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GAUGING RISK WITH HIGHER MOMENTS: HANDRAILS IN MEASURING AND OPTIMISING CONDITIONAL VALUE AT RISK

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Abstract

The aim of the paper is to study empirically the influence of higher moments of the return distribution on conditional value at risk (CVaR). To be more exact, we attempt to reveal the extent to which the risk given by CVaR can be estimated when relying on the mean, standard deviation, skewness and kurtosis. Furthermore, it is intended to study how this relationship can be utilised in portfolio optimisation. First, based on a database of 600 individual equity returns from 22 emerging world markets, factor models incorporating the first four moments of the return distribution have been constructed at different confidence levels for CVaR, and the contribution of the identified factors in explaining CVaR was determined. Following this the influence of higher moments was examined in portfolio context, i.e. asset allocation decisions were simulated by creating emerging market portfolios from the viewpoint of US investors. This can be regarded as a normal decision-making process of a hedge fund focusing on investments into emerging markets. In our analysis we compared and contrasted two approaches with which one can overcome the shortcomings of the variance as a risk measure. First of all, we solved in the presence of conflicting higher moment preferences a multi-objective portfolio optimisation problem for different sets of preferences. In addition, portfolio optimisation was performed in the mean-CVaR framework characterised by using CVaR as a measure of risk. As a part of the analysis, the pair-wise comparison of the different higher moment metrics of the mean-variance and the mean-CVaR efficient portfolios were also made. Throughout the work special attention was given to implied preferences to the different higher moments in optimising CVaR. We also examined the extent to which model risk, namely the risk of wrongly assuming normally-distributed returns can deteriorate our optimal portfolio choice.

JEL Classification: G11, G15, C61

Keywords: Emerging Markets, Higher Moments, Factor Model, CVaR, Portfolio Choice

1 Introduction

Since Markowitz (1952)[1] formulated his famous mean-variance criterion, virtually everyone agrees upon the mean but challenges the variance. The reason is that the conditions which qualify the variance as an appropriate risk measure are not fulfilled in practical applications. Nevertheless, due to its simple and intuitive characteristics, the mean-variance framework marks the quasi-standard for investment professionals nowadays. In this paper we compare and contrast two approaches with which one can overcome the shortcomings of the variance as a risk measure.

The ongoing discussion on risk measures is not merely a matter of academic debate: it is, rather, at the centre of empirical research. There is ample evidence that financial return distributions are asymmetric, leptokurtic, and, hence, non-normal. In such a real case, the variance exhibits two drawbacks. First, it weighs upper and lower deviations from the mean equally, and its application is in contradiction with the notion that investors only regard those returns as risky which are lower than an expected target value. Second, due to model risk (the risk of wrongly assuming normally-distributed returns) a mean-variance investor can greatly underestimate those extreme events which cause the heaviest losses. More precisely, Chamberlain (1983) show that the more general class of elliptic distributions, which includes normal distribution as a special case, is a precondition for using the standard deviation[2] as an exact measure of risk. In addition, Szegö (2002, 2005) concludes that the elliptic distribution of returns is necessary for the applicability of any risk measure

[1] See also Markowitz (1991).
[2] Within the mean-variance framework, the standard deviation can be regarded as an equivalent risk measure with the variance.

which relies only on the linear correlation coefficient as a measure of dependence between the random returns - which is true for the variance.[3]

Looking at recent suggestions on how to deal with non-normally distributed returns, two distinct approaches emerge. One is to improve the accuracy of the mean-variance framework by involving explicitly the third and fourth higher moments into the portfolio selection process. Lai (1991) solves such a resulting multi-objective optimisation problem by polynomial goal programming[4] (PGP) and derives optimal portfolios of domestic stocks in the presence of skewness preference. Chunhachinda et al. (1997) apply this approach for internationally diversified portfolios. Davies et al. (2006) extend it to the first four moments, including the kurtosis of the return distribution also. The higher precision of this approach, however, comes at the cost of higher model complexity. While, in the original mean-variance framework, only one risk aversion parameter is needed, the PGP requires two or three preference parameters[5] and this makes the mapping and interpreting of optimal portfolios both peremptory and highly sophisticated. In fact, questioning an investor about his/her marginal rates of substitution between the different pairs of higher moments is certainly as demanding as directly aiming for his/her utility function.

Alternatively, one can replace the standard deviation by a different, more suitable, risk measure. One prominent candidate is the CVaR, since it has very attractive properties. First of all, it is a downside measure of risk and, hence,

[3] See Joe (1997) and also Embrechts-McNeil-Straumann (2002).
[4] Tayi and Leonard (1988) introduced polynomial goal programming to solve multi-objective optimisation problems. They applied it to optimal bank balance-sheet management.
[5] Portfolio selection via PGP incorporates a two-step procedure. In the first step, the conflicting and competing objectives are optimised independently in order to obtain a set of non-dominated solutions. In the second step, a polynomial is minimized containing the deviations of the different objectives from their optimum level. For a four moment optimisation, one requires, in total, three different preference parameters to weigh the investor's preference for mean-variance, skewness-variance and kurtosis-variance efficiency.

consistent with the intuitive notion of risk, since it takes into account only the unfavourable part of the return/loss distribution. Secondly, it is a coherent risk measure in the sense of the Artzner-Delbaen-Eber-Heath (1999) axioms. Thirdly, it also accounts for losses beyond Value at Risk (VaR), which is especially important in case of fat-tail distributions. Finally, it has two favourable technical properties: it is continuous with respect to the confidence level and convex with respect to the control variables, the latter being very relevant in portfolio optimisation. In order to optimise within the mean-CVaR framework, as it was shown by Rockafellar and Uryasev (2000), one has to solve a simple linear programming problem. This makes CVaR very appealing in asset allocation.

Although both approaches have the same objective - that is, to extend the mean-variance framework for the case of non-normally distributed returns, nobody has yet (to our knowledge) explicitly created a link between them to analyse how they differ or coincide. Indeed, one can raise the question, in applying CVaR as a risk measure, of the extent to which the information given by higher moments of the return/loss distribution can be utilised. In order to answer this question, we construct factor models based on cross-section return data of 600 individual equities from 22 emerging world markets and determine the explanatory power of the first four moments on CVaR at different confidence levels.

In fact, our main contribution to the literature is that we show that, when CVaR is minimized, we can count on implied preferences in favour of higher skewness, higher mean, lower kurtosis and lower standard deviation. This property of CVaR makes it possible to apply the linear programming proposed

by Rockafellar and Uryasev for CVaR optimisation as a simple and effective alternative to PGP in portfolio allocation.

We have attempted to organise the remainder of the paper in a logical way. Section 2 gives an insight into the methodology of conditional value at risk and describes the factor model. Section 3 introduces the mean-CVaR as well as the higher moment optimisation framework. The results of the empirical analysis are discussed in Section 4. Here, first of all, the specification of data is given, and this is followed by the results provided by the factor analysis. Finally, the findings of portfolio optimisation are presented and analysed, whilst Section 5 offers some concluding remarks.

2 VaR and CVaR as Risk Measure

2.1 The Definition of VaR and CVaR

The investor's perception of risk is naturally associated with the probability of future returns falling below a threshold, which is investor-specific and related to the investment objective. Such a threshold can, for instance, be a minimum required rate of return, the expected return, a stochastic interest rate, or simply the zero level, which distinguishes positive from negative returns. Both the Value at Risk (VaR) and the Conditional Value at Risk (CVaR) support this notion of risk.

