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Working Paper

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CSIO Working Paper, No. 0021

Provided in Cooperation with:

Department of Economics - Center for the Study of Industrial Organization (CSIO), Northwestern University

Suggested Citation: Dana, James D.; Spier, Kathryn E. (2001) : Entry and entry deterrence in a duopoly market, CSIO Working Paper, No. 0021, Northwestern University, Center for the Study of Industrial Organization (CSIO), Evanston, IL

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THE CENTER FOR THE STUDY
OF INDUSTRIAL ORGANIZATION
AT NORTHWESTERN UNIVERSITY

Working Paper #0021

Entry and Entry Deterrence in a Duopoly
Market*

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July 2001

*We would like to thank Mike Whinston for helpful comments.

Abstract

In a homogeneous good, Cournot duopoly model, entry may occur even when the potential entrant has no cost advantage (or even a cost disadvantage) and no access to distribution – the entrant must instead convince one of the vertically integrated incumbents to distribute its output. By sinking its costs of production before negotiations with the incumbents begin, the entrant commits to selling its output which induces the incumbents to bid more aggressively for it. Each incumbent is willing to pay up to the incremental profit earned from the additional output *plus* the incremental loss avoided by keeping the output away from its rival. So the incumbents may be willing to pay *up to the market price* for each unit of available output. A sequential game in which incumbents can produce first is analyzed and conditions under which entry is deterred in equilibrium by preemptive capacity expansions are derived.

1. Introduction

In many industries, distribution is controlled by a small number of vertically integrated firms and entrants must rely upon one of its rivals to distribute its product. The ability of these incumbents to deny entrants access to distribution is a potential source of market power, yet there are many examples of vertically integrated firms distributing rivals' products. The motion picture industry, for example, has many independent filmmakers (including foreign productions) who produce films that they later distribute through large U.S. movie studios. Similarly, small drug producers often rely upon large vertically integrated pharmaceutical companies to market and distribute their products. Small airlines, such as Spirit Airlines at O'Hare airport in Chicago, have successfully entered markets where a few dominant firms control access to terminal gates and baggage carousels. And Ben and Jerry's, the second largest producer of superpremium ice cream in the U.S., recently began distributing its ice cream through Pillsbury, maker of the leading superpremium ice cream brand, Haagen-Dazs. Ben and Jerry's announced the switch after a dispute with their former distributor, Dreyer's, a premium brand ice-cream producer who had announced plans to enter the superpremium ice cream market.¹

A monopolist will distribute a rivals product only if it increases its own profits. This would happen if and only if the entrant creates additional value, either through lower production costs, higher product quality, or valuable product differentiation. Absent additional value creation, a monopolist would refuse to deal with the entrant. We argue that this logic breaks down when there is more than one incumbent and the entrant, by

strategically sinking its cost before entering into contract negotiation with the incumbents, can enter even when entry destroys value.

Specifically, we examine whether vertically integrated Cournot duopoly incumbents will exclude or accommodate an entrant. We assume the entrant is no more efficient than the incumbents and does not benefit from product differentiation of any kind, so a monopolist would credibly refuse to deal with it. However we show that when more than one incumbent has access to distribution then the entrant benefits from a negative externality created by its commitment to sell additional output: each incumbent realizes that it will be harmed if the other incumbent acquires (and subsequently sells) the entrant's output.²

The intuition is easiest to see when incumbents naively ignore the threat of entry. Suppose the incumbents produce the Cournot duopoly output and the entrant subsequently makes a commitment to sell a small amount of additional output. By construction, the incumbents are indifferent with respect to a small increase in their own production, however they are willing to pay up to the market price for the entrant's output! Since the entrant's output will be sold by one firm or the other *and each firm will distribute the extra output if it acquires it*, each firm correctly ignores the impact of the extra output on the price of its inframarginal production (the price will decrease by the same amount regardless of who buys the entrant's output) and realizes that the marginal revenue of the entrant's extra output is equal to the market price.

However, the Cournot production levels are not optimal for the incumbents. In particular, we show that when their costs are sufficiently small, the incumbents will deter entry by symmetrically expanding their output. For a larger range of costs, entry is still

deterred, but one incumbent produces more than the other. For an even larger range of costs, entry is accommodated and the firms' outputs are the same as they would have been if the entrant had independent access to distribution. There is also an intermediate range of costs between the second and third ranges in which both entry deterrence and entry accommodation equilibria exist.

Since the examples we offered include many industries in which products are differentiated, incumbents in these industries might choose to preempt entry through product line expansions rather than capacity expansions. So the entry deterrence strategies might be either capacity expansion or product line extensions.

The U.S. beer industry is a particularly complex and interesting potential example. Although they are not vertically integrated, Anheuser Busch and the Philip Morris Company's Miller Brewing Company dominate the U.S. beer industry and have been able to gain tight control of their distribution networks through exclusive contracts, dubbed "100% share of mind" contracts by Anheuser Busch Chairman August Busch III. Anheuser Busch has aggressively entered the specialty beer segment, a move that has been closely linked to the exclusion of many independents from Anheuser Busch's distribution network and a significant loss in overall market share of independent specialty beers.³ We believe our paper helps to explain Anheuser Busch's decisions, but note that our analysis is limited by the extent to which Anheuser Busch's relationships with its distributors are analogous to vertical integration and product proliferation is analogous to capacity expansion. Note that U.S. beer makers also distribute many foreign beers, even while introducing brands designed to compete head-to-head with these imports.

It is interesting to note that the incumbents are harmed by their inability to commit not to deal with the entrant. The incumbent players compete for the right to distribute the entrant's output, even though the new output will reduce the margin on their existing products. The incumbents would be better off if they could collectively refuse to deal with the entrant or could otherwise restrict the entrant's access to distribution. Furthermore, the incumbents also fail to coordinate their entry deterrence strategies. Interestingly, this leads to over-deterrence. For some parameter values, entry deterrence occurs even though the incumbents' joint profits would have been higher if they accommodated entry. This happens because the entry deterrence equilibrium is asymmetric and the larger firm harms its rival when expands its output to deter entry.

This analysis also highlights the role of strategic commitments in auctions with externalities. If the entrant tried to sell output to the incumbents before sinking its costs it would be unable to do so (even if the incumbents engaged in no entry deterrence). However if the entrant could commit to any selling mechanism (much of the literature on optimal contracts with externalities is summarized in Segal, 1999) there would be no role for strategic commitments such as sinking costs. We believe, however, that the ability of a potential entrant to commit to a selling mechanism is unrealistic in this context. We instead explore a more realistic selling mechanism, a simple auction, and find that the entrant is strictly better off if it sinks its costs before the auction, even though this commits it to inefficient ex post trade (both on and off the equilibrium path). This point is analyzed in more detail in Section 8.

Our paper contributes to the game theoretic literature on entry deterrence begun by Spence (1977) and Dixit (1980). They show that by building extra capacity,

incumbents can credibly commit to respond aggressively to new entry. Because the cost of capacity is sunk, the threat to lower price if entry occurs is credible. In our paper, incumbents make Spence-Dixit style capacity commitments *even though the entrant cannot sell its output directly to consumers*. The incumbents need to make capacity commitments in order to make it credible that neither firm will buy the entrant's capacity.

Gilbert and Vives (1986) and Waldman (1987) extend this literature to consider multiple incumbents. Their papers examine the hypothesis that non-cooperative oligopolists free ride on their rivals entry deterrence with the result that total entry deterrence is diminished relative to cooperating firms. Gilbert and Vives argue against this hypothesis citing other offsetting effects while Waldman argues that in the presence of uncertainty free riding will occur. While our model is quite different and includes no uncertainty, we demonstrate that under some conditions our incumbents would be strictly better off if they agreed to accommodate entry.

Rasmusen (1988) extends the Spence-Dixit models by allowing the incumbent to "buy out" the vertically-integrated entrant. He shows that the Spence-Dixit result is only valid if the incumbent can commit not to acquire the entrant. In his model the incumbent always finds it profitable to buy the entrant when entry occurs (entry doesn't occur unless a buyout is going to occur). So the entrant's decision to enter depends not on the entrant's expected profits from producing (though it must be credible for the entrant to stay in the market after sinking its entry costs if it is not acquired) but on how much the incumbent is willing to pay to acquire him. And this in turn depends on how big an impact the entrant has on the incumbent's profits. But Rasmusen's model is fundamentally different from ours because the entrant's outside option is to sell his

output himself. Rasmusen argues that entry for buyout is less likely in imperfectly competitive markets because buyout becomes a public good. In contrast, in our model the entrant cannot harm a monopoly incumbent, so buyout is more likely in imperfectly competitive markets.

Our paper is also related to the literature on the persistence of monopoly. Gilbert and Newbery (1982) showed that new capacity is more valuable to an incumbent than it is to a new entrant, so monopolists tend to persist. In our model, duopoly in distribution persists by construction. The entrant's value of capacity is only equal to what he can get selling it to the incumbents. Nevertheless we show that for sufficiently low capacity cost the incumbents overproduce to preempt entry in production as well. Krishna (1993) extends Gilbert and Newbery to the case where new capacity becomes available in sequentially. She shows the persistence of monopoly depends on the timing of the arrival of new capacity. See also Kamien and Zang (1990), Reinganum (1983), Lewis (1983), and Chen (1999).

