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# Dualism and the big push

Giovanni Valensisi\*

*We develop a two-sector model with specific factors, in which agriculture is subject to diminishing returns and market-clearing wages, while increasing returns and efficiency wages prevail in industry. The asymmetric interaction of the two sectors, jointly with the dualistic structure of the labor market, is such that the model may display multiple equilibria and a low-development trap under plausible parametrization. Additionally, parametric increases of sectoral TFP may reduce the basin of attraction of the low-equilibrium and increase the steady state level of capital stock (and wages) for the stable equilibrium of full industrialization.*

**Keywords:** dualism, industrialization, poverty traps, Kaldor-Verdoorn law, Engel's effects.

**JEL codes:** O11, O14, O41.

## I. *Introduction*

Poverty traps and multiple equilibria are concepts, which have been used fruitfully since the very dawn of development economics, and can implicitly be traced back even to Adam Smith's "Early draft of part of the Wealth of Nations"<sup>1</sup> and to Malthus's "Principles of political economy"<sup>2</sup>. After the seminal paper of Rosenstein Rodan (1943), the idea that underdevelopment could constitute a state of equilibrium thrived with Nurkse's *vicious circle of poverty* (1953) and Nelson's *low-level equilibrium trap* (1956), and indeed constituted, together with the analysis of dualism, one of the main contributions of the "classical development economics". Several mechanisms, essentially concerning increasing returns coupled with pecuniary externalities or alternatively demographic traps, were from time to time held responsible for creating a multiplicity of equilibria, and possibly preventing the spontaneous industrialization of certain economies, maintaining them at the stage of backward agricultural systems<sup>3</sup>.

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<sup>1</sup>In Smith (1763) page 579 the author argues: "That is easier for a nation, in the same manner as for an individual, to raise itself from a moderate degree of wealth to the highest opulence, than to acquire this moderate degree of wealth."

<sup>2</sup>In 1836 reverend Thomas Malthus observes at page 310 of his Principles of political economy: "...that there are many countries, not essentially different..., which yet, with nearly equal natural capabilities, make very different progress in wealth."

<sup>3</sup>Authors such as Young (1928), Rosenstein-Rodan (1943) and Nurkse (1953) emphasized the importance of increasing returns, while Nelson (1956), Jorgenson (1964) and - later - Dixit (1970) focused on the role of demographic dynamics in creating poverty traps, in which economic growth in absolute terms is balanced by the counteracting dynamics of population, so that GDP per capita remains at a low level.

Despite the deep interest enjoyed by the so-called "classical development economics" in the fifties, its lack of clear successes on the practical side, jointly with the difficulty to reconcile increasing returns with competitive market structures<sup>4</sup> contributed to its decline in favor of the more analytically rigorous paradigm of the neoclassical economy, described first by Solow (1956) and Swan (1956) and later by Cass (1965) and Coopmans (1965) who extended it to endogenize saving decisions into an intertemporal optimization framework. Notwithstanding many important contributions on the role of increasing returns and learning by doing, during the 60's the mainstream approach to growth became that of the neoclassical convex economy converging to a stable and unique steady state. Additionally, attention shifted from the "developmental perspective" - emphasizing the role of structural change and "sectoral balances" - to an aggregate growth perspective - focusing more on reproducible factors' accumulation, and on the determinants of the steady state. It is worth noting here, that the choice of an aggregate model dismisses by definition the role of relative price changes, and overlooks the empirically-founded recognition that economic growth goes hand in hand with structural change<sup>5</sup>. Clearly, this latter flaw can be *a fortiori* misleading when analyzing those economies that are indeed undergoing a process of industrialization and not of "homothetic growth"<sup>67</sup>.

Regardless of the possible limits of aggregate models, the mainstream approach has played a key role in bringing back to the center of the attention the issue of increasing returns, along with their crucial implications for multiple equilibria. The twist away from the traditional paradigm of the convex economy occurred in the mid 80's, when endogenous growth theory started emphasizing the role of knowledge and human capital. Assuming increasing returns to reproducible factors, including knowledge, responded to the need to rationalize two elements of industrial economies that could not but be explained exogenously in Solow's conceptual framework: the persistence of growth even after the capital labor ratio has reached fairly high levels, and the continuity (or possibly even the acceleration) of technical change. Endogenous growth theory was primarily concerned with issues other than explaining the take off of initially poor countries, consequently it initially focused more on the properties of the steady state path, rather than on the possible obstacles to industrialization. Nevertheless, by the mid and late 90's concepts like poverty traps, structural change and multiplicity of equilibria recovered a central role in the debate on economic growth, leading to what has been called a "counter-counterrevolution in development theory"<sup>8</sup>.

On the one hand, it was shown that even in the standard neoclassical set-up (one sector with convex technologies operating under perfect competition), multiple equilibria cannot be excluded *a priori* once empirically significant elements such as heterogeneity in saving behavior, low elasticity of technical substitution, or capital market imperfections are taken into account<sup>9</sup>. On the other hand, advances in the theoretical analysis of non-perfectly competitive market structure, jointly with a new

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<sup>4</sup>Notably during the 50's and 60's only demographic traps had been analyzed in mathematical form, while poverty traps based on increasing returns, specialization and pecuniary externalities were treated only in narrative discursive contributions.

<sup>5</sup>See Pasinetti (1993).

<sup>6</sup>Not surprisingly, the empirical literature has found growing evidence of the limited explanatory power of the so-called "augmented Solow regressions" in the case of poor countries, and has suggested the need to go beyond the common linear specification of the growth process commonly used in cross-country "Barro regressions". See Durlauf and Johnson (1995); Durlauf, Kourtellos and Minkin (2001) and Durlauf, Johnson and Temple (2005).

<sup>7</sup>Furthermore, the common practice in aggregate models of using a linearly homogeneous production function requires even greater caution, because of the subtle but delicate implications of such restriction in terms of positive theory. Solow himself recognized the difficulty in two different articles: Solow (1956) page 67 cautions about applying an aggregate production function, which is linearly homogeneous, to the case in which production depends on a "nonaugmentable resource like land"; Solow (1957) page 314 states the need to net out agricultural contribution to GDP when applying the aggregate production function to the analysis of real economies. Given that in poor economies the primary sector features a larger contribution to GDP, it is plausible to expect the limitations intrinsic in the use of aggregate production functions to be more stringent for the analysis of developing countries.

<sup>8</sup>See Krugman (1992).

<sup>9</sup>See Galor (1996), Azariadis (2005), Easterly (2006), Kraay and Raddatz (2007).

stream of literature on structural change, caused a revival of categories inherited from the "high development theory"<sup>10</sup>.

The renewed interest in poverty traps came also under the pressure of the empirical literature, which increasingly questioned the validity of the neoclassical paradigm of conditional  $\beta$ -convergence, in favor of more complex dynamics able to generate convergence clubs and twin peaked distributions. Cross-country regressions have for long confirmed that economies tend to converge to their own steady state at a rate consistent with the "augmented Solow model", once controlling for the determinants of the steady state itself: typically the saving rate, the initial level of human capital, political stability and degree of price distortion<sup>11</sup>. Despite this, several econometric works accounting for parameters heterogeneity across countries (rather than relying on a common linear specification, as in standard growth regressions) found evidence of *convergence clubs* formation<sup>12</sup>. On the other hand, the existence of convergence clubs seems confirmed also by several studies based on non-parametric inference about the cross-country distribution of GDP per capita, and on the "distribution dynamics" of Markovian growth processes<sup>13</sup>. While not necessarily incompatible with neoclassical growth models, the existence of convergence clubs seems to come at odds with the traditional paradigm of the convex economy à la Solow, while it rationalizes immediately the observed absolute  $\sigma$ -divergence across countries.

In light of the long standing debate summarized above, in this paper we aim at building a theoretical model able to reconcile the neoclassical theory of growth and the "developmental perspective". In particular, we retain from the early development literature the dualistic set-up with its asymmetric treatment of agriculture and industry, in order to highlight the role played by factors' reallocation in the early phases of industrialization. We do so by developing a specific-factor macro model à la Ricardo-Viner-Jones, which may display multiple equilibria and poverty trap under plausible parametrization<sup>14</sup>.

The paper is organized as follows: section II outlines the macro model and the determination of the equilibria, section III explains the effect of exogenous technical progress (here intended as a parametric increase of sectoral TFP) in each of the two sectors, section IV concludes.

## II. *The model*

### PREFERENCES

The economy consists of two sectors, agriculture and industry, producing respectively food - a consumption good - and manufactures, which can be alternatively consumed or invested. In order to allow for well-documented Engel's effect in consumption, a Stone-Geary utility function is used to

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<sup>10</sup>Among the mechanisms proposed to justify the existence of multiple equilibria we may cite: technological non-convexities (see Murphy, Shleifer, Vishny (1989); Azariadis, Drazen (1990); and Ros, Skott (1997)), saving based traps with subsistence consumption (see Ben-David (1998) and Sachs (2003)), learning by doing (see Matsuyama (1992), Stokey (1988)), credit market imperfections (see Galor, Zeira (1993); and Banerjee, Newman (1993)), and institutional traps (see Murphy, Shleifer, Vishny (1993)).

<sup>11</sup>See among others Barro (1991); Mankiw, Romer, Weil (1992); Barro, Sala-i-Martin (1995); Sala-i-Martin (1996); Easterly (2006).

<sup>12</sup>See Durlauf and Johnson (1995); Durlauf, Kourtellos and Minkin (2001) and Durlauf, Johnson and Temple (2005).

<sup>13</sup>See Bianchi (1997); Quah (1993 and 1996); Desdoigt (1999); Azariadis, Stachurski (2005); Azariadis (2005).

<sup>14</sup>For several aspects our set-up resembles the "Rosenstein-Rodan / Leibenstein model" formulated in Ros (2000); however we depart from it in adopting a sociological theory of efficiency wage, eliminating the recourse to the Lewisian labor surplus and allowing for Engel's law in demand composition. These choices allow us to generalize Ros's results while adopting a fully neoclassical formalization - with flexible prices, perfect competition and under the marginal theory of distribution.

describe consumers' preferences across goods:

$$U = (X_a^c - Z)^\alpha (X_i^c)^{1-\alpha};$$

where  $X_a^c$  and  $X_i^c$  are respectively the amount of food and manufactures consumed,  $Z$  is the minimum required amount of food<sup>15</sup>, while  $\alpha$  represents the marginal food expenditure share<sup>16</sup>. Through standard utility maximization under budget constraint, representative consumers' demand can be shown to be:

$$\frac{\alpha}{1-\alpha} \frac{X_i^c}{X_a^c - Z} = \frac{P_a}{P_i}; \quad (1)$$

where  $P_a/P_i$  denotes the agricultural terms of trade. Consistently with the above specification of demand, the price index  $\underline{P}$  is

$$\underline{P} = P_a Z + \psi P_a^\alpha P_i^{1-\alpha}; \quad \psi \equiv \frac{1}{\alpha^\alpha (1-\alpha)^{1-\alpha}}. \quad (2)$$

### TECHNOLOGIES

The agricultural sector produces food employing a backward technology that uses labor and land, but has no scope for reproducible inputs<sup>17</sup>. The agricultural production function is given by

$$X_a^s = A_a L_a^{1-b}; \quad 0 \leq b < 1 \quad (3)$$

where  $X_a^s$  denotes food output,  $L_a$  the labor employed in agriculture,  $(1-b)$  and  $A_a$  are technological parameters describing respectively the degree of returns to labor and the sectoral TFP (which in the case of agriculture summarizes both technological factors but also geographical and climatic conditions). The restriction on  $b$  derives from the hypothesis that land endowment is fixed even in the long-run<sup>18</sup>, and implies decreasing returns to labor ( $b=0$  is a limiting case, representing constant return to labor).

For what concerns the industrial sector, firms utilize labor (in efficiency units) and capital in the production of manufactures. The manufacturing sector is assumed to exhibit increasing returns to scale due to *Marshallian external economies* associated with capital stock, and captured by a *Kaldor-Verdoorn coefficient*, which rationalizes the positive externality stemming from "capital-embodied-knowledge". In other words, we assume that the stock of knowledge is proxied by the average economy-wide stock of capital, and that capital accumulation translates automatically into improvements of the knowledge base and hence of the industrial TFP at the constant rate  $\mu$  (precisely the Kaldor-Verdoorn coefficient). The present formalization is equivalent to assume a learning by doing process, in which the cumulative gross investment represents the index of experience, and knowledge depreciates at the same rate as physical capital<sup>19</sup>.

