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The impact of liquidity shocks through the limit order book

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No. 2008/53

The Impact of Liquidity Shocks Through the Limit Order Book

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The Impact of Liquidity Shocks Through the Limit Order Book*

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January, 2008

Abstract:
This paper analyzes liquidity in an order driven market. We only investigate the best limits in the limit order book, but also take into account the book behind these inside prices. When subsequent prices are close to the best ones and depth at them is substantial, larger orders can be executed without an extensive price impact and without deterring liquidity. We develop and estimate several econometric models, based on depth and prices in the book, as well as on the slopes of the limit order book. The dynamics of different dimensions of liquidity are analyzed: prices, depth at and beyond the best prices, as well as resiliency, i.e. how fast the different liquidity measures recover after a liquidity shock. Our results show a somewhat less favorable image of liquidity than often found in the literature. After a liquidity shock (in the spread or depth or in the book beyond the best limits), several dimension of liquidity deteriorate at the same time. Not only does the inside spread increase, and depth at the best prices decrease, also the difference between subsequent bid and ask prices may become larger and depth provided at them decreases. The impacts are both econometrically and economically significant. Also, our findings point to an interaction between different measures of liquidity, between liquidity at the best prices and beyond in the book, and between ask and bid side of the market.

JEL Classification: G10

Keywords: Liquidity, Resiliency, Limit Order Markets, Limit Order Book Slopes, Liquidity Shocks.

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1 Introduction

In this paper, we analyze the liquidity in an order driven market as provided by the limit order book. Since liquidity refers to how easy and quickly traders can buy or sell large numbers of shares without large price effects or costs, it is clearly determined by several elements. First, the bid-ask spread and the depth available at the best prices are essential. In the literature liquidity is often analyzed by looking only at these so-called "best limits" (i.e. the best prices and depths) in the limit order book (henceforth in short the LOB or simply the book). Although these best limits are important aspects in assessing liquidity, they form nevertheless only a part of the picture. Traders may not only care about the inside spread or the depth at the best prices, but also about the LOB beyond these best limits. If subsequent prices are close to the best ones and if sufficient depth is provided at those prices, large orders can execute at a lower cost, than when the spread is narrower but lower depth is available and/or subsequent prices are further away from the best price. Hence, while the best limits provide an important element, the rest of the LOB may is also relevant when assessing liquidity in a limit order market. Furthermore, for well functioning markets it is also crucial that liquidity is restored quickly when a liquidity shock has taken away a significant part of the liquidity that was available in the book. This dimension of liquidity is called resiliency.

The main objective of this paper is therefore to investigate liquidity provided by the LOB, not only at the best limits, but also beyond in them. We devote particular attention to the resiliency of the book. In the analysis, we take into account different measures of liquidity, in line with the intuition just presented. Each measure then captures an aspect of liquidity. One set of measures focusses on liquidity at the best limits: the bid-ask spread and depth at the best ask and bid price. In measuring liquidity in the LOB beyond the best limits we take two complementary approaches. The first is to use limit order book slopes. Slopes summarize the price and depth dimensions of liquidity in one variable for the ask side and another for the bid side. This has the advantage that a parsimonious model is obtained, but price and depth effects can no longer be separated. In a second, alternative approach, we investigate on the one hand the "spread" between best and subsequent prices, i.e. how far subsequent ask (bid) prices are away from the best ask (bid), expressed in number of ticks. We also look at the depth that is provided at these prices. In all models, we allow for an interaction between the different aspects of liquidity: between liquidity at and beyond the best limits and at ask and bid side of the market, and also between prices and depth. This is important since if e.g. a shock that widens the bid-ask spread would imply an increase in the depth, the impact of the shock on liquidity may be less severe than if depth would decline. In this way, this paper is one of the first to provide a detailed analysis of liquidity and the impact of liquidity shocks at as well as beyond the best limits.

In our empirical methodology, we divide the trading day in 15 minute intervals and compute time-weighted averages of each variable (i.e. liquidity measure) over each interval. We also adjust for intraday patterns exhibited by the data. We develop different specifications of vector autoregressive models (VARs), in each the endogenous variable capture various aspects of liquidity. Starting from these VARs, we subsequently analyze the dynamics of the LOB after shocks, which is the main focus of the paper. For this, we compute, on the basis of the VAR-models, impulse response functions (IRFs) which plot the dynamic evolution of the variables around different types of liquidity shocks. Moreover, these IRFs allow at the same time for unraveling the dynamic relationships between different measures of liquidity, and between ask and bid side of the book. The analysis is performed by using data from the Spanish Stock Exchange SIBE, this is a pure limit order market without market makers[1].

To our knowledge, limit order book slopes have not been extensively analyzed in the literature. Although slopes provide a good measure of the book, their behavior around shocks has not yet been documented. Næs and Skjeltorp (2006) analyze empirically the relation between the shape of the LOB, measured by the average elasticity of demand and supply schedules in the book, and the volume-volatility relation. They find a systematic negative relation between the slope and price volatility and between the slope and the daily number of trades. Kalay, Sade and Wohl (2004) estimate demand and supply elasticity at the opening stage of the Tel Aviv Stock Exchange. The elasticity is largest at the opening. Also, it is larger for the demand schedule than for the supply schedule. An alternative measure, but related to the slope, is the cost of round trip trade (CRT) in Irvine, Benston and Kandel (2000). This measure aggregates the status of the LOB at a given time for a specific transaction size. Suppose that trader wants to buy and sell the same number of shares at the same time. The CRT then determines the cost of such trade, taking into account not only the inside spread and depth at the best prices, but (for larger sizes) also the entire structure of the book. The smaller the CRT, the larger liquidity. They show that the CRT can predict the number of subsequent trades. Moreover, it is correlated with other measures of liquidity, such as the inside spread and depth, but provides additional information. The same information about the LOB is however also used to compute the slopes. Moreover, the latter has the advantage that it is a more general measure since, in contrast with the CRT, it does not refer to a particular transaction size. However, none of the papers above provides a detailed analysis of the relation between different liquidity measures or between prices (or spreads) and depth. Our paper complements and contributes to this literature by doing so.

From a theoretical perspective, also very few papers deal with slopes. Many models are not able to fully capture the behavior of slopes. Parlour (1998) considers the queues

[1]In fact, some smaller stocks have a specialist who basically comits to provide liquidity for that stock. This is, however, not the case for the stocks in our sample.

at the best bid and ask prices in a one-tick market[2]. Foucault, Kadan and Kandel (2005) allow traders to undercut the existing prices in the book, but do not allow for queuing at a given price[3]. This means traders either submit a market order, taking the best prices, or submit an order that undercuts the best price. The spread improvement, which has an impact on the slope, is larger when the proportion of patient traders is larger, the waiting cost is higher and the order arrival rate is smaller. A recent model that is able to formulate predictions about the shape of the LOB is Rosu (2007). Using a continuous time model with only liquidity traders (so without asymmetric information) he shows that, when traders can submit multi-unit market orders, then the book exhibits a hump shape, i.e. limit orders cluster at prices away from the best bid and ask. Moreover, he confirms the results of Foucault, Kadan and Kandel (2005) about resiliency in a limit order market. Finally, he shows that after a market sell order, not only the best bid decreases, but also the best ask, although less than the bid. This leads to a widening of the spread. This fact was also found by Biais, Hillion and Spatt (1995). The latter provide an information-related explanation, but Rosu (2007) shows that even without asymmetric information, this occurs. We motivate the inclusion of variables in the different econometric models below in part by this theoretical literature.

Our results demonstrate a somewhat less sunny picture of liquidity in a limit order market than is often obtained in the literature. First, when the bid-ask spread on average increases in a 15-minute interval, depth at the best prices subsequently decreases. This means that both dimensions of liquidity deteriorate. On the other hand, shocks that increase depth, lower the bid-ask spread. The analysis of the LOB beyond the best limit shows that decreasing liquidity at the best prices leads to less liquidity further in the LOB as well. More specifically, a shock that increases the bid-ask spread, lowers the slope of the book (a flatter book means less liquidity is present), both at bid and ask side. A steeper LOB (a shock to the slopes) in turn implies a smaller spread in the periods after the shock. The analysis of prices and depth in the LOB confirms the results. A shock increasing the bid-ask spread first implies a lower distance between the first and fifth price in the book, but after some periods an increase is observed. Depth beyond the best limits remains relatively unaffected. Inversely, a shock that increases the distance between the first and fifth prices in the book (either at ask or bid side) first causes a decrease in the bid-ask spread, but after some periods, the spread increases. Again depth is less affected but if so, in general decreases. On the other hand, when depth at the best prices increases, also depth beyond does. For all shocks in all models the impact is realized within one to 1.5 hours. Important is that all impacts and interactions

[2]She also briefly discusses a two-tick market, but the exposition does not allow for deriving empirical predictions about slopes.

[3]Limit orders can however queue at different prices since unexecuted limit orders that are undercut, remain in the book. However, traders cannot submit a limit order at an existing price. In other words, new limit orders must always improve the existing best price.

just discussed are not only econometrically significant, but their order of magnitude is also significant from an economic point of view. Finally, three more points are worth stressing. First, it is important to allow for a relation between spreads and depths when analyzing liquidity. Secondly, shocks do not remain confined to the own side, but also cause a response at the other side of the market. If e.g. a shock occurs to slopes or depth at the ask side, this affects not only the ask side of the market, but also the bid side. Thirdly, our results clearly show that liquidity at and beyond the best limits interact.

This paper is structured as follows. Section 2 discusses the empirical methodology used and specifies the different econometric models. Moreover, we outline the procedure for computing the IRFs. In Section 3, we introduce the data set. Section 4 shows a number of descriptive statistics of the data and provides evidence for their intraday patterns throughout the trading day. The results are presented in two sections. Section 5 presents the estimation results of the alternative VAR-models and briefly discusses the most important findings. The main section however is Section 6, which plots the IRFs, i.e. the dynamics of the different variables and their evolution after different kinds of liquidity shocks. Moreover, we point to the interrelationships between different aspects of liquidity and between ask and bid sides of the market. Section 7 provides a number of robustness checks for the results obtained. Section 8 concludes.

2 Empirical Methodology

2.1 Introduction

When modeling intraday time series, two issues should be dealt with: irregular spacing of the data and intraday seasonality. First, the intraday time series are irregularly spaced since successive orders are submitted with irregular durations between them. In the literature, in general two approaches are used to deal with the irregular spacing of observations.[4] The first one is to work in event time and record an observation whenever there is a best limit or blim update (i.e. a change in one of the best prices or depths). However, recall that in this paper we are not only interested in the best limits but also in the LOB beyond. If we would record an observation whenever one of the prices or depths in the LOB changes, most of the other prices and depths typically remain unchanged. These hamper accurate estimations. This is amplified by the fact that the data will contain much microstructure noise. Therefore we opt for the second possibility put forward in the literature: we resample the intraday data at a given frequency (e.g. 15 minutes), such that again regularly spaced data are obtained. Next to econometrically

[4] A third possibililty is to deal directly with the irregularly spaced data by using duration models, or joint models of durations and associated marks such as returns. For an overview of this approach, see e.g. Bauwens and Giot (2001).

more suited, our choice is also motivated by the research question we address. The use of regularly spaced intervals permits us to provide a more aggregated view than would be the case when using event time. It allows for investigating if periods where the book is unexpectedly steeper or flatter (in other words, where liquidity is higher or lower) tend to be reversed quickly or whether they persist during a number of consecutive periods. This provides additional and complementary insights to an analysis of the immediate consequences of liquidity shocks as analyzed in e.g. Degryse, de Jong, van Ravenswaaij and Wuyts (2005) who work in event time.

