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**Working Paper**

## The fog of fraud: Mitigating fraud by strategic ambiguity

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The fog of fraud –  
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# **The fog of fraud – mitigating fraud by strategic ambiguity**

Matthias Lang / Achim Wambach

May 2010

# The fog of fraud – mitigating fraud by strategic ambiguity\*

Matthias Lang<sup>†</sup> and Achim Wambach<sup>‡</sup>

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## Abstract

Most insurance companies publish few data on the occurrence and detection of insurance fraud. This stands in contrast to the previous literature on costly state verification, which has shown that it is optimal to commit to an auditing strategy, as the credible announcement of thoroughly auditing claim reports might act as a powerful deterrent. We show that uncertainty about fraud detection can be an effective strategy to deter ambiguity-averse agents from reporting false insurance claims. If, in addition, the auditing costs of the insurers are heterogeneous, it can be optimal not to commit, because committing to a fraud detection strategy eliminates the ambiguity. Thus strategic ambiguity can be an equilibrium outcome in the market and competition does not force firms to provide the relevant information. This finding is also relevant in other auditing settings and complements the literature on games with ambiguity-averse players.

**JEL classifications:** D8, K4

**Keywords:** Fraud; Commitment; Ambiguity; Strategic Uncertainty; Costly State Verification

## 1 Introduction

Fraudulent claims on insurance policies are an important issue for insurers. The extent of insurance fraud varies widely from small overstatements of claims to deliberately pretending damages that never occurred or that were intentionally arranged. Due to the nature of fraud, estimating the losses for the insurance industry is not an easy task. Nevertheless the Insurance Information Institute, for example, estimates that in both 2004 and 2005 insurance fraud amounted to \$30 billion in the US property and casualty insurance market.<sup>1</sup> This seems consistent with an estimate of \$20 billion for 1994 by the National Insurance Crime Bureau as stated in Brockett et al. (1998). According to Caron and Dionne (1997), in the Canadian province of Quebec 10% of the insurance claims in the automobile insurance are fraudulent to some extent.

Therefore the strategies of insurers to deter insurance fraud do matter. Dionne et al. (2009, p. 69), for example, estimate that in their sample, companies could save up to 41% of the costs due

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<sup>1</sup> See Rees and Wambach (2008), p. 135.

to fraudulent claims by implementing the optimal auditing strategy. Such a strategy has to balance auditing costs and benefits, like exposed fraudulent claims. In the mass market and with small claims it is too costly to audit each claim that is made. Consequently, claim reports are usually scanned for known patterns of fraud and only a certain fraction of these reports is verified in detail. Previous literature, like Picard (1996), who analyzes the canonical model, suggests that there is a commitment problem. The insurer should be interested in announcing the level of auditing to deter insurance fraud. Given the expected level of auditing, the policyholders indeed report only few fraudulent claims. However, as auditing is costly, the insurance company has an incentive to audit only very few claims *ex post*, rendering its *ex ante* announcement not credible. Credible commitment to a certain level of auditing obviously solves this dilemma. Thus, the absence of commitment implies a loss. Yet it is very unusual for insurers to publish their level of auditing and try to overcome the credibility issue by having an industry association scrutinize their level of auditing or using another third-party verification mechanism. Moreover, insurance firms not only announce no data on fraud detection and auditing, but even block access to it. Thus, there are very few empirical studies available.<sup>2</sup>

Therefore we suggest that there is an additional issue. We depart from previous literature by assuming ambiguity-averse agents and uncertainty about the insurer's costs of an audit. This means that the agents do not have a unique probability distribution about the auditing costs. We model the ambiguity on the type space, as the insured do not know which type of insurer they are facing. This leads to ambiguity about the implemented level of auditing. In our model ambiguity-averse agents undertake less fraud due to this uncertainty. Yet commitment dissolves this ambiguity as it makes the level of auditing common information. We show that, even in a competitive market, it can be optimal for the insurers to maintain the ambiguity and forgo commitment.<sup>3</sup> Thus, strategic ambiguity is an equilibrium outcome.<sup>4</sup>

The insurance companies have different reasons to forgo commitment. Insurance companies with high costs save on auditing costs, if they hide their type by abstaining from commitment, as the average auditing probability is higher than their own. Insurance companies with low costs also prefer the uncertainty to commitment, because a higher level of fraud due to the lower average auditing makes their auditing even more profitable. This is caused by the improved ratio between their low costs and the recovered indemnities and the fines imposed on the uncovered fraudsters. Risk aversion is not sufficient for our result which requires ambiguity aversion. If the degree of risk aversion increases, the deterrence of insurance fraud gets easier both with and in the absence of commitment. Ambiguity aversion has only deterrence effects, if there is no commitment. Therefore only ambiguity aversion influences the balance between commitment and no commitment. Consequently, it is impossible to replicate the results of the model in the absence of ambiguity and ambiguity aversion, because it is the uncertainty that makes ambiguity-averse agents less inclined to commit insurance fraud.<sup>5</sup> Therefore,

<sup>2</sup> A notable exception is Dionne et al. (2009). In the context of tax enforcement the Internal Revenue Service in the U.S. defended in several court cases its right to keep auditing probabilities and procedures secret.

<sup>3</sup> Notice that this result requires ambiguity about primitives of the model, here the auditing costs. Uncertainty as a purification of mixed strategies, as proposed by Harsanyi (1973), is not sufficient.

<sup>4</sup> In contrast to Bernheim and Whinston (1998) strategic ambiguity denotes here the strategic choice to withhold information in order to maintain the ambiguity for the other contract party, not the choice of an incomplete contract.

<sup>5</sup> We were encouraged in this view when one insurance executive told us that besides being bad publicity, communicating detailed data on fighting insurance fraud, like the level of auditing, might induce more policyholders to give it a try. Moreover, according to Reinganum and Wilde (1988, p. 794), the IRS confirms that 'one of the tools in the arsenal

all insurers can have an incentive to avoid commitment, even in a setting of low fraud. The combination of cost uncertainty and ambiguity aversion thus leads to a disadvantage of commitment.

In our model, the policyholders are ambiguity averse. Ambiguity denotes uncertainty about probabilities resulting from missing relevant information. We therefore distinguish ambiguity and risk.<sup>6</sup> In the absence of ambiguity, there is a known probability distribution, while under ambiguity the exact probabilities are unknown. This seems to be quite common in economic decision problems. Savage (1954) and Schmeidler (1989) have developed two axiomatized approaches to this problem. The Subjective Expected Utility of Savage requires the decision maker to be ambiguity neutral. This approach has been criticized for various reasons. From a rational point of view, it seems appropriate to take into account the amount of information a decision is based on. This point was first made by Ellsberg (1961). In addition, there are empirical observations, like Kunreuther et al. (1995) or Cabantous (2007), which suggest that the Subjective Expected Utility approach neglects the distinction between risk and ambiguity. Insurers usually request higher premiums and reject to offer an insurance policy in more cases than in the absence of ambiguity. Consequently, Schmeidler's approach allows ambiguity aversion. In the model, we use the representations of preferences with ambiguity aversion by Schmeidler (1989) and Klibanoff et al. (2005). Therefore the decision maker judges situations with missing information more pessimistically than an ambiguity-neutral person.

The problem of costly state verification considered here is not limited to insurance fraud, but also appears in different settings such as financing (Gale and Hellwig, 1985), accounting (Border and Sobel, 1987), principal-agent relationships (Strausz, 1997) or TV licensing schemes (Rinke and Traxler, 2010). The main point is that there is often asymmetric information between the parties of a contract. To avoid the exploitation of these asymmetries, the other side has to use costly state verification technologies, like ticket inspections in public transport. Townsend (1979) began this analysis of the trade-off between auditing costs and losses due to the remaining information asymmetries. Since commitment is optimal in these models, as discussed in, e. g., Baron and Besanko (1984), there have been various proposals to make commitment feasible and credible. Melumad and Mookherjee (1989) introduce delegation as a commitment device and Picard (1996) proposes a common agency financed by lump-sum payments to subsidize auditing costs. This lowers the variable costs of auditing claims in order to solve the credibility problem. Yet we will argue in this paper that in some circumstances it is optimal for firms to avoid commitment to an auditing strategy, even if this is possible and costless.

Previous literature that combines costly state verification and uncertainty about auditing costs often uses a setting of tax evasion. Cronshaw and Alm (1995) analyze this case, but without ambiguity aversion and the possibility of commitment. Therefore, in their model, uncertainty could be counterproductive. Snow and Warren (2005), on the other hand, model ambiguity aversion by a subjective weighting of probabilities. Their paper studies the behavior of taxpayers given this ambiguity, but there is no possibility of commitment. Thus our paper is the first to consider the strategic decision of commitment versus uncertainty.

A second contribution of this paper is to scrutinize a model with ambiguity aversion in a game-  
of the IRS which promotes voluntary compliance is the uncertainty in the minds of the taxpayers.'

<sup>6</sup> Unfortunately, the literature uses various notions. Sometimes ambiguity is called (Knightian) uncertainty or imprecision. The technical details of representations with ambiguity aversion are discussed in Appendix A.1.

theoretic framework. Although many papers deal with the effects of ambiguity aversion in decision making and finance, there are few papers on games with ambiguity-averse players because of problems with the equilibrium concepts. We avoid these problems by modeling the ambiguity on the type space, i. e., the auditing costs of the insurers. This approach is similar to the one used by Levin and Ozdenoren (2004) and Bose et al. (2006) to study auctions with ambiguity-averse bidders. Bade (2010b) uses this approach, too, in order to establish the existence of equilibria in games of multidimensional political competition. It allows to employ common equilibrium concepts, like perfect Bayesian equilibria. Additionally, we choose the information structure, such that beliefs change in a simple and plausible way, when new information arrives. We can thus derive results that are robust to changes in the updating rules. Consequently, we complement this strand of the literature.

The third contribution is to consider whether competition makes firms provide relevant information to consumers and educate them. The argument by, e. g., Becker (1957) and Laibson and Yariv (2007) has been that competitive pressure gives consumers all the relevant information, as a competitor could always reveal the information and win market share. In our model, this is not the case. There is a market equilibrium with perfect competition where firms do not announce their information about auditing levels and ambiguity prevails that allows to mitigate the effects of insurance fraud. In this respect, our model resembles Gabaix and Laibson (2006), who allow firms to shroud the prices of some add-ons to their products.

The rest of the article is organized as follows. Section 2 sets up a stylized model to give an intuition as to how ambiguity about the level of auditing decreases insurance fraud and explains the decision process of the ambiguity-averse policyholders. In Section 3, we take contracts as given and insurers decide on their auditing probabilities and whether or not to commit to their fraud detection strategy. We show that commitment can decrease profits and that insurers do not want to commit, even if they have the possibility to do so. In Section 4, insurers compete in contracts and decide on their auditing strategies. Even in this competitive market firms in some cases want to forgo commitment. Section 5 discusses some extensions of the model and Section 6 contains the concluding remarks.

## 2 Ambiguity in Auditing

To strengthen the intuition of our results, we begin with a stylized model that shows how the ambiguity aversion of the policyholders makes them less inclined to commit insurance fraud. The mechanism for the commitment decision of the insurers requires the full model which is set up in the next section. A risk and ambiguity-averse agent takes out an insurance with a premium  $P$  and coverage  $q$  against a possible loss  $L > 0$ . Without loss of generality we normalize the outside wealth of the agent to 0. The agent's preferences are represented by an increasing and strictly concave utility function  $u$ . A loss  $L$  occurs with probability  $\delta$  and no loss with probability  $1 - \delta$ . Given this loss distribution, a policyholder who reports a loss smaller or higher than  $L$  is immediately recognized as a fraudster. However, if no loss occurs, the policyholder can nevertheless claim a loss of  $L$ , because the occurrence of a loss is private information of the policyholder. As it is common in the literature on costly state verification, the policyholder faces no direct costs or disutility for this behavior.