2. The Model

Three firms, A, B and C, produce output at constant marginal cost $k > 0$. Firms A and B are the "incumbents" and have access to distribution which is assumed to be costless, and Firm C is an "entrant" who cannot independently distribute its output. For simplicity we assume the market demand is $p(z) = 1 - z$ where z is the total amount of output that is distributed to the market by Firms A and B.⁴

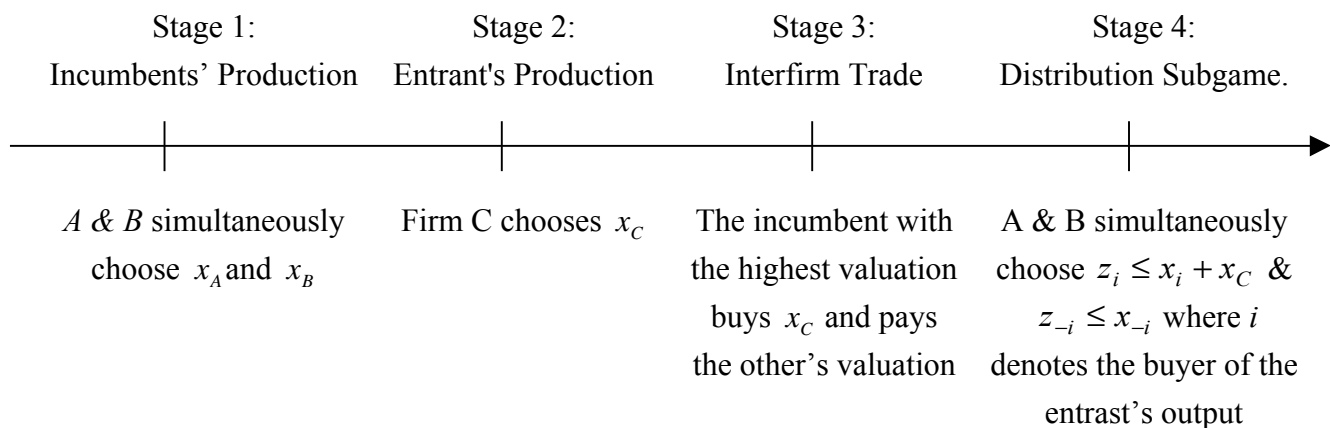


Figure 1: The Timing

The timing of the game is represented in Figure 1. First, the incumbents, Firms A and B, decide simultaneously and non-cooperatively how much to produce, x_A and x_B . Next the entrant, Firm C, decides how much to produce, x_C . Then the entrant sell this block of output to Firms A and B. We assume that the incumbent with the highest valuation acquires the entrant's output and pays Firm C an amount equal to the valuation of the other firm (this is the unique outcome of a first-price auction). When Firm i acquires the entrant's output, the incumbents' interim output endowments are $y_i = x_i + x_C$ and $y_{-i} = x_{-i}$. Finally, each incumbent decides simultaneously and non-cooperatively how much to distribute to the final market, $z_i \leq y_i$, and the equilibrium market price is determined. All actions taken by the players in early stages are observable in subsequent stages.

The incumbent's unconstrained best-response functions are an important piece of notation used in the paper. We use $x_i = R(x_{-i}, c)$ to denote Firm i 's optimal sales as a

function of its expectation of Firm j 's sales and of Firm i 's own cost of sales c (in our analysis c will take on the values k and 0), which is defined as:

$$x_i = R(x_{-i}, c) = \arg \max_{x_i} x_i p(x_i + x_{-i}) - cx_i.$$

Under the assumption that demand is linear this function simplifies to

$$x_i = R(x_{-i}, c) = \frac{1 - x_{-i} - c}{2}.$$

A second useful piece of notation we use is the incumbent's best response function when the entrant enters and produces *as if the entrant could distribute its own output (and the incumbents sell all that they produce)*. So the entrant produces $R(x_A + x_B, k)$ after the incumbents have made their output decision. The incumbents are Cournot oligopolists with respect to each other, but Stackelberg leaders with respect to the entrant. We let $r(x_j)$ denote each incumbent's optimal production as a function of what they expect the other incumbent, x_j , to produce. So

$$r(x_j) = \arg \max_{x_i} x_i p(x_i + x_j + R(x_i + x_j, k)).$$

We also define

$$\hat{x} = R(\hat{x}, 0) \tag{1}$$

and

$$\tilde{x} = \arg \max_{x_A} x_A p(x_A + R(x_A, 0)) - kx_A, \tag{2}$$

so \hat{x} is a Cournot duopolist's output when both firms have zero costs, and \tilde{x} is the Stackelberg leader's output when the leader has cost k and the follower has zero cost.

Lastly, we let $TR_i(z_A, z_B) = z_i p(z_A + z_B)$ denote Firm i 's revenue as a function of each firm's distribution.

Stackelberg Model with Two Leaders and One Follower

In the game in which Firms A and B choose their production in Stage 1, and Firm C chooses its production in Stage 2, and then, in Stage 3, all firms distribute whatever they produce, the unique equilibrium for linear demand $p(x) = 1 - x$ is

$$\left\{ \frac{1-k}{3}, \frac{1-k}{3}, \frac{1-k}{6} \right\},$$

so the total industry output is $x_A + x_B + x_C = 5(1-k)/6$, and the market price is $p = (1-k)/6$ (see the Appendix for a derivation).

Discussion

The results in this paper hinge on there being more than one incumbent. In equilibrium, a monopolist can deter entry by simply producing the monopoly output itself. By producing at the monopoly level, a single incumbent becomes unwilling to pay more than marginal cost k for additional capacity. So an entrant could not profitably enter (unless he had a cost advantage). If the entrant sinks the cost of production then the incumbent monopolist would buy it, but never at a price that justified entry. The same logic does not apply when the incumbents were duopolists. If the monopolist does not buy the entrant's output, then that output never reaches the market, however duopolists realize that if they do not buy the entrant's output their rival may. Hence, if the entrant sinks his costs he is able to exploit a negative externality between the incumbents.

Suppose, as we did in the introduction, that the incumbents naively produce as if there were no threat of entry, so Firms A and B produce $x^* = R(x^*, k)$ in Stage 1. Now suppose the entrant produces a small amount Δ . The most each incumbent is willing to pay for Δ is its profit when it buys (and distributes) Δ less its profit when its rival buys

(and distributes) Δ , or $(x^* + \Delta)p(2x^* + \Delta) - x^*p(2x^* + \Delta)$, which is equal to $\Delta p(2x^* + \Delta)$. So the unit price of the entrant's output will be bid up to the market price for the final good and the entrant's profits will be positive. The negative externality between the two incumbents allows the entrant to extract the full market price for its output. (Note that when Δ is not small it no longer follows that each firm will distribute all of the entrant's output.)

Notice that the incumbents would be better off in this example if they could collectively refuse to deal with the entrant. First, the entrant has forced them to sell beyond their Cournot duopoly output, x^* . Second, the entrant has induced them to pay a premium for this output, $p(2x^* + \Delta) > k$. If the two incumbents could collude and jointly commit not to participate in the auction, then the entrant would have no outlet for its output and the two incumbents would be jointly better off. In fact, both incumbents would be better off if even *just one* of them made an ex ante unilateral commitment not to buy the entrant's output: with a single remaining buyer, the entrant will not receive the price necessary to cover his sunk costs.⁵

3. The Distribution Subgame

Suppose that Firms A and B have output endowments of y_A and y_B at the beginning of the Stage 4 distribution subgame. The incumbents can distribute up to, but not more than, these endowments. How much of their existing endowments will the incumbents choose to distribute to the final market?

Let $z_A(y_A, y_B)$ and $z_B(y_A, y_B)$ denote the Firms' equilibrium distribution decision given y_A and y_B . Given its beliefs about its rival's distribution, Firm A (and by analogy

Firm B) will choose its distribution to maximize its continuation profits, $z_A p(z_A + z_B)$, subject to $z_A \leq y_A$, so $z_A = \min\{R(z_B, 0), y_A\}$. We call $R(z_A, 0)$ and $R(z_B, 0)$ the *distribution best-response functions* or the *zero-cost best-response functions*; they give the profit maximizing level of distribution when the firm has unlimited output. So $\min\{R(z_{-i}, 0), y_i\}$ is Firm i 's constrained best response function. The following Lemma characterizes the incentive to withhold output from the market.

Lemma 1: Given interim endowments y_A and y_B Firm A and B's distribution, $z_A(y_A, y_B)$ and $z_B(y_A, y_B)$, is the unique solution to $z_A = \min\{R(z_B, 0), y_A\}$ and $z_B = \min\{R(z_A, 0), y_B\}$.

Figure 2 graphically depicts the mappings $z_A(y_A, y_B)$ and $z_B(y_A, y_B)$ from the firms' output endowments, y_A and y_B , to their equilibrium distribution quantities, z_A and z_B . The dotted lines represent the production best-response curves. Consider, for example, the interim endowment $y_A = y_B = x^*$. These are the myopic Cournot quantities discussed informally in the last section. Since each firm's endowment is less than their distribution (or zero cost) best-response output it follows that $z_A = y_A$ and $z_B = y_B$: the incumbents will distribute all of their output.

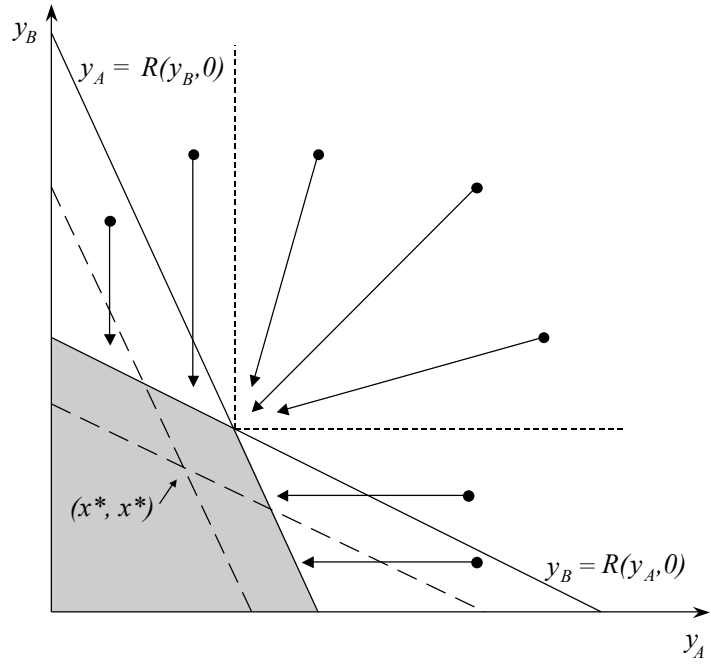


Figure 2: The Distribution Game

More generally, when the firms' endowments define a point in shaded area of Figure 2, the marginal revenue from selling a unit of output is strictly positive for both firms. If only Firm A's interim endowment is greater than its distribution (or zero cost) best-response then in the distribution stage only Firm A will withhold output from the market: $z_A = R(y_B, 0)$ and $z_B = y_B$. Similarly, if only Firm B's interim endowment is greater than its distribution (or zero cost) best-response, but not Firm A's, then only Firm B will withhold output from the market: $z_B = R(y_A, 0)$ and $z_A = y_A$. When both firms' interim endowments are greater than their distribution (or zero-cost) best-responses then one or both will withhold output from the market, but the firm with more output will always withhold more. This insight – that the larger firm has a weakly greater incentive to withhold output – is important for understanding the Stage 3 auction subgame.

