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Financial Frictions, Capital Reallocation, and Aggregate Fluctuations ^{*}

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Abstract

We address an important business cycle fact, i.e., the amplified and hump-shaped responses of output to productivity shocks, in a dynamic general equilibrium model with financial frictions. Models with financial frictions in the current literature have either the amplification mechanism or the propagation mechanism. Our model shows that the dynamic interaction of borrowing constraints, endogenous capital accumulation and capital reallocation among agents with different productivity constitutes a mechanism through which the effects of productivity shock on aggregate output are amplified and propagated, more in line with the empirical evidence than other related models in the literature.

JEL Classification: E32, E44, G3

Keywords: Amplification, Business cycles, Capital reallocation, Financial frictions, Propagation

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1 Introduction

One of the well-known deficiencies of the canonical RBC models is the lack of the sufficient propagation and amplification mechanism¹, as pointed out by Cogley and Nason (1993, 1995) and Andolfatto (1996). Because the aggregate capital stock is the only endogenous state variable in these models, the dynamic structure is essentially $ARMA(1, 1)$ and fails to replicate an important empirical fact, i.e., the amplified and hump-shaped output responses to productivity shocks. Many studies in the literature introduce various frictions into the RBC framework. Additional endogenous state variables help reinforce the internal propagation mechanism. Credit market imperfections are one of these variations.

Carlstrom and Fuerst (1997, 1998) and Kato (2006) introduce financial frictions into the production of capital goods. In addition to the aggregate capital stock, the net worth of capital goods producers becomes another endogenous state variables and is essential for their borrowing capacity. A positive TFP (total factor productivity) shock raises the aggregate demand for capital and the price of capital rises. Although the projects of capital goods producers become more profitable than before, they are subject to credit constraints and cannot expand their investment scale to fully exploit this opportunity. They have to accumulate net worth over time and the supply of capital adapts to the demand in a few periods after the shock. Given that capital is one of the two inputs for the aggregate production of final goods, the delayed and dampened responses of investment result in the hump-shaped and depressed dynamics of aggregate output, in comparison with the frictionless RBC model. Although financial frictions in the production of capital goods help generate the positive autocorrelation of aggregate output qualitatively, aggregate output peaks only two periods after the shock and the maximum value of the output responses is even smaller than in the corresponding *RBC* model. However, Cogley and Nason (1995) show that aggregate output peaks four quarters after the shock in the United States. Furthermore, Andolfatto (1996) shows that the volatility of aggregate output around its trend in the actual U.S. economy is even more than what the frictionless RBC model can predict.

Another line of research on financial frictions, e.g., Kiyotaki and Moore (1997) and Chen (2001), shows that the interaction between credit constraints and the reallocation of productive assets among agents with different productivity can amplify the output responses to exogenous shocks. In these models, the productive asset has a fixed total supply. A TFP shock has the immediate effects on asset reallocation and output peaks one-period after the shock. So, these models are lack of the propagation mechanism in the sense that output does not have the hump-shaped dynamic patterns.

¹“Amplification” refers to the mechanism through which relatively small shocks result in large output fluctuations, while “propagation” refers to the mechanism through which a transitory productivity shock generates positive autocorrelation in aggregate output.

We analyze how the dynamic interactions of borrowing constraints, endogenous capital accumulation and reallocation among agents with different productivity can affect the responses of output, credit, and investment to a transitory TFP shock. We compare our model with a simplified version of models with financial frictions, e.g., Carlstrom and Fuerst (1997, 1998) and Kato (2006), and show that our model outperforms other models in generating both amplified and more delayed output responses, which is more in line with the empirical evidence.

The intuition is explained as follows. We introduce an intermediate good sector into a standard RBC model with capital accumulation. There are two types of agents, households and entrepreneurs. Both agents have projects to produce intermediate goods using capital. Intermediate goods and labor are then employed to produce final goods contemporaneously. Final goods can be either consumed or transformed one-to-one into capital goods without frictions. The entrepreneurs' project is more productive than that of households. If entrepreneurs could fully pledge their project outcomes for external funds, their project investment would be fully financed externally and capital would be all allocated to their projects. However, due to unobservable project choice à la Holmstrom and Tirole (1997), entrepreneurs can pledge only a fraction of their project outcomes for loans and their own funds are required to fill in the gap between total investment and loans. Thus, entrepreneurial net worth is essential for their project investment.

A positive TFP shock raises aggregate demand and pushes up the price of intermediate goods. Extra sales revenues improve entrepreneurial net worth and enable them to acquire more loans and capital goods. The excess demand for loans pushes up the loan rate and induces households to lend more to the entrepreneurial sector as well as reduce their capital holding. There are two positive effects on aggregate output in the next period: the accumulation of the aggregate capital stock and the reallocation of capital goods from the less productive agents (households) to the more productive agents (entrepreneurs).

However, it takes time for entrepreneurs to accumulate net worth and they cannot acquire enough loans to expand their investment scale sufficiently in the shock period. Capital reallocation from households to entrepreneurs is delayed, and output responds to shocks in a delayed fashion, too. Endogenous capital accumulation and gradual reallocation of capital goods among agents with different productivity makes aggregate output peak four periods after the shock and its magnitude is larger than in the corresponding *RBC* model, which are in line with the empirical evidence.

In order to compare the performance of our basic model with that of Carlstrom and Fuerst (1997, 1998) and Kato (2006), we analyze a model with financial frictions in the production of capital goods. Lack of the amplification mechanism, the model with financial frictions in the production of capital goods only generate dampened hump-shaped output responses in the sense that aggregate output peaks two periods after the shock and its magnitude is smaller than in the corresponding *RBC* model. In other words, these

models obtain the propagation mechanism by sacrificing the amplification mechanism.

In order to see whether introducing more financial frictions into our basic model can improve the model performance, we analyze a model with double financial frictions in the production of both capital and intermediate goods. A TFP shock pushes up the price of capital and capital gains improve entrepreneurial net worth. Entrepreneurs can increase loans and capital investment while the rise in the price of capital forces households to reduce their capital stock, both in a larger magnitude than in our basic model. In other words, the amplification mechanism is enhanced qualitatively. However, the output dynamics are almost the same as in our basic model. In this sense, the dynamic interaction of borrowing constraints, endogenous capital accumulation and reallocation in our basic model are sufficient explain the empirical evidence alone.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 discusses the financial contracting problems under alternative scenarios. Section 4 calibrates the model and analyzes the model dynamics. Section 5 concludes.

2 The Model

This section presents the general model with financial frictions in the production sectors of capital and intermediate goods. We can analyze the role of financial frictions in one sector by shutting off the moral hazard problem in the other sector easily.

2.1 Overview

There are three goods in the economy: a capital good, an intermediate good, and a final good. The capital good is durable, while the intermediate good and the final good are perishable. There are three groups of agents: households, entrepreneurs, and capital goods producers, each of unit mass.²

Households are risk averse and infinitely lived. They have a safe project to produce intermediate goods using capital. Entrepreneurs are risk neutral and each has a constant probability of death $\pi \in (0, 1)$. In each period, entrepreneurs of mass $(1 - \pi)$ exit from the economy and new entrepreneurs of the same mass are born, keeping their population constant. Each entrepreneur has three projects for the production of intermediate goods using capital and the projects are subject to idiosyncratic risk: projects have positive output in the case of success and there is no output in the case of failure. Each entrepreneur can choose only one project and his project choice is unobservable to others. It takes one period for households and entrepreneurs to complete their projects. Each agent is endowed with a unit of labor. Final goods are produced using intermediate goods and labor inputs of the three types of agents contemporaneously.

²The relative population size of agents does not matter for our results.

