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**Modeling Electricity Prices:  
From the State of the Art to a Draft  
of a New Proposal**

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IEM – International Energy Markets

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# **Modeling Electricity Prices: From the State of the Art to a Draft of a New Proposal**

## **Summary**

In the last decades a liberalization of the electric market has started; prices are now determined on the basis of contracts on regular markets and their behaviour is mainly driven by usual supply and demand forces. A large body of literature has been developed in order to analyze and forecast their evolution: it includes works with different aims and methodologies depending on the temporal horizon being studied. In this survey we depict the actual state of the art focusing only on the recent papers oriented to the determination of trends in electricity spot prices and to the forecast of these prices in the short run. Structural methods of analysis, which result appropriate for the determination of forward and future values are left behind. Studies have been divided into three broad classes: Autoregressive models, Regime switching models, Volatility models. Six fundamental points arise: the peculiarities of electricity market, the complex statistical properties of prices, the lack of economic foundations of statistical models used for price analysis, the primacy of unequational approaches, the crucial role played by demand and supply in prices determination, the lack of clearcut evidence in favour of a specific framework of analysis. To take into account the previous stylized issues, we propose the adoption of a methodological framework not yet used to model and forecast electricity prices: a time varying parameters Dynamic Factor Model (DFM). Such an eclectic approach, introduced in the late '70s for macroeconomic analysis, enables the identification of the unobservable dynamics of demand and supply driving electricity prices, the coexistence of short term and long term determinants, the creation of forecasts on future trends. Moreover, we have the possibility of simulating the impact that mismatches between demand and supply have over the price variable. This way it is possible to evaluate whether congestions in the network (eventually leading black out phenomena) trigger price reactions that can be considered as warning mechanisms.

**Keywords:** Electricity Spot Prices, Autoregressive Models, GARCH Models, Regime Switching Models, Dynamic Factor Models

**JEL Classification:** C2, C3, Q4

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# **1. Introduction. Some stylized facts on the electric market and the electricity prices**

In the last decades, in order to improve efficiency and reduce electricity prices, following numerous European directives, a progressive liberalization of the electric market has started. This process, which is quite slow due to economies of scale, entry barriers and very high fixed costs faced by those who intend to operate in the energy markets, is in continuous development in some countries, whereas it is already completed in others. Electricity prices will then be determined on the basis of contracts on regular markets, where there is no possibility for arbitrage. In these markets supply will increase or decrease to meet the demand, whose curve results in being inelastic, therefore not much sensitive to price variations.

The large body of literature on electricity prices includes studies with different aims and methodologies depending on the temporal horizon being studied. In the long run the study of the behaviour of electricity spot prices is important for profitability analysis and for power planning, whereas in the medium run it is typically used to obtain a forecast distribution in order to price derivative contracts. The evaluation of derivatives is made on the basis of spot prices, meaning that the price is determined by the market. In this survey we concentrate on those studies whose focus is the determination of trends in electricity spot prices and the forecast of these prices within the short run (day/week-ahead).

Electricity is a particular commodity, characterised by a high variability; this is mainly due to the fact that electric energy cannot be stored, unless through costly and economically unsustainable methods. Only water reserves can be considered as a substitute method to manage the creation of electricity. From the results of numerous studies it does emerge that in Scandinavian countries or in the United States, in which these reserves are abundant, electricity prices show lower peaks due to the possibility of greater flexibility in the creation phase. Therefore, electricity has to be considered as an instantaneous consumption good. A second element capable of influencing prices is the fact that transmission networks are never perfect. Price variation among the different areas occur due to transmission, maintenance and plant costs. Possible overloads and potential faults or technical errors, that could in extreme cases lead to the system blackout, must then be considered in addition to these network problems.

In such a complex framework, the link between price and consumption is extremely difficult to analyse. Consumption, although having a clearly less volatile trend compared to spot prices, presents the same cyclical behaviour. We can therefore state that demand elasticity is very low, but prices are strongly influenced by the level of consumption. High levels of consumption are in fact the determinant of peaks in prices. The increase in demand determines the use of more expensive energetic resources in the production of electricity. In other words, the growth of consumption and therefore of volumes to be produced increases the marginal costs of production, which will rise exponentially depending on the use of nuclear, hydrogen, coal, oil or gas (see Figure 1).

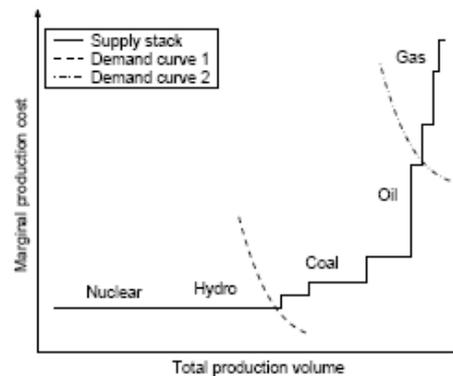


Figure 1. Marginal costs of production

Price cyclicality and electricity demand represent a complex issue.

First of all, electric markets exhibit three different types of seasonality. The first is linked to the greater use of artificial light and heating in the winter, and to the growing use of air conditioning in the summer. The second type of seasonality is weekly and is due to the changes in consumption among weekdays and weekends. Finally, we observe an intra-daily periodicity, which refers to variations between day and night and during the different stages of the day, in which generally we can identify two hot spots. Moreover, it is crucial to take into account that habits and climate conditions change among different countries. Seasonality needs therefore to be continuously focused on each market that has to be analysed. Furthermore, there are other relevant factors of distortion such as extreme temperatures, environmental disasters, particular social events and technical problems previously mentioned, as for example faults in generators.

The combination of the characteristics of the electricity market and the shift from regulated prices to market-determined prices has resulted in a significant increase of electricity price volatility,

exemplified by occasional spikes. In fact, electricity spot prices show an extremely high daily average volatility, which varies between 10% and 50%, depending on the markets considered and on price levels, whereas oil and gas volatilities are 3% and 5%, respectively. The search of the best method to model and explain the trend of spot prices, in order to insure producers and consumers from sudden increases, has become in the last years a very relevant issue for the academic world. However, despite the large number of papers published on this topic, there is no clear empirical evidence supporting a specific theoretical model.

The primary goal of this work is to propose a review of the economic literature on empirical electricity spot price analysis. Attention will be drawn on the methodological aspects, mainly economic and statistical, for evaluating the model performance based on estimation/forecast errors of spot prices. Moreover, it is worth noting that the available models are mostly for univariate analysis and that empirical studies mainly concentrate on the Nord Pool, that is the most mature power market in Europe. Because market structures and price dynamics differ widely across regions, our review will devote special attention to the methodologies applied in different markets. Finally, structural methods of analysis, which are most appropriate for the determination of forward and futures prices, will not be considered here.

The available studies (see Table 1 for a summary) may be classified in terms of the applied methodology. With this respect, three broad classes emerge:

- Autoregressive models, such as ARMA (AutoRegressive Moving Average), ARX (Autoregressive with exogenous inputs), PAR (Periodic AutoRegressive)
- Jumps and regime switching models, such as ARJ including jumps with Poisson or Normal distribution, TAR (Threshold AutoRegressive) having a non linear mechanism, which shifts prices from a normal regime (mean reverting) to one with high prices, whose threshold is predetermined, MS (Markov Switching) at two or three regimes, whose threshold is represented by an unobservable random variable.

- Volatility models, such as ARCH (AutoRegressive Conditional Heteroskedasticity), GARCH (Generalised ARCH), MGARCH (Multivariate GARCH), suitable for describing volatility in a price heteroskedasticity framework (variance changing with time).

In this section we briefly describe the basic model to explain electricity spot price behaviour for each class.

We start with linear autoregression models (AR), followed by their extensions that allow to incorporate exogenous/fundamental factors (ARX). We introduce the second class, regime-switching models that, by construction, should be well suited for modeling the nonlinear nature of electricity prices. This class includes threshold autoregression time series (TAR/TARX) and Markov models with a latent regime-switching variable (RS). Finally, since the residuals of the linear models typically exhibit heteroskedasticity, we discuss implementations of ARCH and GARCH models.

## 1.1. Autoregressive Models

In the engineering context, the standard model that takes into account the random nature and time correlations of the phenomenon under study is the autoregressive moving average (ARMA) model. It is composed of two parts: the autoregressive component and the moving average one.

The autoregressive (AR) model of order  $p$  can be written as  $AR(p)$  and is defined as

$$P_t = \alpha_0 + \alpha_1 P_{t-1} + \dots + \alpha_{t-p} P_{t-p} + \varepsilon_t$$

where  $P_t$  is a time series of electricity price,  $\alpha_0$  is a constant and  $\varepsilon_t$  are the error term, generally assumed to be independent identically-distributed random variables (i.i.d.) sampled from a normal

distribution  $\varepsilon \approx N(0, \sigma^2)$  and  $E(\varepsilon_t) = 0, \text{Var}(\varepsilon_t) = \sigma^2$ . The parameters  $\alpha_1, \alpha_2, \dots, \alpha_p$  are called the *AR coefficients*. The name “autoregressive” comes from the fact that  $P_t$  is regressed on its lagged values.