After having opted for using regularly spaced intervals, it is important to tackle a second issue, and take into account intraday patterns exhibited by the (resampled) series (we will illustrate the presence of such patterns in Section 4). The importance of correctly modeling such intraday seasonality is put forward in a number of studies such as Engle and Russell (1998) or Bauwens and Giot (2001)).

Therefore, we go trough the following steps in our analysis:

1. Define the regularly spaced time intervals (i.e. choose the sampling frequency) and specify regularly spaced variables.

2. Compute the intraday pattern of each variable and deseasonalize each variable by its intraday pattern.

3. Model the deseasonalized variable using the appropriate econometric model.

4. Compute the impulse response functions.

In the next subsections, we discuss each of these points more in detail.

2.2 Sampling Frequency

When resampling the data - by dividing the trading day in equal intervals - a choice must be made on the sampling frequency. The advantage of short intervals (say 1 minute) is that few information is lost in the aggregation of tick-by-tick data in intervals. The disadvantage is that possibly more noise remains. The inverse arguments hold for longer intervals e.g. of one hour (or more). Less noise will be present but much of the dynamics within an interval is lost. Little guidance is provided in the literature in choosing an "optimal" frequency. Aït-Sahalia, Mykland and Zhang (2005) compare various possibilities for resampling tick-by-tick data in the context of realized volatility models. They show that it is optimal to sample as often as possible, but one then needs to correct for microstructure noise. Sparse sampling (e.g. at 5- or 15-minute intervals) is the fourth best solution and sampling at an optimal frequency (to be computed) the third best in estimating realized volatility. Simulations point to an optimal frequency of about 6 to 7 minutes. Using IBM data, Oomen (2005) estimates an optimal sampling

frequency of 2.5 minutes. It is however not clear how these findings extend to other variables, nor to less frequently traded stocks. Almost all stocks in our sample are much less frequently traded than IBM, which is traded almost every second. Some of our stocks even have a duration between updates of one of the five best prices or depths of more than 1.5 minutes[5] (and only part of these updates correspond to actual trades). Sampling at very short frequencies, say 1 to 5 minutes, then implies that the time-weighted average of a variable over the interval is based on just a few observations.

Therefore, we opt for a compromise and divide each trading day in *15-minute intervals*. This is in line with, among others, Ahn, Bae and Chan (2001) and Beltran, Durré and Giot (2004). As a robustness check (see Section 7), we also used a 5-minute sampling frequency, our results remain qualitatively unaltered. In the description of the dataset in Section 3, it is explained that the sample comprises 124 trading days. Moreover, each trading day ranges from 9:00 until 17:30. We thus have 34 intervals per day or 4216 in total. In the remainder of the paper, the time index t refers to 15-minute intervals and each variable has 4216 observations.

For each variable in the models below, we compute time-weighted averages of that variable over each interval. Let m_t^* be a variable in our model (e.g. the bid-ask spread or the slope on the ask side of the LOB), where the star refers to the fact that the variable is not yet corrected for intraday seasonality (see next subsection). Assume there are Υ_t observations in interval t (where Υ_t can differ across intervals) and denote each observation as $m_{t,\tau}^*$, with $\tau = 1, ..., \Upsilon_t$. Assume the observation has a duration of $\omega_{t,\tau}$ seconds.[6] Then the variable m_t^*, the time-weighted average in interval t, is defined as:

$$m_t^* = \frac{\sum_{\tau=1}^{\Upsilon_t} \omega_{t,\tau} m_{t,\tau}^*}{\sum_{\tau=1}^{\Upsilon_t} \omega_{t,\tau}} \tag{1}$$

2.3 Intraday Seasonality

The second step in the procedure involves adjusting each variable for its intraday pattern. Not or not properly implementing this step may often lead to incorrect model estimations. Moreover, an economic reasoning exists for this seasonal adjustment. Market participants in principal know the intraday patterns and thus have expectations for the pattern of a specific variable. They are then only affected by surprises (i.e. deviations) from what was expected. We remove intraday effects by regressing each variable on a series of intraday dummies:

$$m_t^* = \beta_0 + \sum_{j=1}^{33} \beta_j T_j + e_t \tag{2}$$

[5] The average duration between such updates is 34.3 seconds, the minimum is 2.7, the maximum 101.4.

[6] So, it must hold that $\sum \omega_{\tau,t} = 900$ seconds (= 15 minutes).

where m_t^* is the not deseasonalized variable. T_j is one if t refers to period $j = 1, ..., 33$ in the trading day, and zero otherwise. We left out the last period ($j = 34$) to avoid perfect multicollinearity. Equation (2) is then used to generate the fitted (or forecasted) values of m_t^*. Finally, we deseasonalize each variable by taking the difference between m_t^* and its forecasted value. This difference becomes the endogenous variable in the VAR-models of the next section and will be denoted a variable as m_t, so without a star as superscript. From now on, all variables in the models in the next section are to be interpreted as intraday deseasonalized.

2.4 Econometric models

As our main interest lies in investigating the dynamic behavior of spreads and depths at and beyond the best limits and their interrelation, we turn to VAR-models. Their use for analyzing intraday, equally spaced data has been advocated in e.g. Hasbrouck (1999). Below, several alternative models will be specified. Each model, however, starts from the following general VAR-specification:

$$y_t = A_0 + \sum_{l=1}^{L} A_l y_{t-l} + u_t \tag{3}$$

where y_t is the $k \times 1$ vector of endogenous variables, u_t is a k-dimensional white noise process[7] and A_0 and A_l are the conformable coefficient matrices. We consider three alternative specifications of equation (3), each taking a different approach to analyze the LOB. In each model, the vector of endogenous variables is different. Important to recall is that each endogenous variable in the models below is adjusted for intraday seasonality and thus should be understood relative to its intraday pattern.

Equation (3) shows that all VAR-models are specified in reduced form. Instead, we could have used a structural VAR (SVAR), as in Coppejans, Domowitz and Madhavan (2004). We would then need, however, identifying restrictions. Coppejans, Domowitz and Madhavan (2004) do impose identifying restrictions, but their model only includes best prices and depths (comparable to our Model 1 below). Given the Models 2 and 3 below, which include more elaborate variables. Moreover, the main interest of this paper will not lie in the VARs themselves, but rather in the impulse response functions (IRFs). Therefore, we start from the reduced form in (3) and use a particularly adequate method to compute the IRFs. This method does not require a priori identifying restrictions as for SVARs, neither does it require a specific ordering as in a Choleski decomposition. We come back to this in Subsection 2.5 after the three VAR-models have been presented. Finally, robustness checks show that our conclusions do not change when an SVAR is used (see Section 7).

[7] That is $E(u_t) = 0$, $E(u_t u_t') = \Sigma_u$ and $E(u_t u_s') = 0$ for $t \neq s$.

2.4.1 Model 1: Best Limits

In the base case model, we investigate the best limits in the book sampled at 15-minute intervals. More specifically, we set the vector of endogenous variables in (3) equal to:

$$y_t = \{Spr_BA_t, AD1_t, BD1_t\}$$

Spr_BA_t is the time-weighted average inside or bid-ask spread, i.e. the difference between the best ask and bid, in ticks, computed over each 15-minute interval. $AD1_t$ and $BD1_t$ are the time-weighted monetized depth at the best ask and bid, respectively, in euro. We included the spread in the model, instead of separate bid and ask prices (as in e.g. Engle and Patton (2004)). For models in event time, it is natural to allow for divergent evolutions of ask and bid prices. When averaged over 15 minutes, however, both will evolve much in the same way. Therefore, the decomposition of the spread in its evolution on ask and bid side is less meaningful and we include the spread as measure of liquidity. We include both depth at the best ask and bid, however, since Parlour (1998) shows that traders look at both sides of the market when deciding which type of order to submit. This specification is also similar to the one in Coppejans, Domowitz and Madhavan (2004)

2.4.2 Model 2: Limit Order Book Slopes

Just considering the best prices and depths in the LOB only provides a partial picture of liquidity. As argued in the introduction, also the state of the book beyond these best limits is important. In the second VAR-specification, we therefore develop a first approach for investigating the LOB beyond the best limits. A concise way of summarizing the book beyond the best limits is by considering limit order book slopes. Then, the vector of endogenous variables becomes:

$$y_t = \{Spr_BA_t, Slope_A_t, Slope_B_t\}$$

As before, Spr_BA_t is the time-weighted average bid-ask spread in number of ticks. $Slope_A_t$ and $Slope_B_t$ are the slopes of the LOB at the ask and bid side respectively. They are computed in a similar way as in Kalay, Sade and Wohl (2004) and Næs and Skjeltorp (2006). More specifically, we compute the time-weighted average of the slope on the ask side as follows. For each update τ in the book[8] in interval t, we compute the

[8]Recall that in this paper an update is recorded whenever at least one of the five best bid or ask prices or the depth at one of these prices changes.

slope using the following expression for the ask side:

$$Slope_A_{t,\tau} = \frac{1}{5}\left[\frac{ADC1_{t,\tau}}{\frac{AP1_{t,\tau}}{M_{t,\tau}}-1} + \frac{\frac{ADC2_{t,\tau}-ADC1_{t,\tau}}{ADC1_{t,\tau}}}{\frac{AP2_{t,\tau}-AP1_{t,\tau}}{AP1_{t,\tau}}} + \frac{\frac{ADC3_{t,\tau}-ADC2_{t,\tau}}{ADC2_{t,\tau}}}{\frac{AP3_{t,\tau}-AP2_{t,\tau}}{AP2_{t,\tau}}} + \frac{\frac{ADC4_{t,\tau}-ADC3_{t,\tau}}{ADC3_{t,\tau}}}{\frac{AP4_{t,\tau}-AP3_{t,\tau}}{AP4_{t,\tau}}} + \frac{\frac{ADC5_{t,\tau}-ADC4_{t,\tau}}{ADC4_{t,\tau}}}{\frac{AP5_{t,\tau}-AP4_{t,\tau}}{AP5_{t,\tau}}}\right] \quad (4)$$

where $ADC1_{t,\tau}, ..., ADC5_{t,\tau}$ are the *cumulative* depths at the first, ..., fifth ask price in the book[9]. $AP1_{t,\tau}, ..., AP5_{t,\tau}$ are the five first ask prices at time τ, $M_{t,\tau}$ is the midprice defined as the average of the best bid and ask price. All variables are recorded *after* update τ. The expression can be interpreted as the average elasticity for the five first limit prices at the ask side of the market. The slope at for interval t, denoted by $Slope_A_t$, is then the time-weighted average of the slopes $Slope_A_{t,\tau}$ for the 15-minute interval t.

For the bid side of the market, the symmetric procedure is used, although we take *absolute values* of the price differences in the denominator when computing $Slope_B_{t,\tau}$. Note that it follows from the definitions that higher slopes are associated with higher liquidity in the LOB.

2.4.3 Model 3: Prices and Depths in the Limit Order Book

While limit order book slopes offer the advantages of resulting in a concise model, a shortcoming is that it is not possible to distinguish dynamics at the prices or depths. Therefore, we develop a third model allowing for this. We take into account both the difference between subsequent prices at bid and ask side and the depths and specify the vector of endogenous variables in Model 3 as:

$$y_t = \{Spr_BA_t, AD1_t, BD1_t, Spr_A15_t, Spr_B15_t, AD25_t, BD25_t\}$$

The first three variables are defined in the same way as in Model 1. Spr_A15_t is the "spread" between the best ask and the 5th ask price in the book in ticks, Spr_B15_t is the absolute value of the difference between the best bid and 5th bid price. The cumulative depth, in euro, at the second, third, fourth and fifth prices in the book at the ask (bid) side of the market is denoted by $AD25_t$ ($BD25_t$). We thus summarize the book beyond the best limits by four variables, two for prices (Spr_A15_t and Spr_B15_t) and two for depths ($AD25_t$ and $BD25_t$). In this way, we obtain a more parsimonious model than if all five prices and depths at ask and bid side would be included. All variables are time-weighted averages over interval t. The endogenous variables are similar to the ones in Pascual and Veredas (2006). These authors find that, although most of the explanatory

[9]Note that $ADC1_{t,\tau} = AD1_{t,\tau}$.

power of the book concentrates on the best limits, the book beyond them also matters in explaining the order choice of traders. Hence, they are also a determinant of liquidity supply (by limit orders) and demand (by market orders) in a limit order market and are included in the model.

