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**Repeated Cheap-Talk Games  
of Common Interest  
between a Decision-Maker  
and an Expert of Unknown  
Statistical Bias**

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Milano-Bicocca, Italy

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#### Summary

Two agents are engaged in a joint activity that yields a common perperiod payoff at two rounds of play. The expert announces the probability that the current state of the world is low, instead of high, at each stage. Having received the report of the expert, the decision-maker takes action at every period according to his posterior beliefs. At the end of each round of play, the true current state is verifiable. The distinctive assumption of the paper is that the decision-maker makes a subjective appraisal of the expert's reliability: he considers the expert's true forecasts as the outcomes of an experiment of unknown statistical bias. The paper shows that the expert will have instrumental reputational concerns, related to the future estimate of the systematic error associated to his predictions. In contrast with previous work, reputational concerns are shown to enhance the credibility of the initial messages, and to increase both the agents' expected payoff at the first round of play in equilibrium. The equilibrium messages will be noisy, but noisiness will be less costly than it would be in single-stage games.

**Keywords:** Opinion, Expert, Strategic Communication

**JEL Classification:** D81, D84

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# Repeated Cheap-Talk Games of Common Interest between a Decision-Maker and an Expert of Unknown Statistical Bias

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## Abstract

Two agents are engaged in a joint activity that yields a common per-period payoff at two rounds of play. The expert announces the probability that the current state of the world is low, instead of high, at each stage. Having received the report of the expert, the decision-maker takes action at every period according to his posterior beliefs. At the end of each round of play, the true current state is verifiable. The distinctive assumption of the paper is that the decision-maker makes a subjective appraisal of the expert's reliability: he considers the expert's true forecasts as the outcomes of an experiment of unknown statistical bias. The paper shows that the expert will have instrumental reputational concerns, related to the future estimate of the systematic error associated to his predictions. In contrast with previous work, reputational concerns are shown to enhance the credibility of the initial messages, and to increase both the agents' expected payoff at the first round of play in equilibrium. The equilibrium messages will be noisy, but noisiness will be less costly than it would be in single-stage games.

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## 1 Introduction

Many decisions depend on trusting someone. Trust matters especially when a decision-maker is uncertain about some event of interest and asks for advice to someone else called expert. In order to put the advice to use, the very first choice of the decision-maker will concern his subjective assessment of the reliability that he judges to be typical of the honest opinions of the expert. Indeed,

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a decision-maker could not even approach the problem of strategic communication on the expert's side if he did not know how to deal with straight messages. The reliability of an honest advice may not be obvious when the expert makes predictions and the decision-maker has little substantive knowledge of the factors affecting the uncertain event of interest. For instance, the decision-maker may not be aware of the specific process of information extraction connecting signals and beliefs according to the expert. In that case honest forecasts are not reliable per se. The decision-maker may take messages as being truthful and biased at the same time: they can report the opinion of the expert faithfully, and still be subject to some systematic error. The repeated interaction between a decision-maker and an expert of uncertain reliability is the concern of the present paper.

In particular, I consider a simple model of strategic communication between two agents, who play a two-stage cheap-talk game. The agents have the same preferences and aim at maximizing the undiscounted sum of the single-period payoffs of their joint activity. At every round of play the decision-maker is uncertain about the current and binary state of nature, that is identically and independently distributed across time. The expert is a probability assessor and is supposed to believe in the validity of his own predictions. At every stage the expert forms his belief and sends to the decision-maker a message. Having received the message, the decision-maker updates his probability that the current state is low. Then, the decision-maker takes an action that determines the state-contingent single-period payoff, common to both the agents.

Recurring to the statistical literature on the expert problem, the distinctive assumption of the paper is the following: the decision-maker makes a subjective assessment of the honest forecasts of the expert. In particular, the decision-maker takes the expert's true beliefs as the realizations of an experiment with an unknown parameter  $\eta$ . That parameter is a measure of the systematic error in the predictions of the expert. So the expert's true beliefs are considered to be the outcomes of a random variable having a probability density function conditional on the true state and the unknown parameter  $\eta$ . At the initial stage the decision-maker believes the parameter  $\eta$  to be a drawing from some prior distribution function of  $\eta$  with support in the closed unit interval of the real line.

At the end of the first stage, both the agents observe the true current state of nature, and the decision-maker revises his measure of the statistical bias of the expert. At the second round the same stage game is played again with new beliefs and a new message, action and state. The paper focuses on perfect Bayesian equilibria. The equilibrium properties will be valued with respect to the agents' expected payoffs, not with respect to information transmission, since reputation is only subjectively assessed by the decision-maker.

The paper shows that the expert will not reveal his beliefs truthfully in equilibrium at the last round of play. At period 2 the decision-maker's best action will be proportional to his posterior probability that the current state is low. Since the agents share a common single-period payoff, if they hold a common belief that the current state is low, they will agree to the optimal

current action. However, when the decision-maker cannot rule out the possibility of an honest expert being biased, even in the case of genuine reports, the single-period payoff expected by the agents will be different, as well as their best action. So at each round of play the single-period payoff expected by the expert will decrease in the distance between his true belief and the decision-maker's posterior probability that the current state is low. If the decision-maker expects honest messages, the expert will have an incentive to send dishonest messages. Strategic communication will work in the following way: if the expert is more confident that a high state occurs, he will announce a probability that the current state is high greater than his true belief; if the expert is more confident that a low state occurs, he will announce a probability that the current state is low higher than his true belief. In other words, the expert will exaggerate his reports in order to lead the decision-maker to believe what he believes. The worse is the reputation of the expert in the decision-maker's mind, the less reactive will be the decision-maker to messages. As a result, the distortion in the announced probability will be stronger, and the loss expected by the expert will be higher.

At each round the expert will choose a message rule, that is a probability density function of message  $m_t$  conditional on his belief  $p_t$ . The paper shows that in equilibrium the support of the final message rules of the expert will be a finite partition of the closed unit interval of the real line. Unlike the usual outcomes of cheap-talk games, equilibria are proved to exist for every finite cardinality of the equilibrium partition at period 2. The reason is that in the present paper the main problem is the conflict of opinions between the agents: the decision-maker will choose an action that can be higher or lower than the expert's unconstrained best action. That "non-monotonic" conflict between the agents is essential in yielding the result of no finite upper bound to the equilibrium partitions at the second stage.

In equilibrium the expert's incentive to misreport will reduce both the agents' final expected payoffs. At the final stage of play both the agents would be better off if the expert could commit to some message rules the support of which were a finite partition different from the equilibrium one. Moreover, the equilibrium final payoff expected by the expert at the end of period 1 is shown to depend on the final bias expected by the decision-maker. Hence, the message that the expert chooses to send at period 1 will have both a direct effect on his expected current payoff, and an indirect effect on his future reputation and his expected future payoff.

The paper shows that honest initial reports cannot be supported in equilibrium for every equilibrium profile at the final stage of play. As far as the expert's final reputation is concerned, the paper shows the following. If the true state at period 1 is low and the expert has induced the decision-maker to have a lower probability that the initial state is low, the expert's reputation at period 2 will be worse than it would have occurred if he had induced a higher probability. The opposite will obtain if the true state at period 1 is high. Moreover, if the expert induces the decision-maker to have a low probability that the initial state is low, his future reputation will improve with respect to the expected current

bias only if the true state at period 1 is high.

Instrumental reputational concerns at period 1 are shown to imply a trade-off between two conflicting purposes: a) the maximization of the initial payoff expected by the expert, and b) the minimization of the future loss expected by the expert when his reputation falls. In equilibrium the expert's reputational concern at period 1 will partially offset his incentive to misreport at the first round of play. In other words, instrumental reputational concerns will enhance the credibility that the decision-maker can associate to the first-period messages. Consequently, the equilibrium payoff at period 1 expected by both the agents will be higher in repeated games than in single-stage games. In equilibrium all the agents will gain from reputational concerns. That result is in contrast with recent contributions showing that reputational concerns will impair the equilibrium welfare properties.

The rest of the paper is organized as follows. In Section 2, I discuss the related literature, and, in Section 3, I describe the assumptions and the basic model. Section 4 is concerned with the equilibria properties at the last stage of play. Section 5 focuses on the equilibrium reports at the first round of play. In Section 6, I conclude. All proofs are in the Appendix.

## 2 Related Literature

In the economic literature professional advice is often analyzed by way of models of asymmetric information. At least two agents are interested in the value of some parameter or state of the world  $\theta$ . Only one agent, called expert, is going to observe some new data  $Y$ . All the parties agree upon the parameter space, the space of the realizations of  $Y$  and the conditional distribution of  $Y$  given  $\theta$  for all possible values of  $\theta$  (Bayarri-DeGroot (1991)). The consensus upon the distribution of  $Y$  conditional on the unknown parameter makes expertise a special instance of private information to be revealed according to the rules of strategic communication<sup>1</sup>.

In particular, the quality of information exchanged in equilibrium<sup>2</sup> is a major concern in cheap-talk games, along the path initiated by Crawford and Sobel (1982). Ex-ante an agent called sender has observed the value of a random variable, that can be modelled as the sender's type. The sender sends a message to a receiver. The receiver takes an action that determines the welfare of both the agents, jointly with the sender's type. The sender is a "partisan" expert because he is better informed than the decision-maker, and because he has preferences over actions, typically different from the decision-maker. Messages are not verifiable and are "cheap" because their transmission is free of any direct

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<sup>1</sup>A different approach to the issue of professional advice is taken by the contributions concerned with credence goods. In that case, fraud and cheating are the major problems in the interaction between experts and consumers (for an extensive review, see Dulleck and Kerschbamer (2006)).

<sup>2</sup>In principal-agent relationships optimal delegation can be an effective substitute to inefficient mechanisms inducing disclosure of private information (for instance, Demski-Sappington (1987), Li-Suen (2004) and Alonso-Matouschek (2008)).

cost. Equilibria are shown to involve noisy messages because the expert engages in strategic information communication. The present paper takes from Crawford and Sobel the idea of a partisan expert: the expert's expected payoff will depend on the action taken by the decision-maker. However, while in Crawford and Sobel truthful messages will be sent in equilibrium when the interests of the agents coincide<sup>3</sup>, in the present paper noisiness will persist under solidarity in equilibrium.

Ottaviani and Sorensen (2006a) consider a professional expert concerned about appearing to be well informed about the current state of the world. Ex-ante the expert knows his ability, otherwise unobservable. During the game only the expert observes a private signal generated by a multiplicative linear experiment. In such experiment the ability of the expert is the measure of the amount of information that is encoded in the signal. After observing the signal, the expert is free to publicly report any message. At the end of the period an evaluator combines the message from the expert with the ex-post realization of the state to update the belief regarding the ability of the expert. That posterior belief is the reputation of the expert that determines his final payoff. So the correspondent cheap-talk game is static and the expert is perfectly indifferent to the current use of his report. The expert's reputational concern is shown to be incompatible with truthful messages in equilibrium. In particular, if the prior is already concentrated on any particular state, the expert will always wish to bias his report in the same direction. The present paper shares with Ottaviani and Sorensen the idea that reputational concerns can matter; however, the concern for reputation will descend endogenously from the expert's desire to be believed in his repeated interaction with the decision-maker<sup>4</sup>.

Credibility in repeated rounds of information transmission is analyzed by Sobel (1985) for a case in which the receiver must decide whether to trust the sender, whose motives are uncertain. If the sender is a friend, he will share the same preferences of the receiver; if he is an enemy, his best action will be the opposite of the decision-maker's best action. Sender and receiver interact for a finite number of times. At the beginning of every stage, the agents observe a random variable that measures the importance of that current round. Only the sender observes the value of a binary random variable, that is independently and identically distributed across time. Then the sender sends a message to the receiver, who takes a decision affecting the welfare of both the agents. The players maximize the undiscounted sum of their single-period payoffs. The agents' single-period payoff depends on the distance between action and signal. The sender's best action will be the signal if he is a friend, while it will be the opposite of the signal if he is an enemy. Repeated interaction between the agents, coupled with verifiable information at the end of each period of interaction, is shown to make it worthwhile for the receiver to build a reputation

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<sup>3</sup>Crawford and Sobel show that the amount of information revealed in equilibrium will increase as the preferences of the sender and the receiver become more aligned.

<sup>4</sup>Contributions concerned with either the relative evaluation of many experts (Ottaviani-Sorensen (2006b, 2006c), Scharfstein-Stein (1990)) or multiple signals about the same state of the world (Li (2007), Prendergast-Stole (1996)) are less closely related to this paper.

for truthfulness<sup>5</sup>. In equilibrium the sender typically conveys accurate information for the first several periods. An enemy will eventually take advantage of the receiver by misleading him and losing all opportunities for deception in the future. The present paper shares with Sobel the idea of a repeated interaction between expert and decision-maker, both interested in the maximization of the undiscounted sum of the single-period payoffs. However, the uncertainty about the expert's preferences is replaced by the decision-maker's uncertainty about the expert's ability as a probability assessor. Moreover I will consider two-stage games only.

In S. Morris (2001) an informed advisor wishes to convey his valuable information to an uninformed decision-maker in a two-stage cheap-talk game. The decision-maker believes that the advisor may have preferences different from his own, biased in favor of particular decisions. At every round, the advisor observes a noisy signal of the current and binary state of the world, and sends a message to the decision-maker. Having received the message, the decision-maker takes an action from a continuum, affecting the welfare of both the agents. At the end of the period the state is verified and the decision-maker updates his beliefs about the advisor's preferences, given his message and the true state of the world at the first period. Then the game is played again, with the same advisor but a new state, signal, message and action. Here the advisor has no intrinsic reputational concerns as in Ottaviani and Sorensen, but he has instrumental reputational concerns, that arise exclusively from his desire to have his unbiased advice listened to in the future. Morris shows that the advisor will have a reputational incentive to lie at the first stage, if he does not wish to be mistaken for a biased expert. So, when reputational concerns are sufficiently important relatively to the current decision problem, no information will be conveyed in equilibrium at the first period. The present paper shares with Morris the idea that an expert cares about his reputation instrumentally. Once again, the assumption of a conflict of interest between the agents is replaced by different opinions over the probability that a particular state is realized.

