁵Introducing risk aversion does not substantially change the results because the analysis can be developed under a risk neutral probability measure (Cox and Ross, 1976; Harrison and Kreps, 1979).

As for the uncertainty about future payoffs, we assume that x_t evolve over time according to a standard geometric Brownian motion:⁶

$$dx_t = \alpha x_t dt + \sigma x_t dB_t \quad \text{with } \alpha > 0, \sigma > 0 \text{ and } x_0 = x > 0, \quad (1)$$

where dB_t are identically and independently distributed according to a normal distribution with mean zero and variance dt , and both α and σ are constant.

Moreover, we assume that licensing conditions allow the regulator to discretionary revoke the licence, because of inaction, prior to its term, which, for simplicity, we set to infinity.

We model the uncertainty about the actual time to maturity (T) by assuming that T (> 0) is exponentially distributed with intensity parameter λ , and is independent of the process x :

$$\Pr(T \in dt) = \lambda e^{-\lambda t} dt \quad (2)$$

which implies that the expected time to maturity, without taking into account any licensee's investment decision, is $E(T) = \frac{1}{\lambda}$.⁷

Finally, we assume that the industry under consideration may benefit from exogenous developments - such as R&D progress or serialization in the manufacturing chain of particular components that reduce the cost of the

⁶The process x_t may be interpreted as a reduced form of a more general profit function (Dixit and Pindyck, 1994; Grenadier, 2002, Moretto 2008). For instance, (1) is equivalent to consider the present value of revenues accruing from a fixed-scale project. Denoting with V_t the expected present value, this is given by:

$$V_t = E_t \left[\int_t^\infty e^{-r(s-t)} x_t dt \right] = \frac{x_t}{r - \alpha}$$

where $r > \alpha$ is the constant real risk-free rate of interest. Since V_t is a constant multiple of x_t it follows a geometric Brownian motion with the same parameters α and σ (Harrison, 1985, p.44).

⁷This specification allows us to look both at situations where the regulator does not explicitly set time limits, but reserve the right to cancel the licence because of inaction, and at situations where the licensee has a limited amount of time to start using the licence, but the regulator may then discretionary decide not to avail itself of the revocation clause. In the latter case, the regulator may set $\lambda = \frac{1}{T}$, so that the mean maturity corresponds to the desired time to maturity (Carr, 1998).

current technology - implying that the licensee needs to spend less if he/she decide to wait longer before supplying the market.⁸

Since revocation would deprive the licensee of exploiting potential cost savings, to keep things as simple as possible we model K as a decreasing function of the expected time to maturity:^{9 10}

$$K = \bar{K} + \frac{k}{E(T)} \quad (3)$$

where $\bar{K} > 0$ is the long-run capital cost, and $\frac{k}{E(T)}$ is the expected opportunity cost incurred by prematurely investing to avoid losing the licence.

3 The value of the licence and the optimal trigger value

Within the range of x where it is optimal for the licensee to keep the option-to-invest alive, the project value $W(x, K)$, with uncertain time to maturity, is given by the solution of the following ODE (Carr, 1998; Miltersen and Schwartz, 2007):

⁸At the end of Section 4 we will relax this assumption, in order to look at situationsis unaffected by the time to maturity.

⁹This specification is broadly similar to the one used by Miltersen and Schwartz (2007) who consider an R&D project aimed at developing a new product which can be abandoned at any time before manufacturing the product. The Author assume that developing the product requires a per unit of time research expenditure k , and that completion of the project arrives at a random time which is described by a Poisson process with intensity parameter λ . Once the project has been completed, the firm has the option to pay a final (fixed) capital cost, say \bar{K} , to manufacturing the product. In that framework, an increase of λ (i.e. a decrease in the expected time to completion) reduces total capital costs. By contrast, in our framework, since early maturity would prevent the licensee from exploiting potential technological developments, capital costs are modelled as an increasing function of the intensity parameter λ .

¹⁰Equation (3) is also compatible with the case where two opposite trends affect the evolutionary pattern of capital costs. On the one hand, technological developments involving potential input savings and, on the other hand, cost increases attributable to the limited number of licences issued by the regulator (Dixit and Pindyck, 2000; Salsas and Koboldt, 2004). For a more in-depth discussion about alternative micro-foundations of (3), see Appendix A.

$$\frac{1}{2}\sigma^2x^2W_{xx}(x, K)+\alpha xW_x(x, K)-rW(x, K) = \tag{4}$$

$$= \lambda \left[W(x, K) - \max \left(\frac{x}{r - \alpha} - K, 0 \right) \right], \text{ for all } x < \hat{x}$$

where $r > \alpha$ is the constant real risk-free rate of interest, and \hat{x} is the optimal trigger value, i.e. the licensee will invest when x_t hits \hat{x} .

However, if in the meantime the regulator announced that he is about to revoke the licence, its value would fall to $\max \left(\frac{x}{r - \alpha} - K, 0 \right)$, and the licensee will still immediately invest, provided $x > \hat{x}^{NPV}$, where $\hat{x}^{NPV} \equiv (r - \alpha)K$ stands for the standard break-even point at which the value of the discounted cashflow equals the capital cost.

Thus, we get the following system of ODEs:

$$\frac{1}{2}\sigma^2x^2W_{xx}(x, K) + \alpha xW_x(x, K) - (r + \lambda)W(x, K) = 0, \text{ for } 0 < x < \hat{x}^{NPV} \tag{5}$$

and

$$\frac{1}{2}\sigma^2x^2W_{xx}(x, K)+\alpha xW_x(x, K)-(r+\lambda)W(x, K) = -\lambda \left(\frac{x}{r - \alpha} - K \right), \text{ for } \hat{x}^{NPV} \leq x < \hat{x} \tag{6}$$

Equation (5) provides the value of the licence when x is below the *NPV* trigger. In other words, it describes the fact that with probability λ per unit of time, the licence will be revoked, in which case its value would collapse to zero. This reduces the project value $W(x, K)$.

On the other hand, equation (6) provides the value of the licence when x is above the *NPV* trigger. In this case, if the regulator decided to exercise the revocation clause, since the project's *NPV* is positive, the licensee would immediately invest. This increases $W(x, K)$.

