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Institut für Volkswirtschaftslehre

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## Volkswirtschaftliche Diskussionsreihe

Firm Heterogeneity, Credit Constraints,  
and Endogenous Growth

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# Firm Heterogeneity, Credit Constraints, and Endogenous Growth

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## Abstract

This paper is concerned with the role of firm heterogeneity under credit constraints for economic growth. We focus on firm size, innovativeness and credit constraints in a semi-endogenous growth model reflecting recent empirical findings on firm heterogeneity. It allows for an explicit solution for transitional growth and balanced growth path productivity as well as the growth maximizing firm heterogeneity. This enables us to draw inference about the impact of key policy parameters of the model on these quantities and to draw conclusions about firm and capital market related policies.

*Keywords:* Firm heterogeneity, credit constraints, firm size, SME, economic growth

*JEL Classification Number:* E5, O31

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## 1 Introduction

In the recent past new data sets have allowed researchers to detect systematic relations between the characteristics of business firms and their participation in foreign trade and investment. This has spurred an increasing number of theoretical contributions that consider the influence of firm heterogeneity on the pattern of foreign trade and international economic integration. For the empirical evidence on the differences between firms that participate in foreign trade and those that do not, see Bernard et al. (2007). Helpman (2006) reviews the main theoretical lines along which current contributions try to explain these findings. Recently also development studies (e.g. Bond et al. 2008 or Gorodnichenko and Schnitzer 2010) and labor market studies (e.g. Helpman et al. 2008 or Yeaple 2005) are taking firm heterogeneity into account. Less attention has been paid to possible links between firm heterogeneity and other key economic variables, as, for instance, aggregate productivity or growth although data demonstrate that small and medium sized enterprises (SMEs) are responsible for a considerable share in aggregate output and employment.

The focus of this paper is the link between firm size, research and development (R&D), economic growth and long-run productivity in the presence of credit constraints for SMEs on theoretical grounds. To motivate this, we first look at what the existing empirical evidence can tell us about the key mechanisms underlying this relationship. This reveals that there is a tendency for smaller firms to be more innovative than larger ones. However, recent developments in capital market regulations (Basel II) point to a disadvantage for smaller firms in getting the necessary financial leverage. Furthermore, bank surveys indicate that negative shocks on capital markets as e.g. the recent financial crises worsen this disadvantage even more.

The contribution of the present paper to the literature is twofold. First, we develop a semi-endogenous growth model taking account of the influence of firm heterogeneity in innovativeness and credit constraints for SMEs. To the best of our knowledge this hasn't been done before in the literature<sup>1</sup>. From this model we are

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<sup>1</sup>Peretto (1998, 1999a,b) is concerned with firm size but only for homogenous firms. Luttmer (2007) and Baldwin and Robert-Nicoud (2007) analyze the relation between heterogeneity in firm

able to obtain an explicit expression of the optimal degree of firm heterogeneity regarding growth and long-run productivity. The study that comes closest to what we have in mind when considering the link between firm size and growth is Klette and Kortum (2004). However, it takes not account of heterogeneity in innovativeness of firms.

Second, using the results from our model we are able to perform comparative statics on both transitional growth and productivity on the balanced growth path of the economy. This is done with respect to parameters reflecting heterogeneity in credit constraints and innovativeness of firms. Doing so enables us to draw conclusions on how the optimal degree of firm heterogeneity is influenced by policies affecting these parameters and what effects policy interventions have on growth and productivity. We can draw general qualitative conclusions how effectiveness of policies is influenced by the existing degree of heterogeneity in an economy.

The paper is structured as follows. Section 2 provides an overview about the available empirical evidence on firm heterogeneity, innovativeness and credit constraints and motivates our theoretical approach. The model is developed in Section 3 and its results are discussed in Section 4 where we also deduct implications for evaluating the recent EU policy initiatives targeted at SMEs. In Section 5 we finally conclude.