4. The Interfirm Trade Subgame

After the entrant produces x_C , it sells its output as a block to the incumbents.

Assumption: We assume that the incumbent who values the entrant's output more acquires the output and that the price paid is equal to the valuation of the other incumbent.

This is the unique outcome of a first-price auction and an equilibrium outcome of a second-price auction. However it is also the equilibrium outcome of a variety of multi-player bargaining games. In particular, the entrant might announce that he is willing to produce x_C units, invite sealed bid offers, and then accept the highest profitable offer.⁶

The next Lemma establishes that the firm that produced more in Stage one places weakly higher value on the entrant's output than the firm who produced less.

Lemma 2: Given incumbents' production, x_A and x_B , and a block of production x_C available at auction, the incumbent that produced the larger amount in Stage one values the entrant's production, x_C , at least as much as the incumbent that produced less output.

Proof: See Appendix.

If the two incumbents were "required" to distribute the same amount of this additional output, then each firm would be willing to pay the same amount for the entrant's output.⁷ But the larger firm has more to lose from a decrease in the market price, so the incentive to withhold output from the market is always weakly greater for

the larger firm. Therefore, by revealed preference, the larger firm is always willing to pay at least as much as the smaller firm for a block of capacity.

The larger firm may place the same value on the entrant's output as the smaller firm. We will nevertheless *assume* that when the incumbents are asymmetric, the larger firm always takes possession of the entrant's output and pays the smaller firm's willingness to pay. This assumption simplifies the exposition but is not required for the results.

5. The Entrant's Production Decision

The entrant's revenue is the profit the incumbent with smaller output earns when it acquires the entrant's output less the profit the smaller incumbent earns when the larger incumbent acquires the entrant's output. This, together with Lemmas 1 and 2, give us valuable insights into the entrant's output decision.

Lemma 3: Entry will occur if and only if (x_A, x_B) satisfies one of the following conditions:

- 1) $x_A < R(x_B, 0)$, $x_B < R(x_A, 0)$, and $p(x_A + x_B) > k$ (Region A);
- 2) $x_A \geq R(x_B, 0)$ and $x_B < \min\{\tilde{x}, \hat{x}\}$ (Region D_1);
- 3) $x_B \geq R(x_A, 0)$ and $x_A < \min\{\tilde{x}, \hat{x}\}$ (Region D_2),

where \tilde{x} and \hat{x} are defined in (1) and (2) above.

Proof: See Lemma 3A in the Appendix.

Lemma 3 characterizes only whether or not entry will take place. In Lemma 3A in the Appendix, we offer a complete description of Firm C's profit function for all x_A ,

x_B , and x_C , which we use to characterize the size of Firm C's entry decision for all x_A , x_B , and k . So Lemma 3A is more general than Lemma 3.

Figure 3 depicts the three regions described in Lemma 3 for the case where costs are low enough to imply $\tilde{x} \geq \hat{x}$. Figure 4 depicts the three regions described in Lemma 3 when costs are high enough to imply $\tilde{x} < \hat{x}$, but still low enough to imply $p(\hat{x} + \hat{x}) > k$. Figure 5 depicts the three regions when costs are high enough that the condition $p(x_A + x_B) > k$ binds in Region A.

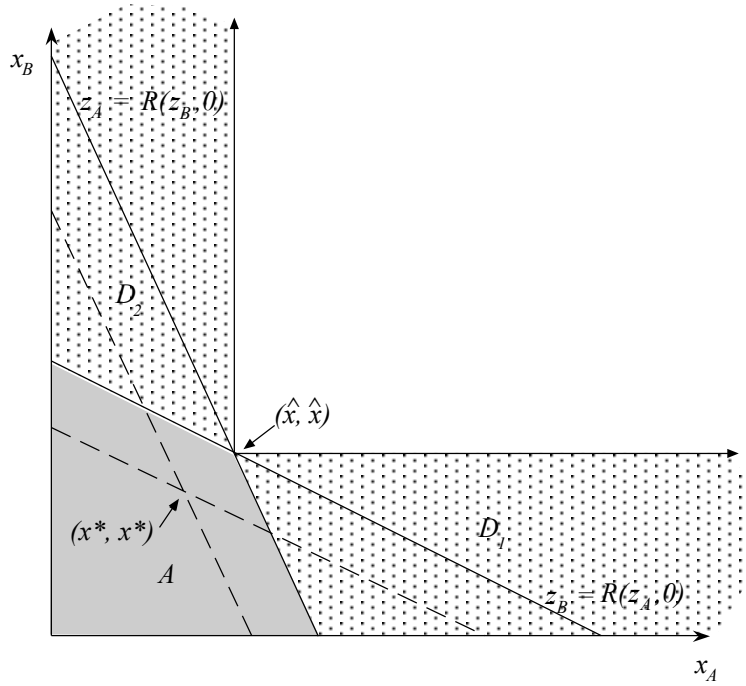


Figure 3: Entry Decision When Costs are Low ($\tilde{x} \geq \hat{x}$ and $p(2\hat{x}) > k$)

Lemma 3 can be described intuitively. If the incumbents' production levels, x_A and x_B , are both greater than or equal to \hat{x} , where \hat{x} is defined by $\hat{x} = R(\hat{x}, 0)$, then Firm C cannot profitably enter because $\partial TR_i(x_A, x_B)/dz_i < 0$ for both incumbents. By Lemma 1, neither firm would distribute any of the entrant's output, so neither firm is willing to

pay anything for it. Neither firm receives any direct value from the additional output and, just as important, *there is no value in keeping the output away from the rival incumbent.*

It follows that if $x_A \geq \hat{x}$ and $x_B \geq \hat{x}$ then Firm C will not produce.

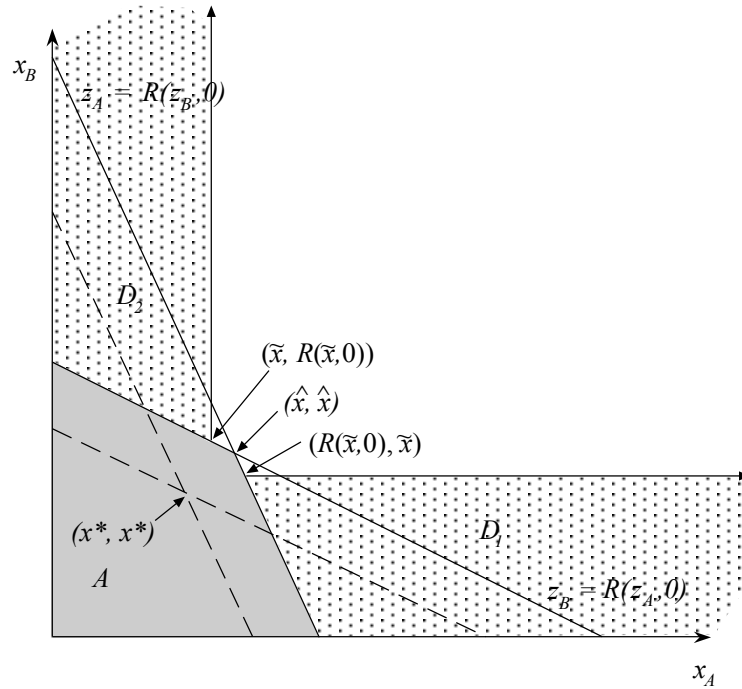


Figure 4: Entry Decision When Costs are Higher ($\tilde{x} < \hat{x}$ and $p(2\hat{x}) > k$)

Second, it is easy to see that if the incumbent's production levels, x_A and x_B , lie within Region A of Figures 3, 4, and 5, then Firm C will produce. In this region both firms would distribute additional output if they had it. For Δ sufficiently small, if Firm C commits to auction Δ additional units of capacity, the additional capacity will be both purchased and distributed, and the unit price will be the market price, which is greater than the k : $p(x_A + x_B + \Delta) > k$. Firm A's revenues are $(x_A + \Delta)p(x_A + x_B + \Delta)$ if it acquires the entrant's output and $x_A p(x_A + x_B + \Delta)$ if it does not, so Firm A's valuation is

$\Delta p(x_A + x_B + \Delta)$ for the entrant's capacity. By the same argument, $\Delta p(x_A + x_B + \Delta)$ is the amount Firm B is willing to pay for entrant's output. So the entrant can profitably produce.

The remaining regions are discussed in the Appendix in the proof of Lemma 3A.