The insurance company cannot observe the loss directly. It just receives the report of the policy-

holder. If the insurer pays out the claim, the policyholder gets  $q$  and therefore in case of fraud ends up with a final wealth of  $q - P$ . However, the insurer has a technology to audit a fraction  $p$  of the reports for their truth. This technology is deterministic. Thus, if the insurance company audits a report, it knows for sure whether it is true or not.<sup>7</sup> In case the insurance company detects a fraud, it pays no indemnity and the policyholder has to pay a fine  $M$  that is determined by law. This is commonly known, but the fraction of audits  $p$  is private knowledge of the insurer. The policyholders only know that some reports will be verified. However the insurer may choose to disclose this fraction  $p$  to the policyholders. Without disclosure there is uncertainty about the level of auditing. We will show that the uncertainty lowers auditing costs, because it deters ambiguity-averse policyholders from fraud.

This uncertainty about probabilities due to the lack of relevant information is called ambiguity. In order to model ambiguity-averse agents, we use the Choquet expected utility as proposed by Schmeidler (1989). A formal introduction to the Choquet expected utility is available in Appendix A.1. Yet the results of this paper do not depend on this specific representation of preferences. In Appendix A.2, we repeat the exercise with smooth ambiguity aversion by Klibanoff et al. (2005). This confirms that additional uncertainty decreases the inclination of the policyholders to commit fraud.<sup>8</sup>

The Choquet expected utility is based on capacities, which capture the information structure and the attitude of an agent towards ambiguity. We assume a capacity  $v$

$$\begin{aligned} \text{claim being audited:} & \quad v(-P - M) = (1 - \mathcal{A})p \\ \text{claim being not audited:} & \quad v(-P + q) = (1 - \mathcal{A})(1 - p) \end{aligned}$$

with a parameter  $\mathcal{A} \in [0, 1]$  for the degree of ambiguity aversion. This captures the assumption that the policyholders know that the auditing probability is around  $p$ , given their estimates of the auditing costs. Thus, the information is the same for every policyholder. Nevertheless, policyholders who are ambiguity neutral,  $\mathcal{A} = 0$ , do not care about the uncertainty and simply take the subjective probability  $p$  into consideration. Ambiguity-averse policyholders,  $\mathcal{A} > 0$ , on the other hand, are more cautious and allow for some margin of error. Consequently, they behave *as if* the probability of getting caught were higher and the level of auditing were in the range of  $[(1 - \mathcal{A})p, 1 - (1 - \mathcal{A})(1 - p)]$ . The lower bound of these intervals is given by the capacity. The upper bound depends on the probability mass that the capacity does not assign to the complementary event, here  $1 - v(-P + q)$ . Accordingly, the probability of not being audited is in the range  $[(1 - \mathcal{A})(1 - p), 1 - (1 - \mathcal{A})p]$ . With ambiguity aversion an agent uses first uses the capacity. However the capacity does not distribute the complete probability mass. In the second step the remaining probability mass is assigned to the worst possible event, here to the event of being detected. Hence for the worst event the upper bound of the corresponding range is used.

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<sup>7</sup> An alternative interpretation would be a stochastic technology in the sense that fraud is exposed only with a certain probability. Yet this does not change the analysis, because we can interpret  $p$  as the reduced probability of a claim being audited and being correctly identified, if it was fraudulent.

<sup>8</sup> Gollier (2009) finds that an increase in ambiguity aversion may actually increase the demand for an ambiguous asset, in contrast to our result. The intuition for his result goes as follows: Rothschild and Stiglitz (1971) show that a higher riskiness does not necessarily lower the demand of risk averse agents for the risky asset. Given ambiguity aversion the agent deems a riskier distribution relevant, compared to an ambiguity-neutral agent. Thus, she may have a higher demand for the uncertain asset.

**Lemma 1.** *Suppose the level of auditing is fixed. If the insurer does not announce the level of auditing and the ambiguity-averse policyholders do not have all the relevant information to determine it exactly, less insurance fraud is committed than in the case of easily available information about the auditing probability.*

**Proof:** Without a loss, the Choquet expected utility  $CEU(\mathcal{A})$  is

$$(1 - \mathcal{A})(1 - p)u(-P + q) + (1 - (1 - \mathcal{A})(1 - p))u(-P - M)$$

for fraudulent claims and  $u(-P)$  without a claim. First we consider the case, if the level of auditing is disclosed. Then the probabilities are known and become objective. Thus, there is no ambiguity and  $\mathcal{A} = 0$ . Therefore the policyholder overstates the loss, if the probability  $p$  of an audit is smaller than

$$p^b = \frac{u(-P + q) - u(-P)}{u(-P + q) - u(-P - M)}.$$

If  $p \geq p^b$ , the policyholder will behave honestly and report only true losses.

In the second case, the insurer does not reveal the probability of auditing a claim. Thus, the policyholder lacks relevant information. The difference to the first case depends on the ambiguity aversion and the amount of ambiguity perceived by the policyholder. An ambiguity-neutral policyholder, i. e., with  $\mathcal{A} = 0$ , takes the same subjective probability into account and evaluates her possible actions as before. With ambiguity aversion  $\mathcal{A} > 0$  the capacity does not distribute the entire probability mass. Thus, the policyholder puts the remaining probability mass on the worst event, here the detection of the insurance fraud. So an ambiguity-averse policyholder acts, as if the probability of detection were  $1 - (1 - \mathcal{A})(1 - p)$ . Once again there is a threshold  $p^*$  for honest reporting, with

$$p^* = \frac{(1 - \mathcal{A})u(-P + q) + \mathcal{A}u(-P - M) - u(-P)}{(1 - \mathcal{A})[u(-P + q) - u(-P - M)]} = p^b - \frac{\mathcal{A}}{1 - \mathcal{A}} \frac{u(-P) - u(-P - M)}{u(-P + q) - u(-P - M)} < p^b.$$

As the last fraction is positive, we can conclude that  $p^* < p^b$  for  $\mathcal{A} > 0$ .  $\square$

Thus, not revealing the probability of an audit decreases the level of auditing that is necessary to deter the policyholders from committing fraud. This means that keeping the policyholders in the dark about the level of auditing reduces their inclination to commit insurance fraud. The next section sets up the main model in the framework of Picard (1996) in order to capture the commitment decision of the insurers.

### 3 The Main Model

The main model characterizes the perfect Bayesian equilibria of the entire game and scrutinizes the conditions that are necessary to make commitment unattractive for the insurers. It has the following timing, which is summarized in Figure 1. First, the degree of ambiguity aversion is assigned. Then  $N > 3$  risk and ambiguity-neutral insurers make contract offers, each providing a quote for a coverage  $q$  and a premium  $P$ . In the next stage, the insured choose a contract from the pool of contract offers. At  $t = 3$ , nature determines the costs of an audit for the insurer. In the extension, we modify

this timing by assuming that the insurance company knows its cost already before the contracting stage. The auditing costs are drawn independently for each insurer and revealed only to the insurers.<sup>9</sup> The policyholders only know the set of possible auditing costs, but not the distribution according to which nature is choosing. Every insurance company has the possibility to commit to some auditing level for each contract that it offers. Commitment is costless for the insurer. If there are costs for communicating the auditing probability and making this announcement credible, it only gets easier to make commitment suboptimal. After that, at  $t = 4$ , the policyholders privately observe the occurrence of a loss. Then, at  $t = 5$ , they decide whether or not to file an insurance claim. At  $t = 6$ , the insurer chooses to what extent to audit the filed claims. Finally, the insurer pays the indemnity  $q$  or gets part of the fine, if an audited claim was fabricated.

- At  $t = 0$  the degrees of ambiguity aversion are realized and revealed to the insured.
- At  $t = 1$  insurers make contract offers  $(q_i, P_i)$ .
- At  $t = 2$  the insured choose contracts.
- At  $t = 3$  auditing costs  $c$  are realized and revealed to the insurer; furthermore, insurers can commit to an auditing probability  $p_i$ .
- At  $t = 4$  losses are realized.
- At  $t = 5$  the policyholders make the insurance claims.
- At  $t = 6$  the insurers decide on the extent of auditing, if no commitment was made.
- At  $t = 7$  indemnities and fines are awarded after auditing the filed claims.

Figure 1: The timing of the model

In this section, periods 1 and 2 are taken as given. Thus, only the stages 3 to 7 of the game are considered. Section 4 solves the full model. The model works as follows:

The loss distribution remains unchanged. A loss  $L$  occurs with probability  $\delta$ . There is a continuum of agents with mass one. Independent of the realization of a loss, which is private information, the policyholder can make an insurance claim. The auditing technology works as before, and auditing a claim costs the insurer an amount  $c > 0$  that is drawn by nature from the set  $\{c_L, c_H\}$  with  $c_H > c_L > 0$ . The exact costs  $c$  are private knowledge of the insurer. The policyholder only knows that the costs can be either low  $c_L$  or high  $c_H$ . Therefore, the uncertainty is modeled, à la Harsanyi (1967), on the type space. A policyholder who does not care about the ambiguity uses subjective probabilities. As she does not know anything, she treats both states of nature symmetrically. Therefore she expects to face each type of insurer with probability 1/2 in line with the principle of insufficient reason.<sup>10</sup> An ambiguity-averse policyholder also treats both states of nature symmetrically, but does not distribute the whole probability mass in order to allow for errors. This safety margin is denoted by  $\mathcal{A}$  as in

<sup>9</sup> The correlation between the auditing costs of different insurers is irrelevant for the model. The analysis remains unchanged with any degree of correlation.

<sup>10</sup> Nevertheless having a subjective probability of 1/2 remains an assumption. However the model does not depend on it. We could use any probabilities  $\pi$  and  $1 - \pi$  with  $0 < \pi < 1$  and adapt the capacity accordingly. This does not change the results except for the threshold values. See also footnote 16.

section 2. We restrict the analysis here to the case of the Choquet expected utility<sup>11</sup> and use the following capacity that follows naturally from the symmetry of the states of the world

$$v(c_i) = (1 - \mathcal{A})\frac{1}{2}, \quad i \in \{L, H\}. \quad (1)$$

Due to the symmetry of the states the capacity assigns both types of insurers the same value. In another approach Gajdos et al. (2008) propose an axiomatic foundation for such a contraction representation. We assume a population of agents with different degrees of ambiguity aversion. The degree of ambiguity aversion  $\mathcal{A}$  is uniformly distributed on  $[0, \bar{A}]$  with  $1 > \bar{A} > 0$ . We denote this distribution by  $U$ . Thus, there are policyholders with little as well as moderate ambiguity aversion in the population.<sup>12</sup> The insurers, who know this distribution, cannot observe the degree of ambiguity aversion of a policyholder. It is important to notice that the information sets are the same for every policyholder, although they differ in their degree of ambiguity aversion and consequently in their capacities. The more ambiguity-averse policyholders only behave, as if their beliefs were more pessimistic.

If a false claim is audited, the policyholder has to pay a fine  $M$ , of which  $m \leq M$  is awarded to the insurer that detected the fraud. The remaining part is lost due, e. g., to litigation. As they are determined by law and legal process,  $M$  and  $m$  are exogenous in the model. This modeling choice is common in the costly state verification literature, like Picard (1996). In the next step we determine the equilibrium of the auditing game beginning after period 4.

### 3.1 Solving the Auditing Game

There are two cases to consider. First, we consider the case in which the insurer commits herself to a certain level of auditing in period 3. We solve the model backwards. If the insurance company committed to a certain level of auditing  $p$ , in period 6 it has to stick to that decision and conduct the audits accordingly. In the next step, we analyze the decision of the insured in period 5 whether or not to report a claim in the absence of a loss. The level of auditing has been announced, so the policyholders do not care about the auditing costs of the insurer. As before, the critical value for the level of auditing is  $p^b = \frac{u(-P+q)-u(-P)}{u(-P+q)-u(-P-M)}$ . If more claims are audited, no fraud is committed. For lower levels of auditing, every policyholder makes a claim. In the third period, the insurers choose  $p_i$ , depending on the costs of auditing  $c_i$ , to maximize their profits.