The portfolio risk, however, is not only affected by the future realisation of asset returns but also by the portfolio allocation decision today. [6] Thus, for a

[6] We omit subscripts of time because our framework is a single-period model.

certain vector of portfolio weights $\underline{x}$ and the random return R with probability density $p(R)$, the cumulative probability distribution of losses $L = f(\underline{x}, R)$ is[7]:

$$\Psi(\underline{x},\zeta) = P\left(L = f(\underline{x},R) \le \zeta\right). \tag{1}$$

Then, based on our portfolio allocation decision $\underline{x}$, the VaR on a confidence level $\alpha \in (0,1)$ is defined as[8]:

$$VaR_{\alpha}(\underline{x}) = \min\left\{\zeta \mid \Psi\left(\underline{x},\zeta\right) \ge \alpha\right\}. \tag{2}$$

The CVaR on a given confidence level α is defined as the expected loss given that the loss L is higher than or equal to the Value at Risk (VaR) on the same confidence level:

$$CVaR_{\alpha} = E\left\{L \middle| L \ge VaR_{\alpha}\right\} \tag{3}$$

For continuous loss distributions the VaR and CVaR on a given confidence level are unique, and their determination is straightforward. In practical applications, when we often have to rely on discrete distributions coming from the series of past returns or finite sampling methods, VaR and CVaR are not necessarily unique. In these cases, we must differentiate between the upper $CVaR^{+}$ and lower $CVaR^{-}$.[9] The $CVaR^{+}$ measures the expected value of losses *strictly* exceeding the VaR, whereas the $CVaR^{-}$ determines the expected value of losses higher than or equal to the VaR as given in formula (3).

[7] One can interpret it as the probability that the loss will not exceed a given threshold ζ. In line with the recent literature of downside risk measures, we define both VaR and CVaR based on the loss, and not directly the return. Please, note that the loss can be obtained by mirroring the return along the y-axis, hence $L = -R$.
[8] See Jorion (1997) and Frey-McNeil (2002) for details.
[9] See Rockafellar and Uryasev (2002).

Rockafellar and Uryasev (2002) proved that, in discontinuous cases, CVaR can be expressed as the weighted average of VaR and $CVaR^{+}$. Using the α-tail distribution

$$\lambda = \frac{\Psi(VaR_\alpha) - \alpha}{1-\alpha} \text{ with } 0 < \lambda < 1, \tag{4}$$

the following equation holds:

$$CVaR_\alpha = \lambda VaR_\alpha + (1-\lambda)CVaR_\alpha^{+}, \tag{5}$$

where Ψ is the cumulative probability distribution of L, so that $\Psi(VaR_\alpha) = P(L \leq VaR_\alpha)$.

2.2 The Influence of Higher Moments on CVaR

The methodology applied for testing the influence of higher moments on CVaR is regression analysis. In the first regression model (the original model) CVaR served as a resultant variable, and the expected return (E), the standard deviation of returns (σ), the skewness (s) and kurtosis (k) were used as explanatory variables.

The expected return, which can be estimated as the (arithmetical) average of returns in a given time period, is a typical measure of "location"[10]. In this case the location of observed values plays a crucial role in the magnitude of the measure. Based on the fact that an increase in the expected return means a decrease in expected loss (given that other conditions are unchanged), it is logical to expect that risk measured by CVaR will decrease in this case.

The standard deviation of returns is the square root of the variance. As it is well-known, the variance can be determined as the squared average of

[10] This expression is used by Pflug (1999, p.1) as he differentiates measures of "dispersion" (such as the variance) and measures of "location" (such as the expected value).

deviations of returns from the mean. As a "volatility" measure, the variance or the standard deviation of the returns respectively has been the traditional measure of risk[11]. As such, it belongs to the category of "location independent" measures, since its value is determined by the relative distance of each return observation from the mean - and not by their absolute location.

Skewness is defined as the normalised third central-moment of a distribution and indicates the degree of asymmetry in the shape of the distribution function (in our case in the return distribution). If the skewness is positive, the distribution function has a longer tail extending to the right (to the direction of large (positive) values) than to the other direction.[12] In the case of negative skewness, precisely the opposite holds, i.e. the distribution function has a longer tail the left, namely to the direction of small (negative) values. Negative skewness suggests the occurrence of extreme negative returns (usually, however, with low probability). Considering that negative return can be interpreted as loss, an increase in the value of skewness – maintaining other conditions unchanged – might cause a decrease in the value of risk measured by CVaR.

Kurtosis is defined as the normalised fourth central-moment of a distribution and intuitively refers to the fact of how the different "scores" are distributed at the different parts of the distribution – namely, in the centre, at the tails, and between the centre and the tails (in the "shoulders"). If we take the bell-shaped normal distribution function as a starting point and replace scores from the area between the centre and the tails (to the centre as well as to the tails) the result is a so-called leptokurtic distribution - which is thinner in the centre and thicker

[11] Cf. Eftekhari-Pedersen-Satchell (2000).
[12] In this case the mean is higher than the median.

at the tails than a normal distribution. Based on these considerations, with an increase in the kurtosis – other conditions remaining unchanged – we can expect intuitively an increase in the CVaR.

The first step was to calculate the expected return, the standard deviation of returns, the skewness, the kurtosis and the CVaR values for each equity. For these calculations we utilized the 600 time series, with 513 weekly return data in each time series. Then we ran a linear regression on the cross-section data. The regression model applied can be written as follows:

$$CVaR_{\alpha} = c_0 + c_E \cdot E + c_{\sigma} \cdot \sigma + c_s \cdot s + c_k \cdot k + \varepsilon \qquad (6)$$

where c_E, c_σ, c_s and c_k are the regression parameters expressing the influence of the particular explanatory variables, the expected return (E), the standard deviation of returns (σ), the skewness (s) and the kurtosis (k) on CVaR, respectively. c_0 is the regression constant and ε denotes the error term. In addition, α serves as a notation for the confidence level chosen in calculating CVaR.

As will emerge from the results presented in Section 4, there is a significant degree of multi-collinearity in the model above (see formula 6). It is a known fact that this restricts the analytical interpretation of the results, namely the regression coefficients given by the model. In particular, the main problem with multi-collinearity is that the effects of the different explanatory variables in explaining the resultant variable cannot be separated.

In order to eliminate multi-collinearity, factor analysis was applied. However, in carrying out the factor analysis, we decided not to reduce the number of variables. Instead of taking this route, our intention was to express the

influence of the original explanatory variables on CVaR in terms of "independent dimensions". As will be seen later, the positive effect of this proved to be the maintenance of the high explanatory power of the original in the new model.

This new model was built on the factors which we derived from the factor analysis. These were used as new explanatory variables and CVaR was kept as a resultant variable. The linear regression model containing the variables mentioned above takes the following form:

$$CVaR_{\alpha} = c_0^* + c_{F_1} \cdot F_1 + c_{F_2} \cdot F_2 + c_{F_3} \cdot F_3 + c_{F4} \cdot F_4 + \varepsilon^* \qquad (7)$$

In both models, i.e. in models 6 and 7, we used a cross-sectional sample of 600, since the sample size was equal to the number of equities considered.

