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**Conference Paper**

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# The St. Petersburg Paradox despite risk-seeking preferences

*An experimental study*

Eike Benjamin Kroll and Bodo Vogt



*Working Paper*

## **Abstract**

The St. Petersburg Paradox is one of the oldest violations of expected utility theory. Thus far, explanations of the paradox aim at small probabilities being perceived as zero and the boundedness of utility. This paper provides experimental results showing that neither risk attitudes nor perception of small probabilities explain the paradox. We find that even in situations where subjects are risk-seeking, the St. Petersburg Paradox exists. This indicates that the paradox lies at the very core of human decision-making processes and cannot be explained by the parameters discussed in previous research so far.

## Introduction

The St. Petersburg Paradox has attracted various researchers so far and the work provides a puzzle to the very core of economic theories (Cox et al. 2008). In the original version of the St. Petersburg Game a fair coin is tossed until it comes up heads for the first time. The game pays  $2^n$  with  $n$  indicating the number of tosses it took for the first occurrence of heads. While the St. Petersburg Game in its original version offers an infinite expected value, people are found not to pay more than \$25 for participating in the game (Hacking 1980). Various researchers have provided explanations for the paradox, but with every explanation a new version of the initial game was constructed that brought the puzzle back (Samuelson 1977).

The first explanation for the observed behavior was decreasing marginal utility of risk-averse agents (Bernoulli 1954), however, the game can be constructed correcting for decreasing marginal utility and the paradox remains. Therefore, the focus shifted towards the question of infinity. Limited time was introduced as the factor putting a bound to the utility of the St. Petersburg Game (Brito 1975; Cowen and High 1988). In contrast it was argued, that the utility of the game could in principle be unbounded but the offer is most probably not considered genuine (Shapley 1977) causing the decision patterns found in experimental investigations. The most straightforward solution of the paradox, however, is that utility is bounded since otherwise one can always create lotteries leading to counterintuitive solutions (Aumann 1977). To avoid infinity the St. Petersburg Game was broken down into a series of finite games, but the paradox still exists (Samuelson 1960). This fact does not indicate that infinity is the underlying cause of the paradox.

Other work argues that the small probabilities cause the paradox since sufficiently small probabilities are regarded as zero (Brito 1975) or small chances for large prices create big risks for the agents (Weirich 1984; Allais 1952). Another approach on using probabilities as an explanation for the phenomenon, more recent work introduced a new weighting function for Cumulative Prospect Theory solving the problem of infinity (Blavatsky 2005).

Despite the different aspects of the discussion about the St. Petersburg Paradox, the most recent work on the issue addresses the origin that was using decisions about St. Petersburg Lotteries to argue in favor of introducing a model of risk-averse behavior. The issue of people showing risk-averse behavior in this type of decisions in the laboratory was stressed recently in a real-payoff setting of the St. Petersburg Game showing that it elicits decisions that are inconsistent with Expected Value Theory (Cox, Sadiraj, and Vogt 2009). Furthermore, it has to be noted that recent work on theories of decisions under risk model risk-averse behavior in a way to accommodate decisions from experiments on the St. Petersburg Paradox.

While the paradox was initially developed to criticize Expected Value Theory inspiring models of risk-averse behavior, this paper will provide results leading to the conclusion that risk preferences do not explain behavior in St. Petersburg Games. An experimental setting is introduced where waiting time is used rather than monetary payoffs. It is shown that preferences over waiting time induce risk-seeking behavior. Then, a St. Petersburg Game is constructed similar to the experiment in (Cox, Sadiraj, and Vogt 2009), but using waiting time instead of monetary payoffs. While the decisions over waiting time elicit different risk-preferences than decisions over monetary payoffs it can be shown that decision patterns in St. Petersburg Games are similar. Therefore, the results of this paper will raise doubt about whether risk preferences can explain behavior in these types of decisions.

## Experiment

The experiment was conducted with 74 students from the Otto-von-Guericke-University Magdeburg from different fields of study. The experiment was conducted in a laboratory environment. The participants were divided into three groups, with one eliciting risk-preferences for waiting time (Treatment 1, 36 participants), and two groups playing St. Petersburg Lotteries with different base waiting times (Treatment 2, 25 participants and Treatment 3, 24 participants). All participants received a show-up fee of 8 Euros before experimental instructions were handed out. Furthermore, it was made clear that there will be no further monetary rewards for the experiment and only the consequences described in the experimental instructions will occur during the rest of the experiment.

