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## Extraction, Fixed Costs and the Hotelling Rule

by

HORST SIEBERT\*

The resource-extraction industry, as exemplified by mining or oil drilling on continental shelves, is characterized by large-scale investments. The capital needed for the extraction, processing and transportation of natural resources must exceed certain critical minimum values. Very often, the capital must be raised on the capital market. Moreover, projects are often of such a size that common ventures between a number of large firms are called for. Long gestation periods are involved in opening up a new coal mine, in constructing a processing facility or in building a transportation system. These characteristics of the extraction industry contrast markedly with the intertemporal theory of resource extraction in which it is normally assumed that the resource-extraction firm can easily adjust the quantities it extracts over time<sup>1</sup>.

This paper analyzes the effect of fixed costs on the intertemporal supply of resources. Though Hotelling's rule that the shadow price of the resource increases with the discount rate still holds, the relevance of the Hotelling paradigm in this case is reduced. Fixed costs reduce the level of the shadow price and influence the intertemporal supply of the resource.

In Section 1, the assumptions of the model are specified. The necessary conditions of an intertemporal profit maximum are discussed in Section 2. In Section 3, the effects of an increase in fixed costs upon the Hotelling rent and upon the supply behavior of the firm are analyzed. Some extensions of the analysis are outlined in Section 4.

### *1. Assumptions*

Let the cost function of a resource-extracting firm be given by

$$(1) \quad \gamma(q) = C(q) + \bar{C} .$$

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<sup>1</sup> The problem was brought to my attention by Ingo Walter, New York University.

Operating costs in each period  $C$  depend on quantities  $q$  of the resource withdrawn and rise progressively, i.e.,  $C_q > 0$ ,  $C_{qq} > 0$ . Capacity costs  $\bar{C}$  are fixed; they are determined by a minimum amount of capital necessary for extraction to take place at all. Note that equation (1) contains a simplified interpretation of fixed costs as arising in each period<sup>2</sup>; it assumes a convex function. The firm takes as given the price  $p_0$  of the natural resource in the initial period. It expects the price to rise at the given exogenous rate  $m$ , so that the price at  $t$  is

$$(2) \quad p(t) = p_0 e^{mt} .$$

Profit in each period must be nonnegative

$$(3) \quad pq - C(q) - \bar{C} \geq 0 .$$

The firm owns a resource stock  $R$  that is nonrenewable so that

$$(4) \quad \dot{R} = -q .$$

The firm has an infinite planning horizon and solves the following problem

$$(5) \quad \text{Max} \int_0^{\infty} e^{-\delta t} [pq - C(q) - \bar{C}] dt \quad \text{s.t. equations (3) and (4)} .$$

The Hamiltonian in current values is defined as

$$(5a) \quad H = [pq - C(q) - \bar{C}] + \lambda(-q)$$

and the Lagrangian is given by

$$(5b) \quad L = H + \pi [pq - C(q) - \bar{C}] .$$

## 2. Profit Maximizing Extraction

Necessary conditions for maximum profit are

$$(5c) \quad \frac{\partial L}{\partial q} = (p - C_q)(1 + \pi) - \lambda \leq 0 \quad q \frac{\partial L}{\partial q} = 0$$

$$(5d) \quad \frac{\partial L}{\partial \pi} = pq - C(q) - \bar{C} \geq 0 \quad \pi \frac{\partial L}{\partial \pi} = 0$$

$$(5e) \quad \dot{\lambda} = \delta \lambda .$$

<sup>2</sup> The assumption of fixed capacity costs per period simplifies the problem because this assumption avoids relating fixed costs to each period or each quantity. In a finite planning problem, fixed period costs can be derived by dividing total costs over all periods by the number of periods. Alternatively, consider fixed costs over an infinite planning horizon and relate them to the individual quantity produced. This, however, creates problems if the mine is not exhausted.

Equation 5d requires that profits be nonnegative. The shadow price  $\pi$  denotes the change in the value of the Lagrangian function for one additional unit of fixed costs<sup>3</sup>.

The shadow price of the resource  $\lambda$  rises with the discount rate  $\delta$ , so that the familiar Hotelling result is yielded.