A summary of all the variables, used in the three models, and their description, is presented in Table 1. Recall also that all variables are adjusted for intraday seasonality.

Table 1: **Summary of Notation**

Note: This table presents a summary of the different variables used in the econometric models, their notation and definition. In the models, all variables are computed as time-weighted averages over 15-minute intervals and adjusted for intraday patterns.

Variable	Description
Spr_BA	Difference between best ask and bid, in # ticks
$AD1$	Depth available at best ask, in 1000 euro
$BD1$	Depth available at best bid, in 1000 euro
$Slope_A$	Slope of the book at the ask side, computed as in equation (4), $(*10^{-6})$
$Slope_B$	Absolute value of the slope of the book at the bid side, computed similar as in equation (4), $(*10^{-6})$
Spr_A15	Difference between best and fifth ask price, in # ticks
Spr_B15	Absolute value of difference between best and fifth bid price, in # ticks
$AD25$	Cumulative depth available at second until fifth ask price, in 1000 euro
$BD25$	Cumulative depth available at second until fifth bid price, in 1000 euro

2.5 Impulse Responses

Although the coefficients in the three VAR models in the previous section already reveal interesting insights, the main goal of this paper is to analyze in detail the dynamic properties of the different variables and their interrelation. Therefore, we compute impulse response functions (IRFs) of the different VAR-models which give the responses of the endogenous variables to different shocks. In order to compute the IRFs, we follow the procedure proposed by Pesaran and Shin (1998). For notational simplicity, we bring together the constant term, the vector of exogenous variables and its lags[10] and denote it by $\mathbf{x}_t$. The general form of the VAR in equation (3) can then be rewritten as:

$$y_t = \sum_{l=1}^{L} A_l y_{t-l} + \mathbf{B}\mathbf{x}_t + u_t \tag{5}$$

[10]Models 1, 2 and 3 do not contain exogenous variables. But as a robustness check, we will include them later in the paper.

with y_t the $(k \times 1)$ vector of endogenous variables and $\mathbf{x}_t$ the vector of all exogenous variables (it thus contains a constant, the exogenous variables and their lags). A_l and $\mathbf{B}$ are conformable coefficient matrices. The following standard assumptions are made (see also e.g. Lutkepohl (1991)): $E(u_t) = 0$, $E(u_t u_t') = \Sigma$, $E(u_t u_s') = 0$ for all s and $E(u_t|\mathbf{x}_t) = 0$. Moreover, all roots of $\left|I_m - \sum_{l=1}^{L} A_l z^l\right|$ fall outside the unit circle. Under these assumptions, we can rewrite (5) in an infinite MA representation:

$$y_t = \sum_{i=0}^{\infty} \zeta_i u_{t-i} + \sum_{i=0}^{\infty} \boldsymbol{\vartheta}_i \mathbf{x}_{t-i}$$

where ζ_i can be determined recursively

$$\begin{aligned} \zeta_i &= A_1\zeta_{i-1} + ... + A_p\zeta_{i-p}, \qquad i = 1, 2, ... \\ \zeta_i &= 0 \text{ for } i \leq 0 \end{aligned}$$

and $\boldsymbol{\vartheta}_i = \zeta_i \mathbf{B}$. Suppose that a $(k \times 1)$ vector ϵ of shocks hits the variables in the model. Then Koop et al. (1996) define the generalized impulse response function of y_t in period $t + n$ as:

$$GIRF_n(\epsilon, I_{t-1}) = E(y_{t+n}|u_t = \epsilon, I_{t-1}) - E(y_{t+n}|I_{t-1}) \tag{6}$$

with I_{t-1} the information set at time $t - 1$. Using the MA representation, we find that:

$$GIRF_n(\epsilon, I_{t-1}) = \zeta_n \epsilon$$

From this expression, it follows that the choice of the vector of shocks is crucial. In practice, there is correlation between shocks to different variables, so a shock in one variable is likely to be accompanied by shocks in other variables. In this case, we cannot attribute movements in a variable to a particular shock. The traditional approach is to solve this problem by using the Choleski decomposition of Σ (see e.g. Lutkepohl (1991)). A main drawback of this decomposition is that the impulse responses can be sensitive to the ordering of the variables imposed in the composition. Moreover, theory does not provide a clear guidance for a specific ordering in our setting. Therefore, we use an alternative methodology, proposed by Pesaran and Shin (1998), which does not impose a particular ordering of the variables.

Their main idea is to start from the generalized IRFs in (6). Instead of shocking all elements in u_t, they shock only one element j and integrate out the effects of other shocks using an assumed or historically observed distribution of the errors. Then:

$$GIRF_n(\epsilon_j, I_{t-1}) = E(y_{t+n}|u_{t,j} = \epsilon_j, I_{t-1}) - E(y_{t+n}|I_{t-1})$$

If u_t has a multivariate normal distribution, it can be shown that:

$$E\left(u_t|u_{t,j}=\epsilon_j\right)=\Sigma\iota_j\sigma_{jj}^{-1}\epsilon_j$$

with ι_j a selection vector with the jth element equal to one and zeros elsewhere; and σ_{jj} is the jj^{th} element of Σ. Hence, the generalized impulse responses to a shock in the jth equation at time t is given by:

$$GIRF_n=\frac{\zeta_n\Sigma\iota_j}{\sqrt{\sigma_{jj}}}\frac{\epsilon_j}{\sqrt{\sigma_{jj}}},\qquad n=0,1,2,...$$

By setting $\epsilon_j=\sqrt{\sigma_{jj}}$ scaled IRFs are obtained as:

$$SGIRF_n=\sigma_{jj}^{-1/2}\zeta_n\Sigma\iota_j,\qquad n=0,1,2,...$$

This formula measures the effect of one standard error shock to the jth equation at time t on expected values of y at time $t+n$.

3 Data

This paper uses data from the Spanish Stock Exchange SIBE, an exchange which operates essentially as a pure order driven market. For the institutional details of SIBE and a description of its main features, we refer to Pardo and Pascual (2007). The sample contains 35 stocks that were part of the IBEX35 stock index during the sample period. The IBEX35 is composed by the 35 most liquid and active stocks, traded on the exchange. Our sample period ranges from July 2000 - December 2000 and thus spans 124 trading days. The data on the LOB contain the five best bid and ask prices and the displayed depth at each of these ten prices. Moreover, we have data on all trades that were executed during the continuous trading session. Preopening or postclosing orders are not included since the trading mechanism during this period is different from the one during the trading day. All changes in the book are timestamped to 100th of a second. The trading data show price and size of each trade. The index numbers and time stamps allow for a perfect matching of trade and LOB data. Because of this matching, it is also possible to detect if hidden depth when executing an order. Since the sample period is before 2001, we do not have to take into account the presence of volatility auctions (see Pardo and Pascual (2007)). These auctions were only introduced at May 14, 2001.

The raw data, as obtained from the different files, first need to be filtered before they can be used in the analysis. The reasoning is that the database contains many typos, as well as other errors, e.g. registers out of sequence and increases in the accumulated volume over the day that are negative. For more details on the procedure, we refer to

Pardo and Pascual (2007), who use the same dataset. Our sample is also the same as the one in Pascual and Veredas (2006).

4 Descriptive Statistics

Table 2 presents the mean, median and standard deviation of each of the variables, used in the three VAR-models. Important to note is that the data in the figures are *not yet adjusted for intraday patterns* (in contrast with the VAR-models). All variables are represented as their unweighted average across the 35 stocks. The average inside spread is almost 6 ticks. A notable result in the table is that liquidity at the ask side of the market is on average larger than liquidity on the bid side. Depth at the best ask, cumulative depth at subsequent ask prices are larger than their counterparts on the bid side, while the spread between the best and fifth ask prices is smaller than the similar difference on the bid side. As a result, also the slope of the LOB is larger on the ask than on the bid side.

Table 2: **Descriptive Statistics**

Note: This table presents the summary statistics, i.e. the mean, median and standard deviation (S.d.), for the different variables used in the econometric models. Data are not adjusted for intraday seasonality. A description of the variables and their notation is presented in Table 1.

	Mean	**Median**	**S.d.**
Spr_BA_t	5.959	5.047	3.809
$AD1_t$	37.459	27.584	40.835
$BD1_t$	34.800	27.008	32.926
Spr_A15_t	10.570	9.201	5.205
Spr_B15_t	10.688	9.114	5.790
$AD25_t$	251.774	179.969	258.240
$BD25_t$	211.566	170.704	162.867
$Slope_A_t$	0.953	0.637	1.301
$Slope_B_t$	0.852	0.614	0.938

Figure 1 draws the mean of the different variables for each of the 34 periods in the day. Obviously, these are again data which are *not yet adjusted for intraday patterns.* In each graph, the x-axis shows the 34 trading periods of a trading day, while the title displays the name of the variable depicted. All graphs draw the unweighted averages over all stocks. Starting with the graphs of Spr_BA, Spr_A15 and Spr_B15, clearly, all are higher at the beginning of continuous trading and decline over the day. At the

end of the trading day, they slightly increase again. These graphs are consistent with earlier results in the literature, documenting U or J-shaped intraday patterns for the spread (see e.g. Biais, Hillion and Spatt (1995)). The evolution of the depth a the best ask and bid ($AD1$ and $BD1$) as well as of cumulative depth at the second until fifth ask and bid prices over the trading day ($AD25$ and $BD25$) show that initially, depth is low and then gradually increases over the day. At the end of trading, depth is highest. Finally, for the the slope of the LOB at the ask and bid side ($Slope_A$ and $Slope_B$), the graphs show that slopes are lower in early periods of the day, and higher later on. Recall that a flatter slope means that liquidity provided by the book is low. Therefore, since slopes increase over the day, liquidity improves.

Figure 1 clearly shows that various measures of liquidity are characterized by intraday patterns. Moreover, liquidity is lowest at the beginning of the trading day, and improves over the day. This holds both in terms of spreads and depths. Limit order book slopes display the same pattern. Given these patterns, it is necessary to correct for intraday seasonality before estimating the different VAR-models to avoid biased estimates. The procedure is outlined in Section 2.

