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## The Dynamic of Bicycle Finals

A Theoretical and Empirical Analysis of Slipstreaming

*Alexander Dilger/Hannah Geyer*

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## **The Dynamic of Bicycle Finals A Theoretical and Empirical Analysis of Slipstreaming**

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### **Abstract**

The finals of bicycle races have certain peculiarities compared to other sports. The leading group in a bicycle race rides comparatively slowly up to a few meters before the finishing line, until one of the competitors tries to shake off his opponents. Only then do all riders perform to the limit. This raises the question of who takes the thankless early lead and why. The rider who is in front just before the final sprint is seldom the one who wins in the end. By means of the relevant physics it can be shown theoretically that on the one hand the better rider will always be able to win the race and, more surprisingly on the other hand, the better rider will definitely be the rider in the slipstream. These findings are confirmed empirically by means of several logistic regressions. 49 final sprints of road races between two up to seven professional racing cyclists with varying performance potentials were analyzed concerning the order of the riders at the beginning of the final sprint and the final outcome of the race. Subsequently, possibilities for further research and implications for sport economics are described.

# **Die Dynamik von Sprintentscheidungen im Radsport**

## **Eine theoretische und empirische Analyse des Windschattenfahrens**

### **Zusammenfassung**

Sprintentscheidungen im Radsport weisen im Vergleich zu anderen Sportarten einige Besonderheiten auf. Die in einem Radrennen führende Gruppe fährt bis wenige Meter vor die Zielinie vergleichsweise langsam, bevor dann ein Fahrer kurz vor dem Ziel versucht, seine Konkurrenten abzuschütteln. Erst dann erreichen die Fahrer ihre Höchstgeschwindigkeit. Der zu Beginn des Sprints in Führung liegende Rennfahrer gewinnt dabei in den seltensten Fällen am Ende auch das Rennen. Daher stellt sich die Frage, wer zu Beginn des Sprints die undankbarere Führungsposition im Wind übernimmt und warum. Zunächst kann mittels grundlegender Physik theoretisch gezeigt werden, dass zum einen der bessere Fahrer immer in der Lage sein wird, das Rennen zu gewinnen, und zum anderen, weniger naheliegend, der bessere Fahrer stets zu Beginn des Sprints im Windschatten fahren wird. Diese theoretischen Ergebnisse werden anschließend empirisch anhand logistischer Regressionen bestätigt. 49 Sprintentscheidungen in Straßenradrennen zwischen zwei bis sieben professionellen Radrennfahrern mit unterschiedlichen Leistungsstärken werden auf die Reihenfolge der Fahrer zu Beginn des Sprints und das Ergebnis des Rennens hin untersucht. Abschließend werden Möglichkeiten für weitere Untersuchungen sowie sportökonomische Implikationen dargestellt.

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# The Dynamic of Bicycle Finals

## A Theoretical and Empirical Analysis of Slipstreaming\*

### 1. Introduction

The finals of bicycle races have certain peculiarities compared to other sports. Thus, in a discussion paper with the telling title “Tour de France – why don’t they ride faster?”, Piorr (1996) posed the question of why the leading group in bicycle races rides relatively slowly up to a few meters before the finishing line, until one of the competitors tries to shake off his opponents. Only then do all riders perform to the limit. The rider who is in front just before the final sprint is seldom the one who wins in the end. That raises the question of who takes the early lead and why. The question can thus be reversed: “Why don’t the competitors stop riding?”, which actually happens in track racing once in a while.

The leading cyclist has the advantage that he has already ridden slightly more than the others. If this lead is large enough immediately before the finishing line, he will win. There would then be no dynamic of the final to be analysed. Admittedly, this is seldom the case, because riding in groups gives riders the advantage of slipstreaming. Kyle (1979) shows that this slipstreaming effect reduces the power needed by 33 %. This is why single riders seldom gain a large lead over the field and maintain it. There are two main advantages for the rider behind, if the lead is only small, say a wheel’s length. On the one hand, there is the abovementioned slipstream effect, and on the other hand, the cyclists behind are able to exploit an element of surprise, as the one in front cannot, of course, constantly observe the others. However, this aspect alone cannot explain the abovementioned peculiarities of bicycle races, since, in order to avoid unwanted surprises, the leading rider could always sprint immediately, thus becoming the first.