Our first proposition is obtained by solving the two ODEs, imposing the boundary condition that $\lim_{x \rightarrow 0} W(x, K) = 0$.

Proposition 1 1) *The option value with uncertain time to maturity is:*

$$W(x, \hat{x}, K) = \begin{cases} m_{11} (x)^{\gamma_1} & \text{for } 0 < x < \hat{x}^{NPV} \\ m_{21} (x)^{\gamma_1} + m_{22} (x)^{\gamma_2} + \frac{\lambda x}{(r-\alpha)(r+\lambda-\alpha)} - \frac{\lambda K}{(r+\lambda)} & \text{for } \hat{x}^{NPV} \leq x < \hat{x} \end{cases} \quad (7)$$

where m_{11} , m_{12} , m_{22} are positive constants, and $\gamma_1 > 1$, $\gamma_2 < 0$ are the positive and negative roots of the auxiliary quadratic equation $\Phi(z) = \frac{1}{2}\sigma^2 z(z-1) + \alpha z - (r+\lambda) = 0$, i.e.:

$$\gamma_1 = \frac{(\frac{1}{2}\sigma^2 - \alpha) + \sqrt{(\frac{1}{2}\sigma^2 - \alpha)^2 + 2(r+\lambda)\sigma^2}}{\sigma^2} > 1$$

$$\gamma_2 = \frac{(\frac{1}{2}\sigma^2 - \alpha) - \sqrt{(\frac{1}{2}\sigma^2 - \alpha)^2 + 2(r+\lambda)\sigma^2}}{\sigma^2} < 0$$

2) *The optimal trigger value is given by:*

$$\frac{\gamma_1 - \gamma_2}{\gamma_1 - 1} m_{22} (\hat{x})^{\gamma_2} - \frac{\hat{x}}{(r+\lambda-\alpha)} + \frac{\gamma_1}{\gamma_1 - 1} \frac{r}{r+\lambda} (\bar{K} + \lambda k) = 0 \quad (8)$$

where $m_{22} = \frac{(r+\lambda-\gamma_1\alpha)}{(\gamma_1-\gamma_2)(r+\lambda-\alpha)} \frac{\lambda K}{(r+\lambda)} (\hat{x}^{NPV})^{-\gamma_2} > 0$

Proof. See Appendix B ■

As shown by the real-option literature, when returns are uncertain, the ability to wait and see before committing a capital outlay always increases the value of the project¹¹. Thus, all the constants m_{11} , m_{12} , m_{22} must be non-negative (see Appendix B).

While $m_{11} (x)^{\gamma_1}$ indicates the value of the option-to-invest in the interval where it is not worth doing so (i.e. $0 < x < \hat{x}^{NPV}$), the second expression in (7) deserves some further explanation.

Keeping in mind that, within the interval $\hat{x}^{NPV} \leq x < \hat{x}$, if the regulator decided to avail himself of the revocation clause, the licensee would immediately invest (with NPV given by $\frac{\lambda x}{(r-\alpha)(r+\lambda-\alpha)} - \frac{\lambda K}{(r+\lambda)}$), the first term $m_{21} (x)^{\gamma_1}$ represents the option value of investing the first time x reaches the optimal trigger \hat{x} , whilst the second term, with the negative root, represents the expected gain due to the ability to keep the option alive if x falls below \hat{x}^{NPV} .

¹¹See Dixit and Pindyck (1994, chs. 6 and 7) for an exhaustive discussion.

From Proposition 1, it is possible to show that at \hat{x}^{NPV} we get (see Appendix B):

$$(m_{11} - m_{21}) (\hat{x}^{NPV})^{\gamma_1} = \frac{(r + \lambda - \gamma_2 \alpha)}{(\gamma_1 - \gamma_2)(r + \lambda - \alpha)} \frac{\lambda K}{(r + \lambda)} > 0 \quad (9)$$

which indicates the increase in the option value when the licensee knows for sure that if the regulator decided to exercise the revocation clause, the project would be immediately carried-out.

Further, taking the derivative of (8) with respect to K (or \bar{K}), it is easy to show that the optimal trigger is monotonically increasing in the investment cost (see Appendix B):

$$\frac{d\hat{x}}{dK} > 0 \quad (10)$$

Equation (7) also allows to derive the option value when the licensee holds a perpetual growth option. Indicating with \bar{x} the optimal trigger when $\lambda = 0$, we obtain:

$$V(x, \bar{x}, \bar{K}) = m(x)^{\beta_1}, \quad \text{for all } x < \bar{x} \quad (11)$$

where $m = \left(\frac{\bar{x}}{r-\alpha} - \bar{K}\right) (\bar{x})^{-\beta_1} > 0$, and $1 < \beta_1 < r/\alpha$ is the positive root of the auxiliary quadratic equation $\Psi(z) = \frac{1}{2}\sigma^2 z(z-1) + \alpha z - r = 0$ ¹².

By (8), when $\lambda = 0$, the trigger reduces to:¹³

$$\bar{x} = \frac{\beta_1}{\beta_1 - 1} (r - \alpha) \bar{K} \quad (12)$$

Note that, when the licensee does not face any risk of losing the licence, the investment rule implies that $V(x, \bar{x}, \bar{K}) \geq \frac{x}{r-\alpha} - \bar{K}$ for all $x \leq \bar{x}$.

In other words, when $\lambda = 0$, the option value is simply equal to the NPV of the project, $\left(\frac{\bar{x}}{r-\alpha} - \bar{K}\right)$, time the probability of investing in the future, given the current level of x , i.e. $\left(\frac{x}{\bar{x}}\right)^{\beta_1}$.

A numerical example will illustrate the relationship between the optimal trigger value and the intensity parameter λ . Suppose $r = 0.05$, $\alpha = 0.03$, $\sigma = 0.2$, $\bar{K} = 30$, and $k = 5$.

¹²That is.:

$$\beta_1 = \frac{\left(\frac{1}{2}\sigma^2 - \alpha\right) + \sqrt{\left(\frac{1}{2}\sigma^2 - \alpha\right)^2 + 2r\sigma^2}}{\sigma^2} > 1$$

¹³Note that \hat{x} in (8) converges to $\bar{x} = \frac{\beta_1}{\beta_1 - 1} (r - \alpha) \bar{K}$ as $\lambda \rightarrow 0$.

