

Broll, Udo; Egozcue, Martín; Wong, Wing-Keung

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## Prospect theory and two moment model: the firm under price uncertainty

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*Dresden Discussion Paper Series  
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UDO BROLL

MARTIN EGOZCUE

WING-KEUNG WONG

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Address of the author(s):

Udo Broll  
Technische Universität Dresden  
Dresden  
Germany

e-mail : [udo.broll@tu-dresden.de](mailto:udo.broll@tu-dresden.de)

Martin Egozcue  
Calle Las Orquideas Chalé Chiche  
Maldonado  
Uruguay

e-mail : [megozcue@yahoo.com](mailto:megozcue@yahoo.com)

Wing-Keung Wong  
Hong Kong Baptist University  
Hong Kong  
China

e-mail : [awong@hkbu.edu.hk](mailto:awong@hkbu.edu.hk)

Editors:

Faculty of Business and Economics, Department of Economics

Internet:

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Working paper coordinator:

Dominik Maltritz

e-mail: [wpeconomics@mailbox.tu-dresden.de](mailto:wpeconomics@mailbox.tu-dresden.de)

# Prospect theory and two moment model: the firm under price uncertainty

*Udo Broll*  
*Technische Universität Dresden*  
*Dresden*  
[udo.broll@tu-dresden.de](mailto:udo.broll@tu-dresden.de)

*Martin Egozcue*  
*Calle Las Orquideas Chale Chiché Maldonado*  
*Maldonado*  
[megozcue@yahoo.com](mailto:megozcue@yahoo.com)

*Wing-Keung Wong*  
*Hong Kong Baptist University*  
*Hong Kong*  
[awong@hkbu.edu.hk](mailto:awong@hkbu.edu.hk)

## Abstract:

Within the prospect theory the paper examines production and hedging decisions of a competitive firm under price uncertainty. We consider the prospect theory for the firm's utility function in the two moment model known as  $(\mu, \sigma)$ -preference. In contrast to the literature our findings show that the production under uncertainty can be larger than in the certainty case. Furthermore, we demonstrate that although the futures markets are unbiased the firm is overhedging.

JEL-Classification: D21, D41, D81

Keywords: Prospect theory, mean-variance model, price uncertainty

## 1. Introduction

The seminal paper by Sandmo (1971) analyzed conditions for optimal production of a competitive firm under price uncertainty. One of the most important results of his paper is that under price uncertainty, a risk averse firm will produce fewer goods than those produced in a price certainty framework. Holthausen (1979), Feder, Just and Schmitz (1980), Kawai and Zilcha (1986) extended this analysis to study firm's hedging behavior. The main result of these papers is known as separation property. This property states that in the presence of future markets, the optimal production is independent of the distribution of random prices and the firm's degree of risk aversion. Safra and Zilcha (1986) have shown that this result holds without the expected utility assumption. In a similar framework Broll and Eckwert (2008) demonstrated how market transparency and information affect the production and hedging decision.

Meyer (1987), Ormiston and Schlee (2001), Eichner and Wagener (2004) extended Sandmo's analysis using two moments decisions models. This tool, was originally developed by Schneeweiss (1967) and Sinn (1983) and further extended by Meyer (1987). One advantage of this framework is that the two moment decision model is a perfect substitute for the expected utility approach if one restrict random variables to belong to a linear distribution class.

Despite these advances, these models assumed that firms maximize an increasing and concave utility function of its profits. However, the study of firm production under uncertainty with other utilities forms others than risk aversion or risk seekers, like prospect theory preferences, to the best of our knowledge has not been studied yet. The purpose of this paper is to analyze whether a firm's optimal production and hedging behavior, under a two moment decision model, differs from risk aversion traditional results, if we assume a firm with prospect theory preferences.

Prospect theory was developed and introduced in economic theory by Kahneman and Tversky (1979) as an alternative to expected utility theory. Starting from experiments, the prospect theory describes how individuals evaluate potential gains and losses. Prospect theory has shown a range of phenomena that could not be otherwise explained within an expected utility framework.

In the context of two moments models, the marginal rate of substitution between expectation and risk can be interpreted as a measure of (absolute) risk aversion. In contrast to the literature we do not restrict the marginal rate of substitution to being positive. This makes our results quite different from the existence in the literature. Unlike the papers by Sandmo (1971) and others, within the prospect theory we can show that production under price uncertainty can be larger than in the certainty case. Furthermore, different from Feder, Just and Schmitz (1980) and Holthausen (1979), we demonstrate that with unbiased futures markets the firm is overhedging, i.e., the management will take risks if the expected loss is negative.

