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Export production, hedging exchange rate risk: the duopoly case

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**Export Production, Hedging Exchange Rate Risk:
The Duopoly Case**

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Dresden Discussion Paper in Economics No. 06/08

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Export Production, Hedging Exchange Rate Risk: The Duopoly Case

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Abstract:

This paper studies a Cournot duopoly in international trade so that the firms are exposed to exchange rate risk. A hedging opportunity is introduced by a forward market where the foreign currency can be traded on. We investigate two settings: First we assume that hedging and output decisions are taken simultaneously. We show that hedging is just done for risk managing reasons as it is not possible to use hedging strategically. In this setting the well-known separation result of the competitive firm holds if both firms have the hedging opportunity. In the second setting the hedging decisions are made before the output decisions. We show that hedging is used not only to manage the risk exposure but also as a strategic device. Furthermore we find that no separation result can be stated.

JEL-Classification: F10, F11, F30, F31

Keywords: Exchange Rate risk; hedging; exports; duopoly

1. Introduction

While the competitive firm under exchange rate risk and with and without a hedging opportunity has been discussed in detail in the literature, there is still a lack of treatment of imperfect markets like oligopolies.¹ This paper is a contribution to fill this lack as we consider a Cournot duopoly under exchange rate risk and with the opportunity to use currency forward contracts.² We investigate two settings: In the first one we assume that hedging and output decisions are taken simultaneously by the firms. In the second setting we assume that the hedging decisions are made before the output decisions. Other studies which are concerned with the duopoly case under uncertainty, too, can also be distinguished by their order of the hedging and output decisions.³ The simultaneous case has only been examined by Eldor/Zilcha (1990) for n identical firms and forward contracts for the homogeneous good. The first paper that was concerned with the second case is from Ungern-Sternberg/Weizsäcker (1990) in which any link between the hedging and output decisions was ruled out.⁴ Allaz (1992), Allaz/Vila (1993) and Broll et al. (1999a) also applied to the sequential case but did not rule out that link. In all of these three particular settings it was shown that hedging can be used as a strategic device. We also show in our setting that hedging is used for strategical reasons besides the incentive for risk reduction. Hence, the firms apply to overhedges in an unbiased forward market. In the simultaneous case the firms apply to full hedges in an unbiased forward market as they are not able to use hedging in a strategical manner. Therefore it is even possible in this case that a firm is harmed if getting the hedging opportunity.

¹For the competitive firm under price risk see e.g. Holthausen (1979) and under exchange rate and price risk see Kawai/Zilcha (1986). In more recent studies these models have been expanded in various ways. For example Broll et al. (1999b) examined the case in which the firm exports to two foreign markets and futures are only available between the foreign currencies. Wong (2003a) and (2003b) determined the case of currency options in different settings. Drees/Eckwert (2003) introduced a signal being correlated with the exchange rate and which is observed prior to the date of decision making.

²If the firms have a motive to hedge risk at all is not a matter of our investigation. For an enfolding discussion concerning this aspect see Froot et al. (1993) and Raposo (1999).

³Another similar distinction is made by Hughes/Kao (1997) comparing the cases of observable and unobservable hedging decisions in the sequential framework.

⁴See Ungern-Sternberg/Weizsäcker (1990), p. 383.

In the sequential case a firm is always better off if receiving an exclusive hedging opportunity.

Furthermore we show that the well-known separation result of the competitive firm⁵ holds in the simultaneous case if both firms are able to hedge whereas in the sequential case no separation property can be found.

The paper is organized as follows. In the 2nd and 3rd section we examine the equilibrium outputs of the Cournot model under certainty and uncertainty but without any hedging opportunities. These cases are used as bases of comparison for the models with hedging. Section 4 is the main part of this paper and is divided into two sections referring to the order of the hedging and output decisions. For both settings we investigate the impact of the hedging opportunity on the optimal outputs and expected utilities of the firms. Thereby we show the strategic incentive of hedging. The last section concludes.

2. The Certainty Case

This section is concerned with export production under certain exchange rates to use it as a basis of comparison in later sections when uncertainty is assumed. The derived equilibrium is the well-known Cournot-equilibrium. In the literature many different settings can be found that assure the existence of an unique equilibrium.⁶ We follow Collie (1992)⁷ and discuss if it is possible to attain, by imposing further assumptions, that the unique equilibrium is solely determined by the first order conditions. This is done in section 2.2. But first of all we have to define the setup of the model which is the basis for extensions in later sections.

⁵See Holthausen (1979), p. 990.

⁶For an enfolding overview see Collie (1992), Friedman (1991), pp. 83-87 and 95-97 and Vives (1999), pp. 93-101.

⁷See Collie (1992), pp. 60-64.

2.1 The Model

Consider a setting with two dates, $t = 0, 1$ (one-period model), and two exporting firms which are domiciled in countries using the same currency.⁸ Between the first and the second date these firms produce a single homogeneous good. The output, $x_i \geq 0$ for $i = 1, 2$, causes costs, $c_i x_i$ with $c_i > 0$ for $i = 1, 2$ and $c_1 \neq c_2$, in $t = 0$, which are denominated in the home currency. In the second date, $t = 1$, both firms sell their entire output in the same foreign country so that the revenues, being received in $t = 1$, are denominated in the foreign currency as well as the price per unit of the good, $p = p(x_1 + x_2)$.⁹ The price is given by the inverse demand of the good which is specified by a downward-sloping function being twice continuously differentiable. Hence, the profit functions in $t = 1$, denominated in the home currency, are defined as follows:¹⁰

$$\Pi_i = (e p - c_i) x_i \text{ for } i = 1, 2. \quad (1)$$

We assume the firms to act as Cournot duopolists who are not influenced by the past behaviour of themselves and the other firm and are only concerned about their profits in $t = 1$. That means they do not care about the implications their behaviour might have for the future. Therefore each firm chooses its output in $t = 0$ in order to maximize its profit in $t = 1$ while assuming that the output of the other duopolist is invariant with respect to its own decision.¹¹ For this reason the choice of the output can be referred to as a best response to the output of the other duopolist and so in an equilibrium the outputs of both firms have to be best responses to each other. Hence, such an equilibrium is called a Cournot-Nash one.¹²

⁸Throughout the paper we call this currency the home one.

⁹It is assumed that the home currency and the foreign one are not the same.

¹⁰The exchange rate between the home and the foreign currency in $t = 1$ is symbolized by e with $e \in (0, e_{\max}]$. The dimension of e is units of the home currency per one unit of the foreign currency. Furthermore we assume a riskless interest rate of zero for the period between $t = 0$ and $t = 1$ so that the costs which occur in $t = 0$ remain the same in $t = 1$. All parameters of the model are common knowledge.

¹¹See Rasmusen (1991), p. 78.

¹²For instance see Rasmusen (1991), pp. 76-78. or Vives (1999), p. 93.

2.2 The Equilibrium

We start with the definition of the two assumptions which assure the existence of an unique equilibrium in the present setting:¹³

(A1) The total revenue, $(x_1 + x_2)p$, is bounded.

(A2) The outputs of the firms are strategic substitutes so that $p' + x_i p'' < 0$ for $i = 1, 2$.¹⁴

Proposition 1. Under the assumptions (A1) and (A2) there exists an unique Cournot-Nash equilibrium.¹⁵

As mentioned earlier, we want to investigate additional assumptions that assure an interior solution so that the unique equilibrium is solely determined by the first order conditions of the duopolists. The following proposition summarizes the results.

Proposition 2. (a) Under no more assumption it is possible that one or both firms do not produce anything in the equilibrium.

(b) Under the assumption that $e p(0) > \min(c_1, c_2)$ at least one firm produces in the equilibrium.

(c) Under the assumptions that $c_1 = c_2$ and $e p(0) > c_1$ both firms produce in the equilibrium.¹⁶

As we want the exporting firms not to be identical we state that $c_1 \neq c_2$ in section 2.1. Thus we have to assume that the unique equilibrium is an interior solution to assure that the equilibrium is solely determined by the following conditions:¹⁷

$$e p(x_1^* + x_2^*) + e x_1^* p'(x_1^* + x_2^*) = c_1, \quad (2)$$

$$e p(x_1^* + x_2^*) + e x_2^* p'(x_1^* + x_2^*) = c_2. \quad (3)$$

¹³(A1) and (A2) are assumed to hold in later sections without stating them explicitly.