While the economic contributions are mostly concerned with the strategic reporting of predictions, it is the use of predictions that is paid particular attention by the statistical literature. French (1986) summarizes the expert problem in the following way: a decision-maker needs to assess his subjectivity probability for an event  $\theta$  of interest; having little substantive knowledge of the factors affecting  $\theta$ , the decision-maker asks another person for advice. An expert is anyone who can give predictions, i.e. anyone who can make probability statements, called judgments or opinions, concerning the event of interest. The problem is: how should the decision-maker incorporate the honest opinion of an expert into his own? P.A. Morris (1974, 1977), Lindley et al. (1979), and French (1980) suggest a Bayesian modeling approach to the use of experts<sup>6</sup>. The decision-maker should look upon the true opinion of the expert as a piece of data: consulting an expert is like performing an experiment, and just as

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<sup>5</sup>Honesty in informal communication is further analysed by Olszewski (2004).

<sup>6</sup>Related works are Morris (1983) and Genest-Schervish (1985).

the results of an experiment are a priori unknown to the experimenter, so the advice of the expert is uncertain to the decision-maker prior to receiving it. According to P.A. Morris (1974), the model of the expert in the decision-maker's mind is a likelihood function  $l(p(\theta) | \theta)$ , that represents the probability of the event that the expert's prior is  $p(\theta)$  given  $\theta$  (not the probability of a probability in the classical sense)<sup>7</sup>. The likelihood function  $l(p(\theta) | \theta)$  summarizes the decision-maker's subjective measure of the expert's reliability<sup>8</sup>. Consequently, a distinction is required between the meaning of an honest probability assessment to the decision maker and the expert himself: the expert looks at his probability assessment as the reflection of his own information, the decision-maker takes the expert's true opinion as information itself. The present paper borrows from the statistical literature the idea that the decision-maker has his own model of the true opinions of the expert. Further, it makes an attempt to reconcile it with strategic behavior on the expert's side, that is typical of the economic literature.

The role of probability assessor played by the expert in the present paper is quite close to the spirit of Harris and Raviv (1993). They consider a population of agents who receive public information but interpret that information differently. That is, it is common knowledge that people have different opinions and believe in the validity of their own judgements. In the present paper, differences of opinions in the population of experts are not analyzed, but they can be consistent with a decision-maker who can only appraise subjectively how much reliable the forecasts of an expert are, when that expert is drawn from a population of opinionated agents.

Finally, the issue of trust between decision-maker and expert can be related to the issue of leadership. In Hermalin (1998) leadership is distinguished from authority because following a leader is a voluntary, rather than coerced, activity of the followers. In particular, in Hermalin leadership is the capacity of the leader to induce rational agents to exert effort when the leader can have incentives to mislead them. But leadership can spring also from the high regard into which subordinates take the competence of a manager. When subordinates receive instructions beyond their scope of direct observation or understanding, they will face the same situation of a decision-maker weighting the opinion of an expert. In this sense, while the theory of yes-men of Prendergast (1993) is

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<sup>7</sup>Morris (1974, p.1238):

Suppose a decision maker is considering the weather to determine the prospects for a picnic. His view is that there is a 50-50 chance of rain....While he is waiting for the weather report he ponders how he will use the weatherman's advice. He first reason that the weatherman will state a probability of rain  $p$ ... The decision maker ... makes a subjective appraisal of the dependence between the expert's advice and the actual weather. Specifically, he asks himself what his assessment of  $p$  would be if an honest clairvoyant told him that it will surely rain on his picnic...For any given value of  $p$  he can calculate the posterior probability of rain...

<sup>8</sup>According to Lindley (1982), an expert is probability calibrated if the decision maker adopts the expert's opinion for his own. Other concepts of calibration are discussed by DeGroot-Fienberg (1983).

concerned with the reliability that a manager expects to characterize the reports from his subordinates, a complementary issue can regard the reliability that subordinates expect to characterize the guidelines issued by their manager.

### 3 Set-up

For two periods a decision-maker and an expert, denoted by  $D$  and  $E$  respectively, are engaged in a joint activity. That joint activity yields a common payoff at every period. The payoff at period  $t$  depends on the state of the world at period  $t$ , denoted by  $\omega_t$ , and the action chosen at period  $t$ , denoted by  $a_t$ . Both agents aim at maximizing the undiscounted sum of the per-period payoffs of their joint activity.

At every period the state of the world can be either  $-1$  (low state) or  $1$  (high state). At every period the feasible action set  $A$  is the closed interval  $[-1, 1]$  of the real line. The common payoff function at period  $t$ , denoted by  $\pi_t(\omega_t, a_t)$ , is the quadratic loss function, i.e.:

$$\pi_t(\omega_t, a_t) = -(\omega_t - a_t)^2 \quad (1)$$

Then a best action is always implementable for every expected current payoff.

At the beginning of every period the expert makes an assessment of the probability that the current state is low. Let  $p_t$  denote the expert's true belief that  $\omega_t$  is low, with  $p_t$  in the closed unit interval of the real line, denoted by  $I$ . During period  $t$  the expert sends a message, denoted by  $m_t$ , to the decision-maker. Every message  $m_t$  belongs to  $I$  and can be interpreted as the expert's announced probability that the current state is low. Having received  $m_t$ , the decision-maker updates his probability that  $\omega_t$  is low, and chooses the current action  $a_t$ .

The beliefs of the expert are assumed to satisfy the following conditions<sup>9</sup>: a) a belief  $p_t$  is formed at period  $t$  before any message  $m_t$  is sent; b) if the expert has belief  $p_t$ ,  $p_t$  will represent his degree of confidence in the event " $\omega_t$  is low", i.e. the expert believes  $p_t$  to be unbiased; and c) at period  $(t - 1)$  the expert expects that in the future he can have any belief  $p_t$  in  $I$  with positive probability. For simplicity, it is assumed that the prior joint probability density function (p.d.f.) of  $p_1$  and  $p_2$  is represented by  $[f(p)]^2$ , where  $f(p)$  is the uniform p.d.f. Hence, the final beliefs of the expert are stochastically independent from his initial beliefs.

The decision-maker believes that each state occurs with equal prior probability, and that the states are stochastically independent across time. The decision-maker considers the true beliefs of the expert as the realizations of an experiment with an unknown parameter labelled  $\eta$ . In particular, the decision-maker considers the true belief  $p_t$  of the expert as the outcome of the random variable  $P_t$  in the sample space  $I$ , having a p.d.f. conditional on the true state

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<sup>9</sup>The process by which the expert comes to have his beliefs is not modelled. The main reasons are the following: a) different experts may have different opinions, and b) the decision-maker is unaware of the inferences that an expert can make.

and the unknown parameter  $\eta$ . Moreover, the decision-maker believes that the true beliefs of the expert at different times form a random sample: provided the true states  $\omega_1$  and  $\omega_2$  are equal to  $\omega$ , the true beliefs  $P_1$  and  $P_2$  are i.i.d. with common likelihood function  $l(p | \omega, \eta)$ .

The parameter  $\eta$  is assumed to belong to the closed unit interval of the real line. The likelihood functions related to the lowest and the highest parameters are the following:

$$\begin{aligned} l(p | -1, 0) &= 2p = l(1 - p | 1, 0) \quad \forall p \in I \\ l(p | -1, 1) &= 1 = l(p | 1, 1) \quad \forall p \in I \end{aligned} \quad (2)$$

When  $\eta$  is zero, the likelihood of  $p$ , conditional on a low (high) state, is linearly increasing (decreasing) in  $p$ . When  $\eta$  is the highest, the likelihood of  $p$  conditional on state  $\omega$  is uniform. For every parameter  $\eta$  in  $(0, 1)$  the likelihood function  $l(p | \omega, \eta)$  is assumed to be the following:

$$l(p | \omega, \eta) = (1 - \eta)l(p | \omega, 0) + \eta l(p | \omega, 1) \quad \omega = -1, 1 \quad (3)$$

i.e. the likelihood function  $l(p | \omega, \eta)$  is a linear mixture of the experiments characterized by the extreme values that  $\eta$  can take. The parameter  $\eta$  is a measure of statistical bias for the following reason. Let  $u(-1 | p, \eta)$  denote the decision-maker's posterior probability that  $\omega$  is low conditional on  $p$  and  $\eta$ , i.e.:

$$u(-1 | p, \eta) = \frac{l(p | -1, \eta)}{l(p | -1, \eta) + l(p | 1, \eta)} \quad (4)$$

That is,  $u(-1 | p, \eta)$  would be the decision-maker's posterior belief that the current state is low, if messages could not be other than honest, and if the decision-maker believed that the expert's forecasts were characterized by parameter  $\eta$  with certainty. From (2)-(3), if  $\eta_j > \eta_i$ , then  $|p - u(-1 | p, \eta_i)| \leq |p - u(-1 | p, \eta_j)|$  for every  $p$  in  $I$ . In other words, the lower is  $\eta$ , the smaller will be the distance between the decision-maker's posterior beliefs and the expert's true beliefs. The higher is  $\eta$ , the greater will be the correction deemed to be right by the decision-maker over the expert's forecasts. The decision-maker would adopt the honest opinions of the expert for his own if he were certain that  $\eta$  is equal to 0 (i.e. the expert is reputed to be perfectly calibrated). The decision-maker would be indifferent to the expert's reports if he were certain that the expert is perfectly unreliable (i.e.  $\eta = 1$ ). It follows that the parameter  $\eta$  measures the systematic error that affects the true beliefs of the expert according to the decision-maker.

To sum up, the uncertainty of the decision-maker is substantial for two reasons. First, the decision-maker cannot be more uncertain about the current state than at the beginning of every period, since his prior probability that the current state is low is 0.5. Second, the decision-maker looks at the expert as a source of forecasts with a systematic and unknown error represented by the parameter  $\eta$ . At the initial period the decision-maker believes the expert's parameter  $\eta$  to be a drawing from the distribution function (d.f.)  $G_1$  of  $\eta$ , with

corresponding generalized probability density function<sup>10</sup> (g.p.d.f.)  $g_1$  of  $\eta$  on  $I$ . It is assumed that under the g.p.d.f.  $g_1$  of  $\eta$  both the expected value of  $\eta$ , denoted by  $\bar{\eta}_1$ , and the variance of  $\eta$ , denoted by  $\sigma^2$ , exist. The support of  $g_1(\cdot)$ , denoted by  $H_1$ , is assumed to be non-singular in  $(0, 1]$ <sup>11</sup>. After action  $a_1$  has been implemented, both the common payoff and the state at period 1 are realized and publicly observed. Then the game is played again by the same agents with new predictions and a new state, a new message, a new g.p.d.f. of  $\eta$  and a new action.

All aspects of the game except  $(p_1, p_2)$  are common knowledge. The game solution concept is that of Perfect Bayesian Equilibria.

## 4 Message Rule for One Period Ahead

I begin the analysis with a discussion of the last stage. Let  $g_2(\eta)$  be the short notation for the g.p.d.f. of  $\eta$  conditional on  $\omega_1$  and  $m_1$ , with support denoted by  $H_2$  and corresponding expected bias denoted by  $\bar{\eta}_2$ . The expert will choose a period-2 message rule that is a g.p.d.f. of  $m_2$  conditional on his true belief  $p_2$ , denoted by  $\mu(m_2 | p_2)$ , with  $\int_I \mu(m_2 | p_2) dm_2 = 1$ . Let  $\beta(m_2 | p_2)$  denote the decision-maker's period-2 belief that the expert announces  $m_2$ , conditional on  $p_2$ , with  $\int_I \beta(m_2 | p_2) dm_2 = 1$ . Finally, let  $\nu(m_2)$  denote the decision-maker's period-2 posterior probability that  $\omega_2$  is low, conditional on message  $m_2$ , i.e.:

$$\nu(m_2) = \frac{\int_{H_2} \int_I l(p_2 | -1, \eta) \beta(m_2 | p_2) g_2(\eta) dp_2 d\eta}{\int_{H_2} \int_I [l(p_2 | -1, \eta) + l(p_2 | 1, \eta)] \beta(m_2 | p_2) g_2(\eta) dp_2 d\eta} \quad (5)$$

For brevity, sometimes I will call  $\nu(m_2)$  the decision-maker's induced posterior belief, meaning that it is the posterior probability that  $\omega_2$  is low to which the decision-maker is lead by the expert sending  $m_2$ . The decision-maker will choose a period-2 action rule that is a g.p.d.f. of  $a_2$  conditional on  $m_2$ , denoted by  $\alpha(a_2 | m_2)$ , with  $\int_A \alpha(a_2 | m_2) da_2 = 1$ .

In equilibrium:

a) if  $\tilde{\mu}(m'_2 | p_2) > 0$ , then  $m'_2$  solves  $\max_{m_2} E_E[\pi_2(\omega_2, \tilde{\alpha}(a_2 | m_2)) | p_2]$ , where  $E_E[\pi_2(\cdot) | p_2]$  denotes the payoff at period 2 expected by the expert having belief  $p_2$ ;

b) if  $\tilde{\alpha}(a'_2 | m_2) > 0$ , then  $a'_2$  solves  $\max_{a_2} E_D[\pi_2(\omega_2, a_2) | \tilde{\nu}(m_2)]$  where  $E_D[\pi_2(\cdot) | \tilde{\nu}(m_2)]$  denotes the payoff at period 2 expected by the decision-maker who has received message  $m_2$ ;

c) if  $\int_I \tilde{\mu}(m'_2 | p_2) dp_2 > 0$ , then  $\tilde{\mu}(m'_2 | p_2) = \tilde{\beta}(m'_2 | p_2)$  for every  $p_2$ .