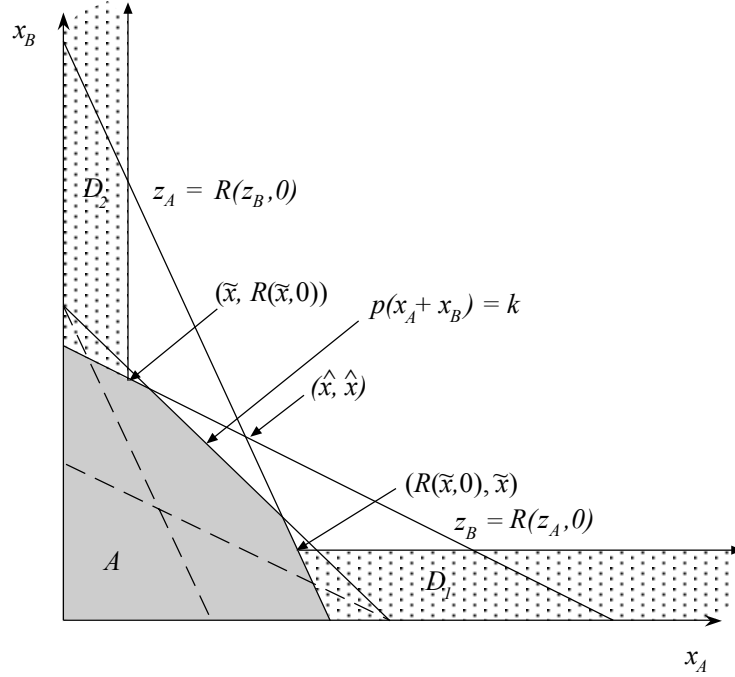


Figure 5: Entry Decision When Costs are Highest ($\tilde{x} < \hat{x}$ and $p(2\hat{x}) < k$)

6. Incumbents' Production Decision

In Section 2, we demonstrated that if the incumbents ignored the threat of entry, then the entrant would be able to produce some additional output and sell it to the incumbents. In this section we examine the incumbents' optimal production decisions. We show that when costs are sufficiently small the incumbents will deter entry by preemptively increasing their output and if the costs are sufficiently large the firms will accommodate entry. There is also an intermediate range in which both entry deterrence and entry accommodation are equilibria. Finally, we also show that when their costs are

sufficiently high the incumbents' production in the entry deterrence equilibrium is asymmetric.

We begin with the case where the cost of capacity is small. When the marginal cost of production is less than $1/6$, there is a symmetric subgame perfect Nash equilibrium in which entry is deterred. Each incumbent produces $\hat{x} = R(\hat{x}, 0)$, as if they had zero cost. Since neither incumbent values additional output, the negative externality is eliminated. So neither firm is willing to pay for C's output and C will not produce. This is shown in the following proposition (uniqueness is shown in Proposition 3).

Proposition 1: A symmetric, pure strategy, subgame perfect Nash equilibrium of the form $\{\hat{x}, \hat{x}, 0\}$ where $\hat{x} = R(\hat{x}, 0)$ exists if and only if $k \leq 1/6$. With the linear demand $\hat{x} = 1/3$, so the equilibrium is $\{1/3, 1/3, 0\}$. The incumbents each sell all that they produce and production by the entrant is deterred.

Proof: See Appendix.

Clearly neither incumbent will produce more than \hat{x} , but why isn't it profitable to produce less than \hat{x} ? Suppose Firm A produced $x_A < \hat{x}$ while Firm B produced $x_B = \hat{x}$. By producing less, Firm A changes both firms' incentives to use the entrant's additional output: since $x + R(x, 0)$ is increasing in x and $R(x, 0)$ is decreasing in x we know that $R(\hat{x}, 0) - x_A > R(x_A, 0) - \hat{x} > 0$. Now neither firm's output is at their distribution best-response function, so the entrant will produce (see Lemma 3) – each firm realizes the other will purchase and distribute Firm C's output if they don't. The negative externality is reintroduced. We show in the proof of Proposition 1 (in the appendix) that for $k < 1/6$

both firms prefer to pay for the output themselves in order to avoid paying $p(x_A + x_B + \Delta)$ for output from the entrant.

When the costs, k , are slightly larger than $1/6$, then if each firm thought the other was producing \hat{x} they would have a unilateral incentive to produce less than \hat{x} . In this case there is no symmetric equilibrium in which entry is deterred, however entry can still be deterred in a non-cooperative equilibrium when the strategies of Firms A and B are asymmetric.

Proposition 2: Asymmetric, pure strategy, subgame perfect Nash equilibria of the form $\{\tilde{x}, R(\tilde{x}, 0), 0\}$, or equivalently $\{R(\tilde{x}, 0), \tilde{x}, 0\}$, where

$\tilde{x} = \arg \max_{x_A} x_A p(x_A + R(x_A, 0)) - kx_A$, exist if and only if $k \in (1/6, 1/(2\sqrt{2})]$. With linear

demand the equilibria are

$$\left\{ \frac{1-2k}{2}, \frac{1+2k}{4}, 0 \right\} \text{ and } \left\{ \frac{1+2k}{4}, \frac{1-2k}{2}, 0 \right\}.$$

Proof: See Appendix.

In the first of the two equilibria Firm A acts as a Stackelberg leader yet is the smaller firm! This paradoxical result occurs because Firm A acts as a leader with costs k , while Firm B acts as a follower with zero costs. Firm C produces zero, and Firm A and B's equilibrium sales are equal to their production.

Firm B deters entry and allow Firm A to “free ride”.⁸ However in this equilibrium Firm B produces more than Firm A, so it earns strictly greater profits. Each firm prefers the equilibrium in which they are the Stackelberg follower. Note also that because Firm B produces more it internalizes more of the benefits of entry deterrence.

Proposition 3: The equilibrium described in Proposition 1 and the two equilibria described in Proposition 2 are the unique entry deterring equilibria of the game.

Proof: See Appendix.

When costs are larger than $1/(2\sqrt{2})$ entry is accommodated. Firms A and B would rather buy all the output that the entrant produces than produce enough output to deter entry.

Proposition 4: Let Firm i 's best response to Firm $j \neq i$ be denoted $r(x_j) = \arg \max_{x_i} x_i p(x_i + x_j + R(x_i + x_j, k))$. A pure strategy, entry accommodating, subgame perfect Nash equilibrium of the form $\{\bar{x}, \bar{x}, R(\bar{x} + \bar{x}, k)\}$, where $\bar{x} = r(\bar{x})$, exists if and only if $k \geq .261$. For linear demand the equilibrium production is

$$\{x_A, x_B, x_C\} = \left\{ \frac{1-k}{3}, \frac{1-k}{3}, \frac{1-k}{6} \right\},$$

so the equilibrium is the same as the equilibrium of the Stackelberg game with two leaders and one follower. No other pure strategy equilibrium exists in which entry is accommodated.

Proof: See Appendix.

Proposition 4 shows that when production costs are sufficiently large, an equilibrium exists in which entry is accommodated. Moreover the entry-accommodation equilibria, if it exists, has the same production as in the game where the entrant does have access to distribution, even though the entrant must distributed its output through one of the incumbents.

7. Welfare

For k greater than $1/4$, the output in the asymmetric entry deterrence equilibrium is greater than the output in the accommodated entry equilibrium. This means that for the range of k in which both the asymmetric entry deterrence equilibrium and the accommodated entry equilibrium exist, i.e., $[\.261204, 1/(2\sqrt{2})]$, welfare is maximized when entry is deterred. Nevertheless for k less than $1/4$, entry deterrence reduces welfare (relative to a requirement that A and B distribute the entrant's output).

8. The Role of Commitment

In this section we show that the entrant benefits from being able to sink its cost of production *before* selling its output to the incumbent firms. Unlike other work on contracting with externalities, we demonstrate a role for making strategic commitments such as sinking costs. For many realistic interim selling mechanisms if the entrant tried to contract with the entrants *before sinking its costs*, then entry would not occur. For example, a mechanism that satisfies our assumptions is one in which the entrant announces a quantity, each incumbent makes alternating offers until both firms decide not to increase their offers any more, and then the entrant chooses whether or not to accept the highest offer on the table (and uses a coin toss if two offers are the same). When the entrant is unable to sink its costs, then the incumbents can produce their Cournot duopoly outputs and entry is deterred; regardless of what quantity the entrant announces, the incumbents will both drop out of the bidding early enough that the unit price is not bid up to k , and so no trade with the entrant will take place.⁹ Sinking costs commits the entrant to trade even when its revenues do not cover sunk costs, by relaxing

the ex post profitability (IR) constraint, which is essential to creating and exploiting the externality.

When the entrant can commit to a first price auction, sinking costs may not be necessary. By committing to trade with the auction winner, *even if the auction price is less than his cost*, the entrant accomplishes the same thing as he does by sinking costs. However, if the entrant can make such powerful commitments, then the entrant can probably commit to even more general mechanisms that earn even higher profits. Segal (1999) derives the optimal direct mechanism for a principal in the presence of contracting externalities. In our model, if $x_A \leq R(x_B, 0)$ and $x_B \leq R(x_A, 0)$, the entrant's optimal direct mechanism is to give Firm A $R(x_B, 0) - x_A$ units when Firm A "accepts" and Firm B "rejects" and to give Firm B of $R(x_A, 0) - x_B$ units when Firm A "rejects" and Firm B "accepts", and to give both firm's zero units (assuming $x_A + x_B > x_M$) when both "accept." Trade is efficient when both firms accept, but inefficient when either incumbent deviates from the Nash equilibrium. The entrant's profits are significantly higher because it is able to lower the outside options of the incumbents maximally without introducing any trade distortion on the equilibrium path.

We find it much more plausible that the entrant is constrained to use mechanisms in which it reserves the option to reject any unprofitable contract. In this case, sinking costs can be useful as a strategic commitment because it relaxes the ex post profitability constraint.

An important area for future work is to look at the class of optimal, renegotiation-proof mechanisms, and examine the role for strategic commitments when these mechanisms are used.

9. Alternative Timings

We have considered a model in which the two incumbents produce before the entrant. An equally reasonable assumption would have been that all the firms produce simultaneously. In this case our discussion in Section 2 of why the duopoly output is *not* an equilibrium is unchanged. If Firms A and B are ignoring the threat of entry Firm C's best response is the same whether he moves simultaneously or subsequently. However, the entry deterrence equilibria we describe are not equilibria of the simultaneous move game. If Firms A and B both believe Firm C will not enter, their best response is to produce the duopoly output.