The paper is organized as follows. In section 2 we start with definitions and notations on prospect theory. Section 3 presents the firm's decision problem under prospect preferences and introduce the two moment decision making model which underlies our analysis. We analyze the decision making under two different scenarios. In section 3.1 there are no risk sharing markets available. In section 3.2 we introduce hedging opportunities and we derive our main results. Section 4 concludes.

## 2. Definitions and notations

Before developing the theory, we first state different types of utility functions as in the following definition.<sup>1</sup>

**Definition 1** For  $j = 1, 2, 3$ ,  $U_j^A$ ,  $U_j^D$ ,  $U_j^S$ ,  $U_j^R$ ,  $U_j^{CS}$ , and  $U_j^{CR}$  are the sets of utility functions  $u$  such that

$$\begin{aligned} U_j^A &= \{u : (-1)^i u^{(i)} \leq 0, i = 1, \dots, j\} \\ U_j^D &= \{u : u^{(i)} \geq 0, i = 1, \dots, j\} \\ U_j^S &= \{u : u_+ \in U_j^A \text{ and } u_- \in U_j^D\} \end{aligned}$$

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<sup>1</sup>We note that one could define “strictly increasing” and “increasing” situations for each of these sets of utility functions. In this paper, we combine both situations into one but, in order to avoid confusion, we assume that for each utility function  $u$ , there is a portion in which  $u^{(j)}$  is not equal to zero. We also note that the theory can be easily extended to include non-differentiable and/or non-expected utility functions. In this paper, we will skip the discussion of non-differentiable or non-expected utility functions. Readers may refer to Wong and Ma (2008) and the references there for the detailed discussion. We also note that in this definition, the reference point for  $U_j^S$  and  $U_j^R$  is zero. One could easily extend the theory to study the non-zero reference point for  $U_j^S$  and  $U_j^R$ .

where  $u^{(i)}$  is the  $i^{\text{th}}$  derivative of the utility function  $u$ ,  $u_+ = \max\{u, 0\}$ ,  $u_- = \min\{u, 0\}$ . Firms in  $U_j^A$  are risk averse whereas firms in  $U_j^S$  with  $S$ -shaped utility functions are risk averse for gains but risk seeking for losses. In this paper, we call firms with utility functions in  $U_j^S$  'prospect firms' or firms with prospect preference. As the utility for prospect firm is concave in the positive domain and convex for the negative domain, they show declining sensitivity in both gains and losses.

We give examples of  $S$ -shaped utility function as follows

$$u(x) = \begin{cases} u_+(x) & x \geq 0 \\ u_-(x) = -\beta u_+(-x) & x < 0 \end{cases} \quad (1)$$

where  $\beta > 0$  and  $u_+(0) = 0$ ,  $u_+^{(1)}(x) > 0$ ,  $u_+^{(2)}(x) < 0$  for  $x \geq 0$ .  $u^{(3)}(x) \geq 0$  for all  $x$ . If  $\beta = 1$ , then  $u(x)$  is a continuous  $S$ -shaped function and  $u(x) \in C^1$  for all  $x$ . In this paper, we define  $u^{(n)}$  be the  $n^{\text{th}}$  derivative of  $u$  for any function  $u$ .

Note that Kahneman and Tversky (1979) propose the following value (utility) function

$$u(x) = \begin{cases} x^{\gamma_G} & \text{if } x \geq 0 \text{ and } \gamma_G \in (0, 1), \\ -\lambda x^{\gamma_L} & \text{if } x < 0, \lambda > 0 \text{ and } \gamma_L \in (0, 1). \end{cases} \quad (2)$$

In addition, Al Nowaihi, Bradley and Dhami (2008) show that under preference for homogeneity and loss aversion prospect theory the value functions will have a power form with identical powers for gains and losses

$$u(x) = \begin{cases} -\lambda(-x)^\alpha & \text{if } x < 0, \\ x^\alpha & \text{if } x \geq 0, \end{cases} \quad (3)$$

where  $\lambda > 0$  and  $\alpha \in (0, 1)$ .