Conditions a) and b) describe a Nash Bayesian equilibrium at period 2. Condition c) says that the decision-maker's period-2 beliefs  $\{\beta(m_2 | p_2)\}$ , and, consequently, his posterior probabilities  $\{\nu(m_2)\}$  are to be consistent with the expert's period-2 message rules in equilibrium.

<sup>10</sup>The adjective "generalized" is used in order to deal with a d.f. of  $\eta$  that can be a discrete distribution, a continuous distribution or a mixture of the two.

<sup>11</sup>Otherwise the d.f. of  $\eta$  could be degenerate.

The following Proposition shows that the decision-maker will never adopt mixed strategies at period 2. Moreover, the expert will not report his true beliefs in equilibrium at the final stage of play.

**Proposition 1** : *the decision-maker's best action  $\hat{a}_2$  conditional on  $m_2$  is a strictly monotonic function of  $\nu(\cdot)$ . Provided  $\bar{\eta}_2 \in (0, 1)$ , truthtelling will never be the expert's period-2 equilibrium message rule.*

Truthtelling cannot be supported in equilibrium at period 2 for the following reason. The decision-maker's best final action depends on his posterior probability that the current state is low. When the decision-maker expects the expert to be honest but biased, the decision-maker's induced posterior belief will typically be different from the belief of that honest expert. If the agents show different degrees of confidence in the event "the current state is low", they will disagree about the expected single-period payoff, and the best action to implement. Consequently, the single-period payoff expected by the expert will be decreasing in the distance between his true belief and the decision-maker's induced posterior belief. In other words, the expert will have an interest in leading the decision-maker to believe what he believes. When the decision-maker conjectures that the messages are honest, the expert will misreport in order to squeeze the gap between his true belief and the decision-maker induced posterior belief.

In particular, misreporting will work in the following direction: if the expert believes that a high state is more likely (i.e.  $p_2 > 0.5$ ), he will send a message  $m_2$  higher than  $p_2$ ; instead, if the expert is confident that a low state will occur, he will announce a probability  $m_2$  lower than  $p_2$ . In other words, if the decision-maker believes that the reports are honest, the expert will exaggerate his announcements in order to make the decision-maker's induced posterior probability closer to his true belief. Moreover, given two distribution functions of  $\eta$ ,  $G'_2$  and  $G''_2$ , if the d.f.  $G'_2$  dominates the d.f.  $G''_2$  in the sense of first order stochastic dominance, then the decision-maker's induced posterior beliefs will be less reactive to the messages sent by the expert under the d.f.  $G'_2$  than under the d.f.  $G''_2$ . As a result, the worse is the reputation of the expert, measured by the bias expected by the decision-maker, the stronger will be the intensity of misreporting.

From Proposition 1 the decision-maker's posterior probability  $\nu(m_2)$  cannot be a continuous function of  $m_2$  in equilibrium at the final stage of play. Now, in order to characterize period-2 equilibria with non-continuous posterior probabilities  $\{\nu(m_2)\}$ , I need to proceed in the following way:

1) fix a g.p.d.f.  $g_2$  of  $\eta$  and a system of period-2 beliefs  $\{\beta(m_2 | p_2)\}$  of the decision-maker that generate  $n_2$  different posterior probabilities  $\{\nu(m_2)\}$ . Let  $V_{n_2}$  denote the correspondent ordered set of period-2 posterior probabilities, i.e.  $V_{n_2} = \{\nu_i | \nu_{(i-1)} < \nu_i < \nu_{(i+1)}\}$ .

2) Once a set  $V_{n_2}$  has been specified, let  $P(V_{n_2})$  denote the set of the elements  $\{Y_i\}$  defined in the following way:

$$Y_i = \{p_2 | |p_2 - \nu_i| \leq |p_2 - \nu_j| \quad \forall \nu_j \in V_{n_2} \setminus \nu_i\} \quad (6)$$

By construction,  $\cup Y_i = I$ . From the proof of Proposition 1, if  $p_2$  belongs to  $Y_i$ , then the payoff at period-2 expected by the expert with belief  $p_2$  from inducing  $\nu_i$  will not be lower than the payoff at period-2 expected by the same agent from inducing any other  $\nu_j$  (i.e.  $E_E[\pi_2(\omega_2, \hat{a}(\nu_i)) - \pi_2(\omega_2, \hat{a}(\nu_j)) | p_2] \geq 0$  for every  $\nu_j \in V_{n_2} \setminus \nu_i$  if  $p_2 \in Y_i$ , where  $\hat{a}(\nu_i)$  stands for the decision-maker's best final action conditional on message  $m_2$  inducing  $\nu_i$ ).

The following Proposition shows that equilibria at the final stage of play always exist. In particular:

**Proposition 2 :**

a) provided  $\bar{\eta}_2 \in (0, 1)$ , for every integer  $n_2$  there will be a unique period-2 equilibrium set  $\tilde{V}_{n_2}$  and a unique equilibrium set  $P(\tilde{V}_{n_2})$ , that is a partition with  $n_2$  elements, thereby simply denoted by  $\tilde{P}_{n_2}$ . For every  $n_2 > 1$ , both  $\tilde{V}_{n_2}$  and  $\tilde{P}_{n_2}$  will be symmetric around 0.5.  $\tilde{V}_{n_2}$  will be a function of  $\bar{\eta}_2$  if  $n_2 > 1$ , while  $\tilde{P}_{n_2}$  will be a function of  $\bar{\eta}_2$  if  $n_2 > 2$ .

b) Provided  $\bar{\eta}_2 = 1$ , there will be a unique period-2 equilibrium set  $\tilde{V}_1$  and a unique equilibrium set  $P(\tilde{V}_1)$  with  $\tilde{V}_1 = 0.5$  and  $P(\tilde{V}_1) = I$ .

Proposition 2 shows that the equilibria at the final stage of play are partitional, similarly to the result of Crawford and Sobel (1982). Since the expert believes in the validity of his own forecast, his reports will depend on his true beliefs only. In equilibrium every posterior probability  $\tilde{v}_i$  in  $\tilde{V}_{n_2}$  can be computed as if the expert chose a uniform message rule for every interval in the equilibrium partition. The equilibrium sets  $\tilde{V}_{n_2}$  and  $\tilde{P}_{n_2}$  with more than one element are shown to be symmetric around 0.5. Moreover, given an interval  $\tilde{Y}_i = [\tilde{y}_{(i-1)}, \tilde{y}_i]$  that belongs to the equilibrium partition  $\tilde{P}_{n_2}$ , its size (i.e.  $(\tilde{y}_i - \tilde{y}_{(i-1)})$ ) will be constant for every  $i$  only if the expert is reputed to be perfectly reliable. Otherwise, the size of each interval  $\tilde{Y}_i$  in  $\tilde{P}_{n_2}$  will be decreasing in  $i$  from 1 to  $\frac{n_2}{2}$ . Consequently, both the equilibrium sets  $\tilde{V}_{n_2}$  and  $\tilde{P}_{n_2}$  will be function of the bias expected by the decision-maker at the final stage of play.

Unlike the usual outcomes of cheap-talk games, Proposition 2 shows that an equilibrium partition with cardinality  $n_2$  exists and is unique for every finite  $n_2$ , as long as the decision-maker does not expect the expert to be totally unreliable (i.e. the g.p.d.f.  $g_2(\eta)$  is not degenerate on 1). The existence of an equilibrium partition for every finite cardinality  $n_2$  is due to the following reason. The action that the expert would freely choose (his unconstrained best action), and the action that the expert both prefers and can actually induce the decision-maker to make are usually different. That difference between the expert's best action and the expert's preferred action within the decision-maker's best actions at period 2 has not always the same sign in the present paper, as it occurs in Crawford and Sobel instead. Cheap-talk games are often marked by some "monotonic" conflict of interest between the agents: for every state of the world the expert's unconstrained best action is always higher or lower than the decision-maker's best action. In the present paper the problem lies with a conflict of opinions

between the agents: the decision-maker will choose an action that can be higher or lower than the expert's unconstrained best action. That feature is essential in yielding the result of no finite upper bound to the equilibrium partitions at the second stage, in contrast with the outcomes of cheap-talk games à la Crawford and Sobel.

Since an equilibrium exists for every finite cardinality  $n_2$ , the higher is the number of messages to which the decision-maker associates a different posterior probability that the current state is low, the finer will be the equilibrium partition at period 2. Thus, provided the expert is reputed to have some reliability at period 2 (i.e.  $\bar{\eta}_2 < 1$ ), multiple equilibria will prevail at period 2. Instead, if the expert is reputed to be totally unreliable (i.e.  $\bar{\eta}_2 = 1$ ), the unique equilibrium will be the babbling equilibrium.

The following Corollary leads to show that at the end of period 1, fixed the cardinality of the partition supporting the expert's message rules, the final payoffs expected by both the agents in equilibrium will never be the highest, if  $n_2 > 2$  and  $\bar{\eta}_2 \in (0, 1)$ . That is:

**Corollary 1** : *let  $E_i [\pi_2 (V_{n_2}, P (V_{n_2}))]$  denote the payoff at period 2 expected by agent  $i$  at the end of period 1 when  $P (V_{n_2})$  is a partition with cardinality  $n_2$  and each  $v_i$  in  $V_{n_2}$  is consistent with a uniform message rule for every interval in  $P (V_{n_2})$ . Provided  $\bar{\eta}_2 \in [0, 1)$ ,  $E_i [\pi_2 (V_{n_2}, P (V_{n_2}))]$  will be maximized if the size of every  $Y_i$  is equal to  $\frac{1}{n_2}$ .*

Proposition 2 and Corollary 1 show that at the final stage of play both the agents would be better off if the expert could commit to message rules different from the equilibrium ones, notwithstanding a partition with  $n_2$  elements. In other words, in equilibrium the expert's incentive to misreport will reduce both the agents' expected final payoffs.

Let  $\tilde{R}_{n_2}$  denote the unique couple of the equilibrium sets  $\tilde{V}_{n_2}$  and  $\tilde{P}_{n_2}$ . Period-2 equilibria characterized by  $\tilde{R}_{n_2}$  can be distinguished only with respect to the decision-maker's beliefs  $\{\beta (m_1 | p_1)\}$  for messages that are never sent by the expert in equilibrium. The following Corollary is concerned with the relationship between the bias expected by the decision-maker at period 2 and the payoff at the final stage expected by the expert at an equilibrium characterized by  $\tilde{R}_{n_2}$ .

**Corollary 2** : *at the end of period 1, the future payoff expected by the expert at every non-babbling equilibrium is strictly decreasing and concave in  $\bar{\eta}_2$ .*

Corollary 2 shows that, if  $n_2$  is greater than 2, the higher is the decision-maker's expected bias at period 2, the greater will be the size of  $\tilde{Y}_1$  and  $\tilde{Y}_{n_2}$  in the equilibrium partition  $\tilde{P}_{n_2}$ , while the the size of all the other intervals will squeeze. As a consequence, every interval  $\tilde{Y}_i$  in the equilibrium partition  $\tilde{P}_{n_2}$  with  $i \neq \{1, n_2\}$ , will shift towards 0.5 as the reputation of the expert deteriorates. The reason is that the expert's incentive to induce the lowest or the highest posterior beliefs,  $v_1$  or  $v_{n_2}$ , will increase with the decision-maker's expected bias at period

2. At the end of period 1, the future payoff expected by the expert at every non-babbling final equilibrium will decrease with the expected distance between his true belief and the decision-maker's induced posterior probability that  $\omega_2$  is low. Hence, the equilibrium future payoff expected by the expert at the end of period 1 will increase if his reputation improves (i.e.  $\bar{\eta}_2$  gets lower than  $\bar{\eta}_1$ ).

Since in correspondence to every non-babbling equilibrium at period 2 the expert's expected payoff depends on  $\bar{\eta}_2$  ultimately, the expert will have an instrumental reputational concern at period 1. The message that the expert chooses to send at period 1 will have both a direct effect on his expected current payoff, and an indirect effect on his expected future payoff. The direct effect comes from message  $m_1$  bringing the decision-maker to choose a particular current action. The indirect effect comes from message  $m_1$  causing a positive or negative change in the expert's reputation, relevant for the second round of play.

## 5 Message Rules for Two Periods Ahead

The expert will choose a period-1 message rule that is a g.p.d.f. of  $m_1$  conditional on his true belief  $p_1$ , denoted by  $\mu(m_1 | p_1)$ , with  $\int_I \mu(m_1 | p_1) dm_1 = 1$ . Let  $\beta(m_1 | p_1)$  denote the decision-maker's period-1 belief that the expert announces  $m_1$ , conditional on  $p_1$ , with  $\int_I \beta(m_1 | p_1) dm_1 = 1$ . Let  $g_2^c(\eta | \omega_1, m_1)$  denote the decision-maker's posterior g.p.d.f. of  $\eta$  at period 2, conditional on  $\omega_1$  and  $m_1$ , and consistent with  $\beta(m_1 | p_1)$ , i.e.:

$$g_2^c(\eta | \omega_1, m_1) = \frac{\int_I l(p_1 | \eta, \omega_1) \beta(m_1 | p_1) g_1(\eta) dp_1}{\int_{H_1} \int_I l(p_1 | \eta, \omega_1) \beta(m_1 | p_1) g_1(\eta) dp_1 d\eta} \quad (7)$$

with corresponding expected bias denoted by  $\bar{\eta}_2^c(\omega_1, m_1)$ . Finally, let  $\lambda(m_1)$  denote the decision-maker's period-1 posterior probability that  $\omega_1$  is low, conditional on message  $m_1$ , i.e.:

$$\lambda(m_1) = \frac{\int_{H_1} \int_I l(p_1 | -1, \eta) \beta(m_1 | p_1) g_1(\eta) dp_1 d\eta}{\int_{H_1} \int_I [l(p_1 | -1, \eta) + l(p_1 | 1, \eta)] \beta(m_1 | p_1) g_1(\eta) dp_1 d\eta} \quad (8)$$

The decision-maker will choose a period-1 action rule that is a g.p.d.f. of  $a_1$  conditional on  $m_1$ , denoted by  $\alpha(a_1 | m_1)$ , with  $\int_A \alpha(a_1 | m_1) da_1 = 1$ .