Assume that the exhaustion date  $T$  is finite. Then the transversality condition  $e^{-\delta T} H(T) = 0$  requires that

$$(5f) \quad e^{-\delta T} \lambda_T q_T = e^{-\delta T} [p_T q_T - C(q_T) - \bar{C}] .$$

The case  $q_T = 0$  clearly violates the transversality condition, i.e., the right side of equation (5f) becomes negative for  $\bar{C} > 0$ . Thus, the assumption of fixed costs excludes one terminal condition discussed by LEVHARI and LIVIATAN ([1977], p. 182). We must have  $q_T > 0$ , "which is probably the most common case in mining economics" (LEVHARI and LIVIATAN [1977], p. 182). From the transversality condition of (5f) we have

$$(5g) \quad \lambda_T = p_T - \frac{\gamma(q_T)}{q_T} .$$

The shadow prices  $\lambda_0$  of the initial situation must be positive. Moreover, the shadow price rises over time. Consequently, the shadow price  $\lambda_T$  must be positive. A positive shadow price  $\lambda_T > 0$  requires a positive profit in  $T$ ; this implies that  $\pi_T = 0$  so that equation (5c) can be rewritten as

$$(5h) \quad \lambda_T = p_0 e^{mT} - C_q(q_T) .$$

Consequently, we have

$$(5i) \quad C_q(q_T) = \frac{\gamma(q_T)}{q_T} .$$

In terminal time  $T$ , marginal operating costs and average costs must be equal. It follows that  $q_T = q^*$  where  $q^*$  denotes the minimum of the average cost curve. Note that if profits are positive in  $T$ , they must also be positive in any  $t < T$ . Thus we have  $\pi(t) = 0$  for  $t < T$ .

Differentiating equation (5c) with respect to time and using equation (5e) we have

$$(5j) \quad \dot{q} + \frac{1}{C_{qq}} (mp - \delta\lambda)$$

and

$$\dot{q} \leq 0 \Leftrightarrow mp - \delta\lambda \leq 0 .$$

<sup>3</sup> Introduce the slack variable -  $z$  in the Lagrangian function. Then we have

$$\frac{dL}{dz} = -\pi .$$

The case  $m > \delta \lambda/p$  can be ruled out, since  $m > \delta$  provides an incentive to leave the resource in the ground<sup>4</sup>. If the market price increases at a lower rate than  $\delta \lambda/p$ , i.e., if the increase in the market price  $mp$  is smaller than the increase in the shadow price, quantities extracted decline over time and converge on  $q^*$ .

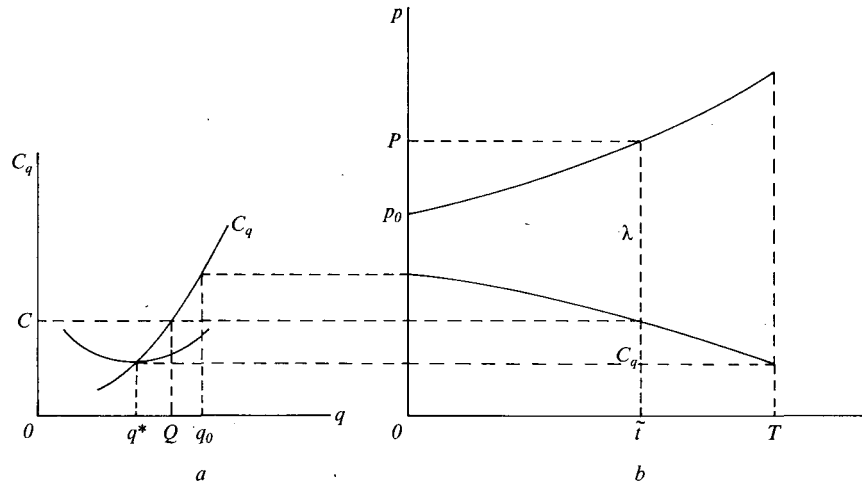


Figure 1

This case is illustrated in Figure 1. Figure 1 b describes the time paths of the market price  $p$ , the shadow price  $\lambda$ , and marginal costs  $C_q$ . The market price rises at the rate  $m$ ; the shadow price of the resource, the firm's user costs, increases with the discount rate. For  $m < \delta \lambda/p$  and for progressively rising costs, the shadow price can only rise if marginal costs of extraction decline. This implies that quantities extracted will fall. The marginal and average cost curves are depicted in Figure 1 a. The firm begins by extracting  $q_0$  at the given initial price  $p_0$ , and works its way down the marginal cost curve until the minimum average costs are reached in time period  $T$ . For a given market price  $0P$  at any moment of time  $\tilde{t}$  (Figure 1 b), the corresponding quantity extracted can be determined by identifying the marginal costs  $0C$  in  $\tilde{t}$  (Figure 1 b) and then looking at the quantities  $0Q$  associated with these costs (Figure 1 a). Note that exogenous price changes  $m=0$  and  $m < 0$  can be interpreted as special cases.

