

Strulik, Holger

**Working Paper**

Patience and prosperity

Diskussionsbeitrag, No. 426

**Provided in Cooperation with:**

School of Economics and Management, University of Hannover

*Suggested Citation:* Strulik, Holger (2009) : Patience and prosperity, Diskussionsbeitrag, No. 426, Leibniz Universität Hannover, Wirtschaftswissenschaftliche Fakultät, Hannover

This Version is available at:

<https://hdl.handle.net/10419/37105>

**Standard-Nutzungsbedingungen:**

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

**Terms of use:**

*Documents in EconStor may be saved and copied for your personal and scholarly purposes.*

*You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.*

*If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.*

# Patience and Prosperity

Holger Strulik\*

This Version: July 2009

**Abstract.** This paper introduces wealth-dependent time preference into a simple model of endogenous growth. The model generates adjustment dynamics in line with the historical facts on savings and economic growth in Europe from the High Middle Ages to today. Along a virtuous cycle of development more wealth leads to more patience, which leads to more savings and even higher wealth. Savings rates and income growth rates are thus jointly increasing during the process of development until they converge towards constants along a balanced growth path. During the transition to modern growth an economy in which the association of wealth and patience is stronger overtakes an otherwise identical economy and generates temporarily diverging growth rates. It is shown how wealth-dependent time preference can explain the existence of a locally stable poverty trap as well as the phenomenon of simultaneously falling interest rates and rising growth rates.

*Keywords:* economic growth, savings, time preference, poverty trap, moral consequences of economic growth.

*JEL:* O11, O41, D90, P48.

---

\* University of Hannover, Department of Economics, Koenigsworther Platz 1, 30167 Hannover, Germany, email: strulik@vwl.uni-hannover.de. I would like to thank Franz Wirl, Volker Grossmann, Alexia Prskawetz, and Uwe Sunde for helpful comments.

*Poverty bears down heavily on all portions of a man's expected life. But it increases the want for immediate income even more than it increases the want for future income.*

(Irving Fisher)

*He that can have patience, can have what he will.*

(Benjamin Franklin)

## 1. INTRODUCTION

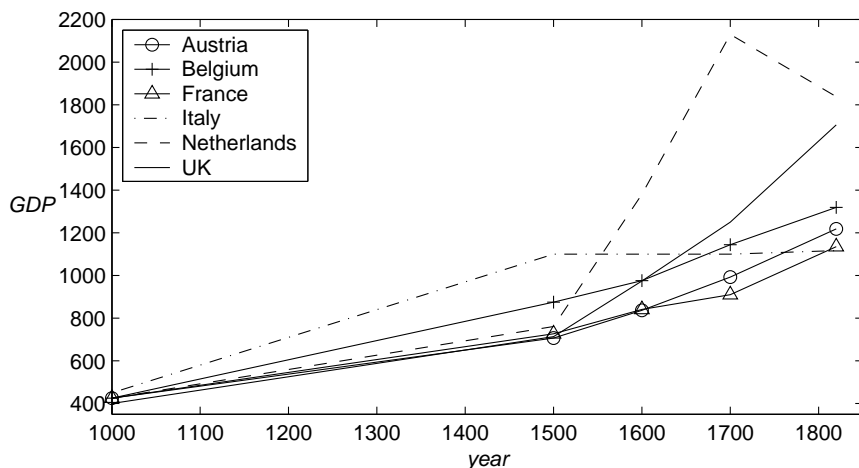
This article proposes a simple preference-driven theory of economic growth. It establishes a savings channel through which the character trait of patience operates simultaneously as cause and consequence of economic growth.

Ever since the work of Barro (1991) and Mankiw, Romer and Weil (1992) empirical studies have found a strong correlation between savings rates and economic growth rates. While standard growth theory usually assumes savings as a cause of economic growth, there is now ample evidence for the reverse causality, i.e. the view of high savings rates as a consequence of high economic growth (see Edwards, 1996, Loayza et al., 2000, and the discussion of the literature in Carroll et al., 2000).

Savings rates are higher for richer people across individuals (Dynan et al, 2004) and for richer countries across countries (Loayza et al., 2000). Within countries, savings rates usually rise with economic development. In England, for example, investment rates have been estimated as 3-6% in 1688 (Dean and Cole), around 8% in 1761-70, and around 14% for 1791-1800 (Feinstein, 1981). In the year 2000 savings rates were above 25 percent in some of the world's richest countries (Heston et al., 2002). Likewise the historical evolution of economic growth rates has been gradual. From the Maddison (2001) data we compute the annual rate of GDP per capita growth for England as 0.0% from year 1 to 1000, 0.1% from 1000 to 1500, 0.2% from 1500 to 1700. In the 19th century growth gets momentum. It is computed as 0.3% from 1700 to 1820, 1.3% from 1820 to 1870, 1.0% from 1870 to 1913, 1.2 % from 1913 to 1960, and 2.1% from 1960 to 2000. Standard models of economic growth have problems with getting the gradual evolution of savings and growth rates right. For reasonable parameterization they predict too high savings and too high growth at low levels of income (Barro and Sala-i-Martin, 2004).

In order to take the empirical regularities into account we augment the well-known linear ( $Ak$ ) growth model with endogenous, wealth-dependent preferences. The model exactly reverses the standard mechanism of development as established by neoclassical growth theory. Neoclassical growth theory assumes a constant rate of time preference rate ( $\rho$ ) and produces the result that – starting from below steady-state – capital productivity decreases along the path of economic development. Consequently, it predicts that the rate of economic growth decreases with the level of development. With contrast, the theory proposed here assumes that capital productivity ( $A$ ) is constant and produces the result that the rate of time preference adjusts from above as the economy develops, i.e. that people become more patient when they get richer. Consequently, and in line with the historical evidence, the model implies that the savings rate and the rate of economic growth increase with the level of development.<sup>1</sup>

FIGURE 1. GDP per capita in Europe Year 1000-1820 ( according to Maddison, 2003)



While the process of gradual take off was roughly the same everywhere in Western Europe (and the Western offshoots) there are also important differences discernable across countries. In particular there was a European “reversal of fortune” observable. Output in Italy, France, and Belgium, was higher than in England and the Netherlands until the 16th century while the opposite was true after the 17th century (see Figure 1). The present model explains the overtaking by cultural differences in the association between wealth and patience. When the development process gets momentum it is further amplified through increasing savings rates in

<sup>1</sup> The proposed theory is not the only extension of the linear growth model that produces reasonable adjustment dynamics. The conclusion compares with the main alternatives, subsistence consumption (Steger, 2000, Strulik, 2009) and habit formation (Carroll et al., 2000).

countries where the association between wealth and patience is stronger.<sup>2</sup> This way, small differences in time preferences can generate temporarily large cross-country differences in economic performance.

The rate of time preference is not a natural constant. As stated by Obstfeld (1990) “Mathematical convenience rather than innate plausibility has always been the main rationale for assuming time-additive preferences in economic modeling.” While this convenience is certainly welcome and justified for many research questions, it is particularly limiting when the focus of investigation is on the evolution of savings rates over time. As a result of this insight there exists already a literature of endogenous time preference to which the present paper relates. Starting with Uzawa (1968) important contributions have been made by Epstein and Hynes (1983), Obstfeld (1990), Drugeon (1996), Palivos et. al. (1997), and Das (2003).

The literature so far focusses mostly on consumption-dependent time preference within the neoclassical growth framework. A major, and somehow counter-intuitive and “unappealing” (Barro and Sala-i-Martin, 1995) result is that, in general, stability of a steady-state requires a positive relationship between the time preference rate and the level of consumption.<sup>3</sup> With contrast to the so far available literature the present article focusses on a negative association between time preference rate and wealth of the representative consumer and on the implied transitional dynamics towards balanced growth.

Becker and Mulligan (1997) have proposed a theory of endogenous time preference according to which individuals can invest resources in order to make future pleasures less remote by imagining them today. The model produces as a result that richer individuals are more patient and may serve as a micro-foundation of a positive association between wealth and time preference.

Empirically it has been found that the time preference rate decreases with income and wealth in panel data analysis (Lawrence, 1991, Samwick, 1998, Ventura, 2003) and large-scale experiments (Dohmen et al., 2009). Unfortunately, many empirical studies on intertemporal consumption behavior impose a constant time-preference rate. In that case we expect the fact of a positive association between wealth and the time preference rate to be reflected in the estimated intertemporal elasticity of substitution. Studies that control for wealth effects have indeed found

---

<sup>2</sup>For example, these countries will be more open towards doctrines that reward savings vis a vis current consumption (the Protestant belief system, Weber, 1904).

<sup>3</sup>Das (2003) has shown that stability of dynamic equilibrium can be obtained if the rate of return to capital falls faster than the rate of patience.

that the intertemporal elasticity of substitution is higher for richer individuals (e.g. Atkeson and Ogaki, 1996, Guvenen, 2006).

Wealth-dependent time preference may be explained – besides Becker and Mulligan’s rational choice approach – as a result of a positive association between wealth and institutions, i.e. through the risk of expropriation (Acemoglu et al., 2001) or between wealth and health, i.e. through life expectancy (Pritchett and Summers, 1996). Both channels, either indirectly or directly, suggest that people are more patient when they populate countries with lower diversity of disease species, i.e. countries of high latitude (Masters and McMillan, 2001, Olsson and Hibbs, 2005). While not denying a direct impact of institutions or health on growth the present paper focusses on the indirect effect through patience on savings and predicts that, *ceteris paribus*, countries at high latitude take off earlier from stagnation to growth (industrialize earlier) than countries in the tropics. The extended model can furthermore produce the result that an economy populated by impatient agents stagnates while an otherwise identical economy of more patient agents converges towards balanced growth.

In his recent book Clark (2007) observes that people became substantially more patient before the onset of the industrial revolution. He discards improvements of institutions and life-expectancy for being not sufficient to having provoked a such substantial drop of time preference and provides instead an evolutionary argument based on Rogers (1994). According to this view the institutionally stable agrarian societies of the Middle Ages created a selective pressure towards lower time preference. More patient people were wealthier which allowed them to have more surviving children and to bequeath more wealth to them, a fact that amplified the genetic transmission of patience.<sup>4</sup> Doepke and Zilibotti (2008) argue in favor of a related yet conceptually different mechanism. There, patience is transmitted through cultural evolution. Patient and thus economically successful parents invest more resources into transferring the character trait of patience to their offspring. Using cultural evolution Doepke and Zilibotti are able to explain why an exogenous change of production technology (an industrial revolution) induced an endogenous change of fortune of different social classes.