Figure 1 about here

Figure 1 shows the solution graphically, and confirms the intuition that, with respect to a situation where the date of investment is left entirely to the licensee's discretion, UOL provisions tend to reduce the optimal trigger.

However, there is an interesting non-monotonic pattern. In our numerical example, the trigger value decreases for λ below 0.5 (i.e. for $E(T)$ above 2 years), but then it increases.¹⁴

Generally speaking, it is possible to show that there exists a critical value of λ above which the risk of losing the licence would induce the licensee to set a trigger higher than the one he/she would have chosen when holding a perpetual proprietary option.

Proposition 2 *There exists a value of the intensity parameter $\tilde{\lambda}$ such that:*

$$\begin{cases} \hat{x} \leq \bar{x} & \text{for } \lambda \leq \tilde{\lambda} \\ \hat{x} > \bar{x} & \text{for } \lambda > \tilde{\lambda} \end{cases}$$

i.e. for $\lambda > \tilde{\lambda}$ the optimal private trigger value with uncertain time to maturity is strictly higher than the one without maturity.

Proof. See Appendix D ■

In words, Proposition 2 states that, if the licensee faces a "very short" expected time to maturity, he/she will maximize the option value by moving up the optimal exercise boundary.

The analogy with financial options may help us to interpret this result.

The real option considered in this paper is equivalent to a call option in a financial asset that gives a constant dividend rate equal to $r - \alpha$.

Therefore, if the dividend rate is positive, there is an opportunity cost of keeping the option alive rather than exercising it.

This opportunity cost is represented by the cash flows that the licensee loses by warehousing the licence. However, as a higher value of the project implies higher dividends, when this value reaches an upper threshold, the opportunity cost of forgone dividends becomes large enough to make worthwhile exercising the option (Dixit and Pindyck, 1994, p.149).

¹⁴Figure 3 in Appendix C shows the relationship between the optimal trigger value and the intensity parameter λ using equation (15) in Appendix A.

Now, if we introduce into this picture a UOL provision which induces the licensee to decide about the project prematurely, the value of the investment opportunity is affected by the parameter λ in three ways.

First, if $x \in (0, \hat{x}^{NPV})$, the effect of early maturity is equivalent to a reduction in the rate of capital gain on x (from α to $\alpha - \lambda$), which increases the dividend rate from $r - \alpha$ to $r + \lambda - \alpha$. In other words, the risk of losing the licence increases the opportunity cost of keeping the option alive. This reduces $W(x, K)$ and, then, the trigger value \hat{x} .

Second, when $x \in [\hat{x}^{NPV}, \hat{x})$, the licensee, although investing prematurely, will receive a positive NPV and the stream of dividends $r - \alpha$ thereafter. As shown by (9), this increases $W(x, K)$, and may lead to an increase in \hat{x} .

Finally, since early maturity would prevent the licensee from exploiting potential cost savings, an increase of λ reduces both the NPV and $W(x, K)$, and this involves an increase in \hat{x} (see (10))

As Proposition 2 states, if the licensee faces a very short expected time to maturity ($\lambda > \tilde{\lambda}$), the overall net effect may be an increase of the optimal exercise boundary above the trigger without maturity.

4 The expected time of investment

We have shown that the risk of losing the licence may either reduce or increase the optimal trigger, depending on the expected time to maturity.

Note that whereas a reduction of the trigger always implies a higher probability of early investment, the reverse does not necessarily apply.

This because, before reaching the optimal trigger, the regulator might warn the licensee that he is about to cancel the licence, and this could induce the licensee to immediately invest, even though x has not already reached the optimal threshold.

In this Section we then look at the expected time of investment by comparing the case where the licensee faces the risk of losing the licence with a situation where the time of investment is entirely left to the licensee's discretion.

By denoting with $E(\hat{\tau}^\lambda)$ the expected time of investment when the licensee faces an uncertain time to maturity, and with $E(\bar{\tau})$ the expected time when the licensee holds a perpetual option, we obtain the following proposition.

Proposition 3 *The difference between the expected time of investment "with"*

and "without time to maturity" may be approximated by the following expression:

$$E(\hat{\tau}^\lambda) - E(\bar{\tau}) \simeq m^{-1} \ln\left(\frac{\hat{x}}{\bar{x}}\right) + E(\hat{\tau}) \frac{[E(\hat{T}) - E(\hat{\tau})]}{E(T)} \quad (13)$$

where $m \equiv (\alpha - \frac{1}{2}\sigma^2) > 0$.

Proof. See Appendix E ■

In (13), $E(\hat{\tau}) = m^{-1} \ln(\frac{\hat{x}}{\bar{x}})$ stands for the expected time for the process x to reach for the first time \hat{x} without taking account of the time to maturity, while $E(\hat{T})$ stands for the expected time to maturity taking the licensee's optimal investment decision into account. Finally, $E(T) = \frac{1}{\lambda} > E(\hat{T})$ stands for the expected time to maturity without taking account of the licensee's optimal investment decision.

Note that since $E(\hat{T})$ accounts for the probability that the regulator will revoke the licence in the interval $(0, \hat{\tau})$, the second term on the r.h.s. of (13) is always negative¹⁵, while the first term is negative when $\hat{x} < \bar{x}$, and positive the other way around.

Thus, when $\lambda < \bar{\lambda}$, i.e. $\hat{x} < \bar{x}$, UOL provisions reduce the expected time of investment, i.e. $E(\hat{\tau}^\lambda) < E(\bar{\tau})$.

However, when the licensee faces a relatively short expected time to maturity ($\lambda > \bar{\lambda}$), the increase in the trigger may be such that $E(\hat{\tau}^\lambda) > E(\bar{\tau})$. In other words, in this case, UOL provisions may involve a potentially perverse effect, by reducing, rather than increasing, the probability of early investment.