The equilibrium<sup>13</sup> in this game is the same as the one described in Proposition 1 of Picard (1996)<sup>14</sup> and depends on the costs of auditing  $c_i$ . If the insurer's costs are above the threshold, i. e.,  $c_i > c' = \frac{(1-\delta)q}{\delta p^b}$ , the insurer of type  $i$  does not audit any claims and all the policyholders claim a loss. Thus, the

<sup>11</sup> The results of this paper are robust to other representations of preferences and, in particular, also hold with smooth ambiguity aversion as proposed by Klibanoff et al. (2005).

<sup>12</sup> The model can be modified, such that all policyholders have the same degree of ambiguity aversion and, accordingly, the same capacity. This does not change the results qualitatively.

<sup>13</sup> To make the equilibrium unique, the insured have to abstain from committing fraud, if the level of auditing is  $p^b$ , although they are indifferent. This seems natural, as the insurer could audit a fraction  $p^b + \epsilon$  of all insurance claims with an arbitrarily small  $\epsilon$  to make this behavior of the policyholders a unique best response. On the other hand, the insurers are indifferent for  $c = c'$ . For uniqueness it is assumed that insurers have a preference for less fraud.

<sup>14</sup> Picard (1996) assumes an exogenously given fraction  $\theta$  of opportunistic policyholders in an otherwise honest population.  $\sigma$  is the fraction of opportunistic policyholders of a certain insurer. Setting  $\theta = \sigma = 1$  resembles our model with credible announcement. However, with commitment the policyholders behave homogeneously in our model, such that there is no adverse selection in contrast to Picard (1996).

premium  $P$  has to be (weakly) higher than the coverage  $q$  to avoid losses for the insurer. In this case, the agents are at best indifferent about entering an insurance contract or not. If the costs of auditing are below the threshold, a fraction  $p^b$  of all claims is audited and no insurance fraud is committed. Thus,  $P \geq \delta(q + p^b c)$ . We now turn to the case in which the insurer decides not to commit.

Solving the model backwards, the analysis begins at  $t = 6$ . As no commitment was made, the insurance company will choose the level of auditing  $p$  to maximize its profits, given that a fraction  $\alpha$  of policyholders without a loss reported a false claim. Consequently, the insurers' expected profit is the premium income  $P$  minus reimbursements  $q$  for claims that are true or not audited minus the auditing costs  $c$  plus the part  $m$  of the fines which the insurer gets from false claims that were audited. Hence the expected profit is

$$P - q[\delta + \alpha(1 - \delta)(1 - p)] - c(\delta + \alpha(1 - \delta))p + m\alpha p(1 - \delta).$$

On the other hand a policyholder with ambiguity aversion  $\mathcal{A}$  acts as if the probability of an audit is

$$\hat{E}[p|\mathcal{A}] = (1 - (1 - \mathcal{A})\frac{1}{2})p_L + (1 - \mathcal{A})\frac{1}{2}p_H$$

with the anticipated auditing probability  $p_L$  of the low-cost insurer respectively  $p_H$  of the high-cost type. If the policyholder is ambiguity neutral, i. e.,  $\mathcal{A} = 0$ , she expects to face both types of insurer with equal probability. However the more ambiguity averse she gets, the more weight she puts on being confronted with a low-cost insurer. Thus she reports truthfully, if

$$(1 - \hat{E}[p|\mathcal{A}])u(-P + q) + \hat{E}[p|\mathcal{A}]u(-P - M) \leq u(-P). \quad (2)$$

Therefore the following program determines the equilibrium, in which the insurers choose the auditing probabilities  $p_L$  and  $p_H$ , after the policyholders decided whether to commit fraud.

$$\begin{aligned} & \max_{p_i \in [0,1]} P - q[\delta + \alpha(1 - \delta)(1 - p_i)] + m\alpha p_i(1 - \delta) - c_i(\delta + \alpha(1 - \delta))p_i, \quad \forall i \in \{L, H\} \\ & \text{subject to } \alpha = U(\mathcal{A}) = \min\{1, \mathcal{A}/\bar{\mathcal{A}}\} \\ & \text{with } \mathcal{A} \text{ such that} \\ & (1 - \hat{E}[p^*|\mathcal{A}])u(-P + q) + \hat{E}[p^*|\mathcal{A}]u(-P - M) = u(-P) \\ & \hat{E}[p^*|\mathcal{A}] = (1 - (1 - \mathcal{A})\frac{1}{2})p_L^* + (1 - \mathcal{A})\frac{1}{2}p_H^* \end{aligned}$$

To calculate the optimal auditing probabilities,  $p_i^*$ , we consider the reasoning of the insurer. The insurer acts after the insured reported their claims. Thus, the level of fraud  $\alpha$  is taken as given. The insurer is indifferent between auditing or not, if the costs are at the threshold  $c^*(\alpha)$ , which depends on the amount of fraud.

$$c^*(\alpha) = \frac{\alpha(1 - \delta)}{\delta + \alpha(1 - \delta)}(q + m) \quad \text{with} \quad \frac{\partial c^*(\alpha)}{\partial \alpha} > 0 \quad \forall \alpha \geq 0. \quad (3)$$

The fraction  $\frac{\alpha(1 - \delta)}{\delta + \alpha(1 - \delta)}$  is just the insurer's belief after period 5 about a claim to be false. This means

that at the threshold the costs of auditing equal the expected benefits of auditing, i. e., the claims  $q$  that need not to be paid and the fines  $m$  awarded to the insurer. If the costs of auditing are lower (resp. higher) than  $c^*(\alpha)$ , all (none of the) claims are audited. Consequently, the policyholder with ambiguity aversion  $\mathcal{A}$  acts as if the probability of auditing is

$$\hat{E}[p|\mathcal{A}] = \begin{cases} 1 & \text{if } c_H < c^*(\alpha) \\ p \in [1 - (1 - \mathcal{A})\frac{1}{2}, 1] & \text{if } c_H = c^*(\alpha) \\ \frac{1}{2}(1 + \mathcal{A}) & \text{if } c_L < c^*(\alpha) < c_H \\ p \in [0, 1 - (1 - \mathcal{A})\frac{1}{2}] & \text{if } c_L = c^*(\alpha) \\ 0 & \text{if } c_L > c^*(\alpha) \end{cases}$$

depending on the auditing costs. This allows to describe the unique perfect Bayesian equilibrium (modulo out-of-equilibrium beliefs and strategies) of the game after period 4 given a contract with premium  $P$  and reimbursement  $q$ .<sup>15</sup> We concentrate here on the more interesting case with attractive fraud, i. e.,  $p^b > 1/2$ , because otherwise the high-cost insurer will never audit any claims, as is shown in Lemma 2 in the appendix.<sup>16</sup> The following proposition distinguishes four cases, which are depicted in Figure 2. If the costs of both types are too high, there will be no auditing and complete fraud. For lower costs there are two cases, (b) and (d), in which one type will be indifferent with respect to auditing. Finally, there remains the case (c) where every type of insurer plays a pure strategy as auditing is valuable for the low-cost type, but not for the high-cost type.

**Proposition 1.** *For given contracts, beliefs and without commitment the equilibrium has the following form with  $\tilde{\alpha} = (2p^b - 1)/\bar{A}$ :*

(a) *If the costs of both types are above the upper threshold,  $c_L > c^*(1) = (1 - \delta)(q + m)$ , there is complete fraud,  $\alpha = 1$ , and no audits,  $p_H = p_L = 0$ .*

(b) *If the costs of the low-cost type are between the two thresholds,  $c^*(\tilde{\alpha}) < c_L \leq c^*(1)$ , there is a high level of fraud  $\alpha = \frac{\delta c_L}{(1 - \delta)(q + m - c_L)}$ , and a low level of audits  $p_H = 0$  and  $p_L = \frac{2p^b}{1 + \alpha\bar{A}}$ .*

(c) *If the costs of both types are separated by the lower threshold  $c_L \leq c^*(\alpha) \leq c_H$ , there is some fraud  $\alpha = \min\{\tilde{\alpha}, 1\}$  and partial audits of  $p_H = 0$  and  $p_L = 1$ .*

(d) *If the costs of both types are below the lower threshold,  $c_H < c^*(\min\{\tilde{\alpha}, 1\})$ , there is little fraud  $\alpha = \frac{\delta c_H}{(1 - \delta)(q + m - c_H)} < 1$ , and a high level of audits  $p_H = \frac{\bar{A}(\tilde{\alpha} - \alpha)}{1 - \alpha\bar{A}}$  and  $p_L = 1$ .*

<sup>15</sup> To have a unique equilibrium firms have to prefer less fraud, ceteris paribus. Moreover, while the insurer's type is unobservable, the policyholders nevertheless correctly anticipate the equilibrium strategy of each type of insurer. Hence the uncertainty only concerns the type space. Therefore the analysis does not require new equilibrium concepts, like Dow and Werlang (1994), Eichberger and Kelsey (2000) or Bade (2010a) that were developed to deal with multiple notions of the support of a capacity and possible updating rules. In addition, several competing notions of a best reply to the strategies of the other players are available. Finally, the interpretation of an equilibrium in beliefs is not equivalent to the notion of a mixed strategy equilibrium as in the absence of ambiguity.

<sup>16</sup> The condition  $p^b > 1/2$  depends on the distribution of types. Changing the distribution can decrease this threshold for  $p^b$ . Thus, for example the threshold would be  $p^b > 1/3$ , if the high-cost type is more likely, i. e.,  $v(c_L) = \frac{1}{3}(1 - \mathcal{A})$  and  $v(c_H) = \frac{2}{3}(1 - \mathcal{A})$ .

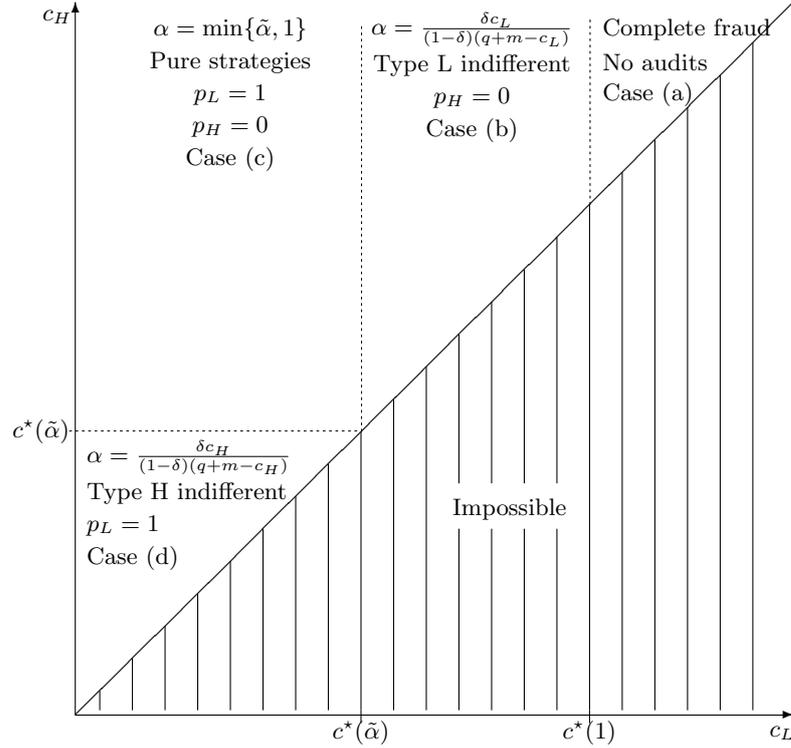


Figure 2: The structure of Proposition 1 with  $\tilde{\alpha} = (2p^b - 1)/\bar{A} < 1$

Suppose for the moment  $(2p^b - 1)/\bar{A} = \tilde{\alpha} \leq 1$ .  $p^b$  is the level of auditing that makes an ambiguity-neutral policyholder indifferent between defrauding and honest behavior. The higher  $p^b$ , the more attractive it is to commit fraud. Thus, a high  $\tilde{\alpha}$  signifies that the level of auditing has to be high to deter fraudsters or that the maximal amount of ambiguity aversion  $\bar{A}$  is low. In both cases there is, ceteris paribus, more fraud at a given level of auditing.