<sup>15</sup>In order for the Stone-Geary utility function to be meaningful, we have to assume that  $X_a^c > Z$ , meaning that in all cases food consumption exceeds the minimum required amount  $Z$  (this hypothesis will be important at a later stage to determine the sign of several magnitudes).

<sup>16</sup>The qualifying adjective "marginal" is used here in the sense that  $\alpha$  is the food expenditure share applying to the supernumerary income, remaining after the consumer has purchased the minimum quantity of food  $Z$ . Clearly, by setting  $Z=0$  one falls back in the homotetic preference case, and the utility function turns into a standard Cobb-Douglas.

<sup>17</sup>The absence of capital among agricultural inputs is evidently inappropriate for high and middle income countries displaying capital-intensive techniques of cultivation, however it represents a suitable approximation for less developed countries (LDC). This widely adopted assumption, however restricts the relevance of the present model to those countries, where subsistence agriculture is especially widespread and the scarce physical capital is employed in non-agricultural activities: above all South Asian and Sub-Saharan African countries.

<sup>18</sup>The fixed argument "land" has been omitted from the production function to lean down the notation.

<sup>19</sup>In this respect, the present model differs from both Arrow's original approach (1962), in which experience is also proxied by cumulative gross investment but without knowledge depreciation, as well as from recent models of structural change that disregard the idea of capital embodied knowledge and relate the learning process to cumulative output (for instance Krugman 1987, Stokey 1988, Matsuyama 1992 and 2002).

In accordance with the previous discussion, the industrial technology is described by a Cobb Douglas production function

$$X_i^s = A_i \tilde{K}^\mu K^\beta (E_{(w_i, w_a)} L_i)^{1-\beta}; \quad \mu > 0, \quad 0 < \beta < 1;$$

where  $X_i^s$ ,  $L_i$  and  $K$  denote respectively manufacturing output, industrial labor and capital stock, the function  $E_{(w_i, w_a)}$  represents labor efficiency, the parameters  $\beta$ ,  $(1 - \beta)$  and  $A_i$  are respectively the capital and labor shares, and the industrial TFP, and finally  $\tilde{K}^\mu$  represents the external positive effect of capital accumulation,  $\tilde{K}$  being the average capital stock of our economy.

The fact that technological economies are *external* to each firm derives from assuming, that the non-rival and non-excludable nature of knowledge is such that the experience acquired by one firm spills over *completely* and *immediately* to the others, exerting a positive externality on all manufacturing producers<sup>20</sup>. In light of this, we can argue that in equilibrium the average capital stock of the economy will match that of the representative firm; accordingly, the industrial production function can be rewritten as

$$X_i^s = A_i K^{\mu+\beta} (E_{(w_i, w_a)} L_i)^{1-\beta}; \quad \mu > 0, \quad 0 < \beta < 1. \quad (4)$$

Clearly, the above production function represents a generalization of the AK technology: as long as  $\mu > 0$  it displays aggregate increasing returns, though not necessarily constant or increasing returns to capital, as typically assumed in AK models or in other models of endogenous growth à la Romer<sup>21</sup>.

Concluding the analysis of technologies, it is straightforward to see that capital accumulation will not trigger a "homothetic growth" for the economy as a whole, precisely because in this set-up reproducible inputs are specific to only one sector: industry. Unlike in aggregate models, here the accumulation of reproducible factors affects asymmetrically the marginal productivity of labor in agriculture and manufacturing, leaving the burden of equilibrium adjustment to labor reallocation, capital-labor substitution (in industry) and eventually to price adjustments. At the same time, resource reallocation across sectors determines a change in output composition and employment shares.

## DISTRIBUTION AND LABOR MARKET

In line with the traditional literature on dualism, distributive issues and "organizational asymmetries" between agriculture and industry play a key role in the present model, especially as concerns the labor market. Our approach, however, departs from the debated hypothesis that wages in the traditional sector are determined à la Lewis by the average productivity of labor; instead, we assume perfect competition among rentiers and laborers, so that the former hire all available workers and pay them at a wage rate equal to their marginal revenue product<sup>22</sup>. Analytically we will thus have:

$$W_a = (1 - b) A_a (L_a)^{-b} P_a; \quad (5)$$

<sup>20</sup>Despite the caveats about some more realistic refinements of the learning by doing process, the hypothesis of immediate and complete spillovers is widely used in the literature (see Krugman 1987, Matsuyama 1992, 2002, Stokey 1988) for it allows to concentrate on the impact of increasing returns without further analytical complications as regards the market structure.

<sup>21</sup>In this way, our generalization of AK models overcomes the problem of excessive sensitivity to restrictive parametrization, as increasing returns to capital arise here only if  $\mu > 1 - \beta$ , with equality yielding constant returns to capital. See Stiglitz (1992) and Solow (1994) for a critique of AK models in this respect.

<sup>22</sup>Maintaining the Lewisian assumption would not change qualitatively the conclusion of our model, but simply reduce the wage gap across sectors (since the average revenue product exceeds the marginal one in agriculture) and the scope for labor re-allocation towards industry, thus shortening the "dualistic" phase. One would however end up postulating the equivalence of rural wage and average labor productivity in agriculture even during the mature phase of the economy, or otherwise need to explain what triggers the change in the distributional rules at a certain point in time.

and

$$R = b A_a (L_a)^{1-b} P_a = \frac{b}{1-b} W_a L_a; \quad (6)$$

where  $W_a$  represents the rural wage in nominal terms and  $R$  the total rents.

Organizational dualism comes into play as regards wage determination in the industrial sector, where we assume the existence of an efficiency mechanism, linking labor productivity with the real wage received<sup>23</sup>. In light of such linkage, the problem faced by industrial entrepreneurs will be

$$\max_{L_i, W_i} [\Pi] = A_i K^{\mu+\beta} (E_{(w_i, w_a)} L_i)^{1-\beta} P_i - L_i W_i; \quad \text{subject to } W_i \geq W_a$$

where upper-case  $W$  indicates wages in nominal terms (lower-case  $w$  are expressed in real terms), and  $E_{(w_i, w_a)}$  is a non-decreasing function relating workers' efficiency with the real wage they receive, and with the real wage they could get if working in agriculture. Notably, the problem faced by industrial entrepreneurs is a constrained maximization, since they cannot hire any worker at a wage lower than the reservation wage the latter could get in agriculture.

The specification of the effort function follows from Akerlof's interpretation of labor contracts as partial gift exchanges, in that  $E_{(w_i, w_a)}$  reflects those sociological considerations (including the real wages paid in the other sector of the economy) that govern the determination of work norms, and hence regulate labor productivity. Consistently with Akerlof, we additionally suppose that the effort function takes the convenient form

$$E(W_i) = \begin{cases} 0; & \text{for } W_i < \omega^{\frac{1}{d}} W_a^\gamma \underline{P}^{1-\gamma} \\ \left[ \frac{W_i/\underline{P}}{(W_a/\underline{P})^\gamma} \right]^d - \omega; & \text{for } W_i \geq \omega^{\frac{1}{d}} W_a^\gamma \underline{P}^{1-\gamma} \quad 0 < d, \gamma < 1; \omega > 0; \end{cases} \quad (7)$$

in which the parameter  $\omega$  implies a minimum threshold to obtain positive effort,  $d$  is a positive parameter and is lower than one to ensure the effort function to be well-behaved (meaning increasing and concave with respect to the real industrial wage  $W_i/\underline{P}$ ), and  $\gamma$  represents the elasticity of industrial real wage to agricultural one. This specification is a generalization of the effort function proposed by Akerlof (1982), and opens the additional possibility of having a less than proportional relationship between the wage received by industrial workers, and the wage they would receive if employed in agriculture<sup>24</sup>.

Under the above assumptions, and as long as  $W_i > W_a$ , the FOC for their profit maximization problem imply the Solow condition of unitary wage elasticity of effort (ensuring cost minimization)

$$W_i = \left( \frac{\omega}{1-d} \right)^{\frac{1}{d}} W_a^\gamma \underline{P}^{1-\gamma}; \quad (8)$$

plus the usual labor demand function

$$L_i = (1-\beta)^{\frac{1}{\beta}} A_i^{\frac{1}{\beta}} (E^*)^{\frac{1-\beta}{\beta}} K^{\frac{\mu+\beta}{\beta}} (W_i)^{-\frac{1}{\beta}} P_i^{\frac{1}{\beta}}; \quad (9)$$

where  $E^* \equiv d\omega/(1-d)$  is the effort level corresponding to  $W_i$ . Given that the second order conditions are met for the assumed well-behaving production and effort functions, the FOC define the solution of the above profit maximization as long as the constraint is satisfied.

<sup>23</sup>While efficiency wage mechanisms do not seem appropriate for the rural sector in LDCs, dominated by casual labor and informal relations, they are indeed much more credible for the formal labor markets of the urban industrial sector. See Mazumdar (1959), Rosenzweig (1988) and Basu (1997).

<sup>24</sup>Note that Akerlof's formalization can be obtained by simply assuming  $\gamma = 1$ , entailing the perfect proportionality of industrial wages and agricultural ones. Apart from this aspect, the rationality for choosing the above specification is the usual one: the threshold  $\omega$  is included to avoid the trivial solution of an optimal zero wage (see Akerlof (1982) for more details), and the restrictions on  $d$  are needed to ensure the existence of a unique internal maximum.

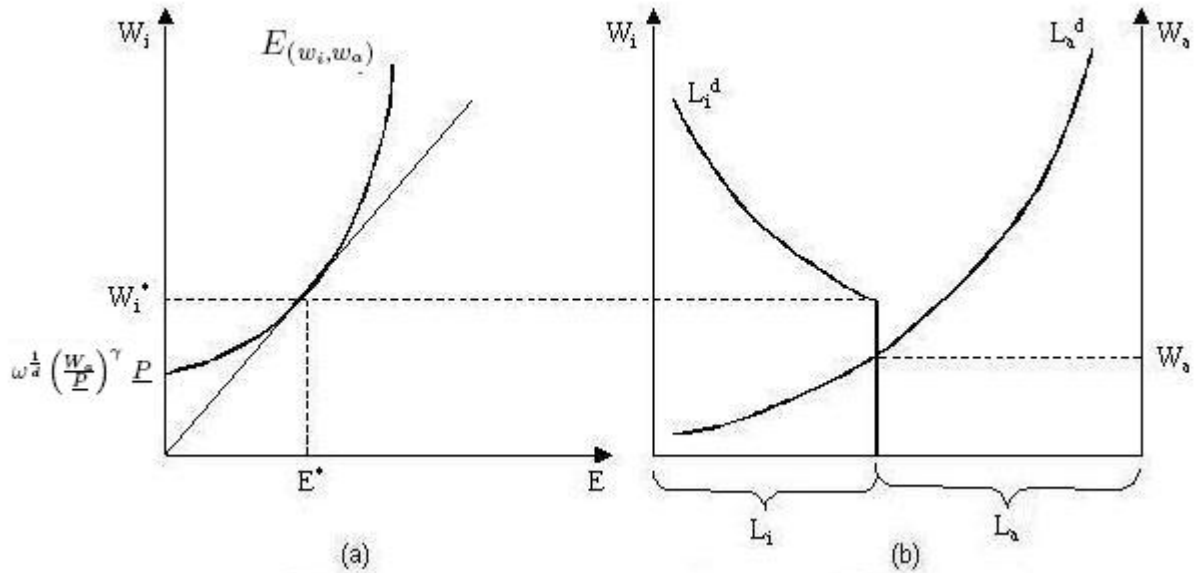


Figure 1: The efficiency wage mechanism

Figure 1(a) represents the diagram corresponding to our specification of effort function on the  $W_i - E$  space. The payroll cost per efficiency unit of labor corresponding to each point of the effort function is given by the slope of the ray from the origin to the same point. Clearly the optimal wage (indicated in the graph as  $W_i^*$ ) corresponds to the point of tangency between the ray and the effort function, since the said coefficient is at its minimum attainable level<sup>25</sup>. Figure 1(b) instead represents the corresponding industrial labor demand on the  $W_i - L_i$  space: at  $W_i^*$  the labor demand schedule has a kink, because entrepreneurs will resist any wage undercutting and keep the wage at its optimal level. Indeed, wages different than  $W_i^*$  would not minimize the cost of labor per efficiency unit and consequently would not be profit maximizing.