The aggregate capital stock depreciates at a constant rate. Capital goods producers are risk neutral and infinitely lived. They have three projects to transform final goods into capital goods contemporaneously and the projects have similar features as those of entrepreneurs. Final goods can also be consumed and is chosen as the numeraire. Newly-produced and existing capital goods are perfect substitutes. Let q_t and v_t denote the price of capital and intermediate goods; let w_t , w_t^e , and w_t^c denote the wage rates of households, entrepreneurs, and capital goods producers, respectively.

The least risky project of entrepreneurs is expected to be more productive than that of households and the least risky project of capital goods producers has an expected rate of return larger than unity. As mutual funds have exclusive technology to (partially) screen the projects of entrepreneurs and capital goods producers, loans are intermediated through mutual funds. There is no direct borrowing and lending among individual agents in our model economy.

Given the time length of their projects, mutual funds provide entrepreneurs with one-period loans and r_t denotes the gross interest rate. If entrepreneurs could credibly choose the least risky project, their project investment would be fully financed by mutual funds. However, due to unobservable project choice, they have to put own funds into the projects so as to let mutual funds be sure that entrepreneurs choose the least risky project.

Given that the project of capital goods producers completes within the period, mutual funds provide them with intra-period loans and the gross loan rate is unity. Due to unobservable project choice, they have to finance part of their project investment using own funds so as to let mutual funds be sure that they choose the least risky project.

Mutual funds can perfectly diversify loan portfolios ex ante and enforce debt repayments ex post. Therefore, mutual funds pool the idiosyncratic project risk of entrepreneurs and capital goods producers, and guarantee a safe rate of return on deposits. Due to perfect competition, mutual funds break even and make no profit.

An exogenous productivity shock realizes at the beginning of each period.

2.2 Households

Households have identical preferences over consumption and leisure,

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma}}{1-\sigma} + \chi \frac{(1-l_t)^{1+\psi}}{1+\psi} \right],$$

where E_t is the expectation operator based on information available in period t and $\beta \in (0, 1)$ denotes the time discount factor. c_t and l_t denote household consumption and labor supply in period t , respectively.

Given that k_{t-1} units of capital goods were invested in the household project in period $t-1$, $G(k_{t-1})$ units of intermediate goods are produced at the beginning of period t .

Assumption 2.1. $G'(k) > 0$ and $G''(k) \leq 0$.

Assumption 2.1 implies that the household project is (quasi-) decreasing return to scale. The sales revenue and wage income of households are $v_t G(k_{t-1})$ and $w_t l_t$, respectively. In addition, they receive $r_{t-1} d_{t-1}$ from mutual funds, where d_{t-1} denotes their inter-period deposits made in period $t - 1$. At the end of period t , they invest k_t units of capital goods in their projects, deposit d_t units of final goods at the mutual funds, and consume c_t units of final goods. Accordingly, their flow-budget constraints are,

$$q_t[k_t - (1 - \delta)k_{t-1}] + d_t + c_t = v_t G(k_{t-1}) + w_t l_t + r_{t-1} d_{t-1},$$

where $\delta \in (0, 1]$ is the depreciation rate of the capital invested in the household project. The optimization over $\{c_t, l_t, k_t, d_t\}$ gives the equilibrium conditions,

$$w_t = \chi(1 - l_t)^\psi c_t^\sigma, \quad (1)$$

$$1 = \beta r_t E_t \left(\frac{c_{t+1}}{c_t} \right)^{-\sigma}, \quad (2)$$

$$r_t q_t = E_t [(1 - \delta)q_{t+1} + v_{t+1} G'(k_t)]. \quad (3)$$

2.3 Unobservable Project Choices

Each entrepreneur can invest capital goods into one of the three projects: “Good”, “Bad”, or “Rotten”. The project generates R_e units of intermediate goods per unit of capital invested and the invested capital depreciates at a rate $\delta' \in (0, 1]$, if the project succeeds; if the project fails, there is no output and the invested capital is fully lost.

Similarly, capital goods producers have another three projects: “Good”, “Bad”, or “Rotten”, with which they transform one unit of final goods into R_c units of capital goods, if the project succeed; otherwise, there is no output of capital goods and the invested final goods are wasted.

The project choices of entrepreneurs and capital goods producers are irreversible and project outcomes are perfectly verifiable at no costs. They also enjoy safe, nonpecuniary private benefits³ during the project process. For convenience of aggregation, we assume that the private benefits from the projects of entrepreneurs (capital goods producers) are proportional to their project investment in terms of capital goods (final goods). The projects differ in the probability of success and private benefits per unit of investment. Let $\{p_m^G, p_m^B, p_m^R\}$ and $\{b_m^G, b_m^B, b_m^R\}$ denote the success probabilities of projects “Good”, “Bad”,

³We follow the Principal-Agent setting in Holmstrom and Tirole (1997). According to Hart (1995), private benefits may refer to any nonpecuniary benefits from running a project, e.g., large offices or luxury business cars. Private benefits are good for the project owners but may reduce the success probability of projects. The trade-off between the success probability and private benefits is a short-cut to capture the divergent objectives between the project owners and outside financiers.

“Rotten” and the private benefits per unit of investment in the respective project, where $m \in \{e, c\}$ denotes the project attributes of entrepreneurs and capital goods producers; $0 < p_m^R = p_m^B < p_m^G < 1$ and $b_m^R > b_m^B > b_m^G = 0$ imply that projects “Rotten” and “Bad” are riskier than project “Good”, but project “Rotten” yields highest private benefits and project “Good” yields lowest private benefits to project owners. Individual agents cannot observe the project choices of others. Mutual funds have expertise in screening out project “Rotten” at no costs but cannot distinguish between project “Good” and project “Bad”. The advantage of mutual funds over individual agents justifies the fact that there is no direct borrowing and lending among individual agents. See Holmstrom and Tirole (1997) for a detailed description.

2.4 Entrepreneurs

As each entrepreneur has a probability of death each period and his project is subject to idiosyncratic risk, entrepreneurs differ in their end-of-period wealth. Due to the linear nature of their preference and technologies, their project investment and loans are proportional to their end-of-period wealth (entrepreneurial net worth in the project), as shown below. In other words, only the first moment of their net worth matters for the economic allocation in the entrepreneurial sector. Thus, we focus only on the economic behavior of an “average” entrepreneur and do not trace the wealth evolution of each entrepreneur.

The “average” entrepreneur, who stays in the economy to the next period, has linear preferences over consumption and private benefits,

$$E_0 \sum_{t=0}^{\tilde{T}} \beta^s [c_t^e + \mathcal{B}_e k_{t-1}^e], \quad (4)$$

where \tilde{T} is the stochastic time of death and $\mathcal{B}_e \in \{b_e^G, b_e^B, b_e^R\}$ denotes private benefits per unit of the capital invested in project “Good”, “Bad”, or “Rotten”, respectively. c_t^e denotes his consumption in period t and k_{t-1}^e denotes the capital invested in period $t - 1$.

Our calibration guarantees that only project “Good” has a positive expected net present value in equilibrium,

$$\frac{p_e^G E_t [R_e v_{t+1} + (1 - \delta') q_{t+1}]}{r_t} > q_t > \frac{p_e^B E_t [R_e v_{t+1} + (1 - \delta') q_{t+1}] + b_e^B}{r_t},$$

Therefore, other projects should not be financed in equilibrium.

For convenience of aggregation, we assume that capital depreciates faster in the household projects than in the entrepreneurs’ projects that turn out to be successful, $\delta = 1 - p_e^G + p_e^G \delta' > \delta'$. In equilibrium, the aggregate capital stock depreciates at the same rate in both household and entrepreneurial sectors, $1 - \delta = p_e^G (1 - \delta')$.⁴

⁴Equalizing the depreciation rates of capital in the households’ project and the entrepreneurs’ project, $\delta' = \delta$, does not affect our results but complicates the notation.

Our calibration also guarantees that project “Good” is expected to be always more productive than the household project, $p_e^G R_e > G'(0)$. Thus, if entrepreneurs could credibly choose project “Good”, they would borrow against all outcomes of project “Good” and intermediate goods would be produced by them only.