Figure 2 depicts the structure of the proposition. If in case (a) the costs of both types are high enough, there is no auditing and therefore complete fraud. If the costs of the low type are lower, case (b) applies and this type of insurer begins to audit with a level of fraud, such that the insurer is indifferent between auditing and not auditing. The high type still abstains from auditing. If the costs of the low type are even lower, the case of the proposition depends on the costs of the high type. In case those are sufficiently high, case (c) applies, where the amount of fraud is such that both types of insurer play a pure strategy, as for the low type it always pays to audit as much as possible due to the recovered indemnities and the fines. For the high type auditing is not worthwhile. If, on the other hand, the costs of the high type are close to the low type, in case (d) the level of fraud makes the high-cost company indifferent between auditing and not auditing. Therefore it joins the low type in doing some auditing. The low type, however, audits every claim. This is the highest level of auditing and the lowest level of fraud that is possible in equilibrium with positive costs of auditing and without commitment. The proof of the proposition is contained in Appendix A.3.

To sum up, in equilibrium smaller costs of auditing reduce the level of insurance fraud by increasing the auditing probabilities. Additionally, the level of auditing and the amount of insurance fraud depend negatively on each other. In general, insurers with high costs do not audit, except for the last case (d) of the proposition. In contrast to Picard (1996) it is possible to have auditing and the insurers employing pure strategies. Therefore the equilibrium differs, if the auditing costs of both types are

not too high; in particular case (c) is impossible without cost heterogeneity. For high costs of both types, auditing is not worthwhile and the results do not change.

Introducing ambiguity aversion decreases the level of auditing, while holding the amount of fraud fixed or even decreasing it. This holds strictly in the cases (b) and (d) of the proposition and weakly in the remaining ones. The effect discussed in Section 2 causes this reduction. To limit the amount of insurance fraud, the policyholders need to expect a lower level of auditing, because the uncertainty about the level of auditing has an additional deterrence effect. Without ambiguity aversion, i. e., for  $\bar{A} = 0$ , the second case (b) is impossible, because one type of insurer cannot deter enough policyholders from filing fraudulent claims as  $p^b > 1/2$ .<sup>17</sup> In addition, the third case (c) always implies complete fraud in the absence of ambiguity. The next section shows that the third case (c) with limited fraud is the most important one for the purpose of this paper. Yet it requires some ambiguity aversion in the population. Therefore ambiguity aversion matters for the results of the paper.

There remain two interesting implications of Proposition 1. First, the profits of the firms vary continuously, as the parameters change, even if the type of equilibrium changes. Second, commitment allows to eliminate insurance fraud completely, which is impossible without commitment. This result is strengthened by the fact that  $c' > c^*(1)$ .<sup>18</sup> Thus, the absence of commitment lowers the threshold for the costs of auditing that are compatible with complete fraud,  $\alpha = 1$ , in equilibrium. Nevertheless, in less extreme cases ambiguity aversion allows to reduce the total costs of the insurers by forgoing commitment.

### 3.2 Comparing Commitment with Non-Commitment

As we have seen in the previous section, commitment implies more auditing and higher auditing costs. Additionally, there is less income for the insurers from fine payments. Therefore non-commitment may lower the total costs of the insurers. If the insurers are of the high-cost type, they need to implement less audits and can profit from the low fraud caused by the high average auditing probability. For insurers of the low-cost type auditing is cheap and they profit from the higher fraud in the population due to the lower average auditing, if the ratio of their costs to the fines is low enough. In order to prove this, we compare the costs due to insurance fraud,  $\alpha(1 - \delta)(1 - p_i)q$ , and auditing,  $(\delta + \alpha(1 - \delta))p_i c_i$ , minus the recovered fines,  $m\alpha p_i(1 - \delta)$ , in the absence of commitment to the costs of auditing under commitment,  $\delta p^b c_i$ . Commitment implies a loss for the insurance firms, if it holds

$$\alpha(1 - \delta)(1 - p_i)q - m\alpha p_i(1 - \delta) + (\delta + \alpha(1 - \delta))p_i c_i \leq \delta p^b c_i. \quad i \in \{L, H\}$$

The next proposition shows that this is possible in the cases (a) and (c) of Proposition 1. However, without ambiguity both cases imply complete fraud,  $\alpha = 1$ . Ambiguity reduces the level of fraud by the mechanism described in Section 2. Consequently, ambiguity aversion is necessary for the results of this paper. The intuition is that ambiguity aversion changes the left hand side of this inequality without touching the right hand side, because it matters only in the case of non-commitment. Risk aversion on the other hand affects both sides.

<sup>17</sup> This changes for  $p^b \leq 1/2$ , as Lemma 2 shows in the appendix.

<sup>18</sup> Due to the definition of  $p^b$  and the concavity of  $u$  it holds  $c' > c^*(1)$ . Formally, this is equivalent to  $q/(\delta p^b) > q + m$ . Consequently, it is enough to show that  $q - p^b(q + m) > 0$ . This is done in Lemma 3 in the appendix.

**Proposition 2.** *In the equilibrium of the game beginning at stage 3 commitment has no advantage for the insurers, if*

- *the costs of auditing are high for both types, i. e.,  $c_L > c'$ . In this case the insurers do no auditing and therefore are indifferent with respect to the commitment decision.*
- *fraud is attractive,  $p^b > 1/2$  and the costs of the low-cost type are low enough, while the costs of the high-cost type are sufficiently large, i. e.,*

$$c_L \leq \frac{m\alpha(1-\delta)}{\delta(1-p^b) + \alpha(1-\delta)} \quad \text{and} \quad c_H \geq \alpha c'$$

with  $\alpha = \min\{\tilde{\alpha}, 1\}$ .

**Proof:** For high costs of auditing, the insurers will abstain from auditing. Consequently, they are indifferent on the commitment issue. Due to Proposition 1, Lemma 3 and the fact that  $c' > c^*(1)$ , this is the case for  $c_L > c'$ . Below  $c'$ , under commitment, audits become worthwhile and no insurance fraud is committed. Without commitment, auditing is still too expensive. Therefore, commitment is necessary to avoid complete fraud. However, once the auditing costs of the low-cost type drop below  $c^*(1)$ , there is auditing even without commitment. We now distinguish the following cases according to Proposition 1.

In case (b) or (d), commitment is always preferable to no commitment, because the insurance company which does partial auditing has an incentive to commit itself, as this decreases its costs. The reason is the concavity of the utility function and that the indifference of the insurer determines the level of fraud. The details can be found in Lemma 3 in the appendix. Consequently, the insurers prefer commitment in these cases.

Now suppose case (c) of Proposition 1 holds with audits of  $p_H = 0$  and  $p_L = 1$ . Then the low-cost type prefers not to commit, if and only if

$$\alpha(1-\delta)(1-p_L)q - m\alpha p_L(1-\delta) + [\delta + \alpha(1-\delta)]p_L c_L \leq \delta p^b c_L.$$

Inserting the value of 1 for  $p_L$ , we get  $-m\alpha(1-\delta) + [\delta + \alpha(1-\delta)]c_L \leq \delta p^b c_L$  and rearranging the terms gives

$$c_L \leq \frac{m\alpha(1-\delta)}{\delta(1-p^b) + \alpha(1-\delta)}. \quad (4)$$

The fraction on the right hand side is obviously positive and does not depend on  $c_L$ . Furthermore the threshold is smaller than  $c^*(\alpha)$ , because by Lemma 3 it holds that

$$\begin{aligned} & q - p^b(q+m) > 0 \\ \iff & \alpha(1-\delta)q + \delta[q - p^b(q+m)] > 0 \\ \iff & \alpha(1-\delta)q[\delta(1-p^b) + \alpha(1-\delta)] - mp^b\delta\alpha(1-\delta) > 0 \\ \iff & \frac{m\alpha(1-\delta)}{\delta(1-p^b) + \alpha(1-\delta)} < \frac{\alpha(1-\delta)(q+m)}{\delta + \alpha(1-\delta)} = c^*(\alpha). \end{aligned}$$

Therefore condition (4) guarantees case (c). Consequently, for  $c_L$  small enough the low-cost type of insurer forgoes commitment. The high-cost type on the other hand avoids to commit, if

$$\alpha(1 - \delta)(1 - p_H)q - m\alpha p_H(1 - \delta) + [\delta + \alpha(1 - \delta)]p_H c_H \leq \delta p^b c_H.$$

Inserting the value of 0 for  $p_H$ , we get  $\alpha(1 - \delta)q \leq \delta p^b c_H$  and this leads to

$$c_H \geq \frac{\alpha(1 - \delta)q}{\delta p^b} = \alpha c'.$$

Moreover, this threshold is higher than the threshold for case (c) as seen by Lemma 3 and

$$\begin{aligned} & q - p^b(q + m) > 0 \\ \Leftrightarrow & \frac{q}{\delta p^b} > \frac{q + m}{\delta} > \frac{q + m}{\delta + \alpha(1 - \delta)} \\ \Leftrightarrow & \alpha c' = \alpha \frac{(1 - \delta)q}{\delta p^b} > \frac{\alpha(1 - \delta)(q + m)}{\delta + \alpha(1 - \delta)} = c^*(\alpha). \end{aligned}$$

Thus, the high-cost company has no incentive to commit, if its costs are high enough. In summary, we have found a range of parameters such that in equilibrium the insurers choose not to commit to an auditing level, even if they have the possibility to do so credibly and free of charge.  $\square$

In the absence of commitment the policyholders do not know which type of insurer they face and there will be some, but not too much, fraud. Both types of insurer could commit to a level of auditing 0 or  $p^b$  and even completely deter the policyholders from filing fraudulent claims, if the auditing costs of the corresponding type are not too high. However, they have an incentive not to do so and prefer an equilibrium without commitment. We now consider the insurance market and characterize the market equilibrium.

## 4 Market Equilibrium

At  $t = 1$ , each insurer  $j$  offers a finite set of  $I_j$  different contracts  $\{(q_{ij}, P_{ij})\}_{i \in \{1, \dots, I_j\}}$ , such that  $0 < P_{ij} \leq q_{ij}$ .<sup>19</sup> The characterization of the market equilibrium requires the definition of two benchmark contracts. The first contract  $(q^{NC}, P^{NC})$  is the utility-maximizing contract that just breaks even, if the insurers avoid commitment.<sup>20</sup> The second contract  $(q^C, P^C)$  is defined accordingly just for the case with commitment. Therefore define the contract  $(q^{NC}, P^{NC})$  as an element of the following set<sup>21</sup>

$$\begin{aligned} & (q^{NC}, P^{NC}) \in \arg \max_{q, P \in \mathbb{R}^+} \delta u(-L + q - P) + (1 - \delta)u(-P) \\ & \text{with } P \geq \delta(q + \frac{1}{2}c_L) + \frac{1}{2}(1 - \delta)(c_L + q - m)\alpha[q, P] \text{ and } \alpha[q, P] = \min \left\{ \frac{2p^b[q, P] - 1}{A}, 1 \right\}, \end{aligned}$$

<sup>19</sup> In order to guarantee that an insurer does not want to offer the same contract several times, we assume that indifferent policyholders are split up equally between the firms offering these contracts. In the next step they are split up equally between the contracts within a firm that yield the same utility.