Unless the constraint forces them to act differently, capitalists set the wage at  $W_i^*$ ; as a result of the downward rigidity of the industrial wage, high-earning jobs will be rationed and only  $L_i^*$  workers will be hired. The remaining workers will be all employed in the rural sector at the market clearing wage, in accordance to equation 5 (which determines the  $L_a^d$  curve in figure 1b); thus a wage gap will arise endogenously across sectors. Clearly, the position of the  $L_i^d$  curve depends, among other factors, on the existing stock of capital, with a higher  $K$  causing *ceteris paribus* an outwards shift of the curve and hence an increase in  $L_i$ .

The adjustment process described so far, follows Kaldor's insights according to which employment creation in the manufacturing sector of typical developing countries is constrained by industrial labor demand and not by supply factors<sup>26</sup>. For this reason, the phase in which  $W_a < W_i$  will be called hereafter *Kaldorian underemployment*<sup>27</sup>. During such phase, "... a faster rate of increase in the demand

<sup>25</sup>It should be noted, however, that the effort function depends on the real agricultural wage ( $W_a/P$ ) and on the price index  $P$ , so that the optimal industrial wage itself is increasing in  $W_a$  and  $P$ .

<sup>26</sup>Quoting Kaldor's own words: "... the supply of labour in the high-productivity, high-earning sector is continually in excess of demand, so that the rate of labour-transference from the low to the high-productivity sectors is governed only by the rate of growth of demand for labor in the latter."(1968) See also Kaldor (1967).

<sup>27</sup>Kaldor actually calls this situation "labor surplus", but we preferred a different definition, in order to avoid confusion between the notion applied here, and Lewis's concept of surplus labor. Clearly, the notion of Kaldorian underemployment is logically tied to that of disguised unemployment, but in the present case the mismatch between the shadow wage (that is the opportunity cost of labor outside the modern sector) and the



for labour in the high-productivity sectors induces a faster rate of labour- transference even when it is attended by *a reduction, and not an increase, in the earnings-differential between the different sectors.*"<sup>28</sup>.

The complete analytical description of the inputs' market during the Kaldorian underemployment phase requires to derive, in addition to equations 5, 6, 7, 8 and 9, total profits and the labor market clearing, which are respectively given by

$$\Pi = \frac{\beta}{1-\beta} W_i L_i; \quad (10)$$

and

$$L_i + L_a = 1. \quad (11)$$

Note that in the last equation we have normalized the labor force to 1, so that  $L_a$  and  $L_i$  respectively represent the employment share of the traditional and of the modern sector; this simplifying normalization, however, comes at the cost of eliminating the effect of demographic variables on our economy.

It should be clear at this point, that Kaldorian underemployment persists only as long as the solution implied by the FOC is admissible, that is as long as  $W_a < W_i$ . Given the hypothesis of diminishing returns to labor in agriculture, however, the withdrawal of labor from the rural sector is bound to increase  $W_a$ ; moreover, since the elasticity of industrial wages to rural ones is lower than one, eventually the latter will reach  $W_i$  and the constraint will become binding. With reference to figure 1b, the expansion of the industrial sector (a shift of the  $L_i^d$  curve toward north-east) tends to close the wage gap, until eventually one uniform wage prevails. Capitalists are then compelled to pay workers a wage equal to the agricultural one, and the Kaldorian underemployment phase gives way to the *economic maturity*: "...a state of affairs where real income per head had reached broadly the same level in the different sectors of the economy."<sup>29</sup>. During the maturity phase employees will be indifferent between working in industry or in agriculture, and thus lack any incentive to increase their effort beyond  $E^*$ , despite any possible increase in the uniform real wage rate.

In light of this reasoning, during maturity wages will be set at

$$W_i = W_a; \quad (12)$$

while industrial labor demand and total profit will continue being determined by equations 9 and 10, with the only caveat that now the uniform wage rate replaces the value of  $W_i$  determined according to efficiency considerations. Obviously, the rural wage and rents determination, and the labor market clearing will hold also during maturity, so equation 5, 6 and 11 complement the description of the labor market.

## MARKET CLEARING

The complete characterization of the economy involves two more equations related to the market clearing for final goods: assuming that the economy is closed to international trade, such conditions are stated directly for food output, and by mean of the consumption expenditure flow identity as concerns manufactures. In determining the proportion of income devoted to personal consumption, we also assume that wage income as well as rents are entirely consumed, while profit-earners save a constant proportion  $s$  of their total income  $\Pi$ . Our system will therefore be completed by the following two equations:

$$X_a^c = X_a^s; \quad (13)$$

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market wage in the industrial sector occurs without any breach of the marginal theory of distribution.

<sup>28</sup>The quotation is taken from Kaldor (1968) page 386, italics in the original.

<sup>29</sup>The quotation is Kaldor's own definition of economic maturity, which he also defined as "the end of the dual economy" (1968).

for the food market (with the  $c$  and  $s$  suffixes meaning respectively consumed and supplied), and

$$P_a X_a^c + P_i X_i^c = W_a L_a + R + W_i L_i + (1 - s) \Pi; \quad (14)$$

for manufactures<sup>30</sup>. We note in passing that Walras law can be used to take manufactures as the numeraire, in order to have the industrial product wage equal to its nominal value, so that

$$P_i = 1. \quad (15)$$

### DYNAMIC OF CAPITAL STOCK

As concerns the dynamic of the state variable  $K$  (hence the long run characterization of the economy), we follow the Ricardian assumption that savings are automatically reinvested into increases of the capital stock. Combining this hypothesis with those underlying equation 14 we can describe the dynamic of the capital stock as

$$\dot{K} = s\Pi - \delta K;$$

where  $\dot{K}$  is the time derivative of the capital stock, and  $\delta$  expresses the depreciation rate of capital. Denoting by  $\hat{K}$  the capital growth rate, the dynamic of the capital stock may be rewritten as

$$\hat{K} = s \frac{\Pi}{K} - \delta. \quad (16)$$

This equation represents the fundamental differential equation of our model, and corresponds to the well-know Solow-Swan equation.

Stated as it is, ours is a "supply-limited model of industrial growth" - using Taylor's jargon - with market-clearing prices and flexible capital labor ratio, as opposed to the fix prices and technological coefficients characterizing the structuralist literature. It is important to emphasize that the choice of a supply-limited model in this context is not meant to undervalue the importance of keynesian arguments concerning the level of effective demand, but only to focus our attention on the *potential* growth path of an economy. Apart from the presence of increasing returns in industry, the distinctive feature of this model is the dualistic characterization of the labor market, leading to a Keynesian-like adjustment of the labor market, in which demand is the driving force of sectoral labor allocation.

### THE EQUILIBRIUM CONFIGURATION

Instead of directly solving the whole system of equations and determine the steady states, we prefer to proceed in three stages to highlight the various economic mechanisms at work in the development process. Holding the capital stock as a pre-determined variable - hence in the short run - the economy is analytically described by a system of fourteen independent equations with fourteen endogenous variables ( $X_a^c, X_i^c, X_a^s, X_i^s, L_a, L_i, P_a, P_i, \underline{P}, W_a, R, W_i, E, \Pi$ ). It is thus possible to determine the nominal industrial wage consistent with the clearing of the goods' market for each given level of capital stock; hereafter the correspondent locus of short-run equilibria in the  $\log W_i - \log K$  space is called **real wage schedule** (indicated as RW). At a second stage, the **locus of stationary capital stock** can be obtained from the dynamic equation 16, to express the value of the nominal industrial wage corresponding to the break-even situation with null net investment. Finally confronting the

<sup>30</sup>We can clarify the reason for closing the model using the consumption flow identity, by making use of some relations explained above: equations 5 and 6, together with food market clearing (equation 13) imply that  $W_a L_a + R = P_a X_a^s = P_a X_a^c$ ; while equation 10 implies that during Kaldorian underemployment  $\Pi = W_i L_i \beta / (1 - \beta)$ . Analogous implications hold during maturity, with the only difference that the wage rate is then common across the two sectors for equation 12. Regardless of the economic phase, hence, equation 14 can be rewritten as  $X_i^s - X_i^c = s\Pi$ , which shows that in equilibrium the excess supply of manufactures shall equate the total amount of savings of the profit-earners.

relative position of the two loci, the necessary conditions for the existence of steady state equilibria and for their stability properties can be determined on the base of relative slopes of the two curves. Clearly, because of the dichotomic working of the labor market before and after the maturity threshold  $W_a = W_i$ , the equilibria shall be derived separately for the two phases.

As emphasized by classical authors (Malthus, Marx and Ricardo above all) and by early development economists of the 50's and 60's, the elasticity of industrial labor supply is the pivotal magnitude summarizing the economic mechanisms at work. Its crucial role is evident once we note that in two-sectors macro models - unlike in aggregate models - this elasticity depends on the interaction between technological conditions (namely the evolution of labor productivity across sectors), demographic variables, and movements in relative prices, while it concurs to determine the speed of labor reallocation across sector, and the effect of such reallocation in terms of profitability.

During Kaldorian underemployment, the fourteen equations composing the system are: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15. From such equations, it can be shown after some algebraic manipulations (see the Mathematical Appendix I.A), that the elasticity of labor supply ( $\epsilon^{LS} \equiv \partial \log L_i / \partial \log W_i$ ) faced by industrial entrepreneurs is equal to

$$\epsilon^{LS} = \frac{(1 - \alpha)(1 - \gamma)(1 - L_i)(X_a - Z)}{\left\{ [\gamma + \alpha(1 - \gamma)] + \frac{Z}{\psi} P_a^{1-\alpha} \right\} [X_a(1 - bL_i) - ZL_a] + \gamma bL_i(X_a - Z) \left( 1 + \frac{Z}{\psi} P_a^{1-\alpha} \right)}; \quad (17)$$

Two observations are important at this stage: labor supply elasticity is non negative for the assumed parametrization - given that all terms in the numerator and denominator are positive - and it is a decreasing function of  $L_i$ , as proved in mathematical appendix I.B<sup>31</sup>. The negative dependency of  $\epsilon^{LS}$  on  $L_i$  arises because *ceteris paribus* a higher industrial labor share turns relative prices in favor of agriculture, hence the nominal wage  $W_i$  will have to grow proportionally more to attract additional workers to industry.

Continuing with a bit of algebra (see the Mathematical Appendix I.C), it can be demonstrated that during Kaldorian Underemployment the equation of the short-run equilibrium locus in log terms is given by

$$\begin{aligned} & (1 - \alpha) \log W_i - \frac{1}{1 - \gamma} \log \Theta + \frac{b\gamma}{1 - \gamma} \log \left[ 1 - \Phi \exp \left( \frac{\mu + \beta}{\beta} \log K - \frac{1}{\beta} \log W_i \right) \right] + \frac{\gamma + \alpha(1 - \gamma)}{1 - \gamma} \\ & \left\{ \log \left[ A_a \left( 1 - \Phi \exp \left( \frac{\mu + \beta}{\beta} \log K - \frac{1}{\beta} \log W_i \right) \right)^{1-b} - Z \right] - \log \left[ \Phi \exp \left( \frac{\mu + \beta}{\beta} \log K - \frac{1}{\beta} \log W_i \right) \right] \right\} = \log \\ & \left\{ 1 + \Xi \exp \left[ (1 - \alpha) \left( \frac{\mu + \beta}{\beta} \log K - \frac{1 - \beta}{\beta} \log W_i \right) \right] \left[ A_a \left( 1 - \Phi \exp \left( \frac{\mu + \beta}{\beta} \log K - \frac{1}{\beta} \log W_i \right) \right)^{1-b} - Z \right]^{\alpha-1} \right\}; \quad (18) \end{aligned}$$

where  $\Phi$ ,  $\Theta$  and  $\Xi$  are constants defined respectively as

$$\begin{aligned} \Phi & \equiv A_i^{\frac{1}{\beta}} (1 - \beta)^{\frac{1}{\beta}} (E^*)^{\frac{1-\beta}{\beta}}; & \Theta & \equiv \left( \frac{\omega}{1 - d} \right)^{\frac{1}{\alpha}} A_a^\gamma (1 - b)^\gamma \psi^{1-\gamma} \left[ \frac{\alpha(1 - s\beta)}{(1 - \alpha)(1 - \beta)} \right]^{\gamma + \alpha(1 - \gamma)}; \\ \Xi & \equiv \Phi^{1-\alpha} \frac{Z}{\psi} \left[ \frac{\alpha(1 - s\beta)}{(1 - \alpha)(1 - \beta)} \right]^{1-\alpha}. \end{aligned}$$

To determine the long-run equilibrium of the system, instead, one simply needs to replace  $L_i$  in equation 16 with its short-run equilibrium value, taken during Kaldorian underemployment from

<sup>31</sup>By setting  $Z$  equal to zero in 17, one can verify that the two above results do not depend on the non-homoteticity of the preferences' structure: simply in the case of Cobb-Douglas utility function the labor supply elasticity would simplify to

$$\epsilon^{LS} \equiv \frac{\partial \log L_i}{\partial \log W_i} = \frac{(1 - \alpha)(1 - \gamma)(1 - L_i)}{\gamma + \alpha(1 - \gamma)(1 - bL_i)}.$$

equation 9, and set  $\hat{K}$  equal to zero. This operation, expressing everything in logarithmic terms, yields

$$\log \left[ \frac{s\beta}{\delta} (1 - \beta)^{\frac{1-\beta}{\beta}} (E^*)^{\frac{1-\beta}{\beta}} \right] + \frac{1}{\beta} \log A_i - \frac{1-\beta}{\beta} \log W_i^{**} + \frac{\mu}{\beta} \log K = 0; \quad (19)$$

where we used the notation  $W_i^{**}$  in order to distinguish the wage compatible with break-even investment from the short-run equilibrium wage.