## 2 Empirical Findings

The importance of SMEs for an economy is generally judged as very high (see e.g. OECD 2009) . This is of course due to their dominance in numbers as compared to large scale enterprises. The German federal bureau for statistics (Statistisches Bundesamt) recently released interesting figures for the German economy in 2005.<sup>2</sup> The study considers the role of small and medium sized enterprises with up to 250 employees and turnover up to 50 m € per year or a balance sheet of up to 43 m € in manufacturing, retail trade, hotel and restaurant industries, transportation,  

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level productivity and aggregate productivity

<sup>2</sup>See Klees and Veldhues (2008)

telecommunication and partly the service sector so that about 80% of all German enterprises are covered. It reveals that 99% of all enterprises belong to the SME sector, they are responsible for 60% of total employment, 35% of total turnover, 40% of all gross investments in structures and 46% of total gross value added. Therefore, it is unsurprising that the SME sector plays so prominent a role in the economic policy debate.<sup>3</sup>

In other industrialized countries the SME sector is on average even more important. In 2003 the world bank released a data base that provides mean values for the time period 1990 to 1999.<sup>4</sup> Extracting the data for the 30 OECD member countries, which build a relevant comparison group for the German economy, reveals that the OECD mean share of SMEs in total employment and value added is 66% and 49%, respectively. The figures for Germany during that period of time are 60% and 43%.

Not directly related to the SME sector but nevertheless interesting in this context is entrepreneurship measured by the fraction of the population aged between 18 and 64 active in running a business. The EIM<sup>5</sup> provides in its Entrepreneurs international (Compendia) data set time series on this measure for 23 OECD countries for 1972 to 2007<sup>6</sup>. This entrepreneurial measure declined during this period in 16 of these countries. The unweighted average decline amounts to 3.7 percentage points while the average value 1972 was 17.1%. Since entrepreneurship is closely related to SME activity <sup>7</sup>, these figures document a clear movement away from the SME sector to an economy governed by larger companies. We should be concerned with this trend, if firm size matters for economic growth. And, indeed, empirical evidence seems to confirm this conjecture.

Pagano and Schivardi (2003) analyze the impact of the firm size distribution on

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<sup>3</sup>See Dannreuther (2007) for the European Union SME policy in general and Jousten (2007) for the taxation of SMEs in particular.

<sup>4</sup>See Ayyagari et al. (2007)

<sup>5</sup><http://www.entrepreneurship-eu.eu>

<sup>6</sup>The 23 countries cover Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Italy, Japan, Luxembourg, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, The Netherlands, U.K. and U.S..

<sup>7</sup>According to Audretsch (2007) today's entrepreneurship yields tomorrow's SMEs.

the growth of labor productivity. In their sectoral study for European countries they find a positive relationship between the concentration of a sector on fewer and larger firms on the one side and higher labor productivity growth on the other side.

Carree and Thurik (1998) come to an opposite result. In their study, they investigate the consequences of the transformation process in Europe's manufacturing industries, particularly in the late 1980s and early 1990s, from large firms toward small firms. Based on a panel-sample of 14 manufacturing industries in 13 European countries they find, on average, that the employment share of relative large firms has a negative impact on output growth. Audretsch and Thurik (2001), in a recent contribution, underpin this result: In their panel study for European countries they find that a concentration towards smaller firms positively affects growth.

Other studies indicate severe differences in the activities of firms of different scale or size with respect to innovations, the driving force of economic growth. Lotti and Schivardi (2005) study the patenting behavior of countries, sectors and firms. They match data of the European Patent Office with the firm level data base AMADEUS (Bureau van Dijk) and find that the probability of patenting rises with firm size, which is not quite surprising. The patenting intensity, i.e. granted patents relative to employment, however, falls with employment. Equating innovative behavior with patenting behavior, the relevance of firm size heterogeneity immediately surfaces.

Yet, there are also different results in the empirical literature. In a firm level analysis Bertschek and Entorf (1996) find a non linear relationship between innovating behavior and firm size. For Germany the relationship depends on the time period considered. Their cross sectional regression with data from 1984 reveals that small and large enterprises are more innovative than medium sized, whereas the same regression with data from 1989 detects that medium sized enterprises are most innovative. For the Belgian economy Bertschek and Entorf (1996) discover a similar pattern as found by Lotti and Schivardi (2005). Acs and Audretsch (1990), Bound et al. (1984), Cremer and Sirbu (1978) and Entorf (1988) provide further results that support the non linear hypothesis for the relationship between firm size and innovative behavior.

Directly relevant to these findings are the recent developments in credit markets, in particular the Advanced Internal Ratings-Based (A-IRB) approach of the Basel II reforms. They lead, besides a more restrictive credit allocation, to higher credit costs for SMEs. Grunert et al. (2002) estimate this mark-up for SMEs to be in the range of 1 to 2.5 percentage points p. a. as compared to large scale companies. Financing constraints are crucial for the R&D sector, since revenues for an R&D project usually occur in the future while costs have to be covered today.

