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**Capital Structure and the Firm under Uncertainty**

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# Capital Structure and the Firm under Uncertainty

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Abstract:

This paper examines the interplay between the real and financial decisions of the competitive firm à la Sandmo. Besides output price uncertainty, the firm faces additional sources of risk which are aggregated into an additive background risk. We show that the firm always chooses its optimal debt-equity ratio to minimize the weighted average cost of capital, irrespective of the risk attitude of the firm and the incidence of the multiple sources of uncertainty. Even though the introduction of the background risk affects neither the optimal debt-equity ratio nor the marginal rate of technical substitution, it does have an adverse effect on the output level of the firm. Furthermore, if capital is a normal input, the presence of the background risk induces the firm to acquire less capital by issuing less debt and equity.

JEL-Classification: D81; G32

Keywords: Background risk; Capital structure; Price uncertainty

## 1. Introduction

The seminal work of Sandmo (1971) has inspired a great many papers examining the theory of the competitive firm under output price uncertainty (see, e.g., Batra and Ullah, 1974; Chavas, 1985; Wong, 1996; and Broll, Chow and Wong, 2001). In all these studies, it is implicitly assumed that the competitive firm is all-equity financed. This assumption may be innocuous in a perfect world in which Modigliani and Miller (1958) assert that the choice of capital structure (i.e., the mix of debt and equity) is a matter of irrelevance to the firm. A corollary to this irrelevance theorem is that the real and financial decisions of the firm are independent and therefore can be made separately.

In the real world, imperfections such as taxation, bankruptcy, information asymmetry, and agency costs are a fact of life, thereby making the celebrated Modigliani-Miller theorem fragile.<sup>1</sup> The purpose of this paper is therefore to reconsider the theory of the competitive firm under output price uncertainty when the real and financial decisions of the firm are *de facto* interdependent. To this end, we modify the tax-adjusted valuation model of Modigliani and Miller (1963) and DeAngelo and Masulis (1980), and place it in the context of the competitive firm under output price uncertainty *à la* Sandmo (1971) and Batra and Ullah (1974). The model is further complicated to shed light on how additional sources of risk, aggregated into an additive background risk, affect the behavior of the firm.

Irrespective of the risk attitude of the firm and the incidence of the multiple sources

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<sup>1</sup>The effects of market imperfections on the Modigliani-Miller irrelevance theorem are studied by a great many papers. Notable examples are Modigliani and Miller (1963), Miller (1977), and DeAngelo and Masulis (1980) on taxation; Kraus and Litzenberger (1973), Scott (1976), and Brennan and Schwartz (1978) on bankruptcy; Myers and Majluf (1984), Narayanan (1988), and Noe (1988) on information asymmetry; Jensen and Meckling (1976), and Barnea, Haugen, and Senbet (1981) on agency costs.

of uncertainty, we show that the optimal input mix depends on the optimal debt-equity ratio, rendering the interdependence of the real and financial decisions of the firm. Myers (1974), Hite (1977), Cooper and Franks (1983), Dotan and Ravid (1985), and Dammon and Senbet (1988) establish similar interactions between corporate investment and financing decisions, albeit without considering the risk attitudes of firms. We further show that the firm always chooses its optimal debt-equity ratio to minimize the weighted average cost of capital. Even though the introduction of the background risk has no effects on the optimal debt-equity ratio and on the marginal rate of technical substitution, it does affect the absolute level of input utilization and the amount of debt and equity issued by the firm. In particular, we show that the firm will acquire less capital by issuing less debt and equity if two reasonable conditions are met. First, capital is a normal input.<sup>2</sup> Second, the utility function of the firm satisfies standard risk aversion in the sense of Kimball (1990, 1993). Finally, we show that the presence of the background risk induces the firm to produce less should the utility function of the firm exhibit standard risk aversion and this result is robust to whether capital is a normal or an inferior input.

The rest of the paper is organized as follows. In the next section, we develop a model of the competitive firm under output price uncertainty which fully integrates the production and capital structure decisions of the firm. Section 3 characterizes the optimal input mix and financing mix when an independent background risk exists. Section 4 examines the economic implications of the background risk on the production and capital structure decisions of the firm. The final section offers some concluding remarks.

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<sup>2</sup>Bear (1965) defines a normal (an inferior) input as one for which an increase in output price results in increased (decreased) utilization of that input.