Key to this result is the assumption that early maturity would deprive the licensee of potential cost savings. For instance, it is possible to show that, if the industry is unlikely to benefit from declining deployment costs, the

¹⁵By (2), the probability that the maturity occurs in the interval $(0, t)$ is $1 - e^{-\lambda t}$. Therefore, substituting the generic unknown time t with $E(\hat{\tau})$, we get

$$E(\hat{T}) \simeq \int_0^{E(\hat{\tau})} t \lambda e^{-\lambda t} dt = \frac{1}{\lambda} [1 - e^{-E(\hat{\tau})}] \simeq \frac{1}{\lambda} E(\hat{\tau})$$

Since $E(\hat{\tau}) = m^{-1} \ln(\frac{\hat{x}}{\bar{x}})$, substituting in the above expression we get $E(\hat{T}) \simeq \frac{1}{\lambda} [1 - (\frac{\hat{x}}{\bar{x}})^{m^{-1}}]$. Note that if $\hat{x} \rightarrow \infty$, $E(\hat{T}) = E(T) = \frac{1}{\lambda}$, i.e. $E(\hat{T})$ converges to the expected time to maturity without taking any private optimal investment decision into account. On the contrary, if $\hat{x} \rightarrow \bar{x}$, the licensee invests immediately, so that $E(\hat{T}) = 0$, i.e. no maturity occurs.

higher is the risk of losing the licence, the lower will be the optimal trigger and, consequently, the lower the expected time of investment.

Figure 2 illustrates the relationship between \hat{x} and λ , when the exercise price is unaffected by the time to maturity.¹⁶

It is worth to note that, in this case, as the intensity parameter λ increases, \hat{x} tends to converge to the trigger without uncertainty, i.e. $\lim_{\lambda \rightarrow \infty} \hat{x} \rightarrow rK$ ¹⁷. In other words, by reducing the expected time to maturity, the regulator can mitigate the effects of uncertainty about future cashflows which slows down the exercise of the proprietary option embedded in the licence.¹⁸

Figure 2 about here

5 Final Remarks

Radio spectrum licences, and other similar exclusive rights of exercise awarded by public authorities, often remain idle.

As long as these rights derive their value from expected commercial returns, licence warehousing may be traced back to the licencees' willingness-to-wait more favorable market conditions, before committing a capital outlay.

However, in an attempt to strike a balance between granting firms with a certain degree of time flexibility and avoiding licences being unused for lengthy periods, regulators have introduced, or are planning to introduce,

¹⁶We use the same values as in the previous numerical example, with the exception of K which is now set constant and equal to 50. Moreover, to illustrate the effects of the volatility parameter, we also consider different values of σ (0, 0.1, 0.2 and 0.3)

¹⁷Since $\lim_{\sigma \rightarrow 0} \gamma_1 = r/\alpha$, it is easy to show that

$$\lim_{\sigma \rightarrow 0} \bar{x} = \frac{r/\alpha}{r/\alpha - 1} (r - \alpha)K \equiv rK$$

¹⁸Note that, in this case, the effect produced by UOL provisions is similar to the one induced by increased competition, when N firms - with private valuation of capital costs - hold a shared option which allows to enter a new market. As shown by Lambrecht and Perraudin (2003), as N increases, since each agent knows almost certainly that at least one of his/her rivals will enter at a lower trigger, he/she will try to preempt the rivals by lowering the trigger as far as possible, and the optimal trigger will converge to the traditional break-even one.

UOL provisions, which allowing to cancel existing licences because of inaction.

In this paper we have looked at the impact of the risk of losing a simple proprietary option upon the private time of investment.

Our findings suggests that while such a risk is likely to affect the time of investment, the effects are not univocal, insofar they appear to depend on the expectations about on-going deployment costs and the expected time to maturity.

The effectiveness of UOL provisions is clearest when licencees do not face declining deployment costs. In other words, in relatively mature industries, UOL provisions appear to be an effective regulatory device to discourage licence warehousing.

By contrast, more caution should be used by regulators in introducing "stringent" UOL provisions in the case of industries which are experiencing rapid technological developments.

For instance, when roll-out costs are likely to decline, shortening the expected time to maturity does not necessarily spur investment at an early date.

Moreover, when licencees face a very high risk of losing the licence, UOL provisions may even involve a perverse effect, by reducing, rather than increasing, the probability of early investment.

A Alternative micro-fundations of eq. (3)

The reduced-form (3) can be supported by a micro-model of innovation technology. Suppose that investment cost at time $t > 0$ is given by $K_t = \bar{K} + I_t$, where $I_t = I_0(1 - \phi)^{y_t} e^{\rho t}$. $\rho > 0$ is the parameter that determines the speed of the increase of the investment cost due to the limited number of licences issued by the regulator, while y_t is a Poisson process, with intensity parameter γ , which measures the number of technology arrivals up to time t , and $\phi \in [0, 1)$ is a constant that captures the magnitude of the innovations (i.e. the higher is ϕ , the more the innovations reduce the cost). It can be shown (Huisman 2001, proposition 3.1) that I_t is obtained as solution of the following process:

$$dI_t = \begin{cases} \rho I_t dt - \phi I_t & \text{with probability } \gamma dt \\ \rho I_t dt & \text{with probability } 1 - \gamma dt \end{cases}$$

where $I_0 > 0$. If we assume that the licensee may experience only one technology innovation in the laps of time before investing, it is easy to show that the expected capital cost is equal to:

$$K = \bar{K} + k(1 - \phi) \int_0^\infty e^{\rho s} \gamma e^{-\gamma s} ds = \bar{K} + I_0(1 - \phi) \frac{\gamma}{\gamma - \rho} \quad (14)$$

However, since an increase in the intensity of licence revocation reduces the magnitude of the innovation the licensee can benefit from (i.e. if λ increases towards ∞ than $\phi \rightarrow 0$), substituting in (14) a simple expression as $\phi = \frac{1}{1+\lambda}$, the above equation reduces to:

$$K = \bar{K} + \frac{k}{1 + E(T)} \quad (15)$$

where $k \equiv I_0 \frac{\gamma}{\gamma - \rho}$. Thus, provided that $\gamma > \rho$, when $\lambda \rightarrow \infty$ the cost of capital becomes $K(\infty) = \bar{K} + k$ while when the licensee is able to exploiting all technological innovations, $\lambda \rightarrow 0$, the cost tends to the long-run capital cost $K(0) = \bar{K}$.

