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by

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Heterogeneous bids in auctions with rational and markdown bidders—Theory and Experiment*

Oliver Kirchkamp[†] J. Philipp Reiß[‡]

25 August 2008

Abstract

We present results from a series of experiments that allow us to measure overbidding and, in particular, underbidding in first-price auctions. We investigate how the amount of underbidding depends on seemingly innocent parameters of the experimental setup.

To structure our data we present and test a theory of constant markdown bids. While a fraction of bidders can be well described by Bayesian Nash equilibrium bids, a larger fraction seems to either use constant markdown bids or seems to rationally optimise against a population where some members use markdown bids and some are rational.

Keywords: Auction, Experiment, Overbidding, Underbidding, Risk-Aversion.

(JEL C92, D44)

1 Introduction

In this paper we present and use a method that allows us to observe overbidding and underbidding in first-price auctions. Using this method we can gain more insight into the bidding heuristics that are used by boundedly rational bidders.

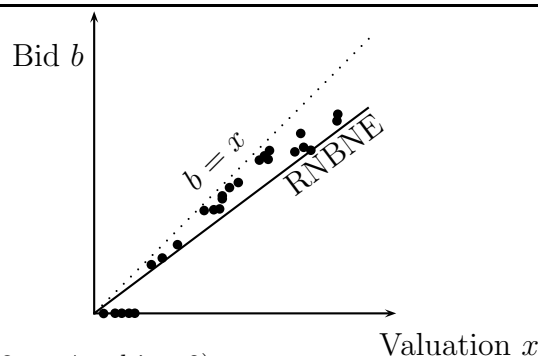
We will see that these heuristics can depend on seemingly innocent parameters of the experimental setup. To organise the data we propose constant markdown bids. In our analysis we consider a heterogeneous population of rational bidders and (boundedly rational) markdown bidders.

We concentrate on first-price auctions. First-price auctions are common and simple institutions. Due to their simplicity they are a useful and well researched workhorse

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Figure 1 An example from an experiment by Cox, Smith, and Walker (1988)

(page 83, figure 8, series 4, exp. 3, $n=4$, subject 2)

in studying general bidding behaviour in auctions. Any theory that aims at explaining bidding behaviour in more complicated auctions should, in particular, perform well in a very simple context.

Let us first have a look at a classic series of experiments with first-price auctions presented by Cox, Smith, and Walker (1983, 1985, 1988). Figure 1 shows data from one of their experiments. Participants repeatedly play a first-price auction with a fixed number of bidders. For each participant valuations are drawn from a uniform distribution. The figure shows normalised valuations¹ that were drawn for a specific bidder on the horizontal axis. Bids are shown on the vertical axis.

The solid line shows the risk neutral Bayesian Nash equilibrium (RNBNE) bidding function for this auction. We see that most bids exceed the risk neutral equilibrium bidding function. This is what we call overbidding and what, since then (see again Cox, Smith, and Walker, 1983, 1985, 1988), was observed in many first-price auction experiments.

Risk aversion is a standard explanation for overbidding in first-price auctions. However, risk aversion does not explain all deviations from risk neutral equilibrium bidding. Cox, Smith, and Walker (1985) find overbidding even in experiments where subjects are paid with lottery tickets—i.e. where subjects with a von Neumann-Morgenstern utility would behave in a risk neutral way. Even worse, in experiments with third-price auctions, risk averse bidders should bid less and not more than the risk neutral equilibrium bid. Kagel and Levin (1993) find, nevertheless, overbidding in experiments with third price auctions.

As an alternative to risk, some studies use the first-price auction to study how regret (see Filiz-Ozbay and Ozbay, 2007 and Engelbrecht-Wiggans and Katok, 2007) and spite (see Morgan, Steiglitz, and Reis, 2003) affect bids. Other studies relax the requirements of consistent expectations (see Eyster and Rabin, 2005 and Crawford and Iriberri, 2007) or correct best replies (see Goeree, Holt, and Palfrey, 2002 and Ockenfels and Selten, 2005). All these approaches give different explanations why bids in the experiment exceed RNBNE bids. But let us have a look at the figure again. For low valuations many bids

¹In this experiment the smallest possible valuation was \$0 or \$0.10 and the largest possible valuation ranges from \$4.90-\$36.10. In the figure the valuations are normalised to $[0, 1]$. Bids that are shown in the figure are normalised correspondingly.

are below, not above, the equilibrium bid. If bidders attach any utility to money these bids cannot be part of an equilibrium. Still, Cox, Smith, and Walker (1988) find in some cases negative intercepts when they approximate bids by linear bidding functions. Further Ivanova-Stenzel and Sonsino (2004) report that 7.4% of the bids in their first-price auction experiments are below the lowest possible valuation.

Despite these findings, underbidding does not receive much attention in the experimental literature. One reason might be that underbidding is often ruled out implicitly or explicitly through the design of the experiment. Choosing zero as the smallest possible valuation looks like an innocent simplification. In this paper we will show that this simplification is not innocent at all. A second reason might be that bids for small valuations are not easy to observe precisely. In this paper we will use a variant of the strategy method that makes it easier to observe these bids.

One aim of this study is methodological: we will compare different ranges for valuations and different restrictions on the lowest possible bid. We will see that innocent changes in these parameters may have a significant impact on bidding behaviour. In that respect we hope that our study might help in the design of future experiments with first-price auctions but also with other institutions.

A second aim is to present a simple theory that organises our data. With our approach we have the possibility to disentangle effects like risk aversion, spite, etc., from limits in the rationality of the bidders. We know already from other experiments that we should expect some heterogeneity among bidders. Finding one model or one bidding function with a fixed functional form that describes all bids reasonably well might not be possible.

Instead, we are proposing three different types of bidders which can be linked to different degrees of rationality. We know from experiments with other games that decision makers apply different levels of reasoning when choosing a strategy in a game (see Bosch-Domenech, Montalvo, Nagel, and Satorra, 2002). In the context of first-price auctions we should mention Crawford and Iriberri (2007) who analyse bidding under different degrees of rationality. In our paper we suggest a specific starting point for such a sequence of different levels of rationality: absolute markdown bids.

Section 2.1 presents the model of an extreme case: a rational bidder who assumes that the opponent is rational, too. In section 2.2 we will look at the opposite extreme: a bidder who is restricted to use absolute markdown bids and assumes that the opponent follows the same restriction. The idea of boundedly rational bidders is not new. Kagel, Harstad, and Levin (1987) have used a more flexible form of markdown bids in the context of affiliated private value auctions and found some explanatory power. In contrast, we introduce an equilibrium foundation for absolute markdown bids that also accommodates heterogeneous bidding behaviour with perfectly rational bidders along with bounded rational markdown bidders. Chen and Plott (1998) also compare several variants of markdown bids with Bayesian Nash equilibrium bids when bidders exhibit constant relative risk aversion (CRRA). They find that CRRA provides a more accurate model than their variants of markdown bids. We do not deny such a possibility. If all bidders must fit a single type of bidding function, then CRRA might be a good choice. In our experiment we want to examine whether some bidders systematically do something else. As a natural next step, we will consider a bidder who is rational but assumes to meet a mix of rational and restricted opponents in section 2.4.

Section 3 then describes the experiment and section 4 presents the results. Section 5 concludes. Anticipating our result we will be able to classify bidders into these three groups that we outlined above. If the experimental setup rules out underbidding then these groups are indistinguishable. Bids can be well explained by established theories, e.g. CRRA. Once underbidding is possible in the experiment, some decision makers continue to bid in line with CRRA, but the majority of decision makers follow a quite different bidding pattern.

2 The theoretical framework

In this section we will derive optimal bidding functions for three different contexts. We will concentrate on a first-price sealed-bid auction with private valuations and two bidders. For comparison we will also refer to the case of a second-price sealed-bid auction.