The theory proposed in the present article is much less involved than the story of Clark or the model of Doepke and Zilibotti. It can, nevertheless, be related to their work. For that purpose we imagine the infinitely living representative agent as an approximation of altruistically linked

---

<sup>4</sup>Galor and Michalopoulos (2006) have developed an evolutionary model of entrepreneurial spirit based on risk aversion.

generations.<sup>5</sup> The modeling of wealth-dependent time-preference then implies that optimizing agents take into account that accumulating wealth (bequests) will make their future selves (their offspring) more patient, which in turn implies that future growth will be more appreciated by their future selves (their offspring), a fact which encourages savings. This effect will be the stronger the higher the increase of patience and economic growth in the future is that can be triggered by higher savings today.<sup>6</sup> This way, the present theory explains the industrial revolution as an endogenous event that occurred when the interaction between patience and growth attained its highest momentum. An endogenous economic evolution from stagnation to balanced growth is shared with unified growth theory (Galor, 2005). However, demographic evolution, a major ingredient of unified growth theory, is not investigated here, a shortcoming that will be addressed in the Conclusions.

The present work is also related to Friedman (2005) who argues (verbally) that people behave morally better when they have “a sense of getting ahead”. With respect to one character trait the present theory is able to prove Friedman’s claim. It will be shown that from the first order conditions for optimal savings behavior follows an unambiguously positive effect of economic growth on patience.

## 2. THE BASIC MODEL

Consider an economy populated by a mass one of individuals who consume  $c$  and hold wealth  $k$ . For the basic version we assume that production operates a linear function,  $y = Ak$ , where  $A$  denotes productivity (net of depreciation). Later we introduce a generalized form of the production function. The constant productivity of  $k$  implies that there is potential for accumulation-driven growth if  $A$  exceeds the time preference rate. As usual the model becomes more realistic when we interpret  $k$  broadly consisting of physical and human capital. In order to be brief we refer to  $k$  in the following as “capital” and to its accumulation as “saving”. The capital stock thus evolves according to

$$\dot{k} = Ak - c. \tag{1}$$

---

<sup>5</sup>It is well-known that if altruism is strong enough to generate an interior solution for bequests, then there is no conceptual difference between a standard growth model of the overlapping generations structure with finite lifetimes and the infinitely living representative agent approach, see e.g. Barro and Sala-i-Martin, 2004.

<sup>6</sup>Dynan et al. (2004) find that richer people save more when young but do not dissave more in old age. The evidence thus rejects the simple life cycle model and supports altruistically linked generations, which are here in a simplifying manner modelled as an infinitely living representative agent.

As usual individuals are assumed to maximize discounted intertemporal utility from consumption. In deviation from the standard model the discount rate depends positively on wealth. As the individuals get richer they become more patient.

$$\max_c V_0 = \int_0^\infty u(c) e^{-\int_0^t \rho(k(v)) dv} dt \quad (2)$$

with  $\rho(k) > 0$ ,  $\rho'(k) < 0$ ,  $\rho''(k) > 0$  and  $\lim_{k \rightarrow \infty} \rho(k) = \bar{\rho}$ . In order to allow for positive long-run growth we assume  $A > \bar{\rho}$ .

Instantaneous utility is assumed to be iso-elastic,  $u(c) = c^{1-\theta}/(1-\theta)$ . Besides the positive correlation between wealth and time preference the model is thus identical to the textbook  $Ak$  growth model. For comparison it is helpful to recall some characteristic features of the standard model. It implies a unique long-run equilibrium which is assumed independently from initial endowments and it predicts a constant savings rate and a constant growth rate at all levels of development (for all  $k$ ). One purpose of the current paper is to show that wealth-dependent time preference is able to repair these counterfactual predictions.

In order to solve the problem of maximizing (2) with respect to (1) we parameterize the time integral and define  $q \equiv \int_0^t \rho(k(v)) dv$  such that  $dq/dt = \rho(k)$ . The time-transformed problem reads

$$\max_c \int_0^\infty \frac{u(c)}{\rho(k)} e^{-q} dq \quad s.t. \quad \frac{dk}{dq} = \frac{Ak - c}{\rho(k)}.$$

Inserting the isoelastic  $u(c)$  form, the current-value Hamiltonian reads  $H = c^{1-\theta}/[(1-\theta)\rho(k)] + \mu(A - c)/\rho(k)$  with costate variable  $\mu$ . The optimal solution fulfils the first order conditions

$$\frac{\partial H}{\partial c} = \frac{c^{-\theta}}{\rho} - \frac{\mu}{\rho} = 0 \quad (3a)$$

$$\frac{\partial H}{\partial k} = \frac{c^{1-\theta}}{1-\theta} \left( \frac{-\rho'}{\rho^2} \right) + \mu \frac{\rho(k) - \rho'(Ak - c)}{\rho^2} = \mu - \frac{d\mu}{dq} \quad (3b)$$

and the transversality condition  $\lim_{q \rightarrow \infty} \mu k e^{-q} = 0$ .

In solving for optimal consumption we begin with reintroducing actual time by substituting  $dq = \rho dt$ . Multiplying (3b) with  $\rho$ , subdividing by  $\mu = c^{-\theta}$ , and inserting  $\dot{\mu}/\mu = -\theta\dot{c}/c$  obtained from log-differentiating (3a) with respect to time, we derive the Ramsey rule (4).

$$\frac{\dot{c}}{c} = \frac{1}{\theta} \left[ A - \rho - \frac{\rho'}{\rho} \frac{c}{1-\theta} - \frac{\rho'}{\rho} (Ak - c) \right]. \quad (4)$$



For  $k \rightarrow \infty$  and thus  $\rho \rightarrow \bar{\rho} > 0$ ,  $\rho' \rightarrow 0$ , the result collapses to the familiar Ramsey rule  $\dot{c}/c = (A - \bar{\rho})/\theta$ . Additionally to the standard model we get two effects of wealth-dependent time preference on growth, a level effect and a growth effect.

The level effect is reflected by the term  $(-\rho'/\rho)c/(1 - \theta)$ . Since  $\rho' < 0$  the term is negative if  $\theta > 1$  and positive if  $\theta < 1$ . Intuitively, if  $\theta$  is large, i.e. the elasticity of intertemporal substitution  $1/\theta$  is low, people expect relatively little utility gain from higher future consumption, which makes them, *ceteris paribus*, less patient today. The opposite is true if  $\theta < 1$ .

To see that the last term in (4) reflects a growth effect on patience recall that  $\dot{k} = Ak - c$ . The growth effect is unambiguously positive. Living in a world of high growth makes people, *ceteris paribus* more patient. As a consequence they choose a steeper profile for consumption growth ( $\dot{c}/c$  is higher) implying that they save more. The growth channel implies that patience can be simultaneously conceptualized as a cause and consequence of economic growth. The model thus supports the observation that higher growth causes higher savings. It relates also to Friedman's (2005) "Moral consequences of Economic Growth". When people have a sense of "getting ahead" they appreciate more potential future gains of consumption and are inclined to save more today, a fact that amplifies further economic growth.

In order to get an explicit solution we introduce a simple functional form for the evolution of time preference.

$$\rho(k) = \bar{\rho} + \rho_0 k^{-\eta}. \quad (5)$$

The parameter  $\eta > 0$  controls the speed with which time preference converges towards  $\bar{\rho}$  when the economy is growing. The fact that  $(\rho'/\rho)\dot{k} = \dot{\rho}/\rho$  allows us to write the partial differential equation (4) as an ordinary differential equation as in (6).

$$\frac{\dot{c}}{c} = \frac{1}{\theta} \left[ A - \rho - \frac{\dot{\rho}}{\rho} \frac{c}{1 - \theta} \frac{1}{Ak - c} - \frac{\dot{\rho}}{\rho} \right]. \quad (6)$$

For equilibrium analysis let  $x$  denote the consumption capital ratio,  $c \equiv c/k$ . Behavior of the economy can then be conveniently discussed in the  $(\rho, x)$  space where  $x$  is the quasi-control variable, and  $\rho$  the state variable. Time preference, tied down by the state of the economy  $k$ , cannot jump instantaneously. From (5) we obtain evolution of time preference

$$\dot{\rho} = -\eta(\rho - \bar{\rho})(A - x). \quad (7)$$

The consumption capital ratio grows at rate  $\dot{x}/x = \dot{c}/c - \dot{k}/k$ . Inserting eqs. (1), (5), and (6) we obtain (8).

$$\dot{x} = \frac{x}{\theta} \left[ A - \rho + \eta(\rho - \bar{\rho}) \left( A - x + \frac{x}{1 - \theta} \right) \right] - (A - x)x. \quad (8)$$

Given an initial state  $k(0)$  and thus  $\rho(0)$ , the evolution of the economy is completely described by the trajectory of (7) and (8) that fulfills the first order conditions and the transversality condition. With contrast to the standard  $Ak$  model the economy shows adjustment dynamics.

### 3. EQUILIBRIUM ANALYSIS

PROPOSITION 1. *There exists a steady-steady-state of stagnation at*

$$\begin{aligned} x = x^* &= A \\ \rho = \rho^* &= \frac{A + \phi}{2} + \sqrt{\frac{(A + \phi)^2}{4} - \phi\bar{\rho}}, \quad \phi \equiv \eta A / (1 - \theta). \end{aligned} \quad (9)$$

*Proof.* Inspect (7) to see that  $\rho$  is constant when  $x = x^* = A$  implying  $c^* = Ak^*$ . Insert  $x = A$  into (8) and solve for  $\rho$  where  $\dot{x} = 0$ . Eliminate the negative root to get  $\rho^*$  in (8). A meaningful solution exists if the radicand is real. To verify that this is always the case, note that  $(A - \phi)^2 > 0$ . From this follows  $A^2 + 2\phi A + \phi^2 > 4\phi A$  and thus  $A^2 + 2\phi A + \phi^2 > 4\phi\bar{\rho}$  because we have  $A > \bar{\rho}$ . Conclude  $(A + \phi)^2 > 4\phi\bar{\rho}$ .  $\square$

PROPOSITION 2. *If  $\theta > 1 - \bar{\rho}/A$ , then there exists a unique balanced growth path along which the economy grows at rate  $(A - \bar{\rho})/\theta$  and where*

$$\begin{aligned} x = x^{**} &= \frac{(\theta - 1)}{\theta} A + \frac{\rho}{\theta} \\ \rho = \rho^{**} &= \bar{\rho}. \end{aligned} \quad (10)$$

*Proof.* Inspect (7) to see that  $\rho$  is constant when  $\rho = \bar{\rho}$ . Insert this information into (8) and solve for  $x$  where  $\dot{x} = 0$  in order to obtain  $x^{**}$ . Verify that  $x^{**}$  is positive for  $\theta > 1 - \bar{\rho}/A$ . Insert  $x = x^{**}$  into  $\dot{k}/k = A - x$  to get the balanced growth rate.  $\square$

Naturally, since  $\rho$  converges towards a constant, the balanced growth is identical with the one obtained for the standard  $Ak$  growth model. The value added of the present model does not lie in its steady-state characteristics but in the new transitional dynamics.