The *MA* models represent time series that are generated by passing the white noise through a non recursive linear filter. The notation  $MA(q)$  refers to the moving average model of order  $q$

$$P_t = \varepsilon_t + \theta_1 \varepsilon_{t-1} + \dots + \theta_q \varepsilon_{t-q}$$

A model which depends only on the previous values of itself is called an autoregressive model *AR* while a model which depends only on the innovation term is called a moving average model *MA*, and of course a model based on both past values and innovation values is an autoregressive moving average model (*ARMA*).

The notation  $ARMA(p, q)$  refers to the model with  $p$  autoregressive terms and  $q$  moving average terms. In fact this model contains the  $AR(p)$  and  $MA(q)$  models,

$$P_t = \alpha_0 + \sum_{i=1}^p \alpha_i P_{t-i} + \sum_{i=1}^q \theta_i \varepsilon_{t-i} + \varepsilon_t$$

These assumptions may be weakened but doing so will change the properties of the model.

To accurately capture the relationship between prices and loads or weather variables, an *ARMAX* (autoregressive moving average with exogenous variables) model can be used. The notation  $ARMAX(p, q, b)$  refers to the model with  $p$  autoregressive terms,  $q$  moving average terms and  $b$  exogenous inputs terms. This model contains the  $AR(p)$  and  $MA(q)$  models and a linear combination of the last  $b$  terms of a known and external time series  $X_t$ . It is given by

$$P_t = \sum_{i=1}^p \alpha_i P_{t-i} + \sum_{i=1}^q \theta_i \varepsilon_{t-i} + \sum_{i=1}^b \eta_i X_{t-b} + \varepsilon_t$$

A number of variations on *ARMA* models are commonly used in econometrics, when the series are integrated or exhibit seasonalities. If multiple time series are used then the  $P_t$  can be thought of as a vector and a *VARMA* (vector autoregressive moving average) model may be appropriate. *PAR* (periodic autoregressive moving average) model is used in a multivariate context with the presence of seasonality in the data.

## 1.2. Jumps and Regime-Switching Models

The “spiky” character of spot electricity prices suggests that there exists a nonlinear switching mechanism between normal and high-price states or regimes. Two broad classes of these models can be distinguished: those where the regime can be determined by an observable variable and those where the regime is determined by an unobservable, latent variable.

The simplest model of the first class is the Threshold Autoregressive model (*TAR*), which assumes that the regime is specified by the value of an observable variable  $v_t$ , generally equal to the price recorded 24 hour before  $v_t = P_{t-d}$ , relative to a threshold value  $T$ , first determined by assumption or by multi-step optimization procedure

$$\begin{cases} P_t = \sum_{i=1}^p \phi_i P_{t-i} + \varepsilon_t, & v_t > T \\ P_t = \sum_{i=1}^p \delta_i P_{t-i} + \varepsilon_t, & v_t < T \end{cases}$$

To simplify the exposition, we have specified a two-regime model only, however, a generalization to multiregime models is straightforward. It is also possible to include exogenous variables (*TARX*) or taking to consideration more sophisticate process.

Given that we can not be certain that a particular regime or jump has occurred at a particular point in time, we can only assign or estimate the probability of its occurrence. Considering a mean-reverting model, that is in fact an autoregressive process of order 1, the most obvious approach is the addition of a stochastic jump process to the mean reverting process (*ARJ*). The most common specifications for the jump are the normal distribution and a compound normal process. In the latter case, the jumps  $J_t$  are each the sum of independently and identically distributed normals  $Z_t$ . The Poisson arrival process for the compound jumps can produce strongly right-skewed jumps.

$$P_t = \varepsilon_t + \alpha_1 P_{t-1} + \sum_{i=0}^{n_t} Z_t \quad \text{with} \quad \begin{cases} Z_t \approx N(\mu, \sigma_Z^2) \\ n_t \approx \text{Poisson}(\lambda) \end{cases}$$

When we let the arrival intensity of the Poisson jumps approach zero, and its multiplication with the expected jump size approach a constant we observe that this model nests a model with normally distributed jumps. Rewriting this in a notation that splits it up in a ‘normal’ process (when there are no jumps) and a spike process (when there is at least one jump), we find that the first state occurs with probability  $q_M = \exp(-\lambda_1)$ , the second with probability  $q_S = 1 - \exp(-\lambda_1)$ . When using this method we consider the jump arrival process as constant through time, whereas in electricity markets we typically observe alternating periods of high and low jump frequency; in fact power prices are time-dependent.

The requirement of stochastic jump arrival probabilities directly leads to regime switching models (*RS*) as natural candidates. In the Markov regime-switching (or simply regime-switching) models, the regime is determined by an unobservable latent variable. The basic (*RS*) model has the following simple specification:

$$P_t = P_t^{R_t}$$

where  $R_t$  is a latent variable representing the regime of the process in time period  $t$ . The price processes  $P_t^{R_t}$ , being linked to each of the regimes  $R_t$ , are assumed to be independent from each other. The distinguishing characteristic is that this latent regime variable is not imposed ex ante like the probability of jump (*ARJ*), but stochastically depends on previously realized price levels.

$$\left\{ \begin{array}{l} \text{Regime } M : P_t = \alpha_2 P_{t-1} + \varepsilon_t, \\ \text{Regime } S : P_t = \alpha_2 P_{t-1} + \sum_{i=1}^{n_t+1} Z_{t,i} + \varepsilon_t, \end{array} \right. \quad \text{with } \begin{cases} Z_t \approx N(\mu_S, \sigma_S^2) \\ n_t \approx \text{Poisson}(\lambda_1) \end{cases}$$

$$\left. \begin{array}{l} \text{Transition matrix } Q : \begin{bmatrix} 1 - q_S & q_S \\ q_M & 1 - q_M \end{bmatrix} \end{array} \right\}$$

At any point in time the price process is either in regime  $M$  (mean reverting) or in regime  $S$  (spike). Contrarily to a stochastic jump model, the probability that a certain state prevails is not constant, but dependent on the previous state, a stochastic entity. The Markov transition matrix  $Q$  contains the probabilities  $q_{M,S}$  of switching from regime  $M$  at time  $t$  to regime  $S$  at time  $t+1$ . Because of the Markov property, the current state  $R_t$  at time  $t$  depends on the past only through the most recent value  $R_{t-1}$ . In practice, the current regime is not directly observable, but determined through an adaptive probabilistic process using Bayesian inference. More precisely, based on the posterior probabilities of the current regime, we can calculate the prior probability of the next regime being of a certain type.

More sophisticated processes can be included in the Markov Switching specification. It is also possible to build a three-regime model that contains a “normal” mean-reverting regime  $M$ , an “up” regime  $U$  and a “down” regime  $D$ , and impose constraint on the transition probability. In this case, with three regimes, the Markov transition matrix is a 3x3 matrix.

### 1.3. Volatility Models

Electricity spot prices, present various forms of non-linear dynamics, the crucial one being the strong dependence of the variability of the series on its own past. Some nonlinearities of these series are a non constant conditional variance and, generally, they are characterized by the clustering of large shocks or heteroskedasticity.

Given an autoregressive model (first class) or a simple regression model, the autoregressive conditional heteroskedasticity model  $ARCH(q)$  considers the variance of the current error term  $Var(\varepsilon_t) = \sigma_t^2$  to be a function of the variances of the previous time period's error terms. Specifically, is assumed that  $\varepsilon_t = \sigma_t u_t$ . Now  $\varepsilon_t$  is time dependent and  $u_t$  is assumed to be independent identically-distributed random variables (i.i.d.) sampled from a normal distribution  $u_t \approx N(0,1)$ . The time varying series  $\sigma_t^2$  are modeled by

$$\sigma_t^2 = \beta_0 + \sum_{i=1}^p \beta_i \varepsilon_{t-i}^2$$

If an autoregressive moving average model ( $ARMA$ ) is assumed for the error variance, the model is a generalized autoregressive conditional heteroskedasticity model  $GARCH(p,q)$ , where  $p$  is the order of the  $GARCH$  terms  $\sigma^2$  and  $q$  is the order of the  $ARCH$  terms  $\varepsilon^2$ . The model is given by

$$\sigma_t^2 = \beta_0 + \sum_{i=1}^q \beta_i \varepsilon_{t-i}^2 + \sum_{i=1}^p \gamma_i \sigma_{t-i}^2$$

Although the existence of large numbers of  $GARCH$  specifications like exponential  $GARCH$ , integrated  $GARCH$ , quadratic  $GARCH$ ,  $GARCH$  in mean, threshold  $GARCH$  and so on,  $GARCH(1,1)$  is the most used volatility model for power spot prices.  $GARCH$  model are also used in multivariate context  $MGARCH$  to understand if the volatility of a market leading the volatility of other markets.