2.1 Bayesian Nash Equilibrium Bids

Deriving the Bayesian Nash Equilibrium for the first-price sealed bid auction is standard and is repeated here to introduce the notation. Consider the case where valuations are distributed uniformly over $[0, 1]$, bidders have constant relative risk aversion (CRRA), i.e. utility is given by $u(x) = x^r$ where r is a parameter of risk tolerance. A risk neutral individual is described by $r = 1$, a risk averse individual has $r < 1$. We will concentrate on the case $r \in (0, 1]$. Let us assume that bidders use a symmetric increasing bidding function $\gamma_I(x)$ with inverse $\gamma_I^{-1}(\cdot)$. In equilibrium all bidders bid according to γ_I . Bidder 1 has valuation x and bids b . Hence, bidder 1 wins the auction if the valuation of bidder 2 is smaller than $z = \gamma_I^{-1}(b)$ which occurs with probability $G(z) \equiv z$. Bidder 1 chooses z to maximise $EU = G(z) \cdot u(x - \gamma_I(z))$ which yields the following first order condition:

$$(x - \gamma_I(z))^r - r \cdot z \cdot (x - \gamma_I(z))^{r-1} \gamma_I'(z) = 0 \quad (1)$$

In the symmetric equilibrium we have $z = x$:

$$(x - \gamma_I(x))^{r-1} \cdot (\gamma_I(x) + x \cdot (r \gamma_I'(x) - 1)) = 0 \quad (2)$$

It is easy to see that in equilibrium $\gamma_I(0) = 0$ which yields the unique solution

$$\gamma_I^*(x) = \frac{x}{1+r}. \quad (3)$$

The second derivative $\partial^2 EU / \partial z^2 = -(rx/(1+r))^{r-1}$ is negative, so we have indeed found a maximum. If valuations are drawn from the interval $[\underline{\omega}, \overline{\omega}]$ instead of $[0, 1]$, one finds similarly that the equilibrium bid is

$$\gamma_I^*(x) - \underline{\omega} = \frac{x - \underline{\omega}}{1+r}. \quad (4)$$

We see that the more risk averse bidders are (the smaller the value of r) the larger is γ_I^* . We also see that for bidders who are not infinitely risk-averse $\gamma_I^*(x) < x$. Bidders “shade their bids” by a fraction of $x - \underline{\omega}$.

For second-price sealed-bid auctions a standard argument shows that it is a weakly dominant strategy to submit a bid equal to valuation x :

$$\gamma_{II}^*(x) = x \quad (5)$$

This holds independently of the degree of risk aversion.

2.2 Equilibrium with markdown bids

In the Bayesian Nash Equilibrium of the first-price auction bidders ‘shade their bids’ proportional to $x - \underline{v}$. This is perhaps not what all bidders do in an experiment. E.g., in a post-experimental questionnaire of another first-price experiment² some participants explained they shade their valuation not by a relative but by a constant amount. Shading by a constant amount might have various reasons:

- It might be difficult to work out the exact form of equation 4. Participants quickly understand that the bid must be somewhat lower than the valuation hence they resort to finding a constant by trial and error.
- Shading by a constant amount could be related to satisficing behaviour. A bidder who wants to gain a certain amount if winning the auction must bid the own valuation minus this amount.
- Shading by a constant amount can also be interpreted as a simple rule given to a bidding agent. If first a principal has to define a bidding rule (before the individual valuation is revealed) and then the agent who follows this rule learns the valuation, then it might be simpler for the principal to require a fixed amount that the agent must gain from each trade.

To formalise this type of bidding behaviour, consider a population of bidders which is restricted to use a bidding function of the following type:

$$\bar{\gamma}_i(x) = \alpha_i + \beta x \quad (6)$$

where β is exogenously given, positive³, and the same for all bidders. We will specifically study the case $\beta = 1$ and refer to it as absolute markdown bids in the following. While β is fixed, bidders can choose different values of α . In the following we consider the equilibrium of the game where both bidders choose their α_i , given a fixed and exogenous β .

Given any two valuations x_i and x_j of two bidders, bidder i wins the auction if $x_j < x_i + (\alpha_i - \alpha_j)/\beta$. If $\alpha_i \leq \alpha_j$ the expected utility for a risk neutral bidder is

$$EU_1 = \int_{\frac{\alpha_j - \alpha_i}{\beta}}^1 \int_0^{x_i - \frac{\alpha_j - \alpha_i}{\beta}} x_i - (\alpha_i + \beta x_j) dx_j dx_i. \quad (7)$$

²Kirchkamp, Poen, and Reiß (forthcoming).

³One can show that the solution we obtain below in equation (9) extends to the case $\beta = 0$.

The first order condition yields

$$\alpha_i^* = \frac{\alpha_j + \beta - 2\beta^2}{1 + 2\beta}. \quad (8)$$

This condition is fulfilled for both players if

$$\alpha_i = \alpha_j = \frac{1}{2} - \beta. \quad (9)$$

In the same way we can study the case $\alpha_i > \alpha_j$ which yields the same condition. The risk neutral Bayesian Nash Equilibrium is a special case ($\alpha = 0, \beta = 1/2$) of equation (9). Equation (7) can be generalised to also account for risk aversion:

$$EU_2 = \int_{\frac{\alpha_j - \alpha_i}{\beta}}^1 \int_0^{x_i - \frac{\alpha_j - \alpha_i}{\beta}} (x_i - (\alpha_i + \beta x_i))^r dx_j dx_i \quad (10)$$

We can follow the same steps as above, however, for general β it is not possible to obtain a closed form solution for α_i as we could with risk neutrality in equation (9). For $\beta = 1$, i.e. the case of absolute markdown bids, which we find particularly interesting, it is straightforward to find the equilibrium

$$\alpha_i = \alpha_j = -\frac{r}{2}. \quad (11)$$

For the case of the second-price sealed-bid auction we see that the weakly dominant bidding strategy from equation (5) already has the structure of equation (6) with $\alpha_i = 0$ and $\beta_i = 1$. A bidder who is constrained to choose a bid that is the valuation shaded by a constant amount will, thus, not deviate from the weakly dominant bid.

2.3 Revenue comparison

Bayesian Nash equilibrium bids: With risk neutral bidders in the Bayesian Nash equilibrium the well known revenue equivalence theorem holds. For a wide range of auctions the expected revenue is the same. In particular the expected revenue in the first-price and in the second-price sealed bid auction do not differ.

It is easy to see that with risk averse bidders in the Bayesian Nash equilibrium the first-price auction generates a higher revenue. From equation (3) we see that bids increase with risk aversion in the first-price auction. If bidders follow the Bayesian Nash equilibrium bidding strategy from equation (3) the expected revenue in the first-price auction is

$$R^I = 2 \int_0^1 \int_x^1 \frac{y}{1+r} dy dx = \frac{2}{3+3r} \quad (12)$$

In the second-price auction it is a dominant strategy to bid the own valuation. Expected revenue in the second-price auction is, hence,

$$R^{II} = 2 \int_0^1 \int_x^1 x dy dx = \frac{1}{3} \quad (13)$$

Thus, $R^{II} < R^I$ for $r < 1$.

Absolute markdown bids: If bidders do not follow the Bayesian Nash equilibrium in the first-price auction, but, instead, follow the markdown bidding function from equation (6) then the expected revenue in symmetric equilibrium is

$$\bar{R} = 2 \int_0^1 \int_x^1 \beta y + \alpha dy dx = \alpha + \frac{2\beta}{3} \quad (14)$$

We see that $R^{\text{II}} < \bar{R}$ iff $\alpha > (1 - 2\beta)/3$. For the case of risk averse markdown bidders that we study in equations (10) and (11) we find that $R^{\text{II}} < \bar{R}$ iff $r < \frac{2}{3}$.

Thus, similar to the Bayesian Nash equilibrium more risk aversion also increases the revenue obtained in the first-price auction with absolute markdown bids. The first-price auction only outperforms the second-price auction if bidders are sufficiently risk averse. In section 4.6 we will compare revenue of the first-price auction and the second-price auction in the experiment.