In this paper, we first consider  $u$  in equation (1) with  $\beta = 1$  or a subset  $U_j^{S0}$  in  $U_j^S$  such that for  $j = 2, 3$

$$U_j^{S0} = \{u \in U_j^S : u_-^{(i)}(-x) = (-1)^{i+1} u_+^{(i)}(x) \text{ for any integer } i \leq j\}. \quad (4)$$

Choosing between  $F$  and  $G$  in accordance with a consistent set of preferences will satisfy the von Neumann-Morgenstern consistency properties. Accordingly,  $F$  is strictly preferred to  $G$ , or equivalently,  $Y$  is preferred to  $Z$  if

$$\Delta Eu \equiv u(F) - u(G) \equiv u(X) - u(Y) \geq 0, \quad (5)$$

where  $u(F) \equiv u(X) \equiv \int_a^b u(z)dF(z)$  and  $u(G) \equiv u(Y) \equiv \int_a^b u(z)dG(z)$ .

### 3. Prospect theory, two moment decision model and price uncertainty

Prospect theory has proven influential in explaining phenomena what could not be properly explained within an expected utility framework. These include the effect of risk on investment, production and finance decisions. First we review and discuss some properties of the two moment model. To distinguish results in the literature from the ones derived in this paper, all cited results will be called propositions and our derived results will be called theorems.

Let the return  $X$  be the random variable with zero mean and variance one, with the location-scale family  $\mathcal{D}_X$  generated by  $X$  such that

$$\mathcal{D}_X = \{Y \mid Y = \mu + \sigma X : \mu \in \mathbb{R}, \sigma \in \mathbb{R}^+\}. \quad (6)$$

It is well-known that any prospect in a location-scale family,  $\mathcal{D}_X$ , can be completely characterized by its mean and variance (see, for example, Meyer (1987), Wong and Li (1999), Wong (2007) and Wong and Chan (2008)). Therefore expected utility rankings can be expressed as a mean-variance preference function as follows

$$V(\mu - \mu_o, \sigma) \equiv \int_{-\infty}^{\infty} u((\mu - \mu_o) + \sigma z)dF(z).$$

For any constant  $\alpha$ , the indifference curve drawn on the  $(\sigma, \mu)$  plane such that  $V(\sigma, \mu)$  is a constant can be expressed as

$$C_\alpha = \{(\sigma, \mu) \mid V(\sigma, \mu) \equiv \alpha\}.$$



In the indifference curve, we follow Meyer (1987) to have:

$$V_\mu(\sigma, \mu) d\mu + V_\sigma(\sigma, \mu) d\sigma = 0,$$

or

$$S(\sigma, \mu) \equiv \frac{d\mu}{d\sigma} = -\frac{V_\sigma(\sigma, \mu)}{V_\mu(\sigma, \mu)}, \quad (7)$$

where

$$\begin{aligned} V_\mu(\sigma, \mu) &\equiv \frac{\partial V(\sigma, \mu)}{\partial \mu} = \int_a^b u^{(1)}(\mu + \sigma x) dF(x), \\ V_\sigma(\sigma, \mu) &\equiv \frac{\partial V(\sigma, \mu)}{\partial \sigma} = \int_a^b u^{(1)}(\mu + \sigma x) x dF(x). \end{aligned}$$

Following the economic literature of modeling behavior of a competitive firm under price uncertainty and risk aversion, we assume that there are no risk sharing opportunities such as commodity or financial futures contracts. The profit function of the firm producing quantity  $Q$  and selling as market price  $P$  is equal to

$$\pi(Q) = PQ - C(Q) - FC,$$

where  $C(Q) + FC$  is the variable and fixed cost, respectively. We assume  $C(0) = 0$ ,  $C^{(1)}(Q) > 0$ ,  $C^{(2)}(Q) > 0$  and  $FC > 0$ .

The market price,  $P$ , of the firm's good is assumed to be stochastic with mean,  $\mu_P$ , standard deviation,  $\sigma_P$ , and density function  $f$  supported on  $[0, b]$ . Thus, the expectation of the profit function under uncertainty follows

$$E[\pi(Q)] = \mu = \mu_P Q - C(Q) - FC,$$

with standard deviation to be  $\sigma = \sigma_P Q$ .

