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A Reappraisal of the Inflation-Unemployment Tradeoff

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11 December 2003

Abstract

This paper offers a reappraisal of the inflation-unemployment tradeoff, based on “frictional growth,” describing the interplay between nominal frictions and money growth. When the money supply grows in the presence of price inertia (due to staggered wage contracts with time discounting), the price adjustments to each successive change in the money supply are never able to work themselves out fully. In this context, temporary nominal rigidities let monetary policy have permanent real effects. Although our theory contains no money illusion, no permanent nominal rigidities, and no departure from rational expectations, there is a long-run inflation-unemployment tradeoff. Our empirical analysis suggests that this Phillips curve may be reasonably flat. We show that the persistence of inflation and unemployment, in response to monetary policy shocks, is related to the slope of the long-run Phillips curve.

Keywords: Inflation, unemployment, Phillips curve, nominal inertia, wage-price staggering, monetary policy, business cycles, forward-looking expectations.

JEL Classifications: E2, E3, E4, E5, J3.

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1. Introduction

This paper proposes a reappraisal of the inflation-unemployment tradeoff, based on a phenomenon we call “frictional growth,” concerning the interaction between growth and frictions. Specifically, we focus on the interplay between monetary growth and nominal frictions arising from time-contingent staggered nominal contracts. We show that frictional growth can nevertheless give rise to a significant downward-sloping long-run relation between inflation and unemployment - even though our theory contains no money illusion, no permanent nominal rigidities, and no departure from rational expectations.

Indeed, the calibrations of our theoretical model and the simulation results from our empirical model indicate that the long-run Phillips curve can be quite flat under plausible circumstances. We also show that the slope of the long-run Phillips curve is related to inflation persistence and unemployment persistence in the aftermath of monetary shocks. In this way, the long- and medium-run movements of inflation and unemployment are closely related.

The intuition underlying our long-run Phillips curve may be summarized as follows. From the microfoundations of staggered nominal contracts,¹ it is well known that, when the time discount rate is positive, current nominal values are influenced more strongly by past than by future nominal values. Consider, for example, two staggered nominal wage contracts, each lasting a year, with one set in January and the other in July.

- As shown in Figure 1, the current (aggregate) price level P_t (say, for Jan-Jun 2004) is a markup over the past contract wage W_{t-1} (for July 2003 - June 2004) and the current contract wage W_t (for Jan-Dec 2004).
- In turn, W_{t-1} depends on P_{t-1} and P_t ; and W_t depends on P_t and P_{t+1} .
- Consequently, the current price level P_t can be viewed as a weighted average of the past and future price levels, P_{t-1} and P_t . Furthermore, under time discounting, the future price level receives less weight than the current one.

¹Regarding Taylor contracts, see for example Helpman and Leiderman (1990), Ascari (2000), and Graham and Snower (2002a, 2002b); for Calvo contracts, see for example Bernanke, Gertler and Gilchrist (2000) and Gali (2002).

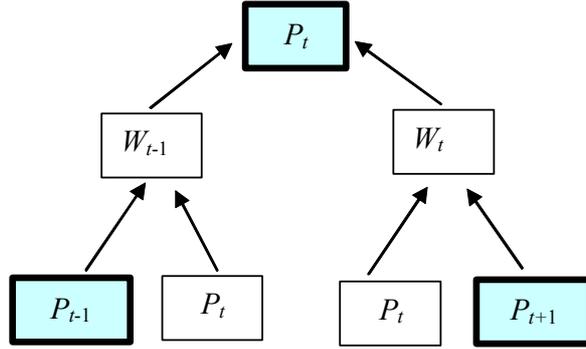


Fig. 1: Intertemporal Relations among Price Levels

In an inflationary environment (with all nominal variables growing), this intertemporal weighting asymmetry has an important implication: *the price level chases after a moving target*. The target price is what the price level would be under instantaneous price adjustment. Since the money supply keeps rising from period to period whereas prices depend more heavily on past prices than future ones, the price adjustments never work themselves out fully. By the time the current price level has begun to respond to the current increase in the money supply, the money supply rises again, prompting a new round of price adjustments.

To analyze the long-run inflation-unemployment tradeoff, we need to consider permanent changes in inflation, associated with permanent changes in money growth (such as those accompanying changes in a central bank's inflation target or other policy rule). Under the intertemporal weighting asymmetry above, a permanent increase in money growth causes the price level to fall further behind its target. As illustrated in Figure 2, an increase in money growth ($\Delta M_t \uparrow$) leads to a proportional increase in the target price ($\Delta P_t^T \uparrow$), but the actual price level increases less than proportionately ($\Delta P_t \uparrow$ by less).² Thus, comparing the initial and final steady states, the level of real money balances rises ($(M/P)_t \uparrow$) and unemployment falls ($u_t \downarrow$) while inflation rises ($\pi_t \uparrow$). In short, the long-run Phillips curve is downward-sloping.

²Specifically, in any given time period, the price level falls relative to the money supply. But in the steady state, the money supply and the price level grow at the same rate.

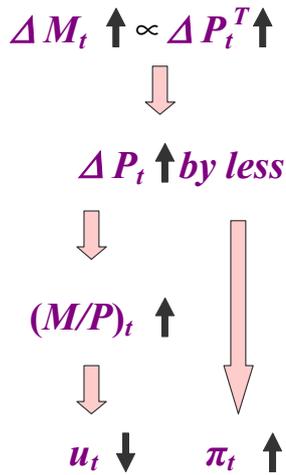


Fig. 2: Prices Chasing a Moving Target

Underlying this long-run tradeoff is a concept of equilibrium that differs from the textbook notions. In these standard notions, an economic equilibrium is attained when all lagged adjustment processes have worked themselves out. (This may mean, for instance, that expectations have become consistent with the underlying stochastic generating processes, or that other temporary adjustment costs - such as production and employment adjustment costs - have been overcome and thus do not influence the steady-state behavior of economic agents.) In our analysis, by contrast, the macro equilibrium is reached when the untrended macro variables (inflation, unemployment) stabilize; but in this equilibrium adjustment costs have not worked themselves out. On the contrary, the equilibrium is the long-run outcome of a race between growing variables and lagged adjustment processes. Specifically, in the context of our analysis, the money supply is continually growing and wages and prices are continually in the process of catching up. In the resulting equilibrium, the adjustment processes - the degree of temporary nominal rigidity (e.g. the length of the contract period) - have a crucial role to play.

Our analysis indicates that the intertemporal weighting asymmetry not only generates a long-run Phillips curve, but also gives rise to plausible impulse responses to money growth shocks. A rise in money growth leads to a quick decline in unemployment, but this unemployment effect dies down with the passage of time (although the effect never disappears entirely). The inflation response is more delayed and gradual. In this way, our analysis is in accord with the main stylized facts concerning inflation persistence: gradual reactions of inflation and unemployment to monetary shocks and the absence of knife-edge inflation behavior, viz., if the

NAIRU is constant, then prolonged swings in unemployment lead to instability in inflation.

It is worth emphasizing that, in practice, the intertemporal weighting asymmetry above depends on more than only time discounting:

- Since future economic conditions are more risky than current ones (i.e. our confidence in our predictions declines as these predictions lie further in the future), risk-averse agents discount the future by a risk premium as well as a time discount rate.
- Furthermore, the higher is the expected separation rate of employees, the less likely it becomes that they will receive the currently negotiated wages that are to accrue in the future. Thus, when the wage contract is negotiated, employees attach greater weight to current remuneration (that depends on wages set in the past) than on future remuneration (that depends on wages set in the future). The average monthly job separation rate in the U.S. has exceeded 3 percent for much of the past decade.³
- In practice, people often tend to be myopic, and thus act as if they had a high intertemporal discount rate.⁴

For brevity, however, these considerations will remain beyond the scope of our formal model, which relies only on time discounting to generate the intertemporal weighting asymmetry. We show that substantial long-run inflation-unemployment tradeoffs can exist even under reasonably low discount rates.

Thus far, downward-sloping long-run Phillips curves have been considered unacceptable on theoretical grounds. In the absence of money illusion - so the conventional argument goes - real economic activities do not depend on the unit of account and, by implication, monetary policy can have no long-term effect on unemployment. Our analysis calls this argument into question. The absence of money illusion implies that real economic activities are unaffected by a proportional change in all nominal variables (past, present, and future). But under

³Using the Bureau of Labor Statistics' Job Openings and labor Turnover Survey (JOLTS), which begins in December 2000, Hall (2003) reports that the monthly separation rate was 3.4 percent in December 2000, 3.2 in December 2001, and 3.0 in December 2002. See also Blanchard and Diamond (1990), who examine household data in the Current Population Survey as well as the manufacturing turnover survey from 1968 through 1981.

⁴In the extreme case where the future is ignored, the discount rate is 100 percent.

circumstances of frictional growth, current nominal variables do not move proportionately to the money supply. As noted, these variables lag behind their target values (which are proportional to the money supply), and the faster the money supply grows (*ceteris paribus*), the further behind they lag. So the absence of money illusion does not imply money super-neutrality. In short, under the standard classical principles, in which all demand and supply functions are homogeneous of degree zero in all nominal variables, it is still possible for monetary shocks to generate a long-run tradeoff between inflation and unemployment.

The paper is organized as follows. In Section 2 we relate our analysis to the existing literature. Section 3 describes the microfoundations of our macro model. Section 4 presents the corresponding macro model.

Section 5 derives the associated forward-looking short-run Phillips curve, in which current inflation depends on expected future inflation and unemployment. Under rational expectations, expected future inflation can be expressed in term of current and past inflation and unemployment. Thus we can derive a closed-form expression for our short-run Phillips curve in which current inflation depends on past inflation and unemployment. The resulting Phillips curve looks remarkably like the traditional backward-looking Keynesian Phillips curve. It thus turns out that the critical difference between the forward-looking New Phillips curve and the traditional backward-looking one does not hinge - as much of the existing literature suggests - on whether current inflation depends on expected future inflation or on past inflation. Instead, that it hinges on theoretical parameter restrictions.

In Section 6 we derive the long-run Phillips curve and explain why it may be reasonably flat in practice. In Section 7 we link the short- and long-run Phillips curves by deriving the impulse-response functions of inflation and unemployment to monetary shocks. We find that the lower is the discount rate, the steeper is the associated long-run Phillips curve (*ceteris paribus*), but the longer it takes for unemployment to converge to its long-run values. Empirically, it may be difficult to distinguish between quick convergence to a flat long-run Phillips curve or slow convergence to a steep one (i.e. between permanent versus very prolonged unemployment effects of money growth shocks).

Section 8 provides an illustrative empirical analysis of the U.S. inflation-unemployment

tradeoff, allowing for frictional growth. We show that the resulting impulse-response functions are broadly in accord with the stylized facts, and the long-run Phillips curve is far from vertical. Finally, Section 9 concludes with some thoughts on the role of monetary policy in accounting for the path of inflation and unemployment in the U.S. over the 1990s.

2. Relation to the Literature

The traditional Keynesian expectations-augmented Phillips curve, in its simplest form, may be expressed as $\pi_t = \pi_{t-1} - \gamma(u_t - u^n) + \varepsilon_t$ (where π is the inflation rate, u is the unemployment rate, u^n is the natural rate of unemployment or NAIRU, γ is a positive constant, and ε_t is white noise). It has been called “a fact in search of a theory,” since it is in accord with prominent empirical regularities, but has proved difficult to rationalize through microfoundations.

The standard textbook version of the New Phillips curve⁵ (NPC) is $\pi_t = E_t\pi_{t+1} - \gamma(u_t - u^n) + \varepsilon_t$,⁶ (where E_t denotes expectations set at time t). It is far less successful in explaining the stylized facts. In particular, it has why inflation is so persistent, with autocorrelations close to unity;⁷ why monetary shocks have delayed and gradual effects on inflation and unemployment;⁸ and why we usually don’t observe “disinflationary booms”⁹. So, with some exaggeration, the New Phillips curve might be called “a theory in search of a fact”! These issues are important, since the Phillips curve is central to our understanding of business cycles and widely used in the analysis of monetary policy.¹⁰

In recent years various attempts have been made to rectify these predictive deficiencies -

⁵It is also known as the “New Keynesian Phillips Curve” or the “New Neoclassical Synthesis.” For surveys see, for example, Gali (2002), Goodfriend and King (1997), Mankiw (2001), and Roberts (1995).

⁶Alternatively, the unemployment term may be replaced by another real variable, such as the output gap. As discussed below, the microfounded version of the NPC, $\pi_t = \beta E_t\pi_{t+1} - \gamma(u_t - u^n) + \varepsilon_t$, where β is the discount factor, is generally considered to have quite similar dynamic implications.

⁷Fuhrer and Moore (1995) argue that although the Taylor model can account for slow adjustment of wages and prices, inflation is a jump variable that can adjust instantly (much like the capital stock adjusts slowly even though investment can adjust instantly).

⁸See, for example, Mankiw (2001).

⁹See Ball (1994). When monetary policy is credible, the announcement of a monetary contraction leads firms to expect disinflation, and thus they moderate their price rises even before the money supply slows down. Consequently, real money balances rise, stimulating aggregate demand and reducing unemployment. Conversely, expansionary monetary policy has a contractionary effect on unemployment. In practice the opposite happens; for a recent appraisal, see for example Ball (1997, 1999).

¹⁰See, for example, Clarida, Gali, and Gertler (1999).

generally by bringing the predictions of the NPC more closely into line with the traditional one - but no consensus on the nature of the Phillips curve has yet been reached.¹¹ Moreover, both the old and new Phillips curves share a further predictive deficiency. It is that *if the NAIRU is reasonably stable through time*, then inflation will change without limit for as long as the unemployment rate differs from this NAIRU.¹² This knife-edge prediction has received little if any empirical support. There is certainly no evidence of limitlessly large deflation when unemployment is high ($u_t > u^n$ in the traditional Phillips curve) or low ($u_t < u^n$ in the NPC). In Europe, the rise in unemployment over much of the 80's and early 90's despite stable inflation is not in accord with this interpretation.¹³ In the US, the fall in both inflation and unemployment during much of the 90's does not fit it either.