The entrepreneur invest k_t^e units of capital goods in either project “Good” or project “Bad” in period t , using his own funds n_t^e and inter-period loans z_t^e . Thus, n_t^e is entrepreneurial net worth in the project. The loan contract specifies a promise to repay $R_t^e k_t^e$ units of final goods in period $t + 1$, if the project succeeds; both parties get zero pecuniary return, if the project fails. The entrepreneur always gets private benefits. In order to motivate the entrepreneur to choose project “Good”, mutual funds must provide him with enough incentives,

$$\{p_e^G E_t[R_e v_{t+1} + (1 - \delta')q_{t+1} - R_t^e] + b_e^G\}k_t^e \geq \{p_e^B E_t[R_e v_{t+1} + (1 - \delta')q_{t+1} - R_t^e] + b_e^B\}k_t^e.$$

The left (right) hand side denotes the expected utility of the entrepreneur if he chooses project “Good” (“Bad”). As the expected rate of return on project “Good” exceeds the interest rate, the entrepreneur prefers to borrow to the limit. The incentive constraints are binding and is simplified to be

$$R_t^e = E_t[R_e v_{t+1} + (1 - \delta')q_{t+1}] - b_e, \quad \text{where} \quad b_e \equiv \frac{b_e^B - b_e^G}{p_e^G - p_e^B} > 0. \quad (5)$$

Any promise to repay more than $R_t^e k_t^e$ to the mutual funds in the case of success is not credible. $p_e^G E_t[R_e v_{t+1} + (1 - \delta')q_{t+1}]$ and $p_e^G R_t^e$ are the expected full value and external value per unit of the capital invested in project “Good”, respectively. The difference between the two values, $p_e^G b_e$, is used to motivate the entrepreneur to choose project “Good” despite the lower private benefits it provides, $b_e^G < b_e^B$.

The mutual funds are expected to break even in lending to the entrepreneur in period t , $r_t z_t^e = p_e^G R_t^e k_t^e$. This implies a credit constraint for him,

$$z_t^e = \Gamma_t^e n_t^e, \quad \text{where} \quad \Gamma_t^e \equiv \frac{p_e^G R_t^e}{r_t q_t - p_e^G R_t^e} \quad (6)$$

is the credit multiplier. As we are interested in the case where entrepreneurs finance their project using both own funds and external funds, our calibration guarantees that the cost per unit of capital invested in the entrepreneur’s project is larger than the discounted external value per unit capital invested, $q_t > \frac{p_e^G R_t^e}{r_t}$, around the steady state. Therefore, the entrepreneur has to put own funds in the project and Γ_t^e is positive. Otherwise, entrepreneurs could finance their projects using external funds only. As Γ_t^e is independent of n_t^e , loans are proportional to entrepreneurial net worth and so are their capital stock, $k_t^e = \frac{z_t^e + n_t^e}{q_t} = \frac{\Gamma_t^e + 1}{q_t} n_t^e$.

Suppose that entrepreneurs finance their project investment using loans in period $t - 1$. At the beginning of period t , entrepreneurs of mass p_e^G have successful projects and the

rest are failed. After the project completion, entrepreneurs of mass π get the signal of survival and the rest have to exit from the economy.

Entrepreneurs who have successful projects and receive the signal of death are of mass $p_e^G(1-\pi)$. They repay their liabilities, sell off their capital stock, and consume all proceeds before they exit from the economy. Entrepreneurs who have failed projects and receive the signal of death are of mass $(1-p_e^G)(1-\pi)$. They are released from the debt obligation and exit from the economy without consumption.

As the expected rate of return on their net worth in project “Good” exceeds the interest rate, the newcomers and the surviving entrepreneurs supply their labor endowment inelastically $l_t^e = 1$ to the production of final goods and their wage income is w_t^e . At the end of period t , the entrepreneur maximizes his expected utility function defined in (4), subject to his credit constraints specified in equation (6) and period budget constraints,

$$q_t k_t^e - z_t^e = n_t^e, \quad \text{where} \quad n_t^e = \mathcal{N}_t^e - c_t^e,$$

\mathcal{N}_t^e denotes his end-of-period wealth. The newcomers and entrepreneurs who have failed projects and survive to the next period are of mass $(1-\pi) + (1-p_e^G)\pi$ and their end-of-period wealth is $\mathcal{N}_t^e = w_t^e$; entrepreneurs who have successful projects and survive to the next period are of mass $p_e^G\pi$ and their end-of-period wealth is $\mathcal{N}_t^e = w_t^e + [R_e v_t + (1-\delta')q_t - R_{t-1}^e]k_{t-1}^e$. As the marginal rate of return on project “Good” exceeds the interest rate, entrepreneurs invest all wealth, borrow to the limit, and postpone consumption to the period of death.

In the aggregate, per capita consumption c_t^e , net worth n_t , and capital stock k_t^e of entrepreneurs are,

$$c_t^e = (1-\pi)p_e^G[R_e v_t + (1-\delta')q_t - R_{t-1}^e]k_{t-1}^e, \quad (7)$$

$$n_t^e = \pi p_e^G[R_e v_t + (1-\delta')q_t - R_{t-1}^e]k_{t-1}^e + w_t^e, \quad (8)$$

$$k_t^e = \frac{n_t^e + z_t^e}{q_t}. \quad (9)$$

2.5 Capital Goods Producers

Due to idiosyncratic risk on their projects, capital goods producers differ in their end-of-period wealth. Similar as entrepreneurs, the linear preference and technologies of capital goods producers enable us to focus on the economic behavior of an “average” capital goods producer instead of tracing the exact wealth distribution of capital goods producers.

The capital goods producer has linear preferences,

$$E_0 \sum_{t=0}^{\infty} (\gamma\beta)^s (c_t^c + \mathcal{B}_c i_t), \quad (10)$$

where c_t^c and i_t denote his consumption and project investment; $\mathcal{B}_c \in \{b_c^G, b_c^B, b_c^R\}$ denotes private benefits per unit of final goods invested in project “Good”, “Bad” or “Rotten”;

$\gamma \in (0, 1)$ implies that capital goods producers are less patient than households and entrepreneurs. It guarantees that their credit constraints are always binding in equilibrium. As the capital goods producer does not care about leisure, he supplies labor endowment, $l_t^c = 1$, inelastically to the production of final goods. Our calibration guarantees that only project “Good” has a positive expected net present value,

$$p_c^G R_c q_t > 1 > p_c^B R_c q_t + b_c^B,$$

given that the gross rate of intra-period loan is unity. Therefore, only project “Good” should be financed. For simplicity, we assume $p_c^G R_c = 1$, i.e., final goods are transformed one-to-one into capital goods in the aggregate.

After final goods are produced in period t , the total wealth of capital goods producer consists of his wage income, w_t^c and the gross return on his inter-period deposits made in period $t - 1$, d_{t-1}^c . In equilibrium, he uses own funds, $n_t^c = r_{t-1} d_{t-1}^c + w_t^c$, and intra-period loans, z_t^c , to finance his project investment, i_t . Thus, n_t^c is his net worth in the project. According to the loan contract, the capital goods producer promises a repayment of $R_t^c i_t$ units of final goods if the project succeeds; both parties get zero return if the project fails. The capital goods producer always get the private benefits. In order to motivate the capital goods producer to choose project “Good”, mutual funds must provide him with enough incentives,

$$p_c^G (R_c q_t - R_t^c) i_t + b_c^G i_t \geq p_c^B (R_c q_t - R_t^c) i_t + b_c^B i_t.$$

The left (right) hand side denotes the expected utility of the capital goods producer if he chooses project “Good” (“Bad”). As shown below, the expected rate of return on project “Good” exceeds the intra-period loan rate, i.e., unity. The capital goods producer prefers to borrow to the limit. The incentive constraints are binding around the steady state and is simplified to be

$$R_t^c = R_c q_t - b_c, \quad \text{where} \quad b_c \equiv \frac{b_c^B - b_c^G}{p_c^G - p_c^B} > 0. \quad (11)$$

Any promise to repay more than $R_t^c i_t$ to the mutual funds in the case of success is not credible. $p_c^G R_c q_t$ and $p_c^G R_t^c$ are the expected full value and external value per unit of final goods invested in project “Good”, respectively. The difference between the two values, $p_c^G b_c$, is used to motivate the capital goods producer to choose project “Good” despite lower private benefits it provides, $b_c^G < b_c^B$.