B Proof of Proposition 1

The general solutions of the two differential equations take respectively the form:

$$W(x, \hat{x}, K) = \begin{cases} m_{11}(x)^{\gamma_1} + m_{12}(x)^{\gamma_2} & \text{for } 0 < x < \hat{x}^{NPV} \\ m_{21}(x)^{\gamma_1} + m_{22}(x)^{\gamma_2} + \frac{\lambda x}{(r-\alpha)(r+\lambda-\alpha)} - \frac{\lambda K}{(r+\lambda)} & \text{for } \hat{x}^{NPV} \leq x < \hat{x} \end{cases} \quad (16)$$

where $\hat{x}^{NPV} \equiv (r-\alpha)K$. Yet, $\gamma_1 > 1$ and $\gamma_2 < 0$ are the positive and negative roots of the auxiliary quadratic equation $\Phi(z) = \frac{1}{2}\sigma^2 z(z-1) + \alpha z - (r+\lambda) = 0$.

$$\gamma_1 = \frac{(\frac{1}{2}\sigma^2 - \alpha) + \sqrt{(\frac{1}{2}\sigma^2 - \alpha)^2 + 2(r+\lambda)\sigma^2}}{\sigma^2} > 1$$

$$\gamma_2 = \frac{(\frac{1}{2}\sigma^2 - \alpha) - \sqrt{(\frac{1}{2}\sigma^2 - \alpha)^2 + 2(r+\lambda)\sigma^2}}{\sigma^2} < 0$$

Since the value of the investment cannot be below the *NPV*, $\lim_{x \rightarrow 0} W(x, K) = 0$. This implies that $m_{12} = 0$ since $\gamma_2 < 0$. Furthermore, since $m_{11}(x)^{\gamma_1}$ stands for the option to develop the project, $m_{11} > 0$. To determine the constants m_{11} , m_{21} , m_{22} and the critical level \hat{x} , the value-matching and smooth-pasting conditions must be satisfied (Dixit and Pindyck, 1994). At $x_t = \hat{x}^{NPV}$

$$m_{11}(\hat{x}^{NPV})^{\gamma_1} = m_{21}(\hat{x}^{NPV})^{\gamma_1} + m_{22}(\hat{x}^{NPV})^{\gamma_2} + \frac{\lambda \hat{x}^{NPV}}{(r-\alpha)(r+\lambda-\alpha)} - \frac{\lambda K}{(r+\lambda)} \quad (17)$$

$$m_{11}\gamma_1(\hat{x}^{NPV})^{\gamma_1-1} = m_{21}\gamma_1(\hat{x}^{NPV})^{\gamma_1-1} + m_{22}\gamma_2(\hat{x}^{NPV})^{\gamma_2-1} + \frac{\lambda}{(r-\alpha)(r+\lambda-\alpha)} \quad (18)$$

and at $x_t = \hat{x}$

$$m_{21}(\hat{x})^{\gamma_1} + m_{22}(\hat{x})^{\gamma_2} + \frac{\lambda \hat{x}}{(r-\alpha)(r+\lambda-\alpha)} - \frac{\lambda K}{(r+\lambda)} = \frac{\hat{x}}{(r-\alpha)} - K \quad (19)$$

$$m_{21}\gamma_1(\hat{x})^{\gamma_1-1} + m_{22}\gamma_2(\hat{x})^{\gamma_2-1} + \frac{\lambda}{(r-\alpha)(r+\lambda-\alpha)} = \frac{1}{(r-\alpha)} \quad (20)$$

Condition (19) reflects the fact that if an early maturity does not occur, the licensee will find it optimal to invest when x_t hits the trigger \hat{x} . Condition

(20) is the usual smooth-pasting condition at the investment threshold level. On the other hand, conditions (17) and (18) reflect the fact that the project value function should be continuous and differentiable at the point when the option to invest meets the value of the project after the maturity time jumps up. Multiplying (18) by \hat{x}^{NPV} , dividing for γ_1 , and subtracting from (17), yield:

$$\begin{aligned} (m_{11} - m_{21}) (\hat{x}^{NPV})^{\gamma_1} &= m_{22} (\hat{x}^{NPV})^{\gamma_2} + \frac{\lambda \hat{x}^{NPV}}{(r - \alpha)(r + \lambda - \alpha)} - \frac{\lambda K}{(r + \lambda)} \\ (m_{11} - m_{21}) (\hat{x}^{NPV})^{\gamma_1} &= m_{22} \frac{\gamma_2}{\gamma_1} (\hat{x}^{NPV})^{\gamma_2} + \frac{\lambda \hat{x}^{NPV}}{(r - \alpha)(r + \lambda - \alpha)\gamma_1} \end{aligned}$$

$$m_{22} \frac{\gamma_2}{\gamma_1} (\hat{x}^{NPV})^{\gamma_2} + \frac{\lambda \hat{x}^{NPV}}{(r - \alpha)(r + \lambda - \alpha)\gamma_1} = m_{22} (\hat{x}^{NPV})^{\gamma_2} + \frac{\lambda \hat{x}^{NPV}}{(r - \alpha)(r + \lambda - \alpha)} - \frac{\lambda K}{(r + \lambda)}$$

Thus, we can solve for m_{22} and for $(m_{11} - m_{21})$:

$$m_{22} = \frac{(r + \lambda - \gamma_1 \alpha)}{(\gamma_1 - \gamma_2)(r + \lambda - \alpha)} \frac{\lambda K}{(r + \lambda)} (\hat{x}^{NPV})^{-\gamma_2} > 0 \quad (21)$$

$$\begin{aligned} (m_{11} - m_{21}) &= \quad (22) \\ &= \left[\frac{\lambda K}{(r + \lambda)} \left(\frac{\gamma_2}{\gamma_1 - \gamma_2} \right) + \frac{\lambda \hat{x}^{NPV}}{(r - \alpha)(r + \lambda - \alpha)} \frac{1 - \gamma_2}{\gamma_1 - \gamma_2} \right] (\hat{x}^{NPV})^{-\gamma_1} \\ &= \frac{(r + \lambda - \gamma_2 \alpha)}{(\gamma_1 - \gamma_2)(r + \lambda - \alpha)} \frac{\lambda K}{(r + \lambda)} (\hat{x}^{NPV})^{-\gamma_1} > 0 \end{aligned}$$

Note that the constants m_{22} and $(m_{11} - m_{21})$ are always nonnegative (Dixit and Pindyck, 1994, p.189).