5 Results: Estimations

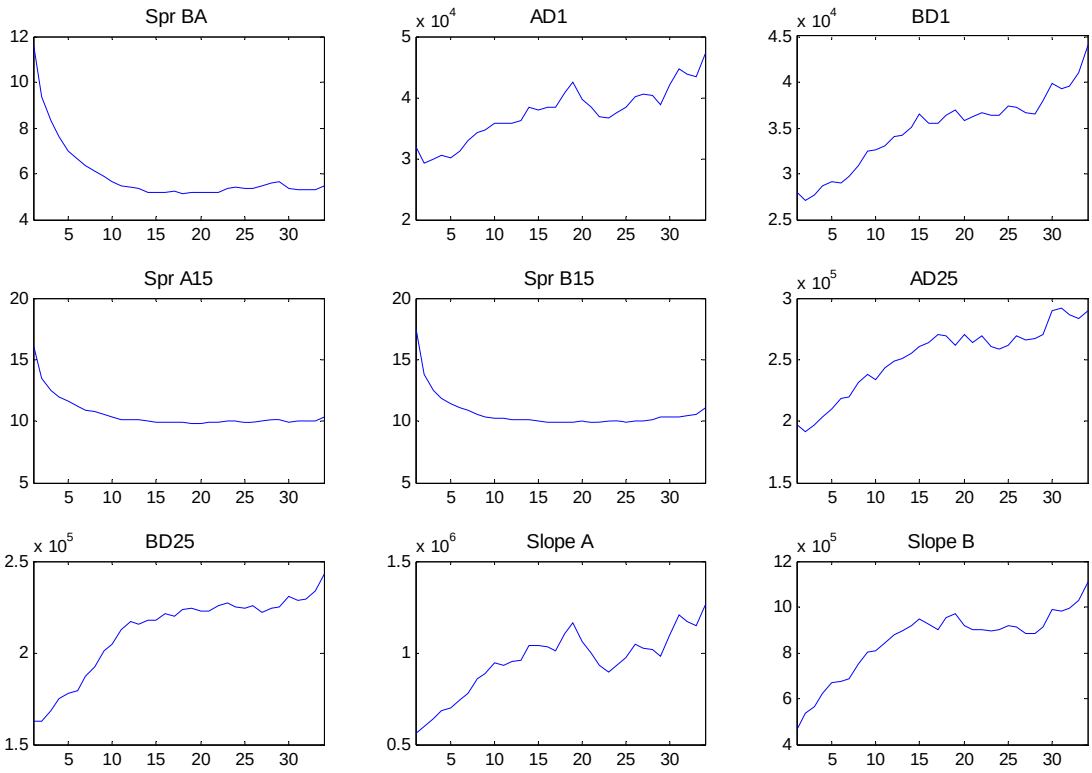
5.1 Introduction

The empirical results of the three VAR-models described above, are presented in two sections. The current section presents the estimation results. The dynamic aspects and the behavior of various variables after shocks are investigated by the IRFs, computed on basis of the VAR-models. An elaborate discussion of them is given in Section 6.

Before estimating the VAR-models, we verified by means of an Augmented Dickey-Fuller test that none of the variables contains a unit root. The results of these tests, not reported, show that in all cases, the null hypothesis of a unit root can be rejected. When subsequently estimating the VAR-models, we include 2 lags of the endogenous variables ($L = 2$). This values is motivated by investigating the AIC-criterion. Moreover, additional lags are in general not significant. The model specification is also in line with the literature. Coppejans, Domowitz and Madhavan (2004) specify a structural VAR-model and include, next to the contemporaneous value, also the first lag. Finally, recall also that each variable is adjusted for intraday patterns and should therefore be interpreted as relative to their pattern in a given period during the trading day.

5.2 Model 1: Best limits

Recall Model 1 includes the inside bid-ask spread and the depth at the best bid and ask. Its estimation results are presented in Table 3. The first column of the table



Note: This figure presents the intraday patterns of the variables of the different VAR-models. All variables are drawn before adjustment for intraday seasonality. The x-axis displays the 34 intervals of each trading day. Unweighted averages across stocks are shown. A description of the variables and their notation is presented in Table 1.

Figure 1: **Descriptive Statistics: Intraday Patterns**

shows the right hand side variables of each equation. The other three columns then display the results for the three equations in the VAR-model, where the header of each column is the endogenous variable. Model 1 was estimated for each stock separately, the table presents the (unweighted) average of the estimated coefficients across the 35 stocks in the sample. Below each coefficient, between brackets, the number of stocks is given for which the coefficient was significantly positive (first element) and negative (second element). Between squared brackets, the 5% and 95% percentile of the estimated coefficient across the 35 stocks are presented. For example, the coefficient of the first lag of the spread (Spr_BA_{t-1}) in the equation of the depth at the best ask ($AD1_t$), averaged over the 35 stocks, is -3.312. For 0 stocks, the coefficient is significantly positive, for 22 significantly negative and for $35 - 0 - 22 = 13$ stocks it is not significant. The 5% and 95% percentiles of the coefficient, computed across the coefficients that are estimated in separate regressions for each of the 35 stocks, are -30.94 and -0.04 respectively.

Several results emerge from Table 3. First, all endogenous variables are autocorrelated, but the autocorrelation decreases quickly. Secondly, lags of the bid-ask spread are significant in the depth equation, with a negative sign. Thirdly, the first lag of ask and bid depth in the spread equation is significant only for some stocks . Fourth, we find some evidence for a significant positive relation between ask and bid depth. In other words, we find evidence for interactions between spread and depth and between ask and bid side of the market. Our results are also in line for the model in Coppejans, Domowitz and Madhavan (2004), both with respect to signs, significance and order of magnitude.[11]

5.3 Model 2: Limit Order Book Slopes

The first approach to account for the LOB behind the best limits, is by means of limit order book slopes as analyzed in Model 2. The estimation results of this model are presented in Table 4. The interpretation of the table is the same as the previous one. We find that the slopes are positively autocorrelated but the magnitude of this autocorrelation decreases quickly. Moreover, the lagged spread has a negative sign in the slope equations. This means that when the spread becomes larger, the slope of the book becomes smaller the next period, ceteris paribus. We also find a positive relation between the slope on the ask and bid side for a majority of the stocks. In other words, not only does the slope at the ask side remains high after a period with steep ask slopes (due to the high autocorrelation), also the slope at the bid side will be higher, ceteris paribus. Finally, there is little evidence for a relation between lagged slopes and the bid-ask spread.

[11] To be precise, note that Coppejans et al. include contemporaneous effects in their VAR and moreover consider the return on the midquote instead of the spread. However, their results for the depth equations are similar to ours.

Table 3: **Model 1: Estimation Results**

Note: This table presents the estimation results of VAR-model 1. The first column shows the right hand side variables. The remaining columns are the equations in the VAR-model, each column header displays the specific endogenous variable on the left hand side of the equation: Spr_BA_t is the bid-ask spread, $AD1_t$ ($BD1_t$) the depth at the best ask (bid). Each variable is the time-weighted average over the 15-minute interval t and adjusted for intraday patterns. Coefficient estimates are reported, as well as (between brackets) the number of stocks, out of 35, for which the coefficient is significantly positive (first element) and negative (second). Between squared brackets, the 5% and 95% percentile of the estimated coefficient across the 35 stocks are presented. Below the table, the adjusted R^2 of each equation is shown, computed as the average of the adjusted R^2 of the individual regressions for each stock.

	Spr_BA_t	$AD1_t$	$BD1_t$
C	0.000	0.000	0.000
	(0,0)	(0,0)	(0,0)
	[0.00,0.00]	[0.01,0.00]	[0.02,0.00]
Spr_BA_{t-1}	0.567	-3.312	-2.669
	(35,0)	(0,22)	(0,19)
	[0.37,0.73]	[-30.94,-0.04]	[-22.62,0.00]
Spr_BA_{t-2}	0.079	-0.896	-1.013
	(26,1)	(2,5)	(1,6)
	[0.00,0.17]	[-5.10,0.21]	[-7.86,0.42]
$AD1_{t-1}$	-0.001	0.452	0.036
	(3,7)	(35,0)	(14,0)
	[-0.01,0.01]	[0.28,0.71]	[-0.01,0.16]
$AD1_{t-2}$	-0.002	0.034	0.009
	(1,7)	(19,6)	(5,2)
	[-0.01,0.00]	[-0.07,0.14]	[-0.06,0.07]
$BD1_{t-1}$	-0.003	0.047	0.456
	(0,13)	(20,0)	(35,0)
	[-0.01,0.00]	[0.00,0.15]	[0.23,0.63]
$BD2_{t-2}$	0.000	0.017	0.057
	(1,5)	(6,2)	(24,2)
	[-0.01,0.01]	[-0.03,0.06]	[-0.07,0.17]
$Adj\ R^2$	0.404	0.25	0.273

Table 4: **Model 2 : Estimation Results**

Note: This table presents the estimation results of VAR-model 2. The first column shows the right hand side variables. The remaining columns are the equations in the VAR-model, each column header displays the specific endogenous variable on the left hand side of the equation: Spr_BA_t is the bid-ask spread, $Slope_A_t$ ($Slope_B_t$) the slope at the ask (bid) side of the market. Each variable is the time-weighted average over the 15-minute interval t and adjusted for intraday patterns. Coefficient estimates are reported, as well as (between brackets) the number of stocks, out of 35, for which the coefficient is significantly positive (first element) and negative (second). Between squared brackets, the 5% and 95% percentile of the estimated coefficient across the 35 stocks are presented. Below the table, the adjusted R^2 of each equation is shown, computed as the average of the adjusted R^2 of the individual regressions for each stock.

	Spr_BA_t	$Slope_A_t$	$Slope_B_t$
C	-0.001	0.000	0.000
	(0,0)	(0,0)	(0,0)
	[-0.01,0.00]	[0.00,0.00]	[0.00,0.00]
Spr_BA_{t-1}	0.565	-0.089	-0.090
	(35,0)	(0,29)	(0,30)
	[0.36,0.73]	[-0.61,0.00]	[-0.82,0.00]
Spr_BA_{t-2}	0.074	-0.036	-0.037
	(24,1)	(0,9)	(0,4)
	[0.00,0.18]	[-0.12,0.02]	[-0.24,0.00]
$Slope_A_{t-1}$	0.039	0.408	0.040
	(1,2)	(35,0)	(17,0)
	[-0.24,0.39]	[0.24,0.73]	[-0.01,0.18]
$Slope_A_{t-2}$	-0.146	0.020	0.014
	(0,10)	(19,7)	(9,1)
	[-0.67,0.09]	[-0.19,0.13]	[-0.03,0.09]
$Slope_B_{t-1}$	-0.014	0.052	0.397
	(2,5)	(22,0)	(35,0)
	[-0.22,0.32]	[-0.01,0.19]	[0.14,0.58]
$Slope_B_{t-2}$	-0.021	0.023	0.060
	(1,5)	(12,2)	(24,4)
	[-0.40,0.38]	[-0.04,0.08]	[-0.06,0.17]
$Adj\ R^2$	0.403	0.235	0.248

5.4 Model 3: Beyond the Best Limits

Model 3 is the second approach to investigate the LOB behind the best limits. The estimation results are presented in Table 5. First note that the conclusions of Model 1 concerning the inside spread and depth at the best bid and ask are robust to the inclusion of new variables. In general, the order of magnitude of the coefficients of the Spr_Ba, $AD1$ and $BD1$ in the respective equations slightly declines, but their sign and significance remain. Secondly, the new variables also exhibit positive autocorrelation, with a rather high $AR(1)$ coefficient. This has important consequences. Suppose the spread between the first and fifth ask price is large in the current period. The high autocorrelation then implies that the difference will indeed decrease, but will remain high from an economic point of view, other things equal. On the positive side, it also means that if depth beyond the best prices is high now, it is likely to remain high the next period, other things equal. Thirdly, Spr_A15_{t-1} and Spr_B15_{t-1} have a positive sign in the equation of the inside spread. This means that a period where subsequent ask (bid) prices are far away from each other, tends to be followed by a period where the inside spread is on average larger. We also find some evidence for a negative relation between Spr_A15_{t-1} (Spr_B15_{t-1}) and $AD25_t$ ($BD25_t$), meaning that not only the inside spread rises, but also the depth in the book decreases. $AD25_t$ ($BD25_t$) are also negatively related to inside spread such that a lower depth beyond the best limits is associated with a larger inside spread. Finally, we find a positive correlation between lagged depth at the best ask (bid) and depth at subsequent ask (bid) prices. These results show that the relation between spreads and depths extends beyond the best limits. Moreover, there is clear evidence, that the LOB beyond the best limits influences the liquidity at the best limits, both in terms of spreads and depth.