### Risk Preference for Waiting Time

To elicit time preferences of subjects, participants were asked to choose between two lotteries, where payoffs were determined as waiting time. The choice of the options is within the random lottery payoff mechanism (Grether and Plott 1979). In our experiment participants were paid a show-up fee of 6 Euros at the beginning of the experiment and told that their decisions were determining a waiting time in the laboratory. This waiting time started after all decisions were made and the chosen lotteries were played out. The participants spent this time in an experimental cabin without any communication devices or books.

The options were chosen in a form that is in line with (Holt and Laury 2002), where Option A offered less risk, but a higher sure waiting time (with a waiting time of either 30 or 40 minutes) and Option B offered a higher risk, but the chance of a much smaller waiting time (with a waiting time of either 5 or 60 minutes). The probabilities of the favorable outcome stayed the same for both options, but varied between .1 and 1.0 as shown in Table 1: Therefore, risk preferences for waiting time could be elicited for each participant by the row in which option B was chosen for the first time. If that point was in row 4 or earlier the choice pattern indicates risk-seeking behavior, if it was in row 6 or later the choice pattern indicates risk-averse behavior. The risk attitude for subjects switching to option B in row 5 cannot be identified since they can either be slightly risk-averse, risk-neutral or slightly risk-seeking.

No.	Option A	Option B	Expected Value difference
1	{.1, 30, .9, 40}	{.1, 5, .9, 60}	-15.5
2	{.2, 30, .8, 40}	{.2, 5, .8, 60}	-11
3	{.3, 30, .7, 40}	{.3, 5, .7, 60}	-6.5
4	{.4, 30, .6, 40}	{.4, 5, .6, 60}	-2
5	{.5, 30, .5, 40}	{.5, 5, .5, 60}	2.5
6	{.6, 30, .4, 40}	{.6, 5, .4, 60}	7
7	{.7, 30, .3, 40}	{.7, 5, .3, 60}	11.5
8	{.8, 30, .2, 40}	{.8, 5, .2, 60}	16
9	{.9, 30, .1, 40}	{.9, 5, .1, 60}	20.5
10	{1.0, 30, 0.0, 40}	{1.0, 5, 0.0, 60}	25

Table 1: Lottery choices determining waiting time

After the choices were made, the experimenter drew a ball from a bingo cage with balls labeled from 1 to 10, determining which choice was selected. Then, the lottery the participant chose for that row was realized and the waiting time started.

### St. Petersburg Game

In a second step participants were offered a series of St. Petersburg Lotteries. All subjects had a base waiting time (Treatment 2, 10 minutes; Treatment 3, 45 minutes) and were offered to participate in a game where this waiting time could be reduced or increased depending on the outcome of the game. This game was designed analogous to the St. Petersburg game used in (Cox, Sadiraj, and Vogt 2009). For participation in the game the waiting time was reduced by  $n$  minutes and a coin is tossed until tails occurs with a maximum of  $n$  tosses. If tails occurs at the  $i$ -th toss, the waiting time was increased by  $2^i$  minutes. Each participant was offered 9 games with only one choice being realized (Grether and Plott 1979), with the games differing by the maximum number of tosses  $n$  (see Table 2:). For example, suppose decision 3 was randomly picked to be realized for a subject. If the subject chose not to play this game, the resulting waiting time was at the base waiting time of 10 (45) minutes. If the subject chose to play the game, the base waiting time was reduced by 3 minutes to 7 (42) minutes. Then, a coin was tossed. If it came up tails at the first toss the waiting time was increased by 2 minutes to 9 (44) minutes. If it came up tails at the

second toss, the waiting time was increased by 4 minutes to 11 (46) minutes. If it came up tails at the third toss, the waiting time was increased by 8 minutes to 15 (50) minutes. If the coin did not come up tails at any of the three tosses, the waiting time remained at 7 (42) minutes.

<i>Tails</i> occurs the first time at toss no.:	Probability	Additional waiting time [in minutes]
1	0,5	2
2	0,25	4
3	0,125	8
4	0,0625	16
5	0,03125	32
6	0,015625	64
7	0,0078125	128
8	0,0039062	256
9	0,0019531	512
Not at all		+/-0

*Table 2: St. Petersburg Game for waiting time.*

After the participants made their choices, the experimenter drew a ball from a bingo cage numbered from 1 through 9 determining which choice was selected for realization. If the participant chose not to play the game offered, the base waiting time was realized and started immediately. If the participant chose to play the game, the experimenter tossed the coin as described above and determined the actual waiting time. All participants spent their waiting time in an experimental cabin without communication devices or other kinds of entertainment possibility. To control for reference-dependence of preferences (Kőszegi and Rabin 2007; Farber 2008), we ran two treatments with different base-waiting times of 10 and 45 minutes.

