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by

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Relating the two Dimensions of Risk Attitudes: An Experimental Analysis

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Abstract

In the framework of expected utility theory, risk attitudes are entirely captured by the curvature of the utility function. In cumulative prospect theory (CPT) risk attitudes have an additional dimension: the weighting of probabilities. With this modification, one question arises naturally: since both utility and probability weighting determine the attitude towards risk, what is the relation between them? We ran a controlled laboratory experiment to answer this question. Our findings suggest that the two dimensions capture different characteristics of individual risk attitude. Though individuals who are risk averse in one dimension are likely to be risk averse in the other, the two dimensions show no significant correlation. Moreover, a significant proportion of subjects are risk averse in one dimension but risk seeking in the other.

Keywords: Risk attitudes, Cumulative prospect theory, Experimental study
JEL classification: C91, D81

*An early version of the paper has been presented at the Cognitive Lunch Series in at the University of Indiana, Bloomington.

1 Introduction

In the expected utility theory (hereafter EU), attitude towards risk is captured by the curvature of utility functions (or the change of marginal utilities). Wakker (1994) argues that the utility function describes an intrinsic appreciation of money, prior to probability or risk, and that it is more natural to see risk attitudes originating from the perception of probabilities. After all risk is primarily about the likelihood of outcomes. In line with this argument, cumulative prospect theory (Kahneman and Tversky, 1979; Quiggin, 1982; Tversky and Kahneman, 1992) (hereafter CPT) separates risk attitudes into two dimensions: the curvature of utility, and a "probabilistic" component, i.e., the transformation of (cumulative) probabilities (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992). In particular, (Tversky and Kahneman, 1992) suggest a four fold patter about risk attitudes: risk aversion for gains and risk seeking for losses of high probability; risk seeking for gains and risk aversion for losses of low probability.

With this modification, one question naturally arises: How to accurately measure these two dimensions of risk? More importantly, since both the curvature of utility and the shape of probability weighting reflect risk attitude, what is the relation between them? Is an individual who is averse in one dimension also likely to be averse in another? Is it possible for an individual to be risk averse in one dimension but risk seeking in another? In this paper, we conducted a laboratory experiment to address these issues.

Several works elicited the utility function and the probability weighting function. Tversky and Kahneman (1992) themselves conducted an experiment to test for the shape of the utility function and the probability weighting function. Later Wakker and Deneffe (1996) and Abdellaoui (2000) developed the trade-off method to elicit the probability weighting function and utility function separately. In this paper we used this trade-off to elicit the utility function and the probability weighting function separately. Yet, our focus is different. Apart from determining and classifying these two functions, we investigate the relation between them.

There have been several theoretical works discussing these two dimensions of risk attitudes.

Hong et al. (1987) investigate the classification of risk attitudes in Rank dependent utility theory, a special case of CPT. They find that risk aversion implies a concave utility function and a convex probability weighting function. In a more recent paper, Schmidt and Zank (2008) investigate similar issue for CPT. They find that in CPT strong risk aversion implies convex probability weighting function but not necessarily a concave utility function. Though the most common finding for the probability weighting function in the literature is an inverse-S shaped curve (see e.g. Abdellaoui (2000); Bleichrodt (2000); Gonzalez (1999); Tversky and Kahneman (1992)). Convex shapes were found to be prevailing in Jullien and Salanié (2000) and van de Kuilen et al. (2007). Also in our experiment we mostly found the convex shape.

Several works explore the nature of risk attitude. van de Kuilen (2008) explores experimentally agents' sensitivity towards probabilities. When agents face repeatedly similar decisions with direct feedback on the consequences, the elicited subjective probability weighting function converges significantly towards linearity. In a similar vein though based on EUT, Schunk and Betsch (2006) explore the connection between decision mode and the curvature of the individual utility function. They find that agents in a deliberate decision mode tend to have a nearly linear utility function. While an intuitive decision mode causes the utility function to be more curved. Also risk attitude has caught the attention of neuroeconomists. It has been shown for instance that risk and reward are processed in different parts of the brain, the dorsal and the ventral MPFC respectively (see Xue et al. (2008)). Though many studies have found a relation between immediate emotions and risky decision making, the evidence for multiple systems is mixed(see Loewenstein et al. (2008) for a review).