Total differentiation of equation 18 yields the coefficient of the real wage schedule for the Kaldorian underemployment interval:

$$\frac{\partial \log W_i}{\partial \log K} = \frac{\mu + \beta}{1 + \beta \epsilon^{LS}}. \quad (20)$$

This coefficient is surely positive, given that the labor supply elasticity is non-negative, and furthermore it is decreasing in  $\epsilon^{LS}$ . Indeed, a given increase in the capital stock will trigger an outflow of labor from agriculture<sup>32</sup>, and the higher the elasticity of industrial labor supply the smaller - *ceteris paribus* - the adjustment in nominal industrial wages required by the expansion  $L_i$ . Besides, since a raise in industrial employment share reduces  $\epsilon^{LS}$ , the real wage schedule will be flatter for low levels of  $L_i$ , and get gradually steeper as industry expands its employment basin. On the other hand, the higher the output elasticity to capital ( $\mu + \beta$ ), the higher the industrial wage in equilibrium, hence the greater the coefficient of the real wage schedule.

As for the stationary capital locus, a close inspection of equation 19 shows that in the  $\log W_i - \log K$  space it represents a straight line sloped

$$\frac{\partial \log W_i^{**}}{\partial \log K} = \frac{\mu}{1 - \beta}. \quad (21)$$

Given the parametrization, the coefficient is positive and increasing in  $\mu$ : the higher the external capital effect, the stronger the positive impact of capital accumulation on the industrial TFP, the higher total profits and the higher the nominal wage compatible with the break-even level of investment. On the other hand, the stationary capital locus is also steeper the greater the capital share, because a higher  $\beta$  means, *ceteris paribus*, a higher level of total profits for the same increase in capital stock<sup>33</sup>, so a higher level of reinvestment.

In plain words, during Kaldorian underemployment higher values of the capital stock trigger the expansion of industries (in terms of labor share and output), leading the agricultural terms of trade to augment; this relative price movement, summed to the withdrawal of labor from agriculture, causes a sharp raise of the rural wage. Both the relative price movement and the rural wage increase drive the upwards adjustment of industrial wages to satisfy the Solow condition. As shown in Mathematical Appendix I.D, the adjustment process required to get the equilibrium in the goods' market is such that higher levels of  $K$  entail a reduction in the wage (and productivity) gap between manufacturing and agricultural activities, to the extent that for sufficiently high capital stock a unique uniform wage (and labor productivity) will prevail in the economy.

Once this happen, and the constraint  $W_a = W_i$  becomes binding, the system enters the maturity phase and the above equilibrium configuration ceases to hold. Indeed, the short-run characterization of the mature economy is still described by equations 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 13, 14, 15, but unlike in the Kaldorian underemployment phase equation 12 now replaces equation 8. As shown formally in Mathematical Appendix II.A, the prevalence of one uniform wage alters significantly the dynamic in the labor market: sectoral labor shares stabilize, regardless of the capital stock, at the level implicitly defined by the equation

$$(1 - \alpha)(1 - \beta) \left[ A_a (1 - L_i)^{1-b} - Z \right] = \alpha(1 - b)(1 - s\beta) A_a L_i (1 - L_i)^{-b}; \quad (22)$$

and meanwhile the labor supply elasticity turns to zero.

<sup>32</sup>Industrial labor demand depends positively on the capital stock  $K$  (see equation 9).

<sup>33</sup>Recall that  $\Pi = W_i L_i \beta / (1 - \beta)$ .

The null elasticity of industrial labor supply during maturity modifies also the real wage schedule, whose equation is then

$$\begin{aligned} \log \frac{(1-\alpha)(1-\beta)}{\Phi\alpha(1-b)(1-s\beta)A_a} + \log \left\{ A_a \left[ 1 - \Phi \exp \left( \frac{\mu+\beta}{\beta} \log K - \frac{1}{\beta} \log W_i \right) \right]^{1-b} - Z \right\} = \\ = \frac{\mu+\beta}{\beta} \log K - \frac{1}{\beta} \log W_i - b \log \left[ 1 - \Phi \exp \left( \frac{\mu+\beta}{\beta} \log K - \frac{1}{\beta} \log W_i \right) \right]; \end{aligned} \quad (23)$$

from which we see that the RW curve on the usual  $\log W_i - \log K$  plane degenerates into a half-line sloped

$$\frac{\partial \log W_i}{\partial \log K} = \mu + \beta; \quad (24)$$

(for a formal proof see the Mathematical Appendix II.B). The slope of the short-run equilibrium locus is now steeper than during the Kaldorian underemployment phase, since the tendency of wages to grow along with capital accumulation (captured by the term  $\mu + \beta$ ) is not mitigated by the effect of elastic labor supply. Considering the whole trend of the RW schedule on the  $\log W_i - \log K$  space, it is first increasing and convex as long as Kaldorian underemployment persists, while after the corner point at the maturity threshold it turns into an upward-sloping half line<sup>34</sup>.

As for the long-run equilibrium of the system, even in maturity the stationary capital locus will continue to be expressed by equation 19, given that equation 9 continues to hold and that nothing alters the differential equation of capital accumulation.

Superimposing the short-run and long-run equilibrium loci we can determine the equilibria, at the interception points, and their stability properties, according to the relative position of the two curves. Ideally, the economy moves along the real wage diagram, with the capital stock growing as long as the short-run equilibrium wage lies below the  $\dot{K} = 0$  locus, and shrinking if the opposite happens. The reason for this is the behavior of total profits, and hence of investment: when the short-run equilibrium wage lies below that compatible with null net investment, reinvested profits will exceed depreciation costs and fuel capital accumulation, while in the opposite situation net investment will be negative and capital stock will fall.

To check the stability properties of the equilibria, differentiate equation 16 with respect to  $\log K$ , make use of the fact that  $\Pi = \beta W_i L_i / (1 - \beta)$ , obtaining

$$\frac{\partial \hat{K}}{\partial \log K} = \frac{s\beta}{1-\beta} \frac{W_i L_i}{K} \left( \frac{\mu}{\beta} - \frac{1-\beta}{\beta} \frac{\partial \log W_i}{\partial \log K} \right);$$

then replace  $\partial \log W_i / \partial \log K$  with the slope of the real wage schedule either for the Kaldorian underemployment phase or for the maturity (equation 20 and 24 respectively)<sup>35</sup>. Since the first two factors are surely non-negative, the sign of the above derivative will be determined by the sign of the term within parentheses. Given that for an unstable equilibrium to emerge  $\partial \hat{K} / \partial \log K$  should necessarily be positive, proposition I can be stated.

<sup>34</sup>While being a piece-wise function, the short-run equilibrium locus is continuous over its whole domain, and continuously differentiable but with the exception of the corner point.

<sup>35</sup>This operation obtains, for Kaldorian underemployment

$$\frac{\partial \hat{K}}{\partial \log K} = \frac{s\beta}{1-\beta} \frac{W_i L_i}{K} \left[ \frac{\mu(1+\beta\epsilon^{LS}) - (1-\beta)(\mu+\beta)}{1+\beta\epsilon^{LS}} \right];$$

and for the maturity phase

$$\frac{\partial \hat{K}}{\partial \log K} = \frac{s\beta}{1-\beta} \frac{W_i L_i}{K} [\mu - (1-\beta)];$$

hence the sign of  $\partial \hat{K} / \partial \log K$  depends on the terms within square brackets.

**Proposition I** *The necessary conditions for the occurrence of an unstable equilibrium are respectively*

$$\mu > \frac{1 - \beta}{1 + \epsilon^{LS}}; \quad (25)$$

*for Kaldorian underemployment ( $\epsilon^{LS}$  being valued in the neighborhood of the equilibrium), and*

$$\mu > 1 - \beta. \quad (26)$$

*for maturity.*

Proposition I can easily be interpreted in terms of slopes of RW and  $\hat{K} = 0$  locus: it simply states that for a generic equilibrium to be unstable, it is necessary that the real wage schedule be flatter than the stationary capital arm<sup>36</sup>.

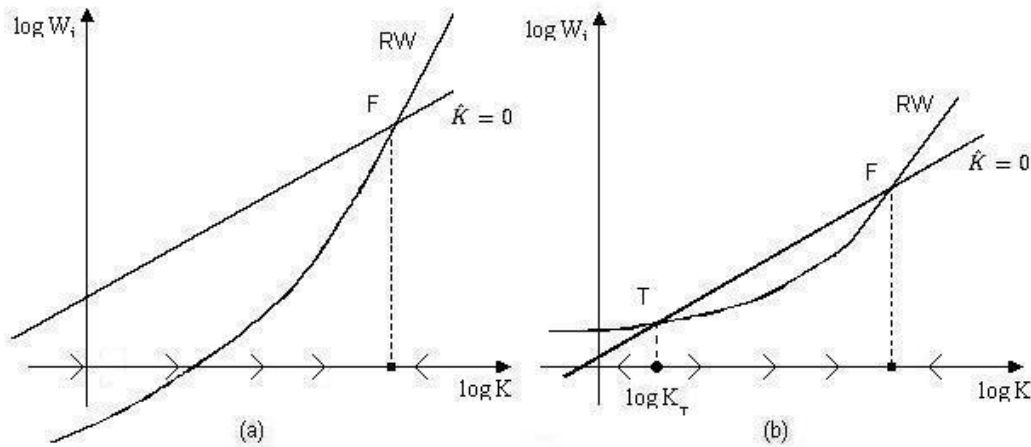


Figure 2: The model

Figure 2 presents two possible configurations of the system characterized by different parametrizations. A third possible configuration is the one in which the RW schedule cuts the stationary capital locus only once and from above (and hence either condition 25 or 26 would be met). In such a case, for capital stocks lower than that corresponding to the interception of the two curves, the economy is stuck in a low equilibrium trap, while for  $K$  higher than the threshold level the system diverges toward an infinite capital stock (and wage rate) with manufacturing production growing indefinitely despite an ultimately stable labor share<sup>37,38</sup>.

Considering at first the case of figure 2a (in which  $\partial \hat{K} / \partial \log K < 0$  over the whole domain), there are two possible equilibria: an unstable equilibrium of pure subsistence at zero capital stock, and an asymptotically stable equilibrium of full-industrialization F, where both sectors coexist. Clearly

<sup>36</sup>The derivation of sufficient conditions for the existence of the poverty trap is not feasible in our macro-model, because it would require an explicit solution of the indeterminate form  $\lim_{K \rightarrow 0} \frac{W_i L_i}{K}$ . Intuitively, the existence of a poverty trap requires the industrial wage to drop so rapidly, as  $K$  shrinks, so as to be insufficient to stimulate from industrial workers a positive efficiency.

<sup>37</sup>A necessary condition for this case to occur is the presence of constant or increasing returns to capital, otherwise in the maturity phase the RW schedule would eventually cut once again the  $\hat{K} = 0$  locus.