From Proposition 2 and Corollary 2, a final equilibrium can depend on the bias expected by the decision-maker at period 2. I underline that dependence with the notation  $R_{n_2}(\bar{\eta}_2^c(\omega_1, m_1))$ . I restrict attention to profiles of period-2 equilibria characterized by cardinality  $n_2$  whatever the realized state at period 1, i.e.  $\tilde{\mathbf{R}}_{n_2}^c(m_1) = \left\{ \tilde{R}_{n_2}(\bar{\eta}_2^c(-1, m_1)), \tilde{R}_{n_2}(\bar{\eta}_2^c(1, m_1)) \right\}$ . Let  $\tilde{\mathbf{R}}_{n_2}^c$  denote the set of the equilibrium profiles  $\left\{ \tilde{\mathbf{R}}_{n_2}^c(m_1) \right\}$ . In equilibrium:

a) if  $\tilde{\mu}(m'_1 | p_1) > 0$ , then  $m'_1$  solves  $\max_{m_1} E_E \left[ \pi_1(\omega_1, \tilde{\alpha}(a_1 | m_1)) + \pi_2 \left( \tilde{R}_{n_2}^c(m_1) \right) | p_1 \right]$ , where  $E_E [\pi_1(\cdot) + \pi_2(\cdot) | p_1]$  denotes the sum of the payoffs expected by the expert who has belief  $p_1$  and sends message  $m_1$ , when the equilibrium final profile is  $\tilde{\mathbf{R}}_{n_2}^c(m_1)$ ;

b) if  $\tilde{\alpha}(a'_1 | m_1) > 0$ , then  $a'_1$  solves  $\max_{a_1} E_D \left[ \pi_1(\omega_1, a_1) + \pi_2 \left( \tilde{R}_{n_2}^c(m_1) \right) | \lambda(m_1) \right]$ , where  $E_D [\pi_1(\cdot) + \pi_2(\cdot) | \lambda(m_1)]$  denotes the sum of the payoffs expected by the decision-maker who has received message  $m_1$ , when the equilibrium final profile is  $\tilde{\mathbf{R}}_{n_2}^c(m_1)$ ;

c) if  $\int_I \tilde{\mu}(m'_1 | p_1) dp_1 > 0$ , then  $\tilde{\mu}(m'_1 | p_1) = \tilde{\beta}(m'_1 | p_1)$  for every  $p_1$ .

Conditions a) and b) describe a Nash Bayesian equilibrium at period 1. Condition c) says that the decision-maker's period-1 beliefs  $\{\beta(m_1 | p_1)\}$  are to be consistent with the expert's period-1 message rules in equilibrium. It follows that the equilibrium posterior probabilities  $\{\tilde{\lambda}(m_1)\}$  and the equilibrium posterior distribution functions  $\{\tilde{G}_2^c(\eta | \omega_1, m_1)\}$  need be consistent with the expert's equilibrium initial message rules as well.

The following Lemma shows that the decision-maker will not adopt mixed strategies even at period 1. Moreover, truthtelling at the first stage of play cannot not be supported in equilibrium.

**Lemma 1** : *the decision-maker's best action  $\hat{a}_1$  conditional on  $m_1$  is a strictly monotonic function of  $\lambda(\cdot)$ . Truthtelling will never be the expert's period-1 equilibrium message rule.*

Lemma 1 shows that at period 1 the decision-maker's best actions will be strictly decreasing in his posterior probability that the state at period 1 is low. Moreover, the decision-maker's best initial actions are independent from the equilibrium partition at the final stage of play. Indeed, it is the expert's future reputation that links his expected payoff at period 1 with the his expected future payoff, but the expert's future reputation is independent from the decision-maker's initial action. Moreover, Lemma 1 shows that honest reports at period 1 cannot be supported in equilibrium for every equilibrium profile at the final stage of play.

In order to characterize equilibria with non-continuous initial action rules, I need to proceed in the following way. Fix a system of period-1 beliefs  $\{\beta(m_1 | p_1)\}$  of the decision-maker, that generates  $n_1$  different posterior probabilities  $\{\lambda(m_1)\}$ . Let  $L_{n_1}$  denote the correspondent ordered set of period-1 posterior probabilities, i.e.:  $L_{n_1} = \{\lambda_i | \lambda_{(i-1)} < \lambda_i < \lambda_{(i+1)}\}$ . The following Lemma shows that the decision maker will expect the same bias at period 2 in correspondence to all the different messages at period 1 that induce an equal posterior probability at period 1, i.e.:

**Lemma 2** : *if  $\lambda(m'_1) = \lambda(m''_1)$ , then  $\bar{\eta}_2^c(\omega_1, m'_1) = \bar{\eta}_2^c(\omega_1, m''_1)$  for every  $\omega_1$ . If  $\lambda(m'_1) > \lambda(m''_1)$ , then  $\bar{\eta}_2^c(1, m'_1) > \bar{\eta}_2^c(1, m''_1)$  and  $\bar{\eta}_2^c(-1, m'_1) < \bar{\eta}_2^c(-1, m''_1)$*

Lemma 2 shows that there is a correspondence between the cardinality of the set of the initial posterior probabilities  $\{\lambda_i\}$  and the cardinality of the sets of the final expected biases. In other words, given  $L_{n_1}$ , there will be  $n_1$  different expected biases at period 2, conditional on each realized state  $\omega_1$ . Let  $\bar{\eta}_2^c(\omega_1, i)$  denote the expected final bias conditional on  $\omega_1$  and any message  $m_1$

inducing  $\lambda_i$ . Let  $\tilde{\mathbf{R}}_{n_2}^c(i)$  denote the period-2 equilibrium profile with cardinality  $n_2$  that would prevail if a message  $m_1$  inducing  $\lambda_i$  were sent at period 1 (i.e.  $\tilde{\mathbf{R}}_{n_2}^c(i) = \left\{ \tilde{R}_{n_2}(\tilde{\eta}_2^c(-1, i)), \tilde{R}_{n_2}(\tilde{\eta}_2^c(1, i)) \right\}$ ). Given a set  $L_{n_1}$  and the corresponding equilibrium set  $\tilde{\mathbf{R}}_{n_2}^c$ , let  $Q(L_{n_1}, \tilde{\mathbf{R}}_{n_2}^c)$  denote the set of the elements  $\{Q_i\}$  defined in the following way:

$$Q_i = \left\{ p_1 \mid E_E \left[ \begin{array}{l} \pi_1(\omega_1, \hat{a}_1(\lambda_i)) + \pi_2(\tilde{R}_{n_2}(\tilde{\eta}_2^c(\omega_1, i))) + \\ -\pi_1(\omega_1, \hat{a}_1(\lambda_j)) - \pi_2(\tilde{R}_{n_2}(\tilde{\eta}_2^c(\omega_1, j))) \\ \forall \lambda_j \in L_{n_1} \setminus \lambda_i \end{array} \right] \geq 0 \mid p_1 \right\} \quad (9)$$

where  $\hat{a}_1(\lambda_i)$  stands for the decision-maker's best initial action conditional on message  $m_1$  inducing  $\lambda_i$ . If  $p_1$  belongs to  $Q_i$  in (9), then the sum of the payoffs expected by the expert with belief  $p_1$  from inducing  $\lambda_i$  will not be lower than the sum of the payoffs expected by the expert from inducing any other  $\lambda_j$ . By construction,  $\cup Q_i = I$ .

Finally, given the same set  $L_{n_1}$ , let  $Q^1(L_{n_1})$  denote the set of the elements  $\{Q_i^1\}$  defined in the following way:

$$Q_i^1 = \left\{ p_1 \mid E_E \left[ \begin{array}{l} \pi_1(\omega_1, \hat{a}_1(\lambda_i)) - \pi_1(\omega_1, \hat{a}_1(\lambda_j)) \geq 0 \mid p_1 \\ \forall \lambda_j \in L_{n_1} \setminus \lambda_i \end{array} \right] \right\} \quad (10)$$

The set  $Q^1(L_{n_1})$  would be relevant if the agents played a single-stage game.

The following Lemma shows that in equilibrium the set  $Q(L_{n_1}, \tilde{\mathbf{R}}_{n_2}^c)$  can be a partition with  $n_1$  elements.

**Lemma 3** : *given  $i < j$ , if  $Q_i$  and  $Q_j$  are non-empty elements of  $Q(L_{n_1}, \tilde{\mathbf{R}}_{n_2}^c)$ , then any  $p_1$  in  $Q_i$  cannot be higher than any  $p_1$  in  $Q_j$ .*

From Lemma 3, if the set  $Q(L_{n_1}, \tilde{\mathbf{R}}_{n_2}^c)$  is a partition with  $n_1$  elements, then every  $\lambda_i$  in  $L_{n_1}$  will be induced with positive probability, and vice versa. Let  $\tilde{\mathbf{S}}_{n_1}(\tilde{\mathbf{R}}_{n_2}^c)$  denote the equilibrium couple  $\{\tilde{L}_{n_1}, Q(\tilde{L}_{n_1}, \tilde{\mathbf{R}}_{n_2}^c)\}$ , where  $Q(\tilde{L}_{n_1}, \tilde{\mathbf{R}}_{n_2}^c)$  is a partition with finite cardinality  $n_1$ . From Proposition 2 there will be a unique equilibrium couple  $\{\tilde{L}_{n_1}^1, \tilde{Q}_{n_1}^1\}$ , denoted by  $\tilde{\mathbf{S}}_{n_1}^1$ . The following Proposition shows that instrumental reputational concerns will never make the equilibrium initial payoffs expected ex-ante by the agents decrease.

**Proposition 3** :

- a)  $E_i \left[ \pi_1(\tilde{\mathbf{S}}_{n_1}(\tilde{\mathbf{R}}_{n_2}^c)) \right] = E_i \left[ \pi_1(\tilde{\mathbf{S}}_{n_1}^1) \right]$  for every  $i = E, D$  if  $n_1 \leq 2$  and/or  $n_2 = 1$ ;
- b)  $E_i \left[ \pi_1(\tilde{\mathbf{S}}_{n_1}(\tilde{\mathbf{R}}_{n_2}^c)) \right] > E_i \left[ \pi_1(\tilde{\mathbf{S}}_{n_1}^1) \right]$  for every  $i = E, D$  if  $n_1 > 2$  and  $n_2 > 1$ .

Proposition 3 is concerned with equilibria characterized by an equilibrium set  $Q(\tilde{L}_{n_1}, \tilde{\mathbf{R}}_{n_2}^c)$  that is partitioned with the same cardinality of the equilibrium set  $\tilde{L}_{n_1}$ . In that case every  $\tilde{Q}_i$  in  $Q(\tilde{L}_{n_1}, \tilde{\mathbf{R}}_{n_2}^c)$  is a proper interval  $[\tilde{q}_{(i-1)}, \tilde{q}_i]$ . Moreover, both the equilibrium sets  $\tilde{L}_{n_1}$  and  $Q(\tilde{L}_{n_1}, \tilde{\mathbf{R}}_{n_2}^c)$  will be symmetric around 0.5. Proposition 3 shows that in equilibrium the posterior distribution functions of  $\eta$  can be ordered according to the criterion of first order statistical dominance. That is, the d.f.  $\tilde{G}(\eta | -1, i)$  will dominate the d.f.  $\tilde{G}^c(\eta | -1, (i+1))$ . Hence, if the true state at period 1 is low and the expert induces  $\tilde{\lambda}_i$ , his reputation at period 2 will be worse than it would have occurred if he had induced  $\tilde{\lambda}_{(i+1)}$ . The opposite will obtain if the true state at period 1 is high. Moreover, if  $\tilde{q}_i < 0.5$ , the d.f.  $\tilde{G}^c(\eta | -1, i)$  will dominate the d.f.  $\tilde{G}^c(\eta | 1, i)$ . In that case, if the expert induces  $\tilde{\lambda}_i$ , his future reputation will improve with respect to the expected current bias if the true state at period 1 is high.