A better understanding of the two dimensions of risk attitudes requires first a careful measurement. There are two ways elicitation: the parametric and the trade-off method. The parametric method assumes a certain form of both functions, then the parameters of these two functions are estimated jointly using experimental data. While the parametric method provided useful insights about the shape of both functions, it has one serious drawback: the estimated parameters may have been confounded by the particular parametric functional form assumed. A second drawback of parametric fitting concerns the joint fitting of utility and probability weighting. The parameter estimates of these functions are interdependent: an overestimation of risk aversion in one component leads to an underestimation in the other, and vice versa. Recognizing this problem, we rely on the second method: the trade-off method. This method was developed by Wakker and Deneffe (1996), and it is so far the only method that allows for a separate measurement of the utility function and the probability weight function. It has been used and further developed by Abdellaoui (2000); van de Kuilen et al. (2007); Abdellaoui et al. (2005); Kobberling and Wakker (2005), in the present paper we mainly rely on the version introduced by Abdellaoui (2000). The detailed procedure is outlined in the following sections.

The paper is organized as follows. Section 2 outlines the method used to obtain utility and weighting functions, i.e., a two-step trade-off methods for utility and weighting functions. Section 3 describes the experimental procedure. The results of this experiment are given in section 4. Finally, section 5 summarizes and discusses the results and implications of the experimental findings.

2 CPT and the trade-off method

The cumulative prospect theory (Kahneman and Tversky, 1992) is a descriptive model for decision making under risk. After 30 years of development, it is now the most prominent alternative to the expected utility theory *(EUT)*. As opposed to EUT outcomes are evaluated relative to a reference point, and both monetary outcomes and probabilities are evaluated subjectively. In this paper we restrict ourselves to risky prospects involving only gains, i.e. for all prospects that probabilities are known and only nonnegative outcomes are possible. As a result, for the rest of the paper the reference point can be normalized to zero. For a discussion of the utility function and the probability weighting function over losses see Kobberling and Wakker (2005) and Abdellaoui et al. (2005).

Formally CPT is defined as follows. Let $P = (x_1, p_1; \ldots; x_n, p_n)$ denote a prospect that assigns probability p_i to outcome x_i, where $x_1 < \ldots < x_r < 0 < x_{r+1} < \ldots < x_n$. The evaluation of this prospect depends two functions: a utility function $u(\cdot)$ and a probability weighting function $w(\cdot)$. The utility function $u(\cdot)$ is assumed to be strictly increasing over the outcome space X, and the function $w(\cdot)$ is a mapping $w : P \rightarrow P$, with $w(0) = 0$ and $w(1) = 1$, where $P = [0, 1]$ is the probability space. Finally, the utility of the prospect P is given by:

$$V(P) = \sum_{i=1}^{r} \pi_i^- u(x_i) + \sum_{i=r+1}^{n} \pi_i^+ u(x_i), \qquad (1)$$

where $\pi_i^- = w^-(\sum_{k=1}^{i} p_k) - w^-(\sum_{k=1}^{i-1} p_k)$ and $\pi_1^- = w^-(p_1)$, and $\pi_i^+ = w^+(\sum_{k=j}^{n} p_k) - w^+(\sum_{k=j+1}^{n} p_k)$ and $\pi_n^+ = w^+(p_n)$.

2.1 The trade-off method

Since the emergence of CPT, various papers have experimentally elicited utility and probability weighting functions. There are two common ways of elicitation. One way is to first assume parametric specifications for both utility and probability weighting, and then estimate the parameters of both. While recognizing the merits of these parametric studies, the initial assumption of the parameters may be a serious drawback. Moreover, since parameters for the utility function and the probability function are estimated at the same time, they may be confounded by each other. In comparison, the other method - the trade-off method (Wakker and Deneffe, 1996)- estimates both functions separately. As a result, the role of probabilities are minimized when estimating the utility function. No assumptions on the shape of the utility function are needed when inferring the probability weighting function.

We now demonstrate the detailed procedure of the trade-off method (hereafter TO method). The TO method first elicits a standard sequence of outcomes. This sequence is used to infer the utility function and later serves as a basis for the elicitation of the probability weighting function.