Mutual funds are expected to break even in lending to the capital goods producer in period t , $z_t^c = p_c^G R_t^c i_t$. This implies credit constraints for the capital goods producer,

$$z_t^c = \Gamma_t^c n_t^c, \quad \text{where} \quad \Gamma_t^c \equiv \frac{p_c^G R_t^c}{1 - p_c^G R_t^c}$$

is the credit multiplier. Our calibration guarantees that the cost per unit of final goods invested in the project is larger than the external value per unit of final goods invested, $p_c^G R_t^c < 1$, around the steady state and thus, $\Gamma_t^c > 0$. Otherwise, he could finance his project investment using external funds only. As Γ_t^c is independent of n_t^c , loans and the project investment are both proportional to his net worth.

Each unit of the net worth of the capital goods producer enables him to acquire Γ_t^c units of intra-period loans and so, he invests $1 + \Gamma_t^c$ units of final goods in project ‘‘Good’’. The expected gross rate of return on his net worth is

$$\xi_t = p_c^G (R_c q_t - R_t^c) (1 + \Gamma_t^c) = \frac{p_c^G b_c}{p_c^G b_c - (q_t - 1)}. \quad (12)$$

The expected one-to-one transformation of final goods into capital goods implies that the price of capital must be no less than unity. Otherwise, the project would make a loss, $\xi_t < 1$. If the price of capital is at unity, $q_t = 1$, the project breaks even by expectation, $\xi_t = 1$. In this case, the capital goods producer does not invest own funds in the project. If the price of capital is larger than unity, $q_t > 1$, the project is profitable by expectation, $\xi_t > 1$, because the rate of return on the project is larger than the intra-period loan rate which is constant at unity. In this case, the capital goods producer puts all own funds in the project and borrows to the limit.

Capital goods producers who have successful projects are of mass p_c^G and their end-of-period wealth is $\mathcal{N}_t^c = (R_c q_t - R_t^c) (1 + \Gamma_t^c) n_t^c$; those who have failed projects are of mass $(1 - p_c^G)$ and their end-of-period wealth is $\mathcal{N}_t^c = 0$. At the end of period t , the capital goods producer chooses consumption and deposits at the mutual funds to maximize his expected utility (10), subject to his period budget constraints,

$$d_t^c + c_t^c = \mathcal{N}_t^c.$$

The linear preference implies that their marginal utility of consumption is one. If they deposit a unit of final goods at the mutual funds in period t , they will get a safe rate of return, $r_t > 1$, in period $t + 1$. They can invest the deposit return in project ‘‘Good’’ for the expected return of $E_t r_t \xi_{t+1}$. The optimization between consumption and deposit at the end of period t gives the equilibrium condition,

$$1 = E_t \gamma \beta r_t \xi_{t+1}. \quad (13)$$

Per capita deposits, net worth, and investment of capital goods producers are,

$$d_t^c = \xi_t n_t^c - c_t^c, \quad (14)$$

$$n_t^c = r_{t-1} d_{t-1}^c + w_t^c, \quad (15)$$

$$i_t = \frac{n_t^c}{1 - p_c^G R_t^c}. \quad (16)$$

The aggregate capital stock K_t evolves over time as follows,

$$K_t = (1 - \delta) K_{t-1} + i_t. \quad (17)$$

2.6 Mutual Funds

Mutual Funds make intra-period loans to capital goods producers after final goods are produced; at the end of the period, the capital goods producers with successful projects repay their liabilities. The mutual funds also make inter-period loans to entrepreneurs at the end of the period; at the beginning of the next period, the entrepreneurs with successful projects repay their liabilities.

Consider the intra-period business of the mutual funds. There is no aggregate uncertainty during the capital goods production. By perfectly diversifying the portfolios of intra-period loans, the mutual funds pool idiosyncratic project risk of capital goods producers. Due to perfect competition, the mutual funds transfer all of the debt repayments to depositors and make zero profit in their intra-period business. Therefore, intra-period deposits have a safe rate of return at unity and we do not have to specify the supply of intra-period deposits explicitly. The results hold in the case of observable project choice.

Consider now the inter-period business of the mutual funds. As specified in subsection 2.4, entrepreneurs with successful projects repay a predetermined amount $p_e^G R_{t-1}^e k_{t-1}^e$ to mutual funds in period t and their end-of-period wealth is

$$p_e^G [R_e v_t + (1 - \delta') q_t - R_{t-1}^e] k_{t-1}^e = p_e^G [R_e (v_t - E_{t-1} v_t) + (1 - \delta') (q_t - E_{t-1} q_t) + b_e] k_{t-1}^e$$

Due to productivity shocks, the prices of capital and intermediate goods differ from their expected values in the previous period, $q_t \neq E_{t-1} q_t$ and $v_t \neq E_{t-1} v_t$. The expected reward to entrepreneurs $p_e^G b_e k_{t-1}^e$ acts as a buffer and enables successful entrepreneurs to repay the promised amount to the mutual funds. By perfectly diversifying loan portfolios, the mutual funds pool the idiosyncratic project risk of entrepreneurs. Thus, the ex post total repayment from the entrepreneurial sector in period t coincides with its expected value in period $t - 1$, i.e., $p_e^G R_{t-1}^e k_{t-1}^e = r_{t-1} z_{t-1}^e$, and the mutual funds pay a safe rate of return on inter-period deposits.

However, these results do not hold in the case of observable project choice. In this case, entrepreneurs finance all of the project investment using loans and they do not have to put own funds in the project in period t . In period $t + 1$, the successful entrepreneurs transfer all of the project outcomes to mutual funds; they only get private benefits and do not have any claim in the project. By lending z_{t-1}^e to entrepreneurs in period $t - 1$, mutual funds get $p_e^G [R_e v_t + (1 - \delta') q_t] k_{t-1}^e$ in period t . The ex post rate of return on inter-period loans is

$$\tilde{r}_t = \frac{p_e^G [R_e v_t + (1 - \delta') q_t] k_{t-1}^e}{z_{t-1}^e} = r_{t-1} \left[1 + \frac{R_e (v_t - E_{t-1} v_t) + (1 - \delta') (q_t - E_{t-1} q_t)}{R_e E_{t-1} v_t + (1 - \delta') E_{t-1} q_t} \right], \quad (18)$$

which is different from its expected value in period $t - 1$, $\tilde{r}_t \neq E_{t-1} \tilde{r}_t = r_{t-1}$. So is the ex post rate of return on inter-period deposits.

2.7 Final Goods Production and Market Equilibrium

Final goods are produced from intermediate goods and labor,

$$Y_t = A_t M_t^\alpha L_t^{(1-\alpha-\alpha_e-\alpha_c)} (L_t^e)^{\alpha_e} (L_t^c)^{\alpha_c}, \quad (19)$$

$$\log A_t = \rho \log A_{t-1} + \epsilon_t, \quad (20)$$

where M_t denotes the input of intermediate goods; L_t , L_t^e , and L_t^c denote the labor inputs of households, entrepreneurs, and capital goods producers. Total factor productivity, A_t , is positively autocorrelated in logarithms, where $\rho \in (0, 1)$. The productivity shock has mean zero, $E_t \epsilon_{t+1} = 0$, and variance, σ_a^2 .