When the difference in the expert's expected current payoff if he induces  $\tilde{\lambda}_i$  instead of  $\tilde{\lambda}_{(i+1)}$  is positive, I will call that difference the expert's expected current advantage from  $\tilde{\lambda}_i$ . Analogously, when the difference in the expert's expected future payoff if he induces  $\tilde{\lambda}_{(i+1)}$  instead of  $\tilde{\lambda}_i$  is positive, I will call that difference the expert's expected future advantage from  $\tilde{\lambda}_{(i+1)}$ . When  $p_1$  belongs to  $\tilde{Q}_i$  and  $\tilde{q}_i < 0.5$ , then the expert's expected current advantage from  $\tilde{\lambda}_i$  will never be lower than the difference in the expert's expected future payoff if he induces  $\tilde{\lambda}_{(i+1)}$  instead of  $\tilde{\lambda}_i$ . When  $p_1 = \tilde{q}_i$ , the expert's expected current advantage from  $\tilde{\lambda}_i$  will be equal to the expert's expected future advantage from  $\tilde{\lambda}_{(i+1)}$ . It means that in equilibrium, when  $p_1$  belongs to  $\tilde{Q}_i$  and  $\tilde{q}_i < 0.5$ , the expert's reputational concern at period 1 will reduce his incentive to announce an exaggaratingly low probability that the current state is low. Instead, when  $p_1$  belongs to  $\tilde{Q}_i$  and  $\tilde{q}_i > 0.5$ , then the expert's expected future advantage from  $\tilde{\lambda}_i$  will never be lower than the difference in the expert's expected current payoff if he induces  $\tilde{\lambda}_{(i+1)}$  instead of  $\tilde{\lambda}_i$ . When  $p_1 = \tilde{q}_i$ , the expert's expected future advantage from  $\tilde{\lambda}_i$  will be equal to the expert's expected current advantage from  $\tilde{\lambda}_{(i+1)}$ . It means that in equilibrium, when  $p_1$  belongs to  $\tilde{Q}_i$  and  $\tilde{q}_i > 0.5$ , the expert's reputational concern at period 1 will partially offset his incentive to misreport an exaggaratingly high probability that the current state is low. So, reputational concerns at period 1 will imply a trade-off between two conflicting purposes:

1) on one side, the current payoff expected by the expert will increase as the gap between the expert's current belief and the decision-maker's posterior belief at period 1 shrinks. That argument would result in noisy current messages, inflated towards the tails of the distribution.

2) On the other side, the more extreme are the initial announcements of the expert, the higher will be his expected future loss driven by a reduced reputation, when he is sufficiently uncertain about the current state.

Proposition 3 shows that the higher is the variance of  $\eta$  at period 1, the stronger will be the impact of reputational concerns at period 1 in equilibrium. So the intervals in  $Q\left(\tilde{L}_{n_1}, \tilde{\mathbf{R}}_{n_2}^c\right)$  will be distributed more evenly than the intervals in  $\tilde{Q}_{n_1}^1$ . From Corollary 1, the initial payoffs expected ex-ante by both the agents will be maximized if all the intervals in the initial partition have the same size. It follows that reputational concerns at period 1 can improve the equilibrium payoff at period 1 expected by both the agents. To sum up, repeated interaction will make the expert concerned with his future reputation. As a result, messages at period 1 will be less noisy than they would be in a single-stage game. All the agents will gain from reputational concerns.

## 6 Conclusions

The paper is concerned with the repeated interaction between a decision-maker and an expert of uncertain reliability. The distinctive assumption of the paper is that the decision-maker makes a subjective assessment of the bias affecting the true opinions of the expert. The distinctive result of the paper is that reputational concerns at the first stage will increase the credibility of the initial messages. Thereby reputational concerns will improve the equilibrium welfare properties at the initial round of play.

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## 8 Appendix

### Proof. of Proposition 1

From (1):

$$\arg \max_A E_D [\pi_2(\omega_2, a_2) \mid \nu(m_2)] = [1 - 2\nu(m_2)] \quad (11)$$

Let  $\widehat{a}_2(\nu(m_2))$  denote the function mapping agent  $D$ 's period-2 posterior probability into his best current action. Hence, if  $\nu(m'_2) < \nu(m''_2)$ , then  $\widehat{a}_2(\nu(m'_2)) > \widehat{a}_2(\nu(m''_2))$ .

Since  $(1 - 2p_2) = \arg \max_A E_E[\pi_2(\omega_2, a_2) | p_2]$ , then, from (11), agent  $E$ 's best choice at the final period, conditional on  $p_2$ , will be the announcement of a message  $m'_2$  that solves  $\min_{m_2} |p_2 - \nu(m_2)|$ .

Suppose that agent  $D$  believes that agent  $E$  will report his true belief at period 2. In that case, agent  $D$ 's period-2 beliefs will be such that:

$$\beta^*(m_2 | p_2) = \begin{cases} 1 & \text{if } p_2 = m_2 \\ 0 & \text{otherwise} \end{cases} \quad (12)$$

Let  $\bar{v}^*(m_2)$  denote agent  $D$ 's period-2 posterior probability that  $\omega_2$  is low, under the system of beliefs in (12).

a) Suppose  $\bar{\eta}_2 \in (0, 1)$ . From (5):

$$1 > \bar{v}^*(m''_2) > \bar{v}^*(m'_2) > 0 \quad \forall m''_2, m'_2 : m''_2 > m'_2 \quad (13)$$

and  $\bar{v}^*(m_2)$  is invertible. From (3):

- )  $\frac{l(p'' | -1, \eta_i)}{l(p'' | -1, \eta_j)} > \frac{l(p' | -1, \eta_i)}{l(p' | -1, \eta_j)}$  if  $\eta_i < \eta_j$  and  $p'' > p'$
- )  $(1 - m_2) \int_{H_2} l(m_2 | -1, \eta) g_2(\eta) d\eta > m_2 \int_{H_2} l(m_2 | 1, \eta) g_2(\eta) d\eta$  if  $m_2 \in [0, 0.5)$ .

The last inequality will be reversed if  $m_2 \in (0.5, 1]$ . So  $\bar{v}^*(m_2)$  will be greater than  $m_2$  if  $m_2 \in [0, 0.5)$ , and it will be lower than  $m_2$  if  $m_2 \in (0.5, 1]$ . Then,  $|p_2 - \bar{v}^*(m_2)|$  will be:

- ) strictly increasing in  $m_2$  if  $p_2 < \bar{v}^*(0)$ ,
- ) zero if  $m_2 = \bar{v}^{*-1}[p_2]$  and  $p_2 \in [\bar{v}^*(0), \bar{v}^*(1)]$ ,
- ) strictly decreasing in  $m_2$  if  $p_2 > \bar{v}^*(1)$ .

Hence, the following period-2 message rule will be optimal for agent  $E$ :

- )  $m_2 = 0$  for every  $p_2 \in [0, \bar{v}^*(0))$ ,
- )  $m_2 = \bar{v}^{*-1}(p_2)$  for every  $p_2 \in [\bar{v}^*(0), \bar{v}^*(1)]$ ,
- )  $m_2 = 1$  for every  $p_2 \in (\bar{v}^*(1), 1]$ .

Accordingly,  $m_2 \neq p_2$  for every  $p_2 \neq \{0, 0.5, 1\}$ . Moreover, if two distribution functions,  $G'_2(\eta)$  and  $G''_2(\eta)$ , are such  $G'_2(\eta)$  dominates  $G''_2(\eta)$  in the sense of first order stochastic dominance, then  $\bar{v}^*(m_2 | g'_2(\eta))$  will be lower than  $\bar{v}^*(m_2 | g''_2(\eta))$  for every  $m_2 < 0.5$ ; the inequality will be reversed for every  $m_2 > 0.5$ .

b) Suppose  $\bar{\eta}_2 = 1$ . From (2) and (4),  $\bar{v}^*(m_2) = 0.5$  for every  $m_2$ . Then,  $|p_2 - \bar{v}^*(m_2)|$  will be invariant with respect to  $m_2$ , and truthtelling can be supported in equilibrium.

c) Suppose  $\bar{\eta}_2 = 0$ . From (2) and (4),  $\bar{v}^*(m_2) = m_2$  for every  $m_2$ . Then,  $\left|p_2 - \bar{v}^*(m_2)\right|$  will be zero only if  $m_2 = p_2$ , and truthtelling can be supported in equilibrium. ■

**Proof. of Proposition 2**

a) Consider a set  $V_{n_2}$  with  $n_2$  finite and greater than 1. From (13):

$$1 > \bar{v}^*(1) \geq v_{n_2} > v_1 \geq \bar{v}^*(0) > 0$$

Since  $(v_i + v_{(i+1)}) < (v_{(i+1)} + v_{(i+2)})$  for every  $i$  from 1 to  $(n_2 - 2)$ , from (6)  $P(V_{n_2})$  will be a partition with  $n_2$  elements such that:

$$\begin{aligned} Y_i &= [y_{(i-1)}, y_i] & i = 1, \dots, n_2 & \quad y_0 = 0 \quad y_{n_2} = 1 & (14) \\ y_{(i-1)} &< y_i = 0.5 [\nu_i + \nu_{(i+1)}] & \forall i \in \{1, \dots, (n_2 - 1)\} \end{aligned}$$

Let  $\mathbf{y}_{V_{n_2}}$  denote the corresponding vector  $(y_1, \dots, y_{(n_2-1)})$ .

b) Properties of period-2 equilibria.

b.1) Suppose that in equilibrium at period 2 the equilibrium set  $\tilde{V}_{n_2}$  has finite cardinality  $n_2$  and is supported by the system of equilibrium message rules  $\{\tilde{\mu}(m_2 | p_2)\}$ . I want to show that the elements of the equilibrium set  $\tilde{V}_{n_2}$  are such that:

$$\begin{aligned} \tilde{v}_i &= \frac{\int_{H_2} \int_{\tilde{Y}_i} l(p_2 | -1, \eta) g_2(\eta) dp_2 d\eta}{\int_{H_2} \int_{\tilde{Y}_i} [l(p_2 | -1, \eta) + l(p_2 | 1, \eta)] g_2(\eta) dp_2 d\eta} & (15) \\ &= \frac{(\tilde{y}_i + \tilde{y}_{(i-1)}) (1 - \bar{\eta}_2) + \bar{\eta}_2}{2} \quad \forall i \in \{1, \dots, n_2\} \end{aligned}$$

Let  $\tilde{M}_i$  denote the subset of messages inducing  $\tilde{v}_i$  under  $\tilde{V}_{n_2}$ . Since  $n_2$  is finite, there must be some  $\tilde{v}_x$  with non-singular  $\tilde{M}_x$ . Let  $\tilde{M}_x^+$  denote the subset of  $\tilde{M}_x$ , collecting all the messages in the support of period-2 equilibrium message rule  $\tilde{\mu}(m_2 | p_2)$  conditional on  $p_2$  in  $\tilde{Y}_x$ . If  $\tilde{M}_x^+$  is singular, then  $\tilde{v}_x$  will satisfy (15). Suppose  $\tilde{M}_x^+$  to be non-singular. In that case, for every  $m_2'$  and  $m_2''$  in  $\tilde{M}_x^+$ :

$$\begin{aligned} \tilde{v}(m_2') - \tilde{v}(m_2'') &= & (16) \\ &= (1 - \bar{\eta}_2) \int_{\tilde{Y}_i} p_2 \left( \frac{\tilde{\beta}(m_2' | p_2)}{\int_{\tilde{Y}_i} \tilde{\beta}(m_2' | p_2) dy} - \frac{\tilde{\beta}(m_2'' | p_2)}{\int_{\tilde{Y}_i} \tilde{\beta}(m_2'' | y) dy} \right) dp_2 \\ &= (1 - \bar{\eta}_2) \int_{\tilde{Y}_i} p_2 \left( \frac{\tilde{\mu}(m_2' | p_2)}{\int_{\tilde{Y}_i} \tilde{\mu}(m_2' | p_2) dy} - \frac{\tilde{\mu}(m_2'' | p_2)}{\int_{\tilde{Y}_i} \tilde{\mu}(m_2'' | y) dy} \right) dp_2 = 0 \\ &\quad \bigcup_{\tilde{M}_x^+} \tilde{\mu}(m_2 | p_2) = 1 \quad \forall p_2 \in (\tilde{y}_{(x-1)}, \tilde{y}_x) & (17) \end{aligned}$$

Now consider the following modified period-2 message rule:

$$\begin{aligned} \hat{\mu}(m_2 | p_2) &= \tilde{\mu}(m_2 | p_2) \quad \forall m_2 \notin \tilde{M}_x^+ \text{ and } p_2 \notin (\tilde{y}_{(x-1)}, \tilde{y}_x) & (18) \\ \hat{\mu}(m_2 | p_2) &= \frac{1}{|\tilde{M}_x^+|} \quad \forall m_2 \in \tilde{M}_x^+ \text{ and } p_2 \in (\tilde{y}_{(x-1)}, \tilde{y}_x) \end{aligned}$$

From (17), under (18) :

$$\frac{\widehat{\mu}(m'_2 | p_2)}{\int_{\widetilde{Y}_i} \widehat{\mu}(m'_2 | p_2) dy} = \frac{\bigcup_{\widetilde{M}_x^+} \widetilde{\mu}(m_2 | p_2)}{\int_{\widetilde{Y}_i} \left( \bigcup_{\widetilde{M}_x^+} \widetilde{\mu}(m_2 | y) \right) dy} \quad \forall m'_2 \in \widetilde{M}_x^+ \text{ and } p_2 \in (\widetilde{y}_{(x-1)}, \widetilde{y}_x)$$

Let  $\widehat{v}(m_2)$  denote agent  $D$ 's period-2 posterior probability that  $\omega_2$  is low conditional on  $m_2 \in \widetilde{M}_x^+$  and (18). Given  $m'_2$  in  $\widetilde{M}_x^+$ , from (16):

$$\begin{aligned} & [\widehat{v}(m'_2) - \widetilde{v}(m'_2)] \frac{\int_{\widetilde{Y}_i} dy}{(1 - \overline{\eta}_2)} \\ &= \bigcup_{m_2 \in (\widetilde{M}_x^+ \setminus m'_2)} \left\{ \int_{\widetilde{Y}_i} \widetilde{\mu}(m_2 | y) dy \left[ \int_{\widetilde{Y}_i} p_2 \left( \frac{\widetilde{\mu}(m_2 | p_2)}{\int_{\widetilde{Y}_i} \widetilde{\mu}(m_2 | y) dy} - \frac{\widetilde{\mu}(m'_2 | p_2)}{\int_{\widetilde{Y}_i} \widetilde{\mu}(m'_2 | y) dy} \right) dp_2 \right] \right\} \\ &= 0 \end{aligned}$$

Consequently, if an equilibrium set  $\widetilde{V}_{n_2}$  exists, it can be supported by the message rule in (18) that implies (15).

b.2) Suppose  $\overline{\eta}_2 \in [0, 1)$  and  $n_2 > 1$ . From (15), if  $\widetilde{V}_{n_2}$  is an equilibrium set, then the equilibrium vector  $\widetilde{\mathbf{y}}_{\widetilde{V}_{n_2}}$  will satisfy the following conditions:

-)  $\widetilde{\mathbf{y}}_{\widetilde{V}_{n_2}}$  is a solution to the system  $\mathbf{f}(\mathbf{y}_{n_2}, \overline{\eta}_2)$  of  $(n_2 - 1)$  equations where the representative equation  $f_i(\mathbf{y}_{n_2}, \overline{\eta}_2)$  is the following:

$$f_i(\mathbf{y}_{n_2}, \overline{\eta}_2) = \left( y_i - \frac{y_{(i-1)} + y_{(i+1)}}{2} \frac{(1 - \overline{\eta}_2)}{(1 + \overline{\eta}_2)} - \frac{\overline{\eta}_2}{(1 + \overline{\eta}_2)} \right) = 0 \quad (19)$$

with  $i \in \{1, \dots, (n_2 - 1)\}$ ,  $y_0 = 0$ ,  $y_{n_2} = 1$ .