The first aspect, the slipstream, is analysed in greater detail in due course. To make matters less complex, it is assumed that only two riders fight out the final. Either the field of other riders is far behind, or it is a sprint race with just two competitors. In the second section, the relevant physics for this specific case will be analysed, especially with respect to riding in the slipstream. The third section contains a simplified model of overtaking, in order to deduce which rider takes the thankless first position. The fourth section presents the results of the

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\* Many thanks to Nadine Beckmann and Claudia Raulf for help with the data. Parts of the theory were published in German in: Dilger, A. (2002). Zur Dynamik im Finale von Radrennen: Analyse des Windschattenfahrens. In: Horch, H.-D., Heydel, J. & Sierau, A. (Eds.), Finanzierung des Sports: Beiträge des 2. Kölner Sportökonomie-Kongresses (pp. 214-223). Aachen: Meyer & Meyer.

empirical analysis and in the final sections, some conclusions are drawn and the outlook for further research discussed.

## 2. The Physics of Slipstreaming

In physics, the following basic relations exist between velocity ( $v$ ), place ( $x$ ), time ( $t$ ), acceleration ( $a$ ), mass ( $m$ ), force ( $F$ ), work ( $W$ ) and power ( $P$ ):<sup>1</sup>

- (1) velocity := change in place per unit of time:  $v = \frac{\Delta x}{\Delta t}$
- (2) acceleration := change in velocity per unit of time:  $a = \frac{\Delta v}{\Delta t}$
- (3) force := mass times acceleration:  $F = m \cdot a$
- (4) work := force times change in place:  $\Delta W = F \cdot \Delta x$
- (5) power := change in work per unit of time:  $P = \frac{\Delta W}{\Delta t}$ .

The maximum performance potential measured in kilowatt ( $kW$ ) is important for describing the riders. For the sake of simplicity it is assumed that just two riders, Riders 1 and 2, take part in the final. They differ only in their performance potential,  $PP_1$  and  $PP_2$ . Moreover, both riders are assumed to be absolutely rational and aware of all relevant parameters, including the respective performance potentials. The following maximum performance applies without loss of generality

$$(6) \quad PP_2 = (1 + q)PP_1.$$

Any differences ( $q \neq 0$ ) in performance can be caused by the riders' basic constitution or the manner in which the competition has proceeded up to that point. It is also assumed that, for the short period of the final,  $PP_1$  and  $PP_2$  and more importantly,  $q$  are constant.

Using (1) to (5), the propulsion  $F_{prop}$  can be calculated from the power ( $P$ ) as:

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<sup>1</sup> These are actually (partly) vectorial parameters. However, in this and the following cases, just one direction of movement is assumed, i.e. on a straight track directly to the finish. Bends and the problems of veering in and out are not considered. The formulas can be found in any physics textbook, e.g. Halliday et al. (2001).

$$(7) \quad F_{prop} = \frac{P}{v}.$$

In addition to the propulsion, two other important forces are at work in cycling. These are rolling friction  $F_{roll}$  and drag  $F_{drag}$ . (See Grappe et al. (1997) or Wilson (2004) and the references they cite.) For the sake of simplification, the track is assumed to be flat. Otherwise, it would be necessary to take gravity into consideration as well. This would be the same for both riders, but with potential differences in acceleration if they had different masses. Additionally, there are of course many other potentially relevant forces and factors, e.g. the rotation of the wheels or energy transmission losses with heat development. However, these relatively minor parameters are ignored here.  $F_{roll}$  can be taken as constant and independent of velocity,<sup>2</sup> such that it is relatively unimportant at higher velocities:<sup>3</sup>