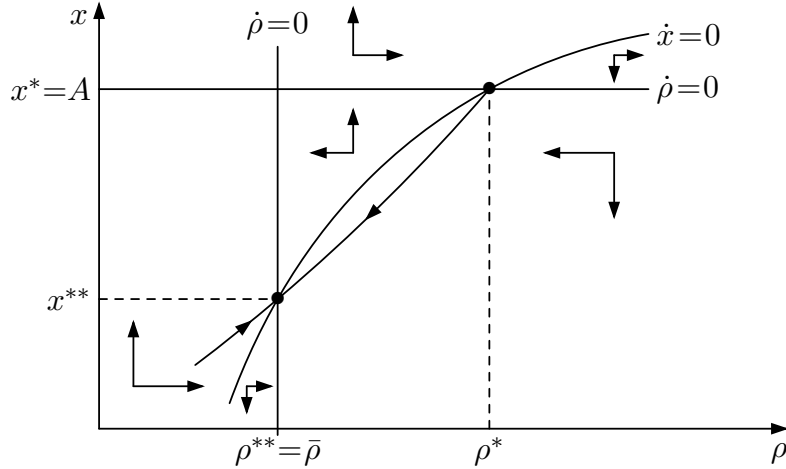
In order to analyze transitional dynamics and stability of equilibria within a phase diagram infer from (7) that there are two loci where  $\dot{\rho} = 0$ . In a  $(\rho, x)$  – diagram one isocline is the horizontal line where  $x = A$ . The other one is the vertical line where  $\rho = \bar{\rho}$ . Next set  $\dot{x} = 0$  in (8) and solve for  $x$  to get the  $\dot{x} = 0$ –isocline.

$$x = \frac{1 - \theta}{\theta} \cdot \frac{\rho^2 - A(1 + \eta - \theta)\rho + \eta\bar{\rho}A}{(1 + \eta - \theta)\rho - \eta\bar{\rho}}. \quad (11)$$

We are now ready to draw the diagram and obtain transitional dynamics.

**PROPOSITION 3.** *If  $\theta > (1 + \eta)$  or if  $\theta < (1 - \eta)$ , then the equilibrium of stagnation is unique and unstable and the balanced growth path is a saddlepoint. In the long-run the economy converges towards the balanced growth path. During transition the time preference rate decreases and the savings rate increases.*

Figure 2: Phase Diagram



*Proof.* Begin with considering the  $\dot{\rho} = 0$  loci in Figure 1. Infer from (7) that  $\dot{\rho} < 0$  in the area right of  $\bar{\rho}$ –line and below  $A$ –line. The opposite is true when  $x$  is above  $A$  and when  $\rho$  is below  $\bar{\rho}$ . Next infer that a meaningful solution requires a positive slope of the  $\dot{x} = 0$ –isocline as drawn in Figure 1. This ensures that the time preference rate at the equilibrium of stagnation ( $\rho^*$ ) is larger than that along the balanced growth path ( $\rho^{**}$ ). A positive slope also ensures that there is a unique intersection of the horizontal  $\dot{\rho} = 0$  curve, i.e. a unique equilibrium of stagnation.

Obtain from (11) the slope of the isocline.

$$\frac{\partial x}{\partial \rho} = \frac{1 - \theta}{\theta} \cdot \frac{(1 + \eta - \theta)\rho - 2\eta\bar{\rho}}{[(1 + \eta - \theta)\rho - \eta\bar{\rho}]^2} \cdot \rho.$$

For  $\theta > 1$  the slope is positive if  $(1 + \eta - \theta)\rho - 2\eta\bar{\rho} < 0$ . A sufficient, not necessary condition for this to hold is  $\theta > 1 + \eta$ . Likewise, for  $\theta < 1$ , the slope is positive if  $(1 + \eta - \theta)\rho - 2\eta\bar{\rho} > 0$ . Since  $\rho \geq \bar{\rho}$ , a sufficient, not necessary condition for this to hold is  $\theta < 1 - \eta$ .

Obtain the arrows of motion in direction of  $x$  by differentiating (8).

$$\frac{\partial(\dot{x}/x)}{\partial x} = \frac{\eta(\rho - \bar{\rho})}{\theta \rho} \left( \frac{1}{1 - \theta} - 1 \right) + 1,$$

which is positive if  $\eta(\rho - \bar{\rho})/(1 - \theta) + \rho > 0$ . For  $\theta < 1$  this requires  $\eta(\rho - \bar{\rho}) + \rho(1 - \theta) > 0$ , which is always true for  $\rho \geq \bar{\rho}$ . Thus  $\partial(\dot{x}/x)/\partial x$  positive. For  $\theta > 1$  a positive slope of  $(\dot{x}/x)$  requires  $\eta(\rho - \bar{\rho}) + \rho(1 - \theta) < 0$ , i.e.  $(1 + \eta - \theta)\rho - \eta\bar{\rho} < 0$ . A sufficient, not necessary condition for this to hold is  $\theta > 1 + \eta$ . The fact that  $\partial(\dot{x}/x)/\partial x > 0$  implies that the arrows of motion point towards larger  $x$  above the  $\dot{x} = 0$ -curve and towards lower  $x$  below. Inspect the arrows of motion in Figure 1 to conclude that the equilibrium  $(x^{**}, \rho^{**})$  is a saddlepoint. The fact that trajectories cannot cross each other eliminates the possibility that the equilibrium  $(x^*, \rho^*)$  is a locally stable spiral. It is thus unstable.

Suppose we start at the equilibrium of stagnation and perturb the economy a little. The only feasible path is then to follow the stable arm of the saddlepoint towards  $(x^{**}, \rho^{**})$ . Any other path leads to  $x = 0$  and thus zero consumption in finite time or to ever expanding  $x$  and thus zero capital stock in finite time implying, again, zero consumption. The paths that involve  $x \rightarrow \infty$ ,  $\rho \rightarrow \infty$  entail running out of capital in finite time and therefore lead eventually to a discrete jump to zero consumption, a fact that violates the first order condition. Intuitively (and analogous to the reasoning within the standard neoclassical growth framework, cf. Barro and Sala-i-Martin, 2004) people save too little in order to sustain positive consumption in the long-run. The paths that approach  $x = 0$  and  $\rho = \bar{\rho}$  violate the transversality condition. This can be seen by re-introducing actual time and rewriting the transversality condition  $\lim_{t \rightarrow \infty} \mu(t)k(t)e^{-\int_0^t \rho(k(v))dv}$ . Intuitively, people save too much and suboptimally react to low current consumption with even higher savings.

The savings rate is defined as  $s = 1 - c/y$ , i.e.  $s = 1 - x/A$ . It is thus rising along the stable manifold that leads from stagnation towards balanced growth.  $\square$

For an intuitive assessment of adjustment dynamics consider an economy that is initially very close to the steady-state of stagnation. In this situation capital stock and production are small and the time preference rate is large. Almost all income is consumed implying that  $c$  is close to  $Ak$ . This in turn means that there are almost no (net) savings and  $\dot{k}$  is close to zero. Having only a relatively short time-window of observations at hand one would conclude that the economy stagnates. At glacier speed, however, capital is accumulated and people get more patient with rising wealth. More patience leads to a higher savings rate (lower  $x$ ) and further rising capital (wealth). Sooner or later economic growth gets momentum and the economy travels with rising savings rates towards the balanced growth path. During the transition both the savings rate and the growth rate are perpetually rising. The feature that produces this realistic outcome is wealth-dependent time preference. Higher economic growth makes people wealthier and thus more patient, which in turn triggers more savings and leads to even higher subsequent growth.

The conditions for convergence towards balanced growth to happen are either  $\theta > 1 + \eta$  or  $\theta < 1 - \eta$ . The range of parameter values ensuring that either one or the other condition is fulfilled is quite broad. Moreover, these conditions are sufficient, not necessary for stability. It is nevertheless interesting to get an intuition of what could happen if they are violated. Since  $\eta > 0$ , one popular value implying a violation is  $\theta \rightarrow 1$ , i.e. the log-utility case. Inspect (8) to see that  $\dot{x}$  has a pole for  $\theta \rightarrow 1$ , i.e. there is convergence with infinite speed. For this special case, as always in the standard  $Ak$  model, there are no transitional dynamics and the economy jumps on the balanced growth path. The level effect of consumption on patience is trumping everything else. Intuitively, the above conditions thus ensure a “moderate” level effect of consumption on patience, either negative if the elasticity of intertemporal substitution is low, i.e.  $\theta > 1 + \eta$ , or positive if the the elasticity high, i.e.  $\theta < 1 - \eta$ .

#### 4. THE SLOW TRANSITION TOWARDS MODERN GROWTH: A CALIBRATION STUDY

We begin with determining the balanced growth path. The savings rate,  $s = 1 - c/y$ , along the balanced growth path is  $s^{**} = 1 - x^{**}/A$  and the implied growth rate is  $(A - \bar{\rho})/\theta$ . For the calibration we assume that the balanced growth rate is 2 percent annually and the savings rate that supports this path is 0.3. Imposing  $\theta = 2$ , a frequently used value in calibration exercises provides the estimates  $A = 0.0667$  and  $\bar{\rho} = 0.0267$ . A real rate of return on capital around 7

percent accords well with the average real return on the stock market for the last century and has been used in other calibration studies (e.g. Jones and Williams, 2000).