## 2. The Model

Consider the competitive firm under output price uncertainty *à la* Sandmo (1971) and Batra and Ullah (1974). The firm produces a single output,  $Q$ , according to a strictly increasing and concave production function,  $Q = F(K, L)$ , where  $K$  and  $L$  are the levels of capital and labor employed by the firm, respectively. Unlike Sandmo (1971) and Batra and Ullah (1974) who assume the output price,  $\tilde{P}$ , being the exclusive source of uncertainty, we introduce the firm to additional sources of uncertainty which are aggregated into an additive background risk,  $\tilde{Z}$ .<sup>3</sup>  $\tilde{P}$  is a strictly positive random variable, while  $\tilde{Z}$  is a zero-mean random variable independent of  $\tilde{P}$ .

The firm acquires capital,  $K$ , with the proceeds from issuing debt,  $D$ , and equity,  $E$ . Shareholders of the firm have initial wealth,  $W_0$ , upon which  $E$  is contributed to the firm as equity capital and the rest,  $W_0 - E$ , is invested in riskless assets earning a rate of return,  $r_e$ . For simplicity, we normalize the price of capital to unity so that the initial balance sheet of the firm can be stated as

$$K = D + E. \tag{1}$$

Furthermore, we assume that the economic rate of capital depreciation equals one, thereby yielding zero salvage value of capital. Labor,  $L$ , is hired at a known wage rate,  $w$ . The total labor costs,  $wL$ , will be paid out of the revenues realized by the firm.

Interest costs of debt are assumed to comprise default risk premiums which are positively related to debt-equity ratios. Throughout the paper we consider only the case where the firm never defaults on its debt (i.e.,  $D$  is sufficiently small). However, due to a lack of bargaining power, the firm has to encounter a pre-specified schedule

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<sup>3</sup>Throughout the paper, a tilde ( $\sim$ ) always signifies a random variable.

of interest rate,  $r_d(\lambda)$ , where  $\lambda = D/E$  is the debt-equity ratio of the firm. We assume that  $r_d$  is weakly increasing and convex with  $r_d(0) = r_e$ .<sup>4</sup> Interest costs of debt are fully tax-deductible.

The firm has to pay corporate income taxes to the government at a constant tax rate,  $t$ . The firm's tax liability is given by

$$\tilde{T} = t(\tilde{P}Q + \tilde{Z} - wL - \delta K - r_d D), \quad (2)$$

where  $\delta$  is the firm-specific rate of capital depreciation for tax purposes. Thus, the wealth of the shareholders can be written as

$$\tilde{W} = (\tilde{P}Q + \tilde{Z} - wL) - \tilde{T} - (1 + r_d)D + (1 + r_e)(W_0 - E), \quad (3)$$

where the first term is the operating profits of the firm, the second term is the tax liability defined in equation (2), the third term is the debt repayment including principal and interest, and the final term is the proceeds from the investment in riskless assets.

Using equations (1) and (2), we can write equation (3) as

$$\tilde{W} = (1 - t)(\tilde{P}Q + \tilde{Z} - wL) - (1 + r_k - t\delta)K + (1 + r_e)W_0, \quad (4)$$

where  $r_k$  is the weighted average cost of capital (WACC) of the firm and is defined by (see, e.g., Brealey and Myers, 1996; Ross, Westerfield, and Jordon, 1998)

$$r_k = (1 - t)r_d \frac{D}{K} + r_e \frac{E}{K}. \quad (5)$$

Since the capital structure weights,  $D/K$  and  $E/K$ , can be expressed as  $D/K = \lambda/(\lambda + 1)$  and  $E/K = 1/(\lambda + 1)$ , inspection of equation (5) reveals that

$$r_k = (1 - t)r_d \frac{\lambda}{\lambda + 1} + r_e \frac{1}{\lambda + 1}, \quad (6)$$

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<sup>4</sup>These assumptions are motivated by the static trade-off theory of capital structure. See, e.g., Kraus and Litzenger (1973), Scott (1976), Brealey and Myers (1996), and Ross, Westerfield, and Jordon (1998).



and thereby  $r_k$  a function of  $\lambda$  only.