We also could have allowed the entrant to produce first. This might be reasonable if the potential entrant were a new product innovator, but had to distribute its product through existing firms and existing firms could choose to imitate and produce themselves. In this case it also matters whether the incumbents buy the entrant's output before or after they produce. If they produce before the auction, it is clear again that the incumbents cannot deter entry by producing the duopoly output. If they produce after the auction, then it is clear that entrant will produce at least the Stackelberg output since (even ignoring the externality created) the entrant can sell the role of Stackelberg leader to the two firms. So the duopoly outcome is not an equilibrium outcome.

10. Conclusion

We present a model in which incumbent firms invest in preemptive production (or capacity expansion) to deter entry even though the entrant cannot sell its produce without distributing it through an incumbent and the entrant cannot produce more efficiently than the incumbents. The entrant exploits a negative externality that is created when it sinks

its cost of production – once an incumbent knows that if it loses the auction its rival will win and distribute the entrant’s output, the willingness to pay for the entrant’s output increases to the market price.

In our model the incumbents have access to distribution but the entrant does not, however this assumption can be motivated in different ways. The simplest motivation is economies of scale in distribution (perhaps spread over multiple products) that blockade the entrant from the distribution market. Another is that the incumbents have brand names that solve an asymmetric information quality-assurance problem with consumers, so the entrant cannot profitably sell to consumers unless it sells the product under the incumbents’ brand name. If we assume additionally that the incumbents can only monitor the entrant’s quality when they control the distribution of the entrant’s product, then the entrant’s only option is to distribute through the incumbents.

Our results have some implications for public policy. First, our results imply that incumbents can generate significant profit increases through agreements not to deal with entrants. Such agreements are already per se illegal. In the telecommunications industry incumbents have historically been required to provide universal access to rival firms. Our results suggest that access requirements could increase welfare when costs are small, but decrease welfare when costs are large. This is because access requirements would cause the incumbents to stop engaging in entry deterrence, which is socially preferred to entry.

Our analysis was restricted to linear demand functions. This assumption was made in order to calculate the profit change resulting from a large deviation in output which was necessary because our model has discontinuous best response functions. We

never explicitly used the linearity of demand. We believe that our analysis can easily be extended to other functional forms, however we are not sure how easily it can be extended to more general demand.

Two important extensions of the model would alter our results. First, if there are more potential entrants, then the incumbents will invest more in entry deterrence (the cost of entry is higher when the entrants produce more). It is likely that the range of costs for which entry accommodation equilibria exist would shrink and the range of costs for which asymmetric entry deterrence equilibria exist would grow (the range of costs for which the symmetric entry deterrence equilibria exists stays the same).

Second, if the incumbents had positive marginal costs of distribution it would be much easier to deter entry. More generally as the proportion of costs that are sunk in the production stage decreases, the importance of the entrant's commitment declines.

Appendix

Derivation of Stackelberg Model with Two Leaders and one follower

As a follower Firm C produces $R(x_A + x_B, k)$. As a Stackelberg leader Firm A produces

$$\arg \max_{x_A} p(x_A + x_B + R(x_A + x_B, k)) - kx_A.$$

Similarly Firm B produces.

$$\arg \max_{x_B} p(x_A + x_B + R(x_A + x_B, k)) - kx_B,$$

For linear demand Firm i 's first order condition, $i \in \{A, B\}$ is

$$\frac{1 - 2x_i - x_{-i} + k}{2} - k = 0$$

so $x_A = x_B = (1 - k)/3$ and $x_C = R(2(1 - k)/3, k) = (1 - k)/6$. So the total industry output is $x_A + x_B + x_C = 5(1 - k)/6$, and the market price is $p = (1 - k)/6$.

Proof of Lemma 2:

Without loss of generality suppose $x_A \geq x_B$. Firm A's valuation for entrant's output, x_C , is

$$TR_A(z_A(x_A + x_C, x_B), z_B(x_A + x_C, x_B)) - TR_A(z_A(x_A, x_B + x_C), z_B(x_A, x_B + x_C)),$$

the total revenue earned if he acquires the entrant's output less the total revenue earned if his rival acquires the entrant's output. Similarly, Firm B's willingness to pay is

$$TR_B(z_A(x_A, x_B + x_C), z_B(x_A, x_B + x_C)) - TR_B(z_A(x_A + x_C, x_B), z_B(x_A + x_C, x_B)).$$

So Firm A will acquire the entrant's output as long as the total industry revenues are weakly higher when Firm A acquires x_C than when Firm B acquires x_C :

$$TR_A(z_A(x_A + x_C, x_B), z_B(x_A + x_C, x_B)) + TR_B(z_A(x_A + x_C, x_B), z_B(x_A + x_C, x_B)) \geq \\ TR_A(z_A(x_A, x_B + x_C), z_B(x_A, x_B + x_C)) + TR_B(z_A(x_A, x_B + x_C), z_B(x_A, x_B + x_C))$$

We can rewrite this expression as:

$$\int_0^{x_C} \left[\frac{\partial TR_A(z_A(x_A + s, x_B), z_B(x_A + s, x_B))}{\partial x_A} + \frac{\partial TR_B(z_A(x_A + s, x_B), z_B(x_A + s, x_B))}{\partial x_A} \right] ds \geq \\ \int_0^{x_C} \left[\frac{\partial TR_A(z_A(x_A, x_B + s), z_B(x_A, x_B + s))}{\partial x_B} + \frac{\partial TR_B(z_A(x_A, x_B + s), z_B(x_A, x_B + s))}{\partial x_B} \right] ds$$

and a sufficient condition for this to be true is that

$$\frac{\partial TR_A(z_A(x_A + s, x_B), z_B(x_A + s, x_B))}{\partial x_A} + \frac{\partial TR_B(z_A(x_A + s, x_B), z_B(x_A + s, x_B))}{\partial x_A} \geq \\ \frac{\partial TR_A(z_A(x_A, x_B + s), z_B(x_A, x_B + s))}{\partial x_B} + \frac{\partial TR_B(z_A(x_A, x_B + s), z_B(x_A, x_B + s))}{\partial x_B} \quad (*)$$

for all s .

Since $x + R(x, 0)$ is increasing in x , it follows that $x_A + R(x_A, 0) \geq x_B + R(x_B, 0)$

and $R(x_B, 0) - x_A \leq R(x_A, 0) - x_B$. We show that (*) holds for all s by considering the

following three cases separately: 1) $R(x_B, 0) - x_A \leq R(x_A, 0) - x_B < s$,

2) $s < R(x_B, 0) - x_A \leq R(x_A, 0) - x_B$, and

3) $R(x_B, 0) - x_A \leq s \leq R(x_A, 0) - x_B$.

Case 1. Suppose that $R(x_B, 0) - x_A \leq R(x_A, 0) - x_B < s$. Then

$\partial z_A / \partial x_A = \partial z_B / \partial x_B = 0$ so

$$\frac{\partial TR_A(z_A(x_A + s, x_B), z_B(x_A + s, x_B))}{\partial x_A} + \frac{\partial TR_B(z_A(x_A + s, x_B), z_B(x_A + s, x_B))}{\partial x_A} = 0$$

and

$$\frac{\partial TR_A(z_A(x_A, x_B + s), z_B(x_A, x_B + s))}{\partial x_B} + \frac{\partial TR_B(z_A(x_A, x_B + s), z_B(x_A, x_B + s))}{\partial x_B} = 0.$$

Hence both sides of (*) are zero and the inequality holds.

Case 2. Next, suppose that $s < R(x_B, 0) - x_A \leq R(x_A, 0) - x_B$. Then

$$\begin{aligned} & \frac{\partial TR_A(z_A(x_A + s, x_B), z_B(x_A + s, x_B))}{\partial x_A} + \frac{\partial TR_B(z_A(x_A + s, x_B), z_B(x_A + s, x_B))}{\partial x_A} \\ &= (x_A + x_B + s)p(x_A + x_B + s) \end{aligned}$$

and

$$\begin{aligned} & \frac{\partial TR_A(z_A(x_A, x_B + s), z_B(x_A, x_B + s))}{\partial x_B} + \frac{\partial TR_B(z_A(x_A, x_B + s), z_B(x_A, x_B + s))}{\partial x_B} \\ &= (x_A + x_B + s)p(x_A + x_B + s) \end{aligned}$$

so both sides of (*) are equal.3

Case 3. Finally, suppose $R(x_B, 0) - x_A \leq s \leq R(x_A, 0) - x_B$. Then

$z_B(x_A + s, x_B) = x_B$ and $z_A(x_A + s, x_B) = R(x_B, 0)$, so the left-hand side of (*) is zero: Firm

A would distribute s , but Firm B, on the other hand would not. Also

$z_B(x_A, x_B + s) = x_B + s$ and $z_A(x_A, x_B + s) = R(x_B + s, 0)$ so the right hand side of (*) is:

$$\begin{aligned} & \frac{dTR_A(R(x_B + s, 0), x_B + s)}{dx_B} + \frac{dTR_B(R(x_B + s, 0), x_B + s)}{dx_B} \\ &= \frac{d(R(x_B + s, 0) + x_B + s)p(R(x_B + s, 0) + x_B + s)}{dx_B} < 0 \end{aligned}$$

This must be negative since $R(x_B + s, 0) + x_B + s > R(0, 0)$ (because $x + R(x, 0)$ is increasing in x) and $xp(x)$ is maximized at $R(0, 0)$, so (*) holds. Q.E.D.

Lemma 3A: We assume $x_A \geq x_B$ and let the reader infer analogous results for the case when $x_B > x_A$. Recall that Firm C's profits are equal to the difference between Firm B's profits if it acquires the output and Firm B's profits if Firm A acquires the output, less Firm C's costs. First consider (x_A, x_B) such that $x_A < R(x_B, 0)$ and $x_B < R(x_A, 0)$. Firm C's profits are:

$$1) (x_B + x_C)p(x_A + x_B + x_C) - x_B p(x_A + x_B + x_C) - x_C k = x_C p(x_A + x_B + x_C) - x_C k$$

when it produces $x_C \leq R(x_B, 0) - x_A$ ("interval A-1");

$$2) (x_B + x_C)p(x_A + x_B + x_C) - x_B p(x_B + R(x_B, 0)) - x_C k$$

when it produces $R(x_B, 0) - x_A < x_C < R(x_A, 0) - x_B$ ("interval A-2");

$$3) R(x_A, 0)p(x_A + R(x_A, 0)) - x_B p(x_B + R(x_B, 0)) - x_C k$$

when it produces $x_C > R(x_A, 0) - x_B$ ("interval A-3").