3 Portfolio Selection with Higher Moments

3.1 Optimisation by Conditional Value at Risk

CVaR was introduced into portfolio optimisation quite recently by Rockafellar and Uryasev (2000, 2002) as an alternative to VaR. Let $\underline{R}_1,\underline{R}_2,...,\underline{R}_T$[13] be a sample set of return vectors. For a particular realisation of asset returns, i.e. for a specific return vector k the loss on a portfolio can be determined as:

$$L_{p_k} = -\underline{x}^T \underline{R}_k = -\sum_{i=1}^{n} x_i R_{k,i}, \qquad (8)$$

where $\underline{x}^T$ is the transpose of the vector of portfolio weights $\underline{x}$.

[13] The number of elements in the sample set equals the number of the return observations in the time series of returns, while the dimension of the vectors is equal to the number of assets in the portfolio.

In order to identify the portfolio with the minimum CVaR at a minimum mean rate of return R^*, as it is shown by Rockafellar and Uryasev (2000), the following linear programming problem has to be solved:

$$\min_{\underline{x},\zeta} CVaR(\underline{x},\zeta) = \zeta + \frac{1}{T(1-\alpha)}\sum_{k=1}^{T} u_k$$

subject to

$$\begin{aligned} &-\frac{1}{T}\sum_{k=1}^{T}(\underline{x}^T \underline{R}_k) \geq R^*, \\ &\underline{x}^T \underline{R}_k + \zeta + u_k \geq 0, \\ &u_k \geq 0, k = 1,2,...,T \\ &\sum_{i=1}^{n} x_i = 1, x_i \geq 0, i = 1, 2,..., n. \end{aligned} \tag{9}$$

By solving (9) we find the optimal portfolio weights $\underline{x}^*$ as well as the corresponding VaR ζ^*[14].

3.2 Higher Moment Portfolio Optimisation

The multi-objective portfolio problem considered here is consistent with Lai (1991), Chunhachinda et al. (1997) and Davies et al. (2006). We argue that the investor has a preference for higher mean and skewness, whilst disliking large variance and kurtosis values. We have, therefore, a multi-objective optimisation problem with four competing objectives but we can simplify this problem by restricting the variance to unity and isolating its effect on the remaining objectives. Our portfolio selection model can, therefore, be

[14] Rockafellar and Uryasev (2000, 2002) pointed out that minimising CVaR requires also identifying the corresponding VaR ζ^*. Usually, but not necessarily, ζ^* is also the global minimum VaR.

formulated in the following way. In the first step, the following multiple objectives have to be optimised independently:

$$\begin{aligned}
&\max Z_1 = E(\underline{x}^T \underline{R}) + x_{n+1} r_f \\
&\max Z_3 = E\left[\underline{x}^T (\underline{R} - E(\underline{R}))\right]^3 \\
&\min Z_4 = E\left[\underline{x}^T (\underline{R} - E(\underline{R}))\right]^4 \\
&\text{subject to}
\end{aligned} \tag{10}$$

$$\underline{x}^T \underline{V} \underline{x} = 1, \sum_{i=1}^{n} x_i = 1 - x_{n+1}$$

$$x_i \geq 0, i = 1, 2, ..., n,$$

where x_{n+1} indicates the proportion of money invested at the risk free rate r_f.

In this step we separately optimise mean return (Z_1), skewness (Z_3) and kurtosis (Z_4). We search for the portfolio with the highest expected return (Z_1^*) in the mean-variance space, the portfolio with the highest skewness (Z_3^*) in the skewness-variance space and the portfolio with the lowest kurtosis (Z_4^*) in the kurtosis-variance space. In all cases we restrict our choice to unit variance portfolios. It should be noted that, usually there exists no single portfolio which is optimal with respect to all the three criteria, and so, as a result, we take the set of non-dominated portfolios for which a more favourable portfolio cannot be found - in the sense that it cannot have a higher mean return at the same level of skewness and kurtosis, a higher skewness at the same level of mean return and kurtosis or a lower kurtosis at the same level of mean return and skewness.

In the second step, given the investor's preferences [α β γ] among the different objectives, the following polynomial has to be minimised:

$$Min\ Z = (1+d_1)^{\alpha} + (1+d_3)^{\beta} + (1+d_4)^{\gamma}$$

subject to

$$\begin{aligned}
&E(\underline{x}^T \underline{R}) + x_{n+1} r_f + d_1 = Z_1^* \\
&E\left[\underline{x}^T (\underline{R} - E(\underline{R}))\right]^3 + d_3 = Z_3^* \\
&-E\left[\underline{x}^T (\underline{R} - E(\underline{R}))\right]^4 + d_4 = -Z_4^* \\
&d_1, d_3, d_4 \geq 0, \underline{x}^T \underline{V} \underline{x} = 1 \\
&\sum_{i=1}^{n} x_i = 1 - x_{n+1}, x_i \geq 0, \quad i = 1, 2, ..., n,
\end{aligned} \tag{11}$$

where d_1, d_3 and d_4 denote the deviations from the optimal policies, Z_1^*, Z_3^* and Z_4^* derived in the first step, respectively. Hence, the objective function in the second step can be interpreted as minimisation of the deviations from each single optimal strategy. Thereby, each deviation is weighted accordingly to its preference parameter α, β and γ, respectively.

4 Empirical Study

4.1 Data

The data for the regression analysis were taken from Standard and Poor's Emerging Market Database (S&P's EMDB). In total, we utilised 600 series of US dollar-based, individual equity returns from, again in total, 22 of the world's emerging markets. All variables in the cross-section regression models were calculated based on the time series of weekly returns on these equities. The time period stretched from the 28th of February 1997 until the 31st of December 2006 and so comprised almost 10 years. The emerging markets

involved in the study (with the number of equities taken from the particular stock market in brackets) were:

Argentina (9), Brazil (35), Chile (27), China (97), Czech Republic (4), Egypt (11), Hungary (7), India (62), Indonesia (16), Israel (23), Korea (61), Malaysia (48), Mexico (23), Morocco (10), Peru (11), Philippine Islands (22), Poland (5), Russia (6), South Africa (27), Taiwan (44), Thailand (30) and Turkey (22).

For portfolio optimisation we relied on DataStream as a source of data, including equity index (total) returns from 21 countries[15]. Here a time series of weekly returns for more than 12 years, covering the period from the 3rd of February 1997 until the 29th of September 2008, was utilised. Asset allocation decisions were simulated by creating emerging market portfolios from the viewpoint of US investors. This can be regarded as a normal decision-making process of a hedge fund focusing on investments in emerging markets.

Table 1 summarises the descriptive statistics for all countries considered. The Jarque-Bera statistics clearly indicate the non-normality of all the market returns.