Productive inputs are priced at their respective marginal products,

$$v_t M_t = \alpha Y_t, \quad (21)$$

$$w_t L_t = (1 - \alpha - \alpha_e - \alpha_c) Y_t, \quad (22)$$

$$w_t^c L_t^c = \alpha_c Y_t, \quad (23)$$

$$w_t^e L_t^e = \alpha_e Y_t. \quad (24)$$

As shown in subsections 2.4 and 2.5, loans and project investment of the two types of agents are proportional to their respective net worth. The assumption of the labor incomes of entrepreneurs and capital goods producers is necessary because it ensures that each entrepreneur and capital goods producer always has a positive level of net worth. In the meantime, we make their wage income very small so that the dynamics of their net worth is not driven by the wage income. Carlstrom and Fuerst (1997) take the same approach.

Markets for capital, intermediate goods, final goods, and credit clear,

$$K_t = k_t + k_t^e, \quad (25)$$

$$M_t = G(k_{t-1}) + p_e^G R_e k_{t-1}^e, \quad (26)$$

$$Y_t = c_t + c_t^c + c_t^e + i_t, \quad (27)$$

$$z_t^e = d_t^c + d_t, \quad (28)$$

Definition 2.1. *Market equilibrium is a set of allocations of households, $\{k_t, l_t, c_t, d_t\}$, entrepreneurs, $\{k_t^e, n_t^e, c_t^e, z_t^e\}$, capital goods producers, $\{n_t^c, i_t, c_t^c, d_t^c\}$, and aggregate variables $\{M_t, Y_t, K_t\}$, given a set of prices $\{v_t, q_t, w_t, w_t^e, w_t^c, r_t, \xi_t, R_t^e, R_t^c\}$ and the exogenous process $\{A_t\}$ satisfying equations (1)-(3), (5)-(9), (11)-(17), (19)-(28).*

3 Three Alternative Models

Section 2 specifies the model with financial frictions in the production of both capital and intermediate goods. We call it Model *DF* (double frictions). This section discusses three alternative models, depending on the side of the moral hazard problem.

3.1 Model FE

In the first case, we assume that the project choice of capital goods producers is observable but that of entrepreneurs is not. Thus, capital goods producers can credibly choose project “Good” and financial frictions exist only in the entrepreneurial sector. It is our basic model and is called model FE (frictions in the entrepreneurial sector).

Capital goods producers can pledge all of the project outcome to mutual funds and finance their project investment using intra-period loans only, $i_t = z_t^c = q_t p_c^G R_c i_t$. Thus, capital goods are priced at $q_t = 1$, and they do not have to put own funds in the project, $n_t^c = 0$. As the deposit rate is less than their time preference rate in equilibrium, $r_t < \frac{1}{\gamma\beta}$, they do not make inter-period deposits at the mutual funds, $d_t^c = 0$. They consume wage income each period, $c_t^c = w_t^c$. For simplicity, we focus on a symmetric equilibrium in which all capital goods producers invest the same amount of final goods i_t in project “Good” and enjoy private benefits, $b_c^G c_t$. Other sectors are same as in model DF .

3.2 Model FC

In the second case, we assume that the project choice of entrepreneurs is observable but that of capital goods producers is not. Thus, entrepreneurs can credibly choose project “Good” and financial frictions exist only in the capital goods production sector. We call it model FC (frictions in the capital goods production).

Entrepreneurs can pledge all of the project outcome to mutual funds, and finance their project investment using inter-period loans only, $q_t k_t^e = z_t^e = \frac{p_e^G [R_e v_{t+1} + (1-\delta') q_{t+1}] k_t^e}{r_t}$. They do not have to put own funds in the project, $n_t^e = 0$. Given that project “Good” is expected to be more productive than that of households, all capital stock is allocated to entrepreneurs and households do not produce intermediate goods. For simplicity, we focus on a symmetric equilibrium in which the newcomers and the surviving entrepreneurs use external funds to invest the same amount of capital goods k_t^e and enjoy private benefits, $b_e^G k_t^e$ in period $t + 1$. They consume their wage income each period, $c_t^e = w_t^e$.

As shown in 2.6, the ex post rate of return on inter-period deposits is different from its expected value in the previous period in the case of observable project choice of entrepreneurs. For uniformity, we use r_t to denote the expected rate of return on inter-period deposits, $r_t \equiv E_t \tilde{r}_{t+1}$. Other sectors remain the same as in the setting with double financial frictions.

3.3 Model RBC

In the last case, we assume that the project choices of entrepreneurs and capital goods producers are observable. Thus, both can credibly choose project “Good” and pledge all the expected outcomes of their projects for external funds. Their project investment is

fully financed by the mutual funds and they do not have to provide own funds. The price of capital is constant at unity, $q_t = 1$ and the projects of capital goods producers earn zero profits, $\xi_t = 1$. Capital is all allocated to entrepreneurs and intermediate goods are produced by entrepreneurs only.

In this case, the model degenerates into a RBC model with a representative agent who has three production technologies: a linear technology to produce intermediate goods using capital, a Cobb-Douglas technology to produce final goods using intermediate goods and labor, and a linear technology to transform final goods into capital goods. So, we call it model *RBC*. The market equilibrium can be defined as the set of two state variables $\{k_t^e, A_t\}$ and eleven control variables $\{c_t, l_t, w_t, c_t^e, w_t^e, c_t^c, w_t^c, i_t, v_t, M_t, Y_t\}$ satisfying equations from (29) to (39),

$$1 = \beta E_t \left(\frac{c_{t+1}}{c_t} \right)^{-\sigma} p_e^G [(1 - \delta') + R_e v_{t+1}], \quad (29)$$

$$w_t = \chi(1 - l_t)^\psi (c_t)^\sigma, \quad (30)$$

$$M_t = p_e^G R_e k_{t-1}^e, \quad (31)$$

$$Y_t = A_t M_t^\alpha L_t^{1-\alpha-\alpha_e-\alpha_c}, \quad (32)$$

$$v_t M_t = \alpha Y_t, \quad (33)$$

$$w_t l_t = (1 - \alpha - \alpha_e - \alpha_c) Y_t, \quad (34)$$

$$c_t^c = w_t^c = \alpha_c Y_t, \quad (35)$$

$$c_t^e = w_t^e = \alpha_e Y_t, \quad (36)$$

$$Y_t = c_t + c_t^e + c_t^c + i_t, \quad (37)$$

$$k_t^e = p_e^G (1 - \delta') k_{t-1}^e + i_t, \quad (38)$$

$$\log A_t = \rho \log A_{t-1} + \epsilon_t. \quad (39)$$

Other variables $\{r_t, \tilde{r}_t, d_t\}$ are inessential to the market equilibrium and can be determined by these eleven variables as shown in subsections 3.1 and 3.2.

4 Dynamic Analysis

4.1 Calibration

We calibrate our basic model (model *FE*) to fulfill some conditions in the non-stochastic steady state. The other three models use the same calibration for consistency.

The quarterly discount factor is set at $\beta = 0.99$, corresponding to an annual interest rate of 4%, while the relative impatience of capital goods producers versus other is set at $\gamma = 0.95$, as in Carlstrom and Fuerst (1997, 1998) and Kato (2006). By convention, we choose the logarithmic preferences for households, $\sigma = 1$ and $\psi = -1$. We set $\chi = 1.92$ so that households work eight hours a day in the final goods production sector in the steady

state, $L = \frac{1}{3}$. Following Carlstrom and Fuerst (1997), we set $\alpha_e = \alpha_c = 0.00001$ so that even the entrepreneurs and capital goods producers with failed projects still have a small wealth to start the projects. We set $\alpha = 0.36$ so that the household wage income accounts for almost 64% of the aggregate output of final goods.