2.4 Optimising against a markdown bidder

In the previous two subsections we have considered a homogeneous population of boundedly rational agents. In an experiment the population might be heterogeneous—some players might be more rational or less restricted than other players. To address this possibility, let us consider the case of a rational bidder who knows that the opponent makes a bid $\bar{\gamma}(x_j)$ according to equation (6) where parameters α_j and β of the opponent are known but where the valuation x_j of the opponent is not known. Now we find the best response $\hat{\gamma}$ in a first-price auction against such a markdown bidder. Here we cannot simplify equation (1) by assuming equilibrium consistency ($x = z$) as we did in the derivation of equation (2). Instead, we have to substitute equation (6) into (1). Solving the first order condition we obtain⁴

$$z = \frac{x - \alpha}{(1 + r)\beta} \quad (15)$$

which, using (6), yields

$$\hat{\gamma}(x) = \frac{x + r\alpha}{1 + r} \quad (16)$$

In a similar way we can study the utility maximization problem of a rational bidder who knows that a share ρ of all bidders in the population of potential opponents are perfectly rational while the remaining population with share $1 - \rho$ consists of markdown bidders. Assume that a rational bidder makes a bid b , then call z the valuation of a markdown bidder with exactly this bid (i.e. all markdown bidders with a valuation smaller z will not win) and call y the valuation of a rational bidder with this bid. The first order condition is

$$\rho \cdot (x - \gamma(y))^{r-1} (x - \gamma(y) - ry\gamma'(y)) + (1 - \rho) \cdot (x - \alpha - z\beta)^{r-1} (x - \alpha - z\beta - rz\beta) = 0. \quad (17)$$

⁴If $\beta = 1$ and α is chosen according to equation (9) we have always $\alpha < 0$ and $\beta > (1 - \alpha)/(1 + r)$. Then the inverse of $\bar{\gamma}$ is in $[0, 1]$ for all values of $\hat{\gamma}(x)$ with $x \in [0, 1]$ which allows us to move from equation (1) to (15). We actually do not need $\beta = 1$. E.g. if $r = 1$ then $\beta > 1/2$ is sufficient.

If all rational bidders use the same symmetric bidding function we have $y = z$. Using $z = (\gamma(x) - \alpha)/\beta$ we can solve for γ and find

$$\gamma(x) = \frac{x}{1+r} + \alpha \cdot (1-\rho) \cdot \frac{r}{1+r \cdot (1-\rho)} \quad (18)$$

which includes for $\rho = 1$ equation (3) and for $\rho = 0$ equation (16) as special cases.

We find it particularly remarkable that even if only a fraction of all bidders uses markdown bids with a negative intercept ($\alpha < 0$) then also rational bidders behave as if they are constrained by using markdown bids with a negative intercept.

3 Experimental setup

The purpose of the experiment is twofold: We want to examine to what extent the existing experimental evidence on first-price auctions is an artefact of the design and we want to find out how far absolute markdown bids are consistent with actual behaviour.

Chen and Plott (1998) study a situation where Bayesian Nash equilibrium bids are not linear. In this setup they do not find much evidence of markdown bids. This does, however, not imply that markdown bids are not relevant for linear bid functions typically studied in the experimental literature since the absence of evidence may also be due to the particular non-linearity in this study. To resolve this issue we deliberately investigate a situation where Bayesian Nash equilibrium bids are linear and markdown bids are clearly distinguishable from them. Of course, our design does not allow us to assess the capability of markdown bids to explain bidding behaviour in all conceivable auctions. However, it allows us to establish whether markdown bids are an element contributing to actual bidding behaviour.

In equations (11,16,18) we see that absolute markdown bids differ from Bayesian Nash equilibrium bids—in particular for low valuations. Absolute markdown bids can be smaller than the smallest valuation while Bayesian Nash equilibrium bids cannot. We will exploit this difference to distinguish absolute markdown bids from Bayesian Nash equilibrium bids. This has two implications for the experiment:

First, we must observe bids also for low valuations in a reliable way. To allow bidders to gain as much experience as possible for low valuations we use a setup with two bidders only. Furthermore, we use the strategy method and play five independent auctions in each round which increases the chance of feedback with low valuations. The idea of this setup is similar to that in Kirchkamp, Poen, and Reiß (forthcoming).

Second, we must provide the realistic possibility for bidders to submit bids that are lower than the lowest valuation. This might be difficult if the lower bound of valuations is equal to zero as in many experimental studies. To this end we want to understand how seemingly innocent changes in the parameters of the experiment affect the choice between Bayesian Nash equilibrium bids and absolute markdown bids. In our experiment we vary the range of valuations and the restriction to submit only positive bids.

In section 2 we determined equilibrium bids and absolute markdown bids for valuations which are distributed uniformly over an interval $[0, 1]$. These bids can be easily generalised to valuations which follow a uniform distribution over any interval $[\underline{v}, \bar{v}]$. Table 1 lists

Table 1 Treatments

Treatment	$[\underline{\omega}, \bar{\omega}]$	restriction of bids	auction
-25	$[-25, 25]$	—	first-price
0	$[0, 50]$	—	first-price
0+	$[0, 50]$	only positive bids	first-price
25	$[25, 75]$	—	first-price
50	$[50, 100]$	—	first-price
50+	$[50, 100]$	only positive bids	first-price
50+II	$[50, 100]$	only positive bids	second-price

these intervals that we study in our experiments and provides treatment names. With our experimental design, we will test the following hypotheses:

Hypothesis 1 (pure Bayesian Nash equilibrium bidding) *If all bidders use Bayesian Nash equilibrium bids we should not find any significant amount of underbidding. Also if bidders are risk averse, or if regret or spite plays a substantial role, we should not find underbidding.*

Hypothesis 2 (partial markdown bidding) *If some bidders use markdown bids or if some bidders believe that there are absolute markdown bids with positive probability we should find underbidding for small and overbidding for large valuations in all treatments where underbidding is possible (i.e. the -25, 25, 50, and 50+ treatment).*

Even with markdown bids we should find no underbidding in the 0+ treatment since there it is not possible to submit negative bids. The 0 treatment where negative bids are allowed is an intermediate case. Some participants might be tempted to assume that bids cannot be smaller than zero, others might not.

Hypothesis 3 (suppression of markdown bidding) *We should find more absolute markdown bids in the 0 treatment than in the 0+ treatment.*

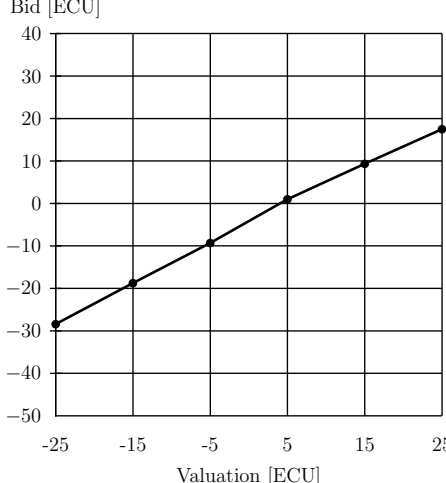
To check whether the restriction to positive bids has any confounding effects even with an interval where the restriction should not matter we compare the 50 to the 50+ treatment leading to hypothesis 4.

Hypothesis 4 (strong negative bids exclusion effect) *We should find more absolute markdown bids in the 50 treatment than in the 50+ treatment.*

While the -25 treatment is theoretically equivalent to the 25 and 50 treatment, the -25 treatment involves negative and positive valuations at the same time. This might be perceived as more difficult and, thus, may give an additional incentive to use (simpler) absolute markdown bids.

Hypothesis 5 (complexity favours markdown bidding) *We should find more absolute markdown bids in the -25 treatment than in the 25 or 50 or 50+ treatments due to increased difficulty.*

Figure 2 A typical input screen in the experiment (translated into English)