<< Table 1 about here>>

4.2 Regression Analysis

In Table 2 the results given by model (6) are summarised and presented at 95 as well as 99 percent confidence level for CVaR. It can be observed that, despite the fact that, at the 99 percent level, the explanatory power is somewhat lower (but still around 90%), at both confidence levels it is high. In addition, all the regression coefficients are significantly different from zero at the 5

[15] Among the countries mentioned above, Israel was omitted since, in this case, the data were not available for the time period prior to 1999.

percent significance level. Moreover, with the exception of the regression constant at the 99 percent confidence level for CVaR, they are also significant at the 1 percent level. It is also worth mentioning that the signs of the regression coefficients are in agreement with the intuitive expectations outlined in Section 2. In particular, the positive sign of the regression coefficient for the standard deviation and the kurtosis indicate that an increase in the value in the respective variable − given that other conditions are unchanged − results in an increase in the value of CVaR. At the same time, the negative sign of the coefficient for the expected return as well as for the skewness refer to the fact that an increase in the value of the above-mentioned variables goes together with a decrease in the value of CVaR. This interpretation, however, has only a limited value due to the presence of multi-collinearity.

<< Table 2 about here>>

There is a high degree of multi-collinearity in model (6)[16]. The presence of multi-collinearity is already suggested by the pair-wise correlation terms between the different explanatory variables. The correlation matrix is presented in Table 3. The high correlation between skewness and kurtosis is most conspicuous with a value above 0.8. At the same time, the correlation terms between mean and standard deviation, standard deviation and skewness as well as that one between standard deviation and kurtosis are also not negligible (values approximately 0.45, 0.46 and 0.33, respectively).

<< Table 3 about here>>

[16] The multi-collinearity was tested by χ^2 test. The value of the test statistics has proved to be $\chi^2 = 436.326$ while the critical value at 5 percent significance level is $\chi^2_{0,05;6} = 12.592$ (the degree of freedom is 6).

The most important results of the factor analysis carried out in order to eliminate multi-collinearity, are presented in Table 4 and Table 5. For extracting the relevant factors Principal Component Analysis (PCA) was applied, and, as a method for rotation, varimax with Kaiser-normalisation was used.

<< Table 4 about here>>

As seen in Table 4, in the four-dimensional space determined by the explanatory variables of model (6), 42% of the total variance is due to the first, 26% to the second, 25% to the third and 7% to the fourth factor, respectively. Despite the fact that only a relatively low proportion of the total variance can be explained by the fourth factor, we decided to retain it with the intention of building a new regression model. As emphasised earlier, this was motivated by the intention of retaining the high explanatory power of the original model.

The factors can be identified based on the rotated component matrix (see Table 5). The correlation terms in the matrix suggests that the first factor embodies the combined effects of skewness and kurtosis. The standard deviation is predominantly represented by the second, while the mean is by the third factor, respectively. The fourth factor shows a noteworthy correlation only with the skewness (0.516), and so it seems obvious to identify it as a factor expressing the skewness effect.

The results given by the new regression model, which was built on the factors provided by the factor analysis, are summarised in Table 6 (see model (7)).[17] In fact, similarly to model (6), two versions are presented, one at 95 percent, and a

[17] We saved the factor loadings as new variables for further analysis. From this we obtained 600 values for each of the 4 factors.

second at 99 percent confidence level for CVaR, respectively. The table shows not only the regression coefficients but also the components of the explanatory power which are attributable to the different factors. The decomposition of the explanatory power in the new model is only possible because of the (linear) independence of factors. The mathematical consequence of this, on one hand, is that the value of the regression coefficient belonging to a specific factor is independent from those of the other factors, whilst, on the other hand, the explanatory power component of each factor does not change by the inclusion or exclusion of different factors into and from the model.

<< Table 5 about here>>

It can be seen in Table 6 that all the regression coefficients of both versions of model (7) are significant at 1% level. In addition, coinciding with our declared aim, it can also be seen that the explanatory power is the same as that of the versions of model (6).[18]

<< Table 6 about here>>

Based on the results presented in Table 6, we can conclude that it is predominantly the second factor, which represents the effect of the standard deviation, which is responsible for the volatility in the value of CVaR (in particular at 95 percent confidence level for CVaR the explanatory power component attributable to it is 91.3%, while at 99 percent level the respective value is 77%). It is not so striking, given that the standard deviation is also a risk measure. It is therefore, understandable that the factor dominated by the standard deviation correlates highly with the risk measured by CVaR. At 95

[18] For the sake of comparison see Table 2.

percent confidence level for CVaR in the magnitude of the explanatory power component, the second factor is followed by the third - which is, however, dominated by the mean, with a much lower contribution than that of the second factor, at 3.8%. At 99 percent confidence level, the fourth factor, in which the skewness effect is "condensed", has the second highest contribution (6.1%). The first factor, which embodies the joint effect of the skewness and kurtosis, together with the skewness dominated fourth factor have contributed to the explanatory power with only about 2% at 95 percent confidence level for CVaR, but with almost 10% at 99 percent confidence level!

Based on the results shown above, we can conclude that, with an increase in the confidence level for calculating CVaR, the explanatory power component of those factors related to the non-normality characteristics of the return distribution, i.e. those ones dominated by the skewness and kurtosis, increases.

4.3 Portfolio Analysis

In our analysis we compare different approaches which take into account higher moments with the standard mean-variance framework. Essentially, we evaluate three different types of portfolio strategy: the classical mean-variance approach, mean-CVaR strategies and higher moment optimisation. In this way, the minimum-variance (MVP) and the tangency portfolio (TP) constitute our base case and benchmark scenario[19]. They are compared to their CVaR counterparts located in the mean-CVaR space, which we call the minimum-CVaR (MCVaR) and the mean-equivalent tangency portfolio (TP-CVaR). For the sake of comparability, the TP-CVaR is constructed so that it provides the same mean return as the tangency portfolio but at the lowest possible CVaR.

[19] For a detailed description of these strategies see e.g. Eun-Resnick (1994).

Both the MCVaR and the TP-CVaR are evaluated at two different confidence levels - 95 and 99 percent. Finally, we account for higher moments explicitly by the multi-objective optimisation and pay special attention to different combinations of preference parameters for the mean (α), skewness (β), and kurtosis (γ).

We analyse the different portfolio strategies in an ex post setting, using all data in our sample for parameter estimation. This procedure counters any effects which may arise from estimation risk and so allows us to focus on important differences or similarities among the portfolio strategies.

4.3.1 Mean-Variance versus Higher-Moment Optimisation

Table 7 presents the higher moments and downside risk metrics of all the optimal portfolio strategies considered. Table 8 shows the corresponding portfolio weights. As we know from the Jarque-Bera test statistics presented in Table 1, the equity index returns from none of the countries are normally distributed. If they were, minimising either the variance, the VaR or the CVaR would make no difference and so the mean-CVaR strategies and the mean-variance strategies would coincide. Consequently, we observe (in the first panel of Table 7) a large variation in the resulting portfolio return distributions.

<<Table 7 about here>>

Looking at the moments of the different portfolio return distributions, it is clear that, by definition, the MVP must provide the lowest portfolio variance among all strategies. Thus, the MCVaR strategies cannot decrease the variance any further in order to reduce the CVaR; instead they must trade mean-variance efficiency for an improvement in CVaR. This effect is the more pronounced,

the more we raise the confidence level. Indeed, the standard deviation then rises from its minimum of 1.93% to 1.97% at 95 percent confidence level and to 2.20% at 99 percent confidence level, respectively. At the same time, the mean return level varies only slightly (by 1 basis point) around the 0.29% level of the MVP. The same applies to the CVaR-counterparts of the tangency portfolio. In these cases there is an increase in the standard deviation from the mean-variance optimum of 2.11% to 2.13% at 95 percent, and to 2.37% at 99 percent confidence level. Interestingly, the improvement in CVaR is achieved mainly by pushing the portfolio distribution to the right. Skewness increases from -0.49 to 0.08 in the minimum risk case and from -0.52 to -0.27 in the tangency portfolio case. Hence, the CVaR strategies support the investors' inherent preference for right skewed return distributions. The effect on the kurtosis, however, is mixed and no clear pattern can be observed, except for a sharp decline in the case of the TP-$CVaR_{99\%}$, where the kurtosis is halved from 3.08 to 1.50.