<sup>20</sup> Given the assumption on the information set, i.e., the capacity defined in (1), the subjective probability of an ambiguity-neutral insurer to have costs  $c_L$  resp.  $c_H$  is  $\frac{1}{2}$ .

<sup>21</sup> We write  $p^b[q, P]$  to make clear that  $p^b$  depends by its definition on the corresponding  $q$  and  $P$ .

which we assume to be a singleton. The expected profits correspond to the behavior of the insurers determined in Proposition 1 (c). The contract  $(q^C, P^C)$  is defined analogously, but the budget constraint is this time

$$P \geq \delta(q + \frac{1}{2}c_L p^b[q, P]) + \frac{1}{2} \min\{\delta c_H p^b[q, P], (1 - \delta)q\}.$$

Furthermore we assume that the conditions in the second case of Proposition 2 are strictly satisfied for both contracts, i. e., for  $\alpha = \alpha[q^{NC}, P^{NC}]$  it should hold  $p^b[q^{NC}, P^{NC}] \geq 1/2$ ,

$$c_L < \frac{m\alpha(1 - \delta)}{\delta(1 - p^b[q, P]) + \alpha(1 - \delta)} \quad \text{and} \quad c_H > \alpha \frac{(1 - \delta)q}{\delta p^b[q, P]}, \quad \forall (q, P) \in \{(q^C, P^C), (q^{NC}, P^{NC})\}. \quad (5)$$

Finally, the auditing costs of the high-cost type should by assumption not be excessively high and  $c_H < \frac{(1 - \delta)q^{NC}}{\delta p^b[q^{NC}, P^{NC}]}$ , which is feasible for  $\alpha < 1$ . The policyholders will choose the utility-maximizing contracts. Any other less attractive contracts will never be chosen. Therefore the equilibrium is determined only modulo such contracts, which we call empty contracts.

**Proposition 3.** *In the unique perfect Bayesian equilibrium of the game every insurer offers exclusively the contract  $(q^{NC}, P^{NC})$  (modulo empty contracts) and avoids commitment.*

The proof relies on three main points. First, due to the assumptions on the auditing costs, in equilibrium it is not sequentially rational to have a contract with complete fraud in the relevant range. In addition, complete fraud would reduce the insurance contract to stochastic redistribution with efficiency losses. This cannot generate a positive surplus. Second, all the policyholders anticipating honest reporting behave homogeneously and therefore receive the same utility in equilibrium. This includes all policyholders with a high degree of ambiguity aversion. Third, in a contract with commitment, the auditing costs of the insurer are independent of the ambiguity aversion of the policyholders.

**Proof:** First we show that this strategy profile is an equilibrium of the game. Given that the other firms offer the contract  $(q^{NC}, P^{NC})$ , firm  $j$  makes zero profits. If it deviates by offering less appealing contracts, we will denote the least attractive non-empty contract by  $(\hat{q}, \hat{P})$ , where the appeal or the attractiveness of a contract is given by  $\delta u(-L + q - P) + (1 - \delta)u(-P)$ .<sup>22</sup> By definition no policyholder who anticipates honest behavior will accept  $(\hat{q}, \hat{P})$ . Thus, only policyholders that anticipate fraudulent behavior might opt for the contract  $(\hat{q}, \hat{P})$ . Therefore there will be complete fraud in the new contract. Yet compared to the equilibrium contract this implies stochastic redistribution financed by the policyholders themselves with efficiency losses due to the auditing costs and the difference  $M - m$  in the fine payments. As agents are risk and ambiguity averse, any contract with this property that does not make a loss offers less utility than the contract  $(q^{NC}, P^{NC})$ . Consequently, the contract  $(\hat{q}, \hat{P})$  will either make a loss or attract no demand at all. Additionally, this loss could not be compensated with gains in the remaining contracts of firm  $j$ , because it will attract only insured anticipating fraudulent behavior, while the policyholders with high ambiguity aversion remain split between the firms with contract  $(q^{NC}, P^{NC})$ . Hence, this deviation is not profitable.

Now consider a deviation with at least one more attractive contract. Let  $(\bar{q}, \bar{P})$  be the most appealing contract offered by firm  $j$ . Then all insured anticipating honest reporting will select the

<sup>22</sup> If there is more than one contract offering the same utility as  $(\hat{q}, \hat{P})$ , we pick one randomly.

contract  $(\bar{q}, \bar{P})$ . However the insurance company makes a loss with every policyholder, if it does not succeed in lowering its costs due to auditing and fraudulent claims by decreasing the level of fraud to  $\alpha < \tilde{\alpha}$  in this contract. In the next three paragraphs, we show that it is impossible to do so.

Assume that the new contract implements commitment. However, given Proposition 2 and our assumptions, commitment makes contracts more expensive for the insurer. Therefore even the best available contract  $(q^C, P^C)$  with commitment is less attractive than  $(q^{NC}, P^{NC})$ , as assumption (5) results in

$$c_L < \frac{\min\{\delta c_H p^b[q^C, P^C], (1 - \delta)q\} - (1 - \delta)q\tilde{\alpha} + (1 - \delta)m\tilde{\alpha}}{\delta(1 - p^b[q^C, P^C]) + \tilde{\alpha}(1 - \delta)}$$

$$\iff \frac{1}{2}\delta c_L + \frac{1}{2}(1 - \delta)(c_L + q - m)\tilde{\alpha} < \frac{1}{2}\delta c_L p^b[q^C, P^C] + \frac{1}{2}\min\{\delta c_H p^b[q^C, P^C], (1 - \delta)q\}.$$

The right hand side of the inequality calculates the expected profit with commitment. It is smaller than the expected profit in the absence of commitment. Consequently, the contract  $(\bar{q}, \bar{P})$  makes losses. Furthermore, the remaining contracts of firm  $j$  attract only policyholders anticipating fraudulent behavior, if they attract any policyholders at all. Thus, it is impossible to make any gains compensating the losses of the new contract and the contract  $(\bar{q}, \bar{P})$  will not implement commitment.

Now, assume that firm  $j$  offers, in addition to contract  $(\bar{q}, \bar{P})$ , some contracts that attract the policyholders with low ambiguity aversion. Then there will be at least partially honest reporting of claims in these contracts. Yet, in this case, some policyholders have acted irrationally and have not chosen the optimal contract. This is again a contradiction.

Finally, the policyholders with a low degree of ambiguity aversion may be attracted to the contract  $(q^{NC}, P^{NC})$  offered by the remaining insurers. This would result in complete fraud in these contracts. Yet, in this case, auditing is profitable for both types of insurers due to the assumptions on  $c_L$  and  $c_H$ , but then complete fraud is never optimal. Thus, some policyholders will report honestly, although they have chosen the contract  $(q^{NC}, P^{NC})$ , which is a clear contradiction. Hence, this deviation is not profitable. Together with Proposition 1 and 2, this completes the first part of the proof and shows that offering the contract  $(q^{NC}, P^{NC})$  without commitment and the low-cost type doing the auditing, i. e.  $p_L = 1$  and  $p_H = 0$ , is an equilibrium of the game.

In order to show that this is the unique equilibrium, assume to the contrary that there is another equilibrium. From the preceding part of the proof, we know that all policyholders anticipating honest behavior will get the same utility in their contracts and that it is impossible to have contracts with complete fraud. If firms make profits on all their contracts in this alternative equilibrium, insurer  $j$  can profitably deviate by offering all market contracts on slightly more attractive terms. Then the insurance company captures the whole market and increases its profits, as the distribution of the policyholders into the contracts remains unchanged.

If, on the other hand, firm  $j$  makes profits in equilibrium on some contracts, but not on all contracts, it can increase its profits by dropping the unprofitable contracts. The contracts of firm  $j$  contained some policyholders anticipating honest reporting, because contracts with complete fraud are impossible in equilibrium. As the utility for policyholders anticipating honest behavior is the same in all equilibrium contracts, this does not change the market share of firm  $j$  for policyholders reporting honestly, but raises its profits as some policyholders with high ambiguity aversion move to

its profitable contracts and its share of fraudulent policyholders decreases at least weakly, lowering its expected costs.

Therefore, in equilibrium, firms make zero profits on all contracts. If these contracts are less attractive than the contract  $(q^{NC}, P^{NC})$ , an insurer may deviate by exclusively offering the contract  $(q^{NC}, P^{NC} + \epsilon)$  with  $\epsilon > 0$  such that the contract is still more attractive than the equilibrium contracts. Then all policyholders with a high degree of ambiguity aversion opt for the new contract. No matter whether the other policyholders follow their lead, this guarantees positive profits for the deviating firm. On the other hand, the definition of contract  $(q^{NC}, P^{NC})$  makes it impossible to offer more appealing contracts in this setting. Consequently, offering the contract  $(q^{NC}, P^{NC})$  is the unique equilibrium of the game.  $\square$

This concludes the analysis of the model, showing that even market pressure does not force insurers to implement commitment. They use the uncertainty created by missing commitment as a deterrence device that makes it possible to offer better contracts. The corollary summarizes this comparison.

**Corollary 1.** *If the insurers are forced to use commitment, they offer the contract  $(q^C, P^C)$  in equilibrium, which is in utility terms less attractive for the policyholders than the contract  $(q^{NC}, P^{NC})$  without commitment. Therefore forgoing commitment implies an ex ante Pareto improvement.*

The next section discusses some extensions of the model.

## 5 Extensions

As already mentioned in Footnote 5, the Internal Revenue Service in the U.S. stated on several occasions that it regards uncertainty about the amount of auditing and auditing procedures as a valuable method to increase tax compliance. Furthermore it went to great lengths to defend this approach in several court cases brought under Freedom of Information Acts. If we assume that taxpayers are mobile to some extent and counties compete for tax revenues, the model in this paper can be modified accordingly. Instead of receiving insurance, agents have to pick one county where they pay taxes. Not declaring their income correctly would correspond to reporting a fraudulent claim. Then the mechanism in this paper might explain why counties stick to the IRS strategy of avoiding commitment. The deviation of attracting many taxpayers with low tax rates financed by committing to an auditing regime is not profitable in the equilibrium of our model.<sup>23</sup>

The next extension goes back to the initial insurance model, but shifts the realization of the cost type after the commitment decision. Therefore, firms do not know which type they are, when they have the possibility to commit to a certain level of auditing. In this case, the considerations of firms change. If the insurance company commits and the auditing costs are high, it has to bear the high auditing costs or the costs of fraud due to the low auditing probability. This threat is weighted against the usual advantages of commitment for the insurer with low costs. The decision about commitment depends on which effect dominates in equilibrium.

Another modification of the timing allows auditing costs to be realized before insurers make their contract offers. Therefore, insurers can signal the auditing costs by their contract offers and there is

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<sup>23</sup> Even if the IRS or the counties are government bodies, they may be unable to change the fines  $M$ , which detected fraudsters have to pay. Otherwise increasing the fines would be a dominant strategy in the model.

two-sided asymmetric information. Thus, at  $t = 0$  nature determines the costs of an audit for the insurer, which is the same for all firms.<sup>24</sup> After that the game is the same as before. Therefore the

- At  $t = 0$  auditing costs  $c$  are realized and revealed to the insurer; furthermore, the degrees of ambiguity aversion are realized and revealed to the insured
- At  $t = 1$  insurers make contract offers  $(q_i, P_i)$
- At  $t = 2$  the insured choose contracts
- At  $t = 3$  insurers can commit to an auditing probability  $p_i$
- $\vdots$

Figure 3: The modified timing of the extended model

analysis of section 3 remains unchanged and there is again a perfect Bayesian equilibrium without commitment. In this equilibrium, a contract  $(\tilde{q}, \tilde{P})$  is offered, which is an element of the following set

$$(\tilde{q}, \tilde{P}) \in \arg \max_{q, P \in \mathbb{R}^+} \delta u(-L + q - P) + (1 - \delta)u(-P)$$

with  $P \geq \delta q + (1 - \delta)q\alpha[q, P]$ .