For analytical purposes let us assume that  $P$  is symmetric about its mean  $\mu_P > 0$ . Then the profits will also be symmetric about the mean. Also defining

$$X = \frac{\pi - \mu}{\sigma}$$

will be symmetric about its zero mean with support on  $[-c, c]$  and  $c > 0$ . Hence, we can express the profit function as follows

$$\pi = \sigma X + \mu = \sigma_P Q X + \mu_P Q - C(Q) - FC.$$

Therefore the expected profit can be expressed as a function of the first two moments of the profit function  $\mu$  and  $\sigma$

$$V(\mu, \sigma) = Eu[\pi(Q)] = \int_{-c}^c u(\mu + \sigma x) f(x) dx.$$

We obtain

**Lemma 1** *For any utility  $u$  satisfying  $u^{(1)}(x) \geq 0$  and  $Y = \mu + \sigma X$  as defined in (6) where  $X$  is a symmetric random variable with zero mean and unit variance, we have  $V_\mu \geq 0$  where  $V_\mu$  is defined in (7).*

**Lemma 2** *For any utility  $u$  satisfying  $u^{(1)}(x) \geq 0$  and  $u^{(2)} \leq (\geq)0$ , and  $Y = \mu + \sigma X$  as defined in (6) where  $X$  is a symmetric random variable with zero mean and unit variance, we have  $V_\sigma \leq (\geq)0$  where  $V_\sigma$  is defined in (7).*

The proof of Lemma 1 and 2 is straightforward. We have to study the convexity of the indifference curve  $C_\alpha$  with the restriction of  $V(\sigma, \mu) \equiv \alpha$ . Under the constraint of  $(\sigma, \mu) \in C_\alpha$ , Wong (2006) obtains the following proposition

**Proposition 1** *If the distribution function of the return with mean  $\mu$  and variance  $\sigma^2$  belongs to a location-scale family and for any utility function  $u$ , if  $u^{(1)} > 0$ , then the indifference curve  $C_\alpha$  can be parameterized as  $\mu = \mu(\sigma)$  with slope*

$$S(\sigma, \mu) = -\frac{V_\sigma(\sigma, \mu)}{V_\mu(\sigma, \mu)}.$$

*In addition,*

- a. *if  $u^{(2)} \leq 0$ , then the indifference curve  $\mu = \mu(\sigma)$  is an increasing and convex function of  $\sigma$ , and*
- b. *if  $u^{(2)} \geq 0$ , then the indifference curve  $\mu = \mu(\sigma)$  is a decreasing and concave function of  $\sigma$ .*

One could rewrite Proposition 1 with the following property.

**Property 1** *If the distribution function of the return on investment with mean  $\mu$  and variance  $\sigma^2$  belongs to a location-scale family possessing the property in (6), then*

- a. *for any risk averter with utility function  $u$ , the indifference curve  $\mu = \mu(\sigma)$  is an increasing and convex function of  $\sigma$ ,*
- b. *for any risk neutral firm with utility function  $u$ , the indifference curve  $\mu = \mu(\sigma)$  is a horizontal function of  $\sigma$ , and*
- c. *for any risk seeker with utility function  $u$ , the indifference curve  $\mu = \mu(\sigma)$  is a decreasing and concave function of  $\sigma$ .*

These properties show that the nature of curvature of the indifference curves for risk averse or risk seeking firms remain the same for different signs of  $\mu$ .

We turn to study the behavior of prospect firms. We consider  $X$  to be a symmetric random variable with zero mean and unit variance with finite support. We also consider the  $S$ -shaped utility functions to satisfy condition stated in (4). This includes utility functions defined in (1) with  $\beta = 1$ . From these conditions, we first obtain the following

**Lemma 3** *For any  $u \in U_2^{S0}$  and  $Y = \mu + \sigma X$  as defined in (6) where  $X$  is a symmetric random variable with zero mean and unit variance with support on  $[-b, b]$ , we have*

- a. *if  $\mu > 0$ , then  $V_\sigma \leq 0$ ,*
- b. *if  $\mu = 0$ , then  $V_\sigma = 0$ , and*
- c. *if  $\mu < 0$ , then  $V_\sigma \geq 0$ .*

**Proof.** As  $u \in U_2^{S0}$ , we have  $u^{(1)}(-x) = u^{(1)(x)}$  and  $u^{(2)}(-x) = u^{(2)(x)}$  for any

$x \geq$ . Also, we have

$$\begin{aligned}
V_\sigma &= \int_{-b}^b u^{(1)}(\mu + \sigma x) x dF(x) \\
&= \int_0^b [u^{(1)}(\mu + \sigma x) - u^{(1)}(\mu - \sigma x)] x dF(x) \tag{8} \\
&= \int_0^{|\frac{\mu}{\sigma}|} [u^{(1)}(\mu + \sigma x) - u^{(1)}(\mu - \sigma x)] x dF(x) \\
&\quad + \int_{|\frac{\mu}{\sigma}|}^b [u^{(1)}(\mu + \sigma x) - u^{(1)}(\mu - \sigma x)] x dF(x) \\
&= I_1 + I_2. \tag{9}
\end{aligned}$$