There are three ways of dealing with the knife-edge problem. One is to assume that the NAIRU varies through time in agreement with the NAIRU hypothesis.¹⁴ Then the NAIRU hypothesis becomes tautologous, lacking explanatory power. The charge of tautology can be avoided only if there is convincing *ex ante* explanatory evidence for the predicted movements of the NAIRU. But such evidence is often hard to come by. For example, if movements in the

¹¹The literature is vast; the following are a few examples. Mankiw and Reis (2001) address them in a model where price information disseminates gradually among economic agents. Roberts (1997) constructs a model in which price expectations are not fully rational. Ball (1995) investigates the effects of monetary policy that is not fully credible. Gali (2002) and Gali, Gertler, and Lopez-Salido (2001) examine inflation persistence in terms of price staggering and the cyclical behavior of marginal costs. Lindbeck and Snower (1999) examine the real effects of monetary shocks in the presence of price precommitment and production lags. Huang and Liu (2002) show that wage staggering is more effective than price staggering in amplifying real persistence of monetary shocks. Helpman and Leiderman (1990) and Erceg, Henderson and Levin (2000) examine the interaction between price- and wage-staggering. Some authors, e.g. Estrella and Fuhrer (1998) focus on rigidities such as habit formation in consumption. Other contributors derive real and nominal persistence from complementarities between wage-price staggering and various real rigidities. For instance, Christiano, Eichenbaum, and Evans (2001) and Dotsey, King, and Wolman (1997) examine the interaction between nominal staggering and variable capital utilization. Jeanne (1998) examines the complementarity between price staggering and real wage rigidity. Bergen and Feenstra (2000) investigate the real effects of monetary shocks under staggered price setting in the context of a translog demand structure and roundabout input-output technologies. Kiley (1997) examines the interaction between price staggering and increasing returns in production. Huang and Liu (2001) analyze price staggering in a vertical input-output structure.

¹²Specifically, the traditional Phillips curve implies that $\Delta\pi_t = -\gamma(u_t - u^n) + \varepsilon_t$, so that inflation falls (rises) without limit when unemployment is high (low), relative to the NAIRU. By contrast, the New Phillips curve implies that $\Delta\pi_{t+1} = \gamma(u_t - u^n) + \varepsilon_{t+1}$ (where $\varepsilon_{t+1} = \pi_{t+1} - E_t\pi_{t+1}$ is an expectational error), so that inflation rises (falls) without limit when past unemployment is high (low).

¹³The rise of European inflation *and* unemployment in the mid-70s and early 80s is not in agreement with the traditional Phillips curve, with a stable NAIRU.

¹⁴In other words, the variations in the NAIRU are such that the resulting difference between the NAIRU and the actual unemployment rate is always inversely proportional to variations in the inflation rate, according to the traditional Phillips curve, or directly proportional to the inflation variations, according to the New Phillips curve.

NAIRU relative to the actual unemployment rate are inversely related to movements in inflation (in accordance with the traditional Phillips curve), then the NAIRU in many continental European countries must have been rising between the mid-70's and early 90's, except for a few years in the late 1980s. But it is far from clear where these NAIRU movements could have come from. The large increases in union density, unemployment benefits and benefit durations, and other welfare state entitlements, the increased stringency of job security legislation, and the big influx of women and young people into the labor force in Europe occurred primarily in the 60's and early 70's. By the 80's and 90's these trends had largely ceased and there were even important moves in the opposite direction.¹⁵ The alleged fall in the U.S. NAIRU in the second half of the 90's is also difficult to explain.¹⁶ With 20-20 hindsight, it is of course possible always to identify new constellations of economic variables that could plausibly have pushed the NAIRU in any direction required by the underlying theory. But the selective nature of this exercise has made a growing number of economists uneasy.

A second way to avoid the knife-edge problem is to suppose that there are long lags in the adjustment of unemployment to macroeconomic shocks, such as the oil price shocks of the mid-70s and early 80s and the interest rate shock of the early 90s. According to this interpretation, the long-run NAIRU in Europe and the U.S. was reasonably stable over the past three decades; the divergent unemployment trajectories in Europe and the U.S. are due to differences in adjustment costs (such as costs of hiring and firing) in the face of some common macroeconomic shocks; and these prolonged unemployment adjustments had little influence on inflation.¹⁷ This approach also has difficulties: the lagged adjustments need to be very long and variable for the explanation to work, and it is not clear why inflation is not sensitive to the prolonged unemployment adjustments.

A third way of avoiding the knife-edge problem is to dispense with the NAIRU altogether. This is the approach is pursued here. Our analysis calls into question the conventional view

¹⁵Rising interest rates and tax rates may well have played a role in driving the NAIRU upwards over the 80's, but the timing of these factors does not always mesh well with the timing of the unemployment increases in various European countries. The relevant literature is voluminous and well-known; an impressive example is Phelps (1994, ch. 17).

¹⁶This literature is also well-known. See, for example, Phelps (1999) and Phelps and Zoega(2001).

¹⁷See, for example, Blanchard and Summers (1986), Lindbeck and Snower (1989, ch. 11), Henry, Karanassou and Snower (2000).

that the long-run Phillips curve is either vertical or nearly vertical and that forward-looking Phillips curves are difficult to reconcile with substantial inflation persistence and unemployment inertia. We show that under plausible empirical assumptions the long-run Phillips curve may be downward-sloping and reasonably flat, and that the flatter this slope is, the more under-responsive is inflation to a money growth shock.

Our analysis is in some respects similar in spirit to the pathbreaking work of Akerlof, Dickens and Perry (1996, 2000), who show that the Phillips curve becomes downward-sloping at low inflation rates when there are permanent downward wage rigidities or departures from rational expectations. But in contrast with these contributions, our analysis indicates that the long-run Phillips curve is downward-sloping even when there are no permanent nominal rigidities, no money illusion, and no departures from rational expectations.

Further notable theoretical analyses of non-vertical inflation-unemployment tradeoffs include Hughes-Hallett (2000) and Holden (2003). Holden shows how an inflation-unemployment tradeoff - again at low inflation rates - can arise in European countries where the nominal wage can only be changed by mutual consent in wage negotiations. Hughes-Hallett's non-vertical Phillips curve is due to aggregation over sectoral/regional Phillips curves with heterogeneous short-run slopes. Our analysis does not rely on such strategic considerations or aggregation issues.

It is worth noting that the strictly microfounded version of the NPC is often expressed as $\pi_t = \beta E_t \pi_{t+1} - \gamma(u_t - u^n) + \varepsilon_t$, where β is the discount factor. Although this Phillips curve is not subject to the knife-edge problem, the conventional wisdom is that since the discount factor β is close to unity, it can usefully be approximated by the textbook version above. On this account, β is commonly set equal to unity when the NPC is used for prediction and policy analysis,¹⁸ and attention in the mainstream literature is focused on explaining inflation persistence rather than avoiding the knife-edge behavior. It is certainly true to say that the conventional analyses of the Phillips curve are broadly compatible with the NAIRU and its knife-edge implications.

The existing empirical evidence on the NAIRU hypothesis and the slope of the long-run

¹⁸See, for example, Gali and Gertler (1999), Romer (1996), Walsh (1998).

Phillips curve is distinctly mixed, and has led major contributors such as Mankiw (2001) to be “agnostic” on the issue. Given economists’ predilection for the classical dichotomy, it is striking how many well-known recent studies reject it. Ball (1997) shows that countries experiencing comparatively large and long declines in inflation tend also to encounter comparatively large increases in their NAIRU’s. Ball (1999) suggests that such a relationship may be due to monetary policy: countries with relatively contractionary monetary policy in the 1980s tended to have relatively large increases in their NAIRU’s. In Bernanke and Mihov (1998) the estimated impulse-response functions of unemployment to monetary shocks do not go to zero (although the estimated influence is statistically insignificant). Akerlof, Dickens and Perry (1996, 2000) find evidence of a long-term tradeoff between inflation and unemployment at low inflation rates. Dolado, Lopez-Salido and Vega (2000) find some evidence of such a tradeoff over the entire range of observations for Spain during 1964-1995. Fisher and Seater (1993), King and Watson (1994) and Fair (2000) find a long-run inflation-unemployment tradeoff as well.

Our analysis will provide theoretical foundation and empirical support for a long-run inflation-unemployment tradeoff. In the process, we will also show that the slope of this tradeoff is closely related to inflation persistence and the dynamic response of unemployment to monetary shocks. We now present a theoretical model which formalizes our central ideas.

3. Microfoundations

Our microfoundations - in the spirit of Ascari (2003), Huang and Liu (2002) and others - are quite standard. The model is based on Graham and Snower (2002b).

Consider an economy in which a continuum of households supplies differentiated labor to a fixed number of identical firms (normalized to one), with the following production function:

$$y_t^s = n_t^d = \left[\int_{h'=0}^1 n_t^d(h')^{\frac{\theta_w-1}{\theta_w}} dh' \right]^{\frac{\theta_w}{\theta_w-1}}, \quad (3.1)$$

where y_t^s is output supplied, n_t^d is aggregate employment demanded, $n_t^d(h)$ is labour of type h demanded, and θ_w is the elasticity of substitution between labour types.

The households are grouped into N wage-setting cohorts, each of which sets the nominal contract wage for N periods. Let $W_t(h)$ be the contract wage received by household h . Then profit maximization subject to the production function above yields the following labor demand function:

$$n_t^d(h) = \left[\frac{W_t(h)}{V_t} \right]^{-\theta_w} n_t^d, \quad (3.2)$$

where n_t is aggregate employment and V_t is the aggregate wage index:

$$V_t = \left[\int_{b'=0}^1 W_t(h)^{1-\theta_w} dh' \right]^{\frac{1}{1-\theta_w}}. \quad (3.3)$$

Each household maximizes the present value of its utility over the contract period, which depends on consumption demand $q_{t+i}^d(h)$, labor supply $n_{t+i}^s(h)$, $i = 0, 1$, and terminal real money balances $\frac{M_{t+1}(h)}{P_{t+1}}$:

$$\max_{V_t(h), q_{t+i}(h), n_{t+i}(h), M_{t+1}(h)} \sum_{i=0}^{N-1} \beta^i \left[\log q_{t+i}^d(h) + \varsigma (1 + \beta) \frac{(1 - n_{t+i}^s(h))^{1-\eta}}{1 - \eta} + v \log \frac{M_{t+1}(h)}{P_{t+1}} \right],$$

where β is the discount factor. This utility function has the desirable long-run properties (see King, Plosser and Rebelo (1988)). The household's intertemporal budget constraint is

$$\begin{aligned} & \sum_{i=0}^{N-1} \beta^i q_{t+i}^d(h) + \beta^1 \frac{B_{t+1}(h) + M_{t+1}(h)}{P_{t+1}} \\ = & \sum_{i=0}^{N-1} \beta^i \left[\frac{W_t(h)}{P_{t+i}} n_{t+i}^s(h) + \frac{T_{t+i}(h)}{P_{t+i}} + \frac{\Pi_{t+i}(h)}{P_{t+i}} \right] + R_t \frac{B_t(h)}{P_t} + \frac{M_t(h)}{P_t}, \end{aligned}$$

where $\Pi_{t+i}(h)$ is its profit income, R_t is the nominal interest factor on its bond holdings $B_t(h)$, and $T_{t+i}(h)$ are lump-sum taxes. The household also faces the labor demand constraint (3.2).

Maximizing the utility function subject to these two constraints, we obtain the consumption function:

$$q_t^d(h) = \frac{1}{v} \frac{M_t(h)}{P_t}; \quad (3.4)$$

and the optimal labor supply:

$$(\theta - 1) \sum_{i=0}^{N-1} \beta^i = \sum_{i=0}^{N-1} \beta^i \theta \varsigma n_{t+i}^s(h) (1 - n_{t+i}^s(h))^{-\eta}. \quad (3.5)$$

Since there are constant returns to labor in aggregate ($y_t = n_t$), the aggregate price level is a unit markup over the aggregate wage:

$$V_t = P_t. \quad (3.6)$$

The product and labor markets clear:

$$y_t^s = q_t^d, \quad (3.7)$$

$$n_t^d = n_t^s, \quad (3.8)$$

where $q_t^d = \int_{h'=0}^1 q_t^d(h') dh'$ and $n_t^s = \int_{h'=0}^1 n_t^s(h') dh'$.

Finally, the government has the following budget constraint:

$$\mathbf{B}_{t+i+1} + \mathbf{M}_{t+i+1} = R_{t+i} \mathbf{B}_{t+i} + \mathbf{M}_{t+i} + \mathbf{T}_{t+i}, \quad (3.9)$$

where \mathbf{B} , \mathbf{M} , \mathbf{T} are the aggregate amounts of bonds, money, net transfers, and $\mathbf{M}_t = \int_{h'=0}^1 \mathbf{M}_t(h') h'$.

To motivate the inflation-unemployment tradeoff as transparently as possible, we now move from these microfoundations to a linear macro model.

4. The Macro Model

To derive our macro model, we log linearize the equations above (around zero inflation)¹⁹ and aggregate them across agents. For simplicity, we assume that there are two wage-setting cohorts.

¹⁹The nonlinear behavior of the general equilibrium system above is described in Graham and Snower (2002b). Due to the linearization here, the predictions of our theoretical macro model are relevant only to low inflation rates. Our empirical analysis below, however, applies to larger variations, since the estimated behavioral equations are associated with the actual variations in money growth.

