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THE CENTER FOR THE STUDY
OF INDUSTRIAL ORGANIZATION
AT NORTHWESTERN UNIVERSITY

Working Paper #0095

The incentive to participate in open source projects: a
signaling approach^{*}

By

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Abstract

This paper examines the incentive of unpaid programmers to contribute to open source software (OSS) projects in order to signal their talents. The analysis shows that if programmers contribute to OSS projects at all, then generically there are multiple equilibria. In these equilibria, an increase in the visibility of performance, an increase in the sensitivity of performance to effort, and an increase in the informativeness of performance about talent may or may not boost the signaling incentive of programmers depending on the stability of equilibrium and on the properties of the probability that successful performance will be observed.

1 Introduction

Open source software (OSS) is a computer program whose source code - the instructions for the program, written in a human readable format - is distributed free of charge and can be modified, extended, adapted, and incorporated into other programs with relatively few restrictions. OSS is a rapidly expanding phenomenon: some OSS such as the Apache web server, dominate their product categories. In the personal computer market, some OSS such as the operating system Linux, the web browser Firefox, and the office suites OpenOffice.org and StarOffice gain rapid popularity.¹

Apart from having millions of OSS users, there are also tens of thousands of participating programmers who contribute to various OSS projects, and there is also a growing number of firms who sell services, support, and documentation for OSS. The majority of the programmers who participate in OSS projects are unpaid volunteers. For example, Hars and Ou (2002) have surveyed 81 individuals involved in open source projects and found that only 16% received any direct monetary compensation for their contribution. This raises obvious questions about the incentives and motivations of the participating programmers. There are three main, mostly complimentary, explanations for the willingness of programmers to contribute to OSS projects. The first two involve intrinsic motivations while the third involves extrinsic motivations.

The first explanation is that programmers simply like to be involved in open source projects, either because they enjoy being creative, or due to a sense of obligation or community related reasons, or simply due to sheer altruism.² Indeed, a web-based survey conducted by Lakhani and Wolf (2003) reveals that the responding programmers were mainly driven by enjoyment-based intrinsic motivations.

The second explanation involves another type of intrinsic motivation. According to this explanation, individual users such as system managers (e.g., users of Apache), who

¹It is estimated that as of March 2005, there were 29 million users of Linux worldwide (see <http://counter.li.org/estimate.php>), and that as of March 2008, over 516 million downloads of Firefox have been registered (see www.spreadfirefox.com), and more than 40 million copies of OpenOffice.org and StarOffice software have been distributed (see <http://www.sun.com/software/star/openoffice/faq.xml>).

²Athey and Ellison (2006) consider a dynamic model of the evolution of open source software projects, in which altruistic programmers who have used the software in the past are motivated to publish their own improvements for the benefit of other users.

make all sorts of software improvements for their own benefit, are willing to share these improvements with other users in their community. A model along these lines is offered by Johnson (2002), who views participation in OSS projects as a private provision of a public good (see Bessen, 2006, and Bitzer, Schrettl, and Schröder, 2007, for related models).

The third explanation, suggested by Lerner and Tirole (2002), is that programmers are willing to contribute to OSS projects in order to signal their ability to potential employers, venture capitalists, or to peers. This enables programmers to boost their human capital or enhance their social status within the programmers' community. Fershtman and Gandal (2007) examine a large data set on programmers' participation in OSS projects and find that the output per contributor is much higher when the OSS is distributed under a less restrictive license and is more commercially oriented. They argue that this result is consistent with the hypothesis that contributing programmers are driven by signaling incentives. Another piece of evidence for this hypothesis is due to Hann et al (2004) who examine a panel data on contributions to three Apache OSS projects for the period 1998 to 2002. They find that credentials earned through the merit-based ranking system within the Apache open source community are associated with a 13% – 27% increase in wages, depending on the rank attained.

Drawing on the “career concerns” literature (e.g., Holmström, 1999), Lerner and Tirole (2002) conjecture that the signaling incentive to participate in OSS projects will become stronger as (i) performance becomes more visible to the relevant audience, (ii) effort has a stronger impact on performance, and (iii) performance becomes more informative about talent. While these conjectures are intuitively appealing, it is also possible to think about the opposite conjectures. For instance, if effort has a greater impact on performance and/or if performance becomes more visible, then even a small amount of effort might enable talented programmers to produce a visible positive signal about their talent.

In this paper I study the signaling incentive to participate in OSS projects in the context of a formal model and then use it to examine the Lerner and Tirole conjectures. In this model, programmers are privately informed about their types: some are “talented” and have high productivity, while others are “untalented” and have low productivity.³ To

³My model therefore differs from Holmström (1999) where agents do not have private information about

signal their talent to prospective employers, programmers participate in OSS projects and each programmer either “succeeds” (i.e., “solves a problem” or “advances within the community’s ranks”) or “fails.” Talented agents can boost their chances to succeed by exerting effort. Prospective employers then imperfectly observe whether specific programmers have succeeded or not and this observation together with their beliefs on the effort of talented agent determine the wages that they offer programmers.

I show that the model always admits a no-effort equilibrium in which firms do not expect programmers to exert effort in OSS projects, and programmers indeed do not exert such effort. However, the model may also admit interior equilibria in which talented programmers exert effort and observed success translates into higher wages. When these equilibria exist, then generically their number is even. Interestingly, this multiplicity of equilibria is not due to out-of-equilibrium beliefs as in Spence style signaling models, because the mapping from effort to success/failure is stochastic. Hence, there are no out-of-equilibrium signals in my model. The analysis shows that conjectures (i)-(iii) may or may not hold depending on whether we start from a stable or an unstable interior equilibrium and depending on the properties of the probability that talented programmers succeed and their success is observed. These results suggest that a-priori, it is hard to say which factors will boost the signaling incentive of talented agents and which factors will weaken it.

There are two closely related papers that also study the signaling incentive of programmers to participate in OSS projects. These papers differ from mine both in terms of their set up and in terms of their main focus. In Lee, Moisa, and Weiss (2003), programmers need to choose between joining closed source software firms and OSS projects. If they join software firms, their wage reflects the expected productivity of all programmers who join closed-source software firms (more and less talented ones). On the other hand, if they join OSS projects, they forgo current wages, but can signal their productivity to software firms and thereby boost their future wages. The main focus of their analysis is on the relative sizes of the closed-source and the open-source systems. They show that an open-source system can exist only if there are sufficiently many talented programmers.

Leppämäki and Mustonen (2004) consider a model in which programmers signal their

their talent.

talent to software firms by choosing how many lines of code to contribute to an OSS project. Talented programmers have a lower cost of writing code and hence they separate themselves from untalented programmers by writing sufficiently many lines of code. Their model departs from the traditional Spence signaling model in that the freely available OSS project imposes either a positive or a negative externality on commercial software, depending on whether the two are substitutes or complements. This externality in turn affects the wages that software firms are willing to offer and hence the marginal benefit to signaling. As a result, talented programmers contribute to the OSS project less (more) if the OSS and commercial software are substitutes (complements) than if the two are independent of each other.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 shows that the model can give rise to multiple equilibria and characterizes them. Section 4 studies the comparative static properties of the model and examines how the incentive to contribute to OSS projects is affected by the visibility of the programmers' performance to prospective employers, by the sensitivity of performance to effort, by the informativeness of performance about talent, and by the fraction of talented programmers in the population of participating programmers. Section 5 examines the effect of intrinsic motivation to contribute to OSS projects. Concluding remarks are in Section 6.

2 The model

Consider a competitive job market with a large number of agents, each of whom is either “talented” and has a high productivity, or “untalented” and has a low productivity. If hired by a firm, the marginal productivity of a talented agent is w , while the marginal productivity of an untalented agent is normalized to 0. Under full information, the wage of each agent is equal to his marginal productivity.