We first prove the case for  $\mu = 0$ . From (8), we have

$$\begin{aligned}
V_\sigma &= \int_0^b [u^{(1)}(\mu + \sigma x) - u^{(1)}(\mu - \sigma x)] x dF(x) \\
&= \int_0^b [u^{(1)}(\sigma x) - u^{(1)}(-\sigma x)] x dF(x) = 0
\end{aligned}$$

as  $u^{(1)}(-\sigma x) = u^{(1)}(\sigma x)$  by assumption that  $u \in U_2^{S0}$ . Thus, the assertion holds for  $\mu = 0$ .

We now prove the case for  $\mu < 0$ . In this situation, we have  $\mu + \sigma x < \sigma x - \mu$  and hence

$$u^{(1)}(\mu + \sigma x) \geq u^{(1)}(\sigma x - \mu) \tag{10}$$

for any  $x \geq \frac{\mu}{\sigma}$  as  $\frac{\mu}{\sigma} < 0$  and  $u^{(2)}(y) \leq 0$  for  $y \geq 0$ . Multiplying (10) by  $x f(x)$  and integrating it from 0 to  $b$ , from (8), we have

$$V_\sigma = \int_0^b [u^{(1)}(\mu + \sigma x) - u^{(1)}(\mu - \sigma x)] x dF(x) \geq 0$$

and thus the assertion holds for  $\mu < 0$ .

We turn to prove the case for  $\mu > 0$ . It can be shown that both integrals  $I_1$  and  $I_2$  are negative in this situation. First we prove that  $I_1$  is negative. As  $x \leq \frac{\mu}{\sigma}$ , we have  $\mu - \sigma x \geq 0$  and hence  $\mu + \sigma x \geq \mu - \sigma x$  for all  $x \geq 0$ . In

addition, because  $u^{(2)}(y) \leq 0$  for  $y \geq 0$ , we have  $u^{(1)}(\mu + \sigma x) \leq u^{(1)}(\mu - \sigma x)$ . Multiplying by  $xf(x)$  and integrating it from 0 to  $\frac{\mu}{\sigma}$ , we get

$$\int_0^{\frac{\mu}{\sigma}} [u^{(1)}(\mu + \sigma x) - u^{(1)}(\mu - \sigma x)] x dF(x) \leq 0.$$

Now, we turn to show that  $I_2 \leq 0$ . As  $u \in U_2^{S0}$ , we have  $u^{(1)}(-x) = u^{(1)}(x)$ , and thus  $u^{(1)}(\mu - \sigma x) = u^{(1)}(\sigma x - \mu)$ . For the second integral  $I_2$ , we have  $x \geq \frac{\mu}{\sigma}$ . That is,  $\sigma x - \mu \geq 0$ . Because  $u^{(2)}(y) \leq 0$  for  $y \geq 0$ , we have

$$u^{(1)}(\mu + \sigma x) \leq u^{(1)}(\sigma x - \mu) = u^{(1)}(\mu - \sigma x)$$

In addition, as  $x \geq \frac{\mu}{\sigma} \geq 0$ , multiplying by  $xf(x)$  and integrating it from  $\frac{\mu}{\sigma}$  to  $b$ , we get

$$\int_{\frac{\mu}{\sigma}}^b [u^{(1)}(\mu + \sigma x) - u^{(1)}(\mu - \sigma x)] x dF(x) \leq 0$$

and thus the second integral  $I_2$  is negative and thereafter, the assertion holds for  $\mu > 0$ .  $\square$

**Proposition 2** *For any  $u \in U_2^{S0}$  and  $Y = \mu + \sigma X$  as defined in (6) where  $X$  is a symmetric random variable with zero mean and unit variance, we have*

- a. *if  $\mu > 0$ , then  $\mu = \mu(\sigma)$  is an increasing function of  $\sigma$ ,*
- b. *if  $\mu = 0$ , then  $\mu = \mu(\sigma)$  is a constant function of  $\sigma$ , and*
- c. *if  $\mu < 0$ , then  $\mu = \mu(\sigma)$  is an decreasing function of  $\sigma$ .*