Similar to the last section, the existence of the equilibrium without commitment requires additional assumptions: We assume that the conditions in the second case of Proposition 2 are satisfied for the contract  $(\tilde{q}, \tilde{P})$  and that the probability of a loss is sufficiently high, respectively low, i. e.,

$$\delta \geq (\leq) \frac{(N - 1)\tilde{P} + \alpha[\tilde{q}, \tilde{P}](c_L - m)}{(N - 1)\tilde{q} + \alpha[\tilde{q}, \tilde{P}](c_L - m) + (Np^b[\tilde{q}, \tilde{P}] - 1)c_L} \quad (6)$$

depending on the sign of the denominator.<sup>25</sup> This condition will guarantee that the advantage of the uncertainty is sufficiently big to restrain the low-cost type from revealing herself and capturing the whole market. Intuitively, for a positive denominator there have to be enough losses to reduce the possible cases of fraud. Thus, the amount of claims to audit is quite high even with commitment, and commitment does not pay for the insurance company, because it loses the deterrence effect and the fine income. If, on the contrary, the denominator is negative, catching fraudulent claims is so attractive for the insurer that a low incidence of losses is necessary to stabilize the equilibrium.

Given these considerations, in equilibrium every insurer will offer the contract  $(\tilde{q}, \tilde{P})$  and avoid commitment.<sup>26</sup> However, this time we have to make sure in the definition of empty contracts not only that they attract no policyholders, but also that they confer no information about the type of the insurers.

<sup>24</sup> See Jost (1996) for a model with heterogeneous costs. However, in the model of Jost (1996), the coverage  $q$  is conditional on the claim being audited, which is not a common feature of insurance contracts.

<sup>25</sup> It can easily be seen that the denominator is bigger than the numerator, such that the fraction is always smaller than one and the constraint set is therefore non-empty. If the denominator is positive, the fraction might be negative, and in this case, the constraint is trivially satisfied. If, on the other hand, the denominator is negative, the fraction is always positive and thus the probability of a loss can be lower than the threshold.

<sup>26</sup> The equilibrium is not unique. There will usually be a continuum of the equilibria, like  $(\tilde{q}, \tilde{P} + \epsilon)$ , of the type described in Proposition 4, depending on the parameter values. Furthermore, there is a separating equilibrium with each cost type offering the best contract that just breaks even, if the type of the insurer is known.

**Proposition 4.** *There is an equilibrium with every insurer offering exclusively the contract  $(\tilde{q}, \tilde{P})$  (modulo empty contracts).*

The proof is found in Appendix A.3. This equilibrium has an interesting feature. When the insurers consider a deviation, both types want to mimic the other type. The reason is that no matter what the off-equilibrium beliefs are, as long as they are independent of the type of the deviating insurer, at least one type has an incentive to deviate. The high-cost type wants to deviate, if the beliefs are tilted towards the low-cost insurer, because there will be little fraud. If, on the other hand, the beliefs are tilted towards the high-cost type, the low-cost insurance company can increase its market share and profits due to the belief of the policyholders about a low auditing probability. Yet the competitors use the commitment decision to signal the type of the deviating firm. This makes the off-equilibrium beliefs dependent on the type of the deviating firm and the deviation is no longer profitable, because once the type of the deviating firm is revealed, by the conditions of Proposition 4 the insurer is worse off than before. This holds, even though the insurer may serve the whole market after a deviation. Thus, the actions of the competitors make this equilibrium possible.

To sum up, in the equilibrium with commitment, the insurance market can break down, i. e., no agent has a utility higher than without an insurance, if the high-cost type is realized and  $c_H \geq c'$ . Ambiguity allows to avoid this fate by making contracts feasible that rely on the deterrence effect of the uncertainty in the absence of commitment. If there is sufficient ambiguity, the level of fraud  $\alpha \leq (2p^b[\tilde{q}, \tilde{P}] - 1)/\bar{A}$  is always smaller than 1.

## 6 Conclusion

In this article, we discuss a costly state verification model with ambiguity about auditing costs. For this purpose, we use an insurance fraud setting. We show that ambiguity aversion reduces the inclination to commit insurance fraud at a given level of auditing. The insurers, on the other hand, can gain by not committing to an auditing probability and maintaining the uncertainty, even if this means abandoning the advantages of commitment. This is the main contribution of this paper, as we prove that uncertainty can be a feasible deterrence device.

The second contribution is to study a model with ambiguity aversion in a game-theoretic framework. Although ambiguity seems even more relevant in a strategic interaction than for a single player, the literature on ambiguity aversion has so far focused on decision theory and finance. We provide a game-theoretic analysis of ambiguity-averse policyholders. Modeling the ambiguity on the type space, i. e., the auditing costs of the insurers, allows to use common equilibrium concepts and makes the results robust to changes in the updating rules.

The third contribution of this article is to consider whether competition forces firms to educate consumers. According to a common line of argument, competitive pressure provides consumers with all relevant information, as competitors have an incentive to reveal the information in order to increase their market shares. In our model, uncertainty prevails and on the equilibrium path no firm has an incentive to make the auditing costs public. Therefore, there is a market equilibrium with perfect competition where firms do not grant access to their information about auditing probabilities and costs and the uncertainty allows to mitigate the effects of insurance fraud.

Finally, we summarize the incentives of insurers to avoid commitment. The insurers benefit from the higher perceived probability of auditing and the resulting lower level of fraud, if their costs of auditing are high enough. However, for low costs, the insurers gain from non-committing, as they catch more fraudsters, thus saving indemnities and earning fines at low costs. In some cases, these effects are so strong that the costs caused by fraud and its deterrence are lower than under credible commitment to an auditing level. Consequently, the insurers will opt to implement strategic ambiguity. The additional uncertainty in the absence of commitment can thus be profit-enhancing. Although commitment can replicate all outcomes without ambiguity and does not imply any additional costs, in some cases commitment is at a disadvantage, because it cannot replicate the ambiguity. This result requires ambiguity aversion. Risk aversion is not sufficient. This finding complements the results of the previous literature which recommended such a commitment, when possible.

## A Appendix

### A.1 Decision Making with Ambiguity Aversion

There are several representations of preferences that allow for ambiguity aversion, like Ghirardato et al. (2004) or Klibanoff et al. (2005). The paper mainly uses the Choquet Expected Utility of Schmeidler (1989). This representation is based on capacities. Capacities mathematically model probability intervals and are therefore non-additive.<sup>27</sup> Let  $\Omega$  be a non-empty set and  $\Sigma$  a  $\sigma$ -algebra on  $\Omega$ . A capacity  $v$  is a function  $\Sigma \rightarrow [0, 1]$  which maps sets into the real numbers with the following properties:

- $v(\emptyset) = 0$  and  $v(\Omega) = 1$
- $\forall A, B \in \Sigma$  it holds  $A \subset B \Rightarrow v(A) \leq v(B)$

Consequently, capacities have values between 0 and 1. Another important property of a capacity is convexity. A capacity  $v$  is convex, if

$$v(A \cup B) \geq v(A) + v(B) - v(A \cap B) \text{ for all } A, B \in \Sigma.$$

The capacities used so far are all at least weakly convex. To calculate expectations regarding a capacity, we need integrals regarding a capacity. Choquet (1953) introduced the integration of a function regarding a capacity. Let  $f$  be a  $\Sigma$ -measurable and bounded function  $\Omega \rightarrow \mathbb{R}$ . Then the Choquet integral of  $f$  regarding  $(\Omega, \Sigma, v)$  is defined as

$$\int_{\Omega} f dv = \int_0^{\infty} v(f \geq t) dt + \int_{-\infty}^0 v(f \geq t) - 1 dt.$$

In this equation,  $v(f \geq t)$  stands for  $v(\omega \in \Omega | f(\omega) \geq t)$ . The Choquet integrals allow to calculate the Choquet Expected Utility. Schmeidler (1989) was the first to propose an axiomatisation of the Choquet Expected Utility. It is based on the common decision-theoretic axioms, except that the independence

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<sup>27</sup> Sometimes capacities are also called non-additive probability measures or Choquet measures.

axiom only has to hold for comonotonic functions. This is less restrictive than independence for all acts. We still need to define agents' attitude towards ambiguity. An ambiguity-neutral agent simply decides according to the subjective expected utility. Schmeidler defined ambiguity aversion as the attitude of an agent who prefers lotteries where she has more confidence in her probability assessment.<sup>28</sup> More formally, ambiguity aversion is defined to be the (weak) preference of a mixture of lotteries compared to lotteries themselves, if there the agent is indifferent between the lotteries. To gain a better insight into ambiguity aversion, we consider some equivalent formulations. For this purpose, we define the core of a capacity as the set of the probability measures, which assign every event the same or a higher probability than the capacity  $v$ :  $\text{Core}(v) = \{P: \mathcal{U} \rightarrow \mathbb{R} \mid P \text{ is an (additive) probability measure and } P(A) \geq v(A) \text{ for all } A \in \mathcal{U}\}$ . Schmeidler (1986) and Schmeidler (1989) prove that the following statements are equivalent:

- The preference relation  $\preceq$  reveals ambiguity aversion.
- The capacity  $v$  is convex.
- For all acts  $f$  it holds  $\int u \circ f dv = \min_{P \in \text{Core}(v)} \int u \circ f dP$ .

In the model, agents update their beliefs according to the Full Bayesian Updating rule of Dempster (1967), if possible. Yet the results do not hinge on this choice.

An alternative representation is the smooth ambiguity aversion proposed by Klibanoff et al. (2005), which goes back to Segal (1987). The agent knows the first- and second-order probability distributions, but does not compute the reduced lottery. The first-order probability distribution is a distribution for the states of the world, i. e., the state space. The second-order probability distribution, on the other hand, reflects the probability for a first-order distribution. In their interpretation, the first-order distribution characterizes risk and the second-order distribution ambiguity. This distinction corresponds to the assumption that the first- and second-order probabilities are based on different information. The intuition is that the agents have some theories or models of the world, that assign probabilities to the states of the world. The trust in each model is denoted by its second-order probability. The preferences  $\succeq$  of the agent are represented by

$$f \rightarrow \int_{\Pi} \phi \left( \int_{\mathcal{U}} (u \circ f) dP \right) d\mu.$$

The function  $\phi$  reveals the attitude of the agent towards ambiguity. An ambiguity-neutral subject with a linear  $\phi$  simply takes the expectation and derives simple probabilities for each state of the world. With ambiguity aversion, the function  $\phi$  is strictly concave. The concavity of this function corresponds to the ambiguity aversion of the policyholders. The function  $u$  is a von-Neumann-Morgenstern utility function, which determines the attitude towards risk. In addition,  $P$  is a probability measure on the state space and  $\Pi$  is the set of first-order probability measures, which are considered relevant.  $\mu$  is an additive measure that corresponds to the second-order distribution. The preference functional may be

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<sup>28</sup> Ambiguity aversion is independent of the attitude towards risk. An agent may be ambiguity-averse and at the same time risk-neutral, and conversely.

interpreted as a double expectation. First the expected utility for every first-order distribution  $P$  is calculated. Then the expected utility for every  $P$  is transformed by the function  $\phi$ . Finally, the mean with respect to the second-order probabilities is calculated.