<sup>38</sup>Given the lack of explicit solution for the steady state, it is not possible to dismiss from a theoretical point of view a fourth case with no interception between the two schedules, and the short-run equilibrium lying entirely above the stationary capital locus (this necessarily requires decreasing marginal productivity of capital). A closer analysis however reveals that such outcome results from extremely low values of the industrial TFP relative to  $\omega$  (the threshold level of real wage necessary to obtain positive effort from workers). Clearly, this rather implausible limiting case implies that no matter how big the capital stock, industrialization will never be self-financing, so that the economy will always be stuck at the purely agrarian stage.

in this case the existence of the equilibrium of full industrialization also requires equilibrium F to be stable, which in turn implies that the industrial production function should display decreasing returns to capital. On the contrary, had the industrial production function been an AK technology ( $\mu = 1 - \beta$ ) or had it exhibited increasing returns to capital ( $\mu > 1 - \beta$ ), there would have been just one unstable equilibrium at  $K = 0$ , but for any positive value of  $K$  capital accumulation could have proceeded indefinitely (in the former case of an AK technology) or even at a growing speed (in the latter case).

The situation depicted resembles closely the results of the standard neoclassical growth model in a two-sectors framework. A new feature is however the convex interval of the RW curve, during which the economy undergoes a process of industrialization (Rostow's take-off), and changes in output and employment composition are fostered by the relatively elastic supply of industrial labor<sup>39</sup>. Moreover, the transference of labor from the low-productive to the high-productive sector entails a double gain in terms of growth: on the one hand, marginal labor productivity in agriculture grows because of diminishing returns to labor, on the other, increasing returns accelerate the growth of productivity in industry, fuelling the expansion of the capital stock and of the whole manufacturing sector<sup>40</sup>.

The structural dynamic described by the Kaldorian Underemployment phase rationalizes several stylized facts often cited in the literature concerning LDCs<sup>41</sup>:

- The "agriculture-industry shift", meaning the declining importance of agriculture in terms of both employment share and percentage contribution to GDP in the course of economic development;
- The existence of wide productivity gaps across economic sectors, with agriculture featuring a much higher employment share than its correspondent GDP share, and hence having a lower average labor productivity than the rest of the economy. Such productivity gaps are mirrored by urban-rural wage gaps, which act as a stimulus to labor reallocation toward city-based industrial employment;
- The progressive reduction of the intersectoral differences in productivity (and wages), as labor reallocation toward industry raises agricultural labor productivity relative to the rest of the economy;
- The well-documented Engel's law, implying a declining importance of food expenditure share as income grows;
- The S-shaped dynamic of saving and investment ratios as GDP grows, with a strong acceleration at low-middle income levels<sup>42</sup>.

As capital accumulation proceeds, however, labor supply turns gradually more inelastic, wage gaps close and the system eventually enters in the maturity phase and stabilizes its employment structure (see equation 22) with the coexistence of both sectors. From that point onwards,  $K$  grows at a slower

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<sup>39</sup>Population growth, which was omitted from our analysis, would basically prolong the Kaldorian underemployment phase by increasing the number of agricultural workers (since industrial jobs are rationed), and reinforcing the tendency of the labor supply to be elastic.

<sup>40</sup>Again Kaldor (1968) expresses this idea very clearly: "... the growth of productivity is accelerated as a result of the transfer at both hands - both at the gaining end and at the losing end; in the first, because, as a result of increasing returns productivity in industry will increase faster, the faster output expands; in the second because when the surplus-sectors lose labour, the productivity of the remainder of the working population is bound to rise."

<sup>41</sup>For a more detailed exposition of these stylized facts see among others Kuznets (1966), Chenery and Syrquin (1975), Syrquin (1989), Taylor (1989) and Bhaduri (1993 and 2003); as regards sectoral wage differential, evidence is often cited in the migration literature, especially for the so-called Todarian models.

<sup>42</sup>Note that the S-shaped dynamic of the investment share of GDP may also shed some light on why capital accumulation is a particularly important engine of growth at low- and middle-income levels, while TFP growth becomes the dominant force at high income levels.

pace, while the combined effect of relative price movements and wage adjustment tends to reduce total profits bringing the system to the stable equilibrium F<sup>43</sup>. Clearly the maturity stage describes the situation of more developed countries, in which the "agriculture-industry shift" has already taken place and structural dynamics typically involve the further expansion of the service sector.

Alternatively, consider the case illustrated in figure 2b, where  $\partial\hat{K}/\partial\log K$  is first positive and then negative. Three equilibria are then possible: (i) a locally stable equilibrium of pure subsistence with zero capital stock, (ii) an unstable low development equilibrium at point T, and (iii) a stable equilibrium of full industrialization at F. For capital stocks lower than  $K_T$  there is an unstable poverty trap causing capital stock to shrink over time while the economy falls back to the state of pure agricultural subsistence. On the other hand, when  $K > K_T$  the effect of increasing returns raises profitability sufficiently to trigger an accelerated growth and a self-fulfilling process of capital accumulation, driving the system to the equilibrium of full industrialization F<sup>44</sup>.

This situation may call for a big push à la Rosenstein Rodan<sup>45</sup>, that is a concerted investment capable of bringing the capital stock beyond  $K_T$ , breaking the poverty trap and making the industrialization process feasible. The relevance of coordination failures and big push policies is further reinforced if we consider the role of the "social overhead capital", and of all sorts of capital characterized by large complementarities, and thus capable to crowd in private investments and stimulate significant supply responses<sup>46</sup>.

In light of the recent wave of criticism against the idea of poverty traps<sup>47</sup>, few words should be spent commenting the situation described in Figure 2b. First of all, it should be pointed out that the poverty trap discussed here is not driven by lack of savings, but by insufficient profitability. Increases in the saving propensity do not alter the necessary condition for the existence of the poverty trap, but only act as a parametric shift of the two curves, and as such may only change the basins of attraction (see Section III for more details). As a consequence, the poverty trap may hold even in presence of international flows of capital, regardless of whether capital markets work perfectly or not. If anything, international capital markets would rather attract resources away from low-yielding national assets, thereby exacerbating the situation.

Secondly, the unstable equilibrium of pure agrarian economy does not necessarily entail a zero growth: the analysis so far has taken sectoral TFP as parameters, however exogenous technical progress acts also in the agricultural sector, and may spur the growth performances even of a completely agricultural economy (in addition to modifying the whole equilibrium configuration, as will be shown later). Thirdly, it is worth noting that the degree of increasing returns required to make the poverty trap a relevant case in our set-up is far lower than in other aggregate models<sup>48</sup>; even a value of  $\mu$  around 0.2 (hence within the estimates cited by Kraay and Raddatz) may be sufficient to make the low equilibrium trap plausible. The reason is that the effect of increasing returns is amplified here by

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<sup>43</sup>These findings seem to confirm the empirical evidence which suggests growth accelerations occurring at middle income level, when capital accumulation is faster and the economy enjoys a double gain from industrialization. See Chenery and Syrquin (1975) and Syrquin 1988.

<sup>44</sup>Again, had the technology displayed increasing returns to K there may have been just one unstable interception, after which the system would diverge indefinitely. This was precisely the third possible configuration we referred to, when commenting figure 2 above.

<sup>45</sup>According to Skott and Ros (1997): "... the essence of a big-push argument is a model with multiple equilibria in which, under certain initial conditions, the economy gets stuck in a poverty trap that can only be overcome through a "big push": No individual firm may have an incentive to expand on its own, even though the coordinated expansion by all firms will be profitable and welfare enhancing."

<sup>46</sup>In the recent literature, the importance of big push considerations in presence of non-tradeable inputs as infrastructures (and more generally of social overhead capital) is emphasized also by Ros and Skott (1997) and Sachs (2005). Despite its relevance, the big push argument should be considered with caution, and not uncritically equated to the so-called "classical aid narrative" (see Easterly 2006), which claims that a sufficient amount of aid would automatically lift countries out of the poverty trap to the take off.

<sup>47</sup>See Kraay and Raddatz 2007 and Easterly 2006.

<sup>48</sup>See for instance equation 11 of Kraay and Raddatz 2007.



the elasticity of industrial labor supply, a factor rather disregarded in aggregate models of growth, although crucial for classical authors.

Finally we note in passing that the above model suggests a theoretical mechanism able to link the multiplicity of equilibria with the structural characteristics of the economy, namely the extent of "agriculture-industry shift". Simulated work based on analogous premises (the "variable returns to scale model") has recently confirmed that this line of reasoning may be empirically fruitful in explaining the poor economic performance of LDCs vis à vis rich nations<sup>49</sup>.

### III. *Comparative statics: the effect of technical progress*

#### PARAMETRIC INCREASE IN AGRICULTURAL TFP

So far, the analysis of the two-sectors economy abstracted from technical progress, and treated the sector-specific TFPs as exogenous parameters. This approach may be convenient from an analytical point of view, but overlooks one of the main forces - if not the main force - behind the long-term increases in income: technical change.

Needless to say, increases in TFP, be it agricultural or industrial, have an unambiguous positive welfare effect, for they allow a greater supply of goods by using more efficiently the given amount of resources. More complex, however, are the effects of technical progress on the equilibrium configuration for the whole dynamic system. Precisely to grasp these effects, we now carry out some comparative statics exercises with regard to sectoral TFPs.

As seen before, any long-run equilibrium, whether stable or unstable, is basically defined by the system between the relevant expression for the real wage schedule (equation 18 for Kaldorian Underemployment and equation 23 for the maturity phase) and the stationary capital locus (equation 19 for the whole domain). To lean down the notation let us rewrite the system as

$$\begin{cases} RW(\log W_i, \log K, A_a, A_i) = 0; \\ G(\log W_i, \log K, A_a, A_i) = 0; \end{cases} \quad (27)$$

where the implicit function  $RW(\cdot)$  is the short-run equilibrium schedule and  $G(\cdot)$  indicates the stationary capital locus.

Besides, recall that the real wage schedule is continuously differentiable with respect to its four arguments (but with the exception of the corner point corresponding to the threshold between Kaldorian Underemployment and maturity), while the  $\dot{K} = 0$  locus is continuously differentiable with respect to the four arguments on its whole domain. In light of this, and provided that the Jacobian of system 27 is non singular, the hypotheses underlying the implicit function theorem are satisfied over the whole domain, excluding the neighborhood of the corner point. With such exception, the implicit function theorem can therefore be applied in the neighborhood of a generic equilibrium (call it point Q) to rewrite system 27 as

$$\begin{cases} RW(\log W_i^Q(A_a, A_i), \log K^Q(A_a, A_i), A_a, A_i) = 0; \\ G(\log W_i^Q(A_a, A_i), \log K^Q(A_a, A_i), A_a, A_i) = 0; \end{cases} \quad (28)$$

in which  $(\log W_i^Q, \log K^Q)$  are the coordinates of the equilibrium point.

As concerns changes in the agricultural total factor productivity, the chain rule theorem can be

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<sup>49</sup>See Graham and Temple (2005).

used to compute the total derivative of each function in system 28 with respect to  $A_a$ , obtaining:

$$\begin{cases} \left. \frac{\partial RW}{\partial \log W_i} \right|_Q \frac{\partial \log W_i^Q}{\partial A_a} + \left. \frac{\partial RW}{\partial \log K} \right|_Q \frac{\partial \log K^Q}{\partial A_a} = -\frac{\partial RW}{\partial A_a}; \\ \left. \frac{\partial G}{\partial \log W_i} \right|_Q \frac{\partial \log W_i^Q}{\partial A_a} + \left. \frac{\partial G}{\partial \log K} \right|_Q \frac{\partial \log K^Q}{\partial A_a} = -\frac{\partial G}{\partial A_a}. \end{cases} \quad (29)$$

Solving this last system for  $\partial \log W_i^Q / \partial A_a$  and  $\partial \log K^Q / \partial A_a$  permits to obtain, from the sign of these derivatives, the direction in which the new equilibrium value (call it Q') resulting from the change in  $A_a$  will lie. Analytically, it can be shown that such solutions are<sup>50</sup>

$$\frac{\partial \log W_i^Q}{\partial A_a} = \frac{\begin{vmatrix} -\frac{\partial RW}{\partial A_a} & \frac{\partial RW}{\partial \log K} \\ -\frac{\partial G}{\partial A_a} & \frac{\partial G}{\partial \log K} \end{vmatrix}}{|J|}; \quad \frac{\partial \log K^Q}{\partial A_a} = \frac{\begin{vmatrix} \frac{\partial RW}{\partial \log W_i} & -\frac{\partial RW}{\partial A_a} \\ \frac{\partial G}{\partial \log W_i} & -\frac{\partial G}{\partial A_a} \end{vmatrix}}{|J|}; \quad (30)$$

While these two expressions hold in general over the whole domain (except in the neighborhood of the corner point), the piecewise nature of the real wage schedule implies that comparative statics should be carried out separately for each phase: Kaldorian underemployment and maturity.