### 3. The Effects of an Increase in Fixed Costs

Compare now the extraction path of two mines with different fixed costs. The resource stock is identical for both mines, but mine 2 has higher fixed costs.

<sup>4</sup> In this case, the user costs are identical to market price, i.e.  $\lambda = p$ .

Assume that the market price rises at the rate  $m$  and that operating cost functions are identical for both mines. Assume also that the fixed costs of mine 2 are not so high as to preclude the extraction of the resources. Call the exhaustion time of mine 2  $T^2$  and leave aside for the moment the problem of whether the terminal time of the two mines are identical or different. The shadow price of mine 2 will approach a lower value since mine 2 has higher average costs in  $T^2$  (Figure 2). The higher costs squeeze the shadow price of the resource which again must rise with the discount rate over time. The higher average costs of mine 2 imply that its supply function is represented by a higher segment of its marginal cost curve. Whereas mine 1 starts with a supply  $q_0$  in the initial period and extracts  $q^*$  in  $T$ , mine 2 supplies  $q_0^2$  initially and produces  $q^{*2}$  in  $T^2$ . Mine 2 is operating on a higher segment of its marginal cost curve and extracts a larger quantity in each period. Since mine 2 is assumed to have the same resource stock as mine 1, exhaustion must occur at an earlier time for mine 2, i.e.  $T^2 < T$ .

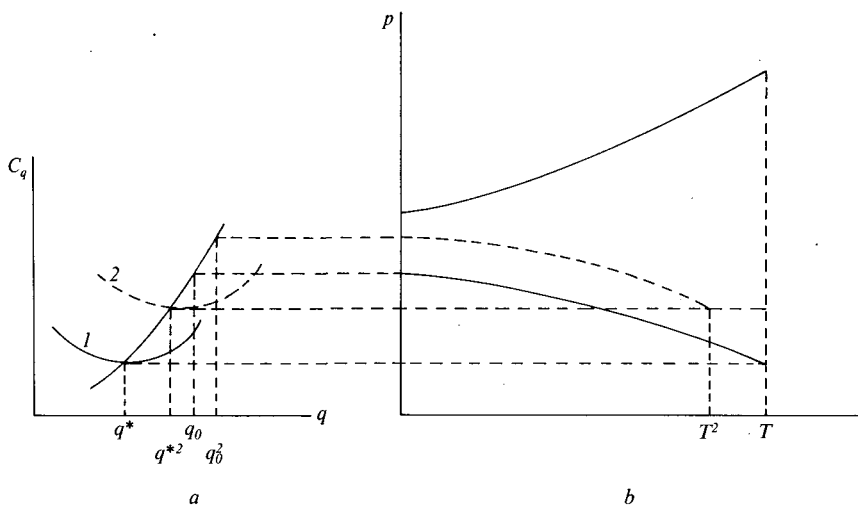


Figure 2

Fixed costs do not invalidate the Hotelling rule, but they reduce the level of Hotelling rents and drive yet another wedge between the market price and the Hotelling rent. Moreover, exhaustion occurs earlier with fixed costs; the time profile of extraction is twisted in favor of the present generation.

Figure 2 may also be used to explain the incidence of an accelerated or immediate write-off for capital outlays. Such a measure reduces fixed costs. This implies that the Hotelling rent rises and that the extraction cost component of price is reduced. The firm will now reach a lower minimum average cost point, and will operate on a lower section of its marginal cost curve. This means that

the quantity of the resource supplied in each period is smaller. Consequently, extraction will occur at a later date: immediate or accelerated write-offs will shift extraction to the future. This result is reached under assumptions which differ from those employed by STIGLITZ ([1979], p. 58), who stresses the role of write-offs in the context of the corporation tax. Within this framework, an immediate write-off of drilling expenditures creates an incentive which attracts capital to the mining sector, if other sectors of the economy cannot also write off capital expenditures immediately.