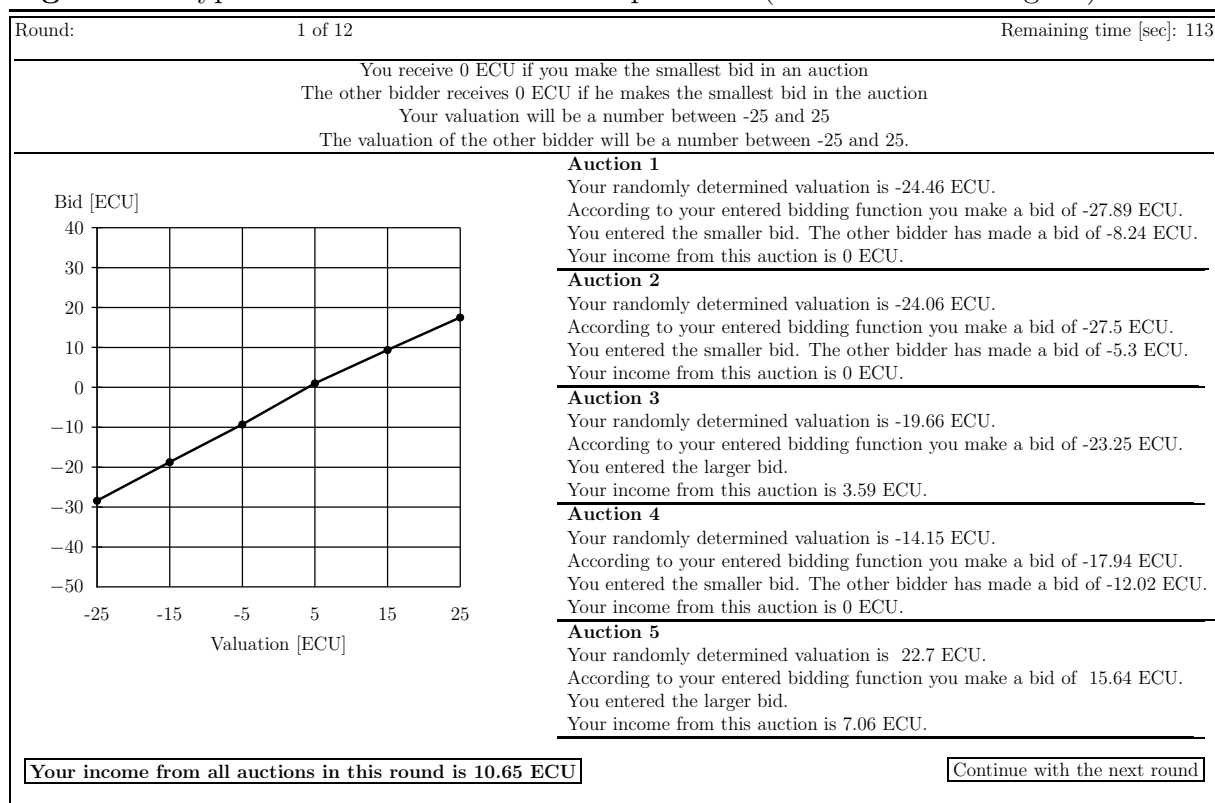
Round:	1 of 12	Remaining time [sec]: 113												
<p>You receive 0 ECU if you make the smallest bid in an auction</p> <p>The other bidder receives 0 ECU if he makes the smallest bid in the auction</p> <p>Your valuation will be a number between -25 and 25</p> <p>The valuation of the other bidder will be a number between -25 and 25.</p>														
<div style="display: flex; justify-content: space-between;"> <div style="width: 45%;">  </div> <div style="width: 50%;"> <p>Please indicate your bidding function depending on the valuation that is still going to be determined</p> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 80%;">For a valuation of -25 ECU I bid:</td> <td style="border: 1px solid black; text-align: center;">-28.4</td> </tr> <tr> <td>For a valuation of -15 ECU I bid:</td> <td style="border: 1px solid black; text-align: center;">-18.74</td> </tr> <tr> <td>For a valuation of -5 ECU I bid:</td> <td style="border: 1px solid black; text-align: center;">-9.3</td> </tr> <tr> <td>For a valuation of 5 ECU I bid:</td> <td style="border: 1px solid black; text-align: center;">1</td> </tr> <tr> <td>For a valuation of 15 ECU I bid:</td> <td style="border: 1px solid black; text-align: center;">9.35</td> </tr> <tr> <td>For a valuation of 25 ECU I bid:</td> <td style="border: 1px solid black; text-align: center;">17.5</td> </tr> </table> <div style="margin-top: 10px; text-align: right;"> <div style="border: 1px solid black; padding: 2px 10px; margin-right: 20px;">Draw bids</div> <div style="border: 1px solid black; padding: 2px 10px;">Finish input stage</div> </div> </div> </div>			For a valuation of -25 ECU I bid:	-28.4	For a valuation of -15 ECU I bid:	-18.74	For a valuation of -5 ECU I bid:	-9.3	For a valuation of 5 ECU I bid:	1	For a valuation of 15 ECU I bid:	9.35	For a valuation of 25 ECU I bid:	17.5
For a valuation of -25 ECU I bid:	-28.4													
For a valuation of -15 ECU I bid:	-18.74													
For a valuation of -5 ECU I bid:	-9.3													
For a valuation of 5 ECU I bid:	1													
For a valuation of 15 ECU I bid:	9.35													
For a valuation of 25 ECU I bid:	17.5													

While for first-price auctions absolute markdown bids differ substantially from Bayesian Nash equilibrium bids there is no such difference for second-price auctions. Underbidding for small valuations can be the result of absolute markdown bids in first-price auctions, but it should disappear (even with absolute markdown bids) in second-price auctions (treatment 50+II).

Hypothesis 6 (no underbidding in second-price auctions) *There should be no significant amount of underbidding in the 50 + II treatment.*

All experiments were conducted between 12/2003 and 12/2004 in the experimental laboratory of the SFB 504 in Mannheim. In total 304 subjects participated in these experiments. A detailed list of the treatments is given in appendix A, instructions are provided in appendix B. The experiments were computerised and we used the software package z-Tree (Fischbacher (2007)).

A typical input screen used in the experiments is shown in figure 2 (translated into English). In each round participants enter bids for six valuations which are equally spaced between \underline{v} and \bar{v} . Bids for all other valuations are interpolated linearly. Upon determination of bidding functions by all participants we draw five random and independent valuations for each participant. Each of these five random draws corresponds to an auction for which the winner is determined and the gain of each player is calculated. The sum of the gains obtained in these five auctions determines the total gain from this round.

Figure 3 A typical feedback screen in the experiment (translated into English)

A typical feedback screen is shown in figure 3. Participants play 12 rounds. Each round consists of a bid input stage and a feedback stage. At the end of these 12 rounds participants complete a short questionnaire and are paid in cash according to their gains throughout in the experiment.

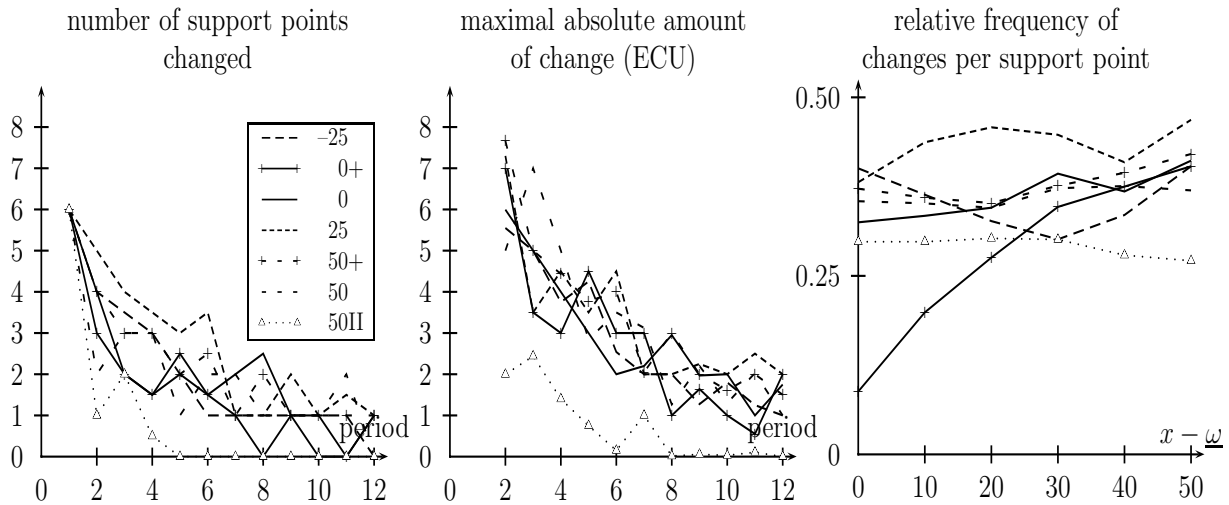
The strategy method has been used before in other auction experiments by Selten and Buchta (1999), Güth, Ivanova-Stenzel, Königstein, and Strobel (2003), Pezanis-Christou and Sadrieh (2003), and Kirchkamp, Poen, and Reiß (forthcoming). From our own experience with this method we know that bids that are observed with the strategy method are very similar to bids observed with alternative methods.

In the context of this paper we should note that the three benchmark solutions we described in sections 2.1, 2.2, and 2.4 can be represented as three different bidding functions which are all straight lines in the experimental interface. It is, however, up to the participants, whether they choose any of these three lines or any other curve.

4 Results

4.1 Convergence of bidding behaviour

Before we look at details of bidding behaviour we have to check whether behaviour stabilises over the course of the experiment. To do this we rely on three indicators of stability. First we count how often participants change support points of their bidding function.

Figure 4 Convergence of bids over time

The figure shows how many of the six support points (hypothetical bids) of a bid function a median bidder changes and the maximal absolute amount of this change.