¹⁴See Bulow et al. (1985), p. 494.

¹⁵See Collie (1992), pp. 60-64. One can easily prove that the required assumptions are met in our setting.

¹⁶The proof can be found in appendix (I).

¹⁷A (*) signifies the optimal choice of a decision variable.

We refer to these conditions in later sections when we compare these results with those under exchange rate uncertainty.

3. The Uncertainty Case

In the present section uncertainty is introduced by assuming that the exchange rate \tilde{e} is a random variable and the firms only know whose probability distribution.¹⁸ So the profit functions are as follows:

$$\tilde{\Pi}_i = (\tilde{e}p - c_i)x_i \text{ for } i = 1, 2. \quad (4)$$

This means that the duopolists are only informed about the probability distribution of \tilde{e} in $t = 1$ when choosing their outputs in $t = 0$. Hence, the decisions of the firms depend on their attitude towards risk. In the following sections we consider two cases: In section 3.1 we assume risk-neutrality and in section 3.2 we introduce von Neumann-Morgenstern utility functions which imply risk-aversion.

3.1 Risk-Neutrality

Introducing a random exchange rate and assuming that both firms are risk-neutral, finding an equilibrium is only slightly different from the certainty case as the firms now maximize in $t = 0$ the expected value of their profits instead of the profits. Therefore we obtain familiar conditions that assure the existence of an unique equilibrium even in the case of uncertainty.

Proposition 3. In the present model there exists an unique Cournot-Nash equilibrium.¹⁹

If we want the unique equilibrium to be solely determined by the first order conditions we could derive a result that is analog to proposition 2.

¹⁸Throughout this paper a tilde ($\tilde{\cdot}$) signifies a random variable. $E(\cdot)$ symbolizes the expectation operator, $\text{Var}(\cdot)$ the variance and $\text{cov}(\cdot, \cdot)$ the covariance.

¹⁹See Collie (1992), pp. 60-64. As $E(\tilde{e}) > 0$ the conditions of Collie are still fulfilled in the present case.

Hence, we have to assume an interior solution to determine the equilibrium by the following conditions:

$$E(\tilde{e}) p(x_1^* + x_2^*) + E(\tilde{e}) x_1^* p'(x_1^* + x_2^*) = c_1, \quad (5)$$

$$E(\tilde{e}) p(x_1^* + x_2^*) + E(\tilde{e}) x_2^* p'(x_1^* + x_2^*) = c_2. \quad (6)$$

Proposition 4. Comparing the equilibria of propositions 1 and 3, we find that the outputs of the firms are the same in both equilibria if $E(\tilde{e}) = e$. Additionally assume that the equilibria are interior solutions, then the optimal total output, $x_{total}^* = x_1^* + x_2^*$, in the equilibrium under uncertainty is greater [smaller] than under certainty if $E(\tilde{e}) > [<] e$.²⁰

Hence, introducing a risky exchange rate does not alter the results if risk-neutrality is assumed and $E(\tilde{e}) = e$ holds. Furthermore we see that the optimal total output rises if $E(\tilde{e}) > e$ while the change in the optimal outputs of the firms is ambiguous. This proposition can also be interpreted as a comparative static analysis considering the question how the optimal outputs are altered by an increase of the exchange rate or expected exchange rate, respectively. This increase causes two effects: First, the firms increase their outputs as the higher exchange rate implies higher marginal revenue. Second, higher outputs decrease the price which in turn lowers the marginal revenue and induces the firms to decrease their outputs. While the effect of an increase of the exchange rate on the individual output decision is ambiguous the entire output increases. Hence, we cannot state whether an increasing exchange rate arises or lowers the optimal profits of the firms.

3.2 Risk-Aversion

In this section the firms are assumed to be risk-avers. Therefore we introduce von-Neumann-Morgenstern utility functions for both firms defined over their particular profits denominated in the home currency. The utility functions are twice continuously differentiable, strictly increasing, $U'_i(\Pi_i) > 0$, and strictly concave, $U''_i(\Pi_i) < 0$. Hence, to find a Cournot-Nash equilibrium the firms maximize their expected utilities by choosing an output and assuming

²⁰The proof can be found in appendix (II).

that the output of the other duopolist is invariant with respect to their own decisions.

Following the procedure that we applied in the case of risk-neutrality, we first state conditions that assure the existence and uniqueness of an equilibrium and afterwards this equilibrium will be compared with that under risk-neutrality.

Proposition 5. In the current model a Cournot-Nash equilibrium exists.²¹ Assuming that all candidate equilibria are interior and that $V_{x_1x_1}^1 V_{x_2x_2}^2 - V_{x_1x_2}^1 V_{x_2x_1}^2 > 0$ holds, there is a unique interior equilibrium.²²

These assumptions despite of being quite restrictive are common in the literature.²³ The reason for this is that it is not possible to find other uniqueness conditions in these kind of models which just restrict the inverse demand function or the cost functions in some ways.

Having set up conditions that assure an unique, interior equilibrium we are able to compare this equilibrium with that under risk-neutrality. The following conditions have to be fulfilled in the equilibrium:

$$E(U_1'(\tilde{\Pi}_1)(\tilde{e}p(x_1^* + x_2^*) + \tilde{e}x_1^*p'(x_1^* + x_2^*) - c_1)) = 0, \quad (7)$$

$$E(U_2'(\tilde{\Pi}_2)(\tilde{e}p(x_1^* + x_2^*) + \tilde{e}x_2^*p'(x_1^* + x_2^*) - c_2)) = 0. \quad (8)$$

Proposition 6. Comparing the equilibria under risk-aversion and risk-neutrality and assuming that they are interior yields that the optimal total output under risk-aversion is smaller than under risk-neutrality.²⁴

Like in proposition 4 we are again able to determine the direction of the change in the optimal entire output but not of the particular optimal outputs. A reason for this is that there are two effects which influence the decision of a risk-avers firm if its revenue becomes stochastic. First, such a firm wants to decrease its exposure towards risk which is only possible by

²¹See Friedman (1991), pp. 95-96. The conditions for existence are fulfilled in this model as the expected utility functions are strict concave in the particular outputs.

²²See Vives (1999), pp. 48 and 98. $V_{x_i x_j}^i$ is a short-cut for $\frac{\partial^2 E(U_i(\tilde{\Pi}_i))}{\partial x_i \partial x_j}$.

²³For instance see Asplund (2002), p. 997.

²⁴The proof can be found in appendix (III).

diminishing the output. Since the firm acts in an oligopoly market its own decision depends on what the other firm does. Hence, if both firms diminish their outputs in order to decrease their exposure towards risk, the price will be higher which in turn is an incentive to increase the output. Whether the first or the second effect dominates depends basically on the relation between the degrees of risk-aversion of the firms.²⁵

Another aspect which should be investigated is how the expected utilities of risk-averse duopolists change from the certainty case to the present one. As we will see in the next section the effect is not clear-cut.

4. Hedging

In this section we extend the model by introducing an opportunity for one or both firms to hedge the exchange rate risk. This is done by an arbitrage-free, competitive currency forward market where the firms can deal with units of the foreign currency at a pre-specified exchange rate of the home currency against the foreign one, $e_f > 0$.²⁶ Before specifying this market in more detail we have to decide whether the optimal hedging positions and outputs are chosen simultaneously or sequentially. In order to constitute hedging as a strategic device Allaz (1992) and Allaz/Vila (1993) argue that a simultaneous setting rules out any strategic link between the forward and the product market which will become evident in the next section.²⁷ Determining the strategical impact of decisions that are taken before choosing the outputs has a long history in the oligopolistic literature. Just to mention a few papers regarding this matter, Brander/Spencer (1983) and Brander/Lewis (1986) examine the strategical impact of R&D expenditures and debt levels

²⁵In the case of a competitive firm only the first effect exists so that the optimal output of the firm under risk-neutrality is greater than under risk-aversion (see Sandmo (1971), pp. 66-67).

²⁶The units that are sold [purchased] on the forward market are denoted by a positive [negative] F_i , $i = 1, 2$. We assume forward rather than futures contracts as the last-mentioned are standardized in their size and delivery date and are marked-to-market. Furthermore we assume that no transaction costs have to be paid. To ensure that it is not optimal to sell or purchase an infinite large amount of the foreign currency we assume that $e_f \in (e_{\min}, e_{\max})$.