Log linearizing the wage index and substituting it into the linearized markup equation, we obtain:²⁰

$$P_t = \frac{1}{2} (W_t + W_{t-1}). \quad (4.1)$$

Log linearizing the consumption function, aggregating across households, and setting the resulting aggregate demand equal to aggregate supply (given by the production function), we find²¹

$$N_t = M_t - P_t \quad (4.2)$$

Substituting the labor demand function and the wage index into the labor supply function and linearizing, we derive the standard Taylor contract equation:²²

$$W_t = \alpha W_{t-1} + (1 - \alpha) E_t W_{t+1} + \gamma (c + \alpha N_t + (1 - \alpha) E_t N_{t+1}) + \omega_t, \quad (4.3)$$

where

$$\alpha = \frac{1}{1 + \beta}$$

is the “discounting parameter”, γ (a positive constant) is the “demand sensitivity parameter” that describes how strongly the contract wage is influenced by changes in labor demand, and c is the “cost-push parameter” representing upward pressure on wages that is independent of demand. E_t denotes expectations formed in period t , and the contract shock ω_t is a white noise error term.²³ We assume that the wage setters have knowledge of nominal wages and employment up to period t .

The aggregate labor supply, defined as the total amount of time available to all households, is constant, so that it can be normalized to zero: $L_t = 0$. The unemployment rate (not in logs)

²⁰ P_t and W_t are the log-linearized forms of the price level \mathbf{P}_t and the contract wage \mathbf{W}_t .

²¹ N_t and M_t are the log-linearized forms of aggregate employment n_t and the money supply \mathbf{M}_t .

²² In order to make this contract equation equivalent to the well-known Taylor contract (Taylor (1980a)), we include an error term, which could be motivated by extending our microfoundations model to include stochastic preferences or productivity shocks unanticipated by the households.

²³ We assume that $E_t \omega_t = 0$.

can be approximated as $u_t = L_t - N_t = -N_t$. Thus, by (4.2), the unemployment equation is

$$u_t = -(M_t - P_t). \quad (4.4)$$

To close the model, we need to specify the process governing the money supply. Since the analysis of the long-run Phillips curve requires that we consider the unemployment rates associated with different long-run inflation rates, we need to consider permanent shocks to money growth. For simplicity, let money growth follow a random walk:

$$\Delta M_t \equiv \mu_t = \mu_{t-1} + \varepsilon_t, \quad (4.5)$$

where M_t is the log of the money supply and ε_t is a white-noise error term. We assume that rational agents at time t know the stochastic process generating money growth, and have information up to the shock ε_t , but do not know future realizations of the money growth shock. It is important to note that our qualitative conclusions do not hinge on this random walk assumption. Any money growth process involving a permanent change in money growth (e.g. an $I(0)$ money growth process with a change in money growth regime, or a permanent change in the monetary authority's reaction function) would do.²⁴

The macro model above comprises four linear equations in four variables (the price level P_t , the contract wage W_t , employment N_t , and the money supply M_t). The supply and demand sides of the economy are equilibrated through the wage contract equation (4.3): a fall in the demand for labor puts downward pressure on the nominal wage W_t . The fall in the nominal wage, in turn, puts downward pressure on the price level (by eq. (4.1)). Thus, given the money supply (4.5), real money balances rise and aggregate demand is stimulated.

In the context of this model, we now proceed to derive the Phillips curve, first in the short-run and then in the long-run.

²⁴Karanassou, Sala and Snower (2002, Appendix 1) show that although the random walk assumption receives some moderate support from the data, the central results can be derived from other money growth processes as well.

5. The Short-Run Phillips Curve

To derive the short-run Phillips curve, we substitute the wage contract equation (4.3) into the price mark-up equation (4.1) to obtain the following price equation:²⁵

$$P_t = \alpha P_{t-1} + (1 - \alpha) (E_t P_{t+1} + \nu_t) + \gamma c + \frac{1}{2} (\omega_t + \omega_{t-1}) + \frac{\gamma}{2} (\alpha N_{t-1} + \alpha N_t + (1 - \alpha) E_{t-1} N_t + (1 - \alpha) E_t N_{t+1}), \quad (5.1)$$

where $\nu_t = E_{t-1} P_t - P_t$ is an expectational error term. Recalling that $\alpha = \frac{1}{1+\beta}$, this equation implies the following *forward-looking short-run Phillips curve*:²⁶

$$\pi_t = \beta E_t \pi_{t+1} - \frac{\gamma}{2} [u_{t-1} + (1 + \beta) u_t + \beta E_t u_{t+1}] + \gamma c (1 + \beta) + \eta_t, \quad (5.2)$$

where $\eta_t = \beta \nu_t + \frac{(1+\beta)}{2} (\omega_t + \omega_{t-1}) - \frac{\gamma\beta}{2} (E_{t-1} u_t - u_t)$ is a random error term. This equation is quite similar to the standard New Phillips curve (given in Section 2), except that inflation depends not just on current unemployment, but also on past and future unemployment.²⁷

In the mainstream literature, it is common to derive conclusions about inflation persistence and the effects of monetary policy from such an equation alone. For example, the influential contribution of Fuhrer and Moore (1995) derives the Phillips curve $\pi_t = E_t \pi_{t+1} + \gamma y_t$ and then states “All of the persistence in inflation derives from the persistence in the driving term y [excess demand](p. 129)”. This approach is misleading, however, since excess demand y - or unemployment u in our model - is an endogenous variable which, along with inflation π , is affected by monetary shocks (and other shocks). Thus, inflation persistence in response to monetary shocks can only be examined in the context of a general equilibrium system, containing both the Phillips curve *as well as* the relation between the real variable (e.g., y or

²⁵To see this, substitute (4.3) into (4.1) and note that $\frac{1}{2} (E_t W_{t+1} + E_{t-1} W_t) = \frac{1}{2} (E_t W_{t+1} + W_t) + \frac{1}{2} (E_{t-1} W_t + W_{t-1}) - \frac{1}{2} (W_t + W_{t-1}) = E_t P_{t+1} + \nu_t$.

²⁶Add the term $-(1 - \alpha) P_t$ to both sides of the previous equation and note that, since we normalize the constant level of the labor force to unity, $N_t = -u_t$.

²⁷It has been argued (e.g. Roberts (1995)) that since unemployment has a high degree of serial correlation, the weighted average of past, current, and future unemployment may be approximated by the current unemployment rate. But this argument runs afoul of the Lucas critique: the degree to which current unemployment depends on past and future unemployment is affected by macro policy (the monetary policy equation (4.5)) and thus cannot be specified *a priori*.

u) and the monetary shock.

For this purpose, we need to embed the Phillips curve (5.2) in our general equilibrium system above, express the expectations of future inflation in terms of current and past macroeconomic variables, and then derive the impulse response functions of inflation and unemployment to money growth shocks. The first step is to find the equilibrium wage and price level in terms of current and past variables. It can be shown²⁸ that the equilibrium nominal wage is

$$W_t = (1 - \lambda)c + \lambda W_{t-1} + (1 - \lambda)M_t + \kappa(1 - \lambda)\mu_t + \omega_t, \quad (5.3)$$

where $\lambda = \frac{\phi_2 - \sqrt{(\frac{\phi_2}{\phi_3})^2 - 4(\frac{\phi_1}{\phi_3})}}{2}$, $\phi_1 = \alpha(1 - \frac{\gamma}{2})$, $\phi_2 = (1 + \frac{\gamma}{2})$, $\phi_3 = (1 - \alpha)(1 - \frac{\gamma}{2})$, $\kappa = \frac{\alpha(1+\lambda)}{1-\alpha-\lambda} > 0$, and $0 < \lambda < 1$.

The equilibrium price level is

$$\begin{aligned} P_t = & (1 - \lambda)c + \lambda P_{t-1} + (1 - \lambda)M_t + (1 - \lambda)\left(\kappa - \frac{1}{2}\right)\mu_t \\ & - \frac{1}{2}\kappa(1 - \lambda)\varepsilon_t + \frac{1}{2}(\omega_t + \omega_{t-1}). \end{aligned} \quad (5.4)$$

Thus the inflation rate is

$$\pi_t = \lambda\pi_{t-1} + (1 - \lambda)\mu_t + \frac{1}{2}(1 - \lambda)(\kappa - 1)\varepsilon_t + \frac{1}{2}\kappa(1 - \lambda)\varepsilon_{t-1} + \frac{1}{2}(\omega_t - \omega_{t-2}). \quad (5.5)$$

Observe that in this equation current inflation (π_t) depends on past inflation (π_{t-1}), the money growth rate, and the monetary and real shocks. It is easy to show that the inflation persistence parameter λ depends inversely on the discount factor β (positively on the discount rate r , where $\beta = \frac{1}{1+r}$, the greater the discount rate the greater the persistence parameter λ). Thus it is clear that the forward-looking Phillips curve (5.2) is compatible with inflation persistence, given the rest of our general equilibrium system. Note that whereas the persistence parameter λ describes the relation between current and past inflation, it does not by itself provide a description of how fast inflation responds to monetary shocks; for the latter purpose, we also need to consider the stochastic structure of the monetary shocks in the inflation equation (5.5).

²⁸The algebraic manipulations underlying these and subsequent steps in this section are given in Karanassou, Sala, and Snower (2002), Appendix 2.

The price equation (5.4) also implies that equilibrium real money balances are

$$\begin{aligned} M_t - P_t &= \lambda(M_{t-1} - P_{t-1}) + (1 - \lambda) \left(\frac{2\alpha - 1}{\gamma} \right) \mu_t \\ &\quad + \frac{1}{2} \kappa (1 - \lambda) \varepsilon_t - (1 - \lambda) c - \frac{1}{2} (\omega_t + \omega_{t-1}). \end{aligned} \quad (5.6)$$

Thus the equilibrium unemployment rate is

$$u_t = (1 - \lambda) c + \lambda u_{t-1} - (1 - \lambda) \left(\frac{2\alpha - 1}{\gamma} \right) \mu_t - \frac{1}{2} \kappa (1 - \lambda) \varepsilon_t + \frac{1}{2} (\omega_t + \omega_{t-1}). \quad (5.7)$$

The Phillips curve may be defined as an equation that translates an impulse response function for inflation (5.5) into an impulse response function for unemployment (5.7), and vice versa. Thus, by equations (5.5), (5.7), and the money supply equation (4.5), we obtain our short-run Phillips curve in closed form:

$$\pi_t = d_0 + d_1 \pi_{t-1} - d_2 u_t - d_3 u_{t-1} + d_4 u_{t-2} + e_t, \quad (5.8)$$

where

$$d_0 = \psi c, \quad d_1 = \frac{\psi \kappa}{2}, \quad d_2 = \frac{\psi(1 + \kappa)}{2}, \quad d_3 = \frac{\psi}{2}, \quad d_4 = \frac{\psi \kappa}{2}, \quad \psi = \frac{1}{\frac{2\alpha - 1}{\gamma} + \frac{\kappa}{2}} \quad (5.9)$$

$$e_t = \frac{\tilde{\omega}_t}{(1 - \lambda B)}, \quad \tilde{\omega}_t = \frac{1}{2} \left[\left(1 + \frac{\psi(1 + \kappa)}{2} \right) \omega_t + \frac{3\psi}{2} \omega_{t-1} - \left(1 + \frac{\psi(\kappa - 1)}{2} \right) \omega_{t-2} \right]. \quad (5.10)$$

The above error term is a moving average process in terms of ω_t , with parameters which are non-linear functions of the theoretical parameters ψ , κ , and λ .²⁹ Inspection of equations (5.9) shows the following relationships among the slope coefficients of (5.8):

$$d_1 = d_4, \quad \text{and} \quad d_3 = d_2 - d_1. \quad (5.11)$$

Note that the closed-form Phillips curve (5.8) looks like the traditional backward-looking Keynesian Phillips curve. Nevertheless, given our macroeconomic model, our closed-form Phillips curve (5.8) is of course equivalent to our forward-looking Phillips curve (5.2). This is noteworthy because the standard way of distinguishing the backward-looking from the forward-

²⁹ ψ , κ , and λ are of course functions of the more basic time-discount parameter α and the demand-sensitivity parameter γ . Equation (5.8) can also be expressed as

$$(1 - \lambda B) (1 - d_1 B) \pi_t = d_0 (1 - \lambda) - (1 - \lambda B) (d_2 + d_3 B - d_4 B^2) u_t + \tilde{\omega}_t.$$

looking Phillips curves is in terms of lags and leads: in the backward-looking curve, current inflation depends on past inflation, whereas in the forward-looking curve it depends on expected future inflation. Our analysis suggests that this distinction is bogus. Since expectations of future inflation can be restated in terms of the current and past values of the variables, any Phillips curve with forward-looking inflation expectations can of course be transformed into a Phillips curve where current inflation depends on past inflation.

What, then, is the relation between the traditional backward-looking, expectations augmented Keynesian Phillips curve and our forward-looking one? In the traditional Phillips curve, the coefficients on past inflation and on unemployment are unrestricted, with one exception: since the traditional expectations-augmented Phillips curves is compatible with the NAIRU, the coefficient on past inflation was restricted to $d_1 = 1$. In our forward-looking Phillips curve, as we have seen, this restriction does not apply.³⁰ Instead, the coefficients of this forward-looking Phillips curve must satisfy the restrictions (5.11) and its error term ($\tilde{\omega}_t$) follows the moving average process given by (5.10).³¹

6. The Long-Run Phillips Curve

In the long-run steady state, $\pi_t = \pi_{t-1}$, $u_t = u_{t-1}$, and the white noises error terms ε_t , and ω_t are zero. Thus, by (5.5), the long-run inflation rate is³²

$$\pi_t^{LR} = \mu_t^{LR}. \quad (6.1)$$

The long-run unemployment rate is (by (5.7))

$$u_t^{LR} = -\frac{(1 - \beta)}{\gamma(1 + \beta)}\mu_t^{LR} + c. \quad (6.2)$$

³⁰In this respect, our forward-looking Phillips curve resembles the old-style Phillips curves prior to the “discovery” of the NAIRU. Our long-run Phillips curve is vertical only when the rate of time discount is zero.

³¹These conditions, however, should not be viewed as restrictions imposed on an estimated Phillips curve equation, for two related reasons. First, the restricted equation may not be estimable. Second, as we argue in Section 7, the phenomenon of frictional growth cannot be captured through single-equation estimation of the inflation-unemployment tradeoff, but requires multi-equation estimation, describing how wages and price depend on the money supply and how unemployment depends on the relation between money and prices (or some other relation between real and nominal variables).