In each period and for each participant this can be a number between zero and six. It is zero if the participant continues to use the bidding function from the last period, and it is six if all bids are changed. The development of the median of this distribution is reported in the left graph of figure 4. By definition all six support points are new in the first period, thus, period 1 must start with 6 changes for all treatments. We see that, after some adjustments during the first few periods, participants apply a stable bidding function. During the second half of the experiment the median bidder does not change more than one or two support points in each period.

Second, the graph in the middle panel of figure 4 shows the absolute amount of these changes over time. For each participant and each period we determine the largest absolute change in the six hypothetical bids from one period to the next. The median of this distribution is shown in the graph. We see that these changes are small compared to the range of the valuation.

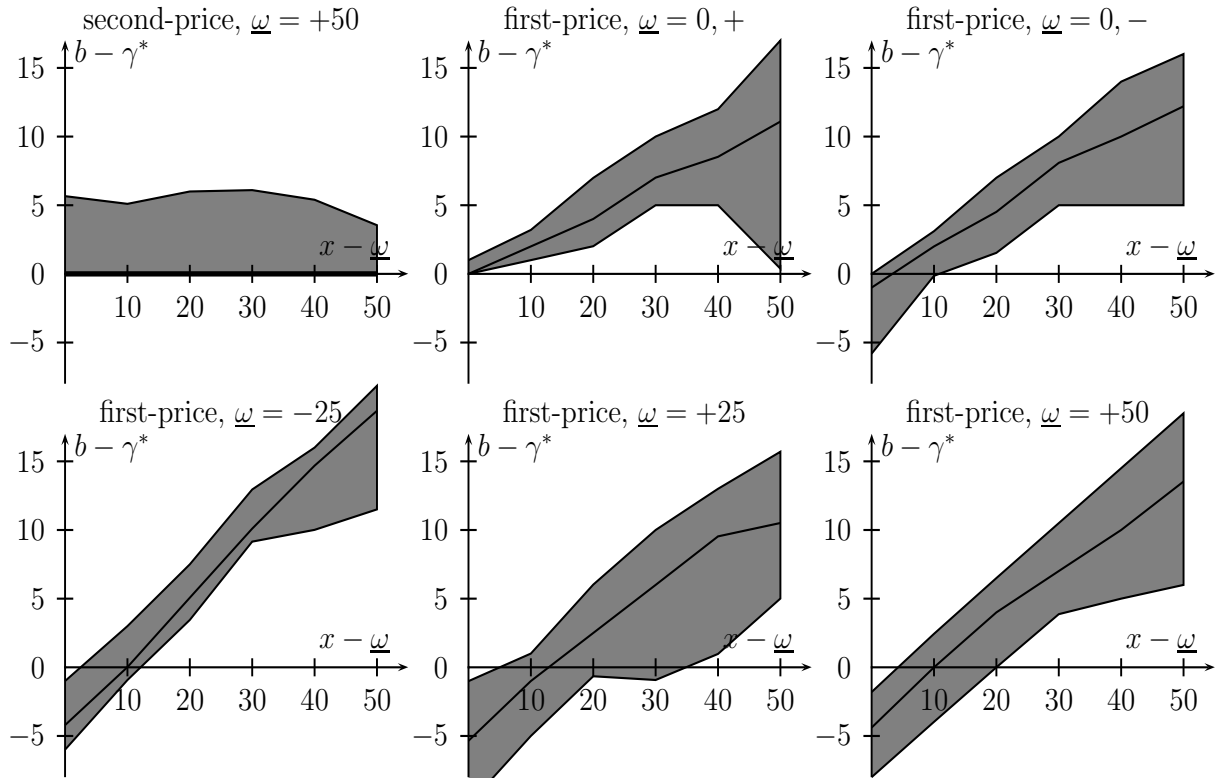
Third, the right panel in the figure shows that changes are distributed fairly evenly over valuations for most treatments. The exception is the 0+ treatment where bidders are, indeed, restricted in their changes for small valuations.

We conclude here that bidding behaviour is stable in the second half of the experiment.

4.2 Visual inspection of aggregate bids

For a first impression of bidding behaviour, figure 5 shows the median and interquartile range⁵ amount of overbidding $b(x) - \gamma^*(x)$ as a function of the valuation x . As above $\gamma^*(x)$ is the Bayesian Nash equilibrium bidding function with risk neutrality ($r = 1$) as given by equation (4). Let us briefly inspect the individual treatments:

⁵Median and quartiles are taken over all bidders and all periods (after period 6) in a given treatment.

Figure 5 Overbidding for different treatments

The figure shows normalised bids on the horizontal and overbidding on the vertical axis. The interquartile range of overbidding is shown as a grey area. The median amount of overbidding is shown as a black line. The first 6 periods from each session are discarded.

Second-price auction: In the second-price treatment bidders have a weakly dominant bidding strategy. Many participants follow this strategy. Overbidding is zero for the 25% quantile and for the median bid. The 75% quantile is, for all valuations, larger than 0, i.e. there are some bidders which bid more than the weakly dominant bidding strategy. This is consistent with the experimental literature. Already Kagel, Harstad, and Levin (1987) find a small amount of overbidding in second-price auctions. Kagel and Levin (1993) confirm that only a small fraction of bidders bids less than the equilibrium strategy while a substantial fraction bids more in second-price auctions.

First price auction, 0+: The traditional first-price treatment prevailing in the experimental literature is characterised by $\underline{\omega} = 0$ and '+', where the sign indicates the restriction to positive bids. The lowest possible valuation is 0, and bids are constrained to be positive. As we should expect we find overbidding in this treatment. Median overbidding and 75% quantile overbidding increase with the valuation. Except for the highest valuation also the 25% quantile increases with the valuation. What we see at the right end of the 0+ graph is a decrease in the amount of overbidding for the 25% quantile. The value of the bid is still increasing for these players, although the slope of the bidding function is now smaller than one. This finding is consistent with risk-aversion and confirms results from several previous experiments, starting with Cox, Roberson, and Smith (1982).

Table 2 Overbidding for different treatments

treatment	n	$b(\underline{\omega}) - \gamma^*(\underline{\omega})$				$b(\overline{\omega}) - \gamma^*(\overline{\omega})$			
		mean	t	$P_{>t}$	P_{bin}	mean	t	$P_{>t}$	P_{bin}
-25 -	4	-4.653	-7.703	.0023	.0625	16.229	20.753	.0001	.0625
0 -	6	-3.453	-2.003	.0508	.0156	10.586	5.798	.0011	.0156
0 +	4	.841	2.009	.9309	1	9.397	14.154	.0004	.0625
25 -	3	-4.953	-5.574	.0154	.125	8.461	4.176	.0264	.125
50 -	4	-7.53	-4.643	.0094	.0625	6.639	2.806	.0338	.0625
50 +	14	-2.597	-1.704	.0561	.212	17.272	7.169	.0000	.0001
all firstprice	29	-4.856	-7.466	.0000	.0000	10.389	13.882	.0000	.0000
secondprice	6	3.549	3.98	.9947	.9844	27.485	36.084	.0000	.0156

The table compares overbidding for the lowest and highest valuation, $\underline{\omega}$ and $\overline{\omega}$. For each treatment n is the number of independent observations. We test whether $b(\underline{\omega}) < \gamma^*(\underline{\omega})$ (underbidding for low valuations) and whether $b(\overline{\omega}) > \gamma^*(\overline{\omega})$ (overbidding for high valuations). Mean deviations from the risk neutral Bayesian Nash equilibrium bid are shown together with results of a parametric t -test ($P_{>t}$) and a non-parametric binomial test (P_{bin}). As in many other experiments we find a significant amount of overbidding for the highest possible valuation $\overline{\omega}$ in all treatments. However, we also find underbidding for the smallest possible valuation $\underline{\omega}$ in all first-price treatments where underbidding is possible, i.e. always, except in the 0+ treatment.

First price auction, all other treatments: All the other treatments allow for bids that are smaller than the smallest possible valuation. Similar to the 0+ treatment, we find overbidding for high valuations. In contrast to the 0+ treatment, we find underbidding for low valuations.