The reason for this can probably be found in the impact of the Basel II reforms on the capital requirements for banks providing credits to SMEs. There is now an increasing number of studies pointing in this direction. Empirical evidence for increasing capital requirements and, hence, increasing refinancing costs can be found in, e.g., Saurina and Trucharte (2004), Altman and Sabato (2005), Jacobson, Lindé and Roszbach (2005) and Berger (2006). Meanwhile, there is evidence that the requirements for banks are tightened further in response to the recent financial crisis (see BIS 2009).

This argument gains in importance since it seems that SMEs are more heavily affected by negative shocks to the capital market. The Bank Lending Survey (BLS) conducted quarterly by the Eurosystem in the Euro area sheds some light on how credit constraints apply to different type of firms. The survey is conducted since 2003 and covers 90 bank groups in all participating countries. Although limited in detail, the reported numbers in the BLS can give insights on the influence of firm size on credit constraints. During the fourth quarter 2008 (see ECB 2009 p. 17), i.e. right after the bankruptcy of Lehman Brothers Holdings Inc., the credit standards applicable for approval of credits or credit lines tightened considerably more for SMEs than for large firms. The percentage point difference of banks reporting tightening standards over banks reporting easing standards due to general economic risks increased for SMEs by 16 and for large firms by 5. For reasons of industry or firm related outlook the numbers were 12 and 2, and for risk of the collateral 12 and 5. It seems therefore that conditions for SMEs deteriorated considerably more due to economic risks. But also increased refinancing costs of the lending banks hit



SMEs harder than large firms. The percentage point difference of banks tightening and banks easing standards due to refinancing costs increased for SMEs on average by 8 and for large firms only by 3.7 points. The increasing finance problems for SMEs in response to the financial crises have also been recognized at the OECD Turin Round Table Meeting (see OECD 2009).

Taking these arguments together, we are tempted to conclude that, first, there is a trade-off between the comparative advantage of SMEs in innovation and a comparative disadvantage in accessing financial leverage. Mata (1996) finds that the majority of young firms, which are typically small, are established below their desired or optimal firm size, mainly due to financial constraints but further due to sunk costs combined with ability constraints of their founders, which are directly linked to human capital, and which is particularly required for developing new ideas. The author concludes that the higher the degree of human capital is, the more efficient a company will be and this will increase firm size towards an optimal level. Second, there might be a tendency in important OECD countries to move away from SMEs towards an economy shaped by large enterprises, particularly when economics of scale matters, as stated by Mata and Machado (1996) for instance. While Audretsch and Thurik (2001) and Mata (1996) conjecture a optimal size of the SME sector, the latter raises the question whether a decreasing size of the SME sector is beneficial or not.

### **3 A Theoretical Model of Firm Heterogeneity and Growth**

We consider a second generation growth model and follow Jones (1995) who assumes a production function for innovations or new blueprints<sup>8</sup>. While this function is identical for all firms engaged in R&D in Jones (1995), we introduce a heterogeneity parameter  $\delta$  that accounts for a size effect.

We assume that the population of firms active in R&D is distributed according

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<sup>8</sup>For a discussion of different types of growth models and their appropriateness see also Jones (1999, 2005)

to

$$h(\delta) = (k + 1)\delta^k, \quad k \geq -1 \quad (1)$$

on  $[\underline{\delta}, 1]$  so that the mass of firms equals  $1 - \underline{\delta}^{k+1}$ . Our derivations below will always consider the limiting case  $\underline{\delta} \rightarrow 0$ , i.e., a unit mass of active R&D firms.<sup>9</sup> The distribution (1) is similar to the Pareto style distribution introduced by Baldwin and Robert-Nicoud (2007).<sup>10</sup> The parameter  $k$  clearly shapes the distribution, which is only meaningful for  $k > -1$ . In the following low values for  $\delta$  will be associated with small and medium sized firms in the R&D sector, while large  $\delta$  characterizes large sale firms. For  $k < 0$  the distribution is skewed to relatively more small firms, for  $k > 0$  we have relatively more large scale firms.