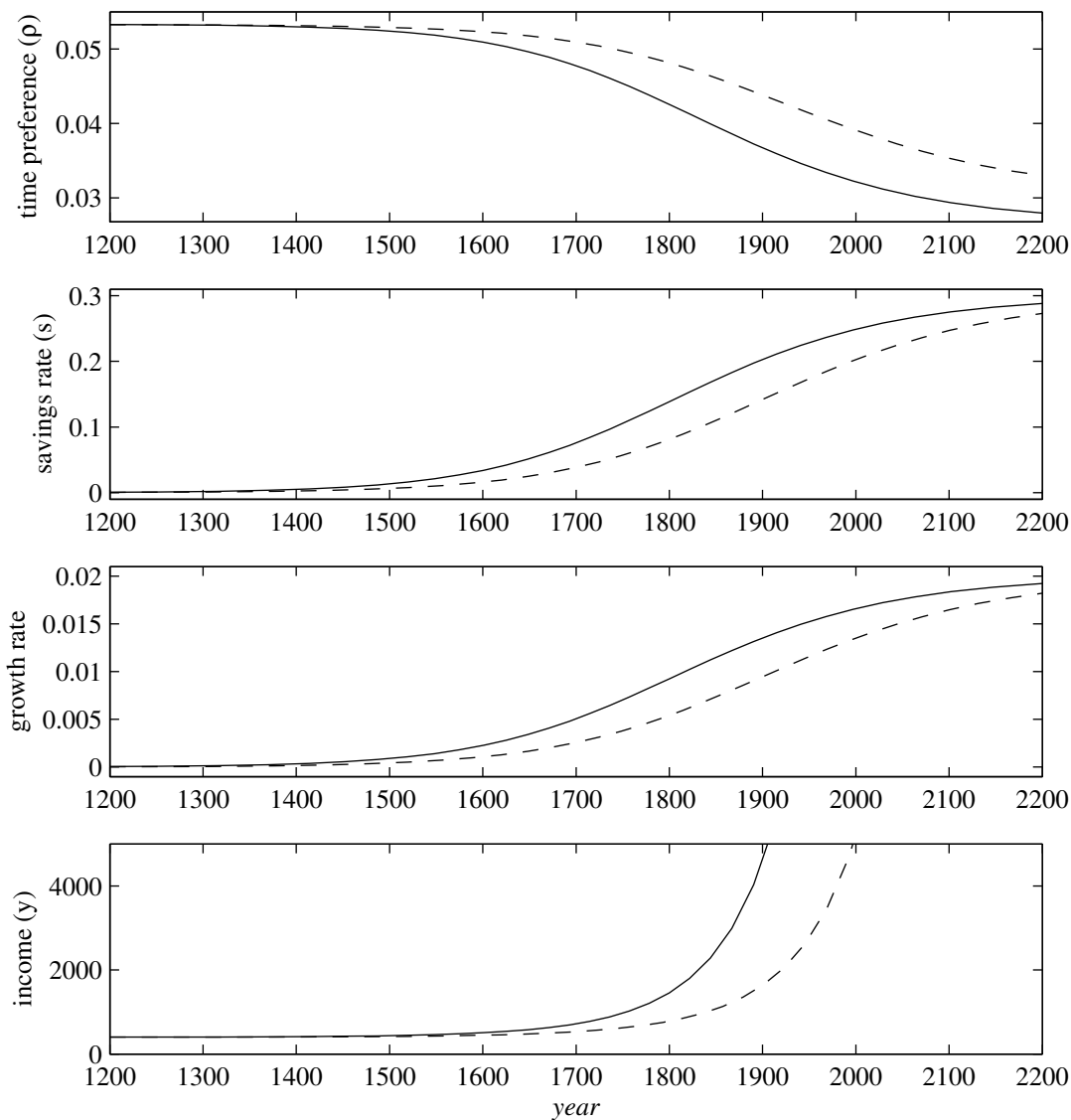
The remaining parameter,  $\eta$ , can be used to control transitional dynamics. Roughly speaking,  $\eta$  determines how strongly patience reacts on changes of wealth. Here, the value of  $\eta$  is adjusted such that the dynamic system generates approximately the historical evolution of savings and income as observed for England, i.e. such that an economy that starts with almost zero growth in the High Middle Ages reaches its highest acceleration of growth in the 19th century. This leads to the estimate of  $\eta = 0.4$ .

In order to extract the path from stagnation towards balanced growth we employ the method of backward integration. The integration starts when the economy is arbitrarily close to the steady-state of growth  $(\bar{\rho}, x^{**})$  and solves the dynamic system (7) and (8) backwards until it comes close to the steady-state of stagnation  $(\rho^*, A)$ . The calculation is terminated when  $\rho = 0.999\rho^*$ . A reversion of time finally renders the adjustment trajectories of the actual economy. The method solves the non-linearized system up to an arbitrarily small error, see Brunner and Strulik (2002) for details.

Figure 2 shows adjustment dynamics. For better comparison with the real data, time has been normalized such that  $t = 1800$  when  $s = 0.14$  (according to Feinstein, 1981) and income has been normalized such that  $y = 400$  when  $t = 1200$  (according to Maddison, 2001). Let us first consider the benchmark case reflected by solid lines. The adjustment process towards modern growth is characterized by little growth in the middle ages. In line with the historical evidence, growth rates are below 0.005 percent before year 1500 and about 0.2 percent around the year 1600. Afterwards, the economy visibly takes off. This means that around Luther's, and Calvin's time wealth is sufficiently high that we, for the first time, observed a pronounced increase of patience and of savings.

While the take-off is visible, compared with modern standards the rate of economic growth is nevertheless small during this period. In line with the empirical observations the model produces a growth rate of about 0.5 percent around 1700, of about 1 percent around 1800 and of about 1.3 percent in 1900. In the early 19th century, when historically the industrial revolution takes place, the model produces the highest momentum, i.e. the *the rates of change* of growth, patience, and savings are the highest. By the year 1900 income has increased to 5000 from 400 in 1200 (actually it increased to 4492 in the year 1900 from 400 in 1200, according to Maddison). In

FIGURE 2. Transitional Dynamics During the 2nd Millennium



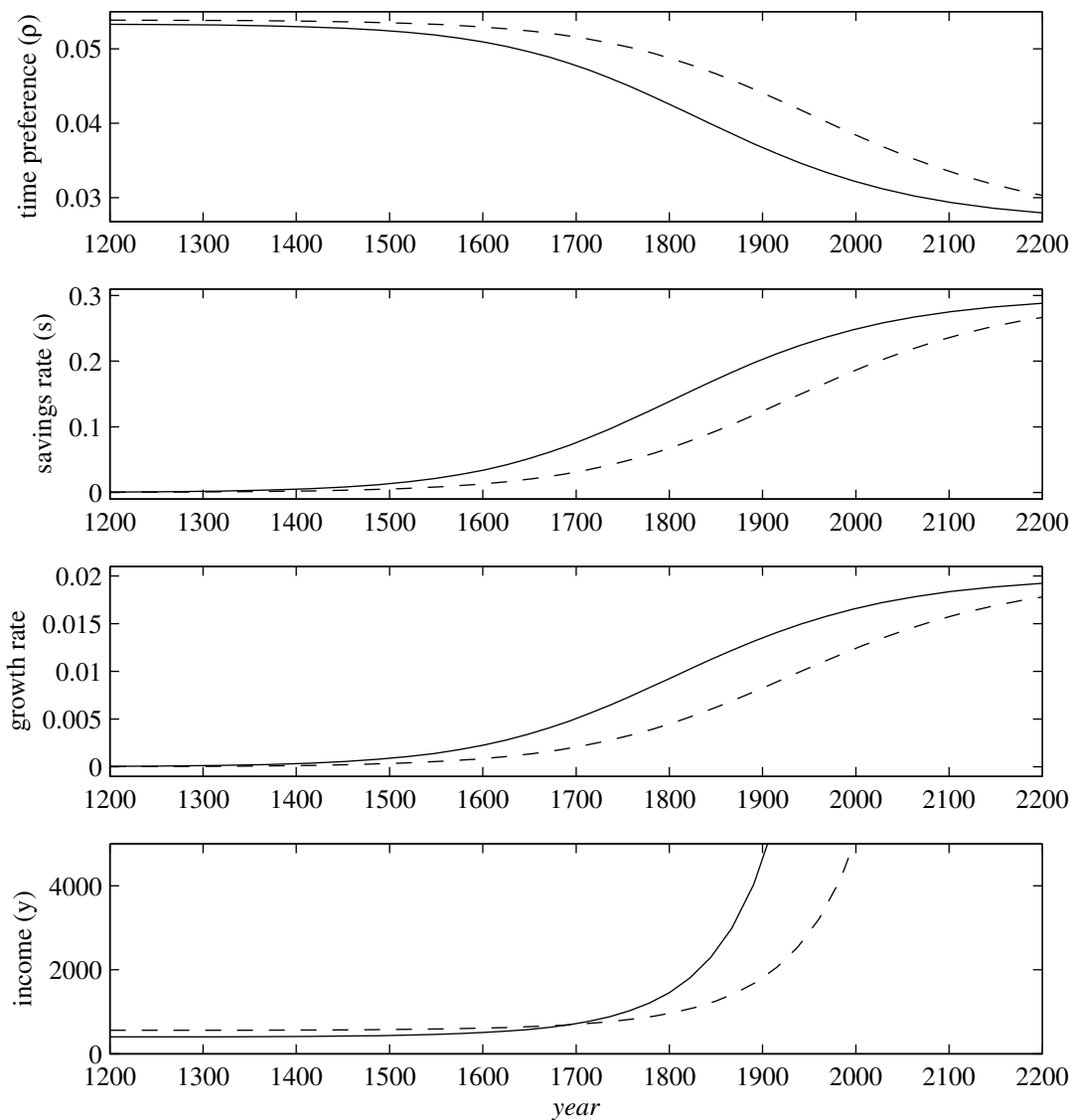
Parameters:  $A = 0.0667$ . Solid lines:  $\eta = 0.4$ ,  $\theta = 2$ ,  $\bar{\rho} = 0.0267$ . Dashed lines:  $\eta = 0.38$ ,  $\theta = 1.81$ ,  $\bar{\rho} = 0.0305$ . "income" is annual income per capita, "growth rate" is annual rate of income per capita.

the late 19th century the acceleration of growth begins to lose momentum while simultaneously, again in line with the historical evidence, the savings rate continues to rise. In year 2000 they growth rate and savings rate are close to their long-run steady-state levels.

The duration of the transition and the qualitative behavior of adjustment dynamics changes depends sensitively on the assumed strength of the wealth–patience association. This can be demonstrated exemplarily by an alternative scenario in which  $\eta = 0.38$  (instead of 0.40). In

order to get the same steady-state of stagnation we have adjusted  $\theta$  to 1.81 (instead of 2) which entails a further adjustment of  $\bar{\rho}$  to 0.0305 (instead of 0.0267) in order to ensure that the economy arrives at the same balanced growth path.

FIGURE 3. Overtaking During the Early Modern Period



Parameters  $A = 0.0667$ ,  $\theta = 2$ ,  $\bar{\rho} = 0.0267$ . Solid lines:  $\eta = 0.4$ . Dashed lines:  $\eta = 0.38$ .