-)  $\widetilde{\mathbf{y}}_{\widetilde{V}_{n_2}}$  is such that:

$$0 < \widetilde{y}_1 < \widetilde{y}_2 < \dots < \widetilde{y}_{(n_2-2)} < \widetilde{y}_{(n_2-1)} < 1 \quad (20)$$

From (19),  $\widetilde{y}_i = (1 - \widetilde{y}_{(n_2-i)})$  and, from (16),  $\widetilde{v}_i = (1 - \widetilde{v}_{(n_2+1-i)})$ . Hence, from (20), both  $P(\widetilde{V}_{n_2})$  and  $\widetilde{V}_{n_2}$  will be symmetric around 0.5.

c) Existence of period-2 equilibria.

c.1) Suppose  $\overline{\eta}_2 \in [0, 1)$ . I want to show that there exists a unique equilibrium set  $\widetilde{V}_{n_2}$  of cardinality  $n_2$  for every finite  $n_2$ . If  $n_2 = 1$ , there will be a unique equilibrium set  $\widetilde{V}_1$ , with  $\widetilde{V}_1 = 0.5$ , and a unique equilibrium partition  $P(\widetilde{V}_1) = I$  (the babbling equilibrium). If  $n_2 > 1$ , since  $\frac{1 - \overline{\eta}_2}{2(1 + \overline{\eta}_2)}$  is positive, the system  $\mathbf{f}(\mathbf{y}_{n_2}, \overline{\eta}_2)$  of  $(n_2 - 1)$  equations from (19) can be written in matrix notation in the following way:

$$\mathbf{Z}\mathbf{y}'_{n_2} = \mathbf{e}' \quad (21)$$

where:

$$\mathbf{Z} = \begin{bmatrix} 1 & -\frac{1 - \overline{\eta}_2}{2(1 + \overline{\eta}_2)} & 0 & 0 & 0 & 0 \\ -\frac{1 - \overline{\eta}_2}{2(1 + \overline{\eta}_2)} & 1 & -\frac{1 - \overline{\eta}_2}{2(1 + \overline{\eta}_2)} & 0 & 0 & 0 \\ 0 & \dots & \dots & \dots & \dots & 0 \\ 0 & 0 & 0 & 0 & -\frac{1 - \overline{\eta}_2}{2(1 + \overline{\eta}_2)} & 1 \end{bmatrix} \quad (22)$$

$$\mathbf{e}' = \left( \frac{\bar{\eta}_2}{1 + \bar{\eta}_2}, \frac{\bar{\eta}_2}{1 + \bar{\eta}_2}, \dots, \frac{1}{2} \right)$$

The matrix  $\mathbf{Z}$  is invertible and every entry  $z_{ij}^{-1}$  of  $\mathbf{Z}^{-1}$  is strictly positive. Hence, there will always exist a unique vector  $\hat{\mathbf{y}}_{n_2}$  such that  $\hat{\mathbf{y}}'_{n_2} = \mathbf{Z}^{-1}\mathbf{e}'$ . Moreover, the vector  $\hat{\mathbf{y}}_{n_2}$  will satisfy (20). It follows that the solution  $\hat{\mathbf{y}}_{n_2}$  is the unique equilibrium vector  $\tilde{\mathbf{y}}_{\tilde{V}_{n_2}}$ , thereby simply denoted by  $\tilde{\mathbf{y}}_{n_2}$ .

Finally,  $(\tilde{y}_i - \tilde{y}_{(i-1)})$  will be decreasing in  $i$  for  $i \leq \frac{n_2}{2}$  if  $\bar{\eta}_2 \in (0, 1)$ . If  $\bar{\eta}_2 = 0$ , then  $\tilde{y}_i = \frac{i}{n_2}$ . Hence,  $\tilde{P}_{n_2}$  will be a function of  $\bar{\eta}_2$  if  $n_2 > 2$ , while  $\tilde{V}_{n_2}$  will be a function of  $\bar{\eta}_2$  if  $n_2 > 1$ .

c.2) Suppose  $\bar{\eta}_2 = 1$ . The only equilibrium set is  $\tilde{V}_1$ , with  $\tilde{V}_1 = 0.5$ , and the unique equilibrium partition is  $P_1 = I$ . ■

### Proof. of Corollary 1

$E_E [\pi_2 (V_{n_2}, P (V_{n_2}))]$  and  $E_D [\pi_2 (V_{n_2}, P (V_{n_2}))]$  will be equal to:

-) when  $n_2$  is an even integer:

$$E_E [\pi_2 (V_{n_2}, P (V_{n_2}))] = o_E + u_E \sum_{j=1}^{\frac{n_2}{2}-1} y_j y_{(j+1)} (y_{(j+1)} - y_j)$$

$$E_D [\pi_2 (V_{n_2}, P (V_{n_2}))] = o_D + u_D \sum_{j=1}^{\frac{n_2}{2}-1} y_j y_{(j+1)} (y_{(j+1)} - y_j)$$

with  $o_E = -\frac{3+\bar{\eta}_2^2}{4}$ ,  $u_E = 2(1 - \bar{\eta}_2^2)$ ,  $o_D = -\frac{(3-\bar{\eta}_2)(1+\bar{\eta}_2)}{4}$ , and  $u_D = 2(1 - \bar{\eta}_2)^2$

-) when  $n_2$  is an odd integer:

$$E_E [\pi_2 (V_{n_2}, P (V_{n_2}))] = -1 + u_E \left( \sum_{j=1}^{\frac{n_2-1}{2}} y_j y_{(j+1)} (y_{(j+1)} - y_j) + y_{\frac{n_2-1}{2}} \left(1 - y_{\frac{n_2-1}{2}}\right)^2 \right)$$

$$E_D [\pi_2 (V_{n_2}, P (V_{n_2}))] = -1 + u_D \left( \sum_{j=1}^{\frac{n_2-1}{2}} y_j y_{(j+1)} (y_{(j+1)} - y_j) + y_{\frac{n_2-1}{2}} \left(1 - y_{\frac{n_2-1}{2}}\right)^2 \right)$$

■

### Proof. of Corollary 2

From Proposition 2,  $\tilde{P}_{n_2}$  will be independent of  $\bar{\eta}_2$  if  $n_2 \leq 2$ , while  $\tilde{V}_{n_2}$  will be independent of  $\bar{\eta}_2$  if  $n_2 = 1$ .

Consider  $\tilde{R}_{n_2}$  with  $n_2 > 2$ . From (19) and (21) – (22):

$$- ) D_{\mathbf{y}} \mathbf{f} (\tilde{\mathbf{y}}_{n_2}, \bar{\eta}_2) = \mathbf{Z}$$

$$- ) D_{\bar{\eta}_2} \mathbf{f} (\tilde{\mathbf{y}}_{n_2}, \bar{\eta}_2) = -\rho^{-2} (1 - \tilde{y}_2, 1 - \tilde{y}_1 - \tilde{y}_3, \dots, 1 - \tilde{y}_{(n_2-3)} - \tilde{y}_{(n_2-1)}, -\tilde{y}_{(n_2-2)})'$$

where  $\rho = (1 + \bar{\eta}_2)$ .

From the implicit function theorem,  $D\mathbf{f} (\tilde{\mathbf{y}}_{n_2}, \bar{\eta}_2) = -\mathbf{Z}^{-1} D_{\bar{\eta}_2} \mathbf{f} (\tilde{\mathbf{y}}_{n_2}, \bar{\eta}_2)$ .

Since  $\tilde{\mathbf{y}}_{n_2}$  is symmetric around 0.5, then:

$$- ) \text{ for every } \tilde{y}_i < 0.5, \frac{d\tilde{y}_{(n_2-i)}}{d\bar{\eta}_2} = -\frac{d\tilde{y}_i}{d\bar{\eta}_2} < 0$$

$$- ) \text{ if } \tilde{y}_i = 0.5, \frac{d\tilde{y}_i}{d\bar{\eta}_2} = 0.$$

Moreover,  $\frac{d\tilde{y}_{(i-1)}}{d\bar{\eta}_2} > \frac{d\tilde{y}_i}{d\bar{\eta}_2}$  for  $i \leq \frac{n_2}{2}$  so that  $(\tilde{y}_i - \tilde{y}_{(i-1)})$  will be decreasing in  $\bar{\eta}_2$  for every  $i$  not smaller than 2 and not greater than  $\frac{n_2}{2}$ .

Let  $\mathbf{F} \left( \frac{d\tilde{\mathbf{y}}_{n_2}}{d\bar{\eta}_2}, \bar{\eta}_2 \right)$  denote the following system of  $(n_2 - 1)$  equations where

the representative equation  $F_i \left( \frac{d\tilde{y}_{n_2}}{d\bar{\eta}_2}, \bar{\eta}_2 \right)$  is the following:

$$F_i \left( \frac{d\tilde{y}_{n_2}}{d\bar{\eta}_2}, \bar{\eta}_2 \right) = -\frac{1-\bar{\eta}_2}{2\rho} \frac{dy_{(i-1)}}{d\bar{\eta}_2} + \frac{dy_i}{d\bar{\eta}_2} - \frac{1-\bar{\eta}_2}{2\rho} \frac{dy_{(i+1)}}{d\bar{\eta}_2} - \frac{1-y_{(i-1)}-y_{(i+1)}}{\rho^2} = 0$$

with  $i \in \{1, \dots, (n_2 - 1)\}$ ,  $y_0 = 0$ ,  $y_{n_2} = 1$ . It follows that:

$$\begin{aligned} & -) D_{\frac{d\tilde{y}}{d\bar{\eta}_2}} \mathbf{F} \left( \frac{d\tilde{y}_{n_2}}{d\bar{\eta}_2}, \bar{\eta}_2 \right) = \mathbf{Z} \\ & -) D_{\bar{\eta}_2} \mathbf{F} \left( \frac{d\tilde{y}_{n_2}}{d\bar{\eta}_2}, \bar{\eta}_2 \right) = \frac{2}{\rho^3} \left( 1 - \tilde{y}_2 + \rho \frac{d\tilde{y}_2}{d\bar{\eta}_2}, \phi, -\tilde{y}_{(n_2-2)} + \rho \frac{d\tilde{y}_{(n_2-2)}}{d\bar{\eta}_2} \right)', \text{ where} \\ & \phi = \left( 1 - \tilde{y}_1 - \tilde{y}_3 + \rho \left( \frac{d\tilde{y}_1}{d\bar{\eta}_2} + \frac{d\tilde{y}_3}{d\bar{\eta}_2} \right), \dots, 1 - \tilde{y}_{(n_2-3)} - \tilde{y}_{(n_2-1)} + \rho \left( \frac{d\tilde{y}_{(n_2-3)}}{d\bar{\eta}_2} + \frac{d\tilde{y}_{(n_2-1)}}{d\bar{\eta}_2} \right) \right) \\ & -) D\mathbf{F} \left( \frac{d\tilde{y}_{n_2}}{d\bar{\eta}_2}, \bar{\eta}_2 \right) = -\mathbf{Z}^{-1} D_{\bar{\eta}_2} \mathbf{F} \left( \frac{d\tilde{y}_{n_2}}{d\bar{\eta}_2}, \bar{\eta}_2 \right) \end{aligned}$$

Since  $\tilde{y}$  is symmetric around 0.5, then  $\frac{d^2\tilde{y}_{(n_2-i)}}{d\bar{\eta}_2^2} = -\frac{d^2\tilde{y}_i}{d\bar{\eta}_2^2} > 0$  for every  $\tilde{y}_i < 0.5$ .