4.3 Results of statistical tests of aggregate behaviour

Table 2 shows mean overbidding for the highest and lowest valuation, $\underline{\omega}$ and $\overline{\omega}$. Overbidding for high valuations is consistent with e.g. CRRA and several other theories of bidding behaviour in auctions that we mentioned above. Underbidding for low valuations is harder to explain. However, underbidding for low valuations is consistent with the absolute markdown bids presented in section 2. The table compares overbidding for the lowest and highest valuation, $\underline{\omega}$ and $\overline{\omega}$. For each treatment n is the number of independent observations. We test whether $b(\underline{\omega}) < \gamma^*(\underline{\omega})$ (underbidding for low valuation) and whether $b(\overline{\omega}) > \gamma^*(\overline{\omega})$ (overbidding for high valuation). Mean bids are shown together with results of a parametric t -test ($P_{>t}$) and a non-parametric binomial test (P_{bin}). To test whether the t -test on the level of the independent observations is appropriate we have applied a Shapiro-Wilk test on means over the independent observations. For none of the treatments the test rejects the assumption of normality.

Hypotheses 1 and 2: Not surprisingly, and as in many other experiments with first-price auctions, we find a significant amount of overbidding for the highest possible valuation $\overline{\omega}$ in all treatments. More interestingly, we find underbidding for the smallest possible valuation $\underline{\omega}$ in all first-price treatments where underbidding is possible, i.e. always, except

Table 3 Slope of bidding function $b(\overline{\omega}) - b(\underline{\omega})$

groups of treatments	independent observations	difference in means	t	$P_{>t}$	z	$P_{>z}$
(0) – (0+)	10	5.482	2.469	0.018	1.492	0.068
(50) – (50+)	12	-2.958	-1.000	0.831	-1.019	0.846
(25–) – (25, 50, 50+)	19	5.059	3.657	0.001	2.500	0.006

The figure compares average slopes of the bidding functions measured as $b(\overline{\omega}) - b(\underline{\omega})$. The column “difference in means” shows the difference between the average slope in the first treatment group and the average slope in the second treatment group. The columns t and $P_{>t}$ show the result of a parametric t -test, the columns z and $P_{>z}$ show the result of a non-parametric Mann-Whitney test.

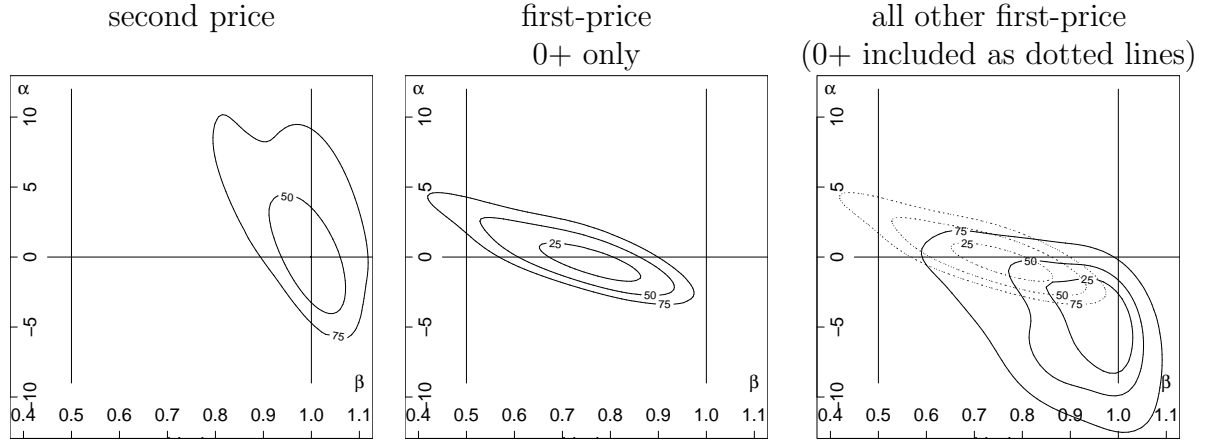
for the 0+ treatment. We find, thus, no support for hypothesis 1 (pure Bayesian Nash equilibrium bidding), but we can confirm hypothesis 2 (partial markdown bidding).

Hypothesis 3: To see if bidding behaviour is affected by the exclusion of negative bids, we compare differences in bids for the smallest and the highest valuation $b(\overline{\omega}) - b(\underline{\omega})$ observed in the 0 treatment to those observed in the 0+ treatment. If the possibility to submit negative bids does not affect behaviour, observed differences in bids should equal zero. The first line in table 3 shows the results. In the 0 treatment the difference in mean bids exceeds that in the 0+ treatment by 5.482 ECU. This difference is significant under a parametric and a non-parametric test, thus, we can confirm hypothesis 3 (suppression of markdown bidding).

Hypothesis 4: The next line of table 3 shows the difference in slopes of the bidding function between the 50 and 50+ treatment. This difference is not significant. Furthermore, not even the sign of the effect is the same as what we expect under a strong treatment effect of explicitly allowing negative bids. Excluding negative bids (and mentioning this fact in the instructions to the experiment) does not per-se trigger behavioural changes towards absolute markdown bids, so that we cannot confirm hypothesis 4 (strong negative bids exclusion effect).

Hypothesis 5: The third line of table 3 shows the difference in slopes of the bidding function between the 25– treatment and the 25, 50, and 50+ treatment. According to hypothesis 5 (complexity favours markdown bidding) we should expect a steeper slope of the bidding function under the 25– treatment. This is confirmed by a parametric and a non-parametric test.

Hypothesis 6: Here we have to go back to table 2. The last line shows the difference between bids in the experiment and equilibrium bids. We see that this difference is positive, not negative. Hence, we do not observe a significant amount of underbidding here which supports hypothesis 6 (no underbidding in second-price auctions).

Figure 6 Individual bidding

Each graph shows contour lines of the kernel density estimate of the distribution of individual bidding functions (see equation 19) with the first six periods of the experiment discarded. Numbers next to the contour lines are estimated percentiles.

4.4 Individual bids

The quartiles of bidding behaviour, as depicted in the left panel of figure 5, suggest heterogeneity among bidders. To better understand individual bidding behaviour we estimate for each bidder a linear bidding function:

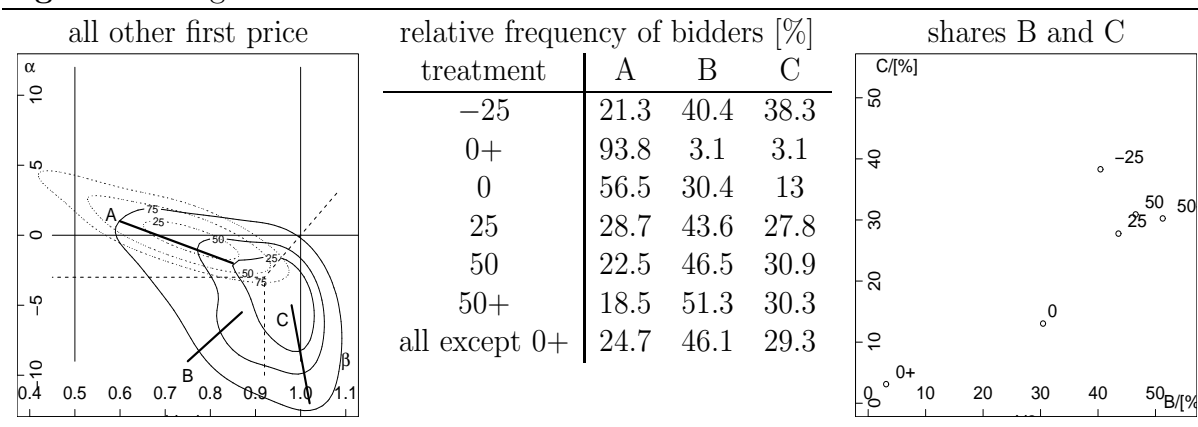
$$b(x) = \underline{\omega} + \alpha + \beta \cdot (x - \underline{\omega}) + u \quad (19)$$

The regression specification normalises valuations and bids such that the point $(\underline{\omega}, \underline{\omega})$ transforms to the origin $(0, 0)$ in valuation-bid space. We conduct the normalisation to facilitate easier comparison of estimated intercepts α across treatments with varying valuation domain. Estimated intercepts and slopes can be interpreted as if the valuation domain is $[0, 50]$ for any treatment. Again we discarded the first six periods of the experiment. Outliers have been eliminated using Hadi's method. The fit of the estimations of equation (19) is very good, e.g., the median R^2 is 0.9918.

Since a scatterplot of all individually estimated coefficients is rather confusing, figure 6 shows the contour lines of the estimated joint distribution of α and β . We aggregate the data in three graphs:

Second price auction: The left panel of figure 6 shows the distribution of estimated bidding functions for second price auctions. In the weakly dominant equilibrium we have $\alpha = 0$ and $\beta = 1$. Indeed, the distribution of estimated values is nicely centered around this value.