Next consider (x_A, x_B) such that $x_A \geq R(x_B, 0)$ and $x_B < \hat{x}$. Firm C's profits are:

$$1) (x_B + x_C)p(R(x_B, 0) + x_B + x_C) - x_B p(R(x_B, 0) + x_B) - x_C k$$

when it produces $x_C \leq \hat{x} - x_B$ ("interval D_A-1");

$$2) \hat{x}p(2\hat{x}) - x_B p(R(x_B, 0) + x_B) - x_C k$$

if $x_C > \hat{x} - x_B$ ("interval D_A-2").

Finally consider (x_A, x_B) such that $x_A > \hat{x}$ and $x_B > \hat{x}$. Firm C's profits are:

$$1) \hat{x}p(\hat{x} + \hat{x}) - \hat{x}p(\hat{x} + \hat{x}) - x_C k = -x_C k < 0 \text{ for all } x_C.$$

Remarks A through C: Necessary and Sufficient Conditions for Entry (Lemma 3):

A) When $x_A < R(x_B, 0)$ and $x_B < R(x_A, 0)$ then Firm C's profit is strictly increasing in x_C when evaluated at $x_C = 0$, so Firm C will enter if and only if $p(x_A + x_B) > k$.

B) When $x_A \geq R(x_B, 0)$ Firm C will enter if and only if $x_B < \tilde{x}$.

Proof: First note that the derivative of Firm C's profit when evaluated at $x_C = 0$,

$$x_B p'(R(x_B, 0) + x_B) \left(\frac{\partial R(x_B, 0)}{\partial x} + 1 \right) + p(R(x_B, 0) + x_B) - k$$

is positive if and only if $x_B < \tilde{x}$ since \tilde{x} maximizes $x_B p(R(x_B, 0) + x_B) - kx_B$. So entry will occur if $x_A \geq R(x_B, 0)$ and $x_B < \tilde{x}$. Next note that when $x_A \geq R(x_B, 0)$ and $x_B > \tilde{x}$ Firm C's profits are strictly decreasing in x_C , so entry will not occur.

C) When $x_A > \hat{x}$ and $x_B > \hat{x}$ Firm C's profits are strictly negative for all $x_C > 0$, so Firm C will not enter.

Remarks 1 through 7: Firm C's Optimal Production (when $x_A < R(x_B, 0)$ and $x_B < R(x_A, 0)$):

1) If $x_A = x_B$ then $R(x_A, 0) - x_B = R(x_B, 0) - x_A$ interval A-2 vanishes.

2) In Interval A-2, where $R(x_B, 0) - x_A < x_C < R(x_A, 0) - x_B$

$$\begin{aligned} & (x_B + x_C)p(x_A + x_B + x_C) - x_B p(x_B + R(x_B, 0)) - x_C k \\ &= x_C p(x_A + x_B + x_C) + x_B (p(x_A + x_B + x_C) - p(x_B + R(x_B, 0))) - x_C k \\ &\leq x_C p(x_A + x_B + x_C) - x_C k \end{aligned}$$

3) Firm C's profit function is continuous in x_C .

4) Firm C will never produce $x_C > \max\{R(x_A, 0) - x_B, R(x_B, 0) - x_A\}$. Its profit at $x_C = \max\{R(x_A, 0) - x_B, R(x_B, 0) - x_A\}$ is strictly higher.

5) If $R(x_A + x_B, k) \leq R(x_B, 0) - x_A$ then Firm C's optimal production is $x_C^* = R(x_A + x_B, k)$.

Proof: By Remark 4, x_C^* is in A-1 or A-2. By Remark 2, the maximal profit in A-2 is less $x_C p(x_A + x_B + x_C) - x_C k$. But since $R(x_A + x_B, k) \leq R(x_B, 0) - x_A$, the maximal profit Interval A-1 is larger than the maximal profit in interval A-2.

6) If $R(x_A + x_B, k) \geq R(x_B, 0) - x_A$ then Firm C's optimal production satisfies $x_C^* \geq R(x_B, 0) - x_A$. More precisely, Firm C's optimal production is the larger of $x_C = R(x_A, k) - x_B$ and $x_C = R(x_B, 0) - x_A$.

Proof: First we find the unconstrained optimal production when Firm C's profits function is $(x_B + x_C)p(x_A + x_B + x_C) - x_B p(x_B + R(x_B, 0)) - x_C k$ (Interval A-2). Using a change of variables, $x_{BC} = x_B + x_C$, it is easy to see that the solution is $x_{BC} = R(x_A, k)$, so its profit is maximized at $x_C = R(x_A, k) - x_B$. However if the constraint, $R(x_B, 0) - x_A < x_C < R(x_A, 0) - x_B$, is binding, i.e., $R(x_A, k) - x_B \geq R(x_B, 0) - x_A$, then Firm C's profits are maximized at $x_C = R(x_B, 0) - x_A$.

7) From Remarks 4 and 6, it follows that when $x_A = x_B$ then $x_C^* \in \{R(x_A + x_B, k), R(x_A) - x_B\}$.

Proof of Proposition 1:

We will demonstrate that no firm has a profitable deviation when $k \leq 1/6$ and at least one firm does when $k > 1/6$.

First consider Firm C. By Lemma 3A (Remark C), if $x_A \geq \hat{x}$ and $x_B \geq \hat{x}$ then Firm C's best response is $x_C = 0$, so in particular $x_C = 0$ is its best response to $x_A = x_B = \hat{x}$.

Next consider Firm A (and by analogy Firm B). Neither Firm A nor Firm B have an incentive to produce more than \hat{x} . By Lemma 3, such a deviation would have no effect on Firm C, and by Lemma 1 it would have no effect on their revenues in the distribution game, but it would increase their costs but not their revenues.

Suppose Firm A (and by analogy Firm B) could increase its profit by producing $x_A < x_B = \hat{x}$. By Lemma 3 as long as $p(x_A + \hat{x}) > k$, which follows from $k \leq 1/6$, Firm C can profitably produce.

Claim: If $x_A < x_B = \hat{x}$ and $k < 1/3$, then Firm C's optimal production satisfies $x_C \geq R(x_A, 0) - \hat{x} > 0$.

Proof of Claim: By Lemma 3A, Remark 5, Firm C will produce $R(x_A + x_B, k)$ if $R(x_A + x_B, k) < R(x_B, 0) - x_A$ and $x_C \geq R(x_A, 0) - x_B$ otherwise. So it is sufficient to show that $R(x_A + x_B, k) > R(x_B, 0) - x_A$. Since

$$R(x_A + x_B, k) = \arg \max_{x_C} x_C p(x_A + x_B + x_C) - x_C k,$$

when $x_B = \hat{x} = 1/3$ and demand is linear, $R(x_A + x_B, k) = 1/3 - x_A/2 - k/2$. Also

$R(x_A, 0) - x_B = 1/2 - x_A/2 - 1/3 = 1/6 - x_A/2$. So $R(x_A + x_B, k) > R(x_B, 0) - x_A$ when

$k < 1/3$.

Since $x_A < x_B$ Firm B will acquire x_C . Since $x_B + x_C = \hat{x} + x_C \geq R(x_A, 0)$ (by the previous claim) Firm B will distribute only $R(x_A, 0)$. So when Firm A deviates to any $x_A < \hat{x}$, Firm A's profits will be $x_A p(x_A + R(x_A, 0)) - kx_A$. The optimal deviation for Firm A solves

$$\max_{x_A} x_A p(x_A + R(x_A, 0)) - kx_A$$

subject to $x_A < \hat{x}$. For linear demand, the unconstrained solution to this maximization is $x_A = \tilde{x} = 1/2 - k$. So for $k \leq 1/6$, $x_A \geq 1/3$ no profitable deviation exists, but for $k > 1/6$, Firm A can increase its profits by producing $x_A = 1/2 - k$.

Proof of Proposition 2:

To prove that $\{\tilde{x}, R(\tilde{x}, 0), 0\}$ (and by analogy $\{R(\tilde{x}, 0), \tilde{x}, 0\}$) is an equilibrium if and only if $k \in (1/6, 1/(2\sqrt{2})]$ we must show that no profitable deviation exists for any player when $k \in (1/6, 1/(2\sqrt{2})]$ and that a profitable deviation does exist when $k \notin (1/6, 1/(2\sqrt{2})]$.

Suppose $k < 1/6$, so $\tilde{x} > R(\tilde{x}, 0)$. It is clear from Lemma 3A that entry is deterred. In the distribution stage Firm B will distribute all of its production, $R(\tilde{x}, 0)$, but Firm A will distribute only $R(R(\tilde{x}, 0), 0)$. If Firm A deviates and produces $R(R(\tilde{x}, 0), 0)$, entry is still deterred and distribution remains the same, but Firm A's costs are lower. So $\{\tilde{x}, R(\tilde{x}, 0), 0\}$ is not an equilibrium when $k < 1/6$. It follows that $\tilde{x} \leq R(\tilde{x}, 0)$.

First, consider possible deviations from equilibrium by Firm C. By Lemma 3, Remark C, it is clear that Firm C will not produce.

Second, consider possible deviations from equilibrium by Firm A. First, suppose Firm A deviates by producing more than \tilde{x} . Then by Lemma 3A Firm C will still produce nothing.

If $x_A < \hat{x}$ (Firm A's deviation is not too large), then in the distribution stage Firm A will distribute x_A and Firm B will withhold some of its output and distribute $R(x_A, 0) < R(\tilde{x}, 0)$. So the optimal deviation for Firm A maximizes $x_A p(x_A + R(x_A, 0)) - kx_A$ subject to $x_A < \hat{x}$. But the solution to this problem is \tilde{x} if $k > 1/6$.