## A.2 The Model with Smooth Ambiguity Aversion

This section shows that the results of Section 2 are valid also in the representation of Klibanoff et al. (2005) with smooth ambiguity aversion. The set  $\Pi$  contains the possible values for the first-order probability  $\hat{p}$  of a claim being audited, while  $\mu(\hat{p})$  denotes the second-order probability of  $\hat{p}$  being the correct probability. We assume that  $\Pi$  and  $\mu$  are such that the value of  $p$  in Section 2 is contained in  $\Pi$  and equals the expected value now, i. e.,  $p = \int_{\Pi} \hat{p} d\mu(\hat{p})$ . Then the utility of the policyholder is

$$\int_{\Pi} \phi [(1 - \hat{p})u(-P + q) + \hat{p}u(-P - M)] d\mu(\hat{p}),$$

if fraud is committed, and  $\phi(u(-P))$  with honest reporting. We show that the policyholders abstain from committing fraud, whenever this is the case in the absence of ambiguity. By Jensen's inequality it holds

$$\begin{aligned} \int_{\Pi} \phi [(1 - \hat{p})u(-P + q) + \hat{p}u(-P - M)] d\mu(\hat{p}) &\leq \phi \left( \int_{\Pi} [(1 - \hat{p})u(-P + q) + \hat{p}u(-P - M)] d\mu(\hat{p}) \right) = \\ &= \phi [(1 - p)u(-P + q) + pu(-P - M)] = \phi(EU) \end{aligned}$$

Hence if the average  $p = \int_{\Pi} \hat{p} d\mu(\hat{p})$  is at least  $p^b$ , no insurance fraud is committed. If the second-order distribution is non-degenerate, this holds even for lower averages. This confirms our earlier result of Lemma 1 and shows that it is robust to the way the ambiguity aversion is modeled.

## A.3 Additional Proofs

Proposition 1 characterizes the equilibrium of the game after period 4 given a contract with premium  $P$  and reimbursement  $q$  with attractive fraud  $p^b > 1/2$ .

**Proof of Proposition 1:** Solving the equilibrium backwards, we consider the insurer setting the level of auditing. Thus, we distinguish the following five cases of average auditing  $p := (p_L + p_H)/2$ : no auditing  $p = 0$ , low partial auditing  $0 < p < 1/2$ , partial auditing  $p = 1/2$ , high partial auditing  $1/2 < p < 1$ , and complete auditing  $p = 1$ . As the problem for the insurer is linear, at least one type has a corner solution and audits all or none of the claims made.

(a) If the policyholders expect no audits,  $p = 0$ , every policyholder will report a claim, even if no loss occurred. Ex post it will still be optimal to abstain from auditing for the insurer, if the costs of auditing  $c$  for both types of insurer are higher than the expected benefit of detecting a fraudster,  $(1 - \delta)(q + m)$ . This is the first case (a) of the proposition with  $c_L > c^*(1)$ . If the costs are lower, this is not an equilibrium as the insurers do some auditing and the expectations of the policyholders are incorrect.

(b) If the level of auditing is low, i. e.,  $0 < p < 1/2$ , the low-cost insurer is exactly indifferent between auditing claim reports or not. Therefore the high-cost insurer will abstain from auditing any claims and we can solve the equilibrium backwards by calculating

$$\alpha = \frac{\delta c_L}{(1 - \delta)(q + m - c_L)}$$

from the definition of  $c^*(\alpha)$  in equation (3) to make the low-cost insurer indifferent. The intuition is that we have the ratio of the costs of falsely targeted audits of valid claims and the gains of auditing the fabricated claims. This ratio needs to be equal to the level of fraud  $\alpha$  in order to make the insurer indifferent between auditing and not auditing. The level of fraud determines by equation (2) the necessary level of auditing

$$p_L = \frac{2}{1 + \alpha \bar{A}} \frac{u(-P + q) - u(-P)}{u(-P + q) - u(-P - M)} = \frac{2p^b}{1 + \alpha \bar{A}}.$$

If fraud were too rewarding, i. e.,  $p^b \geq (1 + \alpha \bar{A})/2$ , the low-cost insurer could not on its own deter enough policyholders from filing false reports, because  $p^b$  is the level of auditing necessary to deter an ambiguity-neutral policyholder from filing a false claim, as we have seen before. Yet  $p^b \geq (1 + \alpha \bar{A})/2$  is equivalent to  $c_L \leq c^*(\tilde{\alpha})$ . If on the other hand the costs,  $c_L$ , are higher than  $c^*(1)$ , it would not be worthwhile to audit any claims for the insurance company. This gives us the second part (b) of the proposition with  $c^*(\tilde{\alpha}) < c_L \leq c^*(1)$ .

(c) In the next step we consider an intermediate level of auditing,  $p = 1/2$ . Here, the low-cost insurer audits every claim made and the high-cost insurer does not audit any claims. Therefore the costs have to be  $c_L \leq c^*(\alpha) \leq c_H$ . Otherwise one of the insurers had an incentive to deviate. If fraud is very attractive, i. e.,  $p^b \geq (1 + \bar{A})/2$ , all insured file claims. Consequently, there is complete fraud. If fraud is less appealing, some policyholders will abstain from filing false claims. Thus, the level of fraud is

$$\alpha = \frac{1}{\bar{A}} \frac{u(-P - M) + u(-P + q) - 2u(-P)}{u(-P + q) - u(-P - M)} = \frac{2p^b - 1}{\bar{A}} = \tilde{\alpha} < 1.$$

Since fraud is still rewarding, i. e.,  $p^b > 1/2$ , there will be some fraud and  $\alpha > 0$ . Part (c) of the proposition describes this equilibrium.

(d) More auditing is achieved, if the low-cost insurer audits every claim and the high-cost insurer some claims, i. e.,  $p_L = 1$  and  $p_H > 0$ . The high-cost insurer has to be indifferent to find this level of auditing optimal. Therefore we solve equation (3) of the definition of the indifference costs for the corresponding level of fraud as in case (b)

$$\alpha = \frac{\delta c_H}{(1 - \delta)(q + m - c_H)}.$$

Thus,  $\alpha$  is smaller than one, if and only if  $c_H < (1 - \delta)(q + m) = c^*(1)$ . Equation (2) determines the level of auditing in equilibrium

$$p_H = \frac{1}{1 - \alpha \bar{A}} \left( \frac{u(-P - M) + u(-P + q) - 2u(-P)}{u(-P + q) - u(-P - M)} - \alpha \bar{A} \right) = \frac{2p^b - 1 - \alpha \bar{A}}{1 - \alpha \bar{A}} = \frac{\bar{A}(\tilde{\alpha} - \alpha)}{1 - \alpha \bar{A}}.$$

If fraud were not attractive enough, i. e., if  $(1 + \alpha\bar{A})/2 > p^b$ , there would not be enough false claims in order to make the high-cost type indifferent with respect to auditing. As in case (b), this condition is equivalent to  $c_H < c^*(\tilde{\alpha})$ . In addition,  $c^*(\alpha)$  is increasing in  $\alpha$  in the relevant range. Thus, the fourth case (d) of the proposition covers the range of low costs of  $c_H < c^*(\min\{\tilde{\alpha}, 1\})$ .

Finally, if every claim is believed to be audited, only true claims are reported. However, then, the best strategy of the insurer ex post is not to audit any reports. Therefore, in the absence of commitment, some policyholders will always report false claims in equilibrium.  $\square$

The appeal of filing a false claim, measured by  $p^b$ , is important for the kind of equilibrium we get. This threshold is determined by the ratio of the utility gains respectively losses of successful and unsuccessful fraud. If fraud is very attractive, there is complete fraud, except for the last case (d) of Proposition 1. The second case (b) is again impossible, as the deterrence effect of the low-cost insurer is not sufficient for  $p^b \geq (1 + \bar{A})/2$  to make this an equilibrium. Thus, the insurer performs either no or full auditing. In case fraud becomes less attractive, the last case (d) occurs less often. This also decreases the level of fraud in the third case (c) and makes the second case (b) feasible. If, finally, fraud is too unrewarding for the policyholders and  $p^b \leq 1/2$ , the cases (c) and (d) do not arise at all. This is shown in Lemma 2. However this case is less interesting, because the high-cost insurer will never do any auditing in the equilibrium without commitment.

**Lemma 2.** *Without commitment and  $p^b \leq 1/2$  the equilibrium for given contracts and beliefs has the following form:*

- (a) Complete fraud,  $\alpha = 1$ , and no audits,  $p_H = p_L = 0$ , if the costs of both types are high,  $c_L > c^*(1) = (1 - \delta)(q + m)$
- (b) A level of fraud  $\alpha = \frac{\delta c_L}{(1 - \delta)(q + m - c_L)}$ , and audits  $p_H = 0$  and  $p_L = \frac{2p^b}{1 + \alpha\bar{A}}$ , for lower costs of the low type,  $c_L \leq c^*(1)$ .

**Proof:** The approach is the same as in the proof of Proposition 1. Solving the equilibrium backwards, we consider the insurer setting the level of auditing. Thus, we distinguish the following three cases of average auditing  $p = (p_L + p_H)/2$ : no auditing  $p = 0$ , low partial auditing  $0 < p < 1/2$  and high partial auditing  $p \geq 1/2$ . As the problem for the insurer is again linear, at least one type has a corner solution.

(a) In this case, the considerations are exactly the same as in Proposition 1.

(b) If the level of auditing is low, i. e.,  $0 < p < 1/2$ , the condition  $c_L \leq c^*(1)$ , the beliefs and the behavior are established in the same way as in Proposition 1. However, with  $p^b \leq 1/2$ , the level of auditing  $p_L$  is always below 1 and therefore the low-cost insurer is able to deter enough policyholders from filing false reports on its own. This gives us the second part of the lemma with  $c_L \leq c^*(1)$ .

Finally, if at least half of the claim reports are believed to be audited, only true claims are reported. For even the ambiguity-neutral policyholders cannot gain by fraudulent claims. They are either too risk-averse or the monetary gains are too small compared to the possible fines. Yet, then the insurer is going to implement no auditing. Consequently, there is no equilibrium with more auditing.  $\square$

The next lemma gives the details of the proof of Proposition 2.

**Lemma 3.** *Assume we are in case (b) or (d) of Proposition 1 and  $i \in \{H, L\}$  denotes the type of insurer made indifferent by the level of false claims. Then the type  $i$  insurer will always prefer to commit to a level of auditing  $p^b$ .*

**Proof:** It will be shown that the costs in the absence of commitment are higher than with it, i. e.,

$$\alpha(1 - \delta)(1 - p_i)q - m\alpha p_i(1 - \delta) + (\delta + \alpha(1 - \delta))p_i c_i \geq \delta p^b c_i.$$

Collecting the  $p_i$  terms we get

$$\alpha(1 - \delta)q - p_i[\alpha(1 - \delta)q + m\alpha(1 - \delta) - (\delta + \alpha(1 - \delta))c_i] \geq \delta p^b c_i.$$

Rearranging the terms in the square brackets gives

$$\alpha(1 - \delta)q - p_i[\alpha(1 - \delta)(q + m - c_i) - \delta c_i] \geq \delta p^b c_i.$$

As  $\alpha = \frac{\delta c_i}{(1 - \delta)(q + m - c_i)}$  the term in square brackets equals 0 and we get  $\alpha(1 - \delta)q \geq \delta p^b c_i$ . This means that the auditing costs,  $(\delta + \alpha(1 - \delta))p_i c_i$ , and cost savings due to exposed frauds, i. e., indemnities not paid out,  $q\alpha p(1 - \delta)$ , and fines received by the insurer,  $m\alpha p(1 - \delta)$ , offset each other. Consequently *only* the possible losses due to falsely stated claims,  $\alpha(1 - \delta)q$ , matter. This effect is caused by the indifference condition (3) of the insurer in equilibrium. By inserting  $\alpha$ , we get  $\frac{\delta c_i}{(1 - \delta)(q + m - c_i)}(1 - \delta)q \geq \delta p^b c_i$ . Multiplying the inequality by  $q + m - c_i$  leads to  $q \geq p^b(q + m - c_i)$ . Finally, arranging the terms for  $c_i$  and dropping  $p^b$  gives us

$$-\frac{q}{p^b} + q + m \leq c_i.$$

It holds  $m \leq M$  and  $\epsilon = [u(-P) - u(-P - M)]q - [u(-P + q) - u(-P)]M > 0$ , because the utility function  $u$  is strictly concave. Therefore,  $q - p^b(q + m) = (1 - p^b)q - p^b m \geq (1 - p^b)q - p^b M = \epsilon[u(-P + q) - u(-P - M)]^{-1} > 0$ . Consequently, the left-hand side of the last inequality is negative. Yet, in this case, the inequalities are always satisfied and the respective insurance company can make itself better off by committing to a level of auditing  $p^b$ .  $\square$

Proposition 4 considers the market equilibrium in the extension of the game.