Under asymmetric information, it is common knowledge that the fraction of talented agents in the population is α , but firms cannot tell the agents' types before hiring them (agents however know their own types). Before the labor market opens up, agents participate in OSS projects in the hope of convincing prospective employers that they are talented.⁴ I

⁴To simplify matters, I assume that the cost of participation is sufficiently low to ensure that all agents

assume that each agent either succeeds (i.e., “solves a problem” or “advances within the community’s ranks”) or fails (i.e., “fails to come up with satisfactory results” or “does not advance within the community’s ranks”). The probability that an untalented agent succeeds is exogenous and equal to p_0 . Talented agents by contrast can boost their probability of success by exerting effort: if a talented agent exerts effort e in OSS projects, his probability of success increases from p_0 to $p(e)$, where $p(e)$ is increasing and strictly concave, with $p(0) = p_0$ and $\lim_{e \rightarrow \infty} p(e) = 1$.

In and of themselves, OSS projects do not benefit the firms nor the agents directly (for now I ignore intrinsic motivations to participate in OSS projects). The only advantage of participation is that it generates a signal on the agents’ talent. Firms cannot directly observe if and how much effort each agent exerts; rather they can only (imperfectly) observe successful performance.⁵ In particular, firms observe a successful performance with probability β ; with probability $1 - \beta$, firms observe nothing and hence cannot tell whether the agent succeeded but his success was not observed, or whether the agent failed. In what follows, β will serve as a measure of the visibility of the agents’ performance to prospective employers.

The payoff of each agent is increasing with his expected wage, EW , and decreasing with his effort level, e :

$$U = EW - e.$$

3 Equilibrium characterization

I now look for a perfect Bayesian equilibrium in which talented agents exert effort, untalented agents do not exert effort, and the beliefs of firms are consistent with the agents’ strategies.⁶ To characterize this equilibrium, suppose that firms believe that the effort of talented agents is \hat{e} and hence expect that talented agents will succeed with probability $p(\hat{e})$. Recalling that the fraction of talented agents is α and that untalented agents succeed with probability p_0 ,

participate.

⁵My model therefore involves “noisy” signaling since the mapping from efforts to observed success is stochastic. This approach differs from the approach in Leppämäki and Mustonen (2004) where prospective employers can perfectly observe the action of each agent which is how many lines of code to write.

⁶Untalented agents do not exert effort because their success probability is independent of their effort and equal to p_0 .

it follows that conditional on observing a success, firms believe that the agent is talented with probability

$$q(\hat{e} | s) = \frac{\alpha p(\hat{e})}{\alpha p(\hat{e}) + (1 - \alpha)p_0}. \quad (1)$$

On the other hand, if firms do not observe a success, then they cannot tell whether (i) the agent is talented, exerted effort, and either failed or his success was not observed, (ii) the agent is untalented and either failed or his success was not observed. The probability of (i) is $\alpha((1 - p(\hat{e})) + (1 - \beta)p_0) = \alpha(1 - \beta p(\hat{e}))$, while the probability of (ii) is $(1 - \alpha)((1 - p_0) + (1 - \beta)p_0) = (1 - \alpha)(1 - \beta p_0)$. Hence, conditional on not observing a positive signal, firms believe that the agent is talented with probability

$$q(\hat{e} | n) = \frac{\alpha(1 - \beta p(\hat{e}))}{\alpha(1 - \beta p(\hat{e})) + (1 - \alpha)(1 - \beta p_0)}. \quad (2)$$

Note that since $p_0 = p(0)$, $q(0 | s) = q(0 | n) = \alpha$: if firms expect talented agents to exert no effort, then success or failure are not informative about talent. Moreover, note that $q(\hat{e} | s)$ approaches 1 as p_0 approaches 0: if untalented agents cannot succeed then a positive signal is a sure sign that the agent is talented.

3.1 The effort choice of talented agents

To characterize the effort that talented agents will exert, note that since the labor market is competitive, the wage of agents is $q(\hat{e} | s)w$ following an observed success and $q(\hat{e} | n)w$ otherwise. Hence, the expected payoff of talented agents given their effort level, e , and given the belief of firms, \hat{e} , is

$$U(e, \hat{e}) = \beta p(e)q(\hat{e} | s)w + (1 - \beta p(e))q(\hat{e} | n)w - e. \quad (3)$$

The first term on the right-hand side of (3) reflects the idea that with probability $\beta p(e)$, a talented agent succeeds and his success is observed by firms. The second term reflects the idea that with probability $1 - \beta p(e)$, a talented agent fails to produce a positive signal about his talent either because his success was not observed by firms or because the agent simply failed. In both cases, firms cannot tell whether the agent is talented or not and hence they

pay him $q(\hat{e} | n)w$. The last term on the right-hand side of (3) is the agent's cost of effort.

Since $p(e)$ is strictly concave, the effort that each talented agent will choose given the firms' beliefs, \hat{e} , is defined implicitly by the following first order condition:

$$\frac{\partial U(e, \hat{e})}{\partial e} = \beta p'(e) \Delta(\hat{e})w - 1 \leq 0, \quad e \frac{\partial U(e, \hat{e})}{\partial e} = 0, \quad (4)$$

where

$$\begin{aligned} \Delta(\hat{e}) &\equiv q(\hat{e} | s) - q(\hat{e} | n) \\ &= \frac{\alpha(1-\alpha)(p(\hat{e}) - p_0)}{(\alpha p(\hat{e}) + (1-\alpha)p_0)(1 - \beta(\alpha p(\hat{e}) + (1-\alpha)p_0))}, \end{aligned} \quad (5)$$

is the increase in the probability that firms assign to an agent being talented following an observed success. The expression $\beta p'(e) \Delta(\hat{e})w$ represents the marginal benefit from effort; it is equal to the marginal effect of effort on the probability of producing a positive signal, $\beta p'(e)$, times the extra expected wage in this event, $\Delta(\hat{e})w$. At an interior optimum, the marginal benefit of effort must be equal to the marginal cost, which is 1. But, if $\beta p'(e) \Delta(\hat{e})w < 1$ for all $e > 0$, then talented agents will not exert any effort. The concavity of $p(e)$ together with the assumption that $\lim_{e \rightarrow \infty} p(e) = 1$ imply that $\lim_{e \rightarrow \infty} p'(e) = 0$; hence, $\frac{\partial U(e, \hat{e})}{\partial e}$ must be negative for sufficiently large values of e . Before proceeding, I establish two important properties of $\Delta(\hat{e})$:

Lemma 1: $\Delta(\hat{e})$ is an increasing function with $\Delta(0) = 0$.

Proof: Straightforward differentiation reveals that

$$\Delta'(\hat{e}) = \frac{\alpha(1-\alpha)p'(\hat{e})[\beta\alpha^2(p(\hat{e}) - p_0)^2 + p_0(1 - \beta p_0)]}{(\alpha p(\hat{e}) + (1-\alpha)p_0)^2(1 - \beta(\alpha p(\hat{e}) + (1-\alpha)p_0))^2} > 0. \quad (6)$$

The assumption that $p_0 = p(0)$ ensures that $q(0 | s) = q(0 | n) = \alpha$, so $\Delta(0) = 0$. ■

Recalling that $\Delta(\hat{e})w$ is the extra expected wage that an agent receives following an observed success, Lemma 1 implies that when firms believe that talented agents exert more effort, they are willing to pay higher wages to agents who were observed to be successful.