²⁷See Allaz (1992), p. 298 and Allaz/Vila (1993), p. 2.

as choice variables, respectively, and Vives (1989) e.g. shows how uncertainty influences this impact in a model where the firms choose the production technology in the first stage. One can also draw a connection to the literature which is concerned with market entry as decisions that are taken before the possible entry are driven by strategic motives.²⁸ Additionally, see chapter 5 of Corchón (2001) who offers a general approach to examining strategical incentives in two-stage games. However, the question if the simultaneous or the sequential setting is the right one cannot be answered in general, in fact this question is an empirical one and depends on the particular industry that is considered. Therefore we examine both settings and begin with the simultaneous one which we refer to as the non-strategical setting. The sequential setting is called the strategical one.

4.1 Non-Strategical Hedging

This section is concerned with the case where the optimal hedging positions and outputs are chosen simultaneously in $t = 0$.²⁹ As the output is sold in $t = 1$ we assume that the delivery date of the forwards is $t = 1$, too. Therefore the forwards offer a perfect hedging opportunity to the firms. In order to find out how this affects the output decisions and the expected utilities of the firms we first allow only one firm to deal with forwards and afterwards both firms, comparing the results with each other, the case without any hedging opportunity and the certainty case.

4.1.1 Hedging Opportunity for one Firm

In this section forward contracts are only available to firm 1. This alters its profit function in $t = 1$:

$$\tilde{\Pi}_1 = (\tilde{e}p - c_1) x_1 + F_1(e_f - \tilde{e}). \quad (9)$$

²⁸See Dixit (1980) and Fudenberg/Tirole (1984), for instance.

²⁹This simultaneous setting has been paid very little attention to in the literature. The almost only article which is worth mentioning is from Eldor/Zilcha (1990). They deal with n identical firms and an inverse demand function that contains a stochastic element. The hedging opportunity is given to all firms and hedging is done by trading the homogeneous good on a forward market.

Before starting the comparisons mentioned above, we state conditions that assure the existence and uniqueness of an equilibrium and characterize some interesting properties of the optimal choices.

Proposition 7. In the present model an interior, unique Cournot-Nash equilibrium (now consisting of x_1^* , x_2^* and F_1^*) exists if assuming that all candidate equilibria are interior and the determinant of the Jacobian of the negative marginal utilities is positive.³⁰ Then the optimal outputs of the firms are solely determined by the following conditions:³¹

$$e_f p(x_1^* + x_2^*) + e_f x_1^* p'(x_1^* + x_2^*) = c_1, \quad (10)$$

$$E(U_2'(\tilde{\Pi}_2))(\tilde{e} p(x_1^* + x_2^*) + \tilde{e} x_2^* p'(x_1^* + x_2^*) - c_2) = 0. \quad (11)$$

Equation (10) shows that firm 1's condition for its optimal output is the same as if the firm was in the certainty case with e_f substituting the deterministic exchange rate. Hence, we can state a weak separation result:

Proposition 8. The optimal output decisions are independent of the optimal hedging decision of firm 1 and its attitude towards risk, but depend on firm 2's attitude towards risk and on the probability distribution of \tilde{e} .³²

The obtained separation result is weaker than in the case of a competitive firm where the optimal output decision is independent of the optimal hedging decision, the probability distribution of the exchange rate and any attitude towards risk.³³ That is not surprising at all, because in the present setting firm 2 has no hedging opportunity so that its optimality condition contains firm 2's utility function and the stochastic exchange rate. The dependencies of firm 2's optimal output decision in turn carry over to firm 1's optimal output decision due to the duopoly product market.

So far we missed to examine the optimal hedging decision of firm 1. In order to do this we have to make some definitions:

³⁰See Vives (1999), pp. 48 and 98. It can be shown that both expected utility functions are concave in their choice variables.

³¹The first condition results from equating both first order conditions of firm 1 with each other. Whereas the second condition is the first order condition of firm 2.

³²These conclusions follow directly from the first order conditions. See Eldor/Zilcha (1990) on page 20 for similar separation results if $e_f = E(\tilde{e})$ holds.

³³For instance see Holthausen (1979), p. 990.

Definition 1. The forward market is called unbiased if $e_f = E(\tilde{e})$. If $e_f > E(\tilde{e})$ [$e_f < E(\tilde{e})$] the forward market is referred to as to be in contango [backwardation].

Definition 2. A hedging position that is equal to [greater than] [smaller than] the revenue in the foreign currency is called a full hedge [overhedge] [underhedge].

The following proposition shows a direct link between the situation on the forward market and the optimal hedging decision:

Proposition 9. The optimal hedging position of firm 1 is a full hedge [overhedge] [underhedge] if the forward market is unbiased [in contango] [in backwardation].³⁴

The obtained link is exactly the same as in the case of a competitive firm.³⁵ The result of the proposition is a direct consequence of the risk-aversion of firm 1. The forward market enables the firm to reduce its exposure towards risk. If the forward market is unbiased this risk reduction is "costless" which induces the firm to dispose of any risk, whereas in a biased forward market risk reduction is not costless so that it is optimal to take some risk.³⁶ The reason why the hedging decision of firm 1 is solely driven by its incentive for risk reduction and not by strategical incentives is that the simultaneous setting rules out any strategic link between the forward and the product market.³⁷ This means that even though the hedging opportunity for firm 1 alters its optimality condition on the product market which in turn induces the optimal outputs of both firms to change, firm 1 is not able to use this connection in a strategical manner as it cannot credibly commit to depart from the stated optimal hedging position. To specify, due to the simultaneous setting and the common knowledge assumption firm 2 knows firm 1's optimality conditions and behaves accordingly so that firm 1's best

³⁴The proof can be found in appendix (IV). See Eldor/Zilcha (1990) on page 19 for the same result in their setting.

³⁵For instance see Holthausen (1979), p. 990.

³⁶In this context "costless" means that risk can be removed at an actuarial fair price.

³⁷See Allaz (1992), p. 298 and Allaz/Vila (1993), p. 2.

responses are determined. As we will see in section 4.2 this is not the case in the sequential setting.³⁸

After examining the optimal decisions of the firms we turn over to the comparisons mentioned at the outset of section 4.1. For convenience we assume the forward market to be unbiased so that firm 1's optimal hedging position is a full hedge under the conditions of Proposition 7. First we compare the current setting with the certainty case.³⁹

Proposition 10. Comparing the equilibria of propositions 1 and 7 we find that:⁴⁰ (i) the optimal output of firm 1 is greater than under certainty as well as its expected utility is; (ii) the optimal output of firm 2 is smaller than under certainty; (iii) the optimal total output is smaller than under certainty which induces the price to be higher.⁴¹

Proposition 10 shows that changing from the certainty case to the current one makes firm 2 act less aggressive due to its exposure towards risk. This in turn enables firm 1 to produce more as its best-response function stays the same in both settings.⁴² This is due to its hedging opportunity which enables firm 1 to manage its risk via its hedging position. Also the expected utility of firm 1 increases. Hence, in this comparison the unbiased hedging opportunity for firm 1 can be referred to as an advantage.

The result concerning firm 2's expected utility is ambiguous as it is determined by two antithetic effects which cannot be generally ordered in terms of their size: On the one hand the decreasing output and risk-aversion cause the expected utility to fall but on the other hand the higher price arises the expected utility.

³⁸Bulow et al. (1985) describe these coherences in a model under certainty in which two firms compete on one market as duopolists and one firm can additionally sell its output on a further market as a monopolist. They also distinguish whether the decision concerning the monopoly market is made before or simultaneous to the decisions concerning the duopoly market (see particularly Bulow et al. (1985) on page 499). Note that we use the term "strategic" in the sense of Allaz (1992) and not in the sense of Bulow et al. (1985) as the latter use this term in the simultaneous setting, too.

³⁹We assume that $e_f = e$ holds.

⁴⁰We assume the equilibrium under certainty to be interior.

⁴¹The proof can be found in appendix (V).

⁴²The best-response function of a firm under certainty is downward-sloping as the outputs of the firms are strategic substitutes (see (A2)).