The firm possesses a von Neumann-Morgenstern utility function,  $U(W)$ , defined over the wealth of its shareholders,  $W$ , with  $U' > 0$  and  $U'' < 0$ , indicating the presence of risk aversion. Before any uncertainty is resolved, the firm chooses an input mix,  $(K, L)$ , and a financing mix,  $(D, E)$ , so as to maximize its expected utility:

$$\max_{K,L,D,E} E[U(\tilde{W})], \quad (7)$$

subject to the balance sheet identity, (1), where  $E$  is the expectation operator with respect to the joint probability distribution of  $\tilde{P}$  and  $\tilde{Z}$ , and  $\tilde{W}$  is defined in equation (4).

### 3. Solution to the Model

Using the balance sheet identity, (1), we can substitute out  $E$  in problem (7). To facilitate the analysis, we reformulate problem (7) as the following two-stage optimization problem.

In the first stage, we derive the optimal leverage:

$$D(K, L) = \arg \max_D E[U(\tilde{W})], \quad (8)$$

for any given input mix,  $(K, L)$ . Based on equation (8), we define the indirect expected utility,  $\bar{U}(K, L)$ , as  $E[U(\tilde{W})]$  evaluated at  $D(K, L)$ . In the second stage, we derive the optimal input mix:

$$(K^*, L^*) = \arg \max_{K,L} \bar{U}(K, L). \quad (9)$$

Substituting equation (9) into equation (8) yields the optimal leverage,  $D^* = D(K^*, L^*)$ , which completes the solution to problem (7).

### 3.1. Optimal Debt-Equity Ratio

Using equation (8),  $D(K, L)$  is defined by the following first-order condition:

$$-E[U'(\tilde{W})]r'_k(\lambda)(\lambda + 1)^2 = 0, \quad (10)$$

where we have used the fact that  $\lambda = D/(K - D)$ . Inspection of equation (10) leads us to the statement of our first proposition.

**Proposition 1.** *The firm chooses its optimal debt-equity ratio to minimize its WACC.*

*Proof.* Since  $E[U'(\tilde{W})] > 0$  and  $\lambda \geq 0$ , inspection of equation (10) reveals that  $r'_k(\lambda) = 0$ . Differentiating equation (6) with respect to  $\lambda$  twice yields

$$r''_k(\lambda) = \frac{1-t}{(\lambda+1)^2}[r''_d(\lambda)\lambda(\lambda+1) + 2r'_d(\lambda)] + \frac{2}{(\lambda+1)^3}[r_e - (1-t)r_d(\lambda)].$$

Since  $r'_d \geq 0$  and  $r''_d \geq 0$ , it follows from the above equation that  $r_k$  is strictly convex for small value of  $\lambda$  such that  $(1-t)r_d(\lambda) < r_e$ . Over this range, equation (6) implies that  $r_k(\lambda) < r_e$ . Otherwise, we have  $r_k(\lambda) \geq r_e$ . Hence, the optimal debt-equity ratio,  $\lambda^*$ , which solves  $r'_k(\lambda^*) = 0$ , must be the unique solution that minimizes  $r_k$ .  $\square$

It should be evident from equation (6) that the optimal debt-equity ratio,  $\lambda^*$ , depends neither on the risk attitude of the firm nor on the input mix chosen by the firm. It is governed solely by the interest rate schedule,  $r_d$ , the corporate tax rate,  $t$ , and the cost of equity,  $r_e$ . However, since  $\lambda = D/(K - D)$ , we have  $D(K, L) = \lambda^*K/(\lambda^* + 1)$ . It follows that the optimal leverage does depend on the amount of capital chosen by the firm.

### 3.2. Optimal Input Mix

Now, we are ready to fully solve problem (7). By Proposition 1, the optimal debt-equity ratio,  $\lambda^*$ , is independent of the input mix chosen by the firm. From equation (6), the optimal WACC is given by  $r_k^* = r_k(\lambda^*)$ , which is also independent of the input mix chosen by the firm. Using this fact and equation (9), the optimal input mix,  $(K^*, L^*)$ , are therefore defined by the following first-order conditions:

$$\mathbb{E}\{U'(\tilde{W}^*)[(1-t)\tilde{P}F_K(K^*, L^*) - 1 - r_k^* + t\delta]\} = 0, \quad (11)$$

$$\mathbb{E}\{U'(\tilde{W}^*)(1-t)[\tilde{P}F_L(K^*, L^*) - w]\} = 0, \quad (12)$$

The second-order conditions are assumed to be satisfied.