3.2 Stable and unstable equilibria

Let $BR(\hat{e})$ denote the solution of (4). This function defines the best response of each talented agent against the firms' beliefs about his effort level. In equilibrium, the firms' beliefs must be consistent with the true efforts of talented agents. Hence, the equilibrium effort level, e^* , is defined implicitly by the equation

$$e^* = BR(e^*). \quad (7)$$

Given its central role in what follows, I will now study the properties of $BR(\hat{e})$ in the next lemma. To establish this lemma, I will impose the following assumption on $p'(0)$:

Assumption (*): The marginal effect of effort on the probability of success is large for $e = 0$:

$$p'(0) > \frac{(\alpha + (1 - \alpha)p_0)(1 - \beta(\alpha + (1 - \alpha)p_0))}{\beta\alpha(1 - \alpha)(1 - p_0)w}. \quad (8)$$

If Assumption (*) fails, then it never pays talented agents to exert effort, no matter how high \hat{e} is, so $BR(\hat{e}) = 0$ for all \hat{e} .

Lemma 2: *Suppose that Assumption (*) holds. Then, the best response of talented agents against the firms' beliefs about their effort levels, $BR(\hat{e})$, has the following properties:*

(i) $BR(\hat{e}) = 0$ for all $0 < \hat{e} \leq \hat{e}_1$ and $BR(\hat{e}) > 0$ for all $\hat{e} > \hat{e}_1$ (talented agents exert effort only if firms expect them to exert a sufficiently large level of effort), where \hat{e}_1 is implicitly defined by $\beta p_0 \Delta(\hat{e}_1) w = 1$.

(ii) For all $\hat{e} > \hat{e}_1$,

$$BR'(\hat{e}) = -\frac{p'(e)\Delta'(\hat{e})}{p''(e)\Delta(\hat{e})} > 0, \quad (9)$$

and $\lim_{\hat{e} \rightarrow \infty} BR'(\hat{e}) = 0$.

Proof: See the Appendix.

To characterize the equilibrium, note from (7) that the equilibrium effort level, e^* , is attained at the intersection of the best-response function, $BR(\hat{e})$, with the 45° line in

the (\hat{e}, e) space (the 45° line reflects the requirement that in equilibrium, firms must hold correct beliefs about the efforts of talented agents). Since $BR(\hat{e})$ passes through the origin, $e^* = 0$ is an equilibrium effort level. Hence, there always exists a no-effort equilibrium in which talented agents are not expected to exert effort and indeed do not exert effort.⁷ The question is whether there also exist interior equilibria with $e^* > 0$.

To address this question, I present $BR(\hat{e})$ in Figure 1, using Lemma 2. As the figure shows, $BR(\hat{e})$ coincides with the horizontal axis for sufficiently small values of \hat{e} . Assumption (*) ensures that there exists a critical value of \hat{e} , denoted \hat{e}_1 , above which $BR(\hat{e})$ becomes positive. Part (ii) of Lemma 2 shows that $BR(\hat{e})$ increases for all $e > \hat{e}$, although eventually its slope becomes flat. Recalling that the equilibrium effort level of talented agents, e^* , is determined by the intersection of $BR(\hat{e})$ with the 45° line, it is clear from Figure 1 that in general, there are two possibilities.

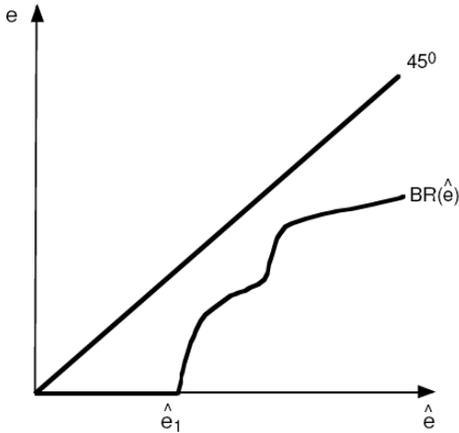


Figure 1a: No interior equilibria

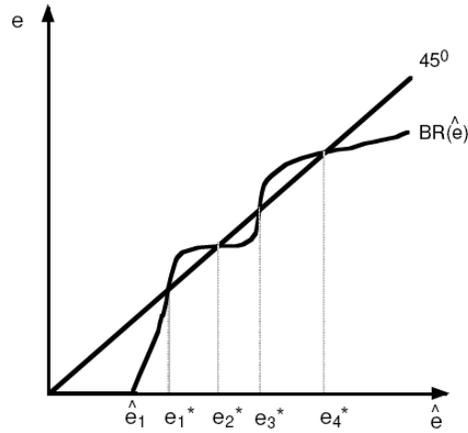


Figure 1b: Four interior equilibria

The first possibility, illustrated in Figure 1a, arises when $BR(\hat{e})$ lies below the 45° line for all $\hat{e} > 0$. In this case, the model does not admit interior equilibria with $e^* > 0$. A sufficient (though not necessary) condition for this case is that $BR'(\hat{e}) < 1$ for all $\hat{e} > \hat{e}_1$. This condition is likely to hold if $p'(\cdot)$ is small relative to $p''(\cdot)$.

⁷Intrestingly, the Athey and Ellison (2006) model also admits a “no-effort” equilibrium in which programmers do not contribute to open source projects. In their dynamic model, this equilibrium is driven by the fact that potential contributors to open source projects are former users. An open source software with 0 quality attracts no users, and hence has no future contributors. As a result, its quality can never improve.

The second possibility, illustrated in Figure 1b, arises when $BR(\hat{e})$ intersects the 45^0 line at least once from below at some $\hat{e} > \hat{e}_1$.⁸ Since $\lim_{\hat{e} \rightarrow \infty} BR'(\hat{e}) = 0$, $BR(\hat{e})$ must also intersect the 45^0 line from above at least once. Hence, if there are interior equilibria with $e^* > 0$, then generically, their number must be even.

A necessary (but not sufficient) condition for the existence of only two interior equilibria (in addition to the no-effort equilibrium) is that $BR''(\hat{e}) < 0$. Using (9), it follows that

$$BR''(\hat{e}) = -\frac{p'(e)}{p''(e)} \frac{d}{d\hat{e}} \left[\frac{\Delta'(\hat{e})}{\Delta(\hat{e})} \right].$$

Since $p'(e) > 0 > p''(e)$, it follows that $BR''(\hat{e}) < 0$ if and only if $\frac{d}{d\hat{e}} \left[\frac{\Delta'(\hat{e})}{\Delta(\hat{e})} \right] < 0$.

Notice that whenever $BR'(e^*) < 1$, the best response of talented agents, evaluated at the equilibrium point, is flatter than the 45^0 line and hence must cut it from above. The resulting interior equilibria (e_2^* and e_4^* in Figure 1b) are then stable in the sense that, starting from any close neighborhood of e^* , a Cournot tatônnement process will converge to e^* . Notice that the no-effort equilibrium is also stable since $BR'(0) = 0$. On the other hand, whenever $BR'(e^*) > 1$, $BR(e^*)$ is steeper than the 45^0 line and hence must cut it from below. Consequently, the resulting equilibria (e_1^* and e_3^* in Figure 1b) are unstable. The following proposition summarizes this discussion:

Proposition 1: *The model always admits a (stable) no-effort equilibrium in which $e^* = \hat{e}^* = 0$. A sufficient (but not necessary) condition for this equilibrium to be unique is that $BR'(\hat{e}) < 1$ for all $\hat{e} > \hat{e}_1$. However, if the model admits interior equilibria with $e^* > 0$, then generically, their number is even, with half being stable and half being unstable. A necessary (but not sufficient) condition for the model to admit only two interior equilibria is that $\frac{d}{d\hat{e}} \left[\frac{\Delta'(\hat{e})}{\Delta(\hat{e})} \right] < 0$.*

⁸It is also possible that $BR(\hat{e})$ is just tangent to the 45^0 line. Such tangency point is also an equilibrium, but this equilibrium is non-generic in the sense that it will vanish if we introduce small perturbations that shift $BR(\hat{e})$ either upward or downward. In the rest of the paper, I will therefore restrict attention to generic equilibria.