The next proposition is concerned with the comparison of the current setting to that under uncertainty but without any hedging opportunity.

Proposition 11. Comparing the equilibria of propositions 5 and 7 we can just state that the optimal output of firm 1 rises if the hedging opportunity is made available to it.⁴³

As stated subsequent to proposition 10 the hedging opportunity enables firm 1 to choose its output like being under certainty which means acting more aggressive. But how this influences firm 2's output and the outputs' sum cannot be stated as we indeed know the slope of firm 2's best-response function under certainty but not under uncertainty.

At a first glance one could think that the expected utility of firm 1 has to rise as the firm employs its hedging opportunity. But as we described subsequent to proposition 9, firm 1 cannot use the hedging decision in a strategical manner but has to follow its altered optimality conditions in order to give a best response to firm 2's decision which knows firm 1's new conditions. Hence, in the simultaneous setting it cannot generally be ruled out that firm 1 is harmed due to its additional choice variable.⁴⁴ In the case of a competitive firm this is not possible as its optimal output increases and the full hedge removes all uncertainty.⁴⁵

4.1.2 Hedging Opportunity for both Firms

Now the forward market is made available to both firms. The conditions for existence and uniqueness of a Cournot-Nash equilibrium are similar to those of the previous section:

Proposition 12. Assuming that all candidate equilibria are interior an unique Cournot-Nash equilibrium (now consisting of x_1^* , x_2^* , F_1^* and F_2^*) exists if the determinant of the Jacobian of the negative marginal utilities is positive.⁴⁶ Then the optimal outputs are solely determined by the compounded

⁴³The proof can be found in appendix (VI).

⁴⁴See Bulow et al. (1985), pp. 490-491., who show an example in which this happens.

⁴⁵See Holthausen (1979), p. 990.

⁴⁶See Vives (1999), pp. 48 and 98.

first order conditions of the firms:

$$e_f p(x_1^* + x_2^*) + e_f x_1^* p'(x_1^* + x_2^*) = c_1, \quad (12)$$

$$e_f p(x_1^* + x_2^*) + e_f x_2^* p'(x_1^* + x_2^*) = c_2. \quad (13)$$

Hence, under the conditions of this proposition the optimal outputs are determined by the same conditions as in the certainty case whereas e_f substitutes e . This enables us to state a stronger separation result than in the preceding section:⁴⁷

Proposition 13. The optimal output decisions are independent of the optimal hedging decisions, the probability distribution of the exchange rate and the attitudes towards risk of both firms.⁴⁸

The separation result is as in the case of a competitive firm.⁴⁹ The optimal outputs are set independent of anything that is influenced by the exchange rate risk. The decision how risky the profit should be is made by choosing the optimal hedging position and is linked to the situation on the forward market in the same manner as it was shown in proposition 9. Again, the firms are not able to use their hedging decisions in a strategical manner so that these decisions are solely driven by risk managing incentives.

As in the foregoing section we want to determine how the hedging opportunity, now being available to both firms, alters the outputs and expected utilities of the firms compared to the certainty case as well as to the cases under uncertainty without hedging and with a hedging opportunity for one firm. Again we will assume an unbiased forward market that induces full hedges for both firms.

Starting with the comparison of the current setting with the certainty case and assuming unique, interior solutions in both cases everything stays the same as long as $e_f = e$ holds. This follows directly from proposition 12. The

⁴⁷See proposition 8.

⁴⁸This follows immediately from the compounded first order conditions that determine the optimal outputs (equation (12) and (13)). See Eldor/Zilcha (1990) on page 20 for similar separation results in the case of an unbiased forward market.

⁴⁹For instance see Holthausen (1979), p. 990.

reason for this is the separation result which states that the firms act, with respect to their outputs, as if they were under certainty. Because managing their risk exposure is done by means of their hedging positions. The results for $e_f \neq e$ can be derived analogous to proposition 4 and can also be interpreted in the same way.⁵⁰ Hence, we turn over to the next comparison.

As the current setting and the certainty case constitute the same results if assuming interior solutions, comparing the current setting to that under uncertainty but without any hedging opportunity is equivalent to the comparison of the last mentioned case with the certainty one. This has already been done for the outputs in proposition 6. Now that result is reversed: The optimal total output increases if forward contracts are made available to both firms.⁵¹

Although our intuition tells us that the hedging opportunity for both firms should increase their expected utilities this cannot be said in general. This is because of two contrary and one ambiguous effects altering the expected utility of a firm: First the increasing optimal total output lowers the price which in turn induces the expected utility to decrease. Second providing a full hedge yields a deterministic profit that increases the expected utility. Third we are not able to decide how the optimal output of the particular firm is altered. For example in the setting of Eldor/Zilcha (1990) it is even possible that both firms are harmed by introducing an unbiased forward market.⁵²

Completing, the change from the case of section 4.1.1 to the current one yields the exact contrary of proposition 10 due to the similarity of the current case and the certainty one: The optimal output of firm 1 and its expected utility decrease whereas the optimal output of firm 2 increases. The optimal total output is higher resulting in a lower price. As described subsequent to proposition 11 receiving the hedging opportunity does not necessarily led to a higher expected utility which is also valid for firm 2 in this comparison.

In the following section we skip the assumption that hedging and out-

⁵⁰As the results derived in proposition 4 for the optimal outputs are ambiguous this ambiguity carries over to the expected utilities.

⁵¹See proposition 6, its proof and its interpretation. The same result is obtained in the setting of Eldor/Zilcha (1999) on pp. 20 and 22.

⁵²See Eldor/Zilcha (1990), p. 21.

put decisions are made simultaneously and turn over to the case where the decisions are made sequentially.

4.2 Strategical Hedging

In order to design a sequential setting we assume that the hedging decisions are made in $t = -1$ and that their delivery date is $t = 1$.⁵³ Hence, in $t = 0$ the optimal forward decisions have already been taken and as we assume them to be common knowledge they have to be taken into account when choosing the optimal outputs. The applied equilibrium concept is therefore the subgame-perfect Nash equilibrium (SPNE) from Selten (1965). A SPNE strategy choice in the current setting consists of the optimal hedging positions and a pair of strategy sets for the outputs that constitute a Cournot-Nash equilibrium for all possible hedging positions, $(x_1^+(F_1, F_2), x_2^+(F_1, F_2))$.⁵⁴

As in the previous section we want to investigate how this hedging opportunity affects the outputs, the market price and the expected utilities of the firms. We first assume that the hedging opportunity is only available to one firm and thereafter to both firms. The derived results are compared with each other, the certainty case and the case without any hedging opportunity.

4.2.1 Hedging Opportunity for one Firm

Assume that the hedging opportunity is only available to firm 1. To derive the SPNE we first have to determine the particular Cournot-Nash equilibrium of the outputs in $t = 0$ for all possible forward positions. Afterwards we go back to $t = -1$ and determine the optimal hedging position taking into account how this choice affects the optimal outputs. The next proposition

⁵³As the expectations concerning the exchange rate in $t = 1$ are assumed to be the same in $t = -1$ and $t = 0$ the forward contracts offer a perfect hedging opportunity to the firms.