Rearranging terms, equation (11) becomes

$$\mathbb{E}[U'(\tilde{W}^*)\tilde{P}](1-t)F_K(K^*, L^*) = \mathbb{E}[U'(\tilde{W}^*)](1 + r_k^* - t\delta). \quad (13)$$

Likewise, rearranging terms in equation (12) yields

$$\mathbb{E}[U'(\tilde{W}^*)\tilde{P}](1-t)F_L(K^*, L^*) = \mathbb{E}[U'(\tilde{W}^*)](1-t)w. \quad (14)$$

Dividing equation (13) by equation (14), we get

$$\frac{F_K(K^*, L^*)}{F_L(K^*, L^*)} = \frac{1 + r_k^* - t\delta}{(1-t)w}. \quad (15)$$

Equation (15) states that, at the optimum, the marginal rate of technical substitution, which is the ratio of the marginal product of capital and the marginal product of labor, equals the ratio of the marginal cost of capital and the tax-adjusted wage rate. Since the marginal cost of capital depends on the optimal debt-equity ratio,  $\lambda^*$ , the real and financial decisions of the firm are indeed integrated. The following proposition summarizes this observation.

**Proposition 2.** *The real and financial decisions of the firm are integrated in that the marginal rate of technical substitution equals the ratio of the marginal cost of capital and the tax-adjusted wage rate.*

#### 4. Implications of Background Risk

In this section, we want to examine the economic implications of the background risk on the behavior of the firm. To this end, we consider a benchmark case in which the background risk does not exist, or equivalently  $\tilde{Z} = 0$ . As such, the decision problem of the firm becomes

$$\max_{K, L, D} \mathbb{E}_P[U(\tilde{W}_0)], \quad (16)$$

where  $\mathbb{E}_P$  is the expectation operator with respect to the probability distribution of  $\tilde{P}$ , and  $\tilde{W}_0$  is defined in equation (4) subject to  $\tilde{Z} = 0$ .

Following Kihlstrom, Romer, and Williams (1981) and Nachman (1982), we define the following derived utility function:

$$V(W) = \mathbb{E}_Z[U(W + \tilde{Z})],$$

where  $\mathbb{E}_Z$  is the expectation operator with respect to the probability distribution of  $\tilde{Z}$ . Using this derived utility function and applying the law of iterated expectations, problem (7) can be written as

$$\max_{K, L, D} \mathbb{E}_P[V(\tilde{W}_0)]. \quad (17)$$

Note that the background risk,  $\tilde{Z}$ , does not appear directly in problem (17). Comparing problem (16) with problem (17) reveals that introducing  $\tilde{Z}$  into problem (16) is equivalent to replacing the utility function,  $U$ , by the derived utility function,  $V$ .

Although  $U$  and  $V$  should be closely related, the theory of risk aversion developed by Arrow (1965) and Pratt (1964) is too weak to yield any meaningful linkage between these two utility functions. To resolve this problem, Kimball (1990, 1993) advocates a stronger, yet canonical, notion of risk aversion known as standard risk aversion.<sup>5</sup> He defines  $U''' > 0$  as prudence and the index,  $-U'''(W)/U''(W)$ , as a local measure of the degree of absolute prudence. He shows that a strictly increasing and concave utility function exhibits standard risk aversion if, and only if, it exhibits decreasing absolute risk aversion in the Arrow-Pratt sense and decreasing absolute prudence. If the underlying utility function,  $U$ , exhibits standard risk aversion, Eeckhoudt and Kimball (1992) and Kimball (1993) show that the derived utility function,  $V$ , will be more risk averse than  $U$  in the usual Arrow-Pratt sense.<sup>6</sup>

Equipped with this intuitive relationship between  $U$  and  $V$ , we recognize that the effects of the background risk on the behavior of the firm should be qualitatively tantamount to those of increased risk aversion. Following Diamond and Stiglitz (1974), we work with a differentiable family of utility functions,  $U(W, \rho)$ , where  $\rho$  is an ordinal index of risk aversion. Given this notation, Diamond and Stiglitz (1974) show that an increase in  $\rho$  represents increased risk aversion if, and only if, the Arrow-Pratt measure of absolute risk aversion increases with  $\rho$ :

$$\frac{\partial}{\partial \rho} \left( -\frac{U_{WW}}{U_W} \right) = \frac{U_{WW}U_{W\rho} - U_W U_{WW\rho}}{U_W^2} > 0. \quad (18)$$

According to Bear (1965), capital is said to be a normal (an inferior) input if an increase in the output price increases the utilization of capital. Mathematically, capital is a normal (an inferior) input if  $F_L F_{KL} - F_K F_{LL} > (<) 0$ . The proof of the

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<sup>5</sup>Loosely speaking, standard risk aversion formalizes the intuitive notion that bearing one risk should make an individual less willing to bear another, even when the two risks are independent.