3.3 An example

To illustrate Proposition 1, suppose that $p(e) = 1 - \left(\frac{1}{2}\right)^{t+e}$. Clearly, $p(e)$ is increasing and concave, with $\lim_{e \rightarrow \infty} p(e) = 1$. Substituting $p(e)$ in equation (4) and rearranging terms, the best-response of talented agents against \hat{e} is given by

$$BR(\hat{e}) = \text{Max} \left\{ \frac{\ln(\beta \ln(2) \Delta(\hat{e}) w) - t \ln(2)}{\ln(2)}, 0 \right\}, \quad (10)$$

where

$$\Delta(\hat{e}) \equiv \frac{\alpha(1-\alpha)\left(\frac{1}{2}\right)^t \left(1 - \left(\frac{1}{2}\right)^{\hat{e}}\right)}{\left[\alpha\left(1 - \left(\frac{1}{2}\right)^{t+\hat{e}}\right) + (1-\alpha)\left(1 - \left(\frac{1}{2}\right)^t\right)\right] \left[1 - \beta\left(\alpha\left(1 - \left(\frac{1}{2}\right)^{t+\hat{e}}\right) + (1-\alpha)\left(1 - \left(\frac{1}{2}\right)^t\right)\right)\right]}.$$

To ensure that $BR(\hat{e}) > 0$ for sufficiently large \hat{e} , I will impose Assumption (*). In the context of the present example, this assumption amounts to

$$w > \frac{\left[1 - (1-\alpha)\left(\frac{1}{2}\right)^t\right] \left[1 - \beta\left(1 - (1-\alpha)\left(\frac{1}{2}\right)^t\right)\right]}{\left(\frac{1}{2}\right)^{2t} \ln(2) \alpha \beta (1-\alpha)}. \quad (11)$$

Setting $\alpha = \beta = 0.5$ and $t = 0.01$, I now show $BR(\hat{e})$ in Figure 2 for two values of w . When $w = 20$, there exist two interior equilibria - a stable equilibrium with $e^* = 2.065$ and an unstable equilibrium with $e^* = 0.008$. When $w = 6$, $BR(\hat{e})$ lies everywhere below the 45° line and hence there do not exist interior equilibria.

4 Comparative statics

Having characterized the equilibrium, I can now examine its comparative statics properties. I begin with the three conjectures of Lerner and Tirole (2002) which state that the signaling incentive of agents is stronger when:

- (i) performance becomes more visible to the relevant audience,
- (ii) effort has a stronger impact on performance, and

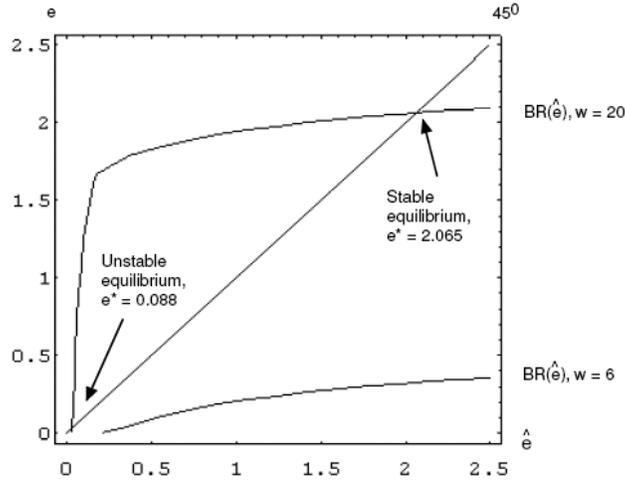


Figure 2: illustrating the example
Parameter values: $\alpha = \beta = 0.5$, $t = 0.01$

(iii) performance becomes more informative about talent.

Then I will proceed by studying how the signaling incentive of talented agents depends on the fraction of talented agents in the population of participating agents. At first blush it may seem that an increase in the fraction of talented agents may create a competitive pressure that will induce talented agents to exert more effort. Using my model I will examine whether this is true.

In all four cases, I will use the following result which is clear from Figure 1b:

Lemma 3: *An upward shift of $BR(\hat{e})$ increases the equilibrium effort level, e^* , in stable equilibria and decreases it in unstable equilibria, and conversely when $BR(\hat{e})$ shifts downward.*

4.1 The effect of increased visibility of performance

To examine conjecture (i), recall that β is a measure of the visibility of the agents' performance to firms. Hence, I examine the effect of visibility on the agents' incentives by looking at how e^* is affected by an increase in β :

Proposition 2: *An increase in β which measures the visibility of the agents' performance*

to firms, induces talented agents to exert more effort in stable interior equilibria and exert less effort in unstable interior equilibria.

Proof: In the case of interior equilibria, the best-response of talented agents, $BR(\hat{e})$, is implicitly defined by

$$\beta p'(e)\Delta(\hat{e})w = 1. \quad (12)$$

Using this equation,

$$\frac{\partial BR(\hat{e})}{\partial \beta} = -\frac{p'(e)}{\beta p''(e)} > 0.$$

Hence, an increase in β shifts $BR(\hat{e})$ upward, so Lemma 3 implies that an increase in β increases e^* in stable interior equilibria and decreases e^* in unstable interior equilibria.

■

Proposition 2 shows that Lerner and Tirole's (2002) conjecture that the signaling incentive of agents will become stronger as their performance becomes more visible to the relevant audience is true only if the model admits interior equilibria and only if these interior equilibria are stable.

Next, I will show that the effect of increased visibility of performance on effort depends not only on the stability of equilibrium, but also on whether effort and visibility are substitutes or complements in the production of positive signals on performance. To this end, note that thus far, I have assumed that the probability that an agent succeeds and his success is observed is $\beta p(e)$ if the agent is talented and βp_0 if the agent is untalented. This assumption implies that the effort of talented agents and the visibility of success are complements in the production of positive signals in the sense that an increase in β raises the marginal productivity of effort in producing positive signals. While this assumption is reasonable, one can also imagine the opposite case, where effort and visibility are substitutes rather than complements. For example, if effort contributes not only to the agents' performance, but is also required to attract attention to their performance, then an exogenous increase in visibility may allow agents to attract the same amount of attention with less effort. I now explore this possibility in the following proposition:

Proposition 3: *Assume that the probability that an agent succeeds and his success is ob-*

served is given by $p(e, \beta)$ if the agent is talented and by $p_0(\beta) \equiv p(0, \beta)$ if the agent is untalented, where $p(e, \beta)$ is increasing in e and β , strictly concave in e , and $\lim_{e \rightarrow \infty} p(e, \beta) = 1$. Moreover, assume that $\frac{\partial^2 p(e, \beta)}{\partial e \partial \beta} < 0$, so that e and β are substitutes in the production of positive signals (an increase in β lowers the marginal productivity of effort in producing positive signals). Then, $p_0(\beta) + \alpha(1 - p_0(\beta)) < \frac{1}{2}$ is sufficient for an increase in β to induce talented agents to exert less effort in stable interior equilibria and exert more effort in unstable interior equilibria.

Proof: See the Appendix.

Proposition 3 shows that the result of Proposition 2 can be reversed if e and β are substitutes in the production of positive signals about success rather than complements. This suggests in turn that whether increased visibility of performance enhances or weakens the signaling incentive of talented agents depends not only on the stability of equilibrium, but also on whether effort and visibility are complements in the production of positive signals (as implicitly assumed in Proposition 2) or substitutes (as assumed in Proposition 3).