Proceeding with a taxonomic logic, suppose first that equilibrium Q occurs during the Kaldorian underemployment phase. In such a case, the partial derivatives in 30 should be replaced with their actual values computed from equations 18 and 19. Indicating with  $J^{KU}$  the Jacobian corresponding to the Kaldorian underemployment phase, this operation yields:

$$\begin{cases} \frac{\partial \log W_i^Z}{\partial A_a} = -\frac{\mu}{\beta A_a} \frac{[(1-\gamma)X_a + \gamma Z] \left(1 + \frac{Z}{\psi} P_a^{1-\alpha}\right) - (1-\gamma)(1-\alpha)X_a}{(X_a - Z) \left(1 + \frac{Z}{\psi} P_a^{1-\alpha}\right)} \frac{1}{|J^{KU}|}; \\ \frac{\partial \log K^Z}{\partial A_a} = -\frac{1-\beta}{\beta A_a} \frac{[(1-\gamma)X_a + \gamma Z] \left(1 + \frac{Z}{\psi} P_a^{1-\alpha}\right) - (1-\gamma)(1-\alpha)X_a}{(X_a - Z) \left(1 + \frac{Z}{\psi} P_a^{1-\alpha}\right)} \frac{1}{|J^{KU}|}; \end{cases} \quad (31)$$

Under the assumed parametrization, 31 implies that the two derivatives under consideration assume the opposite sign of  $|J^{KU}|$  (see Mathematical Appendix III.A for more details).

Moving to the maturity phase, the same procedure shall be followed to carry out the comparative statics, replacing the partial derivatives of equation 30 with their actual values calculated from equations 23 and 19. As shown in Mathematical Appendix III.A, this procedure yields

$$\begin{cases} \frac{\partial \log W_i^Q}{\partial A_a} = -\frac{\mu Z}{\beta(X_a - Z)A_a} \frac{1}{|J^{MA}|}; \\ \frac{\partial \log K^Q}{\partial A_a} = -\frac{(1-\beta)Z}{\beta(X_a - Z)A_a} \frac{1}{|J^{MA}|}; \end{cases} \quad (32)$$

proving that during maturity the two derivatives considered take the opposite sign of  $|J^{MA}|$ .

Furthermore, Samuelson's "correspondence principle between statics and dynamics"<sup>51</sup> can be utilized to prove that

$$|J^{KU}| > 0 \iff \mu > \frac{1-\beta}{1+\epsilon^{LS}};$$

<sup>50</sup>Of course in the following section all partial derivatives should be valued at Q, that is at the value corresponding to the equilibrium; for simplicity we omit this detail from the notation of the text.

<sup>51</sup>The principle was analyzed by Samuelson in 1941 and 1947; for a recent treatment of the principle see Gandolfo (1997).

and

$$|J^{MA}| > 0 \iff \mu > (1 - \beta);$$

meaning that  $|J^{KU}|$  ( $|J^{MA}|$ ) is positive when the corresponding equilibrium point is unstable, and negative in the opposite case<sup>52</sup>. The implications of the correspondence principle for equation 31 and 32 are summarized in proposition II.

**Proposition II** *Parametric increases in agricultural TFP reduce the basin of attraction of the locally stable equilibrium of pure subsistence, provided that an unstable equilibrium exists for positive values of  $K$ <sup>53</sup>. Furthermore increases in  $A_a$  move the stable equilibrium (if any) - be it in the Kaldorian underemployment interval or in the maturity one - towards North-East, increasing the steady state value of  $\log W_i$  and  $\log K$ <sup>54</sup>.*

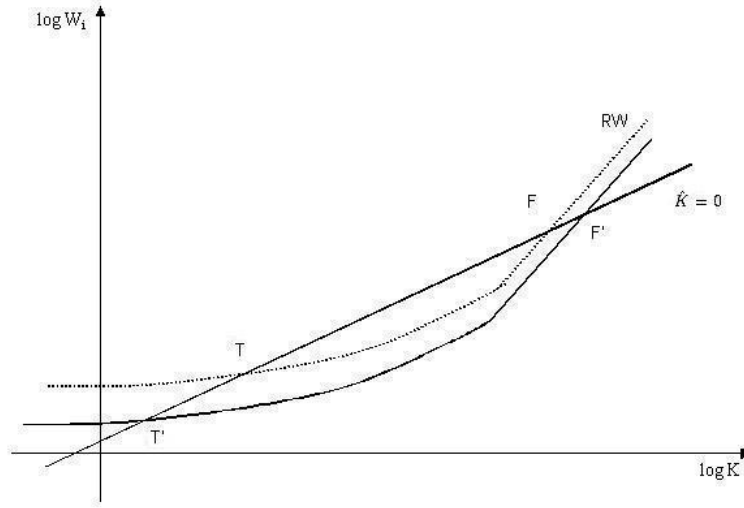


Figure 3: The effect of an increase in agriculture TFP

The results of proposition II are shown diagrammatically in figure 3, representing the case in which a poverty trap occurs during Kaldorian underemployment (dashed schedules represent the equilibrium loci before the TFP increase). The modification of the real wage schedule vis à vis the invariance of the stationary capital locus, reduces the basin of attraction of the low-level equilibrium - from  $(-\infty, \log K^T)$  to  $(-\infty, \log K^{T'})$ , correspondingly lowering the minimum critical level of capital beyond which increasing returns make industrialization self-sustaining. Intuitively, the increase in  $A_a$  leads to a larger availability of food for given agricultural employment share and capital stock. This fact lowers the agricultural terms of trade and in turn raises *ceteris paribus* the real wages of both sectors, thus allowing a higher profitability to capitalist entrepreneurs in industry; moreover, the increase in  $A_a$  helps closing the wage gap (remember that  $\gamma < 1$ ), thus lowering the level of capital stock at which maturity starts.

It is worth noting that in the maturity phase - unlike in the case of Kaldorian underemployment - the positive link between agricultural TRF and the steady state equilibria of full industrialization mentioned in proposition II relies on the non-homoteticity of the preference structure. Had preferences been homotetic, during maturity the increase in  $A_a$  would have only triggered changes in relative

<sup>52</sup>Recall that for the implicit function theorem to hold,  $|J^{KU}|$  and  $|J^{MA}|$  must be different from zero.

<sup>53</sup>Indeed the derivatives in 31 or 32 are both negative. Clearly, whenever the equilibrium of pure subsistence is unstable (as in the case of figure 2a) parametric changes in  $A_a$  will only affect the position of the globally stable equilibrium F.

<sup>54</sup>In this case the derivatives in expression 31 and 32 are both positive.

prices, leaving the industrial labor share, the wage rate and the capital stock in equilibrium unaffected (as can be verified by putting  $Z = 0$  in equation 32). As a matter of fact, this result conforms with existing literature that widely acknowledged the importance of Engel's effects in reinforcing structural change (see for instance in Murphy Shleifer and Vishny (1989), Stokey (1988), Matsuyama (1992), but also in Pasinetti (1993)).

In line with the above results, the importance of the primary sector in the early phases of industrialization is confirmed by the comparison between two emblematic historical cases: USSR in the 20s and China in the late 70s and 80s<sup>55</sup>. In the former case, the land reform of 1917 was unable to stimulate decisive productivity improvements in agriculture leading to sharp increases in food prices and to great social unrest. As a result, industrialization in USSR implied deep conflicts between cities and countryside, and capital accumulation could take place only through forcible surplus extraction from the countryside. In contrast, China under Deng-Xiao-Ping embarked in a program of agrarian reforms, which stimulated large productivity improvements. The increase in grain supplies helped maintaining urban real wages at a competitive level, favoring capital accumulation and fuelling industrial growth, while the rural sector maintained a reservoir of cheap labor for the high-yielding industrial areas on the coast.

Besides, an interesting parallel could be drawn between the role of agriculture in the present model of industrialization and the role of agriculture in the Kaleckian and structuralist interpretation of inflation in developing countries. In the Kaleckian literature, the inability of agricultural productivity to keep the pace with the growing industrial sector leads to the so-called "wage-good-constraint": the increase in food prices exerts upward pressure on nominal wages, thus triggering an inflation spiral<sup>56</sup>. In the present set-up, it could be argued that the efficiency wage mechanism during the Kaldorian underemployment phase acts in a way that turns the "wage-good-constraint" into a potential profitability constraint possibly giving raise to a poverty trap: unless food is available at a sufficiently low price, capital accumulation is simply not self-sustaining for low capital stock, so that the system falls back towards a purely agrarian economy.

#### THE CASE OF PARAMETRIC INCREASE IN INDUSTRIAL TFP

Applying the same procedure used for parametric changes in the agricultural TFP, we can shed some light also on the comparative statics regarding increases in  $A_i$ <sup>57</sup>. Total derivation of system 28 with respect to  $A_i$  yields

$$\begin{cases} \left. \frac{\partial RW}{\partial \log W_i} \right|_Q \frac{\partial \log W_i^Q}{\partial A_i} + \left. \frac{\partial RW}{\partial \log K} \right|_Q \frac{\partial \log K^Q}{\partial A_i} = - \frac{\partial RW}{\partial A_i}; \\ \left. \frac{\partial G}{\partial \log W_i} \right|_Q \frac{\partial \log W_i^Q}{\partial A_i} + \left. \frac{\partial G}{\partial \log K} \right|_Q \frac{\partial \log K^Q}{\partial A_i} = - \frac{\partial G}{\partial A_i}; \end{cases} \quad (33)$$

<sup>55</sup>In this respect more recent evidence concerns of the contrasting experience of Asian and African countries as regards the impact of the Green Revolution in raising agricultural yields (see Sachs 2005): whereas in the former countries agricultural productivity rose steadily along the 70s paving the way for the successive industrialization, in Sub-Saharan Africa food production per capita actually fell. Though suggestive, the picture in this case is however blurred by other factors such as demographic changes, soil depletion, desertification etc.

<sup>56</sup>See for instance Kalecki 1976 and Basu 1997. Note however that this idea was already present in Kaldor 1954, and 1967, with special reference to the burst of inflation crisis in Latin America.

<sup>57</sup>Note that, because of the algebraic properties of Cobb Douglas production functions, all forms of technical change - unbiased, labor augmenting and capital augmenting (also called Hicks neutral, Harrod neutral and Solow neutral) - translate into variations of the parameter  $A_i$ , and are thus essentially indistinguishable from one another.

while solving the above system for  $\partial \log W_i^Q / \partial A_i$  and  $\partial \log K^Q / \partial A_i$  obtains

$$\frac{\partial \log W_i^Q}{\partial A_i} = \frac{\begin{vmatrix} -\frac{\partial RW}{\partial A_i} & \frac{\partial RW}{\partial \log K} \\ -\frac{\partial G}{\partial A_i} & \frac{\partial G}{\partial \log K} \end{vmatrix}}{|J|}; \quad \frac{\partial \log K^Q}{\partial A_i} = \frac{\begin{vmatrix} \frac{\partial RW}{\partial \log W_i} & -\frac{\partial RW}{\partial A_i} \\ \frac{\partial G}{\partial \log W_i} & -\frac{\partial G}{\partial A_i} \end{vmatrix}}{|J|}; \quad (34)$$

Here again all partial derivatives should be values at the equilibrium point, and need to be considered separately for Kaldorian underemployment and for maturity, because of the piecewise nature of the real wage schedule.

Following a conditional line of reasoning, let us suppose first that the generic equilibrium Q occurs during the Kaldorian underemployment phase; accordingly, the relevant expressions for the partial derivatives should be computed from equations 18 and 19. After some algebra (shown with more detail in Mathematical Appendix III.B) the above formulas reduce to

$$\begin{cases} \frac{\partial \log W_i^Q}{\partial A_i} = -\frac{[X_a(1-bL_i) - ZL_a] \left[ \frac{Z}{\psi} P_a^{1-\alpha} + \gamma + \alpha(1-\gamma) \right] + \left(1 + \frac{Z}{\psi} P_a^{1-\alpha}\right) (X_a - Z) \gamma b L_i}{A_i \beta L_a \left(1 + \frac{Z}{\psi} P_a^{1-\alpha}\right) (X_a - Z) |J^{KU}|}; \\ \frac{\partial \log K^Q}{\partial A_i} = -\frac{\left(1 + \frac{Z}{\psi} P_a^{1-\alpha}\right) \{X_a - ZL_a - bL_i [X_a - \gamma(X_a - Z)]\} - (1-\alpha)(1-\gamma)(1-b)X_a L_i}{A_i \beta L_a \left(1 + \frac{Z}{\psi} P_a^{1-\alpha}\right) (X_a - Z) |J^{KU}|}; \end{cases} \quad (35)$$

which imply, under the assumed parametrization, that the derivatives  $\partial \log W_i^Q / \partial A_i$  and  $\partial \log K^Q / \partial A_i$  take the opposite sign of  $|J^{KU}|$ .