³²Since money growth follows a random walk, the long run money growth rate varies through time (μ_t^{LR} has a time subscript) and the long-run inflation rate is time-varying as well.

Substituting equation (6.1) into (6.2), we obtain the long-run Phillips curve:

$$\pi_t^{LR} = -\frac{\gamma(1+\beta)}{(1-\beta)}u_t^{LR} + \frac{\gamma(1+\beta)}{(1-\beta)}c. \quad (6.3)$$

Note that the slope depends on the discount factor β and the demand sensitivity parameter γ .

The intuition underlying this downward-sloping long-run Phillips curve, given in Section 1, is now easy to confirm. The price level under instantaneous adjustment may be found by letting the time span of each wage contract tend toward zero, so that α tends to $1/2$ and μ_t to zero. From the real money balance equation (5.6) we find that the resulting “target price level” (frictionless price) is $P_t = M_t + c$.³³ However, when the money supply grows in the presence of the intertemporal weighting asymmetry in the wage contract ($\alpha > \frac{1}{2}$), then the price level lags behind the moving target price level. Specifically, by (5.6), the steady state price level becomes $P_t = M_t + c - \frac{2\alpha-1}{\gamma}\mu_t$. Clearly, a permanent increase in money growth (a rise in μ_t) causes the price level to fall below the target price, and consequently real money balances ($M_t - P_t$) rise and unemployment falls.

In the textbook literature on the New Phillips Curve,³⁴ the discount factor is set equal to unity ($\beta = 1$, so that $\alpha = 0.5$ in the contract equation (4.3), i.e. there is no intertemporal weighting asymmetry), thereby making the long-run Phillips curve vertical and consistent with the NAIRU hypothesis. The underlying reasoning - that the discount rate is just a few percent and thus can be approximated by zero - is misleading because (a) the slope of the long-run Phillips curve depends nonlinearly on the discount factor and (b) the effect of the discount factor on the slope depends on the value of the demand sensitivity parameter γ .

There is little agreement in the literature about the appropriate value of γ . Taylor (1980b) estimates it to be between 0.05 and 0.1; Sachs (1980) finds it in the range 0.07 and 0.1; Gali and Gertler (1999) estimate it to be between 0.007 and 0.047; calibration of microfounded models (e.g. Huang and Liu (2002)) assigns higher values. Table 1 presents the slope of the long-run Phillips curve associated with various values of the discount rate r (where $\beta = \frac{1}{1+r}$, $\alpha = \frac{1}{1+\beta}$)

³³We evaluate the target price in the absence of the white noise shocks ε_t and ω_t .

³⁴See, for example, Blanchard and Fisher (1989, p. 395). The authors however express discomfort with this: “Even under lognormality of money and the price level (actually, even under certainty) the optimal rule is not one in which the parameter is equal to a half” (p. 420).

and the γ parameter.³⁵

r (%)	β	α	<i>slope</i>				
			$\gamma = 0.01$	$\gamma = 0.02$	$\gamma = 0.05$	$\gamma = 0.07$	$\gamma = 0.10$
1.0	0.990	0.502	-2.01	-4.02	-10.1	-14.1	-20.1
2.0	0.980	0.505	-1.01	-2.02	-5.05	-7.07	-10.1
3.0	0.971	0.507	-0.68	-1.35	-3.38	-4.74	-6.77
4.0	0.962	0.510	-0.51	-1.02	-2.55	-3.57	-5.10
5.0	0.953	0.512	-0.41	-0.82	-2.05	-2.87	-4.10

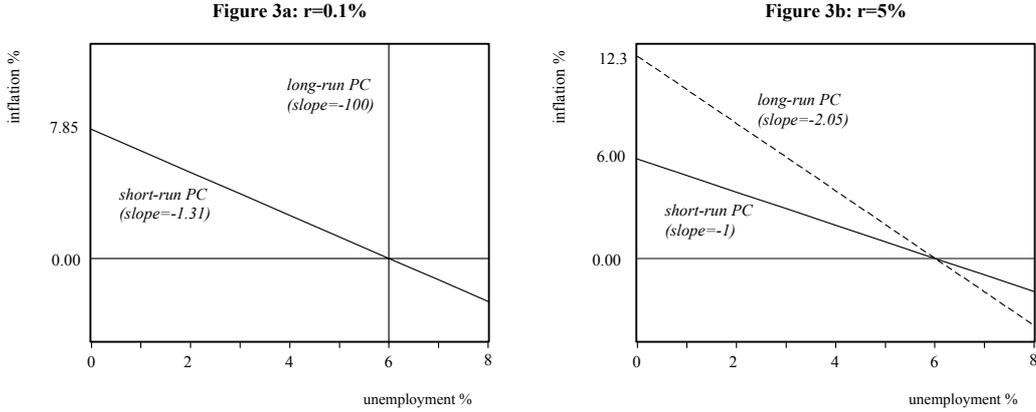
Observe that, except for combinations of particularly low discount rates and particularly high demand sensitivity parameters γ , the slope of the long-run Phillips curve is quite flat. These results, however, are merely suggestive, since the theoretical model above is obviously far too simple to provide a reliable account of the long-run inflation-unemployment tradeoff under frictional growth. For that purpose it would be necessary to examine the role of other growing variables (such as capital and productivity) in conjunction with other frictions (such as unemployment inertia). The illustrative empirical model in Section 8 is a small step in this direction.

It can be shown that, for plausible parameter values, our short-run Phillips curve has a flatter slope and lower intercept than its long-run counterpart.³⁶ Figures 3 provide two examples of associated short- and long-run Phillips curves. Observe that although the long-run Phillips curve is nearly vertical when the discount rate (defined annually) is very low (at 0.1%) and much flatter when the discount rate is high (5%), the short-run Phillips curve remains quite flat in both cases.

³⁵The discount rate applies to a period of analysis which is half the contract span.

³⁶In particular, the slope of the short-run Phillips curve (5.8) is $\frac{\partial \pi_t}{\partial u_t} = d_2 = -\frac{\gamma + \gamma\kappa}{2(2\alpha - 1) + \gamma\kappa}$ whereas the slope of the long-run Phillips curve (6.3) is $\frac{\partial \pi_t^{LR}}{\partial u_t^{LR}} = -\frac{\gamma}{2\alpha - 1}$. It can be shown that if, as is plausible, the long-run slope is less than -1 , the long-run Phillips curve is steeper than the short-run one. (This is a sufficient but not necessary condition, as shown in Karanassou, Sala, and Snower (2002), Appendix 2). The intercept of the short-run Phillips curve (5.8) is given by $d_0 = \left(\frac{2\gamma}{2(2\alpha - 1) + \gamma\kappa}\right) c$, which is smaller than the long-run Phillips curve (6.3) intercept: $\left(\frac{\gamma}{2\alpha - 1}\right) c$. (For the underlying derivations, see Karanassou, Sala, and Snower (2002), Appendix 2.)

Figures 3: The Short- and Long-Run Phillips Curves
($\gamma=0.05$, $c=0.06$)



7. Theoretical Impulse Response Functions

We now examine the connection between the short- and long-run Phillips curves by deriving the impulse response functions of inflation and unemployment to a permanent monetary shock, i.e. a one-off unit shock to money growth (4.5), occurring at time $t = 0$: $\varepsilon_0 = 1$ and $\varepsilon_t = 0$ for $t > 0$. At time $t = 0$, economic agents know the process (4.5) generating money growth, but not the realizations of the error term ε_{t+i} , $i \geq 1$.

Thus the monetary shock ε_0 is known to the wage setters at time $t = 0$, but not at time $t = -1$ (so that the expectations of wage setters at time $t = -1$ are $E_{-1}\varepsilon_0 = 0$). Since the current wage W_0 depends on the past wage W_{-1} , the current wage W_0 does not adjust fully to the shock ε_0 . On this account, the shock has real effects.

Let $R(\pi_t)$ and $R(u_t)$ be the period- t responses of inflation and unemployment (respectively) to the above money growth shock, *ceteris paribus*. By the inflation equation (5.5), we find that the inflation responses through time are:

$$\begin{aligned}
R(\pi_0) &= 1 - \frac{1}{2} \left[\frac{(1-\lambda)(1-\beta)}{\gamma(1+\beta)} + \frac{1+\lambda}{2} \right] < 1, \\
R(\pi_t) &= 1 - \lambda^{t-1} \left(\frac{1-\lambda^2}{2} \right) \left(\frac{1-\beta}{\gamma(1+\beta)} - \frac{1}{2} \right), \\
R(\pi_{LR}) &\equiv \lim_{t \rightarrow \infty} R(\pi_t) = 1 \text{ (long-run response)}.
\end{aligned} \tag{7.1}$$

By the unemployment equation (5.7), the unemployment responses through time are:

$$\begin{aligned}
R(u_t) &= -\frac{(1-\beta)}{\gamma(1+\beta)} - \lambda^t \left(\frac{1+\lambda}{2} \right) \left(\frac{1}{2} - \frac{1-\beta}{\gamma(1+\beta)} \right), \\
R(u_{LR}) &\equiv \lim_{t \rightarrow \infty} R(u_t) = -\frac{(1-\beta)}{\gamma(1+\beta)}, \text{ (long-run response)}.
\end{aligned} \tag{7.2}$$

The impulse-response function for inflation always lies above the initial ($t = 0$) inflation rate, and the impulse-response function for unemployment always lies below the initial ($t = 0$) unemployment rate. Note that *the long-run response of unemployment is simply the inverse of the slope of the long-run Phillips curve*. The equations above indicate that the medium-term responses of inflation and unemployment to the monetary shocks (viz., inflation and unemployment persistence) are closely related to the long-run Phillips curve tradeoff. In particular, the inflation and unemployment responses fall into two broad classes (by the equations above):

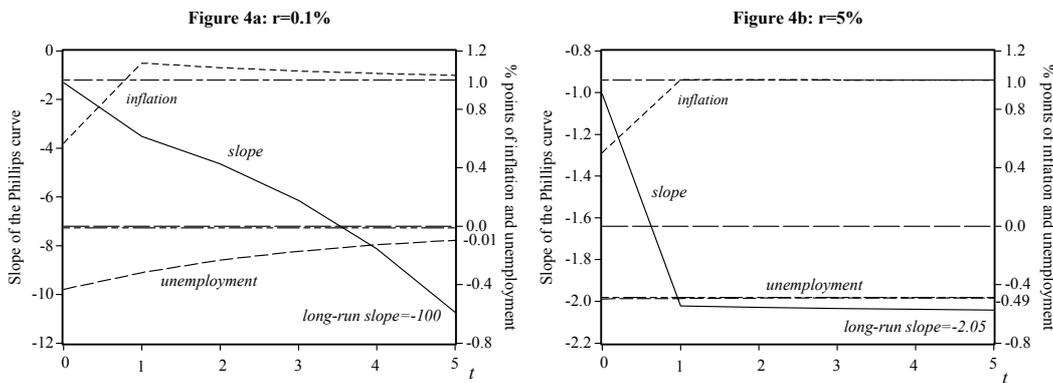
1. *Inflation and unemployment under-shooting*: If $\frac{(1-\beta)}{\gamma(1+\beta)} > \frac{1}{2}$ - so that the absolute value of the slope of the long-run Phillips curve is less than 2 - then inflation gradually rises toward its new long-run equilibrium ($\pi_t < \pi_{LR}$, and $\pi_{t+1} > \pi_t$ for $t \geq 0$); unemployment gradually falls towards its new long-run equilibrium ($|u_t| < |u_{LR}|$ for $t \geq 0$).
2. *Inflation over-shooting slowly and unemployment over-shooting quickly*: If $\frac{(1-\beta)}{\gamma(1+\beta)} < \frac{1}{2}$ - so that the absolute value of the slope of the long-run Phillips curve exceeds 2 - then inflation rises, over-shooting its new long-run equilibrium after one period, and then gradually falls toward this equilibrium ($\pi_0 < \pi_{LR}$, $\pi_t > \pi_{LR}$, and $\pi_{t+1} < \pi_t$ for $t \geq 1$). Unemployment falls, over-shooting its new long-run equilibrium, and then gradually rises toward this equilibrium ($|u_t| > |u_{LR}|$, and $|u_{t+1}| < |u_t|$ for $t \geq 0$). The maximum impact of the monetary shock on unemployment is achieved before the maximum impact on inflation.

For most of the empirically reasonable parameter values given in Table 1, the impulse-response functions can be shown to fall into Class 2, the class that accords with the stylized facts (viz., the inflation responses to monetary shocks are delayed and gradual, the unemployment responses occur more quickly). Figures 4 depict the impulse response functions for inflation, unemployment, and the slope of the Phillips curve for the same parameter values as in Figures 3.³⁷ The horizontal axis measures time; the left-hand vertical axis measures the slope of the Phillips curve; and the right-hand vertical axis measures the inflation and unemployment rates.

Observe when the discount rate is very low ($r = 0.1\%$), in Fig. 4a, the long-run Phillips curve is virtually vertical, but the short-run Phillips curve at time $t = 0$ is very flat, and it takes a very long time for unemployment, inflation, and the Phillips curve slope to reach their long-run values.

By contrast, when the discount rate is higher ($r = 5\%$), the long-run Phillips curve is quite flat, and it takes a short time for unemployment, inflation, and the slope to reach their long-run values.

**Figures 4: Impulse Response Functions
($\gamma=0.05$)**



The shock is a 1 % point increase in the money growth rate at $t=0$

It is easy to show that this pattern holds for the full range of discount rates: *The lower the discount rate (for a given value of the demand-sensitivity parameter γ):*

³⁷The value of c has no effect on the slope of the Phillips curve.