When we fix the level of profit, we can represent the slope,  $S(\mu, \sigma)$ , of an indifference curve as defined in Proposition 1. In our context  $S(\mu, \sigma)$  represents the marginal rate of substitution between expectation  $\mu$  and risk  $\sigma$  of the firm's profit. Unlike other studies, for example, Meyer (1987), Eichner and Wagener (2004), Broll, Wahl and Wong (2006) we do not restrict  $S(\mu, \sigma)$  to be positive only. We obtain

**Theorem 1** *For any  $u \in U_2^{S0}$  and  $Y = \mu + \sigma X$  as defined in (6) where  $X$  is a symmetric random variable with zero mean and unit variance, we have*

- a. if  $\mu > 0$ , then  $S(\mu, \sigma) \geq 0$ ,
- b. if  $\mu = 0$ , then  $S(\mu, \sigma) = 0$ ,
- c. if  $\mu < 0$ , then  $S(\mu, \sigma) \leq 0$ .

The *proof* follows from Proposition 2.

### 3.1 Production under price uncertainty

The seminal paper by Sandmo (1971) establishes the main finding for the optimal production of a competitive firm in which a risk averse firm will produce less than what is produced in a price certain framework. In this paper, we extend his work to study the behavior of a firm in a two moment decision model with prospect preferences such that utility  $u \in U_2^{S0}$ . We first state the following theorem.

**Theorem 2** *If the firm's utility  $u \in U_2^A$  or  $u \in U_2^{S0}$  and if there is a global optimum production such that  $Q^* > 0$ , then the slope of the indifference curve for the manager at the optimum satisfies*

$$S(\mu, \sigma)|_{Q^*} = \frac{\mu_P - C^{(1)}(Q^*)}{\sigma_P}.$$

**Proof.** The firm maximizes

$$\max_Q Eu[\pi(Q)] = \max_Q V(\mu(Q), \sigma(Q)) = \int_{-c}^c u(\mu + \sigma x) f(x) dx.$$

with

$$\mu = \mu_P Q - C(Q) - FC,$$

and

$$\sigma = \sigma_P Q.$$

The first order condition is

$$\begin{aligned} \frac{\delta V(\mu, \sigma)}{\delta Q} &= \int_{-c}^c u^{(1)}(\mu + \sigma x) [\mu_P - C^{(1)}(Q) + \sigma_P x] f(x) dx \\ &= [\mu_P - C^{(1)}(Q)] \int_{-c}^c u^{(1)}(\mu + \sigma x) f(x) dx \\ &= +\sigma_P \int_{-c}^c u^{(1)}(\mu + \sigma x) x f(x) dx \\ &= [\mu_P - C^{(1)}(Q)] V_\mu + \sigma_P V_\sigma = 0. \end{aligned}$$

Rearranging

$$S(\mu, \sigma)|_{Q^*} = -\frac{V_\sigma}{V_\mu} = \frac{\mu_P - C^{(1)}(Q^*)}{\sigma_P}.$$

Under the firm's risk aversion preferences, Sandmo (1971) has shown that under price uncertainty, optimal production,  $Q_U^*$ , will be less than the optimal production in the certainty case,  $Q_C^*$ .

**Proposition 3** *Suppose the manager's utility is  $u \in U_2^A$ . The firm's production under certainty will be larger than under uncertainty, i.e.  $Q_U^* < Q_C^*$ .*

**Proof.** In the certainty case, the optimal production satisfies

$$P = \mu_P = C^{(1)}(Q_C^*).$$

As we have seen uncertainty price optimal production  $Q_U^*$  satisfies

$$S(\mu, \sigma)|_{Q^*} = \frac{\mu_P - C^{(1)}(Q_U^*)}{\sigma_P} \geq 0,$$

$$\mu_P \geq C^{(1)}(Q_U^*).$$

Hence

$$C^{(1)}(Q_C^*) \geq C^{(1)}(Q_U^*).$$

As we have assumed that  $C^{(2)}(Q) > 0$  then  $C^{(1)}(Q)$  is an increasing function therefore,

$$Q_C^* \geq Q_U^*.$$

■

This result holds if the firm is risk averse. But if we consider a firm with prospect preferences, with  $u \in U_2^{S0}$  this result may not hold and we can get the opposite result.