To complete the conditional analysis, suppose instead that the equilibrium point Q belongs to the maturity interval; in such case, the relevant partial derivatives in expression 34 should be computed from equations 23 and 19. After some algebraic manipulation this operation obtains:

$$\frac{\partial \log W_i^Q}{\partial A_i} = -\frac{X_a - Z + L_i(1-b)Z}{\beta A_i (X_a - Z) L_a} \frac{1}{|J^{MA}|}; \quad \frac{\partial \log K^Q}{\partial A_i} = -\frac{X_a - Z + L_i(1-b)Z}{\beta A_i (X_a - Z) L_a} \frac{1}{|J^{MA}|}; \quad (36)$$

in which  $J^{MA}$  indicates the Jacobian corresponding to the maturity interval. Equation 36 implies that the derivatives  $\partial \log W_i^Q / \partial A_i$  and  $\partial \log K^Q / \partial A_i$  take the opposite sign of  $|J^{MA}|$ .

Like in the previous case, the correspondence principle ensures that the sign of  $|J^{KU}|$  and  $|J^{MA}|$  in 35 and 36 can be univocally determined.

It is therefore possible to recap the comparative statics results in proposition III.

**Proposition III** *Regardless of the homoteticity of the utility function, parametric increases in industrial TFP reduce the basin of attraction of the locally stable equilibrium of pure subsistence - provided that an unstable equilibrium exists for positive values of capital stock<sup>58</sup> - and move the stable equilibrium (if any) towards North-East<sup>59</sup>, increasing the steady state value of capital and wages.*

Proposition III is illustrated graphically in figure 4, which considers the case of existence of the poverty trap (dashed schedules represent the equilibrium loci before the productivity increase)<sup>60</sup>. The economic explanation goes as follows, regardless of which phase the economy goes through. The increase in  $A_i$  raises, *ceteris paribus*, the supply of manufactures, leading to a moderate increase in the agricultural terms of trade and in agricultural wages, which in turn trigger an upwards adjustment of

<sup>58</sup>Indeed the derivatives in 35 or 36 are both negative. Clearly, whenever the equilibrium of pure subsistence is unstable (as in the case of figure 2a) parametric changes in  $A_i$  will only affect the position of the globally stable equilibrium F.

<sup>59</sup>In this case the derivatives in expression 35 or 36 are both positive.

<sup>60</sup>In the absence of a poverty trap, the only impact of the industrial productivity increase would be to increase the steady state level of capital and wages for the stable interception.

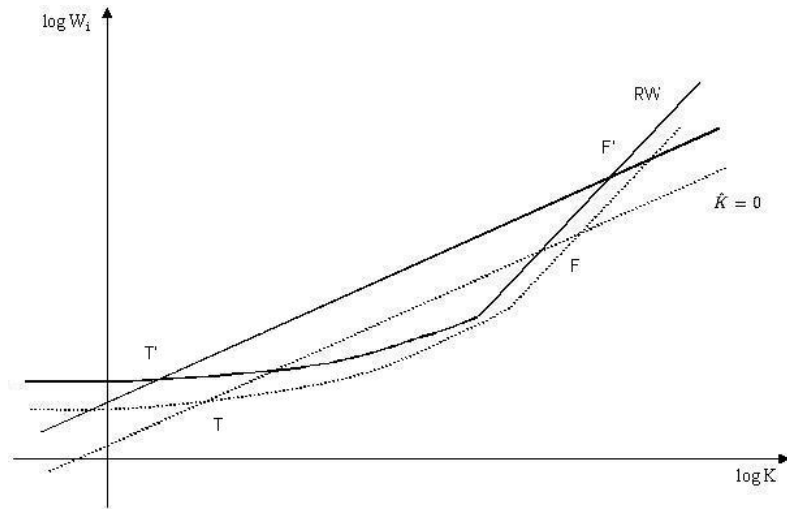


Figure 4: The effect of an increase in industrial TFP

the nominal industrial wages. These factors explain the upwards move of the RW schedule. The raise in industrial productivity brings, however, a much larger gain to entrepreneurs, boosting their profits, and allowing a faster capital accumulation; this is reflected in the upwards shift of the stationary capital locus. Since the vertical movement of the  $\dot{K} = 0$  locus outweighs that of the real wage schedule<sup>61</sup>, the unstable low-development equilibrium (if any) will occur for a lower level of capital stock. Technical progress in industry directly boosts the profitability of entrepreneurs, so that a self-sustaining accumulation of capital becomes viable even for lower capital stocks. For exactly the same reasons, the equilibrium of full industrialization will always be pushed towards higher levels of capital stock by improvements in industrial TFP, regardless of the phase of the economy.

## VI. CONCLUSIONS

In line with our main objective, we have combined in this two-sector macro-model several aspects emphasized by the neoclassical theory of growth and structural change, with other insights drawn from the more dated literature about dual economies and industrialization. Interestingly, the adoption of an efficiency wage mechanism in the urban labor market (unlike in the rural one) and the presence of technological external economies in industry, are sufficient to rationalize a view of the agriculture-industry shift à la Kaldor, and to originate possible poverty traps that may justify policies of concerted investment to bring capital stock up to a minimum critical level.

Of course, Kaldor's structure of causality pivots around the central role of effective demand, while we retain a Ricardian supply-driven framework, resembling in this respect Lewis's model of unlimited supply of labor. Nevertheless, the complex interactions between agriculture and industry, the importance of labor reallocation to the more dynamic sector, and the asymmetric working of the labor market represent common aspects that link the present work to Kaldor's "Strategic factors in economic development", and highlight the crucial role of industrialization and increasing returns in the process of development.

As concerns instead the long debate on the big push argument, the above analysis has shown how - in presence of a dualistic labor market - even moderate degrees of increasing returns in industry may be sufficient to give rise to poverty traps, since the effect of increasing returns is reinforced by the

<sup>61</sup>This can be verified by directly computing  $\partial \log W_i / \partial A_i$  for the real wage schedule and for the stationary capital locus: this derivative in the latter case outweighs the correspondent derivative for RW.

elastic supply of labor for the more dynamic industrial sector. While this result is encouraging for the plausibility of the mechanisms outlined here, our model is surely quite sensitive to the parametrization adopted, and, likewise the majority of poverty trap models, it "tends to be lacking in testable quantitative implications"<sup>62</sup>. Nevertheless, we believe the mechanisms analyzed here may well be relevant for LDCs<sup>63</sup>, and above all for today's Sub-Saharan Africa, the region with the closest conditions to our theoretical framework: extremely capital-poor agricultural sector and widespread areas of subsistence agriculture.

Besides, this work has shown the peculiar relation between sectoral TFP, and the possible bottlenecks to capital accumulation, explaining how increases in the TFP of any of the two sectors, may help making the poverty trap if not less likely at least less stringent. Interestingly, two of the strategies suggested by our model to overcome a poverty trap, also seem at the cornerstone of China's economic success in the last twenty years, and of its growing economic intervention in Sub-Saharan Africa: raising agricultural productivity and accumulating physical capital, with special attention to the "social overhead capital".

## Mathematical Appendix

### I. THE KALDORIAN UNDEREMPLOYMENT PHASE

#### A. DETERMINATION OF THE LABOR SUPPLY ELASTICITY

Substituting in the consumption expenditure flow identity (equation 14), the value of agricultural wages and rents (equations 5 and 6), then using the numeraire (equation 15) the equation of food market clearing (equation 13) and the fact that  $\Pi = \beta W_i L_i / (1 - \beta)$  yields

$$X_i^c = \frac{1 - s\beta}{1 - \beta} W_i L_i. \quad (37)$$

Substituting this expression for  $X_i^c$  into the demand function (equation 1), and solving for  $P_a$  yields

$$P_a = \frac{\alpha(1 - s\beta)}{(1 - \alpha)(1 - \beta)} \frac{W_i L_i}{A_a L_a^{1-b} - Z}. \quad (38)$$

On the other hand, combining equations 8 and 5 obtains

$$W_i = \left( \frac{\omega}{1 - d} \right)^{\frac{1}{d}} [A_a(1 - b)L_a^{-b}P_a]^\omega \left[ \psi P_a^\alpha \left( 1 + \frac{Z}{\psi} P_a^{1-\alpha} \right) \right]^{1-\omega}. \quad (39)$$

Using equation 38 into 39 yields, after some manipulation

$$W_i^{(1-\alpha)(1-\gamma)} = \Theta (1 - L_i)^{-b\gamma} L_i^{\gamma+\alpha(1-\gamma)} [A_a(1 - L_i)^{1-b} - Z]^{-[\gamma+\alpha(1-\gamma)]} \left\{ 1 + \frac{Z}{\psi} \left[ \frac{\alpha(1 - s\beta)}{(1 - \alpha)(1 - \beta)} \right]^{1-\alpha} W_i^{1-\alpha} L_i^{1-\alpha} [A_a(1 - L_i)^{1-b} - Z]^{-(1-\alpha)} \right\}^{1-\gamma}; \quad (40)$$

with

$$\Theta \equiv \left( \frac{\omega}{1 - d} \right)^{\frac{1}{d}} A_a^\gamma (1 - b)^\gamma \psi^{1-\gamma} \left[ \frac{\alpha(1 - s\beta)}{(1 - \alpha)(1 - \beta)} \right]^{\gamma+\alpha(1-\gamma)}.$$

<sup>62</sup>The quotation is taken from Azariadis and Stachurski (2005), recognizing a limit which is common to most models of poverty trap.

<sup>63</sup>In this respect, our judgement seems confirmed by the simulations presented in Graham and Temple (2005), which suggest that the presence of multiple equilibria may play a key role in explaining income dispersion across countries and particularly suitable to characterize the poorest LDCs.

Taking logs obtains

$$(1 - \alpha)(1 - \gamma) \log W_i + b\gamma \log [1 - \exp(\log L_i)] - [\gamma + \alpha(1 - \gamma)] \log L_i + \\ + [\gamma + \alpha(1 - \gamma)] \log \left\{ A_a [1 - \exp(\log L_i)]^{1-b} - Z \right\} = \log \Theta + (1 - \gamma) \log \\ \left\{ 1 + \frac{Z}{\psi} \left[ \frac{\alpha(1 - s\beta)}{(1 - \alpha)(1 - \beta)} \right]^{1-\alpha} \left[ A_a (1 - \exp(\log L_i))^{1-b} - Z \right]^{-(1-\alpha)} \exp[(1 - \alpha)(\log W_i + \log L_i)] \right\};$$

from which total differentiation yields the industrial labor supply elasticity as in equation 17.

#### B. DEPENDENCY OF THE LABOR SUPPLY ELASTICITY ON $L_i$

Instead of directly computing  $\partial \epsilon^{LS} / \partial L_a$ , one can re-arrange the expression of  $\epsilon^{LS}$  as

$$\epsilon^{LS} = \frac{(1 - \alpha)(1 - \gamma)}{\left[ 1 + \frac{(1 - L_a)(1 - b)A_a L_a^{1-b}}{L_a(A_a L_a^{1-b} - Z)} \right] \left[ \gamma + \alpha(1 - \gamma) + \frac{Z}{\psi} P_a^{1-\alpha} \right] + \frac{\gamma b(1 - L_a)}{L_a} \left( 1 + \frac{Z}{\psi} P_a^{1-\alpha} \right)}. \quad (41)$$

and then compute the derivative of each term in the denominator with respect to  $L_a$ . This operation proves that all terms in the denominator of 41 are decreasing in  $L_a$ <sup>64</sup>, hence  $\epsilon^{LS}$  is increasing in  $L_a$ , or - given equation 11 - decreasing in  $L_i$ .