In the case of renewable resources, the relevance of fixed costs may be reduced by letting the resource stock grow over time, i.e., by postponing the harvest for one or several periods. CLARK ([1965], p. 249) has shown that pulse fishing is a possible outcome, if fixed extraction costs are large enough.

#### 4. Market Equilibrium and Fixed Costs

In the preceding sections, the price of the resource was assumed to rise at the rate  $m$ . An alternative approach consists in not predetermining the price but allowing it to vary freely. Differentiating equation (5c) with respect to time and substituting equation (5e) now yields for  $\pi_t = 0$

$$(6) \quad \hat{p} = \delta \frac{p}{\lambda} + \frac{C_{qq}\dot{q}}{p} .$$

Equation 6 represents a relationship between quantities supplied and changes in price. Assume now that an auctioneer sets prices in such a way that resource markets are cleared in each period and over time. In determining the market clearing price the auctioneer considers demand behavior

$$(7) \quad q = f(p) \quad \text{with} \quad f_p < 0 .$$

Note, however, that the firm does not anticipate demand behavior since it is supposed to behave as a perfect competitor. Inserting the demand function into equation (7) yields

$$(7a) \quad \hat{p} = \frac{\delta \lambda}{(1 - C_{qq}f_p)p} .$$

Equation (7a) describes the way in which the market price must change through time. The rate of change of the market price is positive; consequently, quantities demanded will fall. Also the market price increases at a slower rate than the shadow price of the resource, since

$$\frac{\lambda}{(1 - C_{qq}f_p)p} < 1 .$$

The term  $\lambda/p$  rises. If we assume that  $f_p$  is constant<sup>5</sup>, and taking into account that  $C_{qq}$  falls with a reduced  $q$ , the increase of the market price is reduced over time.

The demand side introduces an additional restraint on the previous model. Let the resource be extracted in finite time. The terminal situation is now limited both from the supply side and the demand side. Assume that the commodity cannot be stored, so that at terminal time  $T$  the arbitrage between  $T$  and  $T+1$  can be neglected. For terminal time  $T$ , the condition must hold that the quantity demanded is equal to the quantity supplied, i.e.,

$$f(p_T) = q^*$$

Assume for simplicity that demand falls to zero, once  $p_T = f^{-1}(q^*)$  is reached. Under these simplifying assumptions, Figure 3 illustrates the time pattern of extraction. The terminal conditions are determined by two forces: fixed costs and demand.

It is interesting to analyze the way in which exogenous changes in fixed costs and in demand affect the solution. Higher fixed costs will increase  $q^*$  and reduce the Hotelling rents. A lower demand curve (to the left of the curve shown in Figure (3c)) will reduce terminal price. The time path of the market price must be lower since a lower quantity is demanded at any given price on the original path; the stock equilibrium would be violated if the old price path were adhered to. Consequently, a shift of the demand curve to the left will move the price path downward. Exhaustion will occur at a later terminal time. The increase in fixed costs or the downward shift of the demand curve will eventually make it uneconomical to extract the resource.

### 5. Potential Extensions

The model presented can be extended in a number of ways that are relevant to the real world.