To explore this issue further, I will now modify the example considered in Section 3.3 and assume that the probability of observing a success is $p(e, \beta) = 1 - \left(\frac{1}{2}\right)^{\beta+e}$ if the agent is talented and $p_0(\beta) \equiv p(0, \beta) = 1 - \left(\frac{1}{2}\right)^\beta$ if the agent is untalented. It is easy to verify that this example satisfies the assumptions in Proposition 3; in particular, $\frac{\partial^2 p(e, \beta)}{\partial e \partial \beta} = -(\ln(2))^2 \left(\frac{1}{2}\right)^{\beta+e} < 0$, so e and β are substitutes in the production of positive signals.

Proposition 4: *Suppose that the probability of observing a successful action is $p(e, \beta) = 1 - \left(\frac{1}{2}\right)^{\beta+e}$ if the agent is talented and $p_0(\beta) \equiv p(0, \beta) = 1 - \left(\frac{1}{2}\right)^\beta$ if the agent is untalented. Then, an increase in β induces talented agents to exert less effort in stable interior equilibria and exert more effort in unstable interior equilibria.*

Proof: Substituting $p(e, \beta)$ and $\frac{\partial p(e, \beta)}{\partial e} = \ln(2) \left(\frac{1}{2}\right)^{\beta+e}$ in equation (4) and rearranging terms, the best-response function of talented agents for sufficiently large values of \hat{e} is given by

$$BR(\hat{e}) = \frac{\ln(\ln(2)) \Delta(\hat{e}) w - \beta \ln(2)}{\ln(2)},$$

where

$$\Delta(\hat{e}) \equiv \frac{\alpha(1-\alpha)\left(1-\left(\frac{1}{2}\right)^{\hat{e}}\right)}{\left[1-\alpha+\alpha\left(\frac{1}{2}\right)^{\hat{e}}\right]\left[1-\alpha\left(\frac{1}{2}\right)^{\beta+\hat{e}}-(1-\alpha)\left(\frac{1}{2}\right)^{\beta}\right]}.$$

Differentiating $BR(\hat{e})$ with respect to β ,

$$\frac{\partial BR(\hat{e})}{\partial \beta} = -\frac{1}{1-\alpha\left(\frac{1}{2}\right)^{\beta+\hat{e}}-(1-\alpha)\left(\frac{1}{2}\right)^{\beta}} < 0.$$

Lemma 3 therefore implies the result. ■

Similarly to Proposition 3, Proposition 4 also shows that when effort and visibility are substitutes in the production of positive signals rather than complements, then increased visibility may induce talented agents to exert less effort even when attention is restricted only to stable equilibria.

4.2 The effect of increased sensitivity of performance to effort

Next, I examine the conjecture that the signaling incentive of agents will become stronger when effort has a stronger impact on performance. To this end, I will introduce a new shift parameter, γ , which increases the probability of talented agents to succeed at each effort level. That is, I will assume that the probability that a talented agent will succeed in OSS projects is given by $p(e, \gamma)$, where $\frac{\partial p(e, \gamma)}{\partial \gamma} > 0$. The probability that an untalented agent will succeed remains p_0 . To keep the notation simple, I will continue to denote the derivative of $p(e, \gamma)$ with respect to e by $p'(e, \gamma)$. I can now examine the effect of increased sensitivity of performance to effort on the signaling incentive of talented agents by studying how an increase in γ affects e^* :

Proposition 5: *Let $\varepsilon_{p'\gamma} \equiv \frac{\partial p'(e^*, \gamma)}{\partial \gamma} \frac{\gamma}{p'(e^*, \gamma)}$ be the elasticity of $p'(e^*, \gamma)$ with respect to γ and $\varepsilon_{\Delta\gamma} \equiv \frac{\partial \Delta(e^*, \gamma)}{\partial \gamma} \frac{\gamma}{\Delta(e^*, \gamma)}$ be the elasticity of $\Delta(e^*, \gamma)$ with respect to γ . Then, an increase in γ which implies that effort has a stronger effect on performance induces talented agents to exert more effort in interior equilibria if either (i) $\varepsilon_{p'\gamma} + \varepsilon_{\Delta\gamma} > 0$ and the equilibrium is stable, or (ii) $\varepsilon_{p'\gamma} + \varepsilon_{\Delta\gamma} < 0$ and the equilibrium is unstable. Otherwise, an increase in γ induces talented agents to exert less effort in interior equilibria.*

Proof: Given the shift parameter, γ , the best-response of talented agents for sufficiently large values of \hat{e} is implicitly defined by the following first order condition:

$$\beta p'(e, \gamma) \Delta(\hat{e}, \gamma) w = 1.$$

Using this equation,

$$\begin{aligned} \frac{\partial BR(\hat{e})}{\partial \gamma} &= - \frac{\frac{\partial p'(e, \gamma)}{\partial \gamma} \Delta(\hat{e}, \gamma) + p'(e, \gamma) \frac{\partial \Delta(\hat{e}, \gamma)}{\partial \gamma}}{p''(e, \gamma)} \\ &= [\varepsilon_{p'\gamma} + \varepsilon_{\Delta\gamma}] \times \frac{\Delta(\hat{e}, \gamma) p'(e, \gamma)}{-p''(e, \gamma) \gamma}. \end{aligned}$$

The proof follows by recalling that $p'(e, \gamma) > 0 > p''(e, \gamma)$. ■

Proposition 5 shows that an increase in γ can shift $BR(\hat{e})$ either upward or downward, depending on the sign of $\varepsilon_{p'\gamma} + \varepsilon_{\Delta\gamma}$. Noting that

$$\frac{\partial \Delta(e^*)}{\partial \gamma} = \frac{\alpha(1-\alpha) \frac{\partial p(e^*, \gamma)}{\partial \gamma} [\beta \alpha^2 (p(e^*, \gamma) - p_0)^2 + p_0(1 - \beta p_0)]}{(\alpha p(e^*, \gamma) + (1-\alpha)p_0)^2 (1 - \beta(\alpha p(e^*, \gamma) + (1-\alpha)p_0))^2} > 0,$$

it follows that $\varepsilon_{\Delta\gamma} > 0$. Hence, if $\varepsilon_{p'\gamma}$ is positive (which is the case if $\frac{\partial p'(e^*, \gamma)}{\partial \gamma} > 0$) or not too negative, then $BR(\hat{e})$ shifts upward when γ increases, so e^* increases in stable interior equilibria and decreases in unstable interior equilibria. Otherwise, if $\varepsilon_{p'\gamma} + \varepsilon_{\Delta\gamma} < 0$, then $BR(\hat{e})$ shifts downward and the reverse holds. Hence, an increase in γ can either lead to more effort by talented agents, as Lerner and Tirole's (2002) conjecture, or to less effort, contrary to their conjecture.