$$(8) \quad F_{roll} = r.$$

The drag depends on the square of the relative velocity through the air. Again, for the sake of simplicity and without loss of generality, it is assumed that the wind speed is zero, so that the relative speed through the air is the same as the actual riding speed  $v$ . Furthermore, the drag depends on the air density  $\rho$ , the drag coefficient  $C_d$  and the front surface  $A$  of the rider and his bicycle. In the case of two separately riding, but identical cyclists, these three extra variables can be combined into one constant  $c$ :<sup>4</sup>

$$(9) \quad F_{drag} = \frac{\rho}{2} C_d A v^2 = c v^2.$$

If a rider cycles in the slipstream of the other cyclist, it will be assumed, that his drag  $F_{slipstream}$  decreases for that period of time by the factor  $0.62 - 0.0104d_w + 0.0452d_w^2$  as Olds (1998) constructed from graphical data of Kyle (1979). Therefore

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<sup>2</sup> See di Prampero (1979), p. 202.

<sup>3</sup> To be more exact, it holds that  $F_{roll} = C_r mg$ , with  $C_r$  being the rolling coefficient and  $g$  the gravitational constant. One could substitute  $zm$  for  $r$  in the following with the constant  $z := C_r g$ .

<sup>4</sup> See di Prampero (1979), Formula (3), p. 202.

$$(10) \quad F_{slipstream} = (0.62 - 0.0104d_w + 0.0452d_w^2)F_{drag} = (0.62 - 0.014d_w + 0.0452d_w^2)cv^2,$$

where  $d_w$  is the wheel-to-wheel distance between the two riders. Kyle (1979) further assumes that for a distance  $d_w \geq 3$  meters no slipstream-effect occurs. Therefore beyond this distance, (9) is applied. This entails simplifying since the drag of the first rider also decreases, if the other one rides behind him. The air in front is denser than behind, which causes a wake to the rear. The other rider partly fills the space behind him and thus reduces the wake (the drag coefficient  $C_d$  decreases). Therefore, the front rider benefits, even if he remains in the lead position. For the acceleration of a rider in front it thus follows from (3) and (7) to (9):

$$(11) \quad a = \frac{\left(\frac{P}{v} - r - cv^2\right)}{m}.$$

In order to compute the velocity  $v$  from equation (11), a non-trivial differential equation has to be solved. It is not evident whether it has a closed form and can be analytically solved for  $v$ .

### 3. A Simple Model of Overtaking

Since the general approach seems too complex, from now on, the momentary velocity  $v$  is replaced by the initial velocity  $v_0$  in the equation for acceleration (11), which means no large error in the short term. Ideally, this mistake and those discussed above would cancel each other out. However, the initial velocity must not be too low or even zero. A very low or zero velocity is a problem in general, since the propulsion  $F_{prop}$  approaches infinity for  $v$  approaching zero.

It is assumed that Rider 2 is at the rear end of the slipstream from Rider 1 with the same velocity  $v_0$ . The value of  $v_0$  will be considered further on in the paper. First, it is necessary to consider how (with a given  $v_0$ ), the final's dynamic develops if both riders produce their maximum performance  $PP_1$  or  $PP_2$  respectively, at the same time. The acceleration of Rider 2 then amounts to:

$$(12) \quad a_2 = \frac{(1+q)\frac{PP_1}{v_0} - r - (0.62 - 0.0104d_w + 0.0452d_w^2)cv_0^2}{m}.$$

Differences in the acceleration of Rider 1 arise as a result of the difference in performance potential ( $q$ ) and the slipstream effect ( $0.62 - 0.0104d_w + 0.0452d_w^2$ ). As long as Rider 2 does not overtake and in turn does not give Rider 1 slipstream, the velocity of the latter is:

$$(13) \quad v_1 = \frac{\frac{PP_1}{v_0} - r - cv_0^2}{m}t + v_0.$$

Accordingly, for Rider 2, the following applies:

$$(14) \quad v_2 = v_1 + \frac{qPP_1t}{v_0m} + \frac{(0.38 + 0.0104d_w - 0.0452d_w^2)cv_0^2t}{m}.$$