- *the steeper is the long-run Phillips curve and*
- *the longer it takes for the slope of the Phillips curve to converge to its long-run value.*

Thus, observationally, it may make little difference whether the long-run Phillips curve is flat - so that an increase in money growth permanently reduces unemployment - or near-vertical - so that the effect is not permanent, but very prolonged. In other words, it may be difficult, if not impossible, in practice to distinguish between a world in which there is quick convergence to a flat long-run Phillips curve and one in which there is slow convergence to a steep one. In both cases, money growth shocks have long-lasting effects on unemployment.³⁸

8. Empirical Analysis

Our empirical analysis is based on multi-equation estimation, since the phenomenon of frictional growth cannot be captured through the usual procedure of estimating a single-equation Phillips curve. When we estimate a traditional or New Phillips curve as a single equation, we are unable to assess how the effects of money growth work their way through the wage-price adjustment process and thereby affect unemployment. Money growth does not enter a single-equation Phillips curve at all; it is substituted out when the impulse-response function of inflation is substituted into the impulse-response function for unemployment to derive the Phillips curve. On this account, we estimate a dynamic structural model, with the following building blocks, matching those of our theoretical model: a wage setting equation and a price setting equation, to portray nominal sluggishness (so that changes in money growth lead to changes in real money balances), and the unemployment equation indicates how the changes in real money balances affect the unemployment rate.

³⁸In this context it is also easy to show that we can avoid the counterfactual implication of disinflationary booms, analogously to Mankiw and Reis (2001). In the context of the Calvo model of random nominal adjustment, Mankiw and Reis avoid disinflationary booms by assuming that only a fraction of agents receives updated information in each period. The analogue in the Taylor model of fixed, staggered adjustment is to assume that all agents receive information about monetary shocks with a one-period lag. It is trivial to see that if monetary shocks are announced one period in advance and if agents' information about these shocks is received one period in arrears, then the resulting model generates precisely the same results as the model above. More generally, our model avoids the implication of disinflationary booms whenever the lead time for monetary announcements is not greater than the lag time in agents' information updates.

In most of the current empirical literature, by contrast, the Phillips curve is estimated in a single-equation framework.³⁹ It is customary to use the lead of inflation as a proxy for expected future inflation. Thus the NPC can be consistently estimated by generalized method of moments (GMM) or, since the model is linear in the parameters, two stage least squares (TSLS). Bardsen, Jansen and Nymoen (2002), show that the empirical results are sensitive to the choice and exact implementation of the estimation method. Overall, there is no agreement in the recent literature about the appropriate method of estimating the NPC and how to test it against the traditional Phillips curve. Consequently, there is disagreement about whether the empirical evidence favors the traditional or New Phillips curves.

The choice of the forcing variable is crucial when estimating the inflation dynamics associated with the Phillips curve. Gali and Gertler (1999), Gali, Gertler and Lopez (2001) find evidence in support of the NPC only when they use labor income share as the forcing variable. Rudd and Whelan (2001) propose using a present value term of the forcing variable in the inflation regression and report results that are consistent with a backward-looking (traditional) Phillips curve.⁴⁰

The choice of instruments can have a strong influence on the GMM estimates of the NPC and their significance. It is widely accepted that the test for overidentifying restrictions as a means to detect invalid instruments has low power. In addition, Bardsen, Jansen and Nymoen (2002), and Rudd and Whelan (2001) argue that the results can be significantly biased by using variables as instruments that actually belong in a well-specified inflation regression. Furthermore, if the forcing variable is regarded as endogenous then it should be instrumented in the estimation. Bardsen, Jansen and Nymoen (2002) argue that to derive the dynamic properties of inflation,

³⁹The NPC is simply expressed as

$$\pi_t = \beta E_t \pi_{t+1} + \gamma x_t,$$

where β is the discount factor, and the "forcing variable" x_t is a measure of excess demand (unemployment rate, output gap) or a measure of real marginal costs (like the wage share).

⁴⁰Since rational expectations are also model consistent, they use repeated substitution to express the NPC as

$$\pi_t = \beta^{k+1} E_t \pi_{t+k+1} + \gamma \sum_{j=0}^k \beta^j E_t x_{t+j}.$$

The last term in the above equation is a present value term of the forcing variable and γ is estimated using GMM.

we require an analysis of the system that includes the forcing variable as well as the rate of inflation, and conclude that “as statistical models, both the pure and hybrid NPC⁴¹ are inadequate”.

In this context, it is important to keep in mind our theoretical argument (Section 4) that any forward-looking Phillips curve (containing leads) can be translated into a backward-looking one (containing only contemporaneous and lagged variables) by solving the macro model and expressing expectations of future inflation in terms of present and lagged variables. On this account, as well as the unresolved empirical issues above, the wage and price equations in our empirical model are specified solely in terms of current and past variables. (They can, however, be interpreted as the outcome of decisions by forward-looking agents, because these agents’ expectations of the future depend on their information about current and past variables and the underlying stochastic processes.) Thus, the empirical wage and price equations may be seen at the counterparts of equations (5.3) and (5.4), respectively.⁴²

We solve these three equations as a system and derive the implied inflation-unemployment tradeoff. This empirical exercise merely aims to illustrate how an estimated Phillips curve can be derived from equations describing the interplay between money growth and nominal frictions. The exercise is no more than a preliminary first step towards a full-blown empirical investigation,⁴³ which lies well beyond the scope of this paper.

⁴¹This point is consistent with our argument in Section 4 that inflation persistence in response to monetary shocks can only be evaluated in the context of a general equilibrium system including the Phillips curve, rather than through the Phillips curve alone.

The hybrid specification of the Phillips curve can be expressed as

$$\pi_t = \beta^b \pi_{t-1} + \beta^f E_t \pi_{t+1} + \gamma x_t.$$

⁴²More precisely, the empirical model may be understood as a natural extension of our theoretical model to include staggered contracts of both wage and prices. Thus in our empirical model, past nominal values affect the current wage level differently from the current price level.

⁴³Such an analysis would, for example, contain a wider range of explanatory variables (e.g. dividing the labor force into skilled and unskilled workers, distinguishing between productivity in different sectors of the economy, etc.), a larger number of equations (e.g. the unemployment rate could be derived from labor demand and labor supply equations, the capital stock could be endogenized, etc.), and so on. It would also examine the implications of GMM and 2SLS estimation of wage and price equations containing leads.

8.1. Data and Estimation

We use US annual time series data, obtained from the OECD and Datastream, covering the period 1966-2000. The definitions of the variables are given in Table 2.

<u>Table 2: Definitions of variables</u>	
M_t : money supply (M3)	fw_t : financial wealth $\left(\frac{\text{SP500}}{\text{labor productivity}} \right)$
P_t : price level	o_t : real oil price
W_t : nominal wages	fd_t : real foreign demand (exports-imports)
u_t : unemployment rate	τ_t : indirect taxes as a % of GDP
pr_t : real labor productivity	b_t : real social security benefits
m_t : real money balances ($M_t - P_t$)	ssc_t : real social security contributions
k_t : real capital stock	
All variables are in logs except for u_t , foreign demand, fd_t , and the tax rate, τ_t .	
The variables m_t , ssc_t , b_t , and fd_t have been normalized by working age population.	
The financial wealth variable fw_t is defined as in Phelps and Zoega (2001).	

The price setting, wage setting, and unemployment rate equations of our model were initially estimated individually using the autoregressive distributed lag (ARDL) approach to cointegration analysis developed by Pesaran and Shin (1995), Pesaran (1997), and Pesaran et al. (1996). These papers argue that the traditional ARDL approach justified when regressors are I(0), can also be valid with I(1) regressors. An important implication of this methodology is that, since an ARDL equation can always be reparameterized in an error correction format, the long-run solution of the ARDL can be interpreted as the cointegrating vector of the variables involved.

The dynamic specification of each equation was determined by the optimal lag-length algorithm of the Akaike and Schwarz information criteria. The selected estimated equations are dynamically stable (i.e., the roots of their autoregressive polynomials lie outside the unit circle), and pass the standard diagnostic tests (for no serial correlation, linearity, normality, homoskedasticity, and constancy of the parameters of interest) at conventional significance levels.⁴⁴ In order to take into account potential endogeneity and cross equation correlation, we then estimated the equations as a system using three stages least squares (3SLS). These results are presented in Table 3.⁴⁵ The model tracks the data very well.⁴⁶

⁴⁴See Karanassou, Sala and Snower (2002), Appendix 3.

⁴⁵Constants and trends are omitted for brevity.

⁴⁶The actual and fitted values of the estimated system are pictured in Karanassou, Sala and Snower (2002), Appendix 4.

Table 3: US model, 3SLS, 1966-2000.

Dependent variable: u_t			Dependent variable: P_t			Dependent variable: W_t		
	coef.	std. e.		coef.	std. e.		coef.	std. e.
u_{t-1}	0.43	(0.12)	P_{t-1}	1.19	(0.13)	W_{t-1}	0.24	(0.10)
u_{t-2}	-0.30	(0.11)	P_{t-2}	-0.54	(0.08)	ΔW_{t-2}	0.48	(0.10)
m_t	-0.12	(0.03)	W_{t-1}	0.34	(0.10)	P_t	0.68	(0.09)
fd_t	-0.16	(0.05)	M_t	0.01	(*)	M_t	0.09	(*)
Δk_t	-0.01	(0.002)	u_t	-0.72	(0.16)	u_t	-0.41	(0.17)
o_{t-1}	0.01	(0.003)	pr_t	-0.30	(0.06)	pr_t	0.32	(0.09)
fw_t	-0.01	(0.005)	o_t	0.02	(0.004)	b_t	0.05	(0.02)
ssc_t	0.04	(0.02)	o_{t-1}	0.01	(0.004)			
			o_{t-2}	-0.01	(0.003)			
			τ_t	0.02	(0.006)			

(*) coefficient is restricted so that there is no money illusion.
 Δ denotes the difference operator.

In the unemployment equation, product demand-side influences are captured through real money balances and financial wealth⁴⁷ (affecting domestic demand), as well as net foreign demand. Product supply-side influences are captured through the oil price, capital accumulation, and social security contributions. Observe that the sum of the lagged dependent variable coefficients is small and positive, implying a low degree of unemployment persistence. Since the US unemployment rate is trendless, the explanatory variables in the unemployment equation need to be specified as non-trended series as well. On this account, real money balances, social security contributions and benefits, and foreign demand are normalized by working age population, whereas financial wealth is deflated by productivity.

The price and wage equations are quite standard.⁴⁸ Prices depend on wages and the money supply, and wages depend on prices and the money supply. Productivity has a positive effect on nominal wages and a negative effect on prices. The unemployment moderates the mark-up of prices on wages, and of wages on prices. The lag structure of our price and wage equations is consistent with our theoretical model.⁴⁹ The restriction of no money illusion is imposed on the

⁴⁷See Phelps (1999), Fitoussi et al. (2000), and Phelps and Zoega (2001).

⁴⁸In order for all variables in our price and wage equations to be integrated of the same order, the equations need to be reparameterized before estimation. For instance, consider the price equation in Table 3: $P_t = a_0 + a_1 P_{t-1} + a_2 P_{t-2} + a_3 W_{t-1} + (1 - a_1 - a_2 - a_3) M_t + \beta' x_t$, where β' is a row vector of parameters, and x_t is a column vector of the real variables. The above can be reparameterized as $(P_t - M_t) = a_0 + a_1 (P_{t-1} - M_{t-1}) + a_2 (P_{t-2} - M_{t-2}) + a_3 (W_{t-1} - M_{t-1}) - (a_1 + a_2 + a_3) \Delta M_t - a_2 \Delta M_{t-1} + \beta' x_t$. These two equations are statistically equivalent. We estimate our price equation using the latter equation, and present the Table 3 results in the format of the former equation. The analogous procedure is applied to the wage equation.

⁴⁹In both the theoretical and empirical models, current wages and prices are explained in terms of past wages and prices and the current money supply. The empirical model may be understood as a natural extension of our

price and wage equations, so that each equation is homogeneous of degree zero in all nominal variables. Specifically, we restrict the coefficient of money in each of our nominal equations to be equal to one minus the coefficients of all nominal variables on the right-hand side of that equation.⁵⁰ These restrictions could not be rejected at conventional significance levels.

8.2. Empirical Impulse-Response Functions

In this empirical context, we examine the influence of a money growth shock on inflation and unemployment through time. Specifically, suppose that the economy is initially in a steady state, with the money supply growing at the constant rate μ . Then, at time $t = 0$, the money growth rate increases by a fixed amount to μ' . This shock is unanticipated and may be interpreted as a single realization of the stochastic process generating the money supply.⁵¹ We derive the inflation and unemployment responses to this shock for time $t \geq 0$.⁵²

Figure 5 presents the impulse response functions (IRFs) that correspond to a 10% permanent increase in the growth rate of money supply. The inflation IRF has all the desirable properties, namely, the influence of the monetary shock on inflation is delayed and gradual, and in the long run inflation is equal to money growth. The unemployment IRF also exhibits plausible behavior: the unemployment effect of the monetary shock is also delayed and gradual, but this effect occurs sooner than the inflation effect (e.g. the maximum unemployment effect occurs well before that on inflation.) Also observe that the inflation and unemployment responses take a long time to converge to their long-run values.

The only strikingly unconventional property of the unemployment IRF is that the unemployment effect does not die down to zero; rather, a 10 percent increase in money growth leads to a 2.73 percent fall in long-run unemployment.⁵³ Thus, the slope of the long-run Phillips

theoretical model to include staggered contracts of both wage and prices. Thus in our empirical model, past nominal values affect the current wage level differently from the current price level.

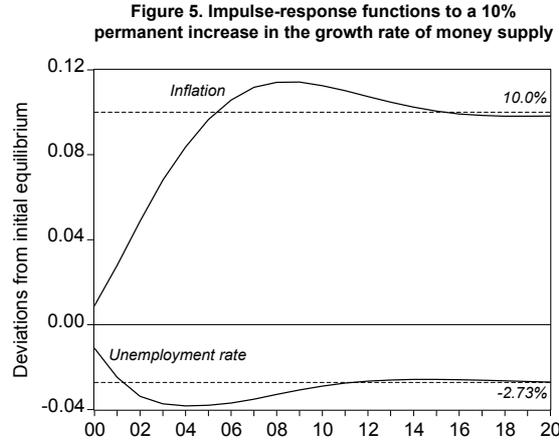
⁵⁰For example, the price equation in Table 3 (first equation in the previous footnote) is clearly homogeneous of degree zero in M_t , P_t , P_{t-1} , P_{t-2} , and W_{t-1} . The analogous restriction is imposed on the wage equation.