Following Carlstrom and Fuerst (1997, 1998), we choose $\rho = 0.95$ for the autocorrelation coefficient of TFP and the standard deviation of the TFP shocks is set at $\sigma_a = 0.007$. As in Carlstrom and Fuerst (1997), a quarterly rate of business failure at 1% implies $p_e^G = p_c^G = 0.99$. Capital invested in the household project depreciates at a quarterly rate of $\delta = 2.5\%$ and capital invested in the entrepreneurs' projects that become successful depreciates at the rate of $\delta' = 1.52\%$. Thus, the aggregate capital stock depreciates at the rate of $\delta = 2.5\%$ in equilibrium.

By assumption, $R_c = \frac{1}{p_c^G} = 1.01$. The expected profitability of the projects of capital goods producers is $\xi = \frac{1}{\gamma} > 1$ in the steady state so that they invest all own funds into the projects and borrow to the limit. We set $b_c = 0.53$ so that capital goods producers finance half of their project investment using intra-period loans, as in Bernanke, Gertler, and Gilchrist (1999).

Our results do not depend on the aggregate capital stock and we normalize it at unity. Capital-output ratio ranges from 2.8 for US (Jones, 2002) to 1.8 for Japan (Hayashi and Prescott, 2002). We set $\frac{K}{Y} = 2.2$. Entrepreneurs finance half of the their project investment using external funds, as in Bernanke, Gertler, and Gilchrist (1999). We calibrate the production functions of entrepreneurs and households in order to match the above conditions. The household production function takes the the following form,

$$G(k) = \begin{cases} gk_t & \text{for } 0 < k \leq \underline{k}, \\ \frac{\epsilon}{1+\lambda} k^{1+\lambda} & \text{for } k > \underline{k}, \end{cases} \quad \text{thus,} \quad G'(k) = \begin{cases} g & \text{for } 0 < k \leq \underline{k}, \\ \epsilon k^\lambda & \text{for } k > \underline{k}, \end{cases}$$

where $g = \frac{\epsilon}{1+\lambda} \underline{k}^\lambda$. We set $\{R_e = 1.34, b_e = 0.76, \epsilon = 0.145, \lambda = -0.2, \underline{k} = 0.0001\}$. In equilibrium, the expected marginal product of project ‘‘Good’’ of entrepreneurs always exceeds that of the household project $p_e^G R_e = 1.33 > g = 1.14$. As the steady state value of the households' capital stock is $k = 0.5$, the household production function is differentiable around the steady state. Note that a simple shift of capital from households to entrepreneurs increases output even without any change in the aggregate capital stock.

4.2 Impulse Responses to Productivity Shocks

The endogenous variables are approximated as the linear functions of the state variables in logarithms around the respective steady states of the four models (*DF*, *FE*, *FC*, and *RBC*), which we solve using the MATLAB codes provided by Schmitt-Grohe and Uribe (2004). We analyze the model dynamics with respect to a transitory TFP shock in period 0, given that the models are in steady state before period 0. We first show how financial

frictions in the capital goods production can result in the dampened hump-shaped output responses to the TFP shock in subsection 4.2.1. Then, we show how our basic model with financial frictions in the entrepreneurial sector can generate more amplified and delayed output responses in subsection 4.2.2. Finally, we show that the model with double financial frictions does not improve the performance of our basic model much in subsection 4.2.3.

4.2.1 Financial Frictions in the Capital Goods Production

Figure 1 shows the impulse responses of model *RBC* (dotted line) and model *FC* (solid line) to a TFP shock, where Agg, FG, HH, EN, and CGP refer to aggregate, final goods, households, entrepreneurs, and capital goods producers, respectively.

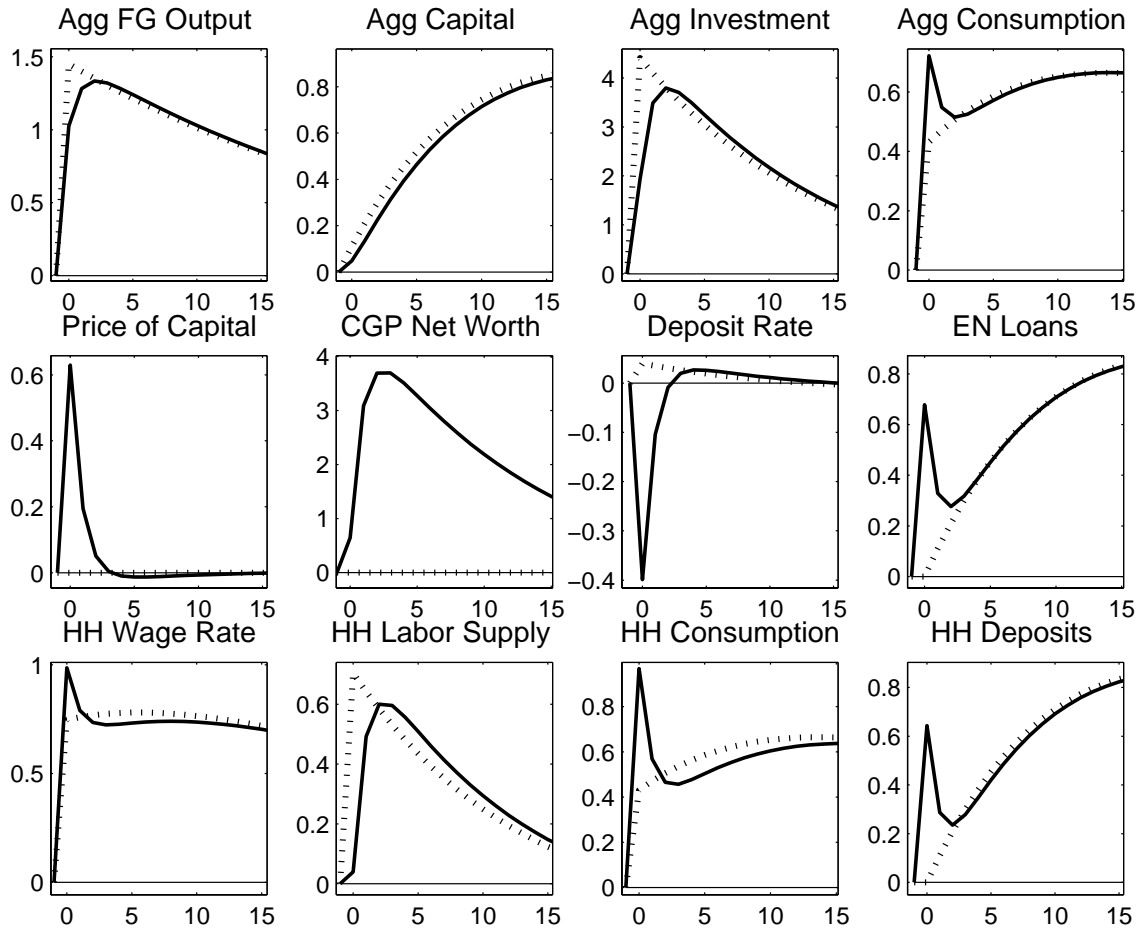


Figure 1: Model *FC* vs. Model *RBC*

Consider model *RBC* first. As capital is the only endogenous state variable, the dynamic structure is essentially $ARMA(1, 1)$ and fails to generate the hump-shaped output dynamics. A 1% TFP shock raises the marginal products of intermediate goods and labor in period 0. The rise in the price of intermediate goods makes the ex post value of the entrepreneurs' project output exceed its expected value. Essentially, entrepreneurs transfer all of the project outcomes to households via the mutual funds and households

benefit from the positive wealth effect. At the same time, the rise in the household wage rate makes households increase labor supply by 0.7%, because they prefer to smooth consumption over time. As the aggregate supply of intermediate goods is determined by the project investment of entrepreneurs made in period -1 , aggregate output of final goods rises by 1.45% in period 0.