The production function for firm  $\delta \in [\underline{\delta}, 1]$  for new blueprints is given by<sup>11</sup>

$$\dot{A}_\delta = \delta^{-\alpha} A^\phi L_\delta^\lambda, \quad (2)$$

where  $\dot{A}_\delta$  denotes new blueprints produced by the firm.  $A$  is the economy wide stock of blueprints and  $L_\delta$  is the amount of R&D labor employed by the firm.  $\phi < 1$ ,  $\lambda < 1$  and  $\alpha > 0$  are productivity parameters. Time is continuous and all variables correspond to the current time  $t$  if not stated otherwise. The total number of blueprints discovered in a particular instance in time is obtained after aggregating all  $\dot{A}_\delta$ , i.e.  $\dot{A} = \int \dot{A}_\delta f(\dot{A}_\delta) d\delta$ , where  $f(\dot{A}_\delta)$  denotes the density function for  $\dot{A}_\delta$ . Total employment in R&D which equals the total exogenous R&D labor supply is defined analogously,  $L = \int L_\delta g(L_\delta) d\delta$ , where  $g(L_\delta)$  denotes the corresponding density for employment.

We interpret the new blueprints as unique ideas to produce differentiated intermediate input factors which are used as an input to final goods production. We therefore follow most of the contributions to the new growth literature which utilize

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<sup>9</sup>The function that we will have to integrate below is continuous except at  $\underline{\delta} = 0$ . Yet, it is well known that the Rieman integral of a function that is continuous on  $[a, b]$  except at finitely many points exists. See, e.g., Capiński and Kopp (2004), p. 9.

<sup>10</sup>For empirical evidence in favor of a Pareto distribution for firm size see, e.g., Luttmer (2007).

<sup>11</sup>We treat time as continuous and whenever no confusion can arise the time index will be dropped in the following.  $\dot{x}$  denotes  $\frac{\partial x}{\partial t}$ .

a love of variety approach in producing final goods with a set of intermediate input factors. Growth in the set of available intermediate input factors motivates growth through a positive externality. However, we do not model explicitly these sectors of the economy as this is not necessary for our results. We rather assume in accordance with the literature that a firm after having created a new blueprint can license for an infinite time horizon its particular variant to the intermediate sector and extracts its flow profits  $\pi_\delta$  in exchange. In addition, we assume as usual that variants are symmetric so that profits for every variant are identical  $\pi_\delta = \pi \forall \delta \in [\underline{\delta}, 1]$ .

R&D firms must pay their workers in advance. We assume a sector of financial intermediaries or banks who raise funds from the household sector and lend to R&D firms at a common rate  $r$ . The present value of the flow of profits over the interval  $[t, \infty)$  is, then, given by:

$$V = \int_t^\infty \pi_s e^{-\int_t^s r(\tau) d\tau} ds.$$

The wage rate  $w$  paid to each worker equals the marginal product of labor, which, in turn, equals the marginal product of producing additional blueprints  $\lambda A^\phi L_\delta^{\lambda-1}$ , times the present value of an additional blueprint  $V$ :

$$w = \lambda \delta^{-\alpha} A^\phi L_\delta^{\lambda-1} V$$

Thus, the wage bill that accrues to each newly produced blueprint is given by

$$\frac{wL_\delta}{\dot{A}} = \lambda V.$$

Without any fixed costs of raising funds, the dividends  $d$  paid to the shareholders equal

$$d = \pi - r\lambda V, \tag{3}$$

where we assume that the firms have to pay back their loans at the end of each period. Thus, while firms are different with regard to their ability to produce blueprints, they produce the same dividend stream so that there is no incentive for shareholders to trade stocks. We will now introduce fixed costs of raising funds that maintain this property of the model.

We assume higher costs of monitoring loans to smaller firms that are a fraction  $1 - \delta^\gamma$ ,  $\gamma > 0$  of the firm's credit requirement  $\lambda V$ . These costs are meant to reflect the credit constraints and financial disadvantages that smaller firms are faced with as explained in Section 2. Both the wage bill and the fixed costs of the credit have to be paid immediately and must be recovered by future profit flows so that

$$\lambda V = \int_t^\infty \lambda \pi e^{-\int_t^s r(\tau) d\tau} ds.$$

For this to hold, the firm must shift the burden of the monitoring costs on its workers so that wages paid per patent shrink to  $\delta^\gamma \lambda V$ . Since both parts of the credit have to be remunerated with the common interest rate  $r$ , the dividends for the owner or shareholder of the firm are given by

$$\begin{aligned} d &= \pi - r\delta^\gamma \lambda V - r(1 - \delta^\gamma)\lambda V \\ &= \pi - r\lambda V \end{aligned}$$

which reproduces the result (3) above. Thus, even with different cost of obtaining credit the value of a patent is identical across shareholders and no trade in patents arises. Labor has to bear the costs for financial leverage completely. Small firms with high financing costs manage to pay the going wage rate only by reducing their employment and thereby increasing the marginal productivity of labor which just outweighs the financing costs.