**Table 1.a: Recent studies on electricity prices**

Article	Class	Model	Market	Frequency	P. Transformation	Exogenous Variable
Cyriel De Jong (2007)	2, 3	Regime Switching, Mean Reverting, Stochastic Jumps, GARCH Models	Nord Pool, EEX, APX, Powernext, EXAA, OMEL, PJM, NEPOOL.	Hourly	Logarithm	None
Adam Misiorek, Stefan Trueck and Rafal Weron (2006)	1, 2, 3	AR, ARMAX, TAR, TARX, GARCH, Regime Switching Models	CalPX (California Power Exchange)	Hourly	Logarithm	System Load, Temperature, Power Plan Availability
Rafał Weron and Adam Misiorek (2006)	1, 2	AR, ARX, Threshold ARX Models	Nord Pool	Hourly	Logarithm	Temperature
Alvaro Cartea and Marcelo G. Figueroa (2005)	2	Mean Reverting and Jump Diffusion Models	England and Wales Market	Daily	Log Return	None
Rafał Weron, Ingve Simonsen and Piotr Wilman (2003)	2	Mean Reverting and Jump Diffusion Models	Nord Pool	Daily	First Difference	None
Andrew C. Worthington, Adam Kay Spratley and Helen Higgs (2002)	3	M GARCH Model	NEM (Australian National Electricity Market): NSV, QLD, SA, SNO, VIC.	Daily	None	None
Graeme Guthrie and Steen Videbeck (2007)	1	PAR Model, Principal Components	NZEM (New Zealand Electricity Market)	Half Hourly	None	None
Graeme Guthrie and Steen Videbeck (2002)	1	PAR, State Space Models	NZEM (New Zealand Electricity Market)	Half Hourly	None	None
Niels Haldrup and Morten Ø. Nielsen (2006)	2	Regime Switching, Structural Models	Nord Pool	Hourly	Logarithm	Congestion
Niels Haldrup and Morten Ø. Nielsen (2004)	2	Regime Switching model	Nord Pool	Hourly	Logarithm	Congestion

Notes. The articles included in the table are classified according to: author, class (1=autoregressive; 2=jump and regime switching; 3=volatility), model applied, reference market, frequency of data, transformation made on prices and exogenous variables used.

**Table 1.b : Recent studies on electricity prices**

Article	Class	Model	Market	Frequency	P. Transformation	Exogenous Variable
Apostolos Serletis and Akbar Shahmoradi (2006)	3	M GARCH Models	Alberta's Power Market	Hourly	First Difference	Natural Gas Price
Julia Popova (2004)	1	Spatial Error Models	PJM	Hourly	None	System Load, Temperature, Forward
Angel Leon and Antonio Rubia (2002)	3	VAR, OGARCH, MGARCH Models	MEM (Argentina Electricity Market)	Hourly	Three Blocks / day	None
Michel Culot, Valérie Goffin, Steve Lawford , Sébastien de Menten (2005)	2	Men Reverting Jump Diffusion, non parametric Models	APX (Amsterdam Power Exchange)	Daily, Hourly	Logarithm	None
Siem Jan Koopmana, Marius Ooms and M. Angeles Carnero (2005)	3	ARFIMA, G ARCH Models	Nord Pool, EEX, APX, Powernext	Monthly	Logarithm	Hydro Reservoir Levels
Michael Bierbrauer, Stefan Truck and Rafa I Weron (2004)	2	Two and three Regime Switching Models	Nord Pool	Daily	Logarithm	None
Ronald Huisman (2007)	2	Temperature Dependent Regime Switching	APX	Hourly	Logarithm	Temperature
Ronald Huisman and Ronald Mahieu (2001)	2	Regime Jump Model	CalPX, UKPX EEX,APX	Daily	None	None
Abdou Kâ Diongue, Dominique Guégan, Bertrand Vignal (2007)	3	G GARCH	EEX	Hourly	Logarithm	None
Bruno Bosco and Matteo Pelagatti (2006)	3	AR GARCH	IPEX (Italian Power Exchange)	Daily	Mean	None

Notes. The articles included in the table are classified according to: author, class (1=autoregressive; 2=jump and regime switching; 3=volatility), model applied, reference market, frequency of data, transformation made on prices and exogenous variables used.

## 2. Modelling Electricity Spot Prices. What Does the Literature Say?

In the following we will summarize the main features of the literature that addresses the issue of evaluating the performance of different models and methodologies applied to the analysis of spot electricity prices and their short run forecasting. Since our goal is to provide a picture of the current state of the art we will illustrate in detail only a selection of recent papers that appear to be very interesting both from the methodological point of view and also in terms of the empirical evidence they provide. On the basis of the evidence coming from the survey we propose in the next section the adoption of a “new” methodological framework for electricity prices analysis and forecasting.

We refer to the Dynamic Factor Models; they have been introduced in the late ‘70s for macroeconomic analysis but they represent a “new” approach to modelling the electricity market in the sense that they never have been used for this purpose.

### 2.1. General Contributions

An overview of all the candidate models suitable to describe the features of the electricity market is provided by Misiorek, Trueck, and Weron (2006). The aim of their paper is to assess the short-term point and interval forecasting performance of different time series models of the electricity spot market during normal (calm), as well as extremely volatile, periods.

Since the authors want to mimic a typical practitioner *praxis*, adopting a truly real time forecasting approach, they choose as test ground the California power market, that offers freely accessible high quality electricity price and load data; moreover this is a quite interesting market, since it provides the ideal framework for studying those behaviours typically leading to a market crash (really occurred in winter 2000/2001).

After reviewing the most diffuse time series based modeling approaches for electricity spot prices the authors specify a set of competitor models:

- AR/ARX: linear autoregression models eventually incorporating (X components) exogenous/fundamental factors (the system load in particular),

- AR/ARX-GARCH,
- TAR/TARX (non-linear, threshold regime-switching)
- Markov models with a latent regime-switching variable

The time series of hourly system prices, system-wide loads and day-ahead load forecasts was constructed using data obtained from the UCEI institute and the California independent system operator CAISO, for the calibration period July 5, 1999 – April 2, 2000; the period April 3 – December 3, 2000 was used for out-of sample testing.

The empirical evidence is again in favour of regime-switching models, but in their simpler form: TAR/TARX models outperform their linear counterparts, both in point and interval forecasting, but simple ARX models reveal a quite encouraging forecasting performance. On the other side, an additional GARCH component generally decreases point forecasting efficiency so that GARCH-inspired specifications do not outperform the relatively simple ARX approach.

The primacy of TARX models emerges both within the point forecasts framework and in the interval forecasting one; in the latter the non-linear Markov regime-switching model systematically underestimated the range of possible next-day electricity prices and yielded the worst results of all tested models.

In the paper by De Jong (2006) the focus is mainly on the existence of typical occasional spikes that are the main source of the large volatility affecting the electricity spot prices and because of their importance are usually incorporated into appropriate pricing, portfolio, and risk management models. Energy markets seem to suffer a level of uncertainty far larger than other commodity markets. Being electricity not storable, spot prices ultimately depend on local and temporal supply and demand conditions. In fact, on one side, large industrial customers usually can not vary their power demand in response to market prices, whereas on the other most power plants can gear up or gear down generation only with a significant time lag.

This low level of flexibility is the main determinant of occasional extreme price spikes, which revert within hours or days to a their “standard” level.

In the light of this, the investigation on the nature of power spikes in a number of different markets becomes a relevant line of research.

In particular the author makes a comparison among different time-series models aimed at capturing the dynamics of these disruptive spot prices:

- standard mean reverting structure, which is a simple AR(1) model;
- stochastic Poisson jumps model;
- Markov switching regime model with stochastic Poisson jumps;
- Markov switching model with three regimes: “Normal”-mean reverting, Up and Down;
- Markov switching regime model with independent spikes;
- threshold model.

All these models have in common that the spot price (actually a day-ahead price),  $P_t$ , is divided into a predictable component,  $f_t$ , and a stochastic component,  $X_t$ :

$$p_t = \ln P_t = f_t + X_t.$$

The first component,  $f_t$ , accounts for predictable regularities, and typically is a deterministic function of time. The stochastic second component  $X_t$ , that is the log spot price from which predictable trends have been removed, is the more interesting and triggers the most of the specification effort by the author.

All the regime switching models above are used to evaluate whether the price spikes should be treated as abnormal and independent deviations from the ‘normal’ price dynamics or whether they form an integral part of the price process.

The empirical application is referred to six day-ahead markets in Europe (Nord Pool Elspot-Scandinavia, EEX-Germany, APX-Netherlands, Powernext-France, EXAA-Austria, OMEL-Spain) and two in the US (PJM-US and New England Pool-US).

As for the empirical evidence the paper concludes that, although they have a limited parameterization, regime-switching models are able to capture the price dynamics significantly

better than a GARCH(1,1) model, a jump-model and a threshold model in the eight different markets.

The regime-switching model that strongly looks like a traditional jump model yields the best fit on average, but it is worth noting that there exist significant differences among the markets probably due to the different shares of hydro-power in the total supply stack: in fact hydro-power serves as an indirect means to store electricity, which has a dampening effect on spikes.