The results above show that the mean-CVaR strategies indeed change the higher moments of the portfolio distribution and the direction of the change is mainly consistent with the investors' preference. Nevertheless, these strategies manipulate the portfolio distribution rather indirectly, since their main objective is to minimise the tail risk at a given return level.

In contrast, the higher moment portfolio optimisation explicitly takes skewness and kurtosis preference into account. The results of these strategies are given in the second panel of Table 7 for a varying set of preference parameters. First, note that the results of the mean-variance tangency portfolio (TP) and those of the [1 0 0] are almost identical. Considering that in the latter case there is

neither preference for a higher skewness nor for a lower kurtosis, the higher moment optimisation framework is expected to provide the portfolio with the highest excess return over the risk-free rate to a unit of variance. This is exactly the tangency portfolio in the mean-variance context. Small differences between TP and [1 0 0] are only due to numerical issues in the portfolio optimisation routine. The other two particular higher moment portfolios are those with the unit of pure skewness/variance preference [0 1 0], and the unit of pure kurtosis/variance preference [0 0 1]. The [0 1 0] portfolio comprises nearly 100% equities from Malaysia and features by far the highest skewness (3.63), but, in return for this, provides the poorest mean (0.10) and the highest kurtosis (42.52). In contrast, the first three portfolio moments of the [0 0 1] strategy deviate less markedly from the mean-variance case but the decrease from 2.95 to 0.06 in the kurtosis is remarkable.

In addition to the three optimal portfolio with respect to a single moment [1 0 0], [0 1 0], and [0 0 1], we consider three reasonable combination of the preference parameters: the case of equal (unit) preference for all moments [1 1 1], higher preference for the mean than for the skewness and kurtosis [2 1 1], and the higher preference for the mean than the skewness and no preference for the kurtosis [2 1 0]. The resulting portfolio moments show that the PGP indeed manipulates the portfolio return distribution in the desired directions. Nevertheless, it is still difficult to conclude how “risky” a certain portfolio strategy is by looking only at the portfolio moments. This is the main drawback of the PGP, since neither calibrating the preference parameters nor interpreting the portfolio moments is intuitive. It is more natural for an investor to think in terms of risk-budgets, that is to say, “how much loss is acceptable”. The downside risk measures support this view of risk. Thus, if we look at the VaR

and CVaR measures, we can see that the PGP strategies are more risky than a simple mean-variance portfolio, although we account for the higher moments explicitly. Hence, the interaction of all moments on the tail risk is hard to evaluate in advance. For instance, while the [1 1 1] and [2 1 1] strategies provide reasonable results, the [2 1 0] portfolio is hardly a sound portfolio strategy. It does offer more favourable skewness values but costs a huge decline in the mean-variance efficiency and increases the kurtosis considerably. Compared to the mean-variance tangency portfolio (TP), its mean return decreases by one third from 0.36% to 0.22%, its variance triples from 2.11% to 6.23%, and its kurtosis is, at 19.25, more than six times larger. The CVaR is also about three times larger than that of the TP. Consequently, accounting for higher moments explicitly, demands a very complex calibration of the preference parameters. In contrast, it is much easier to use the CVaR as a risk measure, which also results in more favourable, but even more balanced, portfolio moments. Furthermore, the optimal portfolio weights in Table 8 show another important difference between the mean-CVaR and the PGP portfolios: the latter are less diversified and invested in only 2 to 4 countries, whilst the mean-CVaR portfolios are invested in 6 to 9.

4.3.2 The Effect of Model Risk

To emphasise the effect of model risk, we calculate the downside risk measures in two different ways. On the one hand, the VaR and CVaR are derived from the empirical portfolio return distribution and, on the other hand, we simply plug the estimated mean and standard deviation into the standard formula for normally distributed returns:

$$VaR_{\alpha}(\underline{x}) = E(L) + N_{\alpha}\sigma(L). \tag{12}$$

$$CVaR_{\alpha}(\underline{x}) = E(L) + \frac{\varphi(N_{\alpha})}{1-\alpha}\sigma(L), \tag{13}$$

where φ(.) denotes the standard normal density function and N_{α} is the α-quantile of the standard normal distribution. In formulae (12) and (13) *E(L)* and $\sigma(L)$ stand for the expected (mean) loss and the standard deviation of loss, respectively.

The later approach indicates the extent to which model risk can deteriorate our optimal choice. For instance, looking at the CVaR, one can see that the risk metrics which were derived under the assumption of normally distributed returns underestimate the historical values in all cases except for the [1 1 1] strategy. Therefore, estimating the CVaR with only the first two moments is not sufficient, since one must also account for the higher moments. The picture changes if we compare the parametrical and empirical VaR. At 99 percent confidence level, the results still tend to support the fact that parametrically derived values underestimate their empirical counterpart. Except for the [0 0 1] strategy, all empirically derived downside risk measures are larger than, or equal to, their parametrical counterparts. In contrast, at the 95 percent confidence level, the results from the mean-variance and mean-CVaR strategies in the first panel show the opposite pattern while the higher moment strategies in the second panel show mixed results. Furthermore, comparing the same downside risk measures at different confidence levels or the VaR with respect to the CVaR at the same confidence level, shows that the downside risk measures increase more sharply in the empirical case than in the parametrical

case. Consequently, the empirical tail distribution is much more risky than the normal distribution indicates.

<<Table 8 about here>>

4.3.3 Mean-Variance versus Mean-CVaR Efficient Portfolios in Respect of Higher Moments

Finally, we present in Figure 1 all mean-variance efficient portfolios and all mean-CVaR efficient frontiers in varying moment spaces. Of course, all mean-variance efficient portfolios in the mean-variance space represent the well known efficient frontier. We can see from the upper left graph that all mean-CVaR efficient portfolios are dominated by the mean variance optimal portfolios and that the mean variance sub-optimality of the mean-CVaR portfolios increases with the confidence level. This is true by definition and would change in favour of the mean-CVaR portfolios if we would plot all portfolios in a mean-CVaR space. This graph, however, does not show the properties of these portfolios in respect of the higher moments. Therefore, we plot all portfolios in the mean-skewness space (upper right graph), the skewness-variance space (lower left graph) and the mean-kurtosis space (lower right graph).[20] The mean-skewness space graph supports our assumption, that the mean-CVaR portfolio trades mean-variance efficiency for more a favourable skewness value and that this effect becomes more pronounced at higher confidence levels. Compared to the mean variance optimal portfolios, the mean-CVaR portfolios shift to the right and provide higher skewness values at the same mean level. Note that the sharp change of direction at the

[20] We omit the kurtosis variance and the kurtosis skewness spaces, because they show no clear pattern.

end of the mean-$CVaR_{99\%}$ portfolios is not due to a sharp decline in volatility (which can be concluded from the upper left graph) but, potentially, to the associated sharp decline of the kurtosis - as the lower right graph indicates. The upper right graph exhibits two interesting features. First, the skewness decreases from the minimum risk portfolios as we move to moderate risk levels but then increases steadily. The latter fact is intuitive, since the degree of diversification decreases with higher portfolio variance, but, with lower diversification, the skewness increases. The reason lies in the sub-additivity property of the skewness and hence diversification decreases the skewness.