⁵⁴We write $x_i^+(F_1, F_2)$, $i = 1, 2$ to point that the hedging positions are not optimal. The only article that is concerned with a similar setting is a working paper from Broll et al. (1999a). They assume one firm to be under certainty all the time. Besides this paper there are articles from Allaz (1992), Allaz/Vila (1993) and Hughes/Kao (1997) that apply to a sequential setting, too, but consider the forward market for the homogeneous good. As will be seen later this difference is crucial for the strategical impact of hedging. Moreover the last three articles assume that the output decisions are taken under certainty.

states conditions that assure the existence and uniqueness of the particular Cournot-Nash equilibrium:

Proposition 14. In the present model and given a hedging position F_1 , an interior, unique Cournot-Nash equilibrium exists in $t = 0$ if assuming that all candidate equilibria are interior and $V_{x_1x_1}^1 V_{x_2x_2}^2 - V_{x_1x_2}^1 V_{x_2x_1}^2 > 0$ holds.⁵⁵

Under the conditions of proposition 14 the optimal outputs are determined by the following conditions for each hedging position:

$$E(U'_1(\tilde{\Pi}_1) (\tilde{e} p(x_1^+(F_1) + x_2^+(F_1)) + \tilde{e} x_1^+(F_1) p'(x_1^+(F_1) + x_2^+(F_1)) - c_1)) = 0, \quad (14)$$

$$E(U'_2(\tilde{\Pi}_2) (\tilde{e} p(x_1^+(F_1) + x_2^+(F_1)) + \tilde{e} x_2^+(F_1) p'(x_1^+(F_1) + x_2^+(F_1)) - c_2)) = 0. \quad (15)$$

Now we go back to $t = -1$ to find the optimal hedging position. Therefore we have to maximize firm 1's expected utility subject to its optimality condition in $t = 0$. F_1^* is determined by the following condition:⁵⁶

$$\frac{E(U'_1(\tilde{\Pi}_1(F_1^*)) \tilde{e})}{E(U'_1(\tilde{\Pi}_1(F_1^*)))} (1 - x_1^*(F_1^*) x_2^{*'}(F_1^*) p'(x_1^*(F_1^*) + x_2^*(F_1^*))) = e_f. \quad (16)$$

From equation (16) we see that the optimal hedging decision depends on the probability distribution of the exchange rate and firm 1's attitude towards risk. These dependencies carry over to the optimal outputs as the outputs depend on the hedging position. Moreover the outputs depend on firm 2's attitude towards risk as can be concluded from equation (15). Hence, it is not possible to state any separation result.

In order to show that hedging can be seen as a strategic device the next proposition deals with the effect an increasing hedging position has on the optimal outputs:

Proposition 15. Assume that (i) the forward market is unbiased, (ii) the outputs are strategic substitutes in the current environment ($V_{x_i x_j}^i < 0$, $i, j =$

⁵⁵See Vives (1999), pp. 48 and 98.

⁵⁶We assume that the optimal hedging position is unique and determined by its first order condition.

1, 2, $i \neq j$) and (iii) the preferences of firm 1 exhibit non-increasing absolute risk-aversion. Then an increase of the hedging position rises the optimal output of firm 1 and lowers firm 2's optimal output.⁵⁷

Proposition 15 shows that firm 1 is able to affect the output decisions via its hedging position. The link between the forward and the product market comes through the income effect under risk aversion which induces the best-response function of firm 1 to increase for an increasing hedging position which means acting more aggressive.⁵⁸ Therefore firm 1's output rises and as firm 2's best-response function is downward-sloping (due to assuming strategic substitutes) firm 2's output falls. To find out if firm 1 uses this link in a strategical manner, we determine firm 1's optimal hedging position in an unbiased forward market.

Proposition 16. If proposition 15 holds and the optimal hedging position is determined by equation (16), then the optimal hedging decision of firm 1 is an overhedge.⁵⁹

Hence, proposition 16 proves that hedging is used by firm 1 for strategical reasons, too, besides its incentive for risk reduction. In contrast to the simultaneous setting this kind of usage is possible due to the sequential progression as here firm 1 can credibly commit to a hedging position that is different from that one which is solely driven by risk managing incentives. Hence, firm 1 chooses its hedging position incorporating the impact this decision has on the optimal outputs of both firms.⁶⁰ Therefore the firm that exclusively receives

⁵⁷The proof can be found in appendix (VII). See Broll et al. (1999a) on page 8 for the same result in their setting. So as to derive "intuitive" results in comparative static analysis it is common to assume the best-response functions to be downward-sloping and the equilibrium to be stable (see Vives (1999), p. 102). Both conditions are met in our setting: The best-response functions are downward-sloping as we assumed the outputs to be strategic substitutes in the current environment (see Vives (1999), p. 36). Then the stability of the unique equilibrium follows from the conditions of proposition 14 (see Vives (1999), p. 51).

⁵⁸See Broll et al. (1999a), pp. 10-13.

⁵⁹The proof can be found in appendix (VIII). See Broll et al. (1999a), p. 13.

⁶⁰Compare the optimality conditions concerning firm 1's hedging position of the simultaneous (23) and the sequential setting (16) with each other.

such a hedging opportunity can never be harmed by this.⁶¹

If firm 1 is risk-neutral the described link of proposition 15 disappears as the hedging position no longer affects the best-response functions of the firms in $t = 0$. Hence firm 1 cannot use hedging in a strategical manner. Furthermore, a risk-neutral firm does not have an incentive to reduce risk in this framework, so it does not apply to hedging in an unbiased forward market at all.⁶² This is in contrast to the results of Allaz (1992), Allaz/Vila (1993) and Hughes/Kao (1997) where a risk-neutral firm uses hedging as a strategical device.⁶³ The reason for this is that the forward market in their articles is for the homogeneous good which induces a strategic link between the forward and spot market as hedging affects the marginal revenue from spot sales which means affecting the best-response functions.⁶⁴

After examining the optimal decisions we turn over to the mentioned comparisons and assume the forward market to be unbiased for convenience. First we compare the current setting with that under uncertainty but without any hedging opportunity.

Proposition 17. Under the conditions of proposition 16 the comparison of the equilibria of propositions 5 and 14 yields that (i) the optimal output of firm 1 is greater than without hedging as well as its expected utility is; (ii) the optimal output of firm 2 is smaller than without hedging.⁶⁵

The result of proposition 17 is a direct consequence of the link between the forward and the product market as the optimal hedging position in the current case is greater than null which induces the described changes in the optimal outputs. The expected utility of firm 1 increases, too, if changing from the case without forward contracts to the current one as otherwise firm 1 would not apply to hedging. Note that this conclusion is only true for

⁶¹See Bulow et al. (1985), particularly on p. 499, who state an analogous result for their setting and generalize it to "any decision [firm] A might take at one time that would affect its marginal profitability at a later time."

⁶²See Broll et al. (1999a), p. 10.

⁶³Note that all of them apply to the sequential setting.

⁶⁴See Allaz (1992), p. 305 or Allaz/Vila (1993), p. 2.

⁶⁵This follows directly from proposition 15 as the optimal hedging position is an over-hedge (see proposition 16).

the sequential setting as we explained subsequent to proposition 16. Hence, receiving exclusively such a hedging opportunity is an advantage for firm 1. If firm 2 is harmed in terms of expected utility cannot be said in general, as the effect of the hedging position on the optimal total output and the price is not clear-cut.

The next comparison is concerned with the certainty case and the current one.⁶⁶

Proposition 18. Under the conditions of propositions 16 the comparison of the equilibria of proposition 1 and 14 yields that the optimal output of firm 1 is greater and firm 2's optimal output is smaller than under certainty.⁶⁷

In this proposition both firms alter their behaviour. While firm 2 acts less aggressive due to uncertainty, firm 1 acts more aggressive as it uses the sequential hedging opportunity in a strategic manner. If firm 1 would use the hedging opportunity just to manage its risk exposure it would have chosen a full hedge and would have induced a behaviour like under certainty.

As in the previous comparison the altering of the optimal total output is not clear-cut so that the investigation how the expected utilities of the firms are changed yields ambiguous results.

4.2.2 Hedging Opportunity for both Firms

Completing the strategical setting we allow both firms to deal with forward contracts. As in the preceding section the SPNE is derived by first determining the Cournot-Nash equilibria of the outputs in $t = 0$ for all possible forward positions of both firms. Thereafter the optimal hedging positions in $t = -1$ are determined while taking into account how this affects the optimal outputs. The conditions that assure the existence and uniqueness of a Cournot-Nash equilibrium are similar to those of proposition 14:

Proposition 19. In the present model and given the hedging positions F_1 and F_2 an interior, unique Cournot-Nash equilibrium exists in $t = 0$ if assum-

⁶⁶We assume that the equilibrium under certainty is interior and that $e_f = e$ holds.