From (19) and (20), we obtain the constant m_{21} and the trigger \hat{x} . Multiplying (20) by \hat{x} , and dividing for γ_1 , yield:

$$\begin{aligned} m_{21} (\hat{x})^{\gamma_1} + m_{22} (\hat{x})^{\gamma_2} + \frac{\lambda \hat{x}}{(r - \alpha)(r + \lambda - \alpha)} - \frac{\lambda K}{(r + \lambda)} &= \frac{\hat{x}}{(r - \alpha)} - K \\ m_{21} (\hat{x})^{\gamma_1} + m_{22} \frac{\gamma_2}{\gamma_1} (\hat{x})^{\gamma_2} + \frac{\lambda \hat{x}}{\gamma_1 (r - \alpha)(r + \lambda - \alpha)} &= \frac{\hat{x}}{\gamma_1 (r - \alpha)} \end{aligned}$$

Then the investment trigger is given by the following implicit function:

$$\frac{\gamma_1 - \gamma_2}{\gamma_1 - 1} m_{22} (\hat{x})^{\gamma_2} - \frac{\hat{x}}{(r + \lambda - \alpha)} + \frac{\gamma_1}{\gamma_1 - 1} \frac{r}{r + \lambda} K = 0 \quad (23)$$

Although equation (23) must be solved numerically, it can be shown that it has a unique positive solution for \hat{x} . Finally, we get the constant m_{21} as:

$$m_{21} = \left[\frac{\hat{x}}{(r + \lambda - \alpha)} \frac{1 - \gamma_2}{\gamma_1 - \gamma_2} + \frac{\gamma_2}{\gamma_1 - \gamma_2} \frac{rK}{(r + \lambda)} \right] (\hat{x})^{-\gamma_1}$$

Further, substituting m_{22} into (23), this can be rewritten as follows:

$$f(\hat{x}, K) \equiv \frac{1}{\gamma_1 - 1} \frac{(r + \lambda - \gamma_1 \alpha)}{(r + \lambda - \alpha)} \frac{\lambda K}{(r + \lambda)} (\hat{x}^{NPV})^{-\gamma_2} (\hat{x})^{\gamma_2} - \frac{\hat{x}}{(r + \lambda - \alpha)} + \frac{\gamma_1}{\gamma_1 - 1} \frac{r}{r + \lambda} K = 0$$

and, by totally differentiating $f(\hat{x}, K)$ with respect to K , we are able to investigate the effect of a change in the investment cost on the optimal trigger:

$$\frac{d\hat{x}_i}{dK} = - \frac{f_K(\hat{x}, K)}{f_{\hat{x}}(\hat{x}, K)} \quad (24)$$

Since $f_{\hat{x}}(\hat{x}, K) = \frac{\gamma_2}{\gamma_1 - 1} \frac{(r + \lambda - \gamma_1 \alpha)}{(r + \lambda - \alpha)} \frac{\lambda K}{(r + \lambda)} (\hat{x}^{NPV})^{-\gamma_2} (\hat{x})^{\gamma_2 - 1} - \frac{1}{(r + \lambda - \alpha)} < 0$, the sign of (24) is given by the numerator:

$$f_K(\hat{x}, K) = \frac{1 - \gamma_2}{\gamma_1 - 1} \frac{(r + \lambda - \gamma_1 \alpha)}{(r + \lambda - \alpha)} \frac{\lambda}{(r + \lambda)} (\hat{x}^{NPV})^{-\gamma_2} (\hat{x})^{\gamma_2} + \frac{\gamma_1}{\gamma_1 - 1} \frac{r}{r + \lambda} > 0$$

This concludes the proof.

C Optimal trigger value using eq. (15)

Figure 3 below shows the relationship between the optimal trigger value and the intensity parameter λ using equation (15), with $r = 0.05$, $\alpha = 0.03$, $\sigma = 0.2$, $\bar{K} = 30$, and $k = 20$.

Figure 3 about here

D Proof of Proposition 3

By Equation (23), let us define $Y_1(\lambda) \equiv \frac{\gamma_1 - \gamma_2}{\gamma_1 - 1} m_{22} (\hat{x})^{\gamma_2}$ and $Y_2(\lambda) \equiv -\frac{\gamma_1}{\gamma_1 - 1} \frac{r}{r + \lambda} K + \frac{\hat{x}}{(r + \lambda - \alpha)}$. The optimal trigger is given by $Y_1(\lambda) = Y_2(\lambda)$ while the trigger without maturity is given by: $Y_2(\lambda = 0) = -\frac{\beta_1}{\beta_1 - 1} \bar{K} + \frac{\bar{x}}{(r - \alpha)} = 0$. However, since $\frac{1}{(r + \lambda - \alpha)} < \frac{1}{(r - \alpha)}$, by comparing $Y_2(\lambda)$ with $Y_2(\lambda = 0)$ to get $\hat{x} > \bar{x}$ it is sufficient that $-\frac{\gamma_1}{\gamma_1 - 1} \frac{r}{r + \lambda} (\bar{K} + \lambda k) < -\frac{\beta_1}{\beta_1 - 1} \bar{K}$, or:

$$\left[\frac{\beta_1}{\beta_1 - 1} - \frac{\gamma_1}{\gamma_1 - 1} \frac{r}{r + \lambda} \right] \bar{K} - \frac{\gamma_1}{\gamma_1 - 1} \frac{r}{r + \lambda} \lambda k < 0 \quad (25)$$

From the auxiliary quadratic equation $\Phi(z) = \frac{1}{2} \sigma^2 z(z - 1) + \alpha z - (r + \lambda) = 0$, we can write:

$$\left[\frac{z}{z - 1} (r - \alpha) + r + \frac{1}{2} \sigma^2 z \right] = \lambda \left[1 - \frac{z}{z - 1} \right]$$

Since β_1 satisfies $\Psi(z) \equiv \left[\frac{z}{z - 1} (r - \alpha) + r + \frac{1}{2} \sigma^2 z \right] = 0$, it is evident that $\gamma_1 > \beta_1 > 1$ and $\gamma_2 < \beta_2 < 0$, from which $\frac{\beta_1}{\beta_1 - 1} > \frac{\gamma_1}{\gamma_1 - 1}$. Therefore, by (25) there may exist a value of λ such that the second term is greater than the first one. This concludes the proof.