We can solve the wage equation

$$w = \frac{wL_\delta}{\dot{A}} \frac{\dot{A}}{L_\delta} = \delta^\gamma \lambda V \frac{\dot{A}}{L_\delta} = \delta^\gamma \lambda V \delta^{-\alpha} A^\phi L_\delta^{\lambda-1} \quad (4)$$

for the demand of labor  $L_\delta$  to obtain

$$L_\delta = \left( \frac{V}{w} \lambda \delta^{\gamma-\alpha} A^\phi \right)^{\frac{1}{1-\lambda}}. \quad (5)$$

Remember that low values of  $\delta$  characterize small firms in our model. If we associate size with employment, the parameters  $\gamma$  and  $\alpha$  must be such that  $\gamma - \alpha > 0$ . Obviously, the financing constraints for small firms have a negative impact on their employment.<sup>12</sup> Inserting the ideas production function (2) into the wage equation

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<sup>12</sup>For a model that elaborates on the relationship between financial constraints and the firm size distribution see Cabral and Mata (2003).

(4) yields:

$$\frac{\dot{A}_\delta}{L_\delta} = \delta^{-\gamma} \frac{w}{\lambda V}.$$

Therefore, the innovation intensity, that is, the number of new blue prints relative to employment, falls with firm size which is in accordance with empirical observations cited above.

Aggregating individual labor demand (5) over all firms,  $\int_{\underline{\delta}}^1 L_\delta h(\delta) d\delta$ , and equating the solution with the given labor supply  $L$  determines the wage rate:

$$w = \lambda A^\phi L^{-(1-\lambda)} V \left( \frac{(1+k)(1-\lambda)}{(1+k)(1-\lambda) + \gamma - \alpha} \right)^{1-\lambda}. \quad (6)$$

The wages paid to R&D workers decrease with  $L$ , which is not surprising. Yet, they also depends on the distribution of firm sizes captured by the parameter  $k$ . As noted above, a higher  $k$  is associated with relatively larger firms and the wage rate increases with  $k$  as long as  $\gamma - \alpha$  is positive, as we assumed above. We note further that the wage rate is directly proportional to the profit value  $V$ . This guarantees that the growth rate of  $A$  is independent of  $V$  and hence independent from the not specified part of the economy<sup>13</sup>.

To obtain an expression for the growth rate of  $A$ , we substitute (6) in (5) and insert the solution for  $L_\delta$  into the production function for ideas (2). Integrating over all firms,  $\dot{A} = \int_{\underline{\delta}}^1 \dot{A}_\delta h(\delta) d\delta$ , yields the desired result:

$$\frac{\dot{A}}{A} = \lambda^\lambda A^{\phi-1} L^\lambda \left( \frac{(1+k)(1-\lambda)}{(1+k)(1-\lambda) + \gamma - \alpha} \right)^{-\lambda} \frac{(1+k)(1-\lambda)}{(1+k)(1-\lambda) + \lambda\gamma - \alpha} \quad (7)$$

There are now opposing forces at work if both  $\gamma - \alpha$  and  $\lambda\gamma - \alpha$  are positive which implies a value for  $\lambda$  that has not to be too small. On the one hand, a higher  $k$  implies more large scale R&D firms and increasing wages (see equation (6)). This pulls employment away from small and medium sized companies with a relatively high innovation intensity. This effect which can be seen in equation (7) by the term in parentheses is harmful for growth. On the other hand, there is a positive effect of

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<sup>13</sup>Note that our model is distinct in that compared to usual growth models as e.g. Jones (1995). The reason for this is that we equate the wage paid in R&D with the marginal product in R&D.

having more large scale firms, if  $\lambda\gamma - \alpha > 0$ . In this case, their advantage through lower credit costs outweighs the disadvantage from having a lower productivity factor  $\delta^{-\alpha}$  in the production function for ideas. This effect is embedded in the rightmost term of equation (7)

It easily can be verified that the above growth rate has a maximum with respect to  $k$  as long as  $\lambda\gamma - \alpha > 0$ . The growth rate rises with  $k$  as long as  $k$  is smaller than

$$k^* = \frac{(\gamma - \alpha)(\lambda\gamma - \alpha)}{\alpha(1 - \lambda)} - 1.$$

Of course, the maximum of the growth rate can be obtained by a value for  $k^*$  smaller than zero which would favor an economy which is biased towards relatively more small firms.