## **2.2. Autoregressive Models**

Guthrie and Videbeck (2002) develop a new approach to understanding the behavior of high frequency electricity spot prices. Their approach treats electricity delivered at different times of the day as different commodities, while recognizing that these commodities may be traded on a small number of intra-day markets. They first present a detailed analysis of the high frequency dynamics of prices at a key New Zealand node. The analysis, which includes the use of a periodic autoregression model, suggests to consider electricity as multiple commodities, and also reveals intrinsic correlation properties that indicate the existence of distinct intra-day markets. Conventional models cannot adequately capture properties that have important implications for derivative pricing and real options analysis. Guthrie and Videbeck therefore extend the literature by introducing a state-space model of high frequency spot prices that preserves this intra-day market structure.

The authors used a periodic autoregression model which impacts on both derivative pricing and real options analysis. The periodic autoregression model is used to value electricity derivatives with payoffs depending on high frequency spot price dynamics. The PAR's principal limitation is the large number of parameters which need to be estimated.

Rather than develop the PAR model, they pursued an alternative approach involving a state-space model. This is easily motivated from the intra-day market structure, and has the additional advantage of requiring a relatively small number of parameters to be estimated. It divides the day into distinct periods based on the correlation structure.

The analysis revealed that daily time series of the prices of these commodities exhibit heterogeneous behavior. Further, the presence of remarkable structure suggests the existence of a small number of intra-day spot markets for electricity. The data suggest also that the structure of intra-day markets varies between weeks and weekends, and across seasons.

Future research will reveal whether these patterns are stable over time and the extent to which they appear in other electricity spot markets. The authors ignored this seasonality in intra-day market

structure when estimating the state-space model in order to keep their model to manageable proportions. If more efficient means of estimating the state-space model can be found, then this extra-level of detail can be incorporated into dynamic models of spot prices.

The contribution by Popova (2004) focuses on the evolution of electricity prices in deregulated market. The author formulates a model that takes into account the spatial features of a network of a market. The model is applied to equilibrium electricity spot prices of the PJM market. This paper addresses the issue of modelling spot prices, because spot prices are one of the key factors in strategic planning and decision support systems of a majority of market players, and are the underlying instrument of a number of electric power derivatives. The goal of the paper is to propose a model for electricity spot price dynamics that takes into account the key characteristics of electricity price formation in the PJM interconnection such as seasonality, weather-dependence, trading in the day-ahead market and spatial attributes of the distribution system.

The novelty of this approach is the utilization of the spatial feature of the PJM market which is divided into twelve transmission zones. The PJM interconnection's pricing mechanism and price data availability is designed in such a way as to allow considering each zone as a hypothetical generating unit. Both forward and spot prices are reported for each hypothetical producer hourly. This facilitates a high-frequency empirical analysis taking into account spatial characteristics of the interconnection. Consequently, the author assumes that the electricity spot price can be represented as a function of its lagged values, the forward price, weather conditions, and demand, which is equal to load. Popova assumes also that there is a unique price generating process, but the disturbances are spatially correlated due to the grid topology and the omitted variables problem.

An empirical analysis indicates that the problem of unobserved spatial correlation in the network can be modelled by the Spatial Error providing an additional insight about the spot electricity prices in this market. The spatial aspect plays an essential role in electricity prices formation and ignoring the spatial characteristics and the grid topology may cause biased results and vague conclusions. The problem of unobserved spatial correlation in the grid can be modelled by the SEM. Strong spatial correlation is supported by the estimating results as well as by the testing procedure. Though the estimation of the "spatial" parameter is of little interest, it helps to bring out consistent estimates of explanatory variables. Therefore, the more robust estimates and inference can be drawn.

Despite its attractiveness, the Spatial Error Model is not the only method available to model the

electricity prices and derivatives. Future of electricity price modelling may be oriented towards models incorporating finer components and an additional information about the network topology, weather conditions and connections between the PJM zones. The additional information can be utilized either by spatial approach or by other modelling methods.

Weron and Misiorek (2006) assess the short-term forecasting power of different time series models in the Nord Pool electricity spot market. Four five-week periods were selected, which roughly correspond to the months of February, May, August and November. Given this choice, the authors are able to evaluate the performance of the models for all seasons of the year and the large out-of-sample interval allows for a more thorough analysis of the forecasting results when compared to the investigations which are typically used in the literature considering single-week test samples.

The models for electricity spot prices considered by the authors include linear and non-linear autoregressive time series with and without additional fundamental variables. The only exogenous information is the air temperature, since generally this is the most influential weather variable on electricity prices. The models were tested on a time series of hourly system prices and temperatures from the Nordic power market.

Weron and Misiorek evaluate the accuracy of both point and interval predictions; the latter are specifically important for risk management purposes, where one is more interested in predicting intervals for future price movements than simply point estimates. The authors investigate the quality of the predictions, both in terms of the Mean Weekly Error (for point forecasts) and in terms of the nominal coverage of the models with respect to the true coverage (for interval predictions). They find evidence that non-linear models outperform their linear counterparts and that the interval forecasts of all models are overestimated in the relatively non-volatile periods. During relatively calm, periods the AR and spike pre-processed AR (p-AR) models generally yielded better point forecasts than their competitors, with p-AR being slightly better than the pure AR specification. However, during volatile weeks of May 2004 for example, the TAR model was the best. Regarding interval forecasts, they found that the estimated 90% and especially the 99% confidence intervals (CI) of the linear models are clearly too narrow for the volatile period.

Better results are obtained for the TAR model, especially for the 90% CI. However, it predicts slightly too narrow 99% intervals and significantly too wide 50% intervals.

Moreover, the authors found that during relatively calm periods for all models almost all confidence intervals include the actual market clearing price (MCP) value. This is especially true for the 90% and 99% intervals, but even for the 50% CIs deviations from the actual MCP are rarely large enough to exclude the price from the interval. This is in contrast to the results for the California power market, where the TAR model yielded acceptable interval forecasts for the whole test sample. A possible reason for such a behavior could be temporal dependence (or “non-whiteness”) in the model residuals. Whether this is true has yet to be investigated.

The study by Guthrie and Videbeck (2007) shows that some important properties of electricity spot prices cannot be captured by the statistical models, which are commonly used to model financial asset prices. Using more than eight years of half-hourly spot price data from the New Zealand Electricity Market, Guthrie and Videbeck find that the half-hourly trading periods fall naturally into five groups corresponding to the overnight off-peak, the morning peak, daytime off-peak, evening peak, and evening off-peak. The starting point for the analysis is to acknowledge that its non-storability means that electricity traded at a particular time of the day is a distinct commodity, quite different from electricity traded at different times. The prices in different trading periods within each group are highly correlated with each other, yet the correlations between prices in different groups are lower. Financial models, which are currently applied to electricity spot prices, are incapable of capturing their behavior. On the contrary, the authors use a periodic autoregression to model prices, showing that shocks in the peak periods are larger and less persistent than those in off-peak periods, and that they often reappear in the following peak period. In contrast, shocks in the off-peak periods are smaller, more persistent, and die out (perhaps temporarily) during the peak periods.

Guthrie and Videbeck illustrate a new approach to modelling electricity prices, the use of periodic autoregressions, because current approaches cannot capture this behavior either. A simple AR process, which ignores the different behavior of prices in different trading periods, can be calibrated to capture the low persistence evident in peak periods, or the greater persistence in off-peak periods, but not both simultaneously. Nor can it capture the reappearance of shocks later in the day when they first appear. The periodic autoregression used in this paper could be used to value electricity derivatives with payoffs depending on high frequency spot price dynamics. The PAR’s main limitation, however, is the large number of parameters to be estimated. For example, with half-hourly trading periods each of the 48 equations has 48 slope coefficients, in addition to the

coefficients of various dummy variables. However, much of the dynamic structure would remain if only a subset of the lagged prices (for example 1, 2, 47 and 48 lags) is used.

Parsimonious specifications might allow to introduce jumps and other relevant properties into the price process, although this line of research is not investigated in this work.

### **2.3. Jumps and Regime-Switching Models**

After reviewing the stylized facts about power markets, Weron, Simonsen and Wilman (2003) build up a model which takes into account the well-known peculiar statistical properties of electricity spot prices.

The first step of their analysis is to remove the seasonal components of the spot price, which are due to the fluctuations in demand, both at the annual level (due to climate condition) and at the weekly level (troughs over the weekends). The second step is to model the stochastic part of the deseasonalised series with a typical Ornstein-Uhlenbeck process, allowing for mean reversion and a volatility regime driven by a standard Brownian motion.

Naturally, it is the “jumpy” characteristics of prices (after a jump they tend to remain high for several time periods, hours, sometimes even days) which requires a regime-switching model. Working on average daily spot prices from the Nord Pool power exchange since January 1, 1997 until April 25, 2000, the authors propose and fit various models that exhibit mean reversion and assess their performance by comparing simulated and market prices. The models are:

- a two-regime model with Gaussian spikes;
- a two-regime model with lognormal spikes;
- a two-regime model with Pareto spikes;
- a three-regime model as in De Jong (2006): standard, jump upward or downward, reversal jump.