There is a condition following from (14) that must be fulfilled, so that Rider 2 does not get shaken off and definitely lose the race. In the slipstream, the rear rider must always be able to cycle faster than the one in front. Otherwise, he falls out of the slipstream, and becomes even slower. If the respective speeds are the same, the rear rider stays in the slipstream, but is never able to overtake and therefore loses as well.

$$(15) \quad v_2 > v_1$$

$\Rightarrow$

$$(16) \quad \frac{qPP_1t}{v_0m} + \frac{(0.38 + 0.0104d_w - 0.0452d_w^2)cv_0^2t}{m} > 0$$

$\Rightarrow$



$$(17) \quad q > \frac{-(0.38 + 0.014d_w - 0.0452d_w^2)cv_0^3}{PP_1}$$

The condition (17) is always fulfilled for positive values of  $q$  as Figure 1 (with  $c = 0.18$  and  $PP_1 = 500$ ) clarifies.

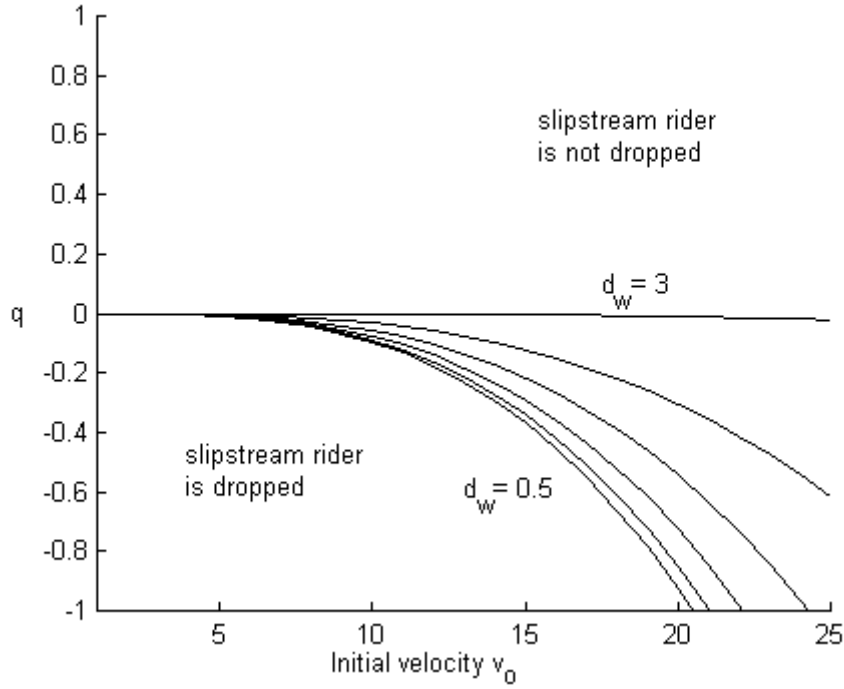


Figure 1: Graphical illustration of condition (17).

The rider with the higher performance potential can thus always catch up with his opponent, as long as he rides in latter's slipstream. Therefore, the second rider has to overcome a distance  $d_b + d_w$  with the velocity difference (between the two riders) stated in (16), such that the integral of this velocity must be equated with  $d_b + d_w$ , where  $d_b$  is the length of the front riders' bicycle and  $t_d$  the time to overtake the leading rider.

$$(18) \quad d_b + d_w = \frac{qPP_1t_d^2}{2v_0m} + \frac{(0.38 + 0.0104d_w - 0.0452d_w^2)cv_0^2t_d^2}{2m}$$

⇒

$$(19) \quad t_d = \sqrt{\frac{2m(d_b + d_w)}{\frac{qPP_1}{v_0} + (0.38 + 0.0104d_w - 0.0452d_w^2)cv_0^2}}$$

If the better performing rider rides in the other one's slipstream, he thus only needs to commence the final sprint a few seconds earlier than  $t_d$ <sup>5</sup> in order to ensure that he wins. Figure 2 depicts the effect of  $d_w$  and  $q$  on  $t_d$  with  $PP_1 = 500$ ,  $m = 70$ ,  $c = 0.18$  and  $d_b = 1.8$ .