<<Figure 1 about here>>

Consequently, we can conclude that mean-CVaR portfolios manipulate not only the tail distribution but the whole portfolio return distribution. Therefore, accounting for higher moments can implicitly and intuitively be achieved by replacing the variance with CVaR.

5 Conclusion

In the paper we attempted to reveal some characteristics of a prosperous risk measure, the conditional value at risk (CVaR), which can be utilised in portfolio optimisation. In particular, the main aim was to study the extent to which the CVaR is determined by the moments of the return distribution and what consequences this relationship has in portfolio allocation.

Firstly, the relationship between the conditional value at risk (CVaR) and the first two central moments of return distribution (namely the mean and the standard deviation) as well as the skewness and kurtosis which can be generated from the third and the fourth moment, was studied empirically. We

relied on a cross-section database including 600 equities from 22 emerging markets of the world. The method applied was linear regression combined with factor analysis. Eventually, a factor model was constructed in order to eliminate multi-collinearity from the original model.

Portfolio optimisation was then performed. On an ex post basis, different approaches which take into account higher moments were compared with the standard mean-variance framework. We considered the minimum variance portfolio (MVP) and the tangency portfolio (TP) as well as their counterparts in the mean-CVaR framework (MCVaR, TP-CVaR), each at different confidence levels (95%, 99%). In addition, we solved in the presence of conflicting higher moment preferences the multi-objective portfolio optimisation problem for different sets of preferences. As a part of the ex post analysis, the pair-wise comparison of the different higher moment metrics of the mean-variance and the mean-CVaR efficient portfolios were also made.

For portfolio optimisation the equity (price) index returns of 21 emerging stock markets were used. Asset allocation decisions were simulated by creating emerging market portfolios from the viewpoint of US investors. This can be regarded as a normal decision-making process of a hedge fund focusing on investments into emerging markets.

We also examined the extent to which model risk can deteriorate our optimal portfolio choice. In doing so, the VaR and CVaR values from the underlying empirical dataset were compared and contrasted with those assuming a normal distribution. The conclusions of the study can be summarised as follows:

Firstly, the explanatory power of the factor model built on the factors given by principal component analysis as explanatory variables and CVaR as a resultant variable proved to be very high for both confidence levels for CVaR. Furthermore, all the regression coefficients were significant at the 1 percent level. However, the explanatory power of this factor model decreased with an increase in the confidence level for calculating CVaR.

Secondly, for the volatility in the value of CVaR, the factor conveying the effect of standard deviation has predominantly proved to be responsible. At the same time, it is remarkable that the strength of the influence of this factor decreased as the confidence level for CVaR increased. In addition, parallel to the decrease in the effect of the factor dominated by the standard deviation, the effect of the factors dominated by the skewness and kurtosis, i.e. those factors representing the non-normality characteristics of the distribution, increased as a result of an increase in the confidence level for CVaR.

Thirdly, considering the effect of model risk, it has been deduced that the empirical tail distribution is much more risky than the normal distribution indicates.

Finally, it has been shown that minimising CVaR can be regarded as a substitute for higher moment portfolio optimisation. This can be explained by the implied preference for a higher skewness (and mean) and a lower kurtosis (and standard deviation). Indeed, it became obvious from our empirical analysis that the portfolios in the mean-CVaR framework clearly trade mean-variance efficiency for more skewness and less kurtosis. In other words, optimising CVaR seems to support the investors' preference for higher skewness and lower kurtosis.

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Appendix

Table 1

Descriptive Statistics of Weekly Emerging Market Returns

The table presents the basic statistics of weekly returns of 21 emerging market indices ranging from the 3rd of February 1997 until the 29th of September 2008. All returns are calculated from the S&P Emerging Market Database (S&P's EMDB) Investible Indices. We report the mean, the standard deviation (Std.), the skewness (Skew.), the kurtosis (Kurt.) and the Jarque-Bera test from in total 608 simple returns.

	Weekly returns in US-Dollar					Weekly returns in local currencies				
	Mean (%)	Std. (%)	Skew.	Kurt.*	Jarque-Bera**	Mean (%)	Std. (%)	Skew.	Kurt.*	Jarque-Bera**
Argentina	0.21	5.26	-0.36	1.97	108.74	0.39	5.17	-0.05	1.68	69.90
Brazil	0.43	5.54	-0.44	0.53	26.08	0.48	4.47	-0.26	2.34	142.30
Chile	0.21	3.20	-0.31	2.08	116.05	0.24	2.73	-0.27	2.15	122.05
China	0.25	5.36	-0.08	4.57	518.07	0.22	5.35	-0.08	4.60	525.69
Czech	0.42	3.96	-0.36	0.98	36.03	0.32	3.54	-0.40	1.56	76.02
Egypt	0.33	4.18	0.25	0.94	27.63	0.41	4.21	0.52	2.33	160.64
Hungary	0.34	4.51	-0.20	2.00	102.74	0.32	4.10	-0.08	2.86	202.73
India	0.31	4.33	-0.48	2.21	144.66	0.35	4.09	-0.45	2.39	161.44
Indonesia	0.34	8.65	1.34	17.08	7443.63	0.37	5.97	0.64	6.14	977.57
Korea	0.34	6.10	0.35	2.45	160.34	0.35	5.23	0.23	1.90	94.10
Malaysia	0.10	5.69	3.64	42.88	47139.12	0.11	4.69	2.97	32.14	26614.78
Mexico	0.35	4.36	-0.24	2.26	131.60	0.38	3.77	-0.26	2.09	114.87
Morocco	0.34	2.62	-0.04	3.43	292.49	0.31	2.39	0.04	4.77	563.81
Peru	0.36	3.55	-0.34	1.98	108.50	0.38	3.44	-0.32	1.73	84.40
Philippines	-0.02	4.73	0.51	5.73	842.17	0.04	3.99	0.28	3.62	332.16
Poland	0.24	4.62	-0.13	1.38	48.61	0.17	3.99	-0.10	1.99	99.11
Russia	0.53	7.37	-0.06	4.90	597.32	0.78	7.61	1.26	11.68	3556.34
South Africa	0.27	4.06	-0.41	2.02	118.05	0.33	3.15	-0.43	3.02	244.88
Taiwan	0.07	4.31	0.01	1.57	60.82	0.09	4.05	0.01	1.68	69.72
Thailand	0.14	6.00	1.48	12.24	3946.47	0.15	5.22	1.36	8.79	2105.95
Turkey	0.48	7.81	0.25	2.29	135.87	0.79	6.73	0.41	2.70	197.31

*Here, we report the excess kurtosis.
**The test hypothesis of normally distributed returns can be rejected at a confidence level less than 1% for all countries.