At the end of period 1, the equilibrium payoff at period 2 expected by agent  $E$ , denoted by  $E_E \left[ \pi_2 \left( \tilde{R}_{n_2} \right) \mid \bar{\eta}_2 \right]$ , will be equal to:

$$\begin{aligned} & 1) -4 \sum_{j=1}^{\frac{n_2}{2}} \left[ \left( \tilde{y}_j^2 - \tilde{y}_{(j-1)}^2 \right) (1 - \tilde{v}_j)^2 + (\tilde{y}_j - \tilde{y}_{(j-1)}) (2 - \tilde{y}_j - \tilde{y}_{(j-1)}) \tilde{v}_j^2 \right] \text{ when} \\ & n_2 \text{ is an even integer,} \\ & 2) -4 \sum_{j=1}^{\frac{n_2-1}{2}-1} \left[ \left( \tilde{y}_j^2 - \tilde{y}_{(j-1)}^2 \right) (1 - \tilde{v}_j)^2 + (\tilde{y}_j - \tilde{y}_{(j-1)}) (2 - \tilde{y}_j - \tilde{y}_{(j-1)}) \tilde{v}_j^2 \right] - \\ & \left( 1 - 2\tilde{y}_{\left(\frac{n_2-1}{2}-1\right)} \right) \text{ when } n_2 \text{ is an odd integer.} \end{aligned}$$

Consequently, if  $n_2 > 1$ , the function  $E_E \left[ \pi_2 \left( \tilde{R}_{n_2} \right) \mid \bar{\eta}_2 \right]$  will be strictly decreasing and concave in  $\bar{\eta}_2$ , with  $\frac{\delta E_E \left[ \pi_2 \left( \tilde{R}_{n_2} \right) \mid \bar{\eta}_2 \right]}{\delta \bar{\eta}_2}$  equal to 0 when  $\bar{\eta}_2 \rightarrow 0$ . ■

### Proof. of Lemma 1

From (1):

$$\arg \max_A E_D \left[ \pi_1(\omega_1, a_1) + \tilde{\pi}_2 \left( \tilde{R}_{n_2}^c(m_1) \mid \lambda(m_1) \right) \right] = [1 - 2\lambda(m_1)]$$

Let  $\hat{a}_1(\lambda(m_1))$  denote the function mapping agent  $D$ 's period-1 posterior probability into his best current action. If  $\lambda(m'_1) < \lambda(m''_1)$ , then  $\hat{a}_1(\lambda(m'_1)) > \hat{a}_1(\lambda(m''_1))$ .

Suppose that agent  $D$  believes that agent  $E$  will announce his true belief at period 1. In that case, agent  $D$ 's period-1 beliefs will be such that:

$$\beta^*(m_1 \mid p_1) = \begin{cases} 1 & \text{if } p_1 = m_1 \\ 0 & \text{otherwise} \end{cases} \quad (23)$$

Let  $\lambda^*(m_1)$  denote agent  $D$ 's posterior probability at period 1 that  $\omega_1$  is low under the system of period-1 beliefs in (23). From (8),  $\lambda^*(m_1) \in (0, 1)$ . Since  $l(p \mid \eta, \omega)$  is always positive for every positive  $\eta$  and for every  $\omega$ , then  $g_2(\eta \mid \omega_1, m_1)$  will never be degenerate under (23) with  $\bar{\eta}_2(\omega_1, m_1) \in (0, 1)$ . From (7) and (23):

$$\begin{aligned} -) \bar{\eta}_2(-1, m_1) &= \bar{\eta}_1 + \sigma^2 \frac{1-2m_1}{2m_1(1-\bar{\eta}_1)+\bar{\eta}_1}, \\ -) \bar{\eta}_2(1, m_1) &= \bar{\eta}_1 - \sigma^2 \frac{1-2m_1}{2(1-m_1)(1-\bar{\eta}_1)+\bar{\eta}_1} \end{aligned}$$

Suppose  $n_2 = 1$  (i.e. babbling equilibrium at period 2). From Corollary 2, the equilibrium payoff at period 2 expected by agent  $E$  will be equal to  $-1$ . Hence:

$$\begin{aligned} & E_E \left[ \pi_1 \left( \omega_1, \hat{a}_1 \left( \lambda^*(m_1) \right) \right) + \pi_2 \left( \tilde{R}_1 \right) \mid p_1 \right] \\ &= -4p_1 \left[ 1 - \lambda^*(m_1) \right]^2 - 4(1-p_1) \left[ \lambda^*(m_1) \right]^2 - 1 \end{aligned}$$

In that case the result of Proposition 1 can be shown to hold with respect to agent  $E$ 's period-1 equilibrium message rule.

Suppose  $n_2 > 1$ . Let  $\tilde{R}_{n_2}^*(m_1)$  characterize a period-2 equilibrium profile where the posterior generalized probability density functions of  $\eta$  are consistent with the system of beliefs in (23). In that case:

$$\begin{aligned} & E_E \left[ \pi_1 \left( \omega_1, \hat{a}_1 \left( \lambda^*(m_1) \right) \right) + \pi_2 \left( \tilde{R}_{n_2}^*(m_1) \right) \mid p_1 \right] = \\ & -4p_1 \left[ 1 - \lambda^*(m_1) \right]^2 - 4(1-p_1) \left[ \lambda^*(m_1) \right]^2 + \\ & + p_1 E_E \left[ \pi_2 \left( \tilde{R}_{n_2}^*(\bar{\eta}_2^*(-1, m_1)) \right) \right] + (1-p_1) E_E \left[ \pi_2 \left( \tilde{R}_{n_2}^*(\bar{\eta}_2^*(1, m_1)) \right) \right] \end{aligned}$$

Hence, at period 1 the sum of the payoffs expected by agent  $E$  will be:

- a) strictly decreasing in  $m_1$  for every  $p_1$  in some proper interval  $[0, p'_1]$
- b) strictly increasing in  $m_1$  for every  $p_1$  in some proper interval  $[p''_1, 1]$ .

Consequently, agent  $D$ 's period-1 beliefs in (23) will be inconsistent with agent  $E$ 's best period-1 message rule. ■

### Proof. of Lemma 2

From (8):

$$\begin{aligned} -) \lambda(m'_1) &= \frac{2(1-\bar{\eta}_1) \int_I p\beta(m'_1|p)dp + \bar{\eta}_1 \int_I \beta(m'_1|p)dp}{2 \int_I \beta(m'_1|p)dp} \\ -) \lambda(m'_1) \geq \lambda(m''_1) &\text{ if } \frac{\int_I p\beta(m'_1|p)dp}{\int_I \beta(m'_1|p)dp} \geq \frac{\int_I p\beta(m''_1|p)dp}{\int_I \beta(m''_1|p)dp} \end{aligned}$$

Since  $|H_1 \cap 0, 1| \geq 2$ , then  $\sigma^2 > 0$ . From (7):

$$\begin{aligned} -) \bar{\eta}_2^c(-1, m'_1) &= \bar{\eta}_2^c(-1, m''_1) \leftrightarrow \bar{\eta}_2^c(1, m'_1) = \bar{\eta}_2^c(1, m''_1) \\ -) \bar{\eta}_2^c(-1, m'_1) &< \bar{\eta}_2^c(-1, m''_1) \leftrightarrow \bar{\eta}_2^c(1, m'_1) > \bar{\eta}_2^c(1, m''_1) \\ -) \bar{\eta}_2^c(-1, m'_1) &\leq \bar{\eta}_2^c(-1, m''_1) \text{ if } \lambda(m'_1) \geq \lambda(m''_1) \quad \blacksquare \end{aligned}$$

### Proof. of Lemma 3

a) Suppose  $n_2 = 1$ . From Corollary 2, given a set  $L_{n_1}$ , every subset  $Q_i$  in (9) will be equal to  $Q_i^1$  in (10). In that case, an analogous version of (14) can be shown to hold.

b) Suppose  $n_2 > 1$ . Given any  $L_1$ , the corresponding set  $Q(L_1, \tilde{\mathbf{R}}_{n_2}^c)$  is  $I$ . Consider a set  $L_{n_1}$  with  $n_1 \geq 2$ . From Corollary 2 and Lemma 2, if  $\lambda_i < \lambda_j$ , then:

$$-) E_E \left[ \pi_2 \left( \tilde{R}_{n_2}(\bar{\eta}_2^c(1, i)) \right) - \pi_2 \left( \tilde{R}_{n_2}(\bar{\eta}_2^c(1, j)) \right) \right] > 0;$$

$$-) E_E \left[ \pi_2 \left( \tilde{R}_{n_2}(\bar{\eta}_2^c(-1, i)) \right) - \pi_2 \left( \tilde{R}_{n_2}(\bar{\eta}_2^c(-1, j)) \right) \right] < 0.$$

$$\text{Let } \delta(\omega_1, i, j) \text{ denote the difference } E_E \left[ \pi_2 \left( \tilde{R}_{n_2}(\bar{\eta}_2^c(\omega_1, i)) \right) - \pi_2 \left( \tilde{R}_{n_2}(\bar{\eta}_2^c(\omega_1, j)) \right) \right].$$

Let  $H(i, j)$  denote  $\left[ \frac{\delta(1, i, j) + 4(\lambda_j^2 - \lambda_i^2)}{\delta(1, i, j) - \delta(-1, i, j) + 8(\lambda_j - \lambda_i)} \right]$ . Then,  $H(i, j) \in (0, 1)$  for every  $i < j$ . Let  $\underline{H}(i)$  denote the lowest  $H(i, j)$  for every  $j > i$ , and let  $\overline{H}(i)$  denote the highest  $H(z, i)$  for every  $i > z$ . It follows that:

$$-) 0 < \underline{H}(1) \leq \overline{H}(n_1) < 1$$

$$-) Q_1 = [0, \underline{H}(1)] \text{ and } Q_{n_1} = [\overline{H}(n_1), 1]$$

-) given  $i$  in  $\{2, \dots, (n_1 - 1)\}$ , then  $Q_i = \emptyset$  if  $\overline{H}(i) > \underline{H}(i)$ ;  $Q_i = [\overline{H}(i), \underline{H}(i)]$  if  $\overline{H}(i) \leq \underline{H}(i)$ , and  $\underline{H}(i) \leq \overline{H}(j)$  if  $Q_i, Q_j \neq \emptyset$  and  $i < j$ .

Consequently, if  $n_1 \geq 2$ , for any set  $L_{n_1}$ , the corresponding set  $Q(L_{n_1}, \tilde{\mathbf{R}}_{n_2}^c)$  will have 2 non-empty elements at least. ■

### Proof. of Proposition 3

a) Suppose  $n_2 = 1$ . From Lemma 3,  $\tilde{\mathbf{S}}_{n_1}(\tilde{\mathbf{R}}_1^c) = \tilde{\mathbf{S}}_{n_1}^1$ .

b) Suppose  $n_2 > 1$ . There always exists a unique  $\tilde{L}_1$ , with  $\tilde{L}_1 = 0.5$  (the period-1 babbling equilibrium).

Suppose that  $n_1 > 1$  and in equilibrium each  $\tilde{\lambda}_i$  in  $\tilde{L}_{n_1}$  is induced with positive probability. In that case, from Lemma 3, the corresponding set  $Q(\tilde{L}_{n_1}, \tilde{\mathbf{R}}_{n_2}^c)$  will be a partition with cardinality  $n_1$  such that:

$$-) \tilde{Q}_1 = [0, \underline{H}(1)] = [0, H(1, 2)] = [0, \tilde{q}_1]$$

$$-) \tilde{Q}_i = [\overline{H}(i), \underline{H}(i)] = [H(i-1, i), H(i, i+1)] = [\tilde{q}_{i-1}, \tilde{q}_i] \text{ for every } i \in \{2, \dots, (n_1 - 2)\}$$

$$-) \tilde{Q}_{n_1} = [\overline{H}(n_1), 1] = [H(n_1 - 1, n_1), 1] = [\tilde{q}_{n_1-1}, 1].$$

Analogously to point b) of the proof of Proposition 2, it can be shown that:

$$\tilde{\lambda}_i = \frac{(\tilde{q}_i + \tilde{q}_{i-1})(1 - \bar{\eta}_1) + \bar{\eta}_1}{2} \quad \forall \tilde{Q}_i, i \in \{1, \dots, n_1\} \quad (24)$$

Moreover, in equilibrium:

$$\tilde{g}_2^c(\eta | \omega_1, i) = \frac{g_1(\eta) \int_{\tilde{Q}_i} l(p_1 | \omega_1, \eta) dp_1}{\int_{H_1} \int_{\tilde{Q}_i} l(p_1 | \omega_1, \eta) g_1(\eta) dp_1 d\eta} \quad (25)$$

Since  $l(p | \eta, \omega)$  is always positive for every positive  $\eta$  and for every  $\omega$ , and since  $|H_1 \cap (0, 1]| \geq 2$ , then  $\tilde{g}_2^c(\eta | \omega_1, i)$  in (25) will never be degenerate with  $\bar{\eta}_2^c(\omega_1, i) \in (0, 1)$ .