<sup>6</sup>Wong (1996) shows further that this result may extend to the case with dependent background risk.

following proposition is relegated to the appendix.

**Proposition 3.** *If capital is a normal input and the firm's utility function exhibits standard risk aversion, then the introduction of the background risk induces the firm to acquire less capital by issuing less debt and equity.*

Proposition 3 implies that the presence of the background risk induces the firm to adopt a lower utilization of capital, thereby a lower degree of leverage, should capital be a normal input. In this case, we would expect the output level of the firm to decrease. However, if capital is an inferior input, we cannot draw such a direct inference about the output level from Proposition 3. The following proposition, where a proof can be found in the appendix, shows that the presence of the background risk is indeed output-reducing.

**Proposition 4.** *If the firm's utility function exhibits standard risk aversion, then the introduction of the background risk induces the firm to produce less.*

Proposition 4 is consistent with the consensus in the literature that uncertainty is output-reducing (see, e.g., Sandmo, 1971; Batra and Ullah, 1974; Chavas, 1985; and Wong, 1996).

## 5. Conclusion

This paper has investigated the interaction between the production and financing decisions of the competitive firm *à la* Sandmo (1971). Besides output price uncertainty, the firm faces additional sources of risk which are aggregated into an additive

background risk. We have shown that the firm always chooses its optimal debt-equity ratio to minimize the weighted average cost of capital, irrespective of the risk attitude of the firm and the incidence of the multiple sources of uncertainty. Even though the introduction of the background risk has no effects on the optimal debt-equity ratio and the marginal rate of technical substitution, it reduces the output level of the firm. Furthermore, if capital is a normal input, the presence of the background risk induces the firm to acquire less capital by issuing less debt and equity.

While this paper focuses on a specific model of production and financing decisions under uncertainty, the methods advanced are completely general. Since multiple sources of risk is a fact of life, these methods should be applicable to many other choice problems under uncertainty (see, e.g., Wong, 1997).

## Appendix

*Proof of Proposition 3.* We replace the utility function in problem (16) with the differentiable family of utility functions,  $U(W, \rho)$ . By Proposition 1, the optimal debt-equity ratio,  $\lambda^*$ , is independent of the presence or absence of the background risk, so is the optimal WACC,  $r_k^*$ . Using this fact, the optimal input mix,  $(K^0, L^0)$ , are therefore defined by the following first-order conditions:

$$H_K = E_P\{U_W(\tilde{W}_0^0, \rho)[(1-t)\tilde{P}F_K(K^0, L^0) - 1 - r_k^* + t\delta]\} = 0, \quad (1)$$

$$H_L = E_P\{U_W(\tilde{W}_0^0, \rho)(1-t)[\tilde{P}F_L(K^0, L^0) - w]\} = 0, \quad (2)$$

where a nought (<sup>0</sup>) indicates an optimal level. Rearranging equations (1) and (2) and dividing yields

$$\frac{F_K(K^0, L^0)}{F_L(K^0, L^0)} = \frac{1 + r_k^* - t\delta}{(1-t)w}. \quad (3)$$

Totally differentiating equations (1) and (2) with respect to  $\rho$  and using Cramer's rule yields

$$\frac{dK^0}{d\rho} = \frac{H_{L\rho}H_{KL} - H_{K\rho}H_{LL}}{H_{KK}H_{LL} - H_{KL}^2}, \quad (4)$$

$$\frac{dL^0}{d\rho} = \frac{H_{K\rho}H_{KL} - H_{L\rho}H_{KK}}{H_{KK}H_{LL} - H_{KL}^2}, \quad (5)$$

where

$$H_{KK} = E_P(U_W \tilde{P})(1-t)F_{KK} + E_P\{U_{WW}[(1-t)\tilde{P}F_K - 1 - r_k^* + t\delta]^2\}, \quad (6)$$

$$H_{LL} = E_P(U_W \tilde{P})(1-t)F_{LL} + E_P[U_{WW}(1-t)^2(\tilde{P}F_L - w)^2], \quad (7)$$