6 Results: Responses to Liquidity Shocks

6.1 Introduction

In this section, Figures 2 until 4 draw the impulse response functions (IRFs) computed on the basis of the four VAR-models. We use the procedure of Pesaran and Shin (1998)[12], as outlined in Section 2.5 and implement one standard deviation shocks to each variable. The IRFs are plotted during 12 intervals after the shocks (the x-axis on each graph), which thus span a period of three hours. We compute the IRFs for each of the 35 stocks separately but all graphs depict, in full lines, the unweighted average of IRFs over the 35 stocks. Dashed lines represent the 5% and 95% percentile of the IRFs, computed across

[12]Recall that the advantage of this procedure is that the ordering of the variables is irrelevant. So we do not need to specify such ordering for the different models below, as would be necessary e.g. when using a Choleski decomposition.

Table 5: **Model 3: Estimation Results**

Note: This table presents the estimation results of VAR-model 3. The first column shows the right hand side variables. The remaining columns are the equations in the VAR-model, each column header displays the specific endogenous variable on the left hand side of the equation: Spr_BA_t is the bid-ask spread, $AD1_t$ ($BD1_t$) the depth at the best ask (bid), Spr_A15_t (Spr_B15_t) the difference (in absolute value) between the best and the fifth ask (bid) price and $AD25_t$ ($BD25_t$) the cumulative depth at the second until fifth ask (bid) price. Each variable is the time-weighted average over the 15-minute interval t and adjusted for intraday patterns. Coefficient estimates are reported, as well as (between brackets) the number of stocks, out of 35, for which the coefficient is significantly positive (first element) and negative (second). Between squared brackets, the 5% and 95% percentile of the estimated coefficient across the 35 stocks are presented. Below the table, the adjusted R^2 of each equation is shown, computed as the average of the adjusted R^2 of the individual regressions for each stock.

	Spr_BA $_t$	*AD1* $_t$	*BD1* $_t$	*Spr_A15* $_t$	*Spr_B151* $_t$	*AD25* $_t$	*BD25* $_t$
C	-0.001	-0.002	0.000	-0.002	-0.002	-0.007	-0.018
	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)
	[-0.01,0.00]	[-0.02,0.00]	[-0.02,0.01]	[-0.01,0.00]	[-0.01,0.00]	[-0.13,0.08]	[-0.09,0.08]
Spr_BA $_{t-1}$	0.533	-1.968	-1.769	0.012	-0.031	-8.376	-4.982
	(35,0)	(0,23)	(0,17)	(7,3)	(2,16)	(0,19)	(1,17)
	[0.28,0.73]	[-16.29,-0.05]	[-15.37,-0.01]	[-0.05,0.07]	[-0.10,0.08]	[-67.02,-0.07]	[-16.90,0.15]
Spr_BA $_{t-2}$	0.044	-0.172	-0.402	0.060	0.091	-1.796	-2.609
	(21,1)	(1,1)	(1,2)	(30,0)	(33,0)	(0,3)	(1,3)
	[-0.02,0.10]	[-1.32,0.19]	[-4.32,0.40]	[0.02,0.10]	[0.03,0.14]	[-14.08,0.61]	[-17.47,0.74]
AD1 $_{t-1}$	-0.002	0.421	0.029	0.002	0.001	0.202	0.047
	(1,5)	(35,0)	(13,0)	(2,1)	(5,2)	(28,1)	(4,1)
	[-0.01,0.00]	[0.22,0.69]	[-0.01,0.10]	[0.00,0.01]	[-0.01,0.01]	[-0.07,0.43]	[-0.04,0.24]
AD1 $_{t-2}$	-0.002	0.008	0.000	0.000	-0.001	0.059	0.009
	(0,6)	(16,7)	(1,3)	(1,2)	(2,2)	(7,4)	(4,2)
	[-0.01,0.00]	[-0.10,0.11]	[-0.07,0.04]	[-0.01,0.01]	[-0.01,0.01]	[-0.12,0.42]	[-0.09,0.14]
BD1 $_{t-1}$	-0.004	0.041	0.427	-0.001	0.002	0.105	0.188
	(0,14)	(16,0)	(35,0)	(1,1)	(3,0)	(10,0)	(27,1)
	[-0.01,0.00]	[0.00,0.11]	[0.15,0.62]	[-0.01,0.00]	[0.00,0.01]	[-0.07,0.57]	[-0.12,0.39]
BD2 $_{t-2}$	0.000	0.007	0.033	-0.001	0.001	-0.008	-0.023
	(2,2)	(1,3)	(18,4)	(2,1)	(4,3)	(0,2)	(1,7)
	[-0.01,0.01]	[-0.04,0.04]	[-0.08,0.11]	[-0.01,0.01]	[0.00,0.01]	[-0.24,0.10]	[-0.17,0.14]

Table 5 (continued)

	Spr_BA_t	$AD1_t$	$BD1_t$	Spr_A15_t	Spr_B151_t	$AD25_t$	$BD25_t$
Spr_A15_{t-1}	0.100	-0.404	-0.443	0.698	-0.003	-5.135	-0.429
	(35,0)	(0,3)	(0,3)	(35,0)	(4,5)	(0,16)	(1,3)
	[0.06,0.14]	[-1.64,0.26]	[-3.67,0.02]	[0.52,0.79]	[-0.06,0.05]	[-28.54,0.19]	[-8.16,3.03]
Spr_A15_{t-2}	-0.023	0.090	-0.068	0.038	0.020	-2.475	0.490
	(2,15)	(3,0)	(0,0)	(20,0)	(11,1)	(2,1)	(1,0)
	[-0.05,0.02]	[-0.49,1.58]	[-1.25,0.30]	[-0.01,0.08]	[-0.01,0.08]	[-23.09,0.98]	[-0.98,4.05]
Spr_B15_{t-1}	0.098	-0.417	-0.268	0.023	0.680	-0.467	-4.718
	(35,0)	(1,4)	(8,2)	(16,1)	(35,0)	(1,2)	(1,9)
	[0.06,0.15]	[-3.44,0.12]	[-2.38,0.34]	[-0.03,0.09]	[0.51,0.81]	[-3.76,11.40]	[-36.72,0.21]
Spr_B15_{t-2}	-0.017	0.536	0.182	0.006	0.053	3.178	-0.321
	(0,14)	(7,1)	(1,0)	(7,2)	(27,0)	(7,0)	(3,1)
	[-0.06,0.02]	[-0.79,4.05]	[-0.33,1.35]	[-0.03,0.04]	[0.01,0.12]	[-0.22,25.45]	[-5.20,4.74]
$AD25_{t-1}$	0.000	0.035	0.002	-0.001	0.000	0.766	0.005
	(0,7)	(35,0)	(5,0)	(0,10)	(4,2)	(35,0)	(2,1)
	[0.00,0.00]	[0.02,0.07]	[-0.01,0.01]	[0.00,0.00]	[-0.01,0.00]	[0.56,0.92]	[-0.01,0.03]
$AD25_{t-2}$	0.000	-0.005	0.001	0.000	0.000	-0.013	0.005
	(2,4)	(4,13)	(3,0)	(2,3)	(3,3)	(10,14)	(2,2)
	[0.00,0.00]	[-0.03,0.02]	[-0.01,0.01]	[0.00,0.00]	[0.00,0.00]	[-0.12,0.09]	[-0.04,0.04]
$BD25_{t-1}$	0.000	0.003	0.029	0.000	-0.001	0.004	0.741
	(1,6)	(1,0)	(33,0)	(1,1)	(2,9)	(5,0)	(35,0)
	[0.00,0.00]	[-0.01,0.02]	[0.00,0.05]	[0.00,0.00]	[-0.01,0.00]	[-0.03,0.05]	[0.44,0.91]
$BD25_{t-2}$	-0.001	-0.001	0.001	0.000	0.000	0.001	0.016
	(0,4)	(0,1)	(6,5)	(2,2)	(0,6)	(1,2)	(14,5)
	[0.00,0.00]	[-0.01,0.01]	[-0.02,0.03]	[0.00,0.00]	[0.00,0.00]	[-0.05,0.03]	[-0.09,0.12]
Adj R^2	0.438	0.272	0.291	0.581	0.567	0.625	0.614

the 35 stocks. The title of each graph shows for which variables the IRF is drawn.

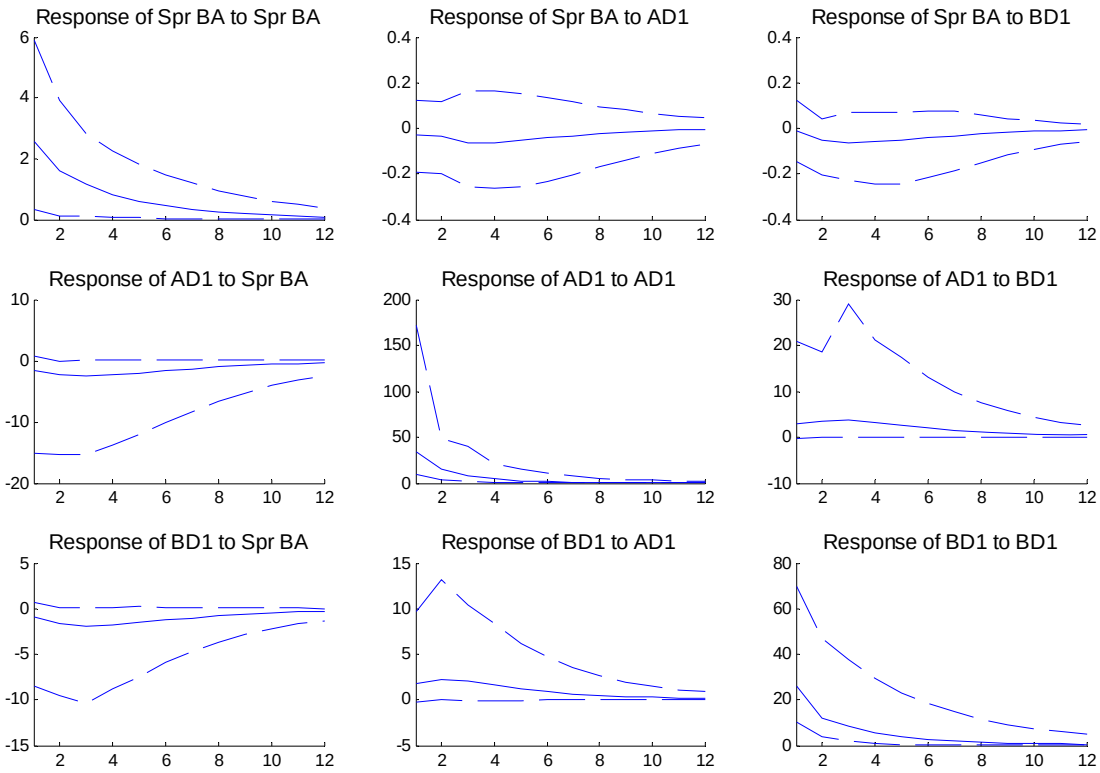
Before presenting the results, it is important to stress (again) the interpretation of the variables. These are adjusted for intraday seasonality and should therefore be interpreted relative to their intraday pattern. Suppose e.g. that a certain shock induces an initial change in the bid-ask spread of 3 ticks. This means that in this case the spread becomes three ticks larger than would be expected on basis of its time-of day patterns alone. Hence, it should not be understood as the spread being equal to three ticks. In the subsections below, a phrase such as "an increase in variable ..." should therefore be read as "an increase in variable ... relative to what is expected on the basis of its intraday pattern". We do not always repeat explicitly this qualification.