To illustrate Proposition 5, I will now modify the example from Section 3.3 by assuming that $p(e, \gamma) = 1 - \left(\frac{1}{2}\right)^{t+\gamma e}$, where $\gamma > 0$. It is easy to verify that $p(e, \gamma)$ is increasing in e and γ , strictly concave in e , and $\lim_{e \rightarrow \infty} p(e, \beta) = 1$. Moreover, $p'(e, \gamma) = \gamma \ln(2) \left(\frac{1}{2}\right)^{t+\gamma e}$ is first increasing and then decreasing with γ . To ensure that $BR(\hat{e}) > 0$ for sufficiently large values of \hat{e} , I will now impose Assumption (*), which can be written in the context of the

example as follows:

$$w > \frac{\left[1 - (1 - \alpha) \left(\frac{1}{2}\right)^t\right] \left[1 - \beta \left(1 - (1 - \alpha) \left(\frac{1}{2}\right)^t\right)\right]}{\left(\frac{1}{2}\right)^{2t} \ln(2) \alpha \beta \gamma (1 - \alpha)}.$$

Substituting for $p(e, \gamma)$ and $p'(e, \gamma)$ in equation (4) and rearranging terms, the best-response function of talented agents for sufficiently large values of \hat{e} , is given by

$$BR(\hat{e}) = \frac{\ln(\beta \gamma \ln(2) \Delta(\hat{e}) w) - t \ln(2)}{\gamma \ln(2)},$$

where

$$\Delta(\hat{e}) \equiv \frac{\alpha (1 - \alpha) \left(\frac{1}{2}\right)^t \left(1 - \left(\frac{1}{2}\right)^{\gamma \hat{e}}\right)}{\left[\alpha \left(1 - \left(\frac{1}{2}\right)^{t + \gamma \hat{e}}\right) + (1 - \alpha) \left(1 - \left(\frac{1}{2}\right)^t\right)\right] \left[1 - \beta \left(\alpha \left(1 - \left(\frac{1}{2}\right)^{t + \gamma \hat{e}}\right) + (1 - \alpha) \left(1 - \left(\frac{1}{2}\right)^t\right)\right)\right]}.$$

To examine the effect of γ on e^* , let $\alpha = \beta = 0.5$, $t = 0.01$, and $w = 20$. Figure 3a shows that when γ increases from 0.5 to 1, $BR(\hat{e})$ shifts upward and the effort of talented agents in the stable interior equilibrium increases from 1.856 to 2.066. However, Figure 3b shows that when γ increases from 0.5 to 2, $BR(\hat{e})$ rotates clockwise, and the effort of talented agents in the stable equilibrium decreases from 1.856 to 1.563. Consequently, the relationship between γ and e^* is non-monotonic.

To examine this nonmonotonicity further, Figure 4 shows e^* as a function of γ for $\alpha = \beta = 0.5$, $t = 0.01$, and $w = 20$. When γ is small, there do not exist interior equilibria. When $\gamma > 0.356$, there exist for each value of γ two interior equilibria: a stable equilibrium with a high e^* and an unstable equilibrium with a low e^* . Focusing on stable equilibria (the upper contour in Figure 4), one can see that e^* increases as γ increases from 0.356 to 0.754. However, once $\gamma > 0.754$, a further increase in γ leads to a decrease in e^* . Hence, the effect of γ on e^* can be positive or negative even when attention is restricted to stable equilibria.

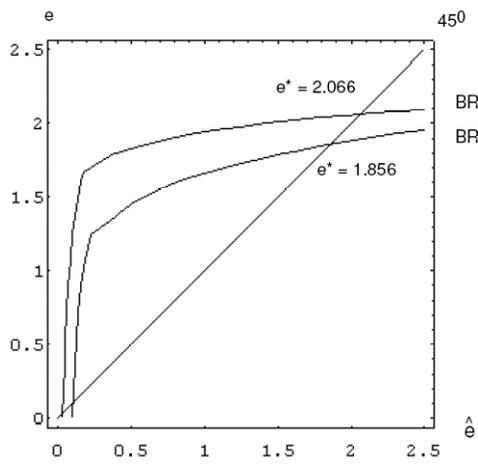


Figure 3a: illustrating the effect of an increase in γ from 0.5 to 1.
 Parameter values: $\alpha = \beta = 0.5$, $t = 0.01$, $w = 20$

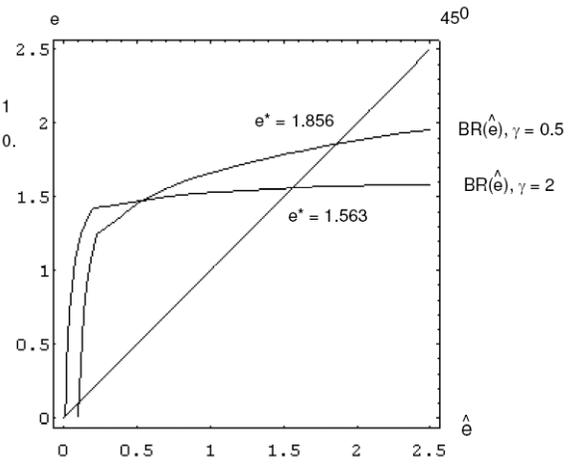


Figure 3b: illustrating the effect of an increase in γ from 0.5 to 2.
 Parameter values: $\alpha = \beta = 0.5$, $t = 0.01$, $w = 20$

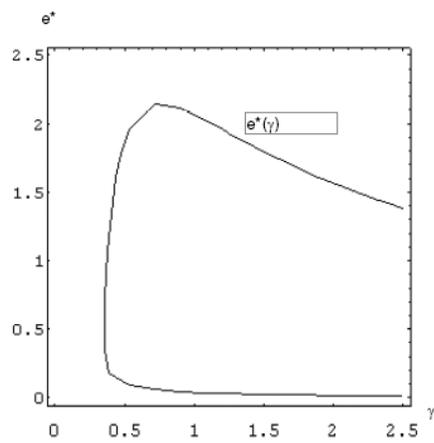


Figure 4: the effect of γ on e^*
 Parameter values: $\alpha = \beta = 0.5$, $t = 0.01$, $w = 20$

4.3 The effect of the informativeness of performance about talent

Conjecture (iii) of Lerner and Tirole states that the signaling incentive of agents will become stronger as performance becomes more informative about talent. This conjecture can be examined by studying the effect of p_0 on the equilibrium effort level of talented agents, e^* , because a decrease in p_0 implies that successful agents are more likely to be talented. That is, when p_0 decreases towards 0, the probability that a successful agent is talented, $q(\hat{e} | s)$, increases towards 1.

Proposition 6: *A decrease in p_0 , which implies that performance is more informative about talent, induces talented agents to exert more effort in stable interior equilibria and exert less effort in unstable interior equilibria.*

Proof: Differentiating equation (12) with respect to e^* and p_0 and recalling that $\Delta(\hat{e}) \equiv q(\hat{e} | s) - q(\hat{e} | n)$ yields,

$$\begin{aligned} \frac{\partial BR(\hat{e})}{\partial p_0} &= -\frac{p'(e) \frac{\partial \Delta(\hat{e})}{\partial p_0}}{p''(e) \Delta(\hat{e})} \\ &= -\frac{p'(e)}{p''(e) \Delta(\hat{e})} \left[\frac{\partial q(\hat{e} | s)}{\partial p_0} - \frac{\partial q(\hat{e} | n)}{\partial p_0} \right] < 0, \end{aligned}$$

where the inequality follows because by assumption, $p'(e) > 0 > p''(e)$ and (1) implies that $\frac{\partial q(\hat{e} | s)}{\partial p_0} < 0$ while (2) implies that $\frac{\partial q(\hat{e} | n)}{\partial p_0} > 0$. Lemma 3 now implies the result. ■

Like Propositions 2, a decrease in p_0 shifts $BR(\hat{e})$ upward, so as a result, e^* increases if the equilibrium is stable but decreases if the equilibrium is unstable. Hence, just like in Proposition 2, the conjecture here is true in stable interior equilibria but not in unstable equilibria.

4.4 The effect of the informativeness of performance about talent

Having considered the three conjectures of Lerner and Tirole, I will now turn to yet another comparative static result - the effect of α on the equilibrium effort of talented agents. The question that I address is the following: suppose that a specific OSS project attracts a

large fraction of talented agents than another OSS project. Which project will provide programmers with stronger incentives to exert effort?