⁵¹Since the shock is a realization of the actual money growth process, this exercise does not run afoul of the Lucas critique.

⁵²We assume that the future values of the exogenous variables are unaffected by the monetary shock (which is obvious, for otherwise these variables would not be exogenous). Thus, given the linearity of our model, the simulation is unaffected by these future variables.

⁵³Also observe that the unemployment IRF overshoots substantially: the maximum effect on unemployment is nearly 4 percent.

curve is -3.66 ($= \frac{10}{-2.73}$).



8.3. Montecarlo Simulations

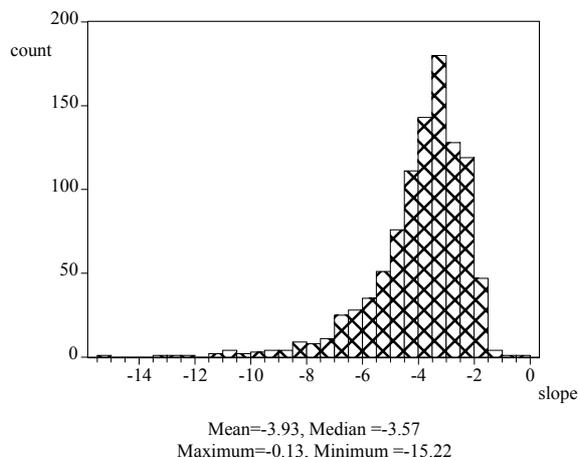
To have confidence that our long-run Phillips curve is indeed not vertical, we need to examine whether our point estimate of the slope (-3.66) is significantly different from infinity. For this purpose, we perform the following Monte Carlo experiment, consisting of 1000 replications. In each replication (i), a vector of error terms $\varepsilon_t^{(i)} = (\varepsilon_{u,t}^{(i)}, \varepsilon_{P,t}^{(i)}, \varepsilon_{W,t}^{(i)})'$, $t = 1, 2, \dots, T$ (of the unemployment rate, price, and nominal wage equations, respectively) is drawn from the normal distribution,⁵⁴ $N(0, \Sigma)$. The vector $\varepsilon_t^{(i)}$ is then added to the vector of estimated equations to generate a new vector of endogenous variables $y_t^{(i)} = (u_t^{(i)}, P_t^{(i)}, W_t^{(i)})$. Next, the equations of the model are estimated using the new vector of endogenous variables $y_t^{(i)}$, and the set of exogenous variables. Finally, the simulation exercise of the previous section is conducted on the newly estimated system to derive a new estimate of the slope of the long-run Phillips curve. In this way, each replication (i) yields a new value for the slope: $S^{(i)}$, $i = 1, 2, \dots, 1000$.

Figure 6 presents the histogram of the 1000 simulated values of the long-run Phillips curve slope. This shows clearly that the estimated slope of the long-run Phillips curve is indeed significantly downward-sloping and reasonably flat, rather than vertical.⁵⁵

⁵⁴We used the normal distribution because the assumption of normality is valid in the estimated system of equations. ($\varepsilon_t \sim N(0, \Sigma)$, where Σ is the variance-covariance matrix of the estimated model.)

⁵⁵See Karanassou, Sala and Snower (2002), Appendix 5 for further evidence in support of this result.

Figure 6
Slope of the long-run Phillips Curve
(1000 observations)



9. Conclusions

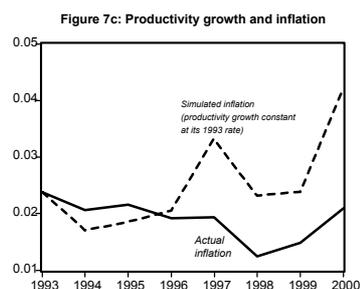
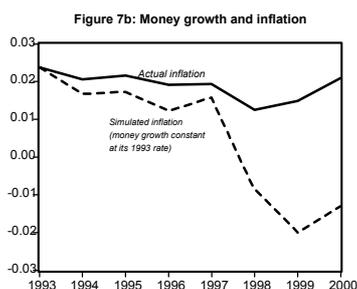
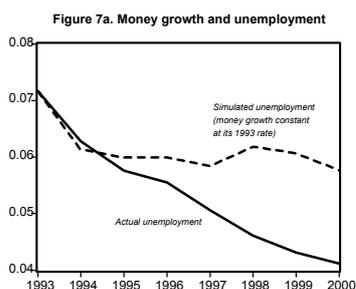
This paper offers a reappraisal of the inflation-unemployment tradeoff on the basis of frictional growth. While the choice between our analysis and the textbook New Phillips curve is an empirical issue, three of our results suggest that our approach is more closely in accord with the established empirical regularities. First, in our analysis movements of inflation and unemployment do not have the knife-edge property; in fact the long-run Phillips curve may be reasonably flat. The available empirical evidence in the OECD does not support the view that inflation falls without limit when unemployment is above some stable NAIRU (implying a vertical long-run Phillips curve); nor does it appear to support the view that there is massive deflation when unemployment is high (implying that the long-run Phillips curve is very steep). Second, our analysis can explain how money growth shocks have a delayed and gradual effect on inflation, so that there is inflation persistence. Third, it shows that monetary shocks usually have a quicker effect on unemployment and the time path of this effect tends to be hump-shaped.

Inevitably, our analysis suggests a reevaluation of the role monetary policy in the macroeconomic system. It shows that since the effects of monetary policy on inflation and unemployment generally take a long time to work themselves out, we cannot expect close correlations between

current money growth (on the one hand) and current inflation and unemployment (on the other), even though monetary policy may have a major influence on these variables over time. Significantly, our analysis indicates that monetary policy can have long-term effects on unemployment. Whether these effects are permanent (along a downward-sloping long-run Phillips curve) or very prolonged (slow adjustment to a near-vertical long-run Phillips curve), may make little observational difference. Indeed, our theoretical model indicates that, in response to variations in the real interest rate, steeper long-run Phillips curves are associated with slower adjustment.

These considerations can have far-reaching implications for our understanding of monetary policy effectiveness. To illustrate briefly, consider the puzzling U.S. macroeconomic developments of the 1990s, when the unemployment rate declined (after 1992) and inflation remained subdued even though the rate of money growth surged. Although our empirical model is merely illustrative of our approach and should not be viewed as a serious tool for evaluating monetary policy, it nevertheless points to a simple story consistent with the facts. Figure 7a depicts the time path of the actual unemployment rate against the one the unemployment rate would have followed, in our model, had money growth remained constant at its 1993 rate. The difference between these two time paths represents the unemployment effect that is attributable to money growth, as an accounting exercise.⁵⁶ Figure 7b illustrates the actual inflation rate against the simulated inflation rate under money growth fixed at its 1993 rate, so that the difference represents the inflation effect attributable to money growth. Finally, Figure 7c depicts the actual inflation rate against the simulated inflation rate under productivity growth fixed at its 1993 rate, so that the difference represents the inflation effect attributable to productivity growth.

⁵⁶The money growth rate was less than 2 percent per annum in 1993, rose steadily to over 8 percent in 1998, before declining beneath 6 percent in 2000. Increased productivity growth is also associated with reduced unemployment in our model, but the influence is much weaker than that of money growth in our model.



Although these figures are merely suggestive - even in our illustrative model, inflation and unemployment are explained by a lot more than just money growth and productivity growth - they make three simple points: First, the surge of money growth over the second half of the 1990s can account for about two thirds of the decline in unemployment over this period (Fig. 7a). Second, the money growth surge was of course associated with a rise in inflation (Fig. 7b). But, third, this inflationary influence was substantially undone by the fall in inflation associated with the increase in productivity growth over the period (Fig. 7c). This is of course a highly selective, impressionistic account of what happened, but it highlights some significant features of our analysis. In particular, since it can take a long time for the long-run inflation effect of a monetary growth shock to manifest itself, a surge in money growth need not be accompanied promptly by a surge in inflation. Inflation does not rise indefinitely when unemployment is low. Finally, monetary policy can have a long-term influence on unemployment and, over a period of half a decade or more, it is hard to tell whether this influence is permanent or prolonged, since the unemployment trajectory reflects the cumulative influence of lengthy impulse-response functions from an ongoing stream of monetary shocks. In any case, monetary policy may play a more important and durable role in the real economy, and with respect to unemployment in particular, than the mainstream theories allow for.

Our analysis is of course just a first step towards a thorough reevaluation of the inflation-unemployment tradeoff in terms of frictional growth. Much remains to be done, both in exploring the microfoundations of time-contingent price adjustment and in building reliable empirical models of how monetary shocks affect real economic activity.

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APPENDICES

Appendix 1a: Time-Series Properties of the Money Supply

The following table presents the results of unit root tests on the US money supply. Observe that we cannot reject the hypothesis that the growth rate of money supply follows an $I(1)$ process at the 5% size of the test.

Table A1: Unit root tests, US money supply, 1966-2000			
	Dickey-Fuller	Phillips-Perron	5% critical value
M_t	$\text{ADF}_{(c,t)} = -0.77$	$\text{PP}_{(c,t)} = -0.35$	-3.54
ΔM_t	$\text{ADF}_{(c)} = -2.80$	$\text{PP}_{(c)} = -2.72$	-2.95
$\Delta^2 M_t$	$\text{ADF} = -7.40$	$\text{PP} = -7.55$	-1.95

ADF_(c,t), and PP_(c,t) denote the unit root tests with constant and trend.
The lag truncation for Bartlett kernel in the PP tests is three.
The order of augmentation in the ADF tests is one.

Appendix 1b: Alternative Specification of the Money Supply Process

Suppose that money growth μ_t follows a stationary autoregressive process and the monetary authority pursues the following mixed strategy: with probability ρ it follows

$$\mu_t = g + \psi_1 \mu_{t-1} + \varepsilon_t, \quad (9.1)$$

and with probability $(1 - \rho)$ it follows

$$\mu_t = g + \psi_2 \mu_{t-1} + \varepsilon_t, \quad (9.2)$$

where ε_t is white noise, $0 < \psi_1, \psi_2 < 1$, and $\psi_1 < \psi_2$.

Thus the money supply rule is

$$\mu_t = g + \xi \mu_{t-1} + \varepsilon_t, \quad (9.3)$$

where $\xi = \rho \psi_1 + (1 - \rho) \psi_2$.

Consequently the equilibrium nominal wage is given by⁵⁷

$$W_t = (1 - \lambda_1) c + \lambda_1 W_{t-1} + (1 - \lambda_1) M_t + \sigma (1 - \lambda_1) \mu_t + \left(\frac{1 - \lambda_1}{1 - \xi} \right) (\kappa - \sigma) g + \omega_t, \quad (9.4)$$

⁵⁷The algebraic steps in the derivation of W_t are given in Appendix 2.

where⁵⁸

$$\sigma = \frac{\xi}{1-\xi} - \frac{\alpha\xi(\lambda_2-1)}{(\lambda_2-\xi)} - \frac{\xi^2(\lambda_2-1)}{(1-\xi)(\lambda_2-\xi)} > 0. \quad (9.5)$$

The price equation is

$$\begin{aligned} P_t &= (1-\lambda_1)c + \lambda_1 P_{t-1} + (1-\lambda_1)M_t \\ &+ \left(\sigma - \frac{1}{2}\right)(1-\lambda_1)\mu_t + \left(\frac{1-\lambda_1}{1-\xi}\right)(\kappa - \sigma)g \\ &- \frac{1}{2}\sigma(1-\lambda_1)\varepsilon_t + \frac{1}{2}(\omega_t + \omega_{t-1}) \end{aligned} \quad (9.6)$$

The long-run solution of the first difference of above equation gives the long-run inflation rate:

$$\pi_t^{LR} = \mu_t^{LR} = \frac{g}{1-\xi}. \quad (9.7)$$

The real money balances equation is given by

$$\begin{aligned} M_t - P_t &= -(1-\lambda_1)c + \lambda_1(M_{t-1} - P_{t-1}) \\ &+ \left[\frac{1}{2}(1+\lambda_1) - \sigma(1-\lambda_1)\right]\mu_t + \left(\frac{1-\lambda_1}{1-\xi}\right)(\sigma - \kappa)g \\ &+ \frac{1}{2}\sigma(1-\lambda_1)\varepsilon_t - \frac{1}{2}(\omega_t + \omega_{t-1}). \end{aligned} \quad (9.8)$$

The unemployment rate equation is

$$\begin{aligned} u_t &= (1-\lambda_1)c + \lambda_1 u_{t-1} - \left[\frac{1}{2}(1+\lambda_1) - \sigma(1-\lambda_1)\right]\mu_t \\ &- \left(\frac{1-\lambda_1}{1-\xi}\right)(\sigma - \kappa)g - \frac{1}{2}\sigma(1-\lambda_1)\varepsilon_t + \frac{1}{2}(\omega_t + \omega_{t-1}). \end{aligned} \quad (9.9)$$

The long-run unemployment rate is

$$\begin{aligned} u_t^{LR} &= c - \left[\frac{1}{2}\left(\frac{1+\lambda_1}{1-\lambda_1}\right) - \sigma\right]\mu_t - \left(\frac{\sigma - \kappa}{1-\xi}\right)g \\ &= c - \pi_t^{LR} \left(\frac{2\alpha - 1}{\gamma}\right). \end{aligned} \quad (9.10)$$

where the long-run inflation rate is $\pi_t^{LR} = g/(1-\xi)$. Changes in the policy parameters ρ , ψ_1 , and ψ_2 move the economy along this long-run Phillips curve by changing the parameter ξ .