The results obtained so far are valid at any point in time. We will now obtain the level of ideas on a balanced growth path of our economy. As usual, we obtain this path by assuming a constant growth rate of ideas  $g_A := \dot{A}/A$ . Equation (7) implies that  $g_A$  is constant, if the term  $A^{\phi-1}L^\lambda$  does not change over time. This in turn requires

$$g_A = \frac{\lambda}{1 - \phi} g_L,$$

where  $g_L := \dot{L}/L$  denotes the growth rate of R&D labor supply. The finding that the economy's growth rate on a balanced growth path is solely determined by the (exogenously) given growth rate of labor supply characterizes our model as a second generation new growth model.<sup>14</sup> However, even on the balanced growth path the distribution of firm size still influences the economy, which can be seen by equating equation (7) with  $\frac{\lambda}{1-\phi}g_L$  and solving for  $A$ . This exercise gives the level for the stock of ideas valid on the balanced growth path

$$A^* = \left( \frac{\lambda}{1 - \phi} g_L \right)^{-\frac{1}{1-\phi}} L^{\frac{\lambda}{1-\phi}} \left( \frac{(1+k)(1-\lambda)}{(1+k)(1-\lambda) + \gamma - \alpha} \right)^{-\frac{\lambda}{1-\phi}} \times \left( \frac{(1+k)(1-\lambda)}{(1+k)(1-\lambda) + \lambda\gamma - \alpha} \right)^{\frac{1}{1-\phi}}.$$

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<sup>14</sup>Of course, if we assume that  $g_L$  can be influenced by some policy measure the growth rate is not exogenous within a more elaborate model.

From this result we see the implications of growth rates off the balanced growth path for levels on the balanced growth path. Of course, higher growth rates caused by an appropriate distribution of firm size manifest themselves in higher levels for the stock of ideas. The different channels through which this works are the same as those identified in the discussion of the effects of  $k$  on the growth rate of  $A$ . An appropriate distribution fosters growth off the balanced growth path and yields finally higher levels in ideas on the balanced growth path. As  $A$  in usual growth models influences the level of production of final goods in the economy, this result is very important.

#### 4 Discussion and Policy Implications

Our model demonstrates that there exists an optimal firm size distribution with respect to the growth rate of ideas off the balanced growth path and with respect to the level of ideas on the balanced growth path. Audretsch and Thurik (2001) already followed these ideas partly in their empirical country study. However, their approach was not motivated by a detailed (semi-)endogenous growth model. They rather assumed that there exists an optimal relative number of SMEs by specifying their empirical model. If a country deviates from this optimum, growth will be negatively affected.

In the light of the theoretical results above, we must note however, that focusing on growth rates is not sufficient. As our model predicts growth rates are only affected by the distribution of firm size off the balanced growth path. On the balanced growth path the level of production is influenced and not the growth rate. This is important especially if long time series are considered in an empirical assessment. In this case we would expect the economy to fluctuate around the balanced growth path and effects on the growth rate might be hidden away. Therefore levels have to be considered as well when analyzing the impact of the firm size distribution.

Another point worth mentioning in the light of our results is their relevance in the political debate. We saw that there is an optimal distribution of firm size captured by the distribution parameter  $k$ . However, the optimal value of  $k$  is certainly not

exogenous. Rather it depends on the model parameters, i.e.  $\lambda$ ,  $\alpha$  and  $\gamma$ . They capture the elasticity of innovating with respect to labor in R&D, the comparative advantage of small firms and lower financial constraints for larger firms. We think that at least the last two parameters can be influenced by economic policy. The Basel II agreement is one of the ways through which these parameters might have been influenced in the past. Concluding this paragraph it might not be necessary to shift  $k$  to its optimal level but it might be also possible to shift the optimal value towards the existing  $k$ .