⁶⁷The proof can be found in appendix (IX).

ing that all candidate equilibria are interior and $V_{x_1 x_1}^1 V_{x_2 x_2}^2 - V_{x_1 x_2}^1 V_{x_2 x_1}^2 > 0$ holds.⁶⁸

Hence, under the conditions of proposition 19 the optimal outputs are unambiguously determined by the following conditions for each tuple (F_1, F_2) :⁶⁹

$$E(U'_1(\tilde{\Pi}_1) (\tilde{e} p(x_1^+ + x_2^+) + \tilde{e} x_1^+ p'(x_1^+ + x_2^+) - c_1)) = 0, \quad (17)$$

$$E(U'_2(\tilde{\Pi}_2) (\tilde{e} p(x_1^+ + x_2^+) + \tilde{e} x_2^+ p'(x_1^+ + x_2^+) - c_2)) = 0. \quad (18)$$

The optimal hedging positions in $t = -1$ are found by maximizing the expected utility of both firms subject to the optimality conditions in $t = 0$. F_1^* and F_2^* are determined by the following conditions:⁷⁰

$$\frac{E(U'_1(\tilde{\Pi}_1(F_1^*)) \tilde{e})}{E(U'_1(\tilde{\Pi}_1(F_1^*)))} (1 - x_1^* x_2^{*'} p'(x_1^* + x_2^*)) = e_f, \quad (19)$$

$$\frac{E(U'_2(\tilde{\Pi}_2(F_2^*)) \tilde{e})}{E(U'_2(\tilde{\Pi}_2(F_2^*)))} (1 - x_2^* x_1^{*'} p'(x_1^* + x_2^*)) = e_f. \quad (20)$$

Again a separation result cannot be stated as the optimal hedging positions of the firms depend on the probability distribution of the exchange rate and the particular attitudes towards risk. These dependencies carry over to the optimal outputs as the outputs in turn depend on the hedging positions.

Furthermore the comparative static results of proposition 15 are still valid and can be repeated for F_2 in an analogous manner: A higher forward position of firm i , $i = 1, 2$, increases its own best-response function and therefore promotes its own output and deters the output of the other firm. Just like in proposition 16 it can be shown that both firms apply to overhedges in an unbiased forward market. Hence, the firms use forward contracts not only to reduce risk but for strategic reasons, too, since otherwise their optimal hedging positions would be full hedges.⁷¹

⁶⁸See Vives (1999), pp. 48 and 98.

⁶⁹For convenience we write x_i^+ instead of $x_i^+(F_1, F_2)$, $i = 1, 2$.

⁷⁰We assume that the optimal hedging positions are unique and determined by their particular first order conditions. For convenience we write x_i^* instead of $x_i^*(F_1^*, F_2^*)$ and $x_i^{*'}$ instead of $\frac{\partial x_i^*}{\partial F_j^*}$, $i \neq j$ and $i, j = 1, 2$.

⁷¹See section 4.1.2.

In the following comparisons the forward market is assumed to be unbiased. First we compare the case without any hedging opportunity to the current one.

Proposition 20. If the comparative static results mentioned above hold for both firms and if the optimal hedging positions are determined by equations (19) and (20), the comparison of the equilibria of propositions 5 and 19 yields that the total optimal output becomes larger when changing from the case without hedging to the current one.⁷²

Both firms act more aggressive due to the availability of the forward contracts as they increase their hedging positions. Therefore the results concerning the optimal outputs are ambiguous which carries over to the expected utilities of the firms. Possibly both firms are better off if neither hedging opportunity exists as it was shown for risk-neutral firms in the setting of Allaz.⁷³

Comparing the current case with that from section 4.2.1 yields ambiguous results for the optimal outputs, because it is indeed possible to state that firm 2 becomes more aggressive as its hedging position increases, but firm 1 can increase its hedging position and act more aggressive, too. It is even ambiguous if firm 2 benefits from receiving the hedging opportunity as the hedging position has to be chosen simultaneously to firm 1's position.⁷⁴

The last comparison is concerned with the current case and the certainty one.⁷⁵

Proposition 21. If the comparative static results mentioned above hold for both firms and if the optimal hedging positions are determined by equations (19) and (20) the comparison of the equilibria of propositions 1 and 19

⁷²The proof can be found in appendix (X).

⁷³See Allaz (1992), p. 305.

⁷⁴To see an example in which the second firm is harmed, modify Bulow et al. (1985), pp. 490-491, in the following manner: The outputs for the monopoly market are chosen before those for the duopoly one and the price on the monopoly market is 59,95 regardless the quantity which is sold by one or both firms. If you compare the case where only firm A is able to sell on the monopoly market with that one where both can do so you will see that firm B's profit declines.

⁷⁵We assume that the equilibrium under certainty is interior and that $e_f = e$ holds.

yields that the optimal total output in the current case is greater than under certainty.⁷⁶

If the firms did not use the forward contracts as strategic devices their optimal hedging positions would have been full hedges and their outputs and expected utilities would have stayed unaltered in comparison with the certainty case.⁷⁷ The strategical usage of the contracts makes the firms act more aggressive on the product market yielding a higher total output and stochastic profits. Whether the expected utilities increase or decrease due to the strategical usage is not clear-cut.

5. Concluding remarks

The purpose of our study was to extend the literature of a Cournot duopoly under risk and with an opportunity to hedge this risk. We first examined the case where the hedging and output decisions are taken simultaneously. Even though the hedging positions influence the optimal outputs it is not possible to use them in this setting in a strategical manner.⁷⁸ Hence, having an exclusive, unbiased hedging opportunity indeed induces that the firm's optimal output is higher than in all other cases but its expected utility does not have to rise in comparison to the uncertainty case without hedging. Furthermore in the case of an exclusive hedging opportunity for one firm the other one does not necessarily benefit from receiving this opportunity, too. Whether the firms prefer the case in which both have the unbiased hedging opportunity to the uncertainty case without hedging or not is ambiguous.

Second we showed that in the sequential setting hedging is done by the firms for two reasons: First they use them to reduce their exposures towards risk which implies full hedges in an unbiased forward market. Second the firms use the forward contracts as a strategic device because a higher hedging position promotes the own output and deters the output of the competitor

⁷⁶The proof can be found in appendix (XI).

⁷⁷See section 4.1.2 for the result in the non-strategical setting.

⁷⁸Therefore we were able to state the separation result in proposition 13.

which implies overhedges in an unbiased forward market.⁷⁹ Furthermore, the sequential setting makes it impossible to state a separation result.

If a firm has an exclusive hedging opportunity its expected utility as well as its optimal output is higher than in the case without hedging. Against, it was not possible to state that this exclusive hedging opportunity makes the firm better off if compared to the certainty case or the case where hedging is available to both firms. As in the simultaneous setting whether the firms prefer the situation where the unbiased hedging opportunity is available to both of them to the case without hedging or not is ambiguous.

Appendix

(I) Proof of Proposition 2

Before we start the proof we have to define the best-response functions and some special output levels:

Definition 3. The best-response function of firm i , $f_i(x_j)$ with $i \neq j$ and $i, j = 1, 2$, defines the best output firm i can choose if firm j produces x_j .⁸⁰

Under the assumptions of proposition 2 the best-response functions are well-defined.⁸¹ Following the proof of Collie (1992) we define output levels \bar{x}_i , $i = 1, 2$ so that $f_j(x_i) = 0 \forall x_i \geq \bar{x}_i$ since $p(\bar{x}_i) = c_j$.⁸² Now we are able to proof proposition 2:

- (a) If $f_j(0) = x_j$ with $x_j \geq \bar{x}_j$ then $f_i(x_j) = 0$ for $i, j = 1, 2$ and $i \neq j$. Hence, the optimal output of firm i in the unique equilibrium is null.⁸³ To assure that both duopolists do not produce in the equilibrium consider the case where $ep(0) < \min(c_1, c_2)$. As the inverse demand

⁷⁹As we argued subsequent to proposition 16, the strategical usage of the forward contracts is only possible in the sequential setting.

⁸⁰See Rasmusen (1991), pp. 77-78.

⁸¹See Collie (1992), p. 62 or Friedman (1991), p. 84. Otherwise we would not have a best-response function but a mapping as the image of $f_i(x_j)$ could be a set.

⁸²See Collie (1992), pp. 61-63.

⁸³For example, let $c_j < ep(0) < c_i$.

function is downward-sloping both firms are better off if producing nothing.