Appendix 2: Theoretical Model and Results

Our model may be summarized as follows:

⁵⁸ $\kappa, \lambda_1, \lambda_2$ are given in Appendix 2.

$$N_t = M_t - P_t, \quad (9.11)$$

$$L_t = 0, \quad (9.12)$$

$$u_t = L_t - N_t = -(M_t - P_t), \quad (9.13)$$

$$\Delta M_t \equiv \mu_t = \mu_{t-1} + \varepsilon_t, \quad (9.14)$$

$$P_t = \frac{1}{2} (W_t + W_{t-1}), \quad (9.15)$$

$$W_t = \alpha W_{t-1} + (1 - \alpha) E_t W_{t+1} + \gamma [c + \alpha N_t + (1 - \alpha) E_t N_{t+1}] + \omega_t, \quad (9.16)$$

2.1: Wage Equation

Substitute (9.11) and (9.15) into (9.16) to get:

$$\begin{aligned} W_t &= \alpha W_{t-1} + (1 - \alpha) E_t W_{t+1} + \gamma \alpha \left[M_t - \frac{1}{2} (W_t + W_{t-1}) \right] \\ &\quad + \gamma (1 - \alpha) \left[E_t M_{t+1} - \frac{1}{2} (E_t W_{t+1} + W_t) \right] + \gamma c + \omega_t. \end{aligned} \quad (9.17)$$

Apply the expectations operator E_t on the above equation, recall that $E_t \omega_t = 0$, collect terms together, so that

$$\begin{aligned} \phi_1 E_t W_{t-1} - \phi_2 E_t W_t + \phi_3 E_t W_{t+1} &= -\gamma [\alpha E_t M_t + (1 - \alpha) E_t M_{t+1}] \\ &\quad - \gamma c, \end{aligned} \quad (9.18)$$

where

$$\phi_1 = \alpha \left(1 - \frac{\gamma}{2} \right), \quad \phi_2 = \left(1 + \frac{\gamma}{2} \right), \quad \phi_3 = (1 - \alpha) \left(1 - \frac{\gamma}{2} \right). \quad (9.19)$$

To obtain the rational expectations solution of the above eq. (9.18), we proceed as follows. Use the backward shift operator B^{59} to rewrite (9.18); then multiply both sides of the resulting equation by B , divide both sides by ϕ_3 , and use $E_t W_t$ as a common factor on the L.H.S.:

$$\left(1 - \frac{\phi_2}{\phi_3} B + \frac{\phi_1}{\phi_3} B^2 \right) E_t W_t = \frac{-B (E_t A_t) - \gamma c}{\phi_3}, \quad (9.20)$$

where

$$E_t A_t = \gamma [\alpha E_t M_t + (1 - \alpha) E_t M_{t+1}]. \quad (9.21)$$

The B polynomial in (9.20) can be expressed as

$$\left(1 - \frac{\phi_2}{\phi_3} B + \frac{\phi_1}{\phi_3} B^2 \right) = (1 - \lambda_1 B) (1 - \lambda_2 B), \quad (9.22)$$

⁵⁹Note that B^1 shifts the variable backward, where B^{-1} shifts the variable forward, i.e.

$$B [E_t W_t] = E_t W_{t-1}, \quad \text{and} \quad B^{-1} [E_t W_t] = E_t W_{t+1},$$

where E_t is in all cases the conditional expectation as of period t .

where $\lambda_{1,2}$ are the roots of the equation

$$\lambda^2 - \frac{\phi_2}{\phi_3}\lambda + \frac{\phi_1}{\phi_3} = 0,$$

i.e.

$$\lambda_{1,2} = \frac{\frac{\phi_2}{\phi_3} \mp \sqrt{\left(\frac{\phi_2}{\phi_3}\right)^2 - 4\left(\frac{\phi_1}{\phi_3}\right)}}{2}, \text{ so} \quad (9.23)$$

$$\lambda_1 + \lambda_2 = \frac{\phi_2}{\phi_3}, \text{ and } \lambda_1\lambda_2 = \frac{\phi_1}{\phi_3} \Rightarrow \lambda_2 = \frac{\alpha}{\lambda_1(1-\alpha)}.$$

It can be shown that one root lies inside the unit circle and the other outside the unit circle. In particular, we can show that when $0 < \gamma < 2$ then $0 < \lambda_1 < 1$ and $\lambda_2 > 1$.

We can rewrite (9.20) using (9.22) as

$$(1 - \lambda_1 B) E_t W_t = \frac{\gamma c}{\phi_3 (\lambda_2 - 1)} - \frac{B(E_t A_t)}{\phi_3 (1 - \lambda_2 B)}. \quad (9.24)$$

Since $|\lambda_2| > 1$, a useful way to express the geometric polynomial $1/(1 - \lambda_2 B)$ is as follows:⁶⁰

$$\frac{1}{1 - \lambda_2 B} = \frac{-(\lambda_2 B)^{-1}}{1 - (\lambda_2 B)^{-1}}.$$

Substitute the above into (9.24) to get:

$$(1 - \lambda_1 B) E_t W_t = \frac{\gamma c}{\phi_3 (\lambda_2 - 1)} + \frac{E_t A_t}{\lambda_2 \phi_3 (1 - \lambda_2^{-1} B^{-1})} \quad (9.25)$$

$$= \frac{\gamma c}{\phi_3 (\lambda_2 - 1)} + \frac{1}{\lambda_2 \phi_3} \sum_{j=0}^{\infty} \left(\frac{1}{\lambda_2}\right)^j E_t A_{t+j},$$

or, using (9.21) and (9.14),

$$(1 - \lambda_1 B) E_t W_t = \frac{\gamma c}{\phi_3 (\lambda_2 - 1)} + \frac{\gamma}{\lambda_2 \phi_3} \sum_{j=0}^{\infty} \left(\frac{1}{\lambda_2}\right)^j (E_t M_{t+1+j} - \alpha E_t \mu_{t+1+j}). \quad (9.26)$$

Further algebraic manipulation leads to

$$(1 - \lambda_1 B) E_t W_t = \frac{\gamma c}{\phi_3 (\lambda_2 - 1)} + \frac{\gamma}{\lambda_2 \phi_3} \left[\frac{\lambda_2 M_t}{\lambda_2 - 1} - \frac{\alpha \lambda_2 \mu_t}{\lambda_2 - 1} + \frac{\lambda_2^2 \mu_t}{(\lambda_2 - 1)^2} \right]$$

$$= (1 - \lambda_1) [c + M_t + \kappa \mu_t],$$

⁶⁰See Sargent (1987).

where⁶¹

$$\kappa = \frac{\lambda_2}{\lambda_2 - 1} - \alpha = \frac{\alpha(1 + \lambda_1)(1 - \alpha)}{\alpha - \lambda_1(1 - \alpha)}. \quad (9.27)$$

(It can be shown that $\kappa > 0$.) So we have

$$(1 - \lambda_1 B) E_t W_t = (1 - \lambda_1) c + (1 - \lambda_1) M_t + \kappa(1 - \lambda_1) \mu_t.$$

A comparison of the above eq. with (9.16) indicates that the rational expectations reduced-form stochastic difference equation for the wage is⁶²

$$W_t = (1 - \lambda_1) c + \lambda_1 W_{t-1} + (1 - \lambda_1) M_t + \kappa(1 - \lambda_1) \mu_t + \omega_t. \quad (9.28)$$

Note that the above is the wage equation given in the text. (In the text the stable root λ_1 is denoted by λ for simplicity.)

2.2: Price Equation

To derive the equation for the price dynamics rewrite the price equation (9.15) as follows:

$$(1 - \lambda_1 B) P_t = \frac{1}{2} (1 - \lambda_1 B) W_t + \frac{1}{2} (1 - \lambda_1 B) W_{t-1},$$

and substitute into it the wage equation (9.28). In the resulting equation, substitute the following expressions (implied by the money supply process (9.14)):

$$M_{t-1} = M_t - \mu_t, \text{ and } \mu_{t-1} = \mu_t - \varepsilon_t.$$

Next, collect terms together to get the price equation given in the text:⁶³

$$\begin{aligned} (1 - \lambda_1 B) P_t &= (1 - \lambda_1) c + (1 - \lambda_1) M_t + (1 - \lambda_1) \left(\kappa - \frac{1}{2} \right) \mu_t \\ &\quad - \frac{1}{2} \kappa (1 - \lambda_1) \varepsilon_t + \frac{1}{2} (\omega_t + \omega_{t-1}). \end{aligned} \quad (9.29)$$

2.3: Inflation Rate Equation

⁶¹Note that

$$(\lambda_2 - 1)(1 - \lambda_1) = \frac{\gamma}{\phi_3},$$

so

$$\frac{\gamma}{\phi_3(\lambda_2 - 1)} = (1 - \lambda_1).$$

⁶²For the solution of linear difference equations under rational expectations see also Blanchard and Kahn (1980), and Sargent (1987).

⁶³Note that $\kappa > \frac{1}{2}$ if $\frac{2\alpha+1}{2\alpha-1} > \lambda_2$.

Let the inflation rate be $\pi_t \equiv \Delta P_t$, and take the first difference of the price dynamics eq. (9.29) to obtain the inflation dynamics equation:

$$(1 - \lambda_1 B) \pi_t = (1 - \lambda_1) \mu_t + \frac{1}{2} (1 - \lambda_1) (\kappa - 1) \varepsilon_t + \frac{1}{2} \kappa (1 - \lambda_1) \varepsilon_{t-1} + \frac{1}{2} (\omega_t - \omega_{t-2}). \quad (9.30)$$

2.4: Real Money Balances, and Unemployment

To obtain the real money balances equation we do the following. Add and subtract on the R.H.S. of the price equation (9.29) the term $\lambda_1 M_{t-1}$, and then rearrange terms so that

$$(1 - \lambda_1 B) (M_t - P_t) = \left[\frac{1}{2} (1 + \lambda_1) - \kappa (1 - \lambda_1) \right] \mu_t + \frac{1}{2} \kappa (1 - \lambda_1) \varepsilon_t - \frac{1}{2} (\omega_t + \omega_{t-1}) - (1 - \lambda_1) c. \quad (9.31)$$

Note that

$$\left[\frac{1}{2} (1 + \lambda_1) - \kappa (1 - \lambda_1) \right] = (1 - \lambda_1) \left(\frac{2\alpha - 1}{\gamma} \right). \quad (9.32)$$

Thus we obtain the real money balances equation given in the text:

$$(1 - \lambda_1 B) (M_t - P_t) = - (1 - \lambda_1) c + (1 - \lambda_1) \left(\frac{2\alpha - 1}{\gamma} \right) \mu_t + \frac{1}{2} \kappa (1 - \lambda_1) \varepsilon_t - \frac{1}{2} (\omega_t + \omega_{t-1}). \quad (9.33)$$

Rewrite the unemployment equation (9.13) as

$$(1 - \lambda_1 B) u_t = - (1 - \lambda_1 B) (M_t - P_t).$$

To obtain the dynamics for aggregate demand, substitute into the above equation the real money balances equation (9.33):

$$(1 - \lambda_1 B) u_t = (1 - \lambda_1) c - (1 - \lambda_1) \left(\frac{2\alpha - 1}{\gamma} \right) \mu_t - \frac{1}{2} \kappa (1 - \lambda_1) \varepsilon_t + \frac{1}{2} (\omega_t + \omega_{t-1}). \quad (9.34)$$

2.5: Short-Run Phillips Curve

Rewrite the unemployment eq. (9.34) and inflation eq. (9.30) as

$$(1 - \lambda_1 B) u_t = (1 - \lambda_1) c - \beta_1 \mu_t - \beta_2 \varepsilon_t + \frac{1}{2} (\omega_t + \omega_{t-1}), \quad (9.35)$$

$$(1 - \lambda_1 B) \pi_t = \delta_1 \mu_t + \delta_2 \varepsilon_t + \beta_2 \varepsilon_{t-1} + \frac{1}{2} (\omega_t - \omega_{t-2}), \quad (9.36)$$

where

$$\begin{aligned}\beta_1 &= (1 - \lambda_1) \left(\frac{2\alpha - 1}{\gamma} \right), \quad \beta_2 = \frac{1}{2} \kappa (1 - \lambda_1), \\ \delta_1 &= 1 - \lambda_1, \quad \delta_2 = \frac{1}{2} (1 - \lambda_1) (\kappa - 1).\end{aligned}$$

Now substitute the money supply eq. (9.14): $(1 - B) \mu_t = \varepsilon_t$ into (9.35) and (9.36) to get

$$(1 - \lambda_1 B) u_t = (1 - \lambda_1) c - \beta_1 \mu_t - \beta_2 (1 - B) \mu_t + \frac{1}{2} (\omega_t + \omega_{t-1}), \quad (9.37)$$

$$(1 - \lambda_1 B) \pi_t = \delta_1 \mu_t + \delta_2 (1 - B) \mu_t + \beta_2 (B - B^2) \mu_t + \frac{1}{2} (\omega_t - \omega_{t-2}). \quad (9.38)$$

Express the (9.37) in terms of μ_t :

$$\mu_t = \frac{(1 - \lambda_1 B) u_t - (1 - \lambda_1) c - \frac{1}{2} (\omega_t + \omega_{t-1})}{\beta(B)}, \quad (9.39)$$

where $\beta(B) = [-(\beta_1 + \beta_2) + \beta_2 B]$.

Substitution of (9.39) into (9.38) leads to the short-run Phillips curve

$$\begin{aligned}(1 - \lambda_1 B) \beta(B) \pi_t &= (1 - \lambda_1 B) \delta(B) u_t - \delta(B) (1 - \lambda_1) c \\ &\quad + \frac{\beta(B) (\omega_t - \omega_{t-2}) - \delta(B) (\omega_t + \omega_{t-1})}{2}, \text{ or} \\ \beta(B) \pi_t &= \delta(B) u_t - \delta_1 c + \frac{\beta(B) (\omega_t - \omega_{t-2}) - \delta(B) (\omega_t + \omega_{t-1})}{2(1 - \lambda_1 B)},\end{aligned}$$

where $\delta(B) = [(\delta_1 + \delta_2) + (\beta_2 - \delta_2) B - \beta_2 B^2]$.