Dashed lines in Figure 2 show the transitional dynamics. The time series for income is again normalized. In order to compare adjustment paths income is measured in units of  $y(0)$  of the benchmark run, i.e. in units of the “England”-case. In order to keep the analogy to the historical development of countries, the economy represented by dashed lines could be called



“Model-Italy”. In the 17th century (after reformation) patience rises visibly in Model-England but stays almost constant in Model-Italy for another century. As a consequence, take-off and industrial revolution are delayed in Model-Italy. Levels of patience, savings, and growth that are reached by England in 1800 are reached about a century later.

By considering two economies that originate from the same steady-state of stagnation and converge towards the same steady-state of balanced growth, Figure 2 demonstrates the role of transitional dynamics for economic performance. With respect to the historical data, however, it seems to be more realistic to allow for alternative steady-states of stagnation while keeping the same balanced growth rate. This would allow for an overtaking during the transitional period. In order to construct such a case we next consider two economies that differ “only” in their  $\eta$ 's (i.e. we refrain from recalibrating the equilibrium of stagnation). More specifically, we assume that Model-England and Model-Italy share the parameter values  $A = 0.0667$ ,  $\theta = 2$ , and  $\bar{\rho} = 0.0267$  and thus a balanced growth rate of 2 percent annually while  $\eta = 0.4$  for Model-England and  $\eta = 0.38$  for Model-Italy.

The fact that  $\eta$  is smaller for Model-Italy implies that its citizens are a little less patient not only along the transition path but also at the beginning, close to the equilibrium of stagnation. Formally,  $\rho^*$  is higher in Model-Italy. Intuitively, the propensity to consume is somewhat higher in less patient Model-Italy, i.e.  $c'(k)$  is larger at all levels of  $k$ . Because  $c''(k) < 0$  and  $c/k = A$  at the steady-state of stagnation, this implies that citizens of Model-Italy have to be somewhat wealthier in the neighborhood of the stagnation.

The implied adjustment dynamics are shown in Figure 3. Model-England (represented by solid lines) is starting somewhat behind in the High Middle Age but its citizens initiate a pronounced fall of the time preference rate earlier in the 17th century, a trait that enables them to overtake Model-Italy's income per capita in the late 17th century, in line with the historical observation (see Figure 1). During the 19th century, the period of the industrial revolution, the gap of growth rates between Model-England and Model-Italy gets largest and is slowly closed afterwards. While the model again predicts income per capita for England reasonably well, it underestimates the speed at which Italy is catching up during the 20th century. For the year 2000 it predicts an income per capita of about 5000, a value that was actually reached in 1957 (according to Maddison, 2003).

## 5. THE GENERAL CASE

While the model so far manages to generate reasonable paths for savings and growth it fails to predict the downward trend of interest rates documented by Clark (2007). This shortcoming is an inevitable yet unwanted side-effect of the linear production function, which provides a forever constant return to capital. In order to get the interest rate dynamics right we generalize towards a Jones-Manuelli (1990) type of production function. The notation follows largely the exposition of this model in Barro and Sala-i-Martin (2004). Production is assumed now to consist of a neoclassical part with diminishing returns and – as before – of a linear part with constant returns, i.e.  $y = Ak + Bk^\alpha \ell^{1-\alpha}$  where  $\ell$  is labor input. The implied interest rate and wage rate are  $r = A + \alpha Bk^{\alpha-1} \ell^{1-\alpha}$  and  $w = (1 - \alpha)Bk^\alpha \ell^{-\alpha}$ . The budget constraint for households is given by  $\dot{k} = rk + w\ell - c$ .

If we imagine that manufacturing operates with constant returns to scale with respect to the accumulable factors (physical and human capital) while agriculture is subject to diminishing returns because of limiting land, then the setup can be interpreted as a toy model of industrialization. In this sense the indicator  $\lambda \equiv Bk^\alpha/y$  measures the relative contribution of the diminishing-returns part (i.e. agriculture) to GDP.<sup>7</sup>

Assuming that households supply one unit of labor inelastically, maximizing (2) w.r.t.  $\dot{k} = rk + w\ell - c$ , and solving the problem as proposed for the benchmark model leads to consumption growth according to the following Ramsey rule.

$$\frac{\dot{c}}{c} = \frac{1}{\theta} \left[ r - \rho - \frac{\rho'}{\rho} \frac{c}{1-\theta} - \frac{\rho'}{\rho} (rk + w - c) \right].$$

Inserting equilibrium factor prices we obtain consumption growth (12), which replaces (4).

$$\frac{\dot{c}}{c} = \frac{1}{\theta} \left[ A + \alpha Bk^{\alpha-1} - \rho - \frac{\rho'}{\rho} \frac{c}{1-\theta} - \frac{\rho'}{\rho} (Ak + Bk^\alpha - c) \right]. \quad (12)$$

For  $\rho = \bar{\rho}$  this expression collapses to the Ramsey rule derived from the standard Jones Manuelli setup (as discussed in Barro and Sala-i-Martin, 2004).

Inserting factor prices into the budget constraint we get the equation of motion (13), which replaces (1).

$$\dot{k} = Ak + Bk^\alpha - c. \quad (13)$$

---

<sup>7</sup>This toy-model approach is only justified because industrialization in the sense of structural change is not the focus of the present paper. See, for example, Strulik and Weisdorf (2008) for a proper model on structural change during the industrial revolution.

In order to transform (12) – (13) into a system of ordinary differential equations, we again consider the particular form (5) for  $\rho(k)$  and introduce the auxiliary variable  $z \equiv k^{\alpha-1}$ . The economy can then be represented by the three-dimensional system (14).

$$\dot{\rho} = -\eta(\rho - \bar{\rho})(A + Bz - x). \quad (14a)$$

$$\dot{x} = \frac{x}{\theta} \left[ A + \alpha Bz - \rho + \eta(\rho - \bar{\rho}) \left( A + Bz + \frac{\theta x}{1 - \theta} \right) \right] - (A + Bz - x)x. \quad (14b)$$

$$\dot{z} = (\alpha - 1)(A + Bz - x)z. \quad (14c)$$

The ordinary Jones–Manuelli model is included as the special case of  $\rho = \bar{\rho}$  for all  $k$ .

Because of the diminishing influence of the non-linear part of capital productivity, i.e.  $z \rightarrow 0$  in a growing economy, the long-run growth path is, of course, the same as for the simple  $Ak$  growth model. At the steady-state of stagnation  $x = A + Bz$ . Using this in (14b) evaluated at a steady-state we get

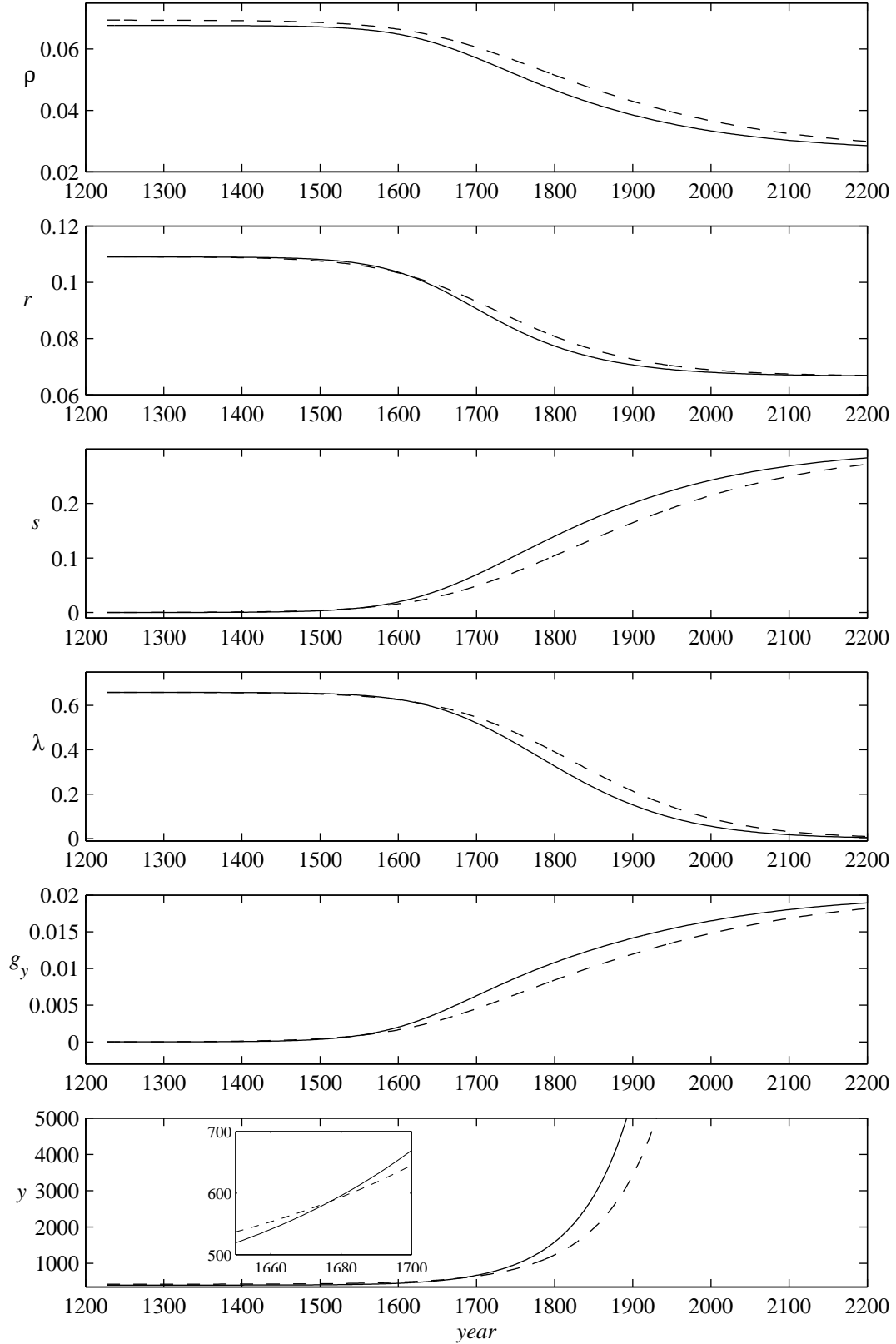
$$0 = A\rho + \alpha Bz\rho - \rho^2 + \eta(\rho - \bar{\rho})\frac{A + Bz}{1 - \theta}. \quad (15)$$

Inserting  $\rho = \bar{\rho} + \rho_0 k^{-\eta}$  and  $z = k^{\alpha-1}$  the equation can be solved numerically for  $k^*$ . From  $k^*$  we can infer  $z^*$  and  $\rho^*$ . Employing once again the method of backward integration the system (14) is solved by starting arbitrarily close to the balanced growth path until it arrives at the steady-state of stagnation. Since both,  $\rho$  and  $z$  are state variables, adjustment dynamics evolve along a two-dimensional stable manifold. In order to approximate the unique optimal trajectory on the manifold the initial deviation from the balanced growth path is adjusted until the trajectory computed by backward integration ends at  $(\rho^*, z^*)$ , see Brunner and Strulik (2002) for details.