Proposition 7: *An increase in α , which implies that the pool of agents is more talented on average, induces talented agents to exert more effort in stable interior equilibria and exert less effort in unstable interior equilibria if α is relatively small and conversely if α is relatively large.*

Proof: Differentiating equation (12) with respect to e^* and α , yields,

$$\begin{aligned} \frac{\partial BR(\hat{e})}{\partial \alpha} &= -\frac{p'(e) \frac{\partial \Delta(\hat{e})}{\partial \alpha}}{p''(e) \Delta(\hat{e})} \\ &= -\frac{p'(e)}{p''(e) \Delta(\hat{e})} \left[\frac{(p(\hat{e}) - p_0) T(\alpha)}{(\alpha p(\hat{e}) + (1 - \alpha) p_0)^2 (1 - \beta (\alpha p(\hat{e}) + (1 - \alpha) p_0))^2} \right], \end{aligned}$$

where

$$T(\alpha) \equiv (1 - \alpha)^2 p_0 (1 - p_0) - \alpha^2 p(\hat{e}) (1 - p(\hat{e})).$$

Since $p'(e) > 0 > p''(e)$ and $p(\hat{e}) > p_0$ for all $\hat{e} > 0$, the sign of $\frac{\partial BR(\hat{e})}{\partial \alpha}$ depends on the sign of $T(\alpha)$. Clearly, $T'(\alpha) < 0$ and $T(0) > 0 > T(1)$. Hence, for each \hat{e} , there exists a unique value of α , denoted $\bar{\alpha}(\hat{e})$, where $\bar{\alpha}(\hat{e}) \in (0, 1)$, such that $\frac{\partial BR(\hat{e})}{\partial \alpha} > 0$ for $\alpha \in [0, \bar{\alpha}(\hat{e})]$ and $\frac{\partial BR(\hat{e})}{\partial \alpha} < 0$ for $\alpha \in (\bar{\alpha}(\hat{e}), 1]$. Lemma 3 now implies the result. ■

An interesting implication of Propositions 7 is that agents may have a stronger incentive to exert effort in an OSS project when the pool of participants is on average less talented. Therefore if an OSS project wants to provide participants with a strong signaling incentive, then it is better off not attracting too many high talented participants.⁹ Moreover, Proposition 7 suggests the following interesting dynamics: suppose that an OSS project starts with a relatively talented pool of programmers. Over time, some will succeed and will be hired away by commercial software companies. Since talented programmers are more likely to produce positive signals and be hired away, the remaining pool of programmers will

⁹Of course, if programmers enjoy interacting with talented programmers and if there are complementarities among programmers (talented programmers create positive externalities), then the more talented the pool of programmers is, the more productive other participants are going to be. These considerations however are outside my model as I focus on the signaling incentive of programmers.

have a lower fraction of talented programmers. Proposition 7 implies that the faster attrition rate of talented programmers will first induce the remaining talented programmers to exert more effort and this will increase their probability of success and therefore accelerate their rate of attrition. Once the fraction of talented programmers drops below a critical level, the process will be reversed since the faster attrition of talented agents will now induce the remaining talented programmers to induce less effort and hence will lower their probability of producing positive signals and hence their rate of attrition.

5 Intrinsic motivation for participation in OSS projects

Up to now I have only considered extrinsic motivations for participation in OSS projects: talented agents take part in OSS projects in the hope of producing positive signals about their talent and thereby boosting their prospects in the labor market. However, this view is obviously too narrow given that many programmers contribute to OSS projects for other reasons, like their sense of creativity, their desire to solve problems that they face in performing daily tasks (e.g., system managers who fix bugs or add new functions to an existing software), or acquiring programming skills. The question is how such intrinsic motivations affect matters.¹⁰

To address this question, suppose that apart from their ability to boost their prospects in the labor market, agents also draw a positive utility v from successful contributions to OSS projects. This utility is independent of whether success is or is not observed by firms.¹¹ Given v , the utility of talented agents becomes

$$U(e, \hat{e}) = p(e) [v + \beta q(\hat{e} | s)w] + (1 - \beta p(e)) q(\hat{e} | n)w - e.$$

The effort level that each talented agent will choose given the firms' beliefs, \hat{e} , is now defined

¹⁰Bitzer, Schrettl, and Schröder (2006) study a dynamic model that involves both intrinsic and extrinsic motives (i.e., signaling) for participation in open source projects and explore the interaction between them.

¹¹Obviously, agents can also benefit from unsuccessful participation. However, in that case their utility will simply increase by a constant and hence their effort level will not be affected. To make things more interesting, I therefore assume that agents receive v only when their actions succeed.

implicitly by the following first order condition:

$$\frac{\partial U(e, \hat{e})}{\partial e} = p'(e) [v + \beta \Delta(\hat{e})w] - 1 \leq 0, \quad e \frac{\partial U(e, \hat{e})}{\partial e} = 0. \quad (13)$$

It is easy to see from (13) that v raises the marginal benefit from effort. Hence, other things being equal, v expands the set of parameters for which the model admits interior equilibria. Moreover, an increase in v shifts the best-response function of talented agents upward, so Lemma 3 implies that it will induce talented agents to exert more effort in stable interior equilibria but less effort in unstable equilibria.

6 Conclusion

The main finding in this paper is that the signaling incentive of programmers to contribute to OSS projects is more complex than it might seem at first glance. First, there always exists a no-effort stable equilibrium, and moreover this equilibrium may be unique if, for example, the marginal effect of effort on the probability of success is relatively small. This implies in turn that OSS projects may never take off. Second, when interior equilibria exist, there are generically an even number of them. This multiplicity of equilibria suggests that a given OSS project may induce a small level of effort or even no effort at all even though a seemingly identical project induces a large level of effort. Third, the comparative static properties of interior equilibria may in general go either way. In particular, shifts in exogenous parameters, like an increase in the visibility of performance and an increase in the marginal productivity of effort, may either boost or weaken the signaling incentive of talented agents depending on whether the equilibrium is stable or unstable and depending on the properties of the probability that successful performance will be observed by prospective employers. Therefore, a-priori it is in general impossible to tell whether increased visibility of performance and increased sensitivity of performance to effort will induce talented agents to exert more or less effort.

7 Appendix

Following are the proofs of Lemma 2 and Proposition 3.

Proof of Lemma 2: (i) Since $p''(e) < 0$, it is easy to see from equation (4) that $\frac{\partial U(e, \hat{e})}{\partial e}$ is a strictly decreasing function of e for all $\hat{e} > 0$. Moreover, the concavity of $p(e)$ and the assumption that $\lim_{e \rightarrow \infty} p(e) = 1$ imply that $\lim_{e \rightarrow \infty} p'(e) = 0$, so $\lim_{e \rightarrow \infty} \frac{\partial U(e, \hat{e})}{\partial e} = -1$ for all $\hat{e} > 0$. Since $\frac{\partial U(e, \hat{e})}{\partial e}$ is continuous in e , this implies that there exists a unique value of e at which $\frac{\partial U(e, \hat{e})}{\partial e} = 0$ if and only if

$$\frac{\partial U(0, \hat{e})}{\partial e} = \beta p'(0) \Delta(\hat{e}) w - 1 > 0. \quad (14)$$