#### C. DETERMINATION OF THE REAL WAGE SCHEDULE

To determine the real wage schedule, replace  $L_i$  in equation 40 with its value from 9, which after a bit of algebra obtains

$$W_i^{(1-\alpha)(1-\gamma)} = \left( 1 - \Phi W_i^{-\frac{1}{\beta}} K^{\frac{\mu+\beta}{\beta}} \right)^{-b\gamma} \left\{ \Phi W_i^{-\frac{1}{\beta}} K^{\frac{\mu+\beta}{\beta}} \left[ A_a \left( 1 - \Phi W_i^{-\frac{1}{\beta}} K^{\frac{\mu+\beta}{\beta}} \right)^{1-b} - Z \right]^{-1} \right\}^{\gamma+\alpha(1-\gamma)} \\ \left\{ 1 + \Xi \left( W_i^{-\frac{1-\beta}{\beta}} K^{\frac{\mu+\beta}{\beta}} \right)^{1-\alpha} \left[ A_a \left( 1 - \Phi W_i^{-\frac{1}{\beta}} K^{\frac{\mu+\beta}{\beta}} \right)^{1-b} - Z \right]^{-(1-\alpha)} \right\}^{1-\gamma} \Theta;$$

where the constant  $\Theta$  is defined as above, while  $\Phi$  and  $\Xi$  are other constants defined as

$$\Phi \equiv A_i^{\frac{1}{\beta}} (1 - \beta)^{\frac{1}{\beta}} (E^*)^{\frac{1-\beta}{\beta}}; \quad \Xi \equiv \Phi^{1-\alpha} \frac{Z}{\psi} \left[ \frac{\alpha(1 - s\beta)}{(1 - \alpha)(1 - \beta)} \right]^{1-\alpha}.$$

Expressed in log terms, this equation is exactly the RW schedule mentioned in the text (equation 18).

#### D. THE EVOLUTION OF THE WAGE GAP DURING KALDORIAN UNDEREMPLOYMENT

<sup>64</sup>The derivatives of each term in the denominator of 41 with respect to  $L_a$  are respectively

$$-\frac{(1 - b) X_a [(X_a - Z) L_a + X_a (1 - b) L_i]}{(X_a - Z)^2 L_a^2} < 0;$$

for the first term,

$$-\frac{Z\alpha(1 - s\beta)P_a^{-\alpha}}{\psi(X_a - Z)^2(1 - \beta)} \left[ \frac{1 + W_i \epsilon^{LS}}{\epsilon^{LS} L_a^2} (X_a - Z) + (1 - b) A_a L_a^{-b} W_i L_i \right] < 0;$$

for the second and fourth term, and

$$-\frac{\gamma b}{L_a^2} < 0;$$

for the third one.



Consider the expression for the agricultural wage relatively to the industrial one<sup>65</sup>

$$\frac{W_a}{W_i} = \frac{(1-b) A_a (L_a)^{-b} P_a}{W_i};$$

using equations 38, and then making use of equation 11 and 9, the wage ratio can be rewritten as

$$\frac{W_a}{W_i} = \frac{\alpha(1-b)(1-s\beta) A_a}{(1-\alpha)(1-\beta)} \frac{\left(1 - \Phi W_i^{-\frac{1}{\beta}} K^{\frac{\mu+\beta}{\beta}}\right)^{-b} \Phi W_i^{-\frac{1}{\beta}} K^{\frac{\mu+\beta}{\beta}}}{A_a \left(1 - \Phi W_i^{-\frac{1}{\beta}} K^{\frac{\mu+\beta}{\beta}}\right)^{1-b} - Z};$$

Taking logs, the wage ratio becomes

$$\begin{aligned} \log \frac{W_a}{W_i} &= \log \frac{\alpha(1-b)(1-s\beta) A_a}{(1-\alpha)(1-\beta)} - b \log \left[ 1 - \Phi \exp \left( \frac{\mu+\beta}{\beta} \log K - \frac{1}{\beta} \log W_i \right) \right] + \\ &+ \log \left[ \Phi \exp \left( \frac{\mu+\beta}{\beta} \log K - \frac{1}{\beta} \log W_i \right) \right] - \log \left\{ A_a \left[ 1 - \Phi \exp \left( \frac{\mu+\beta}{\beta} \log K - \frac{1}{\beta} \log W_i \right) \right]^{1-b} - Z \right\}; \end{aligned}$$

from which total differentiation, yields the following relation:

$$\frac{\partial \log \frac{W_a}{W_i}}{\partial \log K} = \frac{X_a - Z(L_a + bL_i)}{\beta L_a (X_a - Z)} \left( \mu + \beta - \frac{\partial \log W_i}{\partial \log K} \right) > 0;$$

which is greater than zero, given that the coefficient of the real wage schedule during Kaldorian underemployment is equal to  $(\mu + \beta) / (1 + \beta\epsilon^{LS})$ .

Since this derivative is strictly positive for the parametrization assumed above, the wage ratio tends to grow along with increases in the capital stock, from values lower than one (by construction) ultimately reaching one when the system enters the maturity phase and wage gap disappear. To see this, note that the logarithm is a monotonically increasing transformation of the wage ratio and of the capital stock, hence the sign of the log-derivative  $\partial \log \frac{W_a}{W_i} / \partial \log K$  equals the sign of the simple derivative of the wage ratio to capital stock.

## II. THE MATURITY PHASE

### A. DETERMINATION OF THE LABOR SUPPLY ELASTICITY

Equations 37 and 38 continue to hold even in maturity, and additionally the uniformity of wage then ensures that  $W_i = A_a(1-b)L_a^{-b}P_a$ . Combining these three equations yields equation 22 and the fact that

$$\epsilon^{LS} = 0.$$

### B. DETERMINATION OF THE REAL WAGE SCHEDULE

To obtain the equation of the real wage schedule during the maturity phase, combine the industrial labor demand with the implicit equation determining the labor shares in the maturity phase (respectively equation 9, 22), and recall that labor efficiency in the maturity phase will still be given by  $E^*$ . This yields

$$\frac{(1-\alpha)(1-\beta)}{\Phi\alpha(1-b)(1-s\beta)A_a} \left[ A_a \left( 1 - \Phi W_i^{-\frac{1}{\beta}} K^{\frac{\mu+\beta}{\beta}} \right)^{1-b} - Z \right] = W_i^{-\frac{1}{\beta}} K^{\frac{\mu+\beta}{\beta}} \left( 1 - \Phi W_i^{-\frac{1}{\beta}} K^{\frac{\mu+\beta}{\beta}} \right)^{-b}$$

Taking logs, obtains from this expression the real wage schedule as given in the text (equation 23).

<sup>65</sup>Note that the absolute wage gap is tied to the wage ratio by the following relation  $W_i - W_a = (1 - W_a/W_i) W_i$ .

## III. THE EFFECT OF TECHNICAL CHANGE

## A. COMPARATIVE STATICS: AGRICULTURAL TFP

From the previous analysis it should be clear that in system 28 the relevant equations during Kaldorian underemployment are actually 18 for RW and 19 in place of G. Accordingly, the following magnitudes are of interest for comparative statics in the Kaldorian underemployment phase:

$$J^{KU} \equiv \begin{pmatrix} \frac{\partial RW}{\partial \log W_i} & \frac{\partial RW}{\partial \log K} \\ \frac{\partial G}{\partial \log W_i} & \frac{\partial G}{\partial \log K} \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{[X_a(1-bL_i)-ZL_a]\left[\frac{Z}{\psi}P_a^{1-\alpha}+\gamma+\alpha(1-\gamma)\right]+(X_a-Z)\left[\gamma bL_i\left(1+\frac{Z}{\psi}P_a^{1-\alpha}\right)+(1-\gamma)(1-\alpha)\beta L_a\right]}{\beta L_a(X_a-Z)\left(1+\frac{Z}{\psi}P_a^{1-\alpha}\right)} & -\frac{1-\beta}{\beta} \\ -\frac{\mu+\beta}{\beta} \frac{[X_a(1-bL_i)-ZL_a]\left[\frac{Z}{\psi}P_a^{1-\alpha}+\gamma+\alpha(1-\gamma)\right]+(X_a-Z)\left(1+\frac{Z}{\psi}P_a^{1-\alpha}\right)\gamma bL_i}{L_a(X_a-Z)\left(1+\frac{Z}{\psi}P_a^{1-\alpha}\right)} & \frac{\mu}{\beta} \end{pmatrix}^T;$$

and<sup>66</sup>

$$\frac{\partial RW}{\partial A_a} = \frac{[(1-\gamma)X_a + \gamma Z] \left(1 + \frac{Z}{\psi}P_a^{1-\alpha}\right) - (1-\gamma)(1-\alpha)X_a}{A_a(X_a - Z) \left(1 + \frac{Z}{\psi}P_a^{1-\alpha}\right)}; \quad \frac{\partial G}{\partial A_a} = 0.$$

Replacing the partial derivatives of equation 30 with the corresponding values as determined here, obtains after some manipulation 31.

As concerns the sign of  $|J^{KU}|$ , its direct calculation shows after some algebra that

$$|J^{KU}| > 0 \iff \epsilon^{LS} > \frac{1-\beta-\mu}{\mu};$$

which basically verifies the correspondence principle between statics and dynamics.

During maturity, instead, the relevant equations for system 29 are number 23 (for RW) and 19 (for G); accordingly we have the following magnitudes

$$J^{MA} \equiv \begin{pmatrix} \frac{\partial RW}{\partial \log W_i} & \frac{\partial RW}{\partial \log K} \\ \frac{\partial G}{\partial \log W_i} & \frac{\partial G}{\partial \log K} \end{pmatrix} = \begin{pmatrix} \frac{X_a-Z+L_iZ(1-b)}{\beta L_a(X_a-Z)} & -\frac{(\mu+\beta)[X_a-Z+L_iZ(1-b)]}{\beta L_a(X_a-Z)} \\ -\frac{1-\beta}{\beta} & \frac{\mu}{\beta} \end{pmatrix};$$

$$\frac{\partial RW}{\partial A_a} = \frac{Z}{X_a-Z} \frac{1}{A_a}; \quad \frac{\partial G}{\partial A_a} = 0.$$

Replacing the partial derivatives in equation 30 with the corresponding values determined here for the maturity phase, directly obtains 32.

On the other hand, the direct calculation of  $|J^{MA}|$  verifies the correspondence principle, establishing precisely that

$$|J^{MA}| > 0 \iff \mu > (1-\beta);$$

and with this last condition the sign of the derivatives  $\partial \log W_i^Q / \partial A_a$  and  $\partial \log K^Q / \partial A_a$  can be univocally determined, as done in the text.

## B. COMPARATIVE STATICS: INDUSTRIAL TFP

<sup>66</sup>Note that in the last row we expressed  $J^{KU}$  as a transposed matrix only for notational purposes.

Starting with Kaldorian underemployment, the relevant equations for system 33 are 18 (for RW), and 19 (for G). Hence, in addition to the matrix  $J^{KU}$  defined above, the magnitudes of interest for the comparative statics regarding  $A_i$  in the Kaldorian underemployment phase are:

$$\frac{\partial RW}{\partial A_i} = - \frac{[X_a(1 - bL_i) - ZL_a] \left[ \frac{Z}{\psi} P_a^{1-\alpha} + \gamma + \alpha(1 - \gamma) \right] + (X_a - Z) \left( 1 + \frac{Z}{\psi} P_a^{1-\alpha} \right) \gamma b L_i}{\beta L_a (X_a - Z) \left( 1 + \frac{Z}{\psi} P_a^{1-\alpha} \right)} \frac{1}{A_i};$$

$$\frac{\partial G}{\partial A_i} = \frac{1}{\beta} \frac{1}{A_i}.$$

Replacing these values for the corresponding partial derivatives in equation 34, obtains after some manipulation 35. Recalling, finally, the condition for a positive determinant of  $J^{KU}$ , yields the comparative statics result mentioned in the text.

As concerns the maturity phase, instead, the relevant Jacobian is  $J^{MA}$  defined above, and from equations 23 (for RW) and 19 (for G), it is possible to compute the magnitudes

$$\frac{\partial RW}{\partial A_i} = - \frac{1}{\beta} \frac{X_a - Z + L_i(1 - b)Z}{(X_a - Z)L_a} \frac{1}{A_i}; \quad \frac{\partial G}{\partial A_i} = \frac{1}{\beta} \frac{1}{A_i}.$$

Substituting these expressions in equation 34 obtains after a bit of algebra equation 36.

Finally, using the correspondence principle to determine whether  $|J^{MA}|$  is positive or negative, one can univocally establish the sign of the derivatives  $\partial \log W_i^Z / \partial A_i$  and  $\partial \log K^Z / \partial A_i$ .

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