- (b) Assuming that both firms do not produce in the equilibrium we can determine the following conflict: As $ep(0) > \min(c_1, c_2)$, w.l.o.g. we can assume that $ep(0) > c_1$. Hence, $f_1(0) > 0$ as the inverse demand function is continuous.
- (c) Assume that $f_1(x_2) = 0$. Therefore we can conclude that $x_2 \geq \bar{x}_2$. And as $ep(\bar{x}_2) = c_1 = c_2 = ep(\bar{x}_1)$ we find that $\bar{x}_1 = \bar{x}_2 > 0$.⁸⁴ For this reason $ep(x_2) \leq c_2 \forall x_2 \geq \bar{x}_2$. Hence, firm 2 is better off if choosing an output $x_2 \in (0, \bar{x}_2)$. Furthermore, if $x_2 \in (0, \bar{x}_2)$ then $f_1(x_2) \in (0, \bar{x}_1)$. Therefore the outputs of both firms are positive in the unique equilibrium. □

(II) Proof of Proposition 4

The proof of the first part of the proposition follows directly as the condition $E(\tilde{e}) = e$ causes the functions the firms maximize to be identical in the case of certainty and uncertainty.

Since in the second part we assume interior solutions in both cases the effect of an inequality between $E(\tilde{e})$ and e can be examined by comparing the first order conditions of both cases with each other ((2), (3) and (5), (6)). Hence, if $E(\tilde{e})$ is greater [smaller] than e , $(p(x_1^* + x_2^*) + x_i^* p'(x_1^* + x_2^*))$, $i = 1, 2$ have to fall [rise] if changing from the certainty case to the current one. As these terms depend on both outputs we have to consider the total differentials of

⁸⁴ $\bar{x}_1 > 0$ follows directly from the assumption that $ep(0) > c_1$ and the fact that the inverse demand function is continuous.

them to find out how this affects the optimal outputs:⁸⁵

$$\begin{aligned} d(p + p' x_1) &= \frac{\partial(p + p' x_1)}{\partial x_1} dx_1 + \frac{\partial(p + p' x_1)}{\partial x_2} dx_2 \\ &= (2p' + p'' x_1) dx_1 + (p' + p'' x_1) dx_2, \\ d(p + p' x_2) &= \frac{\partial(p + p' x_2)}{\partial x_1} dx_1 + \frac{\partial(p + p' x_2)}{\partial x_2} dx_2 \\ &= (p' + p'' x_2) dx_1 + (2p' + p'' x_2) dx_2. \end{aligned}$$

To give an example how to determine the changes of the outputs with the help of these equations take a look on the case in which $(p + p' x_1)$ and $(p + p' x_2)$ have to fall.⁸⁶ The possible cases are: (i) both outputs rise, (ii) one output rises and the other one stays unaltered and (iii) one output falls and the other one rises so strong that the total output rises. In all other cases one can find a contradiction. In this manner you can find that the optimal total output under uncertainty is greater [smaller] than under certainty if $E(\tilde{e})$ is greater [smaller] than e . The effect on the particular optimal outputs is ambiguous.

□

(III) Proof of Proposition 6

The first order conditions under risk-aversion ((7) and (8)) can be written as:

$$\left(\frac{\text{cov}(U'_1(\tilde{\Pi}_1), \tilde{e})}{E(U'_1(\tilde{\Pi}_1))} + E(\tilde{e}) \right) (p(x_1^* + x_2^*) + x_1^* p'(x_1^* + x_2^*)) = c_1, \quad (21)$$

$$\left(\frac{\text{cov}(U'_2(\tilde{\Pi}_2), \tilde{e})}{E(U'_2(\tilde{\Pi}_2))} + E(\tilde{e}) \right) (p(x_1^* + x_2^*) + x_2^* p'(x_1^* + x_2^*)) = c_2. \quad (22)$$

You can show that in the equations (21) and (22) the particular terms which are multiplied with $p(x_1^* + x_2^*) + x_i^* p'(x_1^* + x_2^*)$ are positive but smaller than $E(\tilde{e})$. Comparing equations (21) and (22) with those under risk-neutrality ((5) and (6)) and keeping in mind that the marginal costs, c_i , are positive

⁸⁵See Chiang/Wainwright (2005), p. 185. For simplicity we skip the arguments of the inverse demand function.

⁸⁶Note that $(2p' + p'' x_i) < (p' + p'' x_i) < 0$ holds for $i = 1, 2$.

the terms $p(x_1^*+x_2^*)+x_i^* p'(x_1^*+x_2^*)$ have to be greater under risk-aversion than under risk-neutrality. As in the proof of proposition 4 we have to consider the total differentials of them to find out how this affects the optimal outputs:

$$\begin{aligned} d(p + p' x_1) &= (2p' + p'' x_1) dx_1 + (p' + p'' x_1) dx_2, \\ d(p + p' x_2) &= (p' + p'' x_2) dx_1 + (2p' + p'' x_2) dx_2. \end{aligned}$$

From these equations it follows that the total output under risk-aversion is smaller than under risk-neutrality while it is possible that one firm even produces more under risk-aversion. □

(IV) Proof of Proposition 9

Under these assumptions firm 1's optimal hedging decision is solely determined by the following first order condition:

$$e_f = \frac{E(U'_1(\tilde{\Pi}_1)\tilde{e})}{E(U'_1(\tilde{\Pi}_1))}, \quad (23)$$

which can also be written as:

$$e_f = \frac{\text{cov}(U'_1(\tilde{\Pi}_1), \tilde{e})}{E(U'_1(\tilde{\Pi}_1))} + E(\tilde{e}). \quad (24)$$

So as to satisfy condition (24) the $\text{cov}(\cdot, \cdot)$ has to be equal to [greater than] [smaller than] null if $e_f = E(\tilde{e})$ [$e_f > E(\tilde{e})$] [$e_f < E(\tilde{e})$] which is done by choosing the optimal F_1 .

$$\frac{\partial U'_1(\Pi_1)}{\partial e} = U''_1(\Pi_1) (p x_1 - F_1). \quad (25)$$

From the last equation the conclusions of the proposition follow immediately. □

(V) Proof of Proposition 10

Comparing the first order conditions of the certainty case ((2) and (3)) with the relevant conditions in the current setting ((10) and (11)) we obtain that

if changing from the certainty case to the current one $p + p' x_1$ has to stay equal and $p + p' x_2$ has to rise. What this means for the optimal outputs can be read off the total differentials of $p + p' x_i$, $i = 1, 2$:

$$\begin{aligned} d(p + p' x_1) &= (2p' + p'' x_1) dx_1 + (p' + p'' x_1) dx_2, \\ d(p + p' x_2) &= (p' + p'' x_2) dx_1 + (2p' + p'' x_2) dx_2. \end{aligned}$$

The expected utility of firm 1 has to rise in comparison to the certainty case as the unbiased forward market causes a full hedge which induces the firm's profit to be certain and as the deterministic profit of the firm is higher. The second argument infers from the fact that firm 1's optimal output and the price are higher than under certainty and that $e_f = e$ holds. □

(VI) Proof of Proposition 11

In this case we cannot apply to the used way of proving, as comparing the first order conditions of the case under uncertainty without a hedging opportunity ((7) and (8)) with the relevant conditions in the current setting ((10) and (11)) yields that if changing from the case without hedging to the current one $p + p' x_1$ has to decrease whereas the alteration of $p + p' x_2$ is ambiguous. Hence, we apply the optimal outputs of the current case to the optimality conditions of the uncertainty case which yields that firm 2's optimality condition is still fulfilled while $V_{x_1}^1$ is negative. Therefore $V_{x_2}^2$ has to stay equal and $V_{x_1}^1$ has to rise. What this means for the optimal outputs can be read off the total differentials of $V_{x_i}^i$, $i = 1, 2$:⁸⁷

$$\begin{aligned} dV_{x_1}^1 &= V_{x_1 x_1}^1 dx_1 + V_{x_1 x_2}^1 dx_2, \\ dV_{x_2}^2 &= V_{x_2 x_1}^2 dx_1 + V_{x_2 x_2}^2 dx_2. \end{aligned}$$

□

⁸⁷Note that $V_{x_1 x_1}^1 V_{x_2 x_2}^2 - V_{x_1 x_2}^1 V_{x_2 x_1}^2$ is assumed to be positive in proposition 5.