The evidence is in line with De Jong (2006) in supporting the changing regime approach: in fact all the models seem to produce estimates for transition probabilities that can be interpreted according to market behaviour. Simulated price trajectories show high correlation with real price data and the parameters estimates are only slightly biased.

However, a critical point does emerge: in some cases the number of price spikes or extreme events generated by simulation of the estimated models is higher than in real price data. This evidence could be explained in terms of a structural drawback of the two-regime models, which are not able to distinguish a new current spike from the reversion following a past spike. Otherwise, this fact could reveal a kind of hypersensitivity typical of such a model category.

While the number of extreme events is overestimated in all models, their estimated magnitude is smaller than the real one in the normal and lognormal models and greater within the Pareto-based one. To sum up, this exercise seems to provide strong support in favour of the three regime model already pointed out also in De Jong (2006).

A comparison among different regime switching structures for spot prices modelling with reference to the Nordic power market is conducted also by Bierbrauer, Truck, and Weron (2006). The authors analyze and model the logarithm of the deseasonalized average daily spot prices from the Nord Pool power exchange since January 1997 until April 25, 2000 and address the issue of modelling spot electricity prices with different regime switching models. The price behavior of spot electricity prices is modelled by dividing the time series into separate phases or regimes with different underlying processes. A jump in electricity prices is considered as a change to another regime. The switching mechanism is assumed to be governed by a random variable that follows a Markov chain with different possible states. Thus, there exists an unobservable variable in the time series that switches between a certain number of states which themselves are driven by independent stochastic processes. Additionally, there is a probability law that governs the transition from one state to another.

The authors start considering the simplest model with two possible states. The two-regime model distinguishes between a base regime and a spike regime. They assume that base regime is governed by a mean-reverting process and they try different types of distributions for the spike regime. As suggested in the literature, Gaussian and Lognormal distribution are used. Since spikes happen very rarely but they usually are of large magnitude, they consider also heavy-tailed distributions (Pareto) for the spike regime.

Clearly, the variety of regime-switching models is due to the possibility of choosing the number of regimes. Following Huisman and Mahieu (2003), they propose a regime switching model with three

possible regimes. The idea behind their specification differs significantly from the previous two-state models. They identify three possible regimes:

- a regime modeling the “normal” electricity price dynamics;
- an initial jump regime for a sudden increase or decrease in price;
- a regime that describes how prices move back to the normal regime after the initial jump has occurred.

This definition implies that the initial jump regime is immediately followed by the reverting regime and then moves back to the base regime.

While most spikes only last for one day, there are periods where the prices exhibit three or more extreme events in a row, a behaviour that could be considered as consecutive spikes. In contrast to the two-regime models, the three-regime model does not allow for consecutive spikes (or remaining at a different price level for two or more periods after a jump).

Eventually they assess the performance of the models by comparing simulated and market prices.

The main finding is that the models produce estimates for transition probabilities that can be interpreted according to market behavior. Simulated trajectories show close similarities with real price data. However, the number of price spikes or extreme events produced by simulations of the estimated models is higher than what could be observed in real price data. This is especially true for the two-regime models, where consecutive spikes have a higher probability than in the three-regime model.

Cartea and Figueroa (2005) present a mean-reverting jump diffusion model for the electricity spot price and derive the corresponding forward price in closed-form. They have analysed electricity spot prices in the market of England and Wales. The introduction of NETA changed in a fundamental way the behaviour of this market introducing competition and price variations. However, its implementation only took place in March 27, 2001, resulting in not enough data to estimate or test the available models. Driven by this lack of data, the authors proposed a spot-based model from which it is also possible to extract in closed-form the forward curve. Both historical spot data as well as market forwards data are then used to calibrate the parameters of the model. Regarding the calibration of the model, the authors have circumvented a known drawback in

electricity spot-based models, that is the overwhelming dependence on a large number of parameters to estimate. As the market evolves and more data becomes available, it will be possible to estimate all the parameters more robustly, as already pointed out by some papers which have analysed more mature markets. The authors are able to reduce the number of parameters to be estimated, using a 'hybrid' approach which estimates some parameters from historical spot data and the remaining from market forward prices. It can be argued that this is an arbitrary choice, since calibrating to a market curve starting at a different point might yield different parameters. Even if this were the case, this is not a serious flaw. This would imply re-calibrating the forward curve with respect to a different market curve. In a dynamic hedging-strategy, this could be done as many times as necessary, depending on the exposure and the nature of the contract.

As to the output of the model, the simulated price path resembles accurately the evolution of electricity spot prices as observed in this market. With regards to the forward curve shown, it succeeds in capturing changing convexities, which is a serious flaw in models that fail to incorporate seasonality effects or an appropriate number of other determinants.

Finally, the robust evidence of fat tails in the distributions of electricity returns, together with the complexities on the calibration of these spot-based models and the existing problem of the exiguous data in this market, suggests the exploration of different alternatives. An interesting line of work to pursue involves models departing from the Gaussian distribution, as for instance those based on Levy processes.

Since regime switching structures (allowing for mean reversion and long memory) seem to provide a qualified framework in order to correctly model the behaviour of spot prices, some papers tried to provide the different regimes with an economic justification as well.

For this purpose Haldrup and Nielsen (2004, 2006) focus on the multilateral electricity price behaviour across regions with physical exchange of power and check the hypothesis that different regimes reflect congestion and non-congestion periods and that the direction of possible congestion episodes produces significant effects on the price dynamics.

In the authors' view, a situation where no grid congestions (or grid bottlenecks) exist across neighboring interconnectors will be characterised by a single identical price across the areas with no

congestions. However, when the transmission capacity in a sector of the grid is not sufficient, a congestion will arise and the market system will establish different price areas, with the higher price (positive price difference with respect to the other area prices) expressed by the region with excess demand of power.

Price differentials among different areas reflect disequilibria between demand and supply in sub-sectors of the grid: the bidding area with the largest price is the area with excess demand.

As a consequence, an electricity market (for example Nord Pool) may be partitioned into separate bidding areas which become also separate price areas when the contractual flows between them exceed the capacity allocated by the transmission system operators for spot contracts. Three different states can be arise: non-congestion and congestion periods with excess demand in the one or the other region.

To explore this issue the authors focus on separate prices bilaterally across grid points and in particular on the direction of the flow congestion; the referred market is Nord Pool for the period 3 January 2000 - 25 October 2003 (more than 33000 observations).

From the technical point of view, they improve and extend some previous models (Haldrup and Nielsen, 2005) allowing both for fractional integration and for a 3-state regime switching multiplicative SARFIMA simultaneously.

The former feature accounts for the long memory of price series, whereas in accordance with the previous discussion three states are defined for the price behaviour: in the non-congestion state the difference in log prices is zero so that bilateral price are fractionally integrated, in the congestion states 1 and 2 different price dynamics can exist. Only conditioning on different states it is possible to correctly separate and identify different price dynamics, that are otherwise mixed in a complex way into a kind of convex combination of separate state processes.

The empirical evidence shows that for Nord Pool data this particular model is well performing, that stressing that the long memory price behaviour may be depending on the current market conditions. Moreover, this approach can be considered a way to identify grid points with very separate price behaviour in different congestion states.

Culot, Goffin, Lawford and de Menten (2006) propose a model with spikes for daily electricity prices, that incorporates various stylized features of power prices, including mean-reversion and

seasonal patterns. A mean reverting affine jump diffusion (AJD) model with spikes is developed for both spot and forward market prices. Spike behaviour flexibly is modeled using a Markov regime switching process, that enables to replicate the short-duration and extreme nature of price spikes.

The model is estimated in a two-step procedure, where “structural” elements are pre-calibrated, and diffusive parameters estimated using maximum likelihood and the Kalman filter (as in Cartea and Figueroa, 2004).

The performance of the model is illustrated, using daily and hourly data from the Amsterdam Power Exchange over the period 2001–2005. The spot data is appropriate for estimation of short-term shocks, spikes, and intra-week seasonality, while the forward curve is used to estimate medium/long-term shocks, and annual seasonality. The capacity in modelling performance of the model is also illustrated using a simulation-based assessment methodology, which shows, in particular, the ability of the hourly model to reproduce complicated intraday patterns. While some complex exotic products must be priced numerically using simulated data or numerical Fourier techniques, the AJD structure means that closed-form solutions exist for a variety of contracts. In short, the authors proposed a general and flexible treatment of power price modelling, that covers many important stylized features of daily and hourly electricity, and that has been shown to be amenable to efficient derivative pricing and hedging applications.

Various extensions of the research in this paper are possible. We can imagine potential model modifications for a more realistic description of the observed spot series, e.g. by changing the annual pattern to account for multiple annual peaks, adapting the Markov regime-switching to allow for time-dependent spikes, or weakening the restrictions on the AJD coefficient matrices to enable modelling of stochastic volatility, correlations between risk factors, or more subtle stylized features. These modifications could come at the expense of an considerable increase in the computational burden.