Since Lemma 1 implies that $\Delta(0) = 0$, condition (14) clearly fails when $\hat{e} = 0$, and by continuity, it also fails for sufficiently small values of \hat{e} . Hence, $BR(0) = 0$ for small values of \hat{e} . On the other hand, since $\Delta'(\hat{e}) > 0$, it follows that $\frac{\partial U(0, \hat{e})}{\partial e}$ is increasing with \hat{e} and moreover,

$$\begin{aligned} \lim_{\hat{e} \rightarrow \infty} \frac{\partial U(0, \hat{e})}{\partial e} &= \beta p'(0) w \left(\lim_{\hat{e} \rightarrow \infty} \Delta(\hat{e}) \right) - 1 \\ &= \frac{\beta p'(0) w \alpha (1 - \alpha) (1 - p_0)}{(\alpha + (1 - \alpha) p_0) (1 - \beta (\alpha + (1 - \alpha) p_0))} - 1 > 0, \end{aligned} \quad (15)$$

where the second equality follows because by assumption, $\lim_{\hat{e} \rightarrow \infty} p(\hat{e}) = 1$, and the inequality follows by Assumption (*). Therefore, there exists a unique value of \hat{e} , denoted \hat{e}_1 , such that $\frac{\partial U(0, \hat{e})}{\partial e} > 0$ for all $\hat{e} > \hat{e}_1$ and $\frac{\partial U(0, \hat{e})}{\partial e} < 0$ otherwise, where \hat{e}_1 is implicitly defined by the equation

$$\frac{\partial U(0, \hat{e})}{\partial e} = \beta p'(0) w \Delta(\hat{e}) - 1 = 0.$$

In sum, whenever $\hat{e} \leq \hat{e}_1$, $\frac{\partial U(e, \hat{e})}{\partial e} < 0$ for all e , implying that $BR(\hat{e}) = 0$. On the other hand, whenever $\hat{e} > \hat{e}_1$, there exists a unique value of e that solves the equation $\frac{\partial U(e, \hat{e})}{\partial e} = 0$. Hence, $BR(\hat{e}) > 0$ for all $\hat{e} > \hat{e}_1$.

(ii) As part (i) shows, $BR(\hat{e})$ is defined implicitly by the equation $\frac{\partial U(BR(\hat{e}), \hat{e})}{\partial e} = 0$ for all $\hat{e} > \hat{e}_1$. Fully differentiating this equation with respect to \hat{e} and using Lemma 1 and

the fact that $p(e)$ is increasing and concave, reveals that $BR'(\hat{e})$, defined in equation (9), is positive.

Using equations (5) and (6), it follows that

$$\begin{aligned} \lim_{\hat{e} \rightarrow \infty} BR'(\hat{e}) &= -\frac{p'(e)}{p''(e)} \lim_{\hat{e} \rightarrow \infty} \frac{\Delta'(\hat{e})}{\Delta(\hat{e})} \\ &= -\frac{p'(e)}{p''(e)} \lim_{\hat{e} \rightarrow \infty} \left[\frac{p'(\hat{e}) [\beta\alpha^2 (p(\hat{e}) - p_0)^2 + p_0(1 - \beta p_0)]}{(\alpha p(\hat{e}) + (1 - \alpha)p_0)^2 (1 - \beta(\alpha p(\hat{e}) + (1 - \alpha)p_0))} \times \frac{1}{(p(\hat{e}) - p_0)} \right] = 0, \end{aligned}$$

where the last equality follows because the concavity of $p(e)$ and the assumption that $\lim_{e \rightarrow \infty} p(e) = 1$ implying that $\lim_{e \rightarrow \infty} p'(e) = 0$. ■

Proof of Proposition 3: Given the assumptions in the proposition, the increase in the probability that firms assign to an agent being talented following an observed success is given by

$$\begin{aligned} \Delta(\hat{e}, \beta) &\equiv q(\hat{e} | s) - q(\hat{e} | n) \\ &= \frac{\alpha p(\hat{e}, \beta)}{\alpha p(\hat{e}, \beta) + (1 - \alpha)p_0(\beta)} - \frac{\alpha(1 - p(\hat{e}, \beta))}{\alpha(1 - p(\hat{e}, \beta)) + (1 - \alpha)(1 - p_0(\beta))} \\ &= \frac{\alpha(1 - \alpha)M(\hat{e}, \beta)}{Z(\hat{e}, \beta)(1 - Z(\hat{e}, \beta))}, \end{aligned}$$

where $M(\hat{e}, \beta) \equiv p(\hat{e}, \beta) - p_0(\beta)$ and $Z(\hat{e}, \beta) \equiv \alpha p(\hat{e}, \beta) + (1 - \alpha)p_0(\beta)$. Note that $Z(\hat{e}, \beta)$ increases with \hat{e} and hence increases from $Z(0, \beta) = p_0(\beta)$ to $\lim_{\hat{e} \rightarrow \infty} Z(\hat{e}, \beta) = p_0(\beta) + \alpha(1 - p_0(\beta))$.

Assuming that \hat{e} is sufficiently large, the best-response of talented agents, $BR(\hat{e})$, is implicitly defined by

$$\frac{\partial p(e, \beta)}{\partial e} \Delta(\hat{e}, \beta) w = 1. \quad (16)$$

Using equation (16),

$$\frac{\partial BR(\hat{e})}{\partial \beta} = -\frac{\frac{\partial^2 p(e, \beta)}{\partial e \partial \beta} \Delta(\hat{e}, \beta) + \frac{\partial p(e, \beta)}{\partial e} \frac{\partial \Delta(\hat{e}, \beta)}{\partial \beta}}{\frac{\partial^2 p(e, \beta)}{\partial e^2}}.$$

Since $p(e, \beta)$ is strictly concave in e , the sign of $\frac{\partial BR(\hat{e})}{\partial \beta}$ depends on the sign of the numerator

of $\frac{\partial BR(\hat{e})}{\partial \beta}$. The first term in the numerator of $\frac{\partial BR(\hat{e})}{\partial \beta}$ is negative because $\frac{\partial^2 p(e, \beta)}{\partial e \partial \beta} < 0$. To determine the sign of the second term, note that

$$\frac{\partial \Delta(\hat{e}, \beta)}{\partial \beta} = \frac{\alpha(1 - \alpha)}{(Z(\hat{e}, \beta)(1 - Z(\hat{e}, \beta)))^2} \times \left[\frac{\partial M(\hat{e}, \beta)}{\partial \beta} Z(\hat{e}, \beta)(1 - Z(\hat{e}, \beta)) - \frac{\partial Z(\hat{e}, \beta)}{\partial \beta} (1 - 2Z(\hat{e}, \beta)) M(\hat{e}, \beta) \right],$$

where $\frac{\partial Z(\hat{e}, \beta)}{\partial \beta} \equiv \alpha \frac{\partial p(\hat{e}, \beta)}{\partial \beta} + (1 - \alpha)p'_0(\beta) > 0$ and $\frac{\partial M(\hat{e}, \beta)}{\partial \beta} = \frac{\partial p(\hat{e}, \beta)}{\partial \beta} - p'_0(\beta) < 0$ because the assumption that $\frac{\partial^2 p(e, \beta)}{\partial e \partial \beta} < 0$ implies that $\frac{\partial p(\hat{e}, \beta)}{\partial \beta} < \frac{\partial p(0, \beta)}{\partial \beta} \equiv p'_0(\beta)$. Hence, $Z(\hat{e}, \beta) \leq \frac{1}{2}$ is sufficient for $\frac{\partial \Delta(\hat{e}, \beta)}{\partial \beta} < 0$ and hence for $\frac{\partial BR(\hat{e})}{\partial \beta} < 0$. Recalling that $Z(\hat{e}, \beta) < p_0(\beta) + \alpha(1 - p_0(\beta))$, it follows that if $p_0(\beta) + \alpha(1 - p_0(\beta)) < \frac{1}{2}$ then $\frac{\partial BR(\hat{e})}{\partial \beta} < 0$. Lemma 3 now implies the result. ■

8 References

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