At this point it is interesting how the transitional growth rate, the steady state level of ideas  $A^*$  and the optimal  $k$  depend on  $\gamma$  and  $\alpha$ . Straightforward calculations reveal that

$$\frac{\partial \dot{A}}{\partial \alpha} > 0, \quad \frac{\partial A^*}{\partial \alpha} > 0, \quad \frac{\partial k^*}{\partial \alpha} < 0,$$

$$\frac{\partial \dot{A}}{\partial \gamma} < 0, \quad \frac{\partial A^*}{\partial \gamma} < 0, \quad \frac{\partial k^*}{\partial \gamma} > 0.$$

The results indicate that increasing the comparative advantage for small firms in innovation, i.e. increasing  $\alpha$ , is beneficial for both transitional growth and the level of ideas on the balanced growth path. Increasing finance costs which hits small firms hardest due to the chosen formal representation is harmful in both cases. However, the dependencies of  $k^*$  have exactly the opposite signs. This gives rise to an interesting mechanism. If  $k$  is smaller than  $k^*$ , i.e. the economy has more than optimal small firms, increasing  $\alpha$  has a positive direct and indirect effect. The direct effect works through the increased innovative capacity of smaller firms. The indirect effect works through lowering  $k^*$  in the direction of the actual  $k$ , i.e. the trade-off between the comparative advantage and disadvantage of smaller firms is solved more in favor of an economy having a larger SME sector. If, however,  $k$  is larger than  $k^*$ , the indirect effect is negative but not strong enough to overcome the positive direct effect.

Naturally, increasing finance costs or credit constraints which hits smaller firms hardest leads to exactly the opposite effects. Increasing  $\gamma$  has a negative direct and indirect effect for an actual  $k$  smaller than  $k^*$ . If  $k$  is small the economy is



hit hard through increased disadvantage of smaller firms getting financial leverage. Furthermore, the optimal trade-off solution is shifted towards a larger large firm sector. For  $k$  larger than  $k^*$  the indirect effect has the opposite sign and, hence, dampens the negative direct effect but does not overcome it.

From this we can draw important policy conclusions. First, policies directed only towards increasing the SME sector and leaving innovative capacities,  $\alpha$ , and credit constraints unaffected are not always beneficial. There exists an optimal degree of firm heterogeneity as already conjectured by Audretsch and Thurik (2001). Fostering the SME too much in that sense is harmful for growth and long run productivity of the economy.

Second, increasing innovative capacity of firms with a bias towards smaller firms, i.e. increasing  $\alpha$ , is always beneficial for the economy in terms of growth and long-run productivity. The effect first increases in  $k$ <sup>15</sup> and then diminishes as  $k$  grows large. The reason for this is that there is a trade-off between increasing the innovative productivities of SMEs on the one hand, and having to carry the finance costs for the upcoming ideas.

Third, decreasing credit constraints biased towards SMEs is always beneficial for growth and long-run productivity. However, the economy benefits most from such a policy if it is shaped by a large SME sector. The same pattern arises in the case of an increasing  $\alpha$ . Negative capital market shocks that increase  $\gamma$  hit such economies therefore harder.

A recent review of the EU policy targeted at SMEs can be found in Dannreuther (2007). The EU tries to promote small businesses since the European Year of the SME 1982 (EYSME). In 1986 the SME Action Program was launched which addressed market failures and tried to improve the business environment. The second phase of SME policy began in the 1990s and was characterized by more sophisticated efforts towards SMEs. The final phase of SME policy started in 2000 with the Lisbon process. It aims at a stronger coordination of the member countries SME policies. Through the Lisbon process targets were defined which work as bench-

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<sup>15</sup>See the Appendix at the end of the paper for details.

marks for evaluating the policy measures. As Dannreuther (2007) points out, the SME policy of the EU became a very important element of the EU economic policy. This is also reflected in the Small Business Act of 2008 and the increased role of the European Investment Bank in financing small businesses (see EC 2010). Also the responses to the financial crises include specific policies designed for the SME sector. In Germany the public KfW bank set up special credit programs in collaboration with private banks for SMEs where the KfW takes over a considerable fraction of the credit risks. On the European level, the European Investment Bank set up a credit program directed to SMEs with a volume of 30 b €. EUR over the period 2008 to 2010.

We conclude that even 20 years after the introduction of the first programs at the European level, promoting SMEs is still an important topic. This seems to imply that after 20 years the goals have not been reached yet and there is still need for further policy interventions. However, based on our results there is need for a careful evaluation of the programs. Their effects on growth and productivity might be different in different economies, depending on the importance of the SME sector in the different countries. Drawing again on the SME database of Ayyagari et al. (2007) shows that the share of SMEs in total employment ranges from a low 56% in the U.K. and a high of 87% in Greece. Thus, identical policies might have a different impact in every country with also possibly different signs.