Due to the autocorrelation in TFP, the marginal product of intermediate goods stays above its steady state value in period 1 and so does the price of intermediate goods. As entrepreneurs can fully pledge the expected full value of their project to mutual funds, they can acquire more loans and expand their project investment. The excess demand of entrepreneurs for loans pushes up the interest rate and induces households to deposit more at the mutual funds. At the same time, capital goods producers increase their investment expenditure to fully accommodate the entrepreneurs' extra demand for capital goods. Essentially, the model dynamics are driven by the fact that households smooth consumption over time by saving in the form of capital goods.

Consider model *FC*. There are three endogenous state variables, $\{k_t^e, d_t^c, z_t^e\}$ and the dynamic interactions between the price of capital and borrowing constraints of capital goods producers can generate the hump-shaped output responses to productivity shocks, as shown in Carlstrom and Fuerst (1997) and Kato (2006). A 1% TFP shock enables entrepreneurs to demand more capital, as mentioned above. Because the capital goods production is constrained by the net worth of capital goods producers, the entrepreneurs' excess demand for capital cannot be fully accommodated and the price of capital goods rises. In comparison with model *RBC*, the additional capital gains on the entrepreneurs' capital stock raises the ex post value of the entrepreneurs' project outcome more dramatically; the ex post return on deposits also exceeds its expected value to a larger extent. The enhanced positive wealth effect induces households to raise deposits and consumption in a larger magnitude. Due to the unexpected increase in their deposit return, households raise labor supply only by 0.04%, despite the rise in the wage rate. Aggregate output of final goods rises by 1.03% in period 0, much less than 1.45% in model *RBC*.

The unexpected increase in the deposit return also improves the net worth of capital goods producers, $n_0^c = \tilde{r}_0 d_{-1}^c + w_0^c$. At the same time, the rise in the price of capital makes their projects more profitable and the credit multiplier rises, too. As a result, they expand their project investment by 1.94%. However, capital goods producers are credit-constrained and cannot fully exploit the profit opportunity in the shock period. As a result, the increase in their project investment is smaller than the 4.4% in model *RBC*.

Given that the price of capital is above its steady state value in period 1, the project of capital goods producers is still more profitable than in the steady state. In order to have more wealth for investment in period 1, they consume less and deposit more in period 0. As a result, their net worth rises by 3.1% in period 1 and they expand the project investment. As their net worth is not enough, capital goods producers are constrained

and cannot fully accommodate the demand for capital goods in period 1. It also justifies the fact that the price of capital is still above its steady state value in period 1.

Due to the credit-constrained production of capital goods, the aggregate capital stock rises much less than in model *RBC* in period 0; so is aggregate output of intermediate goods in period 1. Meanwhile, the household wage rate is 0.79% above the steady state value in period 1. As the deposit return improves household wealth, households raise their consumption and labor supply in period 1. Thus, aggregate output is around 1.28% above the steady state value, still lower than the 1.4% in model *RBC*.

It takes two periods for capital goods producers to accumulate sufficient net worth and accommodate the aggregate demand for capital. The price of capital converges very closely to the steady state value from period 3 on. The interaction between the price of capital and borrowing constraints of capital goods producers constitutes a dampened propagation mechanism through which aggregate output peaks by 1.33% in period 2, later and smaller than in model *RBC*.

4.2.2 Financial Frictions in the Entrepreneurial Sector

Figure 2 shows the impulse responses of model *FE* (solid line) and model *RBC* (dotted line). There are three endogenous state variables, $\{k_t^e, k_t, R_t^e\}$ in model *FE*. Different from the dampened propagation mechanism in model *FC*, it is now the dynamic interaction of borrowing constraints, endogenous capital accumulation and reallocation between entrepreneurs and households that generates the amplified and hump-shaped output responses. As the capital goods production is not subject to financial frictions, the price of capital is constant at unity, $q_t = 1$.

Consider model *FE*. A 1% TFP shock in period 0 raises the price of intermediate goods. Extra sales revenues improve per capita post-repayment wealth of entrepreneurs,

$$\mathcal{N}_0^e = p_e^G [b_e + R_e(v_t - E_{-1}v_0)] k_{-1}^e > p_e^G b_e k_{-1}^e = E_{-1}\mathcal{N}_0^e. \quad (40)$$

Entrepreneurial net worth rises, too. Due to the autocorrelation of TFP, the price of intermediate goods is above the steady state value in period 1 and so is the expected external value of the entrepreneurs' projects. Thus, entrepreneurs acquire more loans and invest more capital in their project. The deposit rate rises to clear the market.

The rise in the deposit rate induces households to deposit more and invest less in the project in period 0. Extra sales revenues of intermediate goods have the positive wealth effect on the household consumption and labor supply. Although the household wage rate rises by 0.96% in period 0, they increase their labor supply only by 0.12%, much less than 0.7% in model *RBC*. Given the predetermined aggregate supply of intermediate goods, aggregate output of final goods increases only by 1.1% in period 0, less than 1.45% in model *RBC*.

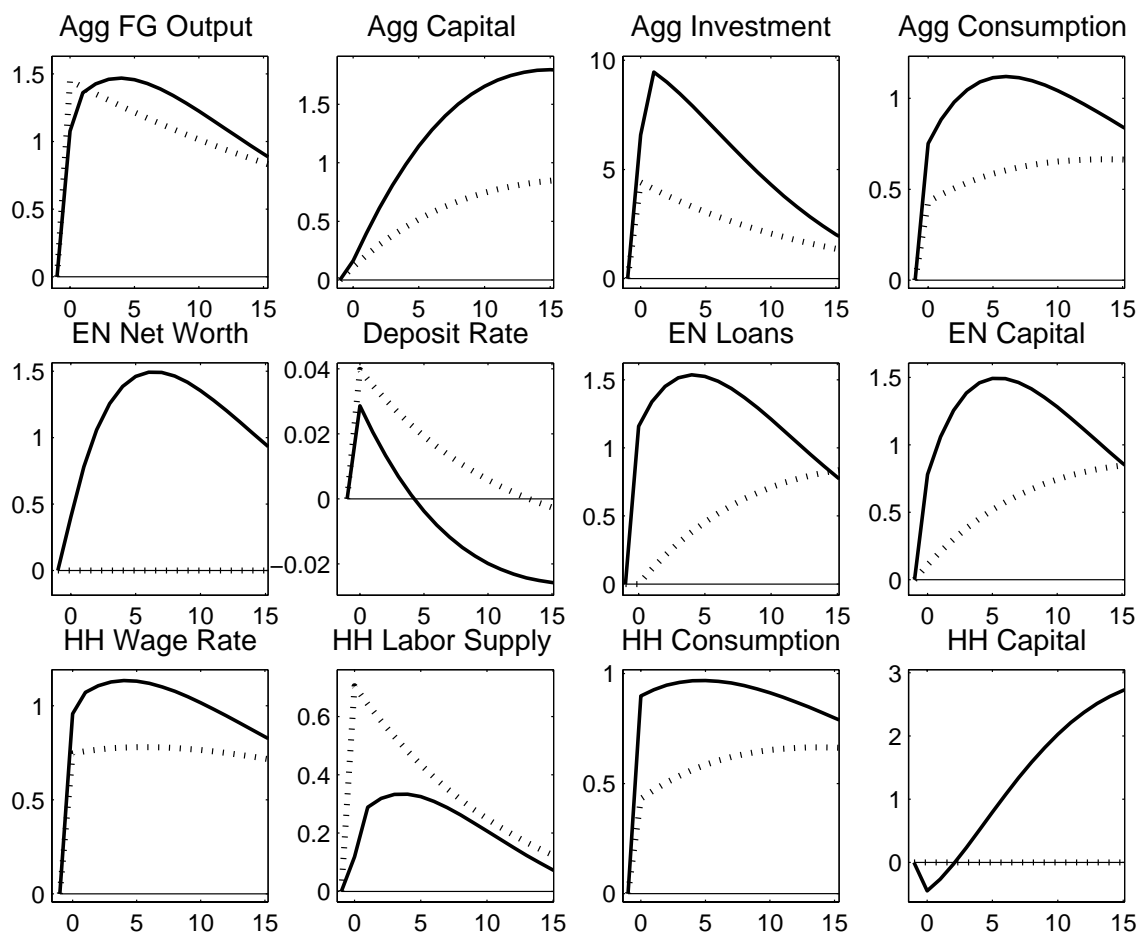


Figure 2: Model *FE* vs. Model *RBC*

It takes time for entrepreneurs to accumulate net worth and expand the project scale. Thus, the entrepreneurs' capital stock peaks five periods after the shock and the reallocation of capital between entrepreneurs and households is also delayed. Given the output of the entrepreneurs' project accounts for more than 86% of aggregate output of intermediate goods in the steady state, the latter almost follows the dynamic pattern of the entrepreneurs' capital stock.