A sequence of outcomes is constructed as follows: Subjects are asked to choose between two lotteries A and B with $A : (p, x_1; 1 - p, r)$ and $B(p, x_0; 1 - p, R)$. While x_0, r, R, p are held fixed with $0 < r < R < x_0$ and $p \in (0, 1)$, x_1 is varied to find an outcome such that subjects are indifferent between the two lotteries. Once the indifference is achieved we proceed to find the next indifference. Here the elicited value of x_1 replaces x_0 in prospect B and x_2 replaces x_1 in prospect A. Then similar x_2 is varied until subjects are indifferent between the prospect $A : (p, x_2; 1-p, r)$ and the prospect $B : (p, x_1; 1-p, R)$. Under CPT, the above two indifference relationships imply

$$[1 - w(p)]u(R) + w(p)u(x_0) = [1 - w(p)]u(r) + w(p)u(x_1), \tag{2}$$

$$(1 - w(p))u(R) + w(p)u(x_1) = [1 - w(p)]u(r) + w(p)u(x_2). \tag{3}$$

Combining (2) and (3), it gives

$$u(x_2) - u(x_1) = u(x_1) - u(x_0), \tag{4}$$

That is, the outcomes x_0, x_1, x_2 distribute with equal distance in the utility axis.

Repeating this procedure $n - 1$ times, we obtain a sequence of outcomes $(x_0, x_1, \cdots, x_n)$ where

$$u(x_{i+1}) - u(x_i) = u(x_{i+2}) - u(x_{i+1}), \forall i = 0, 1, \ldots, n - 2. \tag{5}$$

Assuming that subjects prefer more money to less, $R > r$ implies $x_{i+1} > x_i$. A small value of x_0 is taken initially and then the elicited outcomes $x_1, \ldots, x_n$ increase stepwise, this way of eliciting utility functions is usually called the outward TO method. An alternative method would be to set $R < r$ and with other parameters remaining unchanged. With this modification the sequence of outcomes $x_0, x_1, \ldots, x_n$ decreases, which is called the inward TO method. Fennema and van Assen (1998) compare these two ways of eliciting the utility function and they find that the outward TO method produces results more consistent with Tversky and Kahneman (1992) and Abdellaoui (2000). In order to produce results comparable with previous literat, we also employ the outward TO method.

Having obtained the standard sequence of outcomes $(x_0, x_1, \cdots, x_n)$, in a second step we proceed to determine a sequence of probabilities. Similarly subjects asked to choose a lottery

$$
\begin{array}{cccccc}
Prob. & Gain & & Prob. & Gain & \\
p & x_0 & & p & x_i & \\
& & \sim & & & \\
1-p & x_6 & & 1-p & x_i, & \\
\multicolumn{2}{c}{prospect\ A} & & \multicolumn{2}{c}{prospect\ B} &
\end{array}
\tag{6}
$$

here x_0, x_6 are fixed. For each x_i, $i = 1, \ldots, 5$ p_i is varied until an indifference is achieved. This produces a sequence of p_i $i = 1, \ldots, 5$. In CPT, this indifference relationship imply

$$w(p_i)u(x_n) + (1 - w(p_i))u(x_0) = w(1)u(x_i). \tag{7}$$

After some simple algebraic manipulation, we have

$$w(p_i) = \frac{u(x_i) - u(x_0)}{u(x_n) - u(x_0)}, \quad \forall i = 1, \ldots, n-1. \tag{8}$$

By (4), we know that $u(x_{i+1}) - u(x_i)$ is constant for $i = 1, \ldots, n$. Using this condition the above equation can be simplified into

$$w(p_i) = \frac{i}{n}, i = 1, \cdot, n. \tag{9}$$

Going through $i = 1, \cdot, n$, we would have n points for the probability weighting function, of which the weights are calculated as $\frac{i}{n}$.

Several features of the above procedure are worth some additional remarks. Note that, for the elicitation of the utility function few assumptions are needed. Apart from requiring probability weighting function to be positive and increasing in p, no knowledge about the shape of the probability function, such as continuity or differentiability, is needed. This is a substantial advantage compared to the parametric method, where a specific form for the probability functions needs to be assumed. Second, when eliciting probability weighting functions, we only rely on the property that the sequence of points $(x_0, x_1, \cdots, x_n)$ distribute with equal distance in utility dimension; no assumptions about the form of the utility function is needed. Thus, the above procedure effectively avoids the confounding problem resulting from the simultaneous elicitation of the utility function and the probability weighting function in the parametric method. As a draw back of the TO method it should be mentioned that the reliability of the measurement of probability weighting depends crucially on the accurate assessment of the utility function.

3 The experiment

The experiment was conducted in June 2008 with 124 undergraduate students at the University of Jena. In total we ran 4 sessions, each session lasted about 50 minutes. Altogether the experiment consisted of 4 parts, in this paper we will only present the first two parts that concern the elicitation of the utility function and the probability weighting function. The results of the two other parts will be reported in a different paper. The experiment was programmed with ztree (Fischbacher (2007)). Participants' invitation was managed by ORSEE (Greiner (2004)).