(VII) Proof of Proposition 15

As $x_1^+(F_1)$ and $x_2^+(F_1)$ are implicitly defined by equations (14) and (15) the signs of $\frac{dx_1^+(F_1)}{dF_1}$ and $\frac{dx_2^+(F_1)}{dF_1}$ are found by totally differentiating those equations and applying Cramer's rule:

$$\frac{dx_1^+(F_1)}{dF_1} = \frac{M V_{x_2x_2}^2}{V_{x_1x_1}^1 V_{x_2x_2}^2 - V_{x_1x_2}^1 V_{x_2x_1}^2}, \quad (26)$$

$$\frac{dx_2^+(F_1)}{dF_1} = \frac{-M V_{x_2x_1}^2}{V_{x_1x_1}^1 V_{x_2x_2}^2 - V_{x_1x_2}^1 V_{x_2x_1}^2}. \quad (27)$$

with

$$M = -E(U_1''(\tilde{\Pi}_1)) (\tilde{e}(p + x_1^+ p') - c_1) (e_f - \tilde{e}).$$

$V_{x_1x_1}^1$ and $V_{x_2x_2}^2$ are negative as the expected utility functions are strict concave in their particular outputs, whereas the negativity of $V_{x_1x_2}^1$ and $V_{x_2x_1}^2$ is assumed in the proposition. $V_{x_1x_1}^1 V_{x_2x_2}^2 - V_{x_1x_2}^1 V_{x_2x_1}^2$ is positive due to the conditions of proposition 14 that are still valid. Hence, to prove the conclusions of the proposition we have to show that M is negative. Therefore we decompose M into two parts:

$$M = \frac{1}{p + x_1^+ p'} E(U_1''(\tilde{\Pi}_1)) (\tilde{e}(p + x_1^+ p') - c_1)^2 + \frac{c_1 - e_f (p + x_1^+ p')}{p + x_1^+ p'} E(U_1''(\tilde{\Pi}_1)) (\tilde{e}(p + x_1^+ p') - c_1).$$

The first term of M is negative as firm 1 is risk-averse and $(p + x_1^+ p')$ is positive due to the first order condition (14).

If the preferences of firm 1 exhibit constant absolute risk-aversion (CARA) the second term of M is null: $-\frac{U_1''(\Pi_1)}{U_1'(\Pi_1)} = R(\Pi_1)$ is a positive constant for all possible values of \tilde{e} . Hence, by replacing $U_1''(\tilde{\Pi}_1)$ by $U_1'(\tilde{\Pi}_1)$ (multiplied with a negative constant) in M 's second term and applying the first order condition (14) we obtain that M 's second term vanishes.

If the preferences of firm 1 exhibit decreasing absolute risk-aversion (DARA) the second term of M is non-positive: The first order condition (14) can be written as

$$\frac{\text{cov}(U_1'(\tilde{\Pi}_1), \tilde{e})}{E(U_1'(\tilde{\Pi}_1))} = \frac{c_1 - e_f (p + x_1^+ p')}{p + x_1^+ p'}.$$

Applying this equation to the second term of M and replacing $U_1''(\tilde{\Pi}_1)$ by $-R(\tilde{\Pi}_1) U_1'(\tilde{\Pi}_1)$ yields:

$$-\frac{\text{cov}(U_1'(\tilde{\Pi}_1), \tilde{e})}{E(U_1'(\tilde{\Pi}_1))} E(R(\tilde{\Pi}_1) U_1'(\tilde{\Pi}_1) (\tilde{e} (p + x_1^+ p') - c_1)).$$

Furthermore let $\hat{\Pi}_1$ be the profit where \tilde{e} takes the value $\frac{c_1}{p+x_1^+ p'}$, then the second term of M can be extended in the following manner:

$$-\frac{\text{cov}(U_1'(\tilde{\Pi}_1), \tilde{e})}{E(U_1'(\tilde{\Pi}_1))} E((R(\tilde{\Pi}_1) - R(\hat{\Pi}_1)) U_1'(\tilde{\Pi}_1) (\tilde{e} (p + x_1^+ p') - c_1)). \quad (28)$$

If $p x_1^+ = F_1$ the second term of M is null as in this case the profit becomes deterministic so that $R(\tilde{\Pi}_1) = R(\hat{\Pi}_1)$ holds.

If $p x_1^+ < F_1$, $(R(\tilde{\Pi}_1) - R(\hat{\Pi}_1))$ and $(e (p + x_1^+ p') - c_1)$ have the same signs for all possible values of \tilde{e} and $\text{cov}(U_1'(\tilde{\Pi}_1), \tilde{e})$ is positive. Hence, M 's second term is negative.

If $p x_1^+ > F_1$, $(R(\tilde{\Pi}_1) - R(\hat{\Pi}_1))$ and $(e (p + x_1^+ p') - c_1)$ have opposite signs for all possible values of \tilde{e} and $\text{cov}(U_1'(\tilde{\Pi}_1), \tilde{e})$ is negative. Therefore the second term of M is negative again.

Summing all up, if the preferences of firm 1 exhibit either CARA or DARA expression M is negative so that $\frac{dx_1^+(F_1)}{dF_1}$ is positive and $\frac{dx_2^+(F_1)}{dF_1}$ is negative. \square

(VIII) Proof of Proposition 16

The first order condition (16) for the optimal hedging decision in an unbiased forward market can be written as:

$$\begin{aligned} & E(U_1'(\tilde{\Pi}_1(F_1^*)) \tilde{e}) x_1^*(F_1^*) x_2^{*'}(F_1^*) p'(x_1^*(F_1^*) + x_2^*(F_1^*)) \\ & = E(U_1'(\tilde{\Pi}_1(F_1^*)) \tilde{e}) - E(U_1'(\tilde{\Pi}_1(F_1^*))) E(\tilde{e}). \end{aligned} \quad (29)$$

The left-hand side of equation (29) is positive due to the results of proposition 15 and the right-hand side is $\text{cov}(U_1'(\tilde{\Pi}_1(F_1^*)), \tilde{e})$. For the covariance to be positive the optimal hedging position has to be an overhedge as $\frac{\partial U_1'(\Pi_1)}{\partial e} = U_1''(\Pi_1) (x_1 p(x_1 + x_2) - F_1)$. \square

(IX) Proof of Proposition 18

The optimal hedging position is implicit defined by equation (16). As in $t = 0$ the firms take F_1^* into account when determining the optimal outputs, this is done for firm 1 by inserting equation (16) in its first order condition in $t = 0$ (equation (14)) yielding the following condition:⁸⁸

$$e_f(p + x_1^* p') = c_1(1 - x_1^* x_2^{*'} p'). \quad (30)$$

Now we compare the first order conditions of the certainty case ((2) and (3)) with the relevant conditions in the current setting ((30) and (15)): If changing from the certainty case to the current one $p + p' x_1$ has to decrease whereas $p + p' x_2$ has to increase. What this means for the optimal outputs can be read off the total differentials of $p + p' x_i$, $i = 1, 2$:

$$\begin{aligned} d(p + p' x_1) &= (2p' + p'' x_1) dx_1 + (p' + p'' x_1) dx_2, \\ d(p + p' x_2) &= (p' + p'' x_2) dx_1 + (2p' + p'' x_2) dx_2. \end{aligned}$$

□

(X) Proof of Proposition 20

As in the proof of proposition 18 the relevant conditions in the current case are obtained by inserting equations (19) in (17) and (20) in (18):

$$e_f(p + x_1^* p') = c_1(1 - x_1^* x_2^{*'} p'), \quad (31)$$

$$e_f(p + x_2^* p') = c_2(1 - x_2^* x_1^{*'} p'). \quad (32)$$

Comparing these equations to the relevant conditions in the case without hedging ((7) and (8)) yields that $p + p' x_1$ and $p + p' x_2$ have to decrease if changing from the case without hedging to the current one. How this affects the optimal outputs can be read off the total differentials of $p + p' x_i$, $i = 1, 2$:

$$\begin{aligned} d(p + p' x_1) &= (2p' + p'' x_1) dx_1 + (p' + p'' x_1) dx_2, \\ d(p + p' x_2) &= (p' + p'' x_2) dx_1 + (2p' + p'' x_2) dx_2. \end{aligned}$$

□

⁸⁸The arguments of the functions are omitted for convenience.

(XI) Proof of Proposition 21

The relevant conditions in the current setting are those of the proof of proposition 20 ((31) and (32)). Comparing them with the conditions under certainty ((2) and (3)) yields that $p + p' x_1$ and $p + p' x_2$ have to decrease if changing from the certainty case to the current one. Therefore the same result is obtained as in the proof of proposition 20. □

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