Consider  $i < j$ . From (25):

$$\begin{aligned} -) \frac{\tilde{g}_2^c(\eta|\omega_1, j)}{\tilde{g}_2^c(\eta|\omega_1, i)} &= \frac{\int_{\tilde{Q}_j} l(p_1 | \omega_1, \eta) dp_1}{\int_{\tilde{Q}_i} l(p_1 | \omega_1, \eta) dp_1} \frac{\int_{H_1} \int_{\tilde{Q}_i} l(p_1|\omega_1, \eta) g_1(\eta) dp_1 d\eta}{\int_{H_1} \int_{\tilde{Q}_j} l(p_1|\omega_1, \eta) g_1(\eta) dp_1 d\eta} \text{ for every } \eta \in H_1 \\ -) \frac{\tilde{g}_2^c(\eta_z|-1, j)}{\tilde{g}_2^c(\eta_k|-1, j)} &> \frac{\tilde{g}_2^c(\eta_z|-1, i)}{\tilde{g}_2^c(\eta_k|-1, i)} \text{ for every } \eta_z, \eta_k \in H_1, \text{ for every } \eta_z < \eta_k. \end{aligned}$$

The last inequality will be reversed if  $\omega_1 = 1$ . It follows that  $\bar{\eta}_2^c(-1, j) < \bar{\eta}_2^c(-1, i)$  and  $\bar{\eta}_2^c(1, j) > \bar{\eta}_2^c(1, i)$ . In particular:

$$\begin{aligned} -) \bar{\eta}_2^c(-1, i) &= \bar{\eta}_1 + \sigma^2 \frac{1 - \tilde{q}_{(i-1)} - \tilde{q}_i}{(\tilde{q}_{(i-1)} + \tilde{q}_i)(1 - \bar{\eta}_1) + \bar{\eta}_1}, \\ -) \bar{\eta}_2^c(1, i) &= \bar{\eta}_1 - \sigma^2 \frac{1 - \tilde{q}_{(i-1)} - \tilde{q}_i}{(2 - \tilde{q}_{(i-1)} - \tilde{q}_i)(1 - \bar{\eta}_1) + \bar{\eta}_1}. \end{aligned}$$

So, when  $i < j$ , then  $\bar{\eta}_2^c(1, i) < \bar{\eta}_2^c(1, j) \leq \bar{\eta}_1 \leq \bar{\eta}_2^c(-1, j) < \bar{\eta}_2^c(-1, i)$  if  $\tilde{q}_j \leq 0.5$ , while  $\bar{\eta}_2^c(-1, j) < \bar{\eta}_2^c(-1, i) \leq \bar{\eta}_1 \leq \bar{\eta}_2^c(1, i) < \bar{\eta}_2^c(1, j)$  if  $\tilde{q}_{(i-1)} \geq 0.5$ . Finally:

$$\begin{aligned} \bar{\eta}_2^c(1, i) &= \bar{\eta}_2^c(-1, j) \text{ and } \bar{\eta}_2^c(-1, i) = \bar{\eta}_2^c(1, j) \\ \text{if } \tilde{q}_{(i-1)} &= 1 - \tilde{q}_j \text{ and } \tilde{q}_i = 1 - \tilde{q}_{(j-1)} \end{aligned} \quad (26)$$

b.1) If  $n_1 > 1$  and  $\tilde{L}_{n_1}$  is the support of agent  $E$ 's period-1 equilibrium message rules, the corresponding equilibrium vector  $\tilde{\mathbf{q}}_{\tilde{L}_{n_1}} = (\tilde{q}_1, \dots, \tilde{q}_{n_1-1})$  will satisfy the following conditions:

-)  $\tilde{\mathbf{q}}_{\tilde{L}_{n_1}}$  is a solution to the system  $\mathbf{h}(\mathbf{q}_{n_1}, \mathbf{z})$  of  $(n_1 - 1)$  equations where  $\mathbf{z} = (\bar{\eta}_1, \sigma^2)$ , and the representative equation  $h_i(\mathbf{q}_{n_1}, \mathbf{z})$  is the following:

$$\begin{aligned} h_i(\mathbf{q}_{n_1}, \mathbf{z}) &= \left[ 2(1 - \bar{\eta}_1^2) \frac{(q_{(i+1)} - q_{(i-1)}) - \delta(-1, i, (i+1))}{+\delta(1, i, (i+1))} \right] q_i + \\ &- (1 - \bar{\eta}_1)^2 (q_{(i+1)} - q_{(i-1)}) q_{(i-1)} - (1 - \bar{\eta}_1)^2 (q_{(i+1)} - q_{(i-1)}) q_{(i+1)} + \\ &- 2(1 - \bar{\eta}_1) (q_{(i+1)} - q_{(i-1)}) \bar{\eta}_1 - \delta(1, i, (i+1)) = 0 \end{aligned} \quad (27)$$

with  $i \in \{1, \dots, (n_1 - 1)\}$ ,  $q_0 = 0$ ,  $q_{n_1} = 1$ .

-)  $\tilde{\mathbf{q}}_{\tilde{L}_{n_1}}$  is such that:

$$0 < \tilde{q}_1 < \tilde{q}_2 < \dots < \tilde{q}_{(n_1-2)} < \tilde{q}_{(n_1-1)} < 1 \quad (28)$$

From (27) and (26),  $\tilde{q}_i = (1 - \tilde{q}_{(n_1-1)})$ , and, from (24),  $\tilde{\lambda}_i = (1 - \tilde{\lambda}_{(n_1+1-i)})$ .

Hence, from (28), both  $Q(\tilde{L}_{n_1}, \tilde{\mathbf{R}}_{n_2}^c)$  and  $\tilde{L}_{n_1}$  will be symmetric around 0.5.

b.2) Every  $h_i(\mathbf{q}_{n_1}, \mathbf{z})$  in (27) can be written in the following way:

$$\begin{aligned} & q_i \delta(-1, i, (i+1)) + (1 - q_i) \delta(1, i, (i+1)) \\ &= -2(1 - \bar{\eta}_1) (q_{(i+1)} - q_{(i-1)}) \left( \frac{q_{(i+1)} + q_{(i-1)}}{2} (1 - \bar{\eta}_1) + \bar{\eta}_1 - q_i (1 + \bar{\eta}_1) \right) \end{aligned} \quad (29)$$

Suppose  $n_1 = 2$ . The RHS in (29) will be equal to zero if  $q_1 = 0.5$ . From (26), if  $q_1 = 0.5$ , the LHS in (29) will be equal to zero. Hence,  $q_1 = 0.5$  is the unique solution to the system in (27).

Suppose  $n_1 > 2$ . Consider  $q_i < 0.5$ . Then,  $\bar{\eta}_2^c(-1, i) > \bar{\eta}_2^c(-1, (i+1)) \geq \bar{\eta}_2^c(1, (i+1)) > \bar{\eta}_2^c(1, i)$ . So,  $\bar{\eta}_2^c(\omega_1, (i+1))$  can be written as it follows:

$$\begin{aligned}\bar{\eta}_2^c(-1, (i+1)) &= x\bar{\eta}_2^c(-1, i) + (1-x)\bar{\eta}_2^c(1, i) \\ \bar{\eta}_2^c(1, (i+1)) &= z\bar{\eta}_2^c(-1, i) + (1-z)\bar{\eta}_2^c(1, i)\end{aligned}\quad (30)$$

From Corollary 2:

-)  $E_E \left[ \pi_2 \left( \tilde{R}_{n_2}(\bar{\eta}_2^c(1, i)) \right) - \pi_2 \left( \tilde{R}_{n_2}(\bar{\eta}_2^c(-1, i)) \right) \right] > 0$ . Let that difference be denoted by  $w$ .

-) if  $q_i$  is equal to  $\left(\frac{\lambda_i + \lambda_{(i+1)}}{2}\right)$ , the LHS in (29) will be strictly lower than  $w[-q_i(1-x) + (1-q_i)z]$ , that is negative from (30). Instead, the RHS in (29) will be equal to zero. It follows that in equilibrium  $\tilde{q}_i$  will be lower than  $\left(\frac{\lambda_i + \lambda_{(i+1)}}{2}\right)$  for every  $\tilde{q}_i$  lower than 0.5.

b.3) Let  $\mathbf{z} = (\bar{\eta}_1, 0)$ , and let  $\mathbf{\ddot{q}}_{n_1}$  be such that  $h_i(\mathbf{\ddot{q}}_{n_1}, \mathbf{z}) = 0$  for every  $i \in \{1, \dots, (n_1 - 1)\}$ . From the proof of Proposition 2,  $\mathbf{\ddot{q}}_{n_1}$  exists and is unique. Moreover, from (27), the following matrix  $D_{\mathbf{q}_{n_1}} \mathbf{h}(\mathbf{\ddot{q}}_{n_1}, \mathbf{z})$  is invertible:

$$D_{\mathbf{q}_{n_1}} \mathbf{h}(\mathbf{\ddot{q}}_{n_1}, \mathbf{z}) = \begin{bmatrix} \gamma \ddot{q}_2 & \epsilon \ddot{q}_2 & 0 & 0 & 0 & 0 \\ \epsilon(\ddot{q}_3 - \ddot{q}_1) & \gamma(\ddot{q}_3 - \ddot{q}_1) & \epsilon(\ddot{q}_3 - \ddot{q}_1) & 0 & 0 & 0 \\ 0 & \dots & \dots & \dots & \dots & 0 \\ 0 & 0 & 0 & 0 & \epsilon(1 - \ddot{q}_{(n_1-2)}) & \gamma(1 - \ddot{q}_{(n_1-2)}) \end{bmatrix}$$

where:  $\gamma = 2(1 - \bar{\eta}_1^2)$  and  $\epsilon = -(1 - \bar{\eta}_1)^2$ . Since:

$$D_{\sigma^2} \mathbf{h}(\mathbf{\ddot{q}}_{n_1}, \mathbf{z}) = \begin{bmatrix} -(1 - \ddot{q}_1) \frac{\partial \delta(1,1,2)}{\partial \sigma^2} - \ddot{q}_1 \frac{\partial \delta(-1,1,2)}{\partial \sigma^2} \\ -(1 - \ddot{q}_2) \frac{\partial \delta(1,2,3)}{\partial \sigma^2} - \ddot{q}_2 \frac{\partial \delta(-1,2,3)}{\partial \sigma^2} \\ \dots \\ -(1 - \ddot{q}_{(n_1-1)}) \frac{\partial \delta(1,(n_1-1),n_1)}{\partial \sigma^2} - \ddot{q}_{(n_1-1)} \frac{\partial \delta(-1,(n_1-1),n_1)}{\partial \sigma^2} \end{bmatrix},$$

then  $D\mathbf{h}(\mathbf{\ddot{q}}_{n_1}, \mathbf{z}) = -[D_{\mathbf{q}_{n_1}} \mathbf{h}(\mathbf{\ddot{q}}_{n_1}, \mathbf{z})]^{-1} D_{\sigma^2} \mathbf{h}(\mathbf{\ddot{q}}_{n_1}, \mathbf{z})$ . Hence,  $\frac{d\ddot{q}_i}{d\sigma^2} < 0$  for every  $\ddot{q}_i < 0.5$ .

b.4) Let  $\mathbf{t} = (\bar{\eta}_1, \bar{\eta}_1(1 - \bar{\eta}_1))$ , where  $\bar{\eta}_1(1 - \bar{\eta}_1)$  is the greatest variance compatible with  $\bar{\eta}_1$ . Consider  $\mathbf{t} = (0, 0)$  and let  $\mathbf{\dot{q}}_{n_1}$  be such that  $h_i(\mathbf{\dot{q}}_{n_1}, \mathbf{t}) = 0$  for every  $i \in \{1, \dots, (n_1 - 1)\}$ . From the proof of Proposition 2,  $\mathbf{\dot{q}}_{n_1}$  exists and is unique. Moreover, every  $\dot{q}_i$  in  $\mathbf{\dot{q}}_{n_1}$  is equal to  $\frac{i}{n_1}$ . Let  $r$  denote  $\frac{1}{n_1}$ . The following matrix  $D_{\mathbf{q}_{n_1}} \mathbf{h}(\mathbf{\dot{q}}_{n_1}, \mathbf{t})$  is invertible:

$$D_{\mathbf{q}_{n_1}} \mathbf{h}(\mathbf{\dot{q}}_{n_1}, \mathbf{t}) = \begin{bmatrix} 4r & -2r & 0 & 0 & 0 & 0 \\ -2r & 4r & -2r & 0 & 0 & 0 \\ 0 & \dots & \dots & \dots & \dots & 0 \\ 0 & 0 & 0 & 0 & -2r & 4r \end{bmatrix}. \text{ Since } D_{\eta_1} \mathbf{h}(\mathbf{\dot{q}}_{n_1}, \mathbf{t}) =$$

$$\begin{bmatrix} -4r^2(n_1 - 2) \\ -4r^2(n_1 - 4) \\ \dots \\ -4r^2(2 - n_1) \end{bmatrix}, \text{ then } D\mathbf{h}(\mathbf{\dot{q}}_{n_1}, \mathbf{t}) = -[D_{\mathbf{q}_{n_1}} \mathbf{h}(\mathbf{\dot{q}}_{n_1}, \mathbf{t})]^{-1} D_{\eta_1} \mathbf{h}(\mathbf{\dot{q}}_{n_1}, \mathbf{t}). \text{ Hence}$$

$\frac{d\dot{q}_i}{d\bar{\eta}_1} > 0$  for every  $\dot{q}_i$  lower than 0.5. From b.2) and b.3), in equilibrium

$$\tilde{q}_i \in \left( r, \frac{\tilde{\lambda}_i + \tilde{\lambda}_{(i+1)}}{2} \right).$$

c) Let  $E_i [\pi_1 (L_{n_1}, Q (L_{n_1}))]$  denote the payoff at period 1 expected ex-ante by agent  $i$  in the case of a partition  $Q (L_{n_1})$  with each  $\lambda_i$  in  $L_{n_1}$  satisfying (24). From Corollary 1, both the agents' expected payoffs will be maximized if each  $q_i$  is equal to  $\frac{i}{n_1}$ . Since  $\bar{\eta}_1 \in (0, 1)$ , from b.3) and b.4):

$$\begin{aligned} E_j \left[ \pi_1 (L_{n_1}, Q (L_{n_1})) \mid q_i = \frac{i}{n_1} \forall i \right] &> E_j \left[ \tilde{\pi}_1 \left( \tilde{\mathbf{S}}_{n_1} \left( \tilde{\mathbf{R}}_{n_2}^c \right) \right) \right] > E_j \left[ \tilde{\pi}_1 \left( \tilde{\mathbf{S}}_{n_1}^1 \right) \right] \\ \forall j &= E, D \text{ if } n_1 > 2, n_2 > 1 \end{aligned}$$

■

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