First price auction, 0+: The middle panel depicts coefficient estimates for the 0+ treatment. The risk neutral Bayesian Nash equilibrium predicts $\alpha = 0$ and $\beta = 1/2$. Risk averse equilibrium predicts $\alpha = 0$ and $\beta > 1/2$. As the figure illustrates, the distribution

Figure 7 Categorisation of bidders

of coefficients is concentrated around $\alpha = 0$ and its support includes values for β between $1/2$ and 1 . In this treatment risk averse Bayesian Nash equilibrium and other theories that we mentioned above explain the data quite well.

First price auction, all other treatments: The right panel of figure 6 illustrates coefficient estimates for all the other treatments. To facilitate the comparison we also indicate the distribution estimated for the 0+ treatment as dotted lines. To further facilitate the discussion, figure 7 shows the same graph with additional lines and labels. First consider the right panel of figure 6. It is easy to see that the possibility to make bids smaller than the lower bound of the valuation domain, $\underline{\omega}$, changes bidding behaviour substantially:

- Now a large group of bidders is characterised by $\beta \approx 1$ and $\alpha < 1$. These bidders are, in line with section 2.2, much better described by absolute markdown bids instead of following the Bayesian Nash equilibrium bidding.
- There is still some overlap with the distribution estimated for the 0+ treatment. This region could characterise bidders that are not affected by our treatment conditions and always bid according to the risk averse Bayesian Nash equilibrium (see section 2.1), or are driven by motives like spite or regret.
- Finally, there is a group of bidders with a $\beta < 1$ but still a negative α . In the context of section 2.4 we interpret these bidders as rational bidders who realise that not all bidders are perfectly rational. Alternatively they might be viewed as markdown bidders.

4.5 Categorising individual bidders

When interpreting the contour lines of figure 6 as contour lines of a mountain, the mountain in the graph on the right has three ridges. In the left panel of figure 7 we indicate these ridges approximately with bold lines and letters A, B, and C. These lines are merely intended to clarify the discussion. One could certainly draw them in a slightly different manner.

Ridge A runs parallel to the distribution of bidding functions in the 0+ treatment. We will categorise bidders close to this ridge as rational bidders who assume that they are in a rational world.

Ridge C is close to $\beta = 1$. All bidding functions on this ridge have an $\alpha < 0$. We will categorise bidders close to this ridge as bidders who use absolute markdown bids and who expect their opponents to do the same.

Ridge B is in between (and less pronounced than A or C). As above, we will interpret these bidders as rational bidders who realise that not all bidders are perfectly rational.

We will now categorise bidders into different groups. The dashed lines indicate the borders of the regions that we use. Clearly, there is a certain arbitrariness in choosing threshold values for α and β . We have tried to find a region for the Bayesian Nash (A) bidders which includes almost all of the 0+ bidders. A suitable threshold seemed to be $\alpha = -3$. We have also tried to make region (C) symmetric to $\beta = 1$. This led to a threshold of $\beta = 0.92$.

The table in the center of figure 7 shows the result of this categorisation. The graph on the right hand side of the same figure depicts the sample shares of bidder categories B and C for all treatments. For the 0+ treatment the shares of B and C are negligible. However, for the 0 treatment shares of B and C increase substantially. We attribute this to the possibility to submit negative bids so that markdown bidding is feasible as opposed to the 0+ treatment. While for the remaining four treatments most players are either Bs or Cs, there is still a fraction of about 25% of Bayesian Nash players.

We are now ready to examine our hypotheses at the individual level. Again, we reject hypothesis 1 (pure Bayesian Nash equilibrium bidding) and support hypothesis 2 (partial markdown bidding), except for the 0+ treatment. We also find clear support for hypothesis 3 (suppression of markdown bidding), since there are more Cs in the 0 treatment than in the 0+ treatment. In line with the aggregate data, we cannot support hypothesis 4 (strong negative bids exclusion effect); there is essentially no difference between the 50 and the 50+ treatment. Hypothesis 5 (complexity favours markdown bidding) is supported—the largest share of Cs is found in the –25 treatment.

4.6 Underbidding and Revenue

Standard (e.g. CRRA) models of bidding behaviour explain overbidding in first-price auctions. These models predict a higher revenue in first-price auctions than in second-price auctions. This is confirmed by several experimental studies including Cox et al. (1982).

We found a large amount of underbidding for all treatments with first-price auctions, except for the 0+ treatment. Since we did not find underbidding in second-price auctions, we conclude that underbidding reduces the revenue in first-price auctions compared to second-price auctions.

In the current section we investigate if the drop in revenue in first-price auctions due to underbidding is sufficiently large to eliminate its revenue dominance over the second-price auction. Table 4 reports the excess revenue, i.e. the average difference between the expected revenue in the experiment⁶ R and the risk-neutral equilibrium value R^* . For

⁶To approximate the expected revenue we draw for each pair of bidders in each period not only 5, as

Table 4 Excess revenue for different treatments

treatment	n	$R - R^*$			
		mean	t	$P_{>t}$	P_{bin}
-25	4	10.708	33.693	.0001	.125
0	6	7.918	7.308	.0008	.0312
0 +	4	7.227	47.702	0	.125
25	3	6.071	3.985	.0576	.25
50	4	4.544	3.575	.0374	.125
50 +	8	7.188	11.512	0	.0078
all first-price	29	7.329	16.739	0	0
second-price	6	2.483	5.219	.0034	.0312
		$R^I - R^{II}$ (for $\omega = 50$ only)			
	n	mean	t	$P_{>t}$	$z, P_{>z}$
	18	4.021	5.334	.0001	$z = 2.81, P_{>z} = .005$

The table compares for different treatments the difference between the expected revenue in the experiment R and revenue in the risk-neutral Bayesian Nash equilibrium R^* . n is the number of independent observations. Mean excess revenue is shown together with results of a parametric t -test ($P_{>t}$) and a non-parametric binomial test (P_{bin}), except for the comparison of R^I and R^{II} where we present results of a Wilcoxon rank-sum test. Data from the first 6 periods of each session is discarded.

each treatment n is the number of independent observations. We test whether $R > R^*$ (revenue in the experiment is higher than revenue in equilibrium). This is always the case and in most cases it is significant for the individual treatment. Most importantly, average revenue for all first-price auctions is significantly higher than equilibrium revenue both with parametric and with non-parametric tests. Also in the second-price auction average revenue R^{II} is significantly higher than in equilibrium. In the lower part of table 4 we compare revenue in the first-price R^I and in the second-price auction R^{II} . We find that average revenue is significantly higher in the first-price auction also in spite of underbidding.

5 Concluding remarks

Many first-price auction experiments find that subjects bid more than the risk neutral equilibrium bid, they ‘overbid’. We can confirm this finding. However, the approaches that have been used so far to explain overbidding are not in line with our second finding: underbidding for small valuations.

The idea we are proposing here, namely that some bidders use absolute markdown bids, is independent of the representation of payoffs as lottery tickets or as money and consistent with the traditional experimental evidence. We have seen in section 2 that optimal absolute markdown bids imply underbidding for small valuations and that the

in the experiment, but 100 auctions with random valuations. The average of these revenues (for each bidder and each period) is then used as expected revenue.

presence of a small proportion of bidders with absolute markdown bids is sufficient to make rational bidders behave as if they were constrained in a similar way.

In our experiment we find that a fraction of bidders can still be described well with traditional models, e.g. risk averse Bayesian Nash equilibrium. In many treatments this is a small fraction. We have seen that a substantially larger fraction of bidders follows absolute markdown bids. A third group, finally, behaves like optimisers against such a mixed population.

We can, hence, not conclude that all bidders can be described better with absolute markdown bids. We also cannot say much about the importance of markdown bids in other, perhaps more complicated contexts. We have, however, seen that in our experiment absolute markdown bids become more prominent when the environment becomes more complicated (as in the -25 treatment). Hence, when we find already a fair amount of markdown bids in a very simple auction, we might suspect that markdown bids play an even more important role in more complex auctions.