Table 2

Results of the Multi-Linear Regression Analysis

The table shows the regression parameters of a multi-linear regression model testing the influence of the mean return (*E*), the standard deviation of return (σ), the skewness (*s*) and kurtosis (k) on CVaR: $CVaR_\alpha = c_0 + c_E \cdot E + c_\sigma \cdot \sigma + c_s .s + c_k \cdot k + \varepsilon$ (α is the confidence level for CVaR and ε stands for the error term). The data were taken from S&P's EMDB. In total, 600 series of US dollar-based, individual equity returns were utilised from 22 emerging markets. All variables in the cross-section regression model were calculated based on the time series of weekly returns on these equities. The time period stretched from the 28th of February 1997 until the 31st of December 2006. The results are presented for 95 and 99 percent confidence level in calculating CVaR.

	c_0	c_E	c_σ	c_s	c_k	R^2 (%)
$CVaR_{95\%}$ = f(E,σ, s, k)	0.011**	-0.998**	1.989**	-0.014**	0.001**	96.80
$CVaR_{99\%}$ = f(E,σ, s, k)	0.006*	-1.210**	3.053**	-0.044**	0.003**	89.90

* Significant at 5% level.
** Significant at 1% level.

Table 3

The Correlation Matrix of the Explanatory Variables

Here the pair-wise correlation terms between the mean (E), the standard deviation (σ), the skewness (*s*) and kurtosis (k), i.e. the explanatory variables of the multi-linear regression model are presented.

	Mean (E)	Std. (σ)	Skew. (s)	Kurt. (k)
Mean (E)	1.000	0.445	0.068	0.000
Std. (σ)		1.000	0.464	0.329
Skew. (s)			1.000	0.831
Kurt. (k)				1.000

Table 4

Total Variance Explained

The table summarises the results of the factor analysis carried out in the four-dimensional space determined by the mean (*E*), the standard deviation (σ), the skewness (*s*) and kurtosis (*k*). For extracting the factors Principal Component Analysis (PCA) was applied.

Factor	Eigenvalue	Variance explained (%)	Cum. variance explained (%)
F_1	1.689	42.217	42.217
F_2	1.024	25.591	67.808
F_3	1.014	25.355	93.163
F_4	0.273	6.837	100

Table 5

Rotated Component Matrix

The rotated component matrix given by the factor analysis is shown here. As a method for rotation, varimax with Kaiser-normalisation was used.

	F_1	F_2	F_3	F_4
Mean (E)	-0.014	0.218	0.976	0.009
S.D. (σ)	0.209	0.943	0.248	0.080
Skewness (s)	0.815	0.263	0.018	0.516
Kurtosis (k)	0.990	0.135	-0.016	-0.021

Table 6

Results of the Factor Model

The table presents the regression coefficients of the factor model developed based on factor loadings provided by the factor analysis. The model is given in the form of $CVaR_{\alpha} = c_0^* + c_{F_1} \cdot F_1 + c_{F_2} \cdot F_2 + c_{F_3} \cdot F_3 + c_{F4} \cdot F_4 + \varepsilon^*$ where α is the confidence level for CVaR and ε^* denotes the error term. Instead of getting a reduction in the number of variables, the intention was to express the influence of the original explanatory variables (*E*, σ, *s*, *k*) on CVaR in terms of "independent dimensions". Therefore, in order to keep the high explanatory power of the original multi-linear regression model, we kept all the four factors given by the factor analysis. The decomposition of the explanatory power into the components attributable to the different factors is also reported in the table.

	c_0^*	cF_1	cF_2	cF_3	cF_4	R^2
$CVaR_{95\%} = f(F_1, F_2, F_3, F_4)$	0.131**	0.004**	0.035**	0.007**	-0.003**	96.80
Explanatory power (%)	-	1.00	91.30	3.80	0.70	
$CVaR_{99\%} = f(F_1, F_2, F_3, F_4)$	0.197**	0.011**	0.052**	0.011**	-0.015**	89.90
Explanatory power (%)	-	3.40	76.90	3.50	6.10	

** The respective parameter is significant at 1% level.

Table 7

Higher Moments and CVaR of Ex Post Optimal Portfolio Strategies

The table reports the higher moments and downside risk metrics of 12 different portfolio strategies. The dataset comprises the simple, weekly returns of 21 emerging market indices stretching from the 3rd of February 1997 until the 29th of September 2008. All returns are calculated from the S&P Emerging Database Investible Indices. We use all observations to estimate the parameters. The MCVaR strategies minimises the CVaR at a specific confidence level, while the TP-CVaR strategies minimise the CVaR at the same return level as that of the the tangency portfolio. The sub-index refers to the confidence level which was applied in optimising CVaR. The higher moment optimisation is performed for different preference levels [α β γ], where α denotes the preference for the mean-variance trade-off, β for the skewness-variance trade-off and γ for the kurtosis-variance trade-off. We calculate the mean, the standard deviation (Std.), the skewness (Skew.), the excess kurtosis (Kurt.), the VaR and CVaR at different confidence levels. "Empirical" denotes the VaR and CVaR values from the underlying dataset while "Parametrical" stands for the values assuming a normal distribution.

					Empirical (%)				Parametrical (%)			
	Mean (%)	Std. (%)	Skew.	Kurt.	$VaR_{95\%}$	$VaR_{99\%}$	$CVaR_{95\%}$	$CVaR_{99\%}$	$VaR_{95\%}$	$VaR_{99\%}$	$CVaR_{95\%}$	$CVaR_{99\%}$
μ-σ/CVaR Strategies												
MVP	0.29	1.93	-0.49	2.32	2.77	5.85	4.38	7.20	2.88	4.19	3.69	4.85
$MCVaR_{95\%}$	0.30	1.97	-0.26	2.54	2.59	5.25	4.23	7.27	2.94	4.28	3.76	4.95
$MCVaR_{99\%}$	0.28	2.20	0.08	2.47	3.31	5.62	4.70	6.68	3.33	4.83	4.25	5.57
TP	0.36	2.11	-0.52	3.08	3.10	5.79	4.72	8.11	3.10	4.54	3.98	5.25
$TP\text{-}CVaR_{95\%}$	0.36	2.13	-0.42	3.17	2.89	5.95	4.62	8.16	3.13	4.58	4.02	5.30
$TP\text{-}CVaR_{99\%}$	0.36	2.37	-0.27	1.50	3.36	6.12	5.15	7.55	3.54	5.15	4.53	5.96
Higher Moment Strategies												
[1 0 0]	0.36	2.11	-0.50	2.95	3.05	5.86	4.70	8.03	3.10	4.54	3.98	5.25
[0 1 0]	0.10	5.62	3.63	42.52	7.35	14.73	12.41	18.57	9.14	12.97	11.49	14.87
[0 0 1]	0.30	3.07	-0.14	0.06	4.94	6.76	6.35	8.32	4.75	6.84	6.03	7.88
[1 1 1]	0.28	3.62	0.21	0.48	5.73	8.25	7.39	9.07	5.67	8.14	7.18	9.36
[2 1 1]	0.29	3.06	-0.05	0.13	4.95	6.83	6.32	8.02	4.74	6.82	6.01	7.85
[2 1 0]	0.22	6.23	1.77	19.25	8.37	14.70	12.98	22.15	10.03	14.27	12.63	16.38

Table 8

Ex Post Optimal Portfolio Weights

We allow for weekly rebalancing and also utilise all observations to estimate the parameters. The dataset comprises weekly returns of 21 emerging market indices ranging from the 3rd of February 1997 until the 29th of September 2008. All returns are calculated from the S&P Emerging Database investible indices. The MCVaR strategies minimises the CVaR at a specific confidence level while the TP-CVaR strategies are constructed to minimise the CVaR at the same return level as the tangency portfolio. The sub-index refers to the confidence level which was applied in optimising CVaR. The higher moment optimisation is performed for different preference levels [α β γ], where α denotes the preference for the mean-variance trade-off, β for the skewness-variance trade-off and γ for the kurtosis-variance trade-off. We calculate the mean, the standard deviation (Std.), the skewness (Skew.), the kurtosis (Kurt.), the VaR and CVaR at different confidence levels.