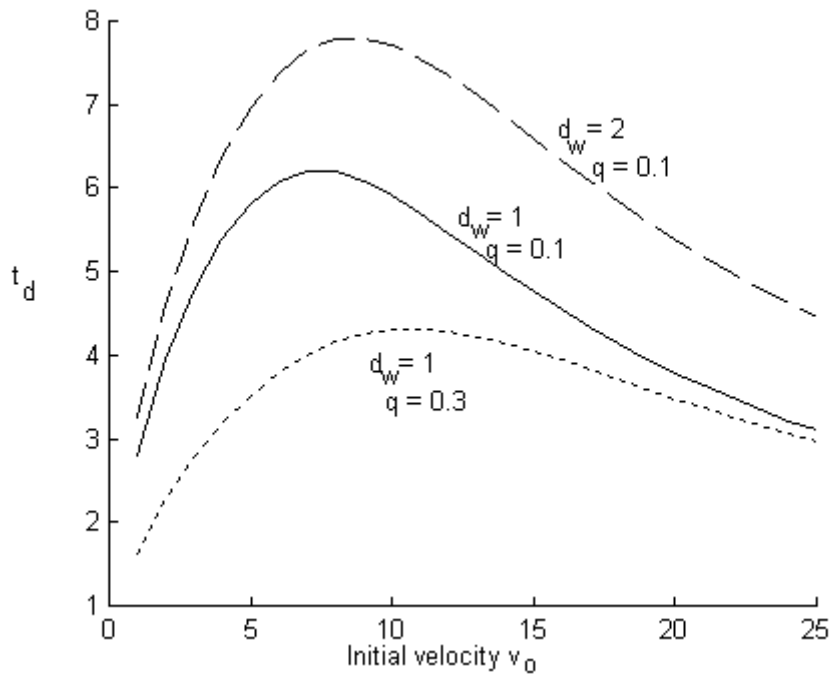


Figure 2: Time  $t_d$  Rider 2 needs to overtake subject to the initial velocity.

More surprisingly, from (17), it can additionally be deduced that the better performing cyclist will definitely be the rider in the slipstream, who will therefore always win as he breaks free of the slipstream in the final spurt. It is not surprising that the better sportsman has a greater chance of winning or even that he will always win. However, slipstreaming not only changes this, but brings it about in a peculiar way, with the better cyclist riding behind the weaker one in his slipstream until the final sprint. While this condition can also be fulfilled for negative

<sup>5</sup> Through integration of (13) or (14) and setting in  $t_d$ , the distance to the finish can be converted in dependence on  $v_0$ .

values of  $q$  with a high initial velocity  $v_0$ , it is regularly not fulfilled with a small  $v_0$ .<sup>6</sup> If the front rider is the better performing one, he thus only needs to lower the common velocity<sup>7</sup> during the slipstream-cycling period, sufficiently far ahead of the finish. Then, he either manages to shake off the rear rider completely with maximum performance, because (17) is not given, or the rear rider has to ride faster and overtake. Accordingly, the better performing rider benefits from the slipstream of the other and wins in the final sprint.

#### 4. Empirical Results

In this section three hypotheses following from the theoretical implications should be tested:

*(H1) The better rider will win the race.*

*(H2) The better rider will be the rider in the slipstream.*

*(H3) The rider in the slipstream will win more often than the rider who starts the sprint out of the front position.*

In order to test these hypotheses empirically, a logistic regression model is used.<sup>8</sup> The model can detect whether the performance potential or the position at the beginning of the final sprint influences the results of the race and whether the performance potential has an influence on the position at the beginning of the sprint.

The empirical data are the results of professional road races from the 2002 season to the 2006 season.<sup>9</sup> In 49 finals of road races in which a small group of riders<sup>10</sup> sprinted in an attempt to

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<sup>6</sup> For a  $v_0$  close to or equal to zero, the simplification in the third section is not allowed as mentioned above. This analysis cannot be used if the differences in performance potential are only marginal.