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A List of independent observations

date	group	\underline{w}	\underline{b}	second-price	participants
20040518-17:41	0	-25	-125	0	8
20040518-17:41	1	-25	-125	0	8
20040519-10:37	0	-25	-125	0	8
20040519-10:37	1	-25	-125	0	8
20040518-10:19	0	0	-100	0	8

continued on next page

date	group	$\underline{\omega}$	\underline{b}	second-price	participants
20040518-10:19	1	0	-100	0	6
20040518-12:15	0	0	-100	0	6
20040518-12:15	1	0	-100	0	6
20040519-17:39	0	0	-100	0	8
20040519-17:39	1	0	-100	0	6
20040518-15:55	0	0	0	0	8
20040518-15:55	1	0	0	0	8
20040519-12:33	0	0	0	0	8
20040519-12:33	1	0	0	0	8
20040602-14:11	0	25	-75	0	8
20040602-14:11	1	25	-75	0	8
20040602-16:03	0	25	-75	0	10
20040517-12:21	0	50	-50	0	8
20040517-12:21	1	50	-50	0	6
20040517-17:17	0	50	-50	0	8
20040517-17:17	1	50	-50	0	8
20031211-18:23	0	50	0	0	14
20031212-10:45	0	50	0	0	14
20040519-15:53	0	50	0	0	8
20040519-15:53	1	50	0	0	10
20050414-08:55	0	50	0	0	10
20050414-08:55	1	50	0	0	10
20050414-13:17	0	50	0	0	10
20050414-13:17	1	50	0	0	10
20041130-17:41	0	50	0	1	10
20041130-17:41	1	50	0	1	10
20041201-14:09	0	50	0	1	10
20041201-14:09	1	50	0	1	10
20041201-15:57	0	50	0	1	10
20041201-15:57	1	50	0	1	8

The parameter \underline{b} is the smallest possible bid. In the +treatments $\underline{b} = 0$, otherwise $\underline{b} = \underline{\omega} - 100$. The highest bid that participants could enter was always $\overline{\omega} + 100$.

B Conducting the experiment and instructions

Participants were recruited by email and could register for the experiment on the internet. At the beginning of the experiment participants drew balls from an urn to determine their allocation to seats. Being seated participants then obtained written instructions in German. These instructions vary slightly depending on the treatment. In the following we give a translation of the instructions.

After answering control questions on the screen subjects entered the treatment described in the instructions. After completing the treatment they answered a short questionnaire on the screen and were then paid in cash. The experiment was done with the

help of z-Tree (Fischbacher (2007)).

B.1 General information

You are participating in a scientific experiment that is sponsored by the Deutsche Forschungsgemeinschaft (German Research Foundation). If you read the following instructions carefully then you can—depending on your decision—gain a considerable amount of money. It is, hence, very important that you read the instructions carefully.

The instructions that you have received are only for your private information. **During the experiment no communication is permitted.** Whenever you have questions, please raise your hand. We will then answer your question at your seat. Not following this rule leads to exclusion from the the experiment and all payments.

During the experiment we are not talking about Euro, but about ECU (Experimental Currency Unit). Your entire income will first be determined in ECU. The total amount of ECU that you have obtained during the experiment will be converted into Euro at the end and paid to you in **cash**. The conversion rate will be shown on your screen at the beginning of the experiment.

B.2 Information regarding the experiment

Today you are participating in an experiment on auctions. The experiment is divided into separate rounds. We will conduct **12 rounds**. In the following we explain what happens in each round.

In each round you bid for an object that is being auctioned. Together with you another participant is also bidding for the same object. Hence, in each round, there are **two bidders**. In each round you will be allocated randomly to another participant for the auction. Your co-bidder in the auction changes in every round. The bidder with the highest bid has obtained the object. If bids are the same the object will be allocated randomly.

For the auctioned object you have a valuation in ECU. This valuation lies between x and $x + 50$ ECU⁷ and is determined randomly in each round. The range from x to $x + 50$ will be shown to you at the beginning of the experiment on the screen and is the same in each round.⁸ **From this range you will obtain in each round new and random**

⁷In the 0+ and 50+ treatments the valuation would be announced precisely: “This valuation lies between 0 and 50 ECU” in the 0+ treatment and “This valuation lies between 50 and 100 ECU” in the 50+ treatment. Whenever x is mentioned in the remainder of the instruction the same comment applies: In the 0+ and 50+ treatments the valuation is always announced precisely.

⁸This sentence was not shown in the 0+ and 50+ treatments, though in all treatments the range was shown on the screen.

Figure 8 Screen from the instructions for the 1. stage of the experiment

Round:	1 of 12	Remaining time [sec]: 113																																																																																																												
<p>You receive 0 ECU if you make the smallest bid in an auction</p> <p>The other bidder receives 0 ECU if he makes the smallest bid in the auction</p> <p>Your valuation will be a number between x and $x + 50$</p> <p>The valuation of the other bidder will be a number between x and $x + 50$.</p>																																																																																																														
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[[In the 0+ and 50+ treatments the interval was already shown exactly in the instructions and consistently also in the figures in the instructions. In the other treatments the interval x to $x + 50$ was, as you see in the figure, described as x to $x + 50$. From the first round of the experiment on the current numbers were given.]]

valuations for the object. The other bidder in the auction also has a valuation for the object. The valuation that the other bidder attributes to the object is determined by the same rules as your valuation and changes in each round, too. All possible valuations of the other bidder are also in the interval from x to $x + 50$ from which also your valuations are drawn. All valuations between x and $x + 50$ are equally probable. Your valuations and those of the other player are determined independently. You will be told your valuation in each round. You will not know the valuation of the other bidder.

B.2.1 Experimental procedure

The experimental procedure is the same in each round and will be described in the following. Each round in the experiment has two stages.

1. Stage

In the first stage of the experiment you see the following screen: [[see figure 8]]

At that stage **you do not know your own valuation for the object in this round.** On the right side of the screen you are asked to enter a bid for **six hypothetical valuations** that you might have for the object. These six hypothetical valuations are x ,

Figure 9 Screen from the instructions for the 2. stage of the experiment

Round:	1 of 12	Remaining time [sec]: 113																																																																																																												
<p>You receive 0 ECU if you make the smallest bid in an auction</p> <p>The other bidder receives 0 ECU if he makes the smallest bid in the auction</p> <p>Your valuation will be a number between x and $x + 50$</p> <p>The valuation of the other bidder will be a number between x and $x + 50$.</p>																																																																																																														
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<div style="border: 1px solid black; padding: 5px; display: inline-block;"> Continue with the next round </div>																																																																																																														

$x + 10$, $x + 20$, $x + 30$, $x + 40$, and $x + 50$ ECU. Your input into this table will be shown in the graph on the left side of the screen when you click on “draw bids”. In the graph the hypothetical valuation is shown on the horizontal axis, the bids are shown on the vertical axis. Your input in the table is shown as six points in the diagram. **Neighbouring points are connected with a line automatically.** These lines determine your bid for all valuations between the six points for those you have made an input. For the other bidder the screen in the first stage looks the same and there are as well bids for six hypothetical valuations. The other bidder cannot see your input.

2. Stage

The actual auction takes place in the second stage of each round. In each round we will play not only a single auction but **five auctions**. This is done as follows: **Five times a random valuation is determined** that you have for the object. Similarly for the other bidder five random valuations are determined. You see the following screen: [[see figure 9]]

For each of your five valuations the computer determines your bid according to the graph from stage 1. If a valuation is precisely at x , $x + 10$, $x + 20$, $x + 30$, $x + 40$, or $x + 50$ the computer takes the bid that you gave for this valuation. If a valuation is between these points your bid is determined according to the joining line. In the same way the bids of

the other bidder are determined for his five valuations. Your bid is compared with the one of the other bidder. The bidder with the higher bid has obtained the object.

Your income from the auction:

For each of the five auctions the following holds:

- The bidder with the higher bid obtains the valuation he had for the object in this auction added to his account minus his bid for the object.
- If the bidder with the higher bid has a negative valuation for the object, the ECU account is reduced by this amount.⁹
- If the bid of bidder with the higher is a negative number, the amount is added to his ECU account.¹⁰
- The bidder with the smaller bid obtains **no income** from this auction.

You total income in a round is **the sum of the ECU income from those auctions in this round where you have made the higher bid.**

This ends one round of the experiment and you see in the next round again the input screen from stage 1.

At the end of the experiment your total ECU income from all rounds will be converted into Euro and paid to you in cash together with your Show-Up Fee of 3.00 Euro.

Please raise your hand if you have questions.

⁹This item is not shown in the 0+ and 50+ treatments.

Note that, in order to be able to use same instructions for all treatments we mention the possibility of negative valuations in all, except the 0+ and 50+ treatments, even if subjects learn later that their valuation is drawn from an interval that contains only positive numbers.

¹⁰This item not shown in the 0+ and 50+ treatments.