	μ-σ/CVaR (%)						Higher Moment (%)					
	MVP	Min-$CVaR_{95\%}$	Min-$CVaR_{99\%}$	TP	TP-$CVaR_{95\%}$	TP-$CVaR_{99\%}$	[1 0 0]	[0 1 0]	[0 0 1]	[1 1 1]	[2 1 1]	[2 1 0]
Argentina	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Brazil	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Chile	18.89	12.60	1.88	0.00	0.00	0.00	0.00	0.00	28.45	0.00	22.85	0.00
China	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Czech	1.82	0.00	0.00	12.49	9.88	5.05	13.86	0.00	0.00	0.00	0.00	0.00
Egypt	10.32	9.02	0.00	10.83	6.81	0.00	8.59	0.00	44.26	70.27	49.28	0.00
Hungary	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
India	2.39	0.00	0.00	0.26	0.00	0.00	2.36	0.00	0.00	0.00	0.00	0.00
Indonesia	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.23	38.63
Korea	0.00	1.06	6.82	2.31	2.08	17.78	3.97	0.00	27.30	0.00	19.59	0.00
Malaysia	2.86	6.52	13.09	0.00	0.00	0.00	0.00	98.78	0.00	0.00	0.00	0.00
Mexico	0.00	0.00	0.00	3.08	0.00	0.00	3.96	0.00	0.00	0.00	0.00	0.00
Morocco	46.32	54.94	45.67	49.58	55.85	50.92	47.41	1.22	0.00	0.00	0.00	0.00
Peru	8.50	12.82	19.50	16.62	19.67	17.02	15.93	0.00	0.00	0.00	0.00	0.00
Philippines	1.22	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Poland	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Russia	0.00	0.00	0.00	3.71	3.56	1.03	1.92	0.00	0.00	0.00	0.00	0.00
South Africa	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Taiwan	7.68	2.62	8.80	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Thailand	0.00	0.43	4.24	0.00	0.00	0.00	0.00	0.00	0.00	29.73	8.05	61.37
Turkey	0.00	0.00	0.00	1.12	2.16	8.20	2.00	0.00	0.00	0.00	0.00	0.00

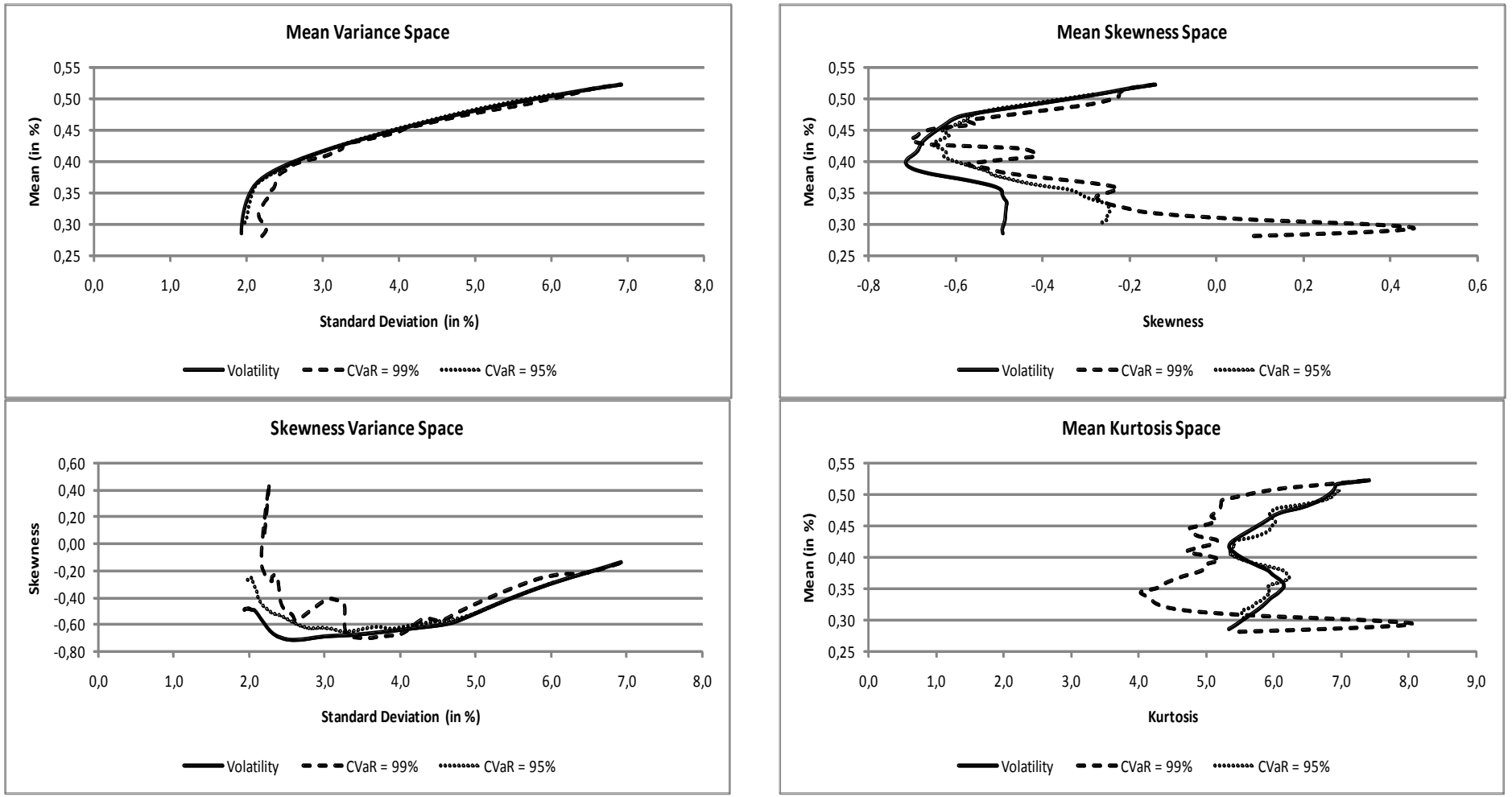

Figure 1: Mean-Variance and Mean-CVaR Efficient Frontiers in Higher Moment Spaces

This figure depicts all mean variance efficient portfolios and all mean-CVaR efficient frontiers in varying moment spaces. These are the mean-variance space (upper left graph), the mean skewness space (upper right graph), the skewness variance space (lower left graph), and the mean-kurtosis space (lower right graph). The various moments are calculated using a two-step procedure. Firstly, we calculate the mean-variance and mean-CVaR efficient portfolio weights. Secondly, using these portfolio weights, we calculate the corresponding moments (mean, standard deviation, skewness and kurtosis) in respect of the historical return distribution.

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