<sup>7</sup> That does not mean abrupt braking, which would give the other rider a chance to shake off his opponent (at least initially). The front cyclist could rather lower the speed gradually by pedalling slower. By doing so, the respective speeds will remain (nearly) the same and the possibility to commence the final sprint with maximum performance is still assured.

<sup>8</sup> For an overview of the logistic regression method see Menard (2002).

<sup>9</sup> Source for results of the races is [www.radsport-news.com](http://www.radsport-news.com) (last viewed on January 22, 2009).

win, the order of the riders at the beginning of the final sprint, the course of the sprint and the results were analysed, using videos from the internet.<sup>11</sup> 26 of the considered finals were duels, in 13 cases, three riders competed against each other and in the last 10 races, a small group of four to seven riders competed. Altogether, 140 riders were involved in the final sprints.

In order to test whether the best performing rider in a group wins more races than a weaker performing rider, and whether a better performing rider starts more races out of the slipstream, a performance measure had to be found. With the aim of deciding which of the riders in the sprint group has the greatest performance potential, their sprint quality was rated on a scale from 1 (lowest level) to 5 (highest level). The scale is based on the ratings given in [www.radsportdaten.de](http://www.radsportdaten.de)<sup>12</sup>, which evaluates more than 6,600 professional riders in eight categories. The categories beyond sprint qualities were performance in stages and one-day races, performance in small and major stage-races (races over several weeks), in cycling against the clock, in championships, as well as climbing qualities. However, it turned out that sprint quality was the most reliable measurement for this analysis.

By means of this performance measurement, the best rider in each group was identified and characterised as “stronger”, and his weaker performing opponent as “weaker”. In groups of more than two riders, each rider, apart from the best performing one, was characterised as “weaker”.

First of all, in 24 out of 49 races, the rider who was in front at the beginning of the final sprint won the race. In the remaining 25 races, one of the riders in the slipstream finally succeeded in winning. 29 races were won by the rider with the greatest performance potential in the group and in only 23 races, did the better performing rider start the sprint from the slipstream. An explanation of this surprisingly low number of victories from the slipstream is given below.

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<sup>10</sup> In the second paragraph above, it was assumed that only two riders take part in the final. To obtain a greater quantity of data for the empirical tests, small groups of riders up to seven were also included in the analysis.

<sup>11</sup> Source for the videos are e.g.: [www.eurosport.de](http://www.eurosport.de), [www.zdf.de/mediathek](http://www.zdf.de/mediathek), [www.sf.tv](http://www.sf.tv) or [www.tds.ch](http://www.tds.ch) (last viewed on February 27, 2007).

<sup>12</sup> Last viewed on January 22, 2009.

	N	Percentage of races with		
		wins by stronger rider	wins by front rider	stronger rider in slipstream
Races with two riders	26	50.00 %	50.00 %	46.15 %
Races with three riders	13	61.54 %	53.85 %	53.85 %
Races with more than four riders	10	80.00 %	40.00 %	40.00 %
All races	49	59.18 %	48.98 %	46.94 %

*Table 1: Descriptive Data*

*(H1) The better rider will win the race.*

The initial results of the logistic regression model show that performance potential has a significantly positive influence on the final result of the race. The rider with the greater performance potential wins significantly more often against his lower performing rival, than the other way round. Table 2 shows that the probability of winning is five times greater for a better performing rider than for a weaker one ( $Exp(B) = 5.002$ ).

*(H2) The better rider will be the rider in the slipstream.*

Instead of simply analysing whether the better performing rider starts the final sprint out of the slipstream significantly more often than an inferior rider, this question is differentiated, depending on whether the following peloton is more or less than 10 seconds away. This discrimination is necessary, since the behaviour of the leading group of riders depends on the distance between them and the peloton. The reason for the different behaviour is obvious: If the peloton is only a few seconds away, the better performing rider is forced to ride as fast as he can to avoid a field sprint. Therefore, he cannot ride in the slipstream of a weaker performing rider.