We used the TO method to elicit separately the utility and the probability weighting function. As explained in the above section, we first constructed a standard sequence of outcomes (hereinafter TO experiment), and then used this standard sequence of outcomes to elicit a sequence of probabilities (hereinafter PW experiment). All outcomes and probabilities were obtained through a series of choice questions. Each question consisted of an choice between two prospects, and subjects were asked to choose the prospect they prefer. For payment one pair of prospects in TO experiment and one pair of prospects in PW experiment was randomly selected, the preferred lotteries were played and subjects were paid accordingly. In the experiment, all outcomes were in the unit of ECU (experimental currency unit). The exchange rate between ECU and Euro was 1 ECU = 0.05 Euro. The detailed instructions are given in the appendix of the paper.

3.1 Eliciting a standard sequence of outcomes for utility functions: TO experiment

For the TO experiment we specified the values of p, r, R, x_0 in (10) were set as follows: $p = \frac{1}{2}$, $r = 0$, $R = 10$, and $x_0 = 20$. A pair of following prospects or lotteries was first presented to the subjects:

$$\begin{array}{cc} Prob. & Gain \\ 0.5 & x_{i+1} \\ 0.5 & 0 \\ prospect\ A & \end{array} \sim \begin{array}{cc} Prob. & Gain \\ 0.5 & x_i \\ 0.5 & 10, \\ prospect\ B & \end{array} \tag{10}$$

and x_{i+1} was varied to establish a indifference relationship. In total we elicited a sequence of 6 outcomes $x_1, x2, \ldots, x_6$.

While the concept is clear, the practical implementation is not straightforward. For implementation some studies rely on the Becker-DeGroot-Marschak (BDM) mechanism (Becker et al., 1964) (see e.g. Irwin et al. (1998) and Keller et al. (1993)), others rely on the auction method (see, e.g., Coppinger, Smith, and Titus, 1980; Cox, Roberson, and Smith, 1982; Kagel, Harstad, and Levin, 1987; and Kagel and Levin, 1993). Both methods ask subjects to pick their indifference value out of a given range. A sensible choice, however, involves a through understanding of the mechanism, choosing a value out of a continuous range is typically cognitive demanding. Noussair et al. (2004) suggest that subjects are often confused or do not taken the procedure seriously. They show experimentally that compared to the other methods, the choice based method is easier for subjects to understand, and consequently yields more reliable data. Choice based methods, however, have one obvious drawback: A large number of choices is needed to infer a precise indifference value, which is typically difficult due to the time constraint in experiments. Taking above considerations into account, we rely on the (modified) bisection choice procedure.

The detailed algorithm of the (modified) bisection choice procedure is as follows:

1. Given any x_i, we first set a potential range for x_{i+1}'s indifference value. This potential range should be large enough to include potential indifference values for x_i, and it should be small enough to allow for a good inference of the indifference point. In the experiment, this potential range was determined by the following equations:

$$\underline{x} = max\{0, (x_i + R) * 0.5 - r\} \tag{11}$$

$$\bar{x} = (x_i + R) * 1.5 - r. \tag{12}$$

The determination of this range reflects the combined consideration of flexibility and efficiency. Let $x_m = \frac{\underline{x}+\bar{x}}{2}$ denote the middle point of the interval $[\underline{x}, \bar{x}]$. Subjects were first presented a pair of lotteries as in (10), with $x_{i+1} = x_m$. To ease calculations only integers were allowed. When x_i is not a even integer, the closest even integer larger than x_i is taken.

2. If A is preferred, we know that x_{i+1} must be increased in order to achieve indifference. We thus let $x_{i+1} = \frac{x_m+\bar{x}}{2}$. Likewise, if B is preferred, x_{i+1} must be decreased. We then let $x_{i+1} = \frac{x_m+\underline{x}}{2}$.

3. Repeat this procedure 4 more times, the interval containing the indifference point will become rather small. Finally, we choose the middle point of the final interval to be x_{i+1}.

A drawback of the bisection procedure is that it is not entirely incentive compatible. If subjects are aware of the entire experimental procedure from the start, they may have incentive to strategically misreport their choices. To see this, notice that pretending to be overly risk averse-choosing A all the time, raises x_{i+1} and thus increases the mean payoff of prospects B. Since subjects are paid their preferred prospect in one randomly chosen pair, this misreporting strategy may increase their expected experimental payoff. To make it more difficult to fully grasp the bisection procedure, we added two more choices to the elicitation of each indifference point. Therefore in total eight choices were taken to elicit