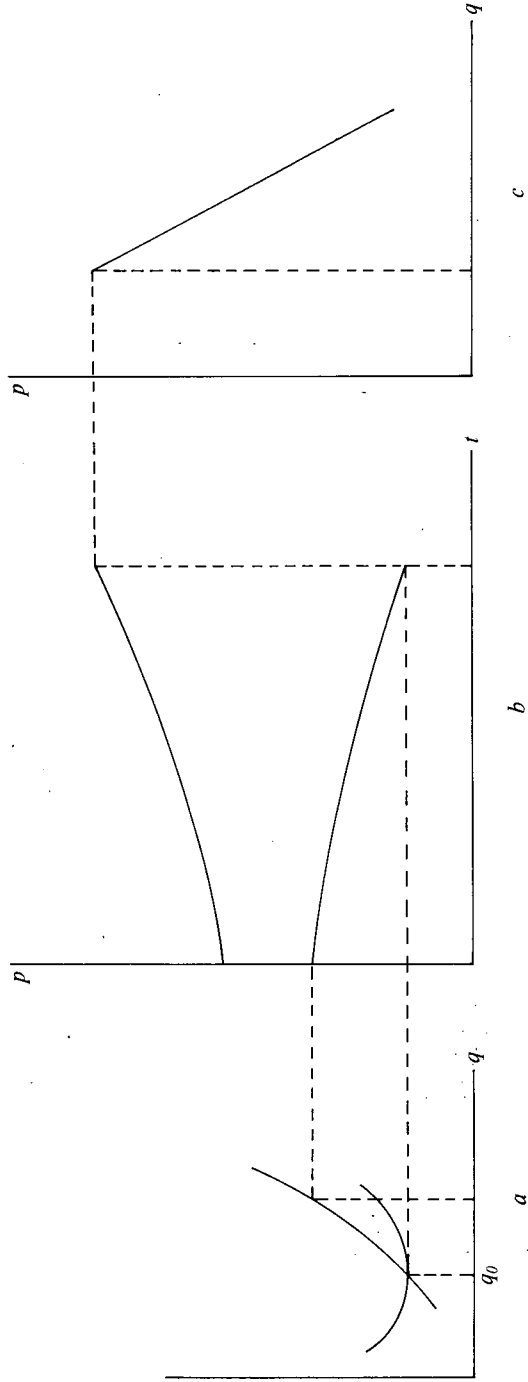
1. Fixed costs have been introduced in this paper as a parameter for each period. Alternatively, fixed capital costs  $\bar{C}_0$  can be related to opening up a mine. Then total costs  $\omega$  for the lifetime of the mine are defined as

$$\omega = \bar{C}_0 + \int_0^t e^{-\delta t} (q(t)) dt$$

In this case, costs (including fixed costs) are defined for a time period  $(0, t)$ . Alternatively, fixed costs  $\bar{C}_0$  can be defined per unit extracted, or, if exhaustion occurs, as  $\bar{C}_0/R$ .

<sup>5</sup> Alternatively, we have to assume that the absolute increase in  $f_p$  does not overcompensate the decline in  $C_{qq}$ .





2. Fixed costs may represent capital costs or clean-up costs originating at the end of extraction (KEMP and LONG [1980]). This type of fixed costs imposes an additional restraint on the shadow price in terminal time  $T$  and thus affects the time profile of extraction.

3. From the point of view of the firm, both the demand behavior and the cost functions contain risk elements so that uncertainty can be explicitly introduced.

4. The venture may be financed to a very large extent by borrowed funds, so that cash-flow constraints enter the decision.

5. The model assumes a given policy environment. Changes in the institutional rules of extracting the resource, i.e. ownership, and alterations in the price parameters such as taxation, price regulation and other interventions, will affect the decision arrived at. Uncertainty in this field may very well bring about a more hasty extraction.

#### *Summary*

This paper starts from the empirical observation that large-scale investments and fixed costs play an important role in the resource-extraction industry. It analyzes the influence of fixed extraction costs and a convex cost curve on the time path of extraction. Fixed costs do not invalidate the Hotelling rule, but they reduce the level of Hotelling rents and drive a wedge between the market price and the Hotelling rent. For any given increase in the market price, an exogenous rise in fixed costs will lead to the extraction of larger quantities in each period, and to earlier depletion. The time path of extraction is also examined under the explicit introduction of demand into the analysis to the assumption that markets clear in each period. It is shown that the way in which large-scale resource ventures are conducted is determined by the two forces of fixed costs and a sufficient level of demand, which may squeeze the Hotelling rent from two sides; these two forces may make large-scale projects uneconomical.

#### *Zusammenfassung*

##### *Ressourcenabbau, fixe Kosten und die Hotelling-Regel*

Ein hoher Kapitalbedarf und fixe Kosten spielen eine große Rolle für ressourcenabbauende Unternehmen. In diesem Beitrag wird der Einfluß von Fixkosten auf das Zeitprofil des Abbaus untersucht. Fixkosten setzen die Hotelling-Regel nicht außer Kraft, aber sie reduzieren die Höhe der Hotelling-Renten und treiben einen Keil zwischen Marktpreis und Hotelling-Rente. Für einen gegebenen Preispfad führt eine Zunahme der Fixkosten zu größeren Abbaumengen in jeder Periode und zu einer früheren Stilllegung. Bei Großprojekten des Ressourcenabbaus kann neben Fixkosten auch ein niedriges Nachfrage-

niveau die Hotelling-Renten drücken, so daß Großprojekte sowohl wegen zu hoher Fixkosten als auch wegen zu niedriger Nachfrage unrentabel werden können.

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