For the calibration we set  $\alpha = 1/3$  and  $B = 1$ . The values of  $\eta$  and  $\rho_0$  are chosen such that the adjustment speed is approximately the same as obtained for the solution of the basic model (under the  $Ak$  technology) and such that the initial interest rate (at the steady-state of stagnation) equals 10.9 percent as documented by Clark (2007) for England in the 13th century. This provides the estimates  $\eta = 0.5$  and  $\rho_0 = 0.1198$ . The other parameters are kept from the last section so that the economy, again, converges towards a steady-state growth rate of 2 percent and a savings rate of 30 percent.

Adjustment dynamics for the benchmark case are shown by solid lines in Figure 4. While adjustment dynamics for  $\rho$ ,  $x$ , and  $s$  are very similar to the simpler  $Ak$  case we additionally observe falling interest rates from historically high values to around 7 percent today. Dotted

FIGURE 4. Patience and Prosperity Dynamics: The General Case



Parameters:  $A = 0.0667$ ,  $\alpha = 0.33$ ,  $\bar{\rho} = 0.0267$ ,  $\theta = 2$ ,  $\rho_0 = 0.1189$  and  $(\eta = 0.35, B = 1)$  for solid lines and  $(\eta = 0.33, B = 1.042)$  for dashed lines.

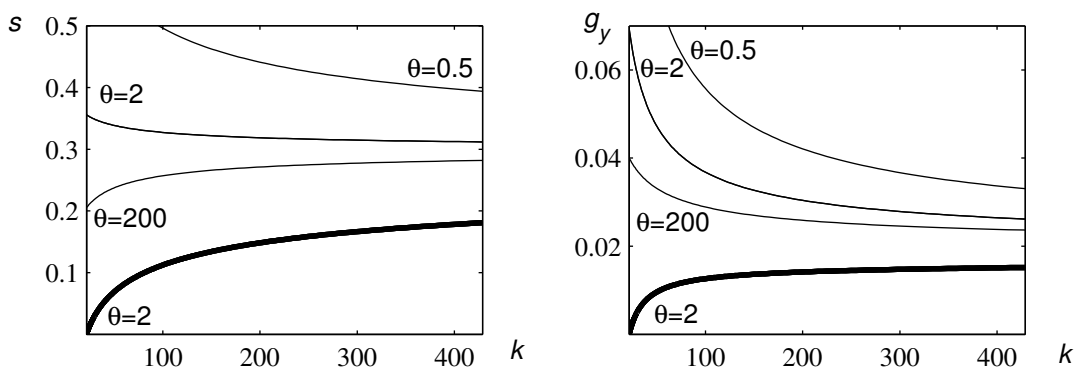
lines consider again an alternative specification of an economy populated by slightly less patient agents ( $\eta = 0.33$  for Model-Italy). The parameterization assumes that both economies share the same interest rate in stagnation and along the balanced growth path. Because the citizens of Model-Italy are less patient everywhere and in particular at the steady-state of stagnation, they generate stagnation at a somewhat higher time-preference rate, i.e. for a somewhat lower capital stock. Since interest rates are assumed to be the same in both countries this implies that total factor productivity has to be somewhat larger in Model-Italy ( $B$  rises from 1 to 1.042). From this follows, as for the basic case, that Model-Italy has to be somewhat richer in stagnation, a fact that will again produce overtaking by the more patient Model-England. A zoom into the respective region reveals that overtaking takes again place at the end of the 17th century.

The numerical solution documents that the model is capable to produce the historical fact of simultaneously falling interest rates and rising growth rates. This interesting result is far from self-evident since falling interest rates per se lead to falling growth rates. More patience (decreasing time-preference), however, is capable to dominate the negative effect of falling capital productivity on growth.

To see the necessity of endogenous time preference for these results it is instructive to compare with the standard Jones Manuelli model. The panel on the left hand side of Figure 5 shows the policy function  $s(k)$  for the benchmark calibration and alternative specifications of the standard Jones-Manuelli model. The policy function relates the savings rate to the state of the economy (subsumed in state variable  $k$ ). The bold line shows the policy function as implied by endogenous time preference. Since time preference is constant in the standard case, the only parameter that can be manipulated in order to get the shape of the policy function right is  $\theta$ , i.e. the degree of relative risk aversion (the inverse of the intertemporal elasticity of substitution)

One sees, that if the patience channel is closed, there is too little curvature of the policy function. Poor people are predicted to save too much. For the benchmark setup of  $\theta = 2$  (and lower values) the standard model even produces adjustment from the “wrong side”, i.e. falling savings rates during economic development. As shown by Barro and Sala-i-Martin (2004) this shortcoming can be prevented by imposing (unrealistically) high values for  $\theta$ , i.e. (unrealistically) low values for the intertemporal elasticity of substitution. But, as the example shows, even if  $\theta = 200$  (or higher) there is still too little curvature of the savings function such that the model,

FIGURE 5. Savings and Growth: Comparison with Standard Model



Parameters:  $A = 0.0667$ ,  $B = 1$ ,  $\alpha = 0.33$ ,  $\bar{\rho} = 0.0267$ ,  $\theta = 2$ ,  $\rho_0 = 0.12$ , and  $\eta = 0.35$ .  $k$  is measured relative to  $k^*$ . Bold line: wealth-dependent preference. Thin lines: standard Jones-Manuelli model, alternative  $\theta$ 's and otherwise same parameters.

in general, predicts too fast adjustment from stagnation to balanced growth as compared with the historical evidence.

The panel on the right hand side of Figure 2 shows the associated growth rates. When the patience channel is closed, falling capital productivity is the only impact on growth and the model counter-factually predicts that richer economies grow at lower rates. For low values of  $\theta$ , the standard model gets the positive correlation between savings and growth right but predicts that both are falling during economic development. For high values of  $\theta$  the standard model counterfactually predicts a negative correlation between savings and growth. Endogenous, wealth-dependent patience is capable to revert these results and generate savings rates rising in accordance with economic growth.

## 6. THE POVERTY TRAP

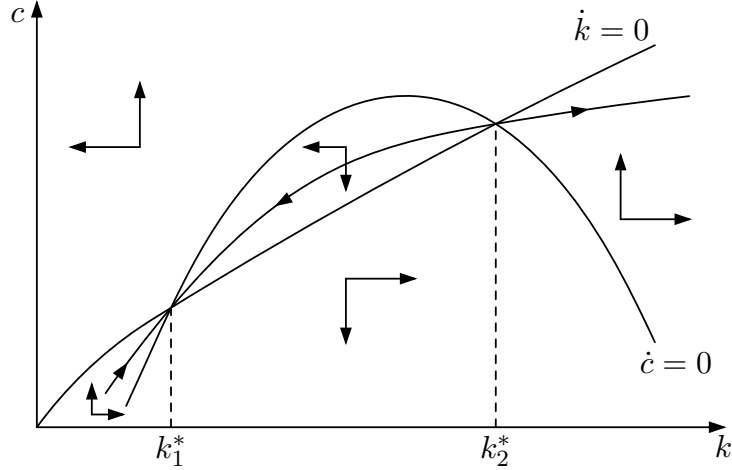
In principle, equation (15) may have two positive roots and the possibility of a locally stable poverty trap arises. In order to investigate this case in more detail in the  $k - c$  space, we derive from (13) the  $\dot{k} = 0$  isocline as  $c = Ak + Bk^\alpha$ . The isocline is positively sloped everywhere and approaches the slope  $A$  as  $k$  goes to infinity. Above the isocline we observe  $\dot{k} < 0$  and vice versa below.

From (12) we get the  $\dot{c} = 0$ -isocline

$$c = \frac{1 - \theta}{\theta} \left[ (A + \alpha Bk^{\alpha-1} - \rho) \frac{\rho}{\rho'} - Ak - Bk^\alpha \right]. \quad (16)$$

As  $k$  goes to infinity and  $\rho'(k)$  goes to zero the first term in brackets vanishes and  $c$  approaches  $+\infty$  if  $\theta > 1$  and  $-\infty$  if  $\theta < 1$ .

FIGURE 6. Phase Diagram



A necessary, not sufficient condition for (15) to have two positive roots is  $\theta < 1$ . This can be seen by noting that the roots solve  $0 = \rho^2 - a\rho + b$  with  $b \equiv \eta\bar{\rho}(A + Bz)/(1 - \theta)$  and that two positive roots necessarily require  $b > 0$ . If  $\theta < 1$ , then coming from low  $k$  the  $\dot{c} = 0$ -curve intersects the  $\dot{k} = 0$ -curve first from below and then from above as shown in Figure 6. Finally, inspect (12) to verify that  $\dot{c} > 0$  above the  $\dot{c} = 0$ -curve and  $\dot{c} < 0$  below and conclude from the resulting arrows of motion that the first equilibrium at  $k_1^*$  is a saddlepoint and the second equilibrium at  $k_2^*$  is unstable.

If an economy starts with capital stock below  $k_2^*$  a movement along any trajectory other than the stable manifold can again be excluded for violating in finite time either the first order conditions or the transversality condition. The unstable equilibrium at  $k_2^*$  separates convergence towards the poverty trap from convergence towards balanced growth. In the Appendix it is shown that as long as there is potential for growth a saddlepoint-stable development trap cannot occur in the standard Jones-Manuelli model augmented by subsistence consumption. The current model thus seems to be the minimum setup that allows for both convergence towards subsistence and convergence towards balanced growth, depending on initial conditions.

Intuitively, if an economy situated at  $k_1^*$  experiences a positive shock of capital and the interest rate falls, consumption decreases and exerts a negative level effect on patience if  $\theta < 1$ . This,

however, is not sufficient to generate stagnation since the level effect has to be strong enough in order to overcompensate the positive growth effect on patience as explained in connection with the Ramsey rule (4).