## Results

### Risk Preferences for Waiting Time

As described in the experimental setting subjects can be sorted as risk-seeking and risk-averse for risky choices on waiting time by looking at the first row in which option B is chosen. In Table 1: it can be seen from the differences in expected values, that risk-seeking individuals would choose option B for the first time in row 4 or earlier, while the switching point from option A to option B would be in row 5 or later for risk-averse subjects. The frequencies for rows in which subjects switched to option B are reported in Table 3:. That means, a subject that chooses option A in rows 1 through 3 and chooses option B in rows 4 through 10 is noted in column 4, while a subject choosing option A in rows 1 through 4 and then switches to option B is noted in column 5. From the expected value differences in Table 1: it can be seen that subjects listed in columns 1 through 4 are risk-seeking and subjects listed in columns 6 through 10 are risk-averse. Subjects that are listed in column 5 cannot be clearly identified as indicated in the section above.

Risk Preference	risk-seeking					risk-averse			
Row of first choosing option B	1	2	3	4	5	6	7	8-10	$\Sigma$
Frequency	1	1	6	19	4	2	2	1	36

Table 3: Risk preference for waiting time

One subject was excluded from analysis, because of switching from option A to B and back to A for a number of times. Assuming a standard utility function this behavior cannot be explained by such a functional form. Furthermore, since it is only one subject showing this behavior it can be assumed as error.

The data set shows 27 subjects showing risk-seeking behavior while 9 subjects made choices showing risk-averse behavior. Therefore we conclude that people show risk-seeking behavior when making decisions about time, where the outcome is subject to risk (1%-level, Binomial-Test).

### St. Petersburg Game

Knowing the results from Treatment 1, one can conclude that the subjects in this study would tend to play all of the offered games. The expected value of the offered gambles on waiting times is equal to the base waiting time. Therefore, a risk-seeking individual would choose to participate in all offered gambles. The results of the St. Petersburg Lotteries show, that while individuals do participate in the gambles for small reductions of the base waiting time, they do not for higher possible reductions of the base waiting time. Therefore, decision patterns are similar to the ones found for real-payoff decisions for the St. Petersburg Game (Cox, Sadiraj, and Vogt 2009).