If Firm A's deviation is large, so $x_A > \hat{x}$, then both firms will distribute \hat{x} . Firm A's profits are no more than $\hat{x}p(\hat{x} + \hat{x}) - k\hat{x}$ which is less than $\tilde{x}p(\tilde{x} + R(\tilde{x}, 0)) - k\tilde{x}$ for $k > 1/6$. So an increase in Firm A's production is never profitable.

Now, suppose that Firm A deviates by producing less than \tilde{x} .

Claim: If $x_A < \tilde{x}$, $x_B = R(\tilde{x}, 0)$, and $k < 1/2$ then Firm C's optimal production satisfies $x_C \geq R(x_A, 0) - x_B > 0$.

Proof of Claim: By Lemma 3A, Remark 5, Firm C will produce $R(x_A + x_B, k)$ if $R(x_A + x_B, k) < R(x_B, 0) - x_A$, and $x_C \geq R(x_A, 0) - x_B$ otherwise. So it is sufficient to show that $R(x_A + x_B, k) > R(x_B, 0) - x_A$. Recall that

$$R(x_A + x_B, k) = \operatorname{argmax}_{x_C} x_C p(x_A + x_B + x_C) - x_C k.$$

Using linear demand, and substituting $\tilde{x} = 1/2 - k$, and $x_B = R(\tilde{x}, 0) = (1 + 2k)/4$ into the objective, this optimization yields $R(x_A + x_B, k) = 3/8 - x_A/2 - 3k/4$. Also using linear demand and $x_B = \hat{x} = 1/3$, $R(x_A, 0) - x_B = 1/2 - x_A/2 - (1 + 2k)/4 = 1/4 - x_A/2 + k/2$, so $R(x_A + x_B, k) > R(x_B, 0) - x_A$ when $k < 1/2$.

Since $x_A < x_B$ Firm B will acquire x_C . Since $x_B + x_C = \hat{x} + x_C \geq R(x_A, 0)$ (by the previous claim) Firm B will distribute only $R(x_A, 0)$. So when Firm A deviates to any $x_A < \hat{x}$, Firm A's profits will be $x_A p(x_A + R(x_A, 0)) - kx_A$. So the most profitable deviation for Firm A is $x_A = \arg\max_{x_A} x_A p(x_A + R(x_A, 0)) - kx_A$ which is equal to \tilde{x} . So no profitable deviation exists for Firm A.

Finally consider possible deviations from equilibrium by Firm B. Clearly Firm B cannot increase its profit by producing *more* than $R(\tilde{x}, 0)$: Firm C would still produce zero. So Firm B would be earning the same revenue at higher cost.

Suppose Firm B produced *less* than $R(\tilde{x}, 0)$.

Claim: If $x_A = \tilde{x}$ then Firm C's best response to $x_B = 1/4$ is $R(x_B + \tilde{x}, k)$ if and only if $k \geq 1/4$.

Proof:

Given that Firm B deviates to $x_B = 1/4$, so $x_B \geq x_A = \tilde{x} = 1/2 - k$, by Lemma 3 Remark 6, Firm C's best response is $R(x_A + x_B, k)$ if $R(x_A + x_B, k) < R(x_A, 0) - x_B$ and $x_C \geq R(x_A, 0) - x_B$ otherwise. Since Firm A produces $\tilde{x} = 1/2 - k$ and Firm B produces $1/4$,

$$R(x_A + x_B, k) = \frac{1 - \frac{1}{4} - \left(\frac{1}{2} - k\right) - k}{2} = \frac{1}{8},$$

and

$$R(x_A, 0) - x_B = \frac{1 - \left(\frac{1}{2} - k\right)}{2} - \frac{1}{4} = \frac{k}{2},$$

so then Firm C's best response is $R(x_A + x_B, k)$ if and only if $k \geq 1/4$.

Consider Firm B's profits when it chooses some deviation $x_B < R(\tilde{x}, 0)$ to which Firm C's best response is $x_C = R(x_A + x_B, k)$. Firm B's profits from such a deviation are $x_B p(x_B + \tilde{x} + R(x_B + \tilde{x}, k))$ which is clearly less than $\max_{x_B} x_B p(x_B + \tilde{x} + R(x_B + \tilde{x}, k))$. For linear demand this upper bound is

$$\max_{x_B} x_B \left(\frac{1 - \tilde{x} - x_B - k}{2} \right),$$

which is maximized at $x_B = 1/4$ where Firm B's profits are

$$x_B \frac{1}{2} \left(\frac{1}{2} - x_B \right) = \frac{1}{32}.$$

But by the claim $x_C = R(x_A + x_B, k)$ is Firm C's best response to $x_C = 1/4$ when $k > 1/4$. So Firm B's most profitable deviation is $x_B = 1/4$ and its profits from that deviation are $1/32$. However there are also deviations to which Firm C's optimal response is satisfies $x_C \geq R(\tilde{x}, 0) - x_B$, and we must also show these do not earn higher profits.

Suppose Firm C's optimal response to a deviation by Firm B satisfies $x_C \geq R(\tilde{x}, 0) - x_B$. If $x_B \in [\tilde{x}, R(\tilde{x}, 0))$ then Firm B will acquire Firm C's output in the auction (because it is larger) and then distribute $R(\tilde{x}, 0)$ in the distribution stage. Firm B's distributes the same amount as when it produces the equilibrium output, $R(\tilde{x}, 0)$, and so earns the same revenue, but its costs are strictly higher (because it pays Firm C more than k for that output). If $x_B \in [0, \tilde{x}]$ then Firm A will withhold some output in Stage 4 and distribute $R(x_B, 0)$. An upper bound for Firm B's payoff from a deviation satisfying $x_B \in [0, \tilde{x}]$ is the profit that solves

$$\max_{x_B} x_B p(x_B + R(x_B, 0)) - kx_B,$$

subject to $x_B \in [0, \tilde{x}]$. This upper bound is achieved at $x_B = 1/2 - k$ (notice that $x_B = \tilde{x}$ so the constraint is not binding). Firm B's profits from this deviation are $(1/2)[1/2 - k]^2$. But Firm B's equilibrium profits are $(1/4)[1/2 - k][1/2 + k]$. It that Firm B will not find it profitable to deviate whenever $k > 1/6$.

Therefore Firm B will deviate to $x_B = 1/4$ if and only if $1/32$ is greater than its equilibrium profits, $1/4(1/4 - k^2)$, or $k > 1/(2\sqrt{2})$.

Proof of Proposition 3:

From Lemma 3A entry is deterred only if $x_A \geq R(x_B, 0)$, $x_B \geq R(x_A, 0)$ or $p(x_A + x_B) \leq k$. However when $p(x_A + x_B) \leq k$ Firm's A and B cannot be producing optimally since profits zero or negative for both firms. Also note that $x_A > R(x_B, 0)$ and $x_B > R(x_A, 0)$ are inconsistent with profit maximization since production could be reduced without changing distribution or revenues. So in any pure strategy, entry deterring equilibrium either $x_A = R(x_B, 0)$ or $x_B = R(x_A, 0)$ must hold. So all the candidates for an equilibrium are of the form $\{\hat{x}, R(\hat{x}, 0), 0\}$ or $\{\hat{x}, R(\hat{x}, 0), 0\}$ where $\hat{x} < R(\hat{x}, 0)$ or equivalently, $\hat{x} < \hat{x}$. Graphically, since $x_A \leq R(x_B, 0)$ and $x_B \leq R(x_A, 0)$, the equilibrium candidates lie on the portion of the Firm's zero cost reaction functions that are on the boundary of Region A in Figures 3, 4, & 5.

Consider $k \leq 1/6$. Suppose there exists an equilibrium $\{\hat{x}, R(\hat{x}, 0), 0\}$ where $\hat{x} < \hat{x}$, so it is different from the equilibrium in Proposition 1. Note that $x_A p(x_A + R(x_A, 0)) - kx_A$ is concave and for that $k \leq 1/6$, $\hat{x} \leq \arg\max_{x_A} x_A p(x_A + R(x_A, 0)) - kx_A$ (see proof of

Proposition 1). If Firm A deviates to \hat{x} , Firm B will distribute only $R(\hat{x},0)$ so Firm A's profits are $\hat{x}p(\hat{x} + R(\hat{x},0)) - k\hat{x}$ which must be higher. So no such equilibrium exists.

Consider $k > 1/6$. Suppose there exists an equilibrium $\{\hat{x}, R(\hat{x},0), 0\}$ where $\hat{x} \neq \operatorname{argmax}_{x_A} x_A p(x_A + R(x_A,0)) - kx_A$, so it is different from the equilibrium in Proposition 2. If $\hat{x} < \operatorname{argmax}_{x_A} x_A p(x_A + R(x_A,0)) - kx_A$ Firm A can deviate to \tilde{x} and earn strictly higher profit since Firm B will distribute $R(\tilde{x},0)$. If $\hat{x} > \operatorname{argmax}_{x_A} x_A p(x_A + R(x_A,0)) - kx_A$ then Firm A can deviate to \tilde{x} which will induce entry and firm B will again distribute $R(\tilde{x},0)$ so Firm A is better off. So no such equilibrium exists. The same argument holds for an equilibrium of the form $\{R(\hat{x},0), \hat{x}, 0\}$. The only candidates for an equilibria are $\{\tilde{x}, R(\tilde{x},0), 0\}$ and $\{R(\tilde{x},0), \tilde{x}, 0\}$, where $\tilde{x} = \operatorname{argmax}_{x_A} x_A p(x_A + R(x_A,0)) - kx_A$ and we showed in Proposition 2 that these candidate equilibria exist if and only if $k \in (1/6, 1/(2\sqrt{2})]$.