6.2 Model 1: Best Limits

The IRFs of Model 1, which only considers the best limits, are drawn in Figure 2. Not surprisingly, the results show that each variable responds most strongly to an own shock. However, we do find clear evidence for interactions, both between spread and depth, and between ask and bid side of the market. A shock in the bid-ask spread (i.e. an increase in the inside spread of one standard deviation) has a negative impact both on $AD1$ and $BD1$. This is in contrast with the results obtained in Degryse et al. (2005) in event time. There, after a shock increasing the spread, depth at the best prices increased. In other words, immediately after an increase in the bid-ask spread, depth increases (Degryse et al. use event time). However, when the average spread over a 15-minute interval increases, depth decreases or in other words, both dimensions of liquidity, spread and depth, deteriorate. This contradiction may be explained as follows. Suppose the spread increases within an interval. If some time after the shock small undercutting of the best prices takes place, the spread on average will remain higher but average depth over the interval decreases. Moreover, as can be seen in the figure, from an economic point of view, the decrease in depth is small but not negligible. If one compares the immediate impact of a spread shock on depth, with the immediate impact of shock to depth on the own or other side, this impact is much smaller than in the latter case.

Further, when a shock occurs that increases the depth at the ask or bid side, the spread decreases slightly on average, but results are mixed across stocks (as can be seen from the 5% and 95% percentiles). When depth at the best prices is higher, the execution probability of an additional limit order that joins the queue is rather small and traders will prefer to undercut the existing prices to obtain execution.

Finally, we find that the impact of a shock is stronger on the ask side than on the bid side. A shock in Spr_BA has a larger impact on $AD1$ than on $BD1$. Moreover, the impact of a shock in $BD1$ on $AD1$, is bigger than the other way around. Also, the response of $AD1$ to its own shock is larger than is the case for $BD1$.



Note: This figure presents the IRFs, computed on basis of VAR-model 1. The x-axis shows the number of 15-minute intervals after the shock (1..12). The title of each graph gives the IRF that is computed. A description of the variables and their notation is presented in Table 1.

Figure 2: **Model 1: Impulse Response Functions**

6.3 Model 2: Limit Order Book Slopes

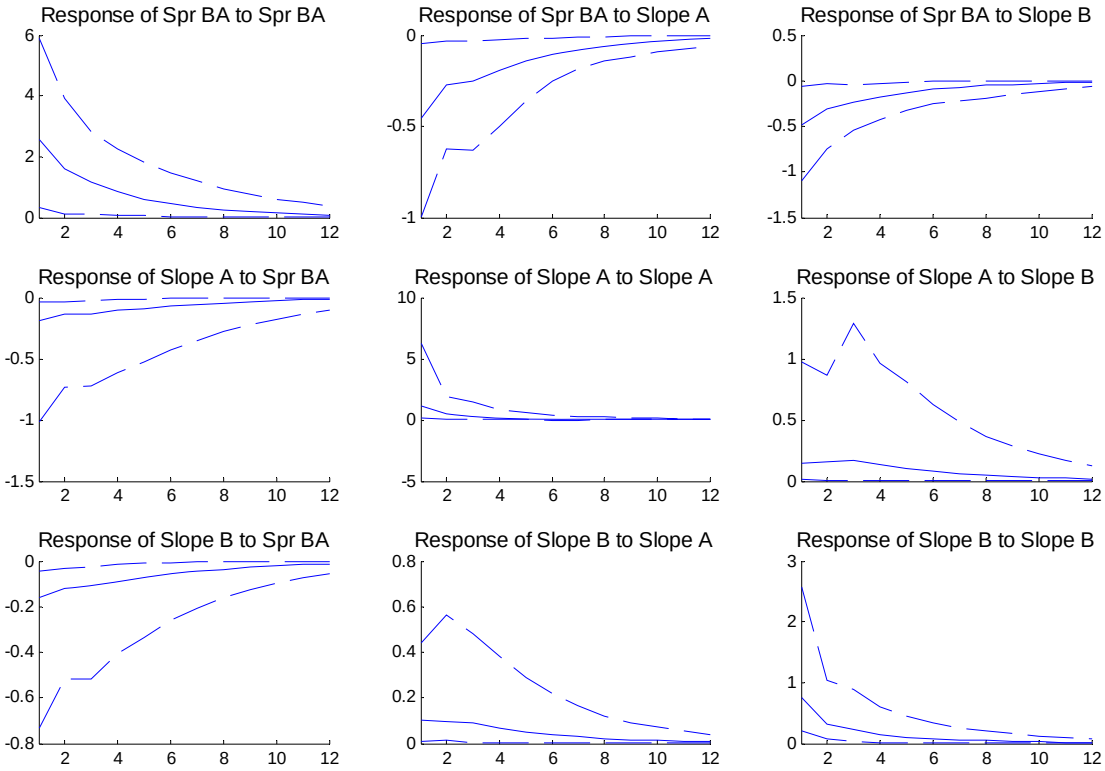
Figure 3 presents the IRFs for VAR-model 2, modeling limit order book slopes. We find clear evidence of the existence of a relationship between the bid-ask spread and slopes, and between slopes on the ask side and bid side. First, the response of the spread to shocks on ask or bid slopes is negative. In other words, if the slope on bid or ask side increases, the spread decreases. The reasoning is as follows. If the slope is higher, either the prices on ask side are closer together (for the bid side, the reasoning is identical), or the depth provided at given prices is larger. For a trader, it is then either less easy to undercut a price beyond the best ask (i.e. to specify a price in between the first and fifth best), or less attractive to join the queue at a particular price since the execution probability is relatively small if their are a number of shares before her in the queue. Therefore, in order for such trader to increase the execution probability is to undercut the best prices, as a result of which the bid-ask spread decreases.

The response of the slopes to a shock in the spread is negative, i.e. when the difference between the best ask and bid increases, the slope of the book at ask and bid side becomes smaller. Confirming the results of the previous model, in general, the response of the ask-slope tends to be larger than the response of the bid-slope.

Finally, we also find evidence for co-movement between ask and bid sides since a positive shock to the ask slope goes together with a positive response of the bid slope and vice versa.

Computed (not reported) confidence intervals around the shocks indicate that the effects are econometrically significant for a large majority of the stocks. Moreover, and probably more important, the effects are also economically significant. This can be seen by comparing the magnitude of the response of a variable to an own shock and a shock in another variable. A shock of one standard deviation to the slope on ask or bid side induces the spread to be initially about 0.5 ticks lower than expected during the next interval of the trading day. If one compares this 0.5 ticks with the response of the spread to a shock in the spread itself of just above 2 ticks, it is clear that 0.5 ticks is not a negligible amount. A similar ratio between both remains when the effect of the respective shocks die out. Also the response of the bid or ask slope to a shock in the spread or to a shock to the slope at the other side of the markets cannot be considered as immaterial, since it on average amounts to about 20% of the response to an own shock.

Summarizing, both econometrically and economically, our results point to the possibility that several dimensions of liquidity can deteriorate at the same time. The evolution of the limit order book slopes moreover indicate that this conclusion is not restricted to the best limits, but extends to the rest of the book as well. However, slopes do not allow to disentangle price and depth effects in the book beyond the best limits. Therefore, we now turn to the results of the IRFs of VAR-model 3.



Note: This figure presents the IRFs, computed on basis of VAR-model 2. The x-axis shows the number of 15-minute intervals after the shock (1..12). The title of each graph gives the IRF that is computed. A description of the variables and their notation is presented in Table 1.

Figure 3: **Model 2: Impulse Response Functions**

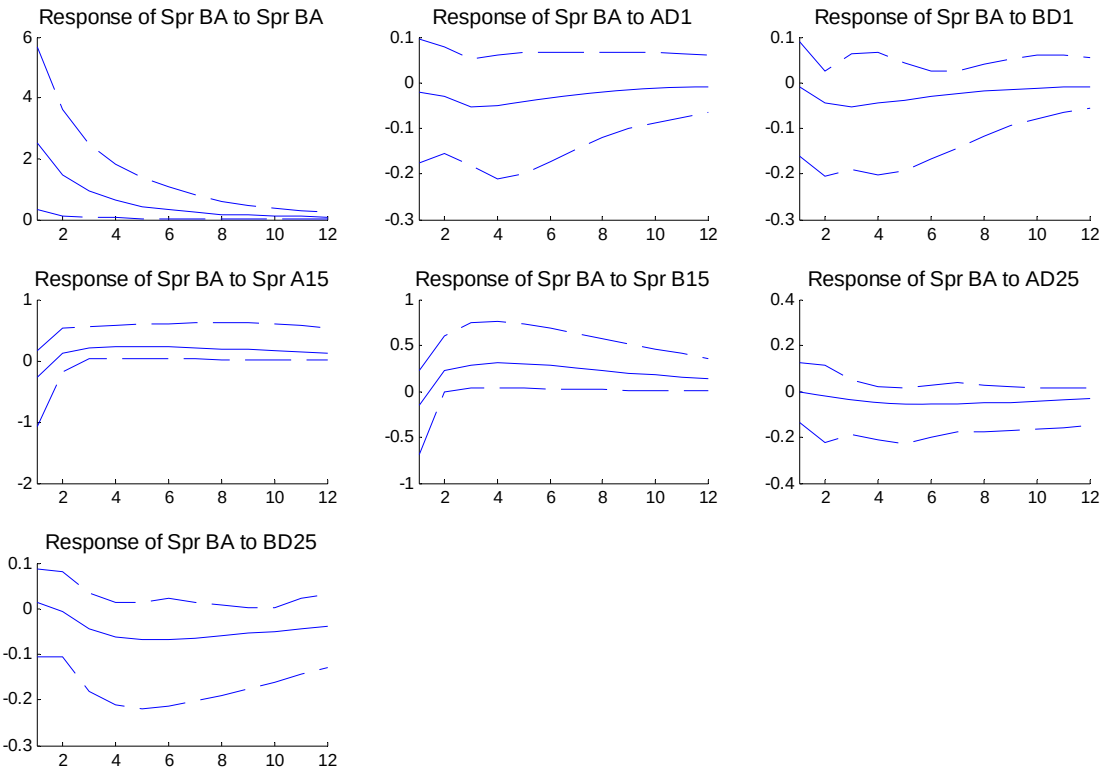
6.4 Model 3: Beyond the Best Limits

The impulse response functions based on Model 3 are shown in Figure 4. We discuss in turn the consequences of a shock in each of the endogenous variables, starting with a shock of one standard deviation in the spread. The graphs show that all variables respond significantly to a shock in the bid-ask spread. The spread itself declines quickly after an increase. All measures of depth decrease after an increase in the spread. Finally, the difference between the best and fifth bid and ask prices first reacts negatively, meaning that the subsequent prices in the book are closer together. This is beneficial for liquidity. Although the inside spread increase, subsequent prices are closer to the best ones, such that an order that walks up in the book still has a relatively small impact. After some periods, as the bid-ask spread declines again, prices become more dispersed, i.e. the difference between the first and fifth best price increases.

Secondly, we turn to a shock in the depth at the best ask. After such shock, the spread declines slightly on average, in line with theoretical predictions. Since the queue of outstanding limit orders is longer if depth is higher, traders will prefer to undercut best prices in order to obtain faster and more certain execution (see e.g. Parlour (1998) or Rosu (2004)). However, results vary across stocks as can be seen from the 5% and 95% percentiles in the graph. Interesting is that the depth beyond the best ask increases as well after an increase in $AD1$. So when depth at the best limits becomes larger, also depth further in the book benefits from this. Moreover, also depth at the best bid rises. Finally, the other spread measures (Spr_A15 and Spr_B15) do not respond significantly to a shock in $AD1$ for a majority of the stocks. A shock in the depth at the best bid has symmetric consequences.