$$\begin{aligned} H_{KL} &= E_P(U_W \tilde{P})(1-t)F_{KL} \\ &+ E_P\{U_{WW}(1-t)[(1-t)\tilde{P}F_K - 1 - r_k^* + t\delta](\tilde{P}F_L - w)\}, \end{aligned} \quad (8)$$

$$H_{K\rho} = E_P\{U_{W\rho}[(1-t)\tilde{P}F_K - 1 - r_k^* + t\delta]\}, \quad (9)$$

$$H_{L\rho} = E_P[U_{W\rho}(1-t)(\tilde{P}F_L - w)]. \quad (10)$$

Using equations (3), (9), and (10), we have

$$H_{K\rho}F_L = H_{L\rho}F_K. \quad (11)$$

Substituting equations (3), (7), (8), and (11) into equation (4) yields

$$\frac{dK^0}{d\rho} = \frac{H_{L\rho}E_P(U_W \tilde{P})(1-t)(F_L F_{KL} - F_K F_{LL})}{(H_{KK}H_{LL} - H_{KL}^2)F_L}. \quad (12)$$

Likewise, substituting equations (3), (6), (8), and (11) into equation (5) yields

$$\frac{dL^0}{d\rho} = \frac{H_{L\rho}E_P(U_W \tilde{P})(1-t)(F_K F_{KL} - F_L F_{KK})}{(H_{KK}H_{LL} - H_{KL}^2)F_L}. \quad (13)$$

By the second-order conditions, we know that  $H_{KK}H_{LL} - H_{KL}^2 > 0$ . Hence, the signs of  $dK^0/d\rho$  and  $dL^0/d\rho$  depend on the signs of  $H_{L\rho}$  and  $F_{KL}$ .

To find the sign of  $H_{L\rho}$ , let  $R(W, \rho) = U_{W\rho}(W, \rho)/U_W(W, \rho)$ . Note first that

$$R_W(W, \rho) = \frac{U_W U_{WW\rho} - U_{W\rho} U_{WW}}{U_W^2} < 0,$$

where the inequality follows from equation (18). Define  $\bar{W}_0^0$  as  $\tilde{W}_0^0$  evaluated at  $\tilde{P} = w/F_L$ . Using equation (2), we can write equation (10) as

$$E_P\{[R(\tilde{W}_0^0, \rho) - R(\bar{W}_0^0, \rho)]U_W(\tilde{W}_0^0, \rho)(1-t)[\tilde{P}F_L(K^0, L^0) - w]\}.$$

Since  $R(W, \rho)$  is decreasing in  $W$  and  $\tilde{W}_0^0$  is increasing in  $P$ , the sign of  $R(\tilde{W}_0^0, \rho) - R(\bar{W}_0^0, \rho)$  is opposite to that of  $\tilde{P}F_L(K^0, L^0) - w$ . Thus, the above expression must be negative, implying that  $H_{L\rho}$  is unambiguously negative.

If capital is a normal input, we have  $F_L F_{KL} - F_K F_{LL} > 0$ . Equation (12) implies that  $dK^0/d\rho < 0$ . From Proposition 1, the optimal debt-equity ratio is the



same irrespective of the presence or absence of the background risk. Thus, we have  $dD^0/d\rho < 0$  and  $dE^0/d\rho < 0$ . This completes our proof.  $\square$

*Proof of Proposition 4.* Totally differentiating  $F(K^0, L^0)$  with respect to  $\rho$  yields

$$\frac{dF(K^0, L^0)}{d\rho} = F_K(K^0, L^0) \frac{dK^0}{d\rho} + F_L(K^0, L^0) \frac{dL^0}{d\rho}.$$

Substituting equations (12) and (13) into the right-hand side of the above equation yields

$$\frac{H_{L\rho} E_P(U_W \tilde{P})(1-t)(2F_K F_L F_{KL} - F_K^2 F_{LL} - F_L^2 F_{KK})}{(H_{KK} H_{LL} - H_{KL}^2) F_L}.$$

Since  $F(K, L)$  is strictly concave, we have  $2F_K F_L F_{KL} - F_K^2 F_{LL} - F_L^2 F_{KK} > 0$  (Silberberg, 1990). It follows from the above expression that  $dF(K^0, L^0)/d\rho < 0$ . This completes our proof.  $\square$

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