It is well known that prices in day-ahead electricity markets exhibit frequent spikes. The paper by Huisman (2007) focuses on how temperature influences the probability on these spikes in APX market. As temperature information is widely available, both actual values as forecasts, it provides timely information to all market participants at all times. Using a regime switching model in which the regime transition probabilities are time dependent it is shown that deviations from expected temperature influences the probability of a spike to occur. To describes the behaviour of daily

average day-ahead prices, as in Bierbrauer, Truck and Weron (2004), the author assumes in the model that the electricity market can be in one out of two regimes. Regime 1 reflects a normally behaving market. Regime 2 reflects a non-normal market due to a shock in demand and/or supply that results in a spike. Here, the transition probabilities are assumed to be a function of temperature. Temperature is assumed to influence the probability on a spike in the case when the actual temperature differs from the expected temperature. As is assumed that consumption volume depends on temperature, an unexpected change in temperature might lead to an unexpected change in consumption volume. This might then lead to a spike, if power producers are not flexible enough to adjust their volumes to the new consumption level. A further assumption is that the impact of temperature on spike probability depends on the season, as in summer months unexpected higher temperature might lead to an increase in demand (air-conditioning), whereas in winter months unexpected lower prices might lead to an increase in consumption (heating).

In line with the theory the results indicates that temperature elasticities are different for the summer and the winter. For summer months, the elasticity parameter is positive and significantly different from zero. This implies that on days where the temperature is higher than what was expected, the price of power is higher. This is opposite for winter months as the parameter is negative and significantly different from zero. That is, on days where the temperature is lower than expected, the price of power is higher. These temperature effects only affect the mean price level. Adding the estimate for the mean spike to the mean price level implies that on average the mean price level during a spike increase, furthermore, the standard deviation in the spike regime is about three times higher. The stationary transition probability implies that every day a spike may occur with a probability of 3.5%. If the temperature is 1 degree Celsius higher than expected, the probability of a spike to occur increases to 4.3%. When it is 5 degrees warmer than expected, the probability equals 10,1%. The estimate for the transition probability from the spike regime to the normal regime corresponds with a probability of 82.3%. That is, in about 82 of 100 spikes, the power price is back in the normal regime after one day and it stays in the spike regime in 18 cases.

The results of this paper can be used for many situations in which practitioners need to manage the risks of spikes. Spikes are forecastable for a certain extend and the model above make it possible to simulate spikes and to better predict and model spike occurrence. In addition, as weather derivatives are traded, the known impact of temperature on spike occurrence can be used to optimize hedging spike risk using weather derivatives.

Using data from various electricity markets, natural gas and oil markets, Huisman and Mahieu (2001) examine the performance of two different models in order to describe the behaviour of electricity prices. The authors assume that electricity prices exhibit mean reversion, but considering that large jumps occur frequently, they propose a new model, where mean reversion is used to control jumps, because jump models that have been applied in previous studies generally suffer from a potential problem with identifying the true mean reversion within the process.

Two ways of modelling electricity price jumps are presented: first, an autoregressive jump model (ARJ), that has been used in various studies, secondly, a new approach based on regime switching models (RS) in order to account for price jumps. The advantage of the latter model is that jumps are modelled separately from mean reversion, which reduces a potential identification problem.

The first result is that mean reversion parameter estimates change from negative to positive after adding a stochastic jump process. Positive signs are not consistent with the nature of mean reversion; they imply a further move away from the long-term mean. Therefore, the intuition is that stochastic jump models indeed lead to misspecification of the true mean reverting behaviour. In an attempt to disentangle the jump modelling from mean-reversion, the authors propose a regime jump model that disentangles mean reversion from jump behaviour. This model identifies three different regimes, a normal one and two that control for a jump and a reversal back to the normal process. The results indicate the existence of mean reversion in the normal process with consistent parameter estimates. This fact implies that the second model resembles more closely the true price path of electricity prices.

In conclusion, the evidence of this work is that autoregressive jump processes are not a proper way to model electricity price jumps as lead to problems with identifying the true mean reverting process in the data. From that perspective, the regime switching model is a first attempt to disentangle mean reversion from jump modelling. These results lead to differences in results when implemented in forward pricing and risk management frameworks.

## **2.4. Volatility Models**

Worthington, Spratley and Higgs (2002) examine the transmission of spot electricity prices and price volatility among the five Australian electricity markets in the National Electricity Market (NEM): namely, New South Wales (NSW), Queensland (QLD), South Australia (SA), the Snowy Mountains Hydroelectric Scheme (SNO) and Victoria (VIC). All of these spot markets are member

jurisdictions of the recently established National Electricity Market (NEM). At the outset, contrary to evidence from studies in North American electricity markets, unit root tests confirm that Australian electricity spot prices are stationary. A multivariate generalised autoregressive conditional heteroskedasticity (MGARCH) model is used to identify the source and magnitude of spillovers. The estimated coefficients from the conditional mean price equations indicate that despite the presence of a national market for electricity, the state-based electricity spot markets are not integrated. In fact, only two of the five markets exhibit a significant own mean spillover. This would suggest that Australian spot electricity prices could not be fruitfully forecasted using lagged price information from either each market itself or from other markets in the national market. However, own-volatility and cross-volatility spillovers are significant for nearly all markets, indicating the presence of strong ARCH and GARCH effects. Strong own- and cross-persistent volatilities are also evident in all Australian electricity markets. This indicates that while the limited nature of the interconnectors between the separate regional spot markets prevents full integration of these markets, shocks or innovations in particular markets still exert an influence on price volatility.

The results indicate the presence of positive own mean spillovers in only a small number of markets and no mean spillovers between any of the markets. This appears to be directly related to the limitations of the present system of regional interconnectors. The full nature of the price and volatility interrelationships between these separate markets could be either under- or over-stated depending on the specific transformation applied to the original data. One possibility is that by averaging the half-hourly prices throughout the day, the speed at which innovations in one market influence another could be understated. For instance, with the data as specified the most rapid innovation allowed in this study is a day, whereas in reality innovations in some markets may affect others within just a few hours.

The analysis could also be extended in a number of other ways. One approach would be to estimate a system of non-symmetrical conditional variance equations for an identical set of data.

This would allow the analysis of cross-volatility innovations and persistence to vary according to the direction of the information flow. Unfortunately, strict computing requirements do not allow the application of this model with the five electricity markets specified in the analysis.

Another useful extension would be to examine each of the five electricity markets individually and in more detail. Finally, the Sydney Futures Exchange (2000) has offered electricity futures contracts for two of Australia's NEM jurisdictions, NSW and Victoria, since September 1997. An

examination of the relationship between Australian spot and derivative electricity prices using, say, cointegration techniques would then be interesting.

Serletis and Shahmoradi (2006) specify and estimate a multivariate GARCH-M model of natural gas and electricity price changes, and test for causal relationships between natural gas and electricity price changes and their volatilities, using data over the deregulated period from January 1, 1996 to November 9, 2004 from Alberta's spot power and natural gas markets. For natural gas, AECO is the most liquid intra-provincial index and daily spot prices were obtained from Bloomberg. The model allows for the possibilities of spillovers and asymmetries in the variance-covariance structure for natural gas and electricity price changes, and also for the separate examination of the effects of the volatility of anticipated and unanticipated changes in natural gas and electricity prices.

In the context of a VARMA-GARCH-in-mean specification, the authors jointly model the conditional variance-covariance process underlying natural gas and electricity price changes. Their model provides a good statistical description of the conditional mean and conditional variance-covariance processes characterizing natural gas and electricity price changes.

The conditional variance of the electricity price seems to be higher on average than that of the natural gas price. For natural gas, volatility appears highest (on average) in 1997, whereas for electricity the period of greatest volatility appears between 1999 and 2001, a period of increased demand, no excess capacity, and considerable uncertainty about future prices. Moreover, the model indicates that there is a bidirectional (linear and nonlinear) Granger-type causality between natural gas and electricity prices. Thus, the existence of bidirectional causality between natural gas and electricity prices means that there are empirically effective arbitraging mechanisms in Alberta's natural gas and power markets, raising questions about the efficient markets hypothesis.

This paper rules out alternative volatility models that do not allow for the possibilities of spillovers and asymmetries in the variance-covariance matrix for natural gas and electricity price changes.

An empirical analysis of daily spot prices for four European electricity markets using periodic seasonal Reg-ARFIMA-GARCH models is presented by Koopman and Ooms (2005) to explain the dynamics in the conditional mean and variance of log prices.

The time series of daily spot electricity prices are examined from four European emerging markets:

- Nord Pool exchange market in Norway;
- European Energy Exchange (EEX) in Germany;
- Powernext in France;
- Amsterdam Power Exchange (APX) in The Netherlands;

The periodic seasonal regression ARFIMA model with seasonal heteroskedasticity and GARCH disturbances combines ideas from different strands of the statistical, geophysical and econometric literature.