Thirdly, we consider a shock in Spr_A15. In general, two variables show a significant response for a majority of the stocks. The first one is the bid-ask spread. When the difference between the best price and the fifth ask price rises, the spread first declines, but from the second period onwards, it starts increasing again. Secondly, also the depth at prices further in the book ($AD25$) declines. This results is similar to the one obtained for the inside bid-ask spread and depth at the best prices. Not only are prices further away from each other, but also the depth in the book decreases. A possible consequence is that larger order thus have a larger price impact and traders submitting such order face higher trading costs. A shock in Spr_B15 has similar affects on the spread and on $BD25$.

Finally, we consider a shock to depth in the book beyond the best limits, i.e. $AD25$ and $BD25$ respectively. When depth behind the best limits enhances, this has also a beneficial effect on the depth at the best prices. Moreover, also the bid-ask spread declines, implying an additional improvement of liquidity.



Note: This figure presents the IRFs, computed on basis of VAR-model 3. The x-axis shows the number of 15-minute intervals after the shock (1..12). The title of each graph gives the IRF that is computed. A description of the variables and their notation is presented in Table 1.

Figure 4: **Model 3: Impulse Response Functions**

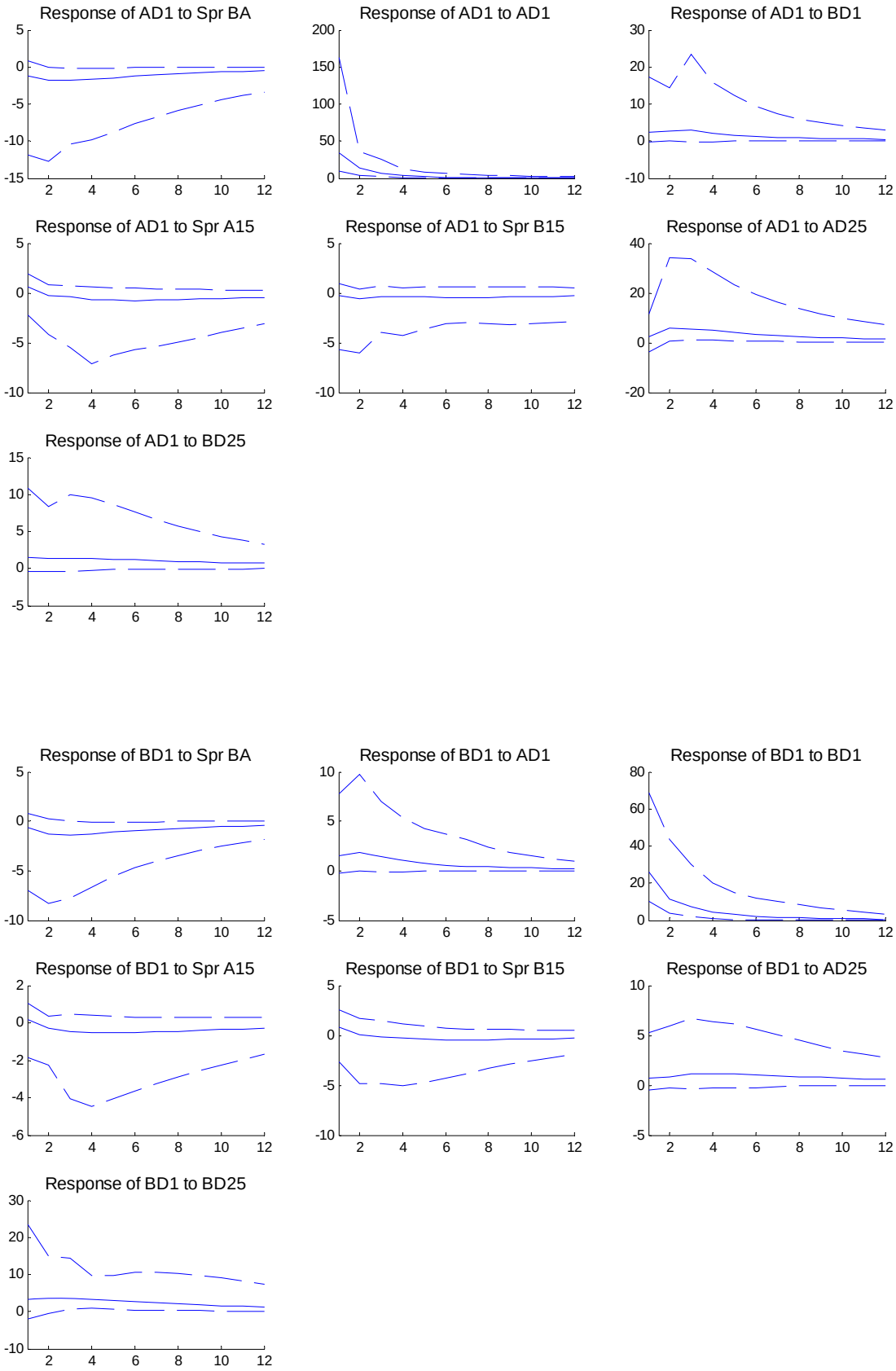


Figure 4 (continued)

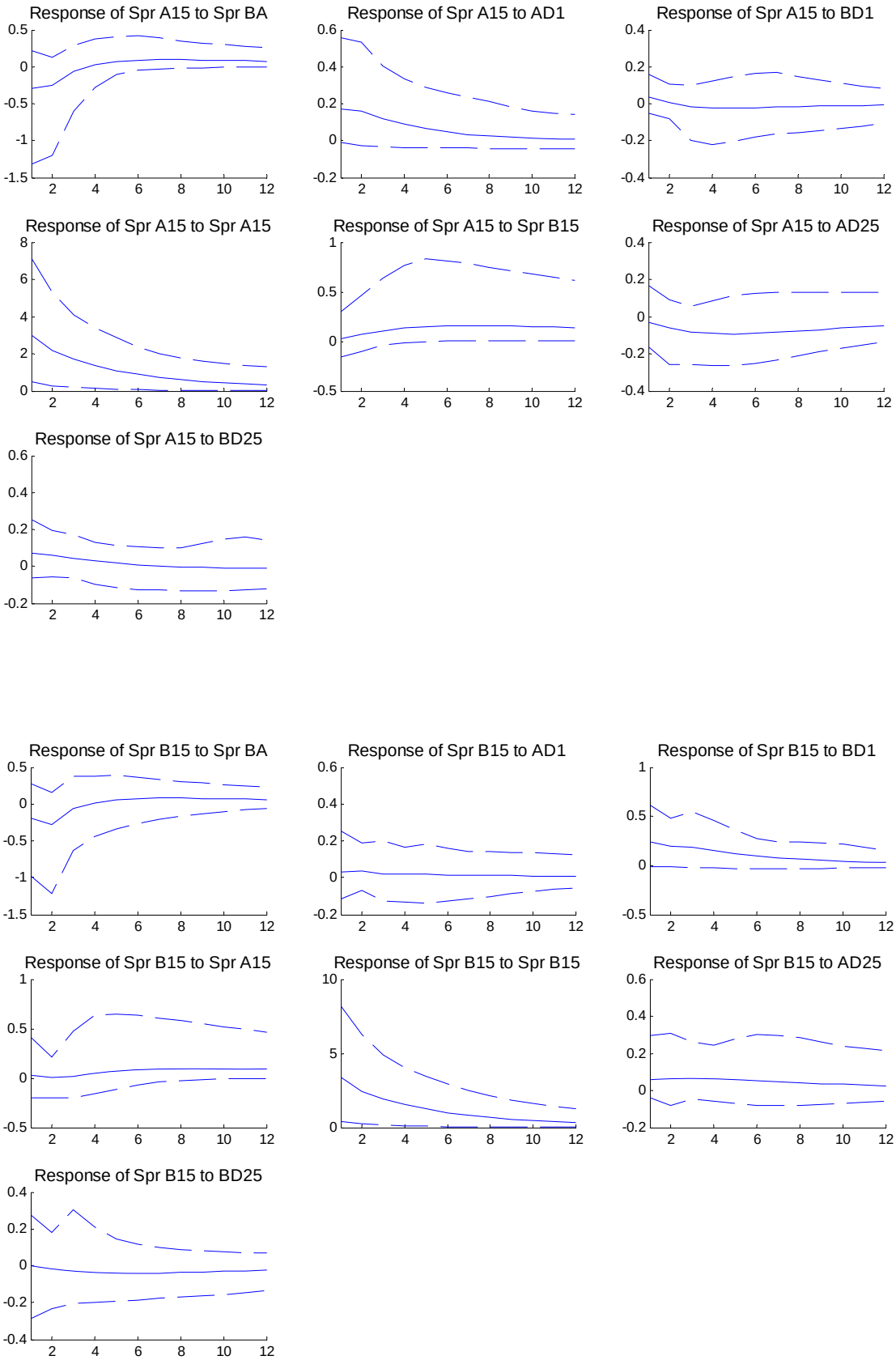


Figure 4 (continued)

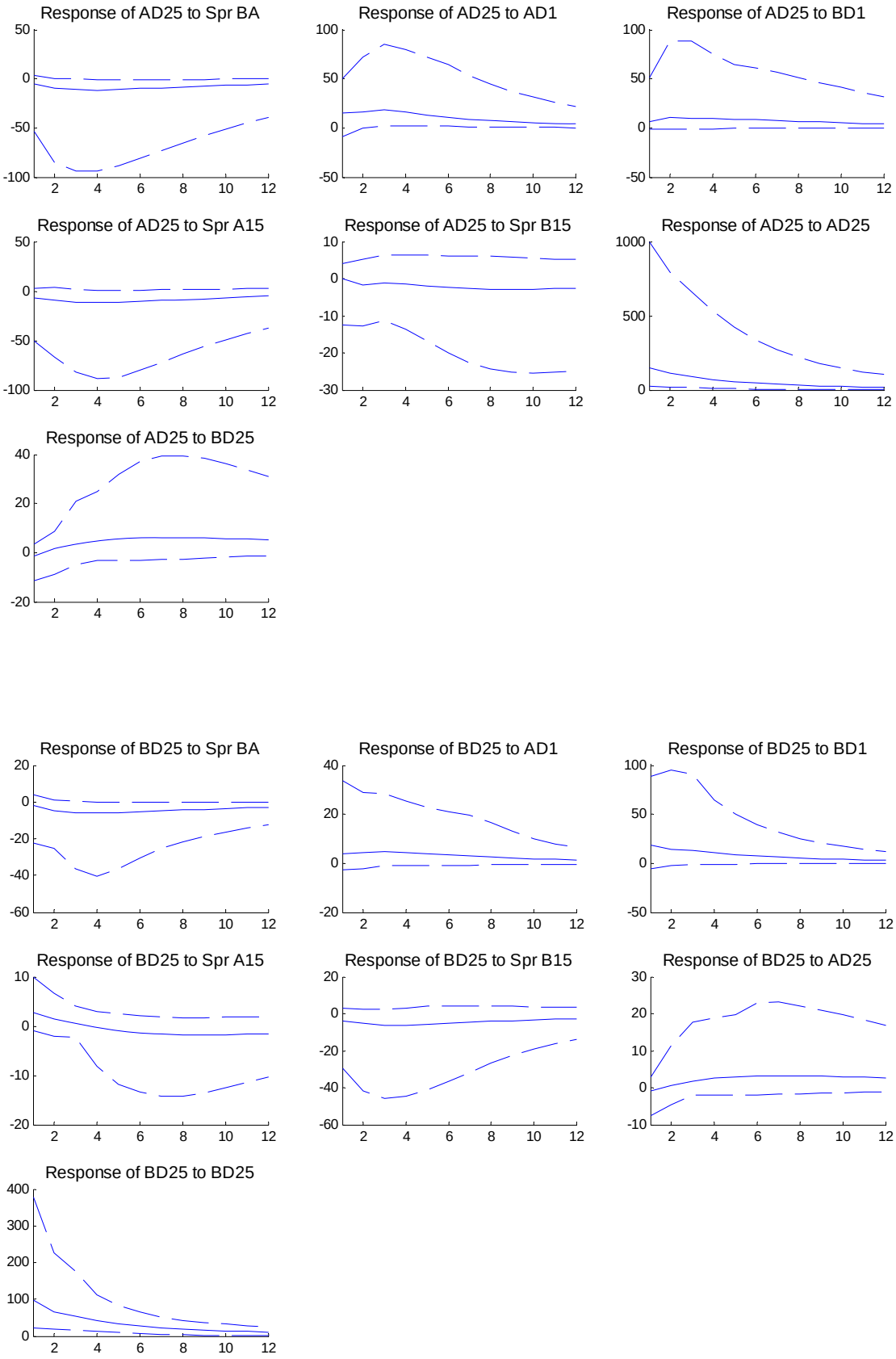


Figure 4 (continued)