In cases where the following group of riders is only a few seconds behind the leading group, the best performing rider is significantly more often in the first position of his sprint group than in the slipstream. Conversely, he rides significantly more often in the slipstream of a weaker performing rider, if the peloton is 10 or even more seconds away. Even for a longer time span of up to 18 seconds, this significant difference persists.

*(H3) The rider in the slipstream will win more often than the rider who starts the sprint from the front position.*

A final implication of the theoretical results is that the influence of the position at the beginning of the sprint on the result of the race should be analysed. Therefore, the number of wins by the rider in second position at the beginning of the sprint is compared to the number of wins by the leading rider and, in the case of a group with more than two riders, the riders in the various positions behind the second rider. In a similar manner to the first two results, this test also confirms the theory.

	<i>(H1)</i>	<i>(H2)</i>	<i>(H3)</i>
Exogenous variable	<i>sprint quality</i> (weaker vs. stronger)	<i>distance of peloton</i> (peloton 10 or more seconds away vs. less than 10 seconds away)	<i>position</i> (on second position vs. in front or positions three to seven)
Endogenous variable	<i>result</i> (probability of winning)	<i>position of better rider</i> (probability of being in slipstream)	<i>result</i> (probability of winning)
Regression coefficient B	1.560	-1.391	7.62
Significance	0.000	0.000	0.039
S	0.388	0.377	0.369
Exp (B)	5.002	0.249	2.143

*Table 2: Results of the Logistic Regression Models*

The rider in the second position wins significantly more often than the other riders in his group. More precisely, Table 2 shows that the probability of winning the races more than doubles for the rider in second position, compared to those in other positions ( $Exp(B) = 2.143$ ). Therefore, the logistic regression models confirm all three implications of the theoretical model. A comparable analysis, with a different type of measurement for performance potential, yielded similar results. In this analysis, performance was measured according to the points gained in the world ranking over the last 12 months before the race took place.

## 5. Conclusions

The main result is that, with respect to the slipstream effect alone, the better performing rider will always win. Either he manages to shake off the other rider so that he finishes first, due to his higher maximum velocity, or he follows the other rider in his slipstream in order to overtake him immediately before the finish.

The underlying reason for this result is the fact that the slipstream effect decreases at a low velocity. If sufficiently low, the effects from the difference in performance potential outweigh the slipstream effect to such an extent that the better performing rider can gain a lead in excess of  $d$ . His competitor will be unable to catch up with him again, because his superior performance enables a higher speed all the way to the finish. Therefore, the weaker performing rider is forced to ride in front, if he does not want to lose his competitor at that point in time, well before the finish. However, this only postpones the defeat, because in the final sprint, the other rider not only benefits from his superior performance potential, but also from the slipstream.

If the difference in performance potential is sufficiently great, the better performing rider shakes off the other one at any point. Therefore, he has to reach a sufficiently high velocity in his opponent's slipstream in order to overtake over a distance greater than  $d$ , before the other rider enters his slipstream and reaches the same velocity. In principle, the necessary condition can be calculated by using and transforming (13) to (19). However, the resulting term is so complex, that no conclusions can be drawn from it.

In order to determine the velocity  $v_0$ , which the rider with lower performance potential should try to reach before the final sprint, the following tentative conclusions can be drawn. This velocity should not be too low, since the other rider could then slowly overtake and, according to the earlier considerations, subsequently shake him off. There is, however, no upper limit,

since the other rider can always drop back further, in order to accelerate later to his maximum velocity and re-enter the slipstream. After overtaking, the weaker performing rider can only remain in the slipstream if he has already reached his maximum velocity.

Therefore, the question of why the cyclists do not ride faster before the final sprint, cannot be answered fully within the context of this pure slipstream model. On the other hand, it is possible to explain why it comes to a final sprint and why the front rider regularly loses. The model also answers the question of who takes the front position in the first place and why. Specifically, the weaker performing rider does so, because he does not want to be left behind even earlier.