**Theorem 3** *Suppose the firm's utility function is  $u \in U_2^{S0}$ . If there is an interior solution, when  $E(\pi) = \mu > 0$  for all  $Q$ , then production under price uncertainty is smaller than under certainty  $Q_C^* \geq Q_U^*$ . If  $E(\pi) = \mu < 0$  for all  $Q$ , then production under price uncertainty is larger than under certainty  $Q_C^* \leq Q_U^*$ .*

**Proof.** This follows from Theorem 1 and Proposition 3. For  $\mu > 0$  then  $S(\mu, \sigma) \geq 0$ , therefore

$$S(\mu, \sigma)|_{Q_U^*} = \frac{\mu_P - C^{(1)}(Q_U^*)}{\sigma_P} \geq 0,$$

hence  $C^{(1)}(Q_C^*) \geq C^{(1)}(Q_U^*)$ . Again as  $C^{(1)}(Q)$  is an increasing function then  $Q_C^* \geq Q_U^*$ . Similarly, if  $\mu < 0$  then  $S(\mu, \sigma) \leq 0$ , hence  $Q_C^* \leq Q_U^*$ . ■

Theorem 3 shows the difference to traditional results under price uncertainty. Taking prospect theory into account of a well known model of a firm there are situations with price uncertainty where optimal production with a stochastic price is larger than optimal production in the certainty case. This result holds when the expected profit is negative for all values of the endogenous variable.

*Example* We offer some calculations for a specific example in which the producing firm has the utility function

$$\begin{aligned} u(\pi) &= \sqrt{\pi} \text{ for } \pi \geq 0, \\ u(\pi) &= -\sqrt{-\pi} \text{ for } \pi < 0. \end{aligned}$$

In the economy output prices could be  $P = 7$  or  $P = 1$  with equal probability. The cost function is  $C(Q) = Q^2$  and  $FC = 5$ . As the numerical example demonstrates the expected profit is negative for all  $Q \geq 0$ . The firm's optimum production level  $Q^*$  for a certain price equal to  $\mu_P = 4$  is  $C^{(1)}(Q^*) = \mu_P$  and will be  $Q^* = 2$ . Meanwhile, under price uncertainty we obtain

$$Eu[\pi(Q^* = 2.12)] = -0.2 \geq -1.$$

This example shows that the expected marginal revenue is less than the marginal cost,

$$\mu_P = 4 < C^{(1)}(Q = 2.12) = 4.49.$$

Note that in the general case with arbitrary preferences the proposition remains valid. One necessary condition of this result is that fixed costs are not zero.

**Corollary 1** *Suppose the firm's utility is  $u \in U_2^{S0}$  and that the fixed costs are zero. If there is an interior solution the firm's optimum production satisfies  $Q_C^* \geq Q_U^*$ .*



**Proof.** If  $FC = 0$ , then we have  $E(\pi) = \mu \geq 0$ . By Theorem 3 we get  $Q_C^* \geq Q_U^*$ . ■

### 3.2 Hedging price risk

Holthausen (1979), Feder, Just and Schmitz (1980) have analyzed optimal production and hedging for a risk averse competitive firm under price uncertainty. Their main result of these works is what is called separation property. This property states that the existence of commodity futures markets makes optimal production independent by changes in the distribution of prices or by the firm's degree of risk aversion. Without the expected utility theory, Safra and Zilcha (1986) have proved that this property holds for all increasing preferences.

Assuming an existing level of output stock  $\bar{Q} > 0$ , the risk averse firm can choose its optimum hedging policy  $H$ , restricted to  $H \geq 0$ , at a given forward price  $P_f$ . The firm's profit function is as follows

$$\pi(H) = P(\bar{Q} - H) + P_f H - FC.$$

We consider fixed costs  $FC > 0$ . As before we express the expected utility of profits as  $V(\mu, \sigma) = Eu(\pi)$ . The firm's decision problem is

$$\max_{H \geq 0} Eu(\pi)$$

with

$$\mu = \mu_P(\bar{Q} - H) + P_f H - FC,$$

and

$$\sigma = \sigma_P(\bar{Q} - H).$$

**Proposition 4** *Suppose the firm's utility is  $u \in U_2^{S0}$ . If there is an interior optimum, the firm's hedging decision satisfies*

$$S(\mu, \sigma)|_{H^*} = \frac{(\mu_P - P_f)(\bar{Q} - H^*)}{\sigma}.$$

**Proof.** The first order condition for hedging is

$$V_\mu \frac{\partial \mu}{\partial H} + V_\sigma \frac{\partial \sigma}{\partial H} = V_\mu (-\mu_P + P_f) - V_\sigma \sigma_P = 0.$$