After some algebraic manipulation, the above short-run Phillips curve can be written as

$$\pi_t = \frac{1}{\beta_1 + \beta_2} [(1 - \lambda_1) c + \beta_2 \pi_{t-1} - (\delta_1 + \delta_2) u_t - (\beta_2 - \delta_2) u_{t-1} + \beta_2 u_{t-2}] + e_t,$$

where

$$e_t = \frac{\tilde{\omega}_t}{(1 - \lambda_1 B)}, \quad \text{and} \quad \tilde{\omega}_t = \frac{\delta(B) (\omega_t + \omega_{t-1}) - \beta(B) (\omega_t - \omega_{t-2})}{2(\beta_1 + \beta_2)}.$$

Through some algebraic manipulation we get:

$$\begin{aligned}\pi_t &= \frac{1}{\beta_1 + \beta_2} [(1 - \lambda_1) c + \beta_2 \pi_{t-1} - (\delta_1 + \delta_2) u_t - (\beta_2 - \delta_2) u_{t-1} + \beta_2 u_{t-2}] + e_t \\ &= \left(\frac{1 - \lambda_1}{\beta_1 + \beta_2} \right) \left[c + \frac{1}{2} \kappa \pi_{t-1} - \frac{1}{2} (1 + \kappa) u_t - \frac{1}{2} u_{t-1} + \frac{1}{2} \kappa u_{t-2} \right] + e_t \\ &= \psi \left[c + \frac{1}{2} \kappa \pi_{t-1} - \frac{1}{2} (1 + \kappa) u_t - \frac{1}{2} u_{t-1} + \frac{1}{2} \kappa u_{t-2} \right] + e_t,\end{aligned} \quad (9.40)$$

where⁶⁴ $\psi = \frac{1-\lambda_1}{\beta_1+\beta_2}$. In addition, the error term can be written as

$$e_t = \frac{\tilde{\omega}_t}{(1-\lambda_1 B)}, \quad \tilde{\omega}_t = \frac{1}{2} \left[\left(1 + \frac{\psi(1+\kappa)}{2} \right) \omega_t + \frac{3\psi}{2} \omega_{t-1} - \left(1 + \frac{\psi(\kappa-1)}{2} \right) \omega_{t-2} \right]. \quad (9.41)$$

Note that the above error term is a moving average process in terms of ω_t , with parameters which are non-linear functions of the theoretical parameters ψ , κ , and λ_1 .⁶⁵

Express equation (9.40) as

$$\pi_t = d_0 + d_1 \pi_{t-1} - d_2 u_t - d_3 u_{t-1} + d_4 u_{t-2} + e_t, \quad (9.42)$$

where

$$d_0 = \psi c, \quad d_1 = \frac{\psi \kappa}{2}, \quad d_2 = \frac{\psi(1+\kappa)}{2}, \quad d_3 = \frac{\psi}{2}, \quad d_4 = \frac{\psi \kappa}{2}.$$

Thus we have the following relationships among the d 's:

$$d_4 = d_1, \quad \text{and} \quad d_3 = d_2 - d_1. \quad (9.43)$$

Alternatively, (9.42) can be written as

$$(1 - \lambda_1 B)(1 - d_1 B) \pi_t = d_0(1 - \lambda_1) - (1 - \lambda_1 B)(d_2 + d_3 B - d_4 B^2) u_t + \tilde{\omega}_t. \quad (9.44)$$

Recall that both the autoregressive and moving average parameters in the above Phillips curve equation are functions of the only two theoretical parameters α and γ . It is useful to have an overall picture of all the relationships among the parameters of the above short-run PC:

$$\begin{aligned} d_0 &= \psi c, \quad d_1 = \frac{\psi \kappa}{2}, \quad d_2 = \frac{\psi(1+\kappa)}{2}, \quad d_3 = \frac{\psi}{2}, \quad d_4 = \frac{\psi \kappa}{2}, \\ \psi &= \frac{2\alpha - 1}{\gamma} + \frac{\kappa}{2}, \quad \kappa = \frac{\alpha(1+\lambda_1)}{\frac{\alpha}{1-\alpha} - \lambda_1}, \quad \lambda_1 = \frac{\frac{\phi_2}{\phi_3} - \sqrt{\left(\frac{\phi_2}{\phi_3}\right)^2 - 4\left(\frac{\phi_1}{\phi_3}\right)}}{2}, \\ \phi_1 &= \alpha \left(1 - \frac{\gamma}{2} \right), \quad \phi_2 = \left(1 + \frac{\gamma}{2} \right), \quad \phi_3 = (1 - \alpha) \left(1 - \frac{\gamma}{2} \right), \quad \text{and} \\ \tilde{\omega}_t &= \frac{1}{2} \left[\left(1 + \frac{\psi(1+\kappa)}{2} \right) \omega_t + \frac{3\psi}{2} \omega_{t-1} - \left(1 + \frac{\psi(\kappa-1)}{2} \right) \omega_{t-2} \right]. \end{aligned}$$

The above equations make it clear that the restrictions we need to impose on the parameters of the short-run PC (9.44) are highly complicated non-linear functions of the theoretical underlying parameters α and γ . Therefore, (9.44) may not be estimable.

2.6: Long-Run Unemployment, Inflation, and the Phillips Curve

To get the long-run solution of the unemployment equation (9.34) we set the backshift operator equal to unity ($B = 1$), and set equal to zero all the error terms (ε 's, ω 's). This gives

⁶⁴Note that $\frac{\beta_1+\beta_2}{1-\lambda_1} = \frac{2\alpha-1}{\gamma} + \frac{\kappa}{2}$.

⁶⁵Recall that ψ , κ , and λ_1 are non-linear functions of the theoretical parameters α and γ of the wage contract equation.

us the following long-run:

$$u_t^{LR} = - \left(\frac{2\alpha - 1}{\gamma} \right) \mu_t^{LR} + c. \quad (9.45)$$

Similarly, the long-run solution of the inflation equation (9.30) is given by

$$\pi_t^{LR} = \mu_t^{LR}. \quad (9.46)$$

To get the long-run Phillips curve we need to substitute (9.46) into (9.45):

$$\pi_t^{LR} = - \left(\frac{\gamma}{2\alpha - 1} \right) u_t^{LR} + \left(\frac{\gamma}{2\alpha - 1} \right) c. \quad (9.47)$$

2.7: Short-Run vs Long-Run Phillips Curve

The slope of the short-run Phillips curve (9.40) is

$$\frac{\partial \pi_t}{\partial u_t} = - \frac{\delta_1 + \delta_2}{\beta_1 + \beta_2} = - \frac{\gamma + \gamma\kappa}{2(2\alpha - 1) + \gamma\kappa}, \quad (9.48)$$

whereas the slope of the long-run Phillips curve (6.3) is

$$\frac{\partial \pi_t^{LR}}{\partial u_t^{LR}} = - \frac{\gamma}{2\alpha - 1}. \quad (9.49)$$

It can be shown that if the (absolute value of the) long-run slope is greater than unity then

$$\left| \frac{\partial \pi_t^{LR}}{\partial u_t^{LR}} \right| > \left| \frac{\partial \pi_t}{\partial u_t} \right|,$$

i.e. the long-run PC is steeper than the short run PC.⁶⁶

The intercept of the short-run Phillips curve (9.40) is

$$\left(\frac{1 - \lambda_1}{\beta_1 + \beta_2} \right) c = \left(\frac{2\gamma}{2(2\alpha - 1) + \gamma\kappa} \right) c > 0, \quad (9.50)$$

⁶⁶This can be shown as follows:

$$\begin{aligned} \left| \frac{\partial \pi_t^{LR}}{\partial u_t^{LR}} \right| > \left| \frac{\partial \pi_t}{\partial u_t} \right| &\Rightarrow \\ \frac{\gamma}{2\alpha - 1} > \frac{\gamma + \gamma\kappa}{2(2\alpha - 1) + \gamma\kappa} &\Rightarrow \\ \gamma(2(2\alpha - 1) + \gamma\kappa) > (2\alpha - 1)(\gamma + \gamma\kappa) &\Rightarrow \\ \gamma((2\alpha - 1) + \gamma\kappa) > (2\alpha - 1)\gamma\kappa &\Rightarrow \\ \frac{\gamma}{2\alpha - 1} > \frac{\gamma\kappa}{(2\alpha - 1) + \gamma\kappa}. \end{aligned}$$

Since the smallest value that α is assumed to take is one half, it follows that the maximum value of right-hand side of the above inequality is unity. Therefore, we can say that a *sufficient* (but not *necessary*) condition for $\left| \frac{\partial \pi_t^{LR}}{\partial u_t^{LR}} \right| > \left| \frac{\partial \pi_t}{\partial u_t} \right|$ is that $\left| \frac{\partial \pi_t^{LR}}{\partial u_t^{LR}} \right| > 1$.

and the intercept of the long-run Phillips curve (9.47) is

$$\left(\frac{\gamma}{2\alpha - 1}\right) c > 0. \quad (9.51)$$

Since both γ and κ are positive, it is not difficult to see that the intercept of the long-run PC is greater than the intercept of the short-run PC:

$$\left(\frac{\gamma}{2\alpha - 1}\right) c > \left(\frac{2\gamma}{2(2\alpha - 1) + \gamma\kappa}\right) c.$$

Appendix 3: OLS Estimates of the Unemployment, Price, and Wage Equations

Table A2: Unemployment equation, OLS, 1966-2000.

Dependent variable: u_t		Misspecification tests*	
	coefficient	s.e.	
u_{t-1}	0.45	(0.14)	SC $[\chi^2(1)]$ 1.51 [0.22]
u_{t-2}	-0.31	(0.13)	LIN $[\chi^2(1)]$ 1.77 [0.18]
m_t	-0.12	(0.04)	NOR $[\chi^2(1)]$ 0.84 [0.66]
η_t	-0.14	(0.06)	ARCH $[\chi^2(1)]$ 0.11 [0.74]
Δk_t	-0.01	(0.002)	HET $[\chi^2(16)]$ 13.9 [0.61]
o_{t-1}	0.01	(0.003)	CUSUM ✓
f_t	-0.01	(0.005)	CUSUMSQ ✓
c_t	0.04	(0.02)	

+ LL=137.77, AIC=-7.36, SC=-6.96

* Probabilities in square brackets
 ✓ Structural stability cannot be rejected at the 5% size of the test
 + Log likelihood (LL), Akaike (AIC) and Schwarz (SC) criteria

Table A3: Price equation, OLS, 1966-2000.

Dependent variable: P_t				
	coefficient	s.e.	Misspecification tests*	
P_{t-1}	0.91	(0.20)	SC[$F(1, 23)$]	7.76 [0.01]
P_{t-2}	-0.37	(0.13)	LIN[$\chi^2(1)$]	2.78 [0.10]
W_{t-1}	0.32	(0.11)	NOR[$\chi^2(2)$]	0.01 [0.99]
M_t	0.05	(0.03)	ARCH[$\chi^2(1)$]	0.00 [0.99]
u_t	-0.65	(0.18)	HET[$\chi^2(22)$]	30.0 [0.12]
θ_t	-0.53	(0.14)	CUSUM	✓
o_t	0.017	(0.005)	CUSUMSQ	✓
o_{t-1}	0.015	(0.006)		
o_{t-2}	-0.006	(0.004)		
τ_t	0.001	(0.007)		

+ LL=141.63, AIC=-7.41, SC=-6.87
++ [$F(1, 23)$] = 4.21 [0.05]

* Probabilities in square brackets
✓ Structural stability cannot be rejected at the 5% size of the test
+ Log likelihood (LL), Akaike (AIC) and Schwarz (SC) criteria
++ Wald test for long-run no money illusion

Table A4: Wage equation, OLS, 1966-2000.

Dependent variable: W_t				
	coefficient	s. e.	Misspecification tests*	
W_{t-1}	0.19	(0.11)	SC[$\chi^2(1)$]	3.04 [0.08]
ΔW_{t-2}	0.47	(0.12)	LIN[$\chi^2(1)$]	1.10 [0.29]
P_t	0.73	(0.12)	NOR[$\chi^2(2)$]	1.76 [0.42]
M_t	0.08	(0.03)	ARCH[$\chi^2(1)$]	0.06 [0.80]
u_t	-0.41	(0.21)	HET[$\chi^2(14)$]	15.1 [0.37]
θ_t	0.35	(0.10)	CUSUM	✓
b_t	0.05	(0.02)	CUSUMSQ	✓

+ LL=127.54, AIC=-6.83, SC=-6.48
++ [$F(1, 27)$] = 0.07 [0.80]

* Probabilities in square brackets
✓ Structural stability cannot be rejected at the 5% size of the test
+ Log likelihood (LL), Akaike (AIC) and Schwarz (SC) criteria
++ Wald test for long-run no money illusion

Appendix 4: Actual and Fitted Values of the Estimated System



Appendix 5: Further Evidence on Whether the Long-Run Phillips Curve is Vertical

In the following table we present the percentage count of slopes within specific class intervals. For example, the probability that the long-run Phillips curve slope lies in the interval $(-6, -1.5)$ is 89%.

Table A5: probability that the PC slope is within a specific interval			
<i>Slope interval</i>	$(-\infty, -6)$	$(-6, -1.5)$	$(-1.5, \infty)$
<i>Probability</i>	10.4 %	89.0 %	0.6 %

We also grouped the values of the generated series $S^{(i)}$, $i = 1, 2, \dots, 1000$, into class intervals of 0.5 units. Using as a cut-off point a 10% count, there is no class interval below $[-4.5, -4.0)$ or above $[-2.5, -2.0)$ that contains at least 10% of the values of slope series S . These class intervals and their respective probabilities are given in the table below.

Table A6: Monte Carlo simulations, 1000 replications class intervals with a count above 10%					
<i>Slope interval</i>	$[-4.5, -4.0)$	$[-4.0, -3.5)$	$[-3.5, -3.0)$	$[-3.0, -2.5)$	$[-2.5, -2.0)$
<i>Probability</i>	11.1 %	14.3 %	18.0 %	12.8 %	11.9 %