E Proof of Proposition 4

Let start with the perpetual case. Denoting with $\bar{\tau} = \inf(t \geq 0 \mid x < \bar{x})$ the optimal investment time, since the instantaneous payoffs are driven by (1), the first passage time $\bar{\tau}$ from x to \bar{x} is a stochastic variable with first moment:

$$E(\bar{\tau}) = m^{-1} \ln\left(\frac{\bar{x}}{x}\right) \quad (26)$$

where $m \equiv (\alpha - \frac{1}{2} \sigma^2)$. So that $\bar{x} = x e^{m E(\bar{\tau})}$, and for the licensee setting $E(\bar{\tau})$ or \bar{x} is the same (Cox and Miller, 1965, p. 221-222). Now, defining with $E(\hat{\tau}^\lambda)$ the expected time to develop the project with uncertain maturity, this is given as the weighted average between the firm's expected time to maturity taking account of its optimal investment decision, say $E(\hat{T})$, and the expected

time to develop the project if the maturity does not occur, say $E(\hat{\tau})$ (where $\hat{\tau} = \inf(t \geq 0 \mid x < \hat{x})$). That is:

$$E(\hat{\tau}^\lambda) = [1 - e^{-\lambda\hat{\tau}}] E(\hat{T}) + e^{-\lambda\hat{\tau}} E(\hat{\tau}) \quad (27)$$

where the weight $[1 - e^{-\lambda\hat{\tau}}]$ indicates the probability that the time to maturity occurs in the interval $(0, \hat{\tau})$. Hence, given that $e^{-\lambda\hat{\tau}} \simeq 1 - \lambda\hat{\tau} + \dots$, it follows that $E(\hat{\tau}^\lambda) \simeq \lambda\hat{\tau}E(\hat{T}) + (1 - \lambda\hat{\tau})E(\hat{\tau})$. Since $\hat{\tau}$ is a stochastic variable we approximate it by its first moment (one can also find the expected time to develop the project with uncertain maturity $E(\hat{\tau}^\lambda)$ by Monte Carlo simulation of the distribution of $\hat{\tau}$). Therefore, we get:

$$\begin{aligned} E(\hat{\tau}^\lambda) &\simeq E(\hat{\tau}) + \lambda E(\hat{\tau}) [E(\hat{T}) - E(\hat{\tau})] \\ &= E(\hat{\tau}) + m^{-1} \ln\left(\frac{\hat{x}}{\bar{x}}\right) + E(\hat{\tau}) \frac{[E(\hat{T}) - E(\hat{\tau})]}{E(T)} \end{aligned} \quad (28)$$

This concludes the proof.

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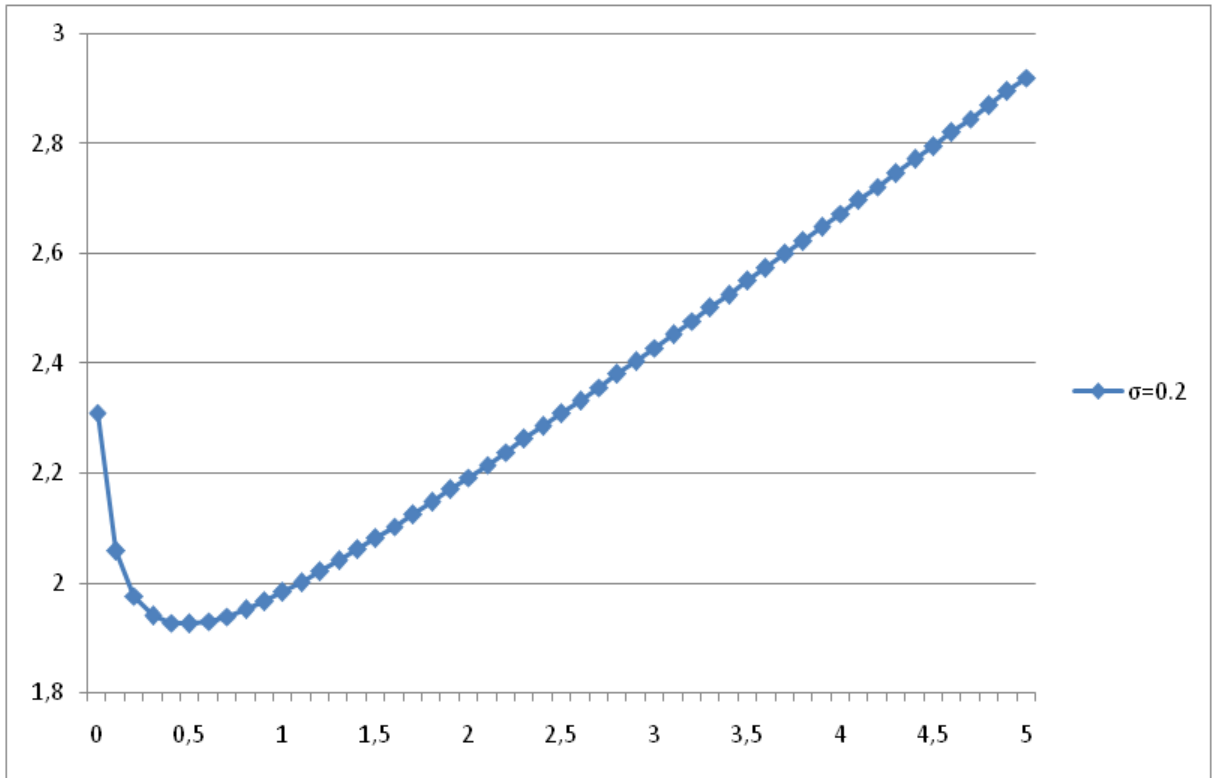


Figure 1: Optimal trigger as a function of the intensity parameter λ (with $K = \bar{K} + \lambda k$).