Rearranging

$$\frac{\mu_P - P_f}{\sigma_P} = -\frac{V_\sigma}{V_\mu}.$$

Now as

$$\sigma_P = \frac{\sigma}{(\bar{Q} - H^*)}.$$

Substituting

$$S(\mu, \sigma) = -\frac{V_\sigma}{V_\mu} = \frac{(\mu_P - P_f)(\bar{Q} - H^*)}{\sigma}.$$

■

Holthausen (1979), Feder, Just and Schmitz (1980) and others have shown that the hedging ratio  $h = H/\bar{Q}$  depend on the difference between future  $P_f$  and expected commodity price  $\mu_P$ .

**Proposition 5** *Suppose the firm's utility is  $u \in U_2^A$ . If there is an interior solution, the optimal hedging  $H^*$  will be, if  $\mu_P > P_f$  then  $\bar{Q} \geq H^*$ . If  $\mu_P < P_f$  then  $\bar{Q} \leq H^*$ .*

**Proof.** The result follows from the first order condition

$$S(\mu, \sigma) = -\frac{V_\sigma}{V_\mu} = \frac{(\mu_P - P_f)(\bar{Q} - H^*)}{\sigma}.$$

By Property 1a) we know that for  $u \in U^A$  implies  $S(\mu, \sigma) \geq 0$  the result follows. ■

This result says that the firms hedging ratio  $h = H/\bar{Q}$  will be less than one under backwardation and larger than one under contango. Does this result also holds for firm's with prospect preference? We analyze the case for backwardation only.

**Theorem 4** *Suppose the firm's utility is  $u \in U_2^{S0}$  and assuming backwardation, i.e.  $\mu_P > P_f$ . Then in a global interior solution, the optimum firm's hedging satisfies:*

- a. if  $\mu > 0$  for all  $H$  then  $\bar{Q} \geq H^*$ ;
- b. if  $\mu < 0$  for all  $H$  then  $\bar{Q} \leq H^*$ .

**Proof.** The proof follows from equality

$$\frac{(\mu_P - P_f)(\bar{Q} - H^*)}{\sigma} = -\frac{V_\sigma}{V_\mu} = S(\mu, \sigma).$$

We have shown that under prospect preference, if  $\mu > 0$  then  $S(\mu, \sigma) \geq 0$ . If  $\mu < 0$  we obtain  $S(\mu, \sigma) \leq 0$ . ■

*Example* We offer some calculations for a specific example in which the producing firm under price risk has the utility function

$$\begin{aligned} u(\pi) &= \sqrt{\pi} \text{ for } \pi \geq 0, \\ u(\pi) &= -\sqrt{-\pi} \text{ for } \pi < 0. \end{aligned}$$

In the economy output prices could be  $P = 7$  or  $P = 1$  with equal probability. The initial inventory is  $\bar{Q} = 2$ ,  $FC = 10$ . and  $P_f = 3.8$ . As the numerical example demonstrates the expected profit is negative for all  $H \geq 0$ . Under prospect preference there is a global maximum at  $H^* = 14.1$  where we obtain

$$Eu[\pi(H^* = 14.1)] = -0.4.$$

Then  $H^* > \bar{Q}$ .

Therefore a competitive firm with prospect preference unlike risk averse firm will hedge depending on the sign of the expected profit. If the expected profit is positive for all  $H > 0$ , the hedging ratio will be less than one as in the risk averse case. However, if the expected profit is negative for all  $H > 0$ , the firm's hedge ratio will be the opposite as in the risk averse case. The intuition is similar than the other result. If the firm has to choose between a certain loss or an uncertain but possible positive profit the management will take the chance.

## 4. Concluding remarks

Prospect theory is a theory that describes decisions between alternative choices that involve risk where the probabilities are known. The theory describes how individuals evaluate potential gains and losses. In the original description the term prospect referred to a lottery. To illustrate the economic

implications of prospect theory we consider a classical decision problem of a competitive firm under price uncertainty.

The paper studies optimum production and hedging decisions by a competitive firm under price uncertainty. The prospect theory is used in a two-moment model to discuss and compare the optimum level of production and hedging under different economic environments. In contrast to the literature, in the case without risk sharing markets, our findings show that the production under uncertainty can be larger than in the case of certainty. Furthermore, we demonstrate that although the futures markets are unbiased the firm is overhedging, i.e., the firm will take risks if the expected profit is negative.

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