Altogether, the interaction of borrowing constraints, capital accumulation and reallocation constitutes a mechanism through which aggregate output of final goods peaks in period 4 by 1.47% above the steady state value, more than the maximum output response of 1.45% in period 0 in model *RBC*; while, aggregate output of final goods peaks in period 2 by only 1.33% in model *FC*. In this sense, model *FE* dominates model *FC* in generating more amplified and delayed output responses to TFP shocks.

4.2.3 Double Financial Frictions

Figure 3 shows the impulse responses of model *DF* (dash-dot line) and model *FE* (solid line). Both entrepreneurs and capital goods producers are subject to financial constraints

and there are six endogenous state variables, $\{k_t, k_t^e, z_t^e, R_t^e, r_t, d_t^c\}$ in model DF . The dynamic interactions between the price of capital and double financial frictions reinforce the amplification mechanism of model FE .

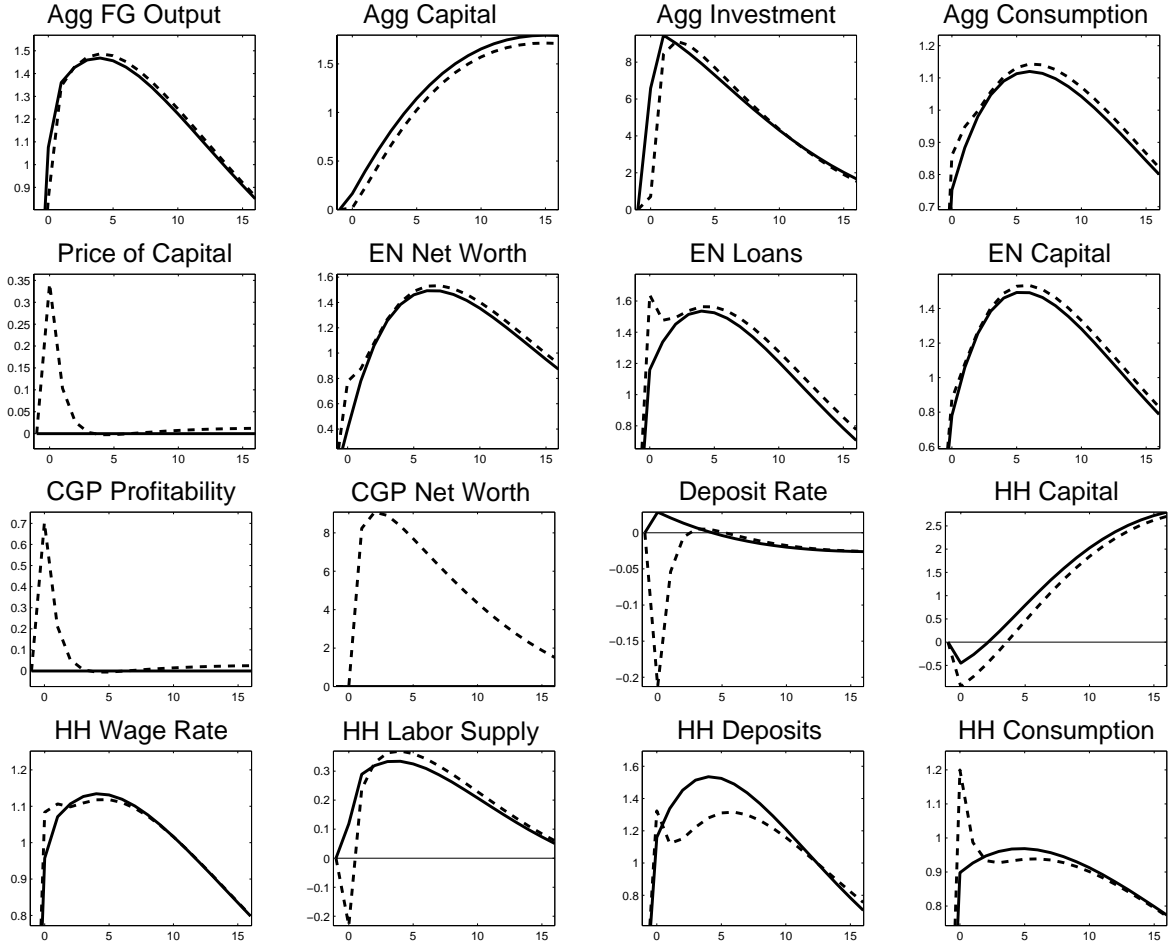


Figure 3: Model FE vs. Model DF

Consider model DF . A 1% TFP shock in period 0 pushes up the price of intermediate goods. Extra sales revenues improve entrepreneurial net worth. It enables entrepreneurs to acquire more loans and invest more capital into their project. Due to the constrained production of capital goods, the price of capital rises in period 0.

Extra sales revenues and capital gains have positive effects on the household wealth in period 0. Households consume and deposit more than in model FE . Despite the rise in the wage rate, households reduce labor supply. Aggregate output of final goods rises by 0.85%, even less than the magnitude of the TFP shock. Meanwhile, the rise in the price of capital depresses the the household project investment more than in model FE .

The rise in the price of capital makes the project of capital goods producers more profitable. They borrow more and expand their investment in period 0. As the price of capital is still above the steady state value in period 1, capital goods producers reduce consumption and increase deposits in period 0 in order to have more wealth for investment

in period 1. The rise in the deposits of households and capital goods producers reduces the interest rate in period 0. In contrast, the interest rate falls in period 0 in model *FE*.

The decline in the interest rate and the further improvement in entrepreneurial net worth due to capital gains enable entrepreneurs to increase their project investment slightly more than in model *FE*. Thus, the rise in the price of capital speeds up the reallocation of capital from households to entrepreneurs in period 0.

However, due to financial frictions, the demand of entrepreneurs for capital is constrained in period 0. It explains that the rise in the price of capital is much smaller than in model *FC*. The increase in the deposits of capital goods producers in period 0 improves their net worth in period 1 and they can expand their project investment by 8.4%, still smaller than in model *FE*. It does not fully accommodate the entrepreneurs' demand for capital and the price of capital is still above the steady state value in period 1.

It takes two periods for capital goods producers to accumulate net worth and accommodate the demand for capital after the shock. The price of capital gets closer to the steady state level rather quickly. The dynamic pattern and the magnitude of the responses of the entrepreneurs' capital stock do not differ much from those in model *FE* after period 3. Aggregate output of final goods peaks by 1.49% in period 4, slightly higher than 1.47% in model *FE*. In this sense, financial frictions and the reallocation of capital in model *FE* can explain the amplified and hump-shaped output responses to TFP shocks alone.

5 Conclusion

We develop a dynamic general equilibrium model with financial frictions to explain one of the important empirical puzzles in the real business cycles literature. Compared to other models with financial frictions in the literature, the dynamic interaction of borrowing constraints, endogenous capital accumulation and reallocation in our model constitutes a robust mechanism through which aggregate output responds to TFP shocks in a more amplified and hump-shaped fashion, in line with the empirical evidence in the literature.

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