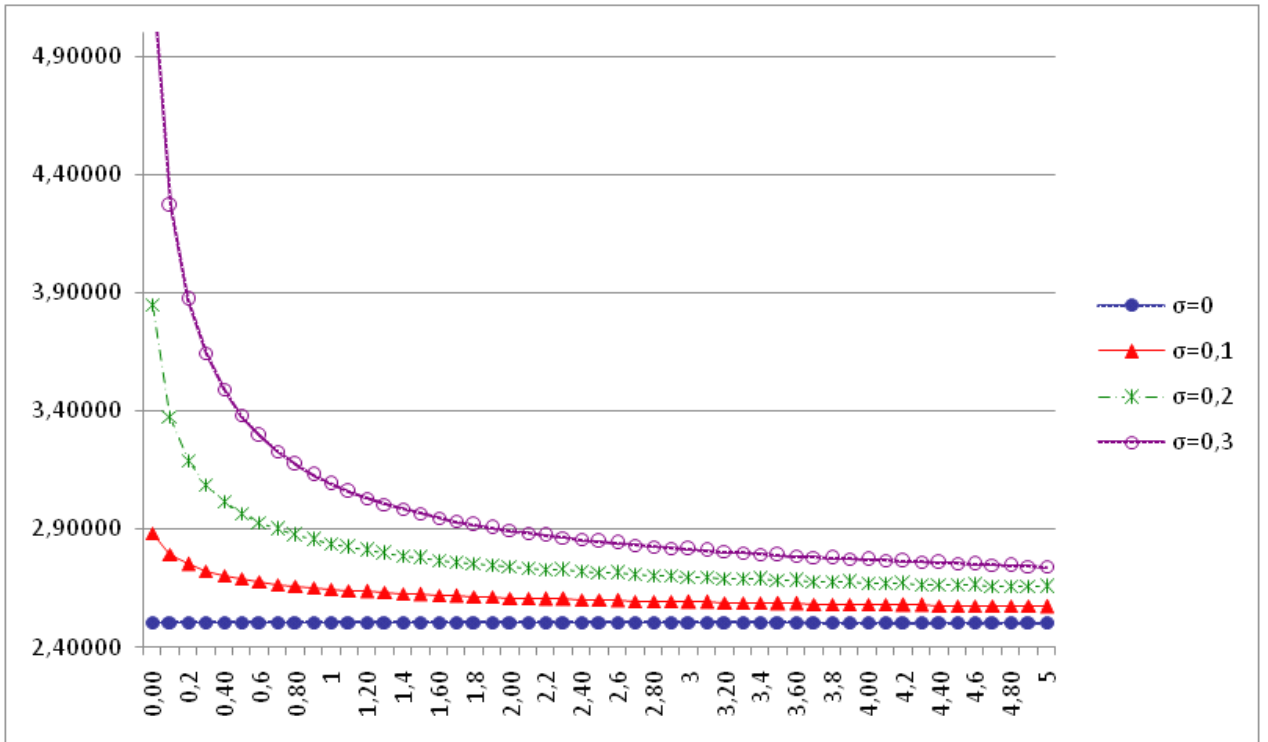


Figure 2: Optimal trigger as a function of λ with K constant.

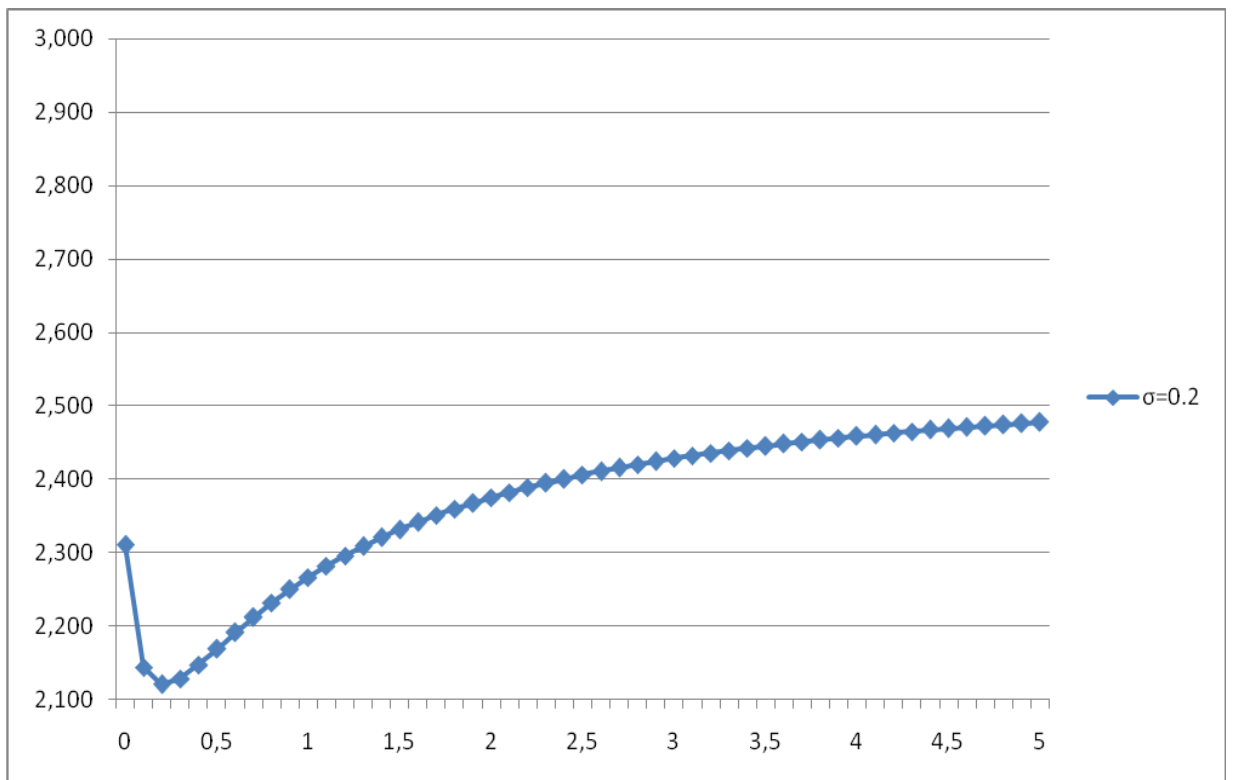


Figure 3: Optimal trigger as a function of the intensity parameter λ (with $K = \bar{K} + \frac{\lambda}{1+\lambda}k$).

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