**Proof of Proposition 4:** First, equilibrium beliefs are defined in such a way that they are correct as far as possible, even off the equilibrium path. Thus, the beliefs of the insured about the type of insurer are the capacities defined in equation (1), if they observe the contract  $(\tilde{q}, \tilde{P})$ , possibly accompanied by less attractive contracts. If, on the other hand, they observe a more appealing contract and at least  $N - 1$  of the insurers commit, they update their beliefs to  $v(c_L) = 1$ . Finally, if the more appealing contract is combined with no firm committing, their beliefs are  $v(c_H) = 1$ . Off the equilibrium path, firms with low costs  $c_L$  commit at  $t = 3$  and firms with high costs avoid commitment, if they observe another firm deviating to a more attractive contract than  $(\tilde{q}, \tilde{P})$ , but for the case of  $c_H \leq \frac{(1 - \delta)\tilde{q}}{\delta p^b[\tilde{q}, \tilde{P}]}$  and their contracts attracting demand. The beliefs of the insurance company about the ambiguity aversion of its policyholders are the general ones.

The low-cost type makes positive profits with the contract  $(\tilde{q}, \tilde{P})$ , because according to Proposition

2, auditing is profitable and the premium is set, such that no auditing gives zero profits. If a firm  $j$  tries to capture the whole market by offering a more attractive contract  $(\hat{q}, \hat{P})$ , agents know the insurance company to be the low-cost type, since the behavior of the competitors reveals its type. Consequently, the insurance company always wants to commit to an auditing level in all its contracts  $\{(q_{ij}, P_{ij})\}_{i \in \{1, \dots, I_j\}}$ . Given assumption (6), the profits with the new contract  $(\hat{q}, \hat{P})$  are lower, because  $\hat{P} - \delta\hat{q} - \delta p^b[\hat{q}, \hat{P}]_{c_L} \leq \tilde{P} - \delta\tilde{q} - \delta p^b[\tilde{q}, \tilde{P}]_{c_L}$  and<sup>29</sup>

$$\begin{aligned} & \tilde{P} - \delta\tilde{q} - \delta p^b_{c_L} \leq \frac{1}{N} \left[ \tilde{P} - \delta\tilde{q} + m\alpha(1 - \delta) - (\delta + \alpha(1 - \delta))c_L \right] \\ \iff & (\tilde{P} - \delta\tilde{q})(N - 1) - N\delta p^b_{c_L} \leq m\alpha(1 - \delta) - (\delta + \alpha(1 - \delta))c_L \\ \iff & (N - 1)\tilde{P} + \alpha(c_L - m) \leq \delta \left[ \tilde{q}(N - 1) + Np^b_{c_L} - m\alpha - (1 - \alpha)c_L \right] \\ \iff & \delta \geq (\leq) \frac{(N - 1)\tilde{P} + \alpha(c_L - m)}{(N - 1)\tilde{q} + \alpha(c_L - m) + (Np^b - 1)c_L}. \end{aligned}$$

The direction of the inequality in the last line depends on the sign of the denominator, as discussed before. Therefore it is a best response for the low-cost insurer to offer  $(\tilde{q}, \tilde{P})$  in this equilibrium. No matter whether the insured go to the deviating insurer or stay with the equilibrium contract, this deviation is not profitable. Indeed, for  $c_H \leq \frac{(1-\delta)\tilde{q}}{\delta p^b[\tilde{q}, \tilde{P}]}$ , all insured opt for the new contract  $(\hat{q}, \hat{P})$ , because the new contract is more attractive and the insured anticipate honest behavior. If  $c_H$  is higher, some policyholders may stay with the old contract. Nevertheless, the strategies of the competitors are sequentially optimal.

The high-cost type, on the other hand, has no incentive to deviate either, because by offering the contract  $(\tilde{q}, \tilde{P})$  with commitment the insurer would make a loss. Similarly, the insurance company would incur a loss by offering more attractive contracts, as its type has been revealed and consequently it faces complete fraud in all contracts. Additionally, all insurers may offer empty contracts in equilibrium. No policyholder will accept these and there is no signaling effect, as long as they are offered by both types. Thus, the market outcome remains the same. Therefore no profitable deviation is possible, as every type will be revealed, if the insurer leaves the pooling equilibrium. In equilibrium, both types of insurers decide to avoid commitment and every firm offers the contract  $(\tilde{q}, \tilde{P})$ .  $\square$

## References

- Bade, S. (2010a). Ambiguous Act Equilibria. *Games and Economic Behaviour*, forthcoming.
- Bade, S. (2010b). Electoral Competition with Uncertainty Averse Parties. *Preprint Max Planck Institute for Research on Collective Goods*, 2010(22).
- Baron, D. P. and Besanko, D. (1984). Regulation, asymmetric information, and auditing. *The RAND Journal of Economics*, 15(4):447–470.
- Becker, G. S. (1957). *The economics of discrimination*. University of Chicago Press.
- Bernheim, B. D. and Whinston, M. D. (1998). Incomplete Contracts and Strategic Ambiguity. *The American Economic Review*, 88(4):902–932.
- Border, K. and Sobel, J. (1987). Samurai Accountant: A Theory of Auditing and Plunder. *The Review of Economic Studies*, 54(4):525–540.

<sup>29</sup> In the next lines, we suppress the dependency on  $[\tilde{q}, \tilde{P}]$  of  $\alpha$  and  $p^b$  for notational convenience.

- Bose, S., Ozdenoren, E., and Pape, A. (2006). Optimal auctions with ambiguity. *Theoretical Economics*, 1(4):411–438.
- Brockett, P. L., Xia, X., and Derrig, R. A. (1998). Using Kohonen’s self-organizing feature map to uncover automobile bodily injury claims fraud. *Journal of Risk and Insurance*, 65(2):245–274.
- Cabantous, L. (2007). Ambiguity Aversion in the Field of Insurance: Insurers’ Attitude to Imprecise and Conflicting Probability Estimates. *Theory and Decision*, 62(3):219–240.
- Caron, L. and Dionne, G. (1997). Insurance fraud estimation: more evidence from the Quebec automobile insurance industry. *Assurances et gestion des risques*, 64(4):567–578.
- Choquet, G. (1953-54). Theory of capacities. *Annales de l’Institut Fourier*, 5:131–295.
- Cronshaw, M. B. and Alm, J. (1995). Tax compliance with two-sided uncertainty. *Public Finance Review*, 23(2):139–166.
- Dempster, A. P. (1967). Upper and lower probabilities induced by a multivalued mapping. *The Annals of Mathematical Statistics*, 38(2):325–339.
- Dionne, G., Giuliano, F., and Picard, P. (2009). Optimal auditing with scoring theory and application to insurance fraud. *Management Science*, 55(1):58–70.
- Dow, J. and Werlang, S. (1994). Nash equilibrium under Knightian uncertainty: Breaking down backward induction. *Journal of Economic Theory*, 64(2):305–324.
- Eichberger, J. and Kelsey, D. (2000). Non-additive beliefs and strategic equilibria. *Games and Economic Behavior*, 30(2):183–215.
- Ellsberg, D. (1961). Risk, ambiguity and the savage axioms. *Quarterly Journal of Economics*, 75(4):643–669.
- Gabaix, X. and Laibson, D. (2006). Shrouded attributes, consumer myopia, and information suppression in competitive markets. *The Quarterly Journal of Economics*, 121(2):505–540.
- Gajdos, T., Hayashi, T., Tallon, J., and Vergnaud, J. (2008). Attitude toward imprecise information. *Journal of Economic Theory*, 140(1):27–65.
- Gale, D. and Hellwig, M. (1985). Incentive-compatible debt contracts: The one-period problem. *Review of Economic Studies*, 52(4):647–663.
- Ghirardato, P., Maccheroni, F., and Marinacci, M. (2004). Differentiating Ambiguity and Ambiguity Attitude. *Journal of Economic Theory*, 118:133–173.
- Gollier, C. (2009). Portfolio choices and asset prices: The comparative statics of ambiguity aversion. *IDEI Working Paper*, 357.
- Harsanyi, J. (1973). Games with randomly disturbed payoffs: A new rationale for mixed-strategy equilibrium points. *International Journal of Game Theory*, 2(1):1–23.
- Harsanyi, J. C. (1967). Games with incomplete information played by ”bayesian” players, i-iii. part i. the basic model. *Management Science*, 14(3):159–182.
- Jost, P.-J. (1996). On the Role of Commitment in a Principal–Agent Relationship with an Informed Principal. *Journal of Economic Theory*, 68(2):510–530.
- Klibanoff, P., Marinacci, M., and Mukerji, S. (2005). A smooth model of decision making under ambiguity. *Econometrica*, 73(6):1849–1892.
- Kunreuther, H., Meszaros, J., Hogarth, R., and Spranca, M. (1995). Ambiguity and underwriter decision processes. *Journal of Economic Behavior and Organization*, 26(3):337–352.
- Laibson, D. and Yariv, L. (2007). Safety in markets: An impossibility theorem for dutch books. *Working Paper*, UCLA Department of Economics.

- Levin, D. and Ozdenoren, E. (2004). Auctions with uncertain numbers of bidders. *Journal of Economic Theory*, 118(2):229–251.
- Melumad, N. and Mookherjee, D. (1989). Delegation as Commitment: The Case of Income Tax Audits. *The RAND Journal of Economics*, 20(2):139–163.
- Mookherjee, D. and Png, I. (1989). Optimal Auditing, Insurance, and Redistribution. *The Quarterly Journal of Economics*, 104(2):399–415.
- Picard, P. (1996). Auditing claims in the insurance market with fraud: The credibility issue. *Journal of Public Economics*, 63:27–56.
- Rees, R. and Wambach, A. (2008). *The Microeconomics of Insurance*. Now Publishers, Boston.
- Reinganum, J. F. and Wilde, L. L. (1988). A Note on Enforcement Uncertainty and Taxpayer Compliance. *The Quarterly Journal of Economics*, 103(4):793–798.
- Rinke, J. and Traxler, C. (2010). Enforcement spillovers. *Review of Economics and Statistics*, forthcoming.
- Rothschild, M. and Stiglitz, J. E. (1971). Increasing risk II: Its economic consequences. *Journal of Economic Theory*, 3(1):66–84.
- Savage, L. J. (1954). *The foundations of statistics*. Wiley, New York.
- Schmeidler, D. (1986). Integral representation without additivity. *Proceedings of the American Mathematical Society*, 97(2):255–261.
- Schmeidler, D. (1989). Subjective Probability and Expected Utility without Additivity. *Econometrica*, 57(3):571–587.
- Segal, U. (1987). The Ellsberg Paradox and Risk Aversion: An Anticipated Utility Approach. *International Economic Review*, 28(1):175–202.
- Snow, A. and Warren, R. S. (2005). Ambiguity about Audit Probability, Tax Compliance, and Taxpayer Welfare. *Economic Inquiry*, 43(4):865–871.
- Strausz, R. (1997). Delegation of Monitoring in a Principal-Agent Relationship. *The Review of Economic Studies*, 64(3):337–357.
- Townsend, R. M. (1979). Optimal Contracts and Competitive Markets with Costly State Verification. *Journal of Economic Theory*, 21(2):265–293.