## **6. Outlook**

An economic principle which emerges from the model is that the riders do not need to cycle faster. This would lead to more effort and thus greater costs (of effort) including real pain, because the final ranking is already certain. Alternative explanations require extending the model, with, for example, the element of surprise that was mentioned in the first section. If the front rider already rides at his maximum velocity, he cannot surprise the other one by increasing his performance and thus shake him off. Another model extension could deal with the building up of energy reserves. However, this presupposes that the riders can recover at different rates, because the better performing rider would otherwise continue to perform better than the other one.

It would also be interesting to loosen the assumed equal characteristics of the two riders. For instance, the riders could differ in their respective rolling friction or drag. It would be particularly interesting to consider a difference in mass  $m$ , accompanied by a parallel difference in performance, due, for example, to greater physical strength. The relevant mass  $m$  includes, in addition to the rider's body, his clothes and bicycle. Consequently, instead of muscles which increase the mass of the rider, one can also assume that a bicycle with better energy transmission is heavier as well.

It is then possible that the heavier rider would accelerate more slowly than the other one, while he can reach a higher maximum velocity at the same time. It would thus be possible for the lighter rider to shake off the heavier one briefly, only to be caught up again and again. This assumption of unequal riders could result in completely new dynamics for the final.



Even with equal masses, it is possible that the better performing rider rides in front, for example, if the field of the remaining riders is following closely. The maximum velocity of the poorer performing rider could be too small to prevent both being caught by the field. If the field of following riders is not far away, the velocity of the two riders in front of them has to be very high, since a greater number of riders can ride faster than a pair of riders. If the two riders in front do not perform fully to their limit, they are “caught”.

Above all, the better performing rider in the front position could be prevented from lowering his velocity sufficiently to shake off the other one completely, or he could force him to overtake. Appropriately, position fights with extremely slow riding or even a total standstill are more common in track cycling with only two competitors, than in big road races.

Another aspect that remains to be analysed is the riders’ subjective knowledge of differences in their respective potential to perform. It has been assumed so far, that the sportsmen were fully aware of all the relevant parameters. If, however, the respective performance potential is confidential information, known only to the individual riders, the value of  $q$  is also unknown to both riders. The results of earlier races are probably common knowledge, but the riders probably do not know the other’s level of exhaustion during the present race. If, for example, the front rider does not know whether he or the other one has the greater performance potential, it is very risky to test this through a reduction in speed. The rider who reduces his speed might be able to move into the slipstream, but it is equally likely that he will be shaken off completely.

In addition, the rear rider could err with respect to the exact moment when he should start the final sprint. This can happen if he does not know all the relevant parameters or does not use them correctly, due to limited rationality. In any event, the rear rider loses if the final sprint starts too late, because he cannot catch up with the other rider in time, before the finish. In addition, he can lose by sprinting too soon, if his opponent remains in his slipstream long enough to overtake him again.

Finally, it would be useful to expand the analysis to more than two riders. The slipstream effect is greater for the third, fourth or fifth driver, while the distance to the front of course is greater. The empirical results indicate that the longer distance could be crucial. Only 3 of 23 races with more than two riders were won by a rider who started the sprint from a position behind the second rider. Furthermore, it would make less sense for an individual rider to start his sprint early, since the others (mutually) provide each other with slipstream. Moreover, the other riders can and do prevent a good competitor from escaping while they ignore the same escaping process from a bad one. Therefore, trying to break away is generally pointless, such

that there are only a few power-consuming attempts and the field rides fairly slowly provided the finishing line is not too close.

This presupposes however, that both the energy consumption and effects of the slipstream on the front rider are modelled as well, since otherwise, a group of slower riders cannot catch up with the fastest rider, irrespective of the size of the group.

An interesting aspect for further empirical analyses could be the question of which rider in the group actually starts the sprint. Theoretically, there are reasons for both the leading rider and the rider in the slipstream to start the sprint. The rider in front could try to use his advantage of being slightly closer to the finishing line, whereas, the rider behind him could use the slipstream and the possibility of surprise and try to overtake his opponent. In order to answer this question, a more exact analysis of the finals must be made, which was not possible from the available videos.

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