Figure 7 shows dynamic behavior of two identical economies, one starting slightly above  $k_2^*$ , the other slightly below. In order to create a locally stable poverty trap  $\theta$  has been set to 0.5. The balanced growth path is kept such that  $s = 0.3$  and  $g_y = 0.02$ . For that purpose the long-run time preference rate has been adjusted to 0.0567, a high yet not completely incomprehensible value. The low value of  $\theta$  generates the “right” curvature of the  $\dot{c}$  isocline but is not sufficient to produce two intersections. For that purpose  $\rho_0$  has been increased to 0.33, a change that does not affect balanced growth. All other parameters are kept from the last scenario.

The time paths for the developing economy are somewhat less satisfying than for the earlier case of  $\theta = 2$ . Adjustment is “too linear”, i.e. there is too little curvature during the time of the historical industrial revolution. Also, income for Model-England is overestimated in the 19th century. On the other hand, the produced time paths are still *comparatively* good approximation of actual behavior given that standard models of economic growth get adjustment dynamics completely wrong when  $\theta$  is below unity (for savings and growth in the Standard Jones Manuelli setup this has been demonstrated in Figure 5). The power of endogenous time preferences is obviously sufficient to override the strong tendency for savings rates and growth rates to fall jointly with interest rates, a result which is usually generated when  $\theta$  is below unity.<sup>8</sup>

Qualitatively the paths for the developing economy differ from those obtained earlier for  $\theta > 1$ . The time preference rate lies now above the interest rate for most parts of the development process (until the 20th century). For an assessment recall that any standard model would not be capable of producing positive growth and successful development if the time preference rate exceeds the interest rate. Intuitively, successful development is possible here, because individuals take into account that time preference rates will be lower in the future such that their future selves (their offspring) will appreciate future income more, a fact that triggers already savings today. The fact that high time preferences rates are compatible with positive growth may help to reconcile the growth literature with the frequently high estimates of  $\rho$  in empirical studies and

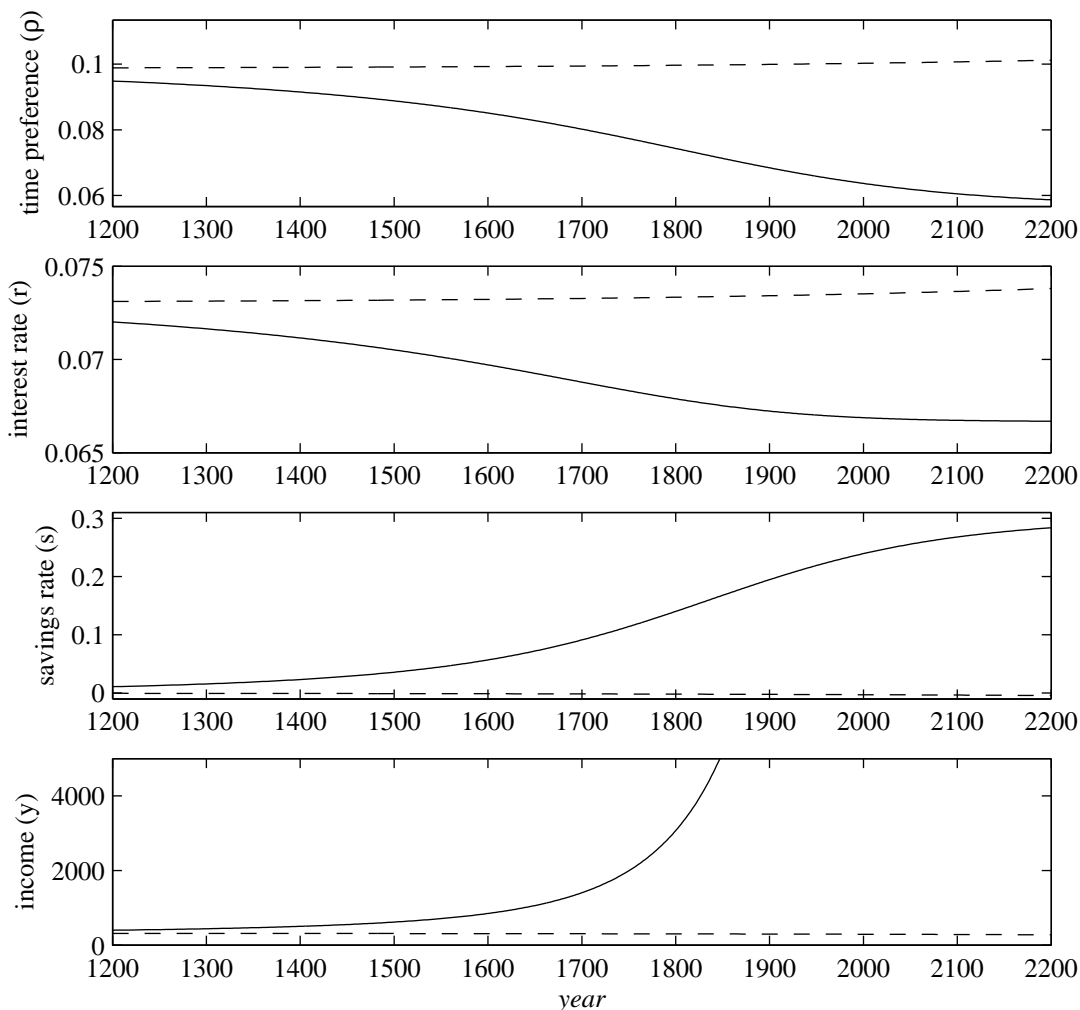
---

<sup>8</sup>At first sight the assumption of  $\theta < 1$  might seem unrealistic since econometric studies (for example Atkeson and Ogaki, 1997, and Guvenen, 2006) as well as macroeconomic considerations (Barro and Sala-i-Martin, 2004) suggest that the elasticity of intertemporal substitution is quite small, implying that  $\theta$  is large and certainly above one. This conclusion, however, has been derived under the assumption that the time preference rate is constant.



experiments conducted in rich and growing economies. For example, in a recent article Andersen et al. (2000) estimate from field experiments in Denmark that  $\rho = 0.10$  (and  $\theta = 0.74$ ).

FIGURE 7. Convergence Clubs



Solid lines: economy starting above  $k_2^*$ . Dashed lines: economy starting below  $k_2^*$ .  
Parameters for both cases:  $A = 0.0667$ ,  $B = 1$ ,  $\alpha = 0.33$ ,  $\bar{\rho} = 0.0267$ ,  $\theta = 0.5$ ,  
 $\rho_0 = 0.33$ , and  $\eta = 0.35$ .

It is telling that  $\rho_0$  has to be adjusted to a high value in order to get the possibility of a poverty trap. This means that in addition to the negative level effect of consumption on patience (stemming from  $\theta$  below unity) time preference has to be assumed to be particularly high at low levels of wealth. In the introduction it has been argued that there are potential determinants of time preference other than individual wealth. In particular geographic location may have an influence either directly through longevity or indirectly through institutions and

the risk or expropriation. This lets us expect that for *given* wealth the rate of time preference is higher at low geographic latitudes (the tropics). Naturally we would expect this effect to show up in  $\rho_0$  rather than in  $\bar{\rho}$ , i.e. we would expect the influence of geography on time preference to vanish when people become infinitely rich. If one is willing to accept this argument, the current model provides a rationale for a local poverty trap at low geographic latitudes.

## 7. DISCUSSION

The current paper has proposed a theory of endogenous patience that explains the long-run evolution of savings and growth in the Western world reasonably well. Possibly the approximation of the actual transition is too good since there is little left for other important drivers of economic development. A reduction of  $\eta$ , for example, would slow down the adjustment speed and open room for factors like structural change, demographic transition, and TFP growth to improve the adjustment dynamics. A reasonable assessment of the results is thus to view endogenous patience as a complementing channel for the explanation of long-run economic development, a channel that has been largely overlooked so far.

Many but not all of the results presented here can alternatively be generated by introducing habit formation or subsistence consumption into the linear growth model. It seems thus worthwhile to compare. A common characteristic of all three approaches is that they produce reasonable adjustment dynamics, i.e. savings and growth are jointly rising during the process of development.

The modeling of habits is technically more involved than the present approach since it requires a second state variable, the habit stock. When augmented with endogenous habit formation already the linear  $Ak$  transforms into a three dimensional dynamic system (Carroll et al., 2000). The higher formal complexity possibly explains why some interesting features obtained here like the savings–interest rate dynamics or the poverty trap have not (yet) been explained within the habit formation model. Although ex post adjustment dynamics look similar, they are conceptually different under habit formation. Adaptive behavior implies that the model generates serial correlation of current savings and consumption decisions with those in the past. With contrast, endogenous patience generates serial correlation with future savings and consumption decisions. If an economy traveling along its balanced growth path is hit by a shock we would

thus expect current consumption to be correlated with past consumption under habit formation but not under endogenous time-preference.

If the strength of wealth-dependent savings is determined by geographic location (either through mortality or expropriation risk) then an explanation of temporarily diverging time preference rates and savings behavior must not rely on culture. In particular, country-specific savings behavior would be compatible with the observation that there is no fundamental difference of savings behavior between natives and immigrants (Carroll et al., 1994). With contrast if savings are driven by habits we would expect savings behavior to differ for immigrants, it least for a while and in particular if habits adjust sluggishly.

The introduction of subsistence consumption in economic models comes in two conceptually different variants. The first variant tries to understand how human energy consumption affects economic choices (for example fertility, effort) and how these choices feed back on macroeconomic behavior and the available diet (Dalgaard and Strulik, 2007). This is, of course, a totally different research agenda than pursued in the present paper. The second variant, which is appears more often in economic modeling, introduces a constant ( $\bar{c}$ ) into the utility function and calls this constant (possibly misleadingly) subsistence consumption. An exploration of subsistence needs, however, is not the purpose of the modeling. The purpose is to get adjustment dynamics right, i.e. to generate savings and growth rates that are jointly increasing with economic development.

Subsistence consumption in form of Stone-Geary utility,  $u(c) = (c - \bar{c})^{1-\theta}/(1 - \theta)$ , implies that the intertemporal elasticity of substitution rises from zero at  $\bar{c}$  to  $1/\theta$  for infinite  $c$ . This produces trajectories similar to the ones for wealth-dependent time preference and the question occurs whether the ad hoc modeling of subsistence is acceptable as a shortcut to endogenous patience. The answer is probably yes on the level of (introductory) macroeconomics textbooks because the “shortcut”-model is formally much less involved. On the level of a serious research question the answer is certainly no. This is instructively demonstrated by Kraay and Raddatz’ (2007) attempts to calibrate the neoclassical growth model with subsistence consumption and in particular to find a proper  $\bar{c}$ . They conclude: “In summary, the model with subsistence consumption could be used to rationalize the persistence of low levels of consumption and capital observed in low income African countries by assuming that they are close to subsistence level, but this requires either to assume that there is significant variation in the levels of subsistence

consumption across African countries or that half of these countries should exhibit high savings and growth rates.”