Proof of Proposition 4:

Proof of Existence:

Consider a deviation by Firm C. By Lemma 3A, Remark 7, since $x_A = x_B = (1-k)/3$, $R(x_B,0) - x_A = R(x_A,0) - x_B$ and Firm C will produce either $R(x_A + x_B, k)$ if $R(x_A + x_B, k) \leq R(x_B,0) - x_A$ and $R(x_B,0) - x_A$ otherwise. So Firm C's optimal production is

$$x_C^* = \begin{cases} R(x_A + x_B, k) = \frac{1}{6}(1-k) & \text{if } k \geq \frac{1}{4} \\ R(x_B, 0) - x_A = \frac{k}{2} & \text{if } k < \frac{1}{4} \end{cases}$$

In particular, when $k > 1/4$, Firm C has no profitable deviation.

Now consider Firm A's production (or equivalently Firm B):

Claim: Given $x_B = (1-k)/3$, Firm A's optimal production is either $r(x_B, k) = (1-k)/3$ or $R(x_B, 0)$.

Proof: Suppose instead that $x_A^* < \frac{1}{3}(1-k)$. Firm C will produce $x_C = R(x_A^* + x_B, k)$ because $R(x_A^* + x_B, k) < R(x_A^*, 0) - x_B$: linear demand and $R(x_A + x_B, k) < R(x_A, 0) - x_B$ at $x_A = x_B = \frac{1}{3}(1-k)$ imply that $R(x_A + x_B, k) < R(x_A, 0) - x_B$ for all $x_A < \frac{1}{3}(1-k)$. So Firm A's profit function is $x_A p(x_A + x_B + R(x_A + x_B, k)) - kx_A$ but this implies its profits are strictly higher at $x_A^* = \frac{1}{3}(1-k)$, which is a contradiction.

Suppose that $x_A^* > (1-k)/3$ and $R(x_A^* + x_B, k) > R(x_B, 0) - x_A^*$. Suppose further that $x_A^* > R(x_B, 0) > \hat{x}$. Then by Lemma 3A, Firm C's profit is independent of x_A . If $x_C \leq \hat{x} - x_B$ Firm C's profit is

$$(x_B + x_C)p(R(x_B + x_C, 0) + x_B + x_C) - x_B p(R(x_B, 0) + x_B),$$

and if $x_C > \hat{x} - x_B$ Firm C's profit is

$$\hat{x}p(R(\hat{x}, 0) + \hat{x}) - x_B p(R(x_B, 0) + x_B).$$

Since this is what Firm A will pay for Firm C's output, Firm A earns strictly greater profit (because its costs are lower) when $x_A^* = R(x_B, 0)$. This is a contradiction.

Suppose as before that $x_A^* > (1-k)/3$ and $R(x_A^* + x_B, k) > R(x_B, 0) - x_A^*$, but now suppose that $x_A^* < R(x_B, 0)$. By Lemma 2, since $x_A^* > x_B$, Firm A will buy Firm C's output for Firm B's valuation. Since Firm B's values Firm C's output at

$$\begin{aligned} (x_B + x_C)p(R(x_B, 0) + x_B) - x_B p(R(x_B, 0) + x_B) \\ = x_C p(R(x_B, 0) + x_B) \end{aligned}$$

when $x_C = R(x_B, 0) - x_A^*$ and $p(R(x_B, 0) + x_B) > k$ it is clear that Firm B's values Firm C's optimal output strictly greater than kx_C . So Firm A's profit, if it produces $x_A^* < R(x_B, 0)$, is strictly less than

$$R(x_B, 0)p(x_B + R(x_B, 0)) - (x_A + x_C)k$$

However if Firm A produces $x_A^* = R(x_B, 0)$, then by Lemma 3A, Remark 6,

$$x_C^* = \max\{R(x_B, 0) - x_A, R(x_A, k) - x_B\},$$

but since $R(x_B, 0) - x_A^* = 0$ and

$$\begin{aligned} R(x_A^*, k) - x_B &= \frac{1 - x_A^* - k}{2} - \frac{1}{3}(1 - k) \\ &= \frac{1 - R(x_B, 0) - k}{2} - \frac{1}{3}(1 - k) \\ &= \frac{1 - \frac{1}{3} - k}{2} - \frac{1}{3}(1 - k) \\ &= -\frac{k}{6} \end{aligned}$$

it follows that $x_C^* = 0$, so if Firm A produce $x_A^* = R(x_B, 0)$ its profits are

$$R(x_B, 0)p(x_B + R(x_B, 0)) - R(x_B, 0)k.$$

Firm A's profits are strictly higher if it produces $x_A^* = R(x_B, 0)$. We conclude that Firm A's optimal production is either $r(x_B, k) = (1-k)/3$ or $R(x_B, 0)$.

So we only need to consider a deviation to $R(x_B, 0)$ to determine whether or not $(1-k)/3$ is Firm A's optimal strategy. Firm A's equilibrium profits are

$$\frac{1}{3}(1-k)\left(1 - \frac{5}{6}(1-k) - k\right) = \frac{1}{18}(1-k)^2 = \frac{1}{18} - \frac{1}{9}k + \frac{1}{18}k^2,$$

and, since $x_C^* = R(x_B, 0) - x_A = 0$, Firm A's profits at $R(x_B, 0)$ are

$$\begin{aligned} & \frac{1}{3}(1+k)\left(1 - \frac{1}{3}(1-k) - \frac{1}{3}(1+k) - k\right) \\ &= \frac{1}{3}(1+k)\left(\frac{1}{3} - k\right) = \frac{1}{9} - \frac{2}{9}k - \frac{1}{3}k^2. \end{aligned}$$

So A will want to deviate to $R(x_B, 0)$ if and only if

$$\frac{1}{9} - \frac{2}{9}k - \frac{1}{3}k^2 > \frac{1}{18} - \frac{1}{9}k + \frac{1}{18}k^2$$

or $2 - 4k - 6k^2 > 1 - 2k + k^2$; $1 - 2k - 7k^2 > 0$; $k < .261204$.

Proof of Uniqueness:

The final part of the proof is to show that no other entry accommodation equilibria exists. Suppose $\{x_A, x_B, x_C\}$ is an entry accommodating equilibrium so $x_C > 0$.

We first claim that in any entry accommodation equilibrium $x_A < R(x_B, 0)$ and $x_B < R(x_A, 0)$. Suppose instead that an entry accommodation equilibrium exists in which $x_A \geq R(x_B, 0)$ (or by analogy that $x_B \geq R(x_A, 0)$). Then by Lemma 3A, Firm C will enter only if $x_B < \tilde{x}$. But if $x_B < \tilde{x}$, Firm B could increase its profits by producing more.

We next claim that in any entry accommodating equilibrium $x_C = R(x_A + x_B, k) \leq \min\{R(x_A, 0) - x_B, R(x_B, 0) - x_A\}$. Suppose instead that $x_C \geq R(x_B, 0) - x_A$ and $x_A \geq x_B$ (or by analogy $x_C \geq R(x_A, 0) - x_B$ and $x_B \geq x_A$). Since

Firm A will buy Firm C's output the equilibrium of the distribution game will be $R(x_B, 0)$ and x_B . If $x_B < \tilde{x}$, Firm B could increase its profits by producing \tilde{x} , so $x_B \geq \tilde{x}$. However, this implies that if Firm A deviated to $x_A = R(x_B, 0)$, entry would be deterred and Firm A would earn strictly higher profits. Since Firm A's production costs are $(R(x_B, 0) - x_A)k$ higher but he saves at least as much as he would have paid Firm C if Firm C produced $x_C = R(x_A, 0) - x_B$, which is Firm B's valuation for $x_C = R(x_A, 0) - x_B$:

$$\begin{aligned} & (x_B + R(x_B, 0) - x_A)p(x_A + x_B + R(x_B, 0) - x_A) - x_B p(R(x_B, 0) + x_B) \\ & = (R(x_B, 0) - x_A)p(x_B + R(x_B, 0)) \end{aligned}$$

So $x_C = R(x_A + x_B, k) \leq \min\{R(x_A, 0) - x_B, R(x_B, 0) - x_A\}$ in all entry accommodating equilibria.

Therefore, since Firm C produces $x_C = R(x_A + x_B, k)$, Firm A maximizes its profits by choosing $x_A = r(x_B)$ and Firm B maximizes its profits by choosing $x_B = r(x_A)$. So the unique entry accommodating equilibrium is

$$\{x_A, x_B, x_C\} = \left\{ \frac{1-k}{3}, \frac{1-k}{3}, \frac{1-k}{6} \right\}.$$

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¹ International markets offer many more examples. U.S. mutual fund providers, Citibank and Solomon/Smith Barney distribute their products in Japan through vertically integrated competitors. Quaker's Gatorade beverages and Anheuser Busch's beers are also distributed by rivals in Japan.

² Molnar (2000) considers a model of horizontal mergers where incentives to merge are shaped by similar negative externalities.

³ "Amid Probe, Anheuser Conquers Turf," The Wall Street Journal, March 9, 1988.

⁴ Our results easily generalize to any linear demand function.

⁵ By construction, when $x_A = x_B = x^*$ each incumbent values an additional unit of output at exactly k . So absent the externality, neither would be willing to pay more than this.

⁶ The potentially important difference between this and a first price auction is that a first price auction might give the entrant the ability to commit to ex post inefficient trade. However this distinction will remain unimportant until in Section 8.

⁷ Similarly, if they were planning to distribute all of their own output plus $w \leq x_C$ of the entrant's output then each would be willing to pay $wp(x_A + x_B + w)$.

⁸ Gilbert and Vives (1986) examine free riding in multiple incumbent entry deterrence model in which the entrant has access to distribution.

⁹ Entry is deterred when $x_A \geq R(x_B, k)$, $x_B \geq R(x_A, k)$, $x_A \geq \tilde{x}$, and $x_B \geq \tilde{x}$ (these four conditions are sufficient for entry deterrence, however only the first two are necessary conditions). It follows that $x_A = x_B = x^*$ is an equilibrium of the game, since it deters entry and the profits the incumbents earn from any deviation in from x^* are bounded above by the profit earned they earn from the same deviation when there is no entrant. This also appears to be the unique equilibrium.