Statistical properties and inference for ARFIMA and other long memory processes are extensively discussed in the monograph by Beran (1994b), in the overview article of Baillie (1996) and, more recently, in the edited volume of Robinson (2003). A novelty in this paper is the introduction of a GARCH process for the variance of a periodic seasonal Reg-ARFIMA model.

The day-of-the-week periodic autocovariances for short run dynamics are modeled by lagged dependent variables and for long run dynamics by seasonal ARFIMA models. Regressors capture yearly cycles, holiday effects and possible interventions in mean and variance. The GARCH-t component takes account of volatility clustering and extreme observations. The model parameters are estimated simultaneously by approximate maximum likelihood methods. Given the persistent changes in volatility, the authors prefer simultaneous estimation of mean and variance parameters above two-step methods. Residual diagnostics show a good fit of the model. The resulting time series models allow for dynamic point forecasting and stochastic simulation.

The Nord Pool market trades hydro power and it is shown that a significant part of the short term price movement can be explained by weekly water reservoir levels and daily electricity consumption. The inclusion of these explanatory variables in the model does not significantly change the estimated periodic heteroskedastic seasonal autocovariance structure in Nord Pool prices.

The basic modeling framework is successful for Nord Pool prices while it can be somewhat improved for prices from other European markets.

Suggestions for future extensions are more flexible distributions for the error term, smoothly time-varying (periodic) parameters and a more extensive specification of the conditional variance equation. More parsimonious periodic AR components can be estimated and tested. The model can also be used for prices at a particular hour of the day. Finally, the strong interrelationships between prices and consumption may lead to multivariate modeling approaches. The empirical findings in this paper may have important consequences for the modeling and forecasting of mean and variance functions of spot prices for electricity and associated contingent assets.

In order to provide robust forecasts for EEX spot electricity prices, Diongue, Guegan and Vignal (2007) propose a new approach based on the k-factor GIGARCH process which allows taking into account a lot of stylized facts observed on the electricity spot prices, in particular stochastic volatility, long memory and periodic behaviours. The authors are principally interested in calculating the conditional mean and variance of the prediction error.

The probabilistic study of this model was introduced in Guégan (2000, 2003). Diongue and Guégan, (2004) developed the parameter estimation of the k-factor GIGARCH process. Here is provided the expression of the forecasts using the k-factor GIGARCH process and given their properties. These marks are applied on the German Wholesale spot electricity market

EEX (European Energy eXchange), providing forecasting hourly electricity spot prices up until a one month-ahead. This goal is completely new in the sense that, in most published papers, the previsions concern mostly the on day-ahead horizon.

The authors apply two models and discuss their capability in forecasting. The tested model are:

- M1: 1-factor GIGARCH model
- M2: 3-factor GIGARCH model

The choice of the number of factors in a k-factor GIGARCH process is not obvious, however it is crucial from a forecasting point of view, see Collet, Guégan and Valdes-Sosa, (2003).

The forecasting results with the two proposed models are highly convening in the sense of RMSE criteria. The conclusion is that the k-factor GIGARCH process is a suitable tool to forecast spot prices, furthermore the model M2 provides better forecasts than model M1, when modelling EEX prices on the period under study.

It is important to see if the methodology proposed in this paper can be fruitfully applied to other spot markets having similar characteristics such as Powernext in France or APX in the Netherlands.

After reviewing the historical facts about Italian Power Exchange, Bosco, Parisio and Pelagatti (2006) analyze the time series of daily average prices generated in this market, which started to operate as a Pool in April 2004. The analysis of these electricity prices carried out in this study permits a good understanding of the most relevant features of the data.

The first finding is the significant change of behaviour that the data generating process has undergone starting from mid January 2005. This fact seems to be due to a learning time needed by the traders involved and by a change of regulation that took place in that period. Another peculiarity of the Italian prices is the relevant drop during Christmas holidays and summer vacations, that makes the use of few sinusoids or monthly dummies not fit for modelling with-year seasonality. An original methodology to deal with this problem has been developed. Furthermore, the interaction of the within-year seasonality with the within-week seasonality has also been modelled. A slow but significant (increasing) linear trend in the prices has also been noted and fitted. The reasons for this may be found in the relevant growth of the prices of hydrocarbon-based energy sources.

To characterize the high degree of autocorrelation and multiple seasonalities in electricity prices, the authors use periodic time series models with GARCH disturbances and leptokurtic distributions and compare their performance with more classical ARMA-GARCH processes. The within-year seasonal variation is modelled using the low frequencies components of physical quantities, which are very regular throughout the sample.

Results reveal that much of the variability of the price series is explained by deterministic multiple seasonalities which interact with each other. Periodic autoregressive GARCH models seem to perform quite well in mimicking the features of the stochastic part of the price process. Leptokurtic PAR-GARCH models fit best the different amount of memory of past observations that each weekday carries, as well as the presence of spikes and some form of volatility clustering.

Although the limited length of the price time series leaves some questions open, the models developed in this paper seem to perform quite well.

The aim of the work by Leon and Rubia (2002) is to forecast the multivariate conditional volatility for portfolios containing intradaily electricity spot price from the Argentine Electricity Market (MEM) by grouping prices in three daily series (block bids). The method proposed could be a useful tool in order to manage the risk implied by the high volatility of the intradaily power price. Some methodologies characterized as simple multivariate conditional volatility models are applied by using orthogonal garch (OGARCH) and multivariate garch (MGARCH) models. Both models have been developed to cope with the time-dependent volatility of portfolios that include a great number of assets in financial and capital markets.

First, the authors estimate the conditional mean of intradaily series by means of the autoregressive vector (VAR) methodology and a deterministic function capturing the strong seasonal behaviour implied in the power commodities. Then, taking the multivariate residual error series from VAR model, they estimate the conditional covariance matrix by the multidimensional GARCH model. More specifically, the models proposed by Alexander (2000) and Engle and Mezrich (1996) are applied. Both models get the estimation of parameters under a feasible computational way. The forecasting performance of OGARCH and MGARCH models, is evaluated in terms of their mean square error, both in sample and out of sample, when these are used for computing the daily block bid volatilities and covariances.

The main conclusion of the forecasting performance comparison between the two approaches is that they give quite similar results. The goal of this paper has been to provide an intuitive tool which could be easily implemented by market agents. Of course, there is an implied trade-off between simplicity and realism in doing so. This methodology could be appropriate to the development of some extensions trying to cover more complex structures, such as the infrequent extreme jumps, pricing and managing basket options taking a block bid portfolio as underlying asset, and also for the valuation of derivatives on intradaily time-blocks of electricity spot prices. These topics are undoubtedly interesting challenges for further research.

### **3. Analyzing and Forecasting the Electricity Prices: a New Proposal**

#### **3.1. The State of the Art. Summarizing Major and Minor Issues**

Six fundamental points arise from the analysis of the theoretical and empirical literature on electricity prices:

- The electricity market retains absolutely peculiar characteristics: it is an auction market that, although liberalised, is not strictly a spot one, but it requires both price and quantity of equilibrium to be defined one day in advance on the basis of expected supply and demand. This guarantees a good match among supply and demand, that, due to the non-storability of electricity, to unexpected peaks in demand and to congestions over the distribution network, could fail, causing jumps in prices and leading in extreme cases to the system blackout.
- The series of electricity prices have complex statistical properties that vary depending on spectral frequency to which data are measured and on sample size. Depending on the cases, it is possible to notice phenomena of seasonality at different frequencies, trends which are more or less linear at low frequencies, phenomena of auto-correlated volatility at high frequencies, and combinations of outliers apparently managed by non standard distributions.
- A wide range of models dedicated to the analysis of the properties of price series follow an approach that can be defined as being agnostic from the point of view of economic interpretation, meaning they do not foster the inference on (economic) factors that influence prices, but they limit the analysis to only their statistical properties.
- However, it seems evident that the evolution of prices over time is driven by the interaction between supply and demand of electricity, that is, from two phenomena not directly measurable and in someway latent. Therefore, in order to effectively model demand and supply it would be suitable to include in the model those factors that determine their trend: for example, climatic factors or the business cycle state that affect demand; productivity, size of the plant and costs of production concerning supply. It is a insidious approach, as these determinants play a role at different frequencies and usually statistical data on them are

characterised by significant measurement errors, which makes more difficult the correct identification of the effects caused by each phenomenon on prices.

- Even for the hidden dangers previously mentioned, the econometric models dedicated to the analysis of electricity prices adopt very simplified specifications, often uniequational, taking into account only a few aspects of the issue at a time.
- Among the models proposed by the literature, none of them seems to be characterised by a uniformly better capability of fitting the data and by an outperforming forecasting behaviour; depending on the market taken as reference, on the sample of data being considered and on the measure of forecasting performance chosen, now prevail very simple autoregressive models, whereas other times Markow switching models with changing regimes.