With respect to the  $Ak$  growth model with subsistence consumption it can be shown that adjustment dynamics of the savings rate and the rate of economic growth are independent from the specification of  $\bar{c}$ . In fact, adjustment dynamics of these variables are completely determined by the specification of the balanced growth path (Strulik, 2009). This implies that either all countries adjust alike toward the same balanced growth path or that countries that lag behind will never catch up. The model cannot explain *temporarily* divergent behavior. With contrast, the current model can explain temporarily different behavior, in particular why originally leading countries are overtaken before and during industrial revolution and why they later catch up again. Additionally, it can explain a patience-driven, locally stable poverty trap, which might be relevant for some of the African countries discussed by Kray and Raddatz.

Finally the concept of endogenous time preference is comprehensive. It directly affects all intertemporal decisions alike whereas the subsistence approach affects other choice variables only through urgent consumption needs. For example, there is no “subsistence education” or “subsistence fertility”. An extension of the present model towards the analysis of other intertemporal decision making is an interesting subject for future research.

## Appendix

PROPOSITION 4. *If there is potential for long-run growth then there cannot simultaneously exist a local (saddlepoint-) stable poverty trap in the Jones Manuelli model with constant time preference and subsistence consumption, i.e. when instantaneous utility is of the Stone-Geary type,  $u(c) = (c - \bar{c})^{1-\theta}$  where  $\bar{c}$  denotes the subsistence level.*

*Proof.* With instantaneous utility of the Stone-Geary type the Ramsey rule in general reads  $\dot{c} = (f'(k) - \rho)(c - \bar{c})/\theta$  and with production of the Jones Manuelli type it becomes

$$\dot{c} = (A + \alpha Bk^{\alpha-1} - \rho) \left( \frac{c - \bar{c}}{\theta} \right).$$

The equation of motion for capital remains to be (13). Generally, equilibria of stagnation could be where  $c = \bar{c}$  or where  $A - \rho - \alpha Bk^{\alpha-1} = 0$ . However, under the requirement that there is potential for long-run, the second option for an equilibrium of stagnation ceases to exist. For positive growth in the long run we must have  $A - \rho > 0$  which is incompatible with  $A - \rho - \alpha Bk^{\alpha-1} = 0$  at some positive  $k$ .

The elements of the Jacobian evaluated at the equilibrium  $c = \bar{c}$ ,  $k = A^\alpha + Bk^{\alpha-1}$  are  $J_{11} = (A + \alpha Bk^{\alpha-1} - \rho)/\theta$ ,  $J_{12} = \alpha(\alpha - 1)Bk^{\alpha-2}(c - \bar{c})/\theta = 0$ ,  $J_{21} = -1$ , and  $J_{22} = A + \alpha Bk^{\alpha-1}$ . The eigenvalues are  $a \equiv A + \alpha Bk^{\alpha-1} > 0$ , and  $a(a - \rho)\theta > 0$  for  $A - \rho > 0$ , i.e. when there is potential for long-run. The equilibrium of stagnation is unstable.  $\square$

## References

- Andersen, S., Harrison, G.W., Lau, M.I., and Rutstroem, E.E., Eliciting risk and time preferences, *Econometrica* 76, 583-618.
- Atkeson, A. and Ogaki, M., 1996, Wealth-varying elasticity of substitution: Evidence from panel data and aggregate data, *Journal of Monetary Economics* 38, 507-537.
- Barro, R.J., 1991, Economic growth in a cross-section of countries, *Quarterly Journal of Economics* 106, 407-443.
- Barro, R.J. and Sala-i-Martin, X., 2004, *Economic Growth*, 2nd ed., MIT Press, Cambridge, MA.
- Becker, G.S. and Mulligan, C.B., 1997, The endogenous determination of time preference, *Quarterly Journal of Economics* 112, 729-758.
- Brunner, M. and H. Strulik, 2002, Solution of perfect foresight saddlepoint problems: A simple method and applications, *Journal of Economic Dynamics and Control* 26, 737-753.
- Carroll, C.D., Rhee, B.-K., and Rhee, C., 1994, Are there cultural effects on savings? Some cross-sectional evidence, *Quarterly Journal of Economics* 109, 685-699.
- Carroll, C.D., Overland, J. and Weil, D.N., 2000, Saving and growth with habit formation, *American Economic Review* 90, 341-355.
- Chakraborty, S., 2004, Endogenous life-time and economic growth, *Journal of Economic Theory* 116, 119-137.
- Clark, G., 2007, *A Farewell to Alms*, Princeton University Press, Princeton.
- Dalgaard, C.J. and Strulik, H., 2007, A bioeconomic foundation of the Malthusian equilibrium: body size and population size in the long-run, University of Hannover, Discussion Paper 373.
- Das, M., 2003, Optimal growth with decreasing marginal impatience, *Journal of Economic Dynamics and Control* 27, 1881-1898.
- Davis, L. E. and Gallman, R.E., 1978, Capital formation in the United States during the nineteenth century, in: P. Mathias and M.M. Postan (eds.), *The Cambridge Economic History of Europe*, Volume VII, Part 2, Cambridge University Press, Cambridge.
- Deane, P. and Cole, W. A., 1969, *British Economic Growth 1688-1959*, Cambridge University Press, Cambridge.
- Doepke, M. and Zilibotti, F., 2008, Occupational choice and the spirit of capitalism, *Quarterly Journal of Economics* 123, 747-793.
- Dohmen, T., Falk, A., Huffman, D., and Sunde, U., 2009, Are Risk Aversion and Impatience Related to Cognitive Ability?, *American Economic Review*, forthcoming.
- Druegeon, J.P., 1996, Impatience and long-run growth, *Journal of Economic Dynamics and Control* 20, 281-313.

- Dynan, K.E., Skinner, J., and Zeldes, S., 2004, Do the rich save more?, *Journal of Political Economy* 112, 397-444.
- Edwards, S., 1996, Why are Latin America's savings rates so low? *An international comparative analysis*, *Journal of Development Economics* 51, 5-44.
- Epstein, L.G., Hynes, J.A., 1983, The rate of time preference and dynamic economic analysis, *Journal of Political Economy* 91, 611-635.
- Feinstein, C. H., 1981, Capital accumulation and the Industrial Revolution, in: R. Floud and D.N. McCloskey (eds.), *The Economic History of Britain Since 1700, Volume 1: 1700-1860* Cambridge University Press, Cambridge.
- Friedman, B.M., 2005, *The Moral Consequences of Economic Growth*, Random House, New York.
- Galor, O., 2005, From stagnation to growth: unified growth theory, in: P. Aghion and S. Durlauf, *Handbook of Economic Growth* Vol 1A, Amsterdam, North-Holland.
- Galor, O., Michalopoulos, S., 2006, Darwinian Evolution of Entrepreneurial Spirit and the Process of Development, Discussion Paper, Brown University.
- Güvenen, F., 2006, Reconciling conflicting evidence on the elasticity of intertemporal substitution: A macroeconomic perspective, *Journal of Monetary Economics* 53, 1451-1472.
- Heston, A., Summers, R., and Aten, B., 2002, Penn World Table Version 6.1, Center for International Comparisons at the University of Pennsylvania.
- Jones, C.I. and Williams, J., 2000, Too much of a good thing? The economics of investment in R&D, *Journal of Economic Growth* 5, 65-85.
- Jones, L.E., Manuelli, R., 1990, A convex model of equilibrium growth: theory and policy implications, *Journal of Political Economy* 98, 1008-1038.
- Kraay, A. and Raddatz, C., 2007, Poverty traps, aid, and growth, *Journal of Development Economics* 82, 315-347.
- Lawrance, E.C., 1991, Poverty and the rate of time preference: evidence from panel data, *Journal of Political Economy* 99, 54-77.
- Loayza, N., Schmidt-Hebbel, K., and Serven, L., 2000a, What drives private saving across the world?, *Review of Economics and Statistics* 82, 165-181.
- Loayza, N., Schmidt-Hebbel, K., and Serven, L., 2000b, Saving in developing countries: an overview, *World Bank Economic Review* 14, 393-414.
- Lucas, R.E., 1990, Supply-side economics: an analytical review, *Oxford Economic Papers* 42, 293-316.
- Maddison, A., 2001, *The World Economy – A Millennial Perspective*, Development Center Studies, OECD, Paris.

- Mankiw, N.G., Romer, D., and Weo., D.N., 1992, A contribution to the empirics of economic growth, *Quarterly Journal of Economics* 107, 407-437.
- Masters W.A., and M.S. McMillan, 2001, Climate and scale in economic growth, *Journal of Economic Growth* 6, 167-186.
- Obstfeld, M., 1990, Intertemporal dependence, impatience, and dynamics, *Journal of Monetary Economics* 26, 45-75.
- Olsson, O., and D.A. Hibbs, 2005, Biogeography and long-run economic development, *European Economic Review*, forthcoming.
- Palivos, T., Wang, P., and Zhang, J., 1997, On the existence of balanced growth equilibrium, *International Economic Review* 38, 205-224,
- Pritchett, L., and Summers, L.H., 1996, Wealthier is healthier, *Journal of Human Resources* 31, 841-868.
- Rogers, A.R., 1994, Evolution of time preference by natural selection, *American Economic Review* 84, 460-481.
- Samwick, A.A., 1998, Discount rate heterogeneity and social security reform, *Journal of Development Economics* 57, 117-146.
- Steger, T.M., 2000, Economic growth with subsistence consumption, *Journal of Development Economics* 62, 343-361.
- Strulik, H., 2009, A Note on economic growth with subsistence consumption, *Macroeconomic Dynamics*, forthcoming.
- Strulik, H. and Weisdorf, J., 2008, Population, food, and knowledge: A simple unified growth theory, *Journal of Economic Growth* 13, 169-194.
- Uzawa, H., 1968, Time preference, the consumption function, and optimum asset holdings, in: Wolfe, J.N. (ed.), *Value, Capital and Growth: Papers in Honour of Sir John Hicks*, Edinburgh University Press, Edinburgh.
- Ventura, L., 2003, Direct measure of time-preference, *The Economic and Social Review* 34, 293-310.
- Weber, M., 1904, *The Protestant Ethic and the Spirit of Capitalism*, reprinted by Routledge, London, 2000.