The issue might get even more complex if sector heterogeneity of an economy is considered. Different sectors of an economy might be characterized by different firm size distributions. This in turn demands policies that are tailored to these different sectors. Policies that do not differentiate with respect to that might lead to very different outcomes across countries as well as across sectors.

## 5 Conclusion

Given the importance policy gives to fostering the SME sector both at the national and international level, one may wonder about the justification for doing so. Empirical evidence suggests that there might be a trade-off between SMEs and large

scale companies defined by innovative capacities and credit constraints. Building on such a trade-off, our theoretical model suggests an optimal firm heterogeneity with respect to growth and long run productivity.

Analyzing the results of this model reveals that it might be a good idea to evaluate the existing policies regarding their appropriateness for the economy to reach this optimum. Caution might be necessary because the same policies might have very different impacts across countries. This is especially important considering harmonized EU wide policies.

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## Appendix

Derivatives of  $\frac{\dot{A}}{A}$

$$\begin{aligned}\frac{\partial \frac{\dot{A}}{A}}{\partial \alpha} &= \frac{\frac{\dot{A}}{A} (1-\lambda)(1+k) + \gamma(1+\lambda) - \alpha}{1-\lambda [(1-\lambda)(1+k) + \gamma - \alpha][(1-\lambda)(1+k) + \lambda\gamma - \alpha]} > 0 \quad \text{for } \lambda\gamma - \alpha > 0, \\ \frac{\partial^2 \frac{\dot{A}}{A}}{\partial \alpha \partial k} &= \frac{\frac{\dot{A}}{A} - (\gamma\lambda)^2 - [(1-\lambda)(1+k) + \gamma(1+\delta) - \alpha][(1-\lambda)(1+k) + \gamma - \alpha]}{A [(1-\lambda)(1+k) + \gamma - \alpha]^2 [(1-\lambda)(1+k) + \lambda\gamma - \alpha]^2} \\ &\quad + \frac{\partial \frac{\dot{A}}{A}}{\partial \alpha} \frac{1}{\frac{\dot{A}}{A}} \frac{\partial \frac{\dot{A}}{A}}{\partial k}.\end{aligned}$$

The cross derivative with respect to  $k$  can be positive or negative. The first term is always negative for  $\gamma - \alpha > 0$ , the second is positive for  $k < k^*$ , zero for  $k = k^*$ , and negative for  $k > k^*$ .

$$\begin{aligned}\frac{\partial \frac{\dot{A}}{A}}{\partial \gamma} &= -\frac{\frac{\dot{A}}{A} \lambda(1-\lambda)\gamma}{A [(1-\lambda)(1+k) + \gamma - \alpha][(1-\lambda)(1+k) + \lambda\gamma - \alpha]} < 0 \quad \text{for } \lambda\gamma - \alpha > 0, \\ \frac{\partial^2 \frac{\dot{A}}{A}}{\partial \gamma \partial k} &= \frac{\frac{\dot{A}}{A} \lambda(1-\lambda)^2\gamma}{A [(1-\lambda)(1+k) + \gamma - \alpha][(1-\lambda)(1+k) + \lambda\gamma - \alpha]} \\ &\quad + \frac{\partial \frac{\dot{A}}{A}}{\partial \gamma} \frac{1}{\frac{\dot{A}}{A}} \frac{\partial \frac{\dot{A}}{A}}{\partial k}.\end{aligned}$$

Here we observe exactly the opposite behavior compared to the  $\alpha$  case above.

Derivatives of  $k^*$

$$\begin{aligned}\frac{\partial k^*}{\partial \alpha} &= -\frac{\alpha(1-\lambda)(\lambda\gamma - \alpha + \gamma - \alpha) + (\gamma - \alpha)(\lambda\gamma - \alpha)}{\alpha^2(1-\lambda)^2} < 0 \quad \text{for } \lambda\gamma - \alpha > 0, \\ \frac{\partial k^*}{\partial \gamma} &= \frac{\lambda(\gamma - \alpha)}{(1-\lambda)\alpha} > 0.\end{aligned}$$

Thus for  $k < k^*$ , increasing  $\alpha$  or decreasing  $\gamma$  moves  $k^*$  in the direction of the existing  $k$ .