7 Robustness

To check for the robustness of the results obtained above, we also estimated a number of alternative models. First, we changed the three VAR-models, used so far. Adding more lags to these models does not change the estimation results; in fact, the additional lags often are not significant. We also added a number of *exogenous variables*, characterizing the order flow during an interval. More specifically, we added the average duration between updates of the book, as well as the average number of updates of the book of a given aggressiveness type that occurred in interval t. The same classification schema as in Biais, Hillion and Spatt (1995) was used to classify updates. Neither of these changes to the three models has an impact on the results presented above.

Secondly, we used different definitions for a number of variables. We used the relative spread, defined as the ratio of the difference between the bid-ask spread and the midprice. We also redid the estimations with depth expressed in number of shares instead of monetized depth (this is the number of shares times the price). Also, we computed weighted slopes using a simple linear scheme:

$$\begin{aligned} Slope_A_{t,\tau} &= \left[\frac{5}{15} \frac{ADC1_{t,\tau}}{\frac{AP1_{t,\tau}}{M_{t,\tau}} - 1} + \frac{4}{15} \frac{\frac{ADC2_{t,\tau} - ADC1_{t,\tau}}{ADC1_{t,\tau}}}{\frac{AP2_{t,\tau} - AP1_{t,\tau}}{AP1_{t,\tau}}} + \frac{3}{15} \frac{\frac{ADC3_{t,\tau} - ADC2_{t,\tau}}{ADC2_{t,\tau}}}{\frac{AP3_{t,\tau} - AP2_{t,\tau}}{AP2_{t,\tau}}} \right. \\ &\quad \left. + \frac{2}{15} \frac{\frac{ADC4_{t,\tau} - ADC3_{t,\tau}}{ADC3_{t,\tau}}}{\frac{AP4_{t,\tau} - AP3_{t,\tau}}{AP4_{t,\tau}}} + \frac{1}{15} \frac{\frac{ADC5_{t,\tau} - ADC4_{t,\tau}}{ADC4_{t,\tau}}}{\frac{AP5_{t,\tau} - AP4_{t,\tau}}{AP5_{t,\tau}}} \right] \end{aligned} \tag{7}$$

instead of using equation (4). Neither of these alternative variables caused our results to change.

Thirdly, we changed the sampling frequency to 5-minute intervals instead of 15-minute intervals. This does not induce any qualitative change to the results found above.

Fourthly, as an alternative to the methodology of Pesaran and Shin (1998) for computing the IRFs, we applied the Choleski decomposition with various orderings. The figures are in general very similar to the one in the text above.

Finally, we estimated a structural VAR-model, imposing similar identification restrictions as in Coppejans, Domowitz and Madhavan (2004). This does not affect the results discussed above. More specifically, Coppejans, Domowitz and Madhavan (2004) start from the following structural VAR-model:

$$\Psi y_t = \sum_{l=1}^{L} A_l y_{t-l} + u_t$$

where Ψ captures contemporaneous effects. Their vector of endogenous variables is specified as $y_t = \{BD_t, AD_t, \Delta m_t\}$, where Δm_t measures the return on the midquote, and AD_t (BD_t) is the depth at the ask (bid) side, six ticks away from the best ask (bid). They include one lag in the estimation. To identify the system, several restrictions are imposed. The variance-covariance matrix of u_t is assumed to be block diagonal. In particular, returns are assumed to be uncorrelated, while shocks to bid and ask side may be contemporaneously correlated. Ψ is specified as:

$$\Psi = \begin{bmatrix} 1 & 0 & -\psi_{13} \\ 0 & 1 & -\psi_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

An additional restriction is imposed on A_1 (the coefficient matrix) in which the coefficient on lagged returns is assumed to be zero. The other elements of A_1 are unrestricted. Economically, this specifications means that neither depth at the bid nor ask side contemporaneously affect returns but only with a lag. On the other hand there may be a contemporaneous and/or effect of returns on depth at both sides of the market. Further, depth on one side of the market does not contemporaneously affect depth on the other side, but only with a lag. However, shocks to depth on the bid and ask sides of the market are correlated.

As a robustness check, we estimated a similar model, but defined the vector of endogenous variables as:

$$y_t = \{Spr_BA_t, AD1_t, BD1_t\}$$

This is the same vector as in Model 1. We also computed impulse responses. These are not different from the ones obtained in Model 1 when using the methodology of Pesaran and Shin (1998). It is however not straightforward to extend this method to VAR-model 2, given the absence of contribution modelling slopes of LOBs. In Model 3, it is again difficult to base restrictions on theory. In this case, a larger number of restriction will be needed in order to be able to estimate the model. Nor theory, nor empirical work is however able to guide a choice of restrictions. For this reason, we opted for the approach of Pesaran and Shin (1998), which avoids this issue.

8 Summary and Conclusion

In this paper, we provided an analysis of liquidity in an order driven market. More specifically, we investigated not only the best limits but also the LOB beyond. We presented two different approaches. First, we computed limit order book slopes, which parsimoniously summarize liquidity provided by the book. Secondly, we measured how

far subsequent prices in the book are away from the best prices (lowest ask and highest bid) and the depth provided at these prices. In both approaches, we separately modeled bid and ask sides of the market. We also specified a model which only include the (inside) bid-ask spread and the depth at the best prices. We estimated different VAR-models and computed impulse response functions after various liquidity shocks, measured as a one standard deviation shock of a variable. The results of these IRFs are summarized in Table 6. The different panels in this table correspond to the various VAR-models in this paper (see Section 2).

From Panel A, it can be seen that a shock that increases the bid-ask spread, decreases depth, meaning that both dimensions of liquidity deteriorate. This is in contrast with the conclusions reached in the literature where often only one dimension of liquidity deteriorates while others improve. We also find that shocks that increase depth, lower the bid-ask spread. Both facts demonstrate the importance of allowing for interactions between dimensions of liquidity. Moreover, a positive shock on depth at one side of the market, also increases depth at the other side.

Panels B and C include also the LOB beyond the best limit in the analysis. They provide interesting additional insights, as compared to a model that only considers the best limits. Panel B shows that after a liquidity shock that decreases liquidity at the best prices, also liquidity in the remainder of the book, as measured by limit order book slopes, deteriorates. These slopes summarize liquidity provided by the book (both at and beyond the best limits). After an increase in the spread, the slopes at bid and ask side decrease, pointing at a deterioration of liquidity. On the other hand, a steeper LOB (a shock to slopes) implies a smaller spread.

Panel C shows that after a liquidity shock that decreases liquidity at the best prices, also liquidity in the remainder of the book deteriorates. More specifically, a shock increasing the bid-ask spread first decreases the distances between the first and fifth price in the book, but after some periods an increase is observed. On the other hand, depth beyond the best limits remains relatively unaffected. Moreover, a shock that increases the distance between the first and fifth prices in the book (either at ask or bid side) first causes a decrease in the bid-ask spread, but after some periods, this spread increases. Again depth is less affected but if so, in general depth decreases as well. On the other hand, shocks that increase depth at the best prices also tend to increase depth in the remainder of the book. Moreover, if depth beyond the best prices increases, also depth at the best prices does so. This holds both for bid and ask sides. Furthermore, a positive shock to depth on the ask side also increases depth on the bid side and vice versa. The results in panel B and C clearly, demonstrate a relation between liquidity at the best limits and liquidity in the rest of the LOB.

Concluding, one can say that these result show a somewhat less sunny picture of

liquidity provision in a limit order market than is often found in the literature. In particular the fact that different dimensions of liquidity deteriorate at the same time is possibly worrying. Nevertheless, most of the impact of a shock is realized within 1 to 1.5 hours.

One reason for the contradiction in the results of the current paper and the literature, can be that we used more aggregated data (15-minute intervals) instead of result in order time. A second explanation may be provided by the possibility that stale limit orders are present in the book. Traders do not monitor the market continuously since this is costly. Immediately after a liquidity shock, depth at the best prices may e.g. increase, but this could mean that such stale limit orders are used. After this, or after other stale orders have been cancelled, the results in this paper may point to the fact that less new liquidity is provided afterwards. Another important difference is the sample period. The SIBE data come from after the bursting of the bubble in asset markets. It is well possible that for the latter period, less traders where left in the market. Moreover, volatility may have been higher in the second half of 2000, and Beltran, Durré and Giot (2004) demonstrate the existence of differences in liquidity between low and high volatility periods.

These possible explanations comprise an interesting road for future research of liquidity provision in limit order markets. A first intriguing topic that certainly deserves future research is the comparison of liquidity in bull and bear markets. A second is the importance of stale limit orders, which are present in the book for a longer time, for the provision liquidity immediately after a shock. As a final point, we want to mention that our sample only contains large, frequently traded stocks. An interesting extension would therefore be to investigate small stocks and assess whether in this case, limit orders are able to provide sufficient liquidity.

Table 6: **Impulse Response Functions: Summary**

Note: This table summarizes the results of the IRFs of the different VAR-models. The rows represent that variables that are shocked by 1 standard deviation, the columns give the responses of the variables. A "+" means a positive response, "-" a positive response and "+/0" ("-/0")a positive (negative) response but only significant for about half of the stocks.

Panel A: Model 1: Best limits

		Shock		
		Spr_BA	AD1	BD1
Response	Spr_BA	+	0/-	0/-
	AD1	-	+	+
	BD1	-	+	+

Panel B: Model 2: Limit order book slopes

		Shock		
		Spr_BA	Slope_A	Slope_B
Response	Spr_BA	+	-	-
	Slope_A	-	+	+
	Slope_B	-	+	+

Panel C: Model 3: Beyond the best limits

		Shock						
		Spr_BA	AD1	BD1	Spr_A15	Spr_B15	AD25	BD25
Response	Spr_BA	+	0/-	0/-	- then +	- then +	0/-	0/-
	AD1	-	+	+	-	-	+	+
	BD1	-	+	+	-	-	+	+
	Spr_A15	- then +	+	0/-	+	+	0/-	0/+
	Spr_B15	- then 0/+	0/+	+	0 then +	+	0/+	-
	AD25	-	+	+	-	0/-	+	0/+ then +
	BD25	-	+	+	0/+ then 0/-	-	0/+ then +	+

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