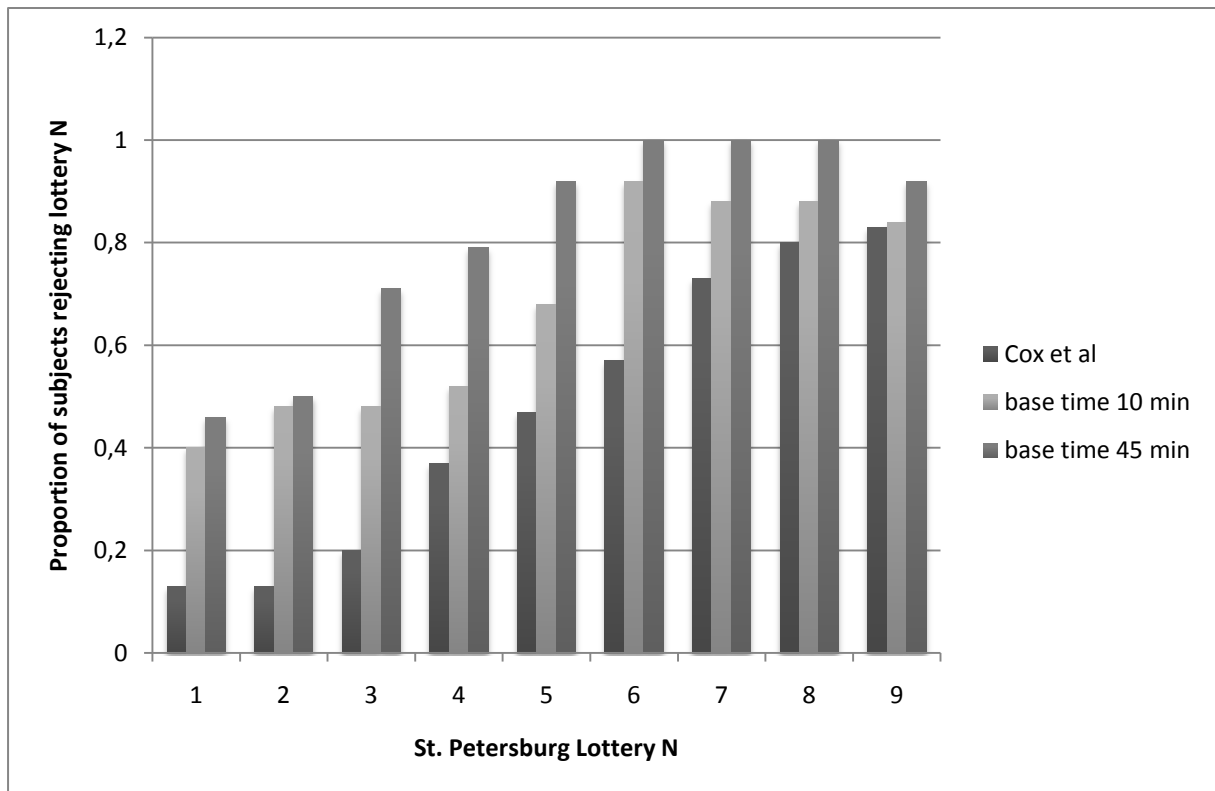


Figure 1: Comparison of rejecting St. Petersburg Lotteries of risk-averse and risk-seeking preferences

In the treatment with a base waiting time of 10 minutes, 2 of the 25 participants choose never to play the game, while the rest mostly starts playing the first game, but switches to answering 'no' along the line. None of the participants chooses to play all offered games.

The data from the treatment with a base waiting time of 45 minutes yields similar results. There is a lower number of subjects to play games where high reductions of the base waiting time than observed in the first treatment. However, the difference is not significant on a statistical level.

## Conclusion

Initially the St. Petersburg Paradox was designed to point out a weakness of Expected Value Theory showing that decision makers are not risk-neutral. As a result the idea of using utilities rather than monetary payoffs introducing decreasing marginal utility for money was developed. From that point on, economists have focused on developing theories of decisions under risk that account for risk-averse behavior as initially found in the investigations of the St. Petersburg Game. Various possibilities of modeling risk-averse behavior in decisions over risky prospects have been proposed in the literature. Risk-averse behavior can be incorporated by weighting of utilities (Bernoulli 1954; Kahneman and Tversky 1979), the weighting of probabilities (Yaari 1987) or a combination of both (Kahneman and Tversky 1992).

While these theories aim at incorporating various anomalies found by experimental economists, the St. Petersburg Paradox still provides puzzles for economists (Cox and Sadiraj



2008; Rieger and Wang 2006). The most recent experiment on decisions on St. Petersburg lotteries raised again the general question of risk preferences in the context of the St. Petersburg Paradox. The experiment in this paper shows, that the same game derives very similar results for risk-seeking and risk-averse preferences. Thus, one has to come to the conclusion that risk preferences are not a conclusive explanation for the behavior found for the St. Petersburg Game.

Risk aversion is, however, not the only solution to the paradox as proposed by the literature. There is also the problem of infinity (Brito 1975; Cowen and High 1988) which is associated with the original form of the game. Therefore, when playing St. Petersburg Lotteries with real consequences a series of finite St. Petersburg Lotteries is used. Furthermore, the question was raised whether participants would regard the offer of the original game as genuine (Shapley 1977). Both types of explanation do not hold for the findings in this paper. While infinity is not the problem in this type of game, it is not possible to control whether participants regarded the offer as genuine or not. However, both explanations would create a behavioral pattern in the experiment of this paper that is the opposite of what was observed. For the point of infinity, if subjects would not believe the experimenter to toss a coin as often as proposed, it would be a safe bet to play the games with a higher number of maximum tosses. Furthermore, if subjects would regard an offer as not genuine it is reasonable to assume that the longer waiting times would not be realized and, therefore, a subject playing the games with low  $n$ , would definitely play the games with high  $n$ .

Another argument explaining the behavioral patterns in St. Petersburg Games is that utility is bounded in general (Aumann 1977). For the version of the game proposed in this paper, an upper bound of utility is the waiting time of zero which is not realized for any of the games proposed. Since the waiting time in this experiment can be interpreted as a loss, the bound of utility would have to occur at the maximum loss that can be perceived. If such a bound exists it would induce the same behavioral pattern as described for the problem of infinity. Therefore, a bound in the dimension of utility cannot explain the same behavioral pattern for monetary payoffs and waiting time.

Other papers argued that very small probabilities are regarded as zero (Brito 1975) or that small probabilities for high wins result in a high risk for the decision maker (Allais 1952; Weirich 1984). These explanations also do not help explaining our results.

In conclusion it can be noted that various issues have been associated with the St. Petersburg Game. However, none of the issues can explain what the core of the paradox is, when it comes to decisions with real outcomes. Whether weighting functions are used for payoffs, probabilities or both, the existing models induce risk-preferences for subjects. As a conclusion of these models, the behavior in the St. Petersburg Game does depend on the risk preference. Therefore, none of the theories can explain the results of this paper.

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