In the light of the previous stylized issues, we consider the necessity of adopting a completely new methodological framework in order to efficiently specify and forecast the behaviour of electricity prices; an eclectic approach is needed which enables the estimate and the effective identification of the unobservable dynamics of demand and supply, the management of extremely wide datasets containing high frequency data, the coexistence of short term determinants of electricity prices with those of long term<sup>1</sup>, the creation of forecasts on future trends as well as simulations of the impacts of structural shocks.

The natural and physiological candidate for the role of this innovative methodological tool is represented by dynamic factor models (henceforth DFM).

These models were introduced in the late '70s and present characteristics which are definitely appropriate for the resolution of the six problems highlighted in the analysis of the literature on modelling and forecasting the electricity prices.

Within the DFM framework it is possible to:

- Produce efficient forecasts on the basis of many predictors and large equation systems.
- Identify, estimate and analyse properties of widespread but unobservable variables, as the economic cycle or market demand and supply (electricity market in this case).

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<sup>1</sup> Extracting the economic signal from the noise

- Clean the data, separating measurement errors and idiosyncratic behaviours from the economic structural signal.

### **3.2. A Brief Survey of DFM Literature. From Theoretical Aspects to Empirical Applications**

In his Ph.D. thesis Geweke (1977) moving from the observation of strong comovements among economic series<sup>2</sup>, introduced the dynamic factor representation, expressing each economic variable as the sum of a distributed lag of a small number of unobserved common<sup>3</sup> factors plus an orthogonal idiosyncratic disturbance. In early applications to macro data Sargent and Sims (1977) and Sargent (1979) find empirical support to the view that a small number of common factors drive a large part of the observed variation in the economic aggregates; the perturbations affecting factors are just the common structural economic shocks the theoretical analysis and the policy makers are interested in, such as demand or supply shocks.

It clearly emerges that dynamic common factors could provide a “natural” way of summarizing in a formal framework the informational content of large economic datasets and provide a sounder statistical basis for the extraction of synthetic measures of complex phenomena from multiple time series. Their great advantage is to efficiently reduce the large dimensional problem of handling tons of variables to identify and estimate a very small number of components.

Finally, composite indexes have attracted a considerable attention due to their capacity of summarizing, describing and identifying not observable economic phenomena hidden in a (large) number of macroeconomic series, like in primis demand and supply.

During the last twenty years the use of DFM has significantly spread in the academic framework as well as at the institutional level and the pioneering empirical applications, all oriented toward the analysis of the business cycle, left space for more differentiated applications in terms of economic content and properties of data.

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<sup>2</sup> As firstly stressed by Burns and Mitchell (1946)

<sup>3</sup> Common to (near)all variables in the dataset

In a sequence of cornerstone papers, Stock and Watson (1989), or SW89, and Stock and Watson (1991, 1992) show how to obtain through the Kalman filter the maximum likelihood estimation of the parameters and the factors in a DFM cast into state space form and within this framework they rationalize and refine the U.S. Business cycle coincident composite index produced by the Conference Board.

Their index is obtained as the unique estimated factor of a low dimensional DFM allowing only for coincident variables. The corresponding  $n$ -period leading index may be obtained as the  $n$ -step ahead forecast of the coincident index based on a linear combination of past values of a group of pre-selected leading indicators. This way Stock and Watson (1999) produced forecasts of US GDP and inflation.

Despite its exceptional innovations, the SW89 proposal suffers three main drawbacks: (a) when  $n$  is very large (the most interesting case), maximizing the likelihood over so many parameters is too much consuming from the computational point of view, (b) the hypothesis that a unique common factor drives most of macroeconomic variables does not fit reality and (c) it is required an ex-ante classification of variables into coincident and leading ones.

Since SW89, a large body of literature has been developed on DFM and forecasting: some lines of research have developed SW89 in an incremental way, whereas others have put forward proposals related but potentially alternative to it.

Stock and Watson (2002a), or SW02a, and Stock and Watson (2002b), or SW02b, address all issues (a), (b) and (c) and show that with large datasets, including both coincident and leading (at all leads) variables, the consistent estimation of  $q > 1$  dynamic factors can be based on static Principal Components Analysis (henceforth PCA), which is equivalent to solve a nonlinear least squares problem. Thus, becomes evident the correspondence between common dynamic factors and composite indexes in the sense that the estimated common factors are just weighted averages (weighted indexes) of variables contained in the original dataset and that the weighting system is an optimal one because minimises a quadratic loss function. SW02a in fact gives the estimated factors an interpretation in terms of “diffusion” indexes developed by NBER analysts to measure business cycles.

In this context, the generation of linear forecasts is directly obtained by using the h-step ahead formulation of the measurement equation of the DFM model.

Following an indirect two step procedure, past values of the previously estimated common factors can also be used within a dynamic linear equation in order to forecast a coincident index, some of its components or any other macroeconomic variable (Marcellino, Stock and Watson, 2003, and Banerjee, Marcellino and Masten, 2003, for the Euro area; Artis, Banerjee and Marcellino (2004) for UK).

To produce iterated h-step ahead forecasts, Favero, Marcellino and Neglia (2004) and Bernanke, Boivin and Elias (2005) proposed an approach that models jointly as a VAR a block of pre-estimated factors (through static PCA) and a set of macroeconomic variables of interest. Such an approach, named Factor Augmented VAR (FAVAR), integrates factor methods into VAR analysis and provide a unified framework for structural VAR analysis using dynamic factors. Forni, Hallin, Lippi and Reichlin (henceforth FHLR) (2003) and Giannone, Reichlin, Sala (2004) constrain the shocks onto the factors equation of VAR to have reduced dimension in a work aimed at forecasting Euro-wide inflation and industrial production. Stock and Watson (2005b) significantly refine the FAVAR approach taking into account in their model the exclusion restrictions implied by the DFM.

FHLR propose two alternative approaches to that of Stock and Watson, for the estimate of a DFM, anyhow based on the use of the principal components.

In the former (FHLR 2003), the loss function to be minimised for the estimation depends on the inverse of the variance and covariance matrix of the idiosyncratic component<sup>4</sup>.

A further line of research, (FHLR, 2000, 2001, 2004), switch from static to dynamic PCA and apply this alternative methodology to the derivation of a composite coincident index for the Euro area. This method allows for a richer dynamic structure than static PCA, but it is based on two-sided filters so its use for forecasting requires trimming the data at the end of the sample. The way for a real time implementation of the dynamic PCA approach was showed by Altissimo et alii (2001b) and it is now adopted by CEPR in order to provide its composite coincident indicator for the Euro area (Eurocoin), which is the single factor estimated from a panel of nearly 1000 economic series. To

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<sup>4</sup> SW (2005a) use the expression “Weighted static PCA” to describe this approach.

construct n-step ahead predictions of the coincident index (Altissimo et al., 2001a) one may project it n-step ahead on its current and past values and on simple averages of the common components of the leading variables contained in the dataset, endogeneously selected on the basis of their lead relation with respect to the coincident index.

A one-sided version of the FHLR filter is used by Giannone, Reichlin and Sala (2004) to produce factor-based predictions of US GDP growth and inflation rate, aimed at miming the US Greenbook forecasts.

Dynamic Factor Models have been extensively used in many different frameworks of analysis as well as the macroeconomic one.

As for high frequency data, as those typical of the electric market, it is possible to find a large number of applications (Connor and Korajczyk, 1988) using factor model methods to estimate unobserved factors and test their consistency with the indications coming from the arbitrage pricing theory. Other works address the analysis of asset prices using approximate factor structures: a survey is in Campbell, Lo and MacKinlay (1996). Recent developments tried to joint macromodels on business cycles and finance models in order to provide both a comprehensive analysis of the term structure of interest rates (Diebold, Piazzesi and Rudebush, 2005) and an inspection of the contribution of the financial system to US business cycles (Compton and da Costa e Silva, 2005)

Nevertheless empirical DFM based applications addressing the analysis of the electricity market, as far as we know, do not seem to exist.

### **3.3. Concluding Remarks and... a New Proposal**

The analysis of the theoretical and empirical literature on the electricity market has confirmed the potential utility of proceeding through lines of research not yet explored, leaving the continuous refinements of models that have already been extensively used.

In the near future we intend to proceed with the analysis and forecasting of electricity prices pursuing a new approach: we will adopt a (time varying parameters) FAVAR model based on an

exact DFM, according to the scheme proposed by SW05b, and fostered by a large dataset containing all measurable variables that, by determining demand and supply of electricity, influence in the short, medium and long run the behaviour of electricity prices.

Once the model is estimated through static PCA, we will identify in both formal and economic sense the unobservable factors, isolating in particular demand and supply of electricity.

This approach will enable us to produce short and medium run forecasts of price trends, eventually classifying them by time slot as well as times of week and times of year. Moreover, we will have the possibility of simulating the impact that shocks caused by mismatches between demand and supply have over the market and over the price variable, evaluating if congestion or saturation of the network, leading black out phenomena, trigger price reactions that can be considered as real warning mechanisms. We will then compare the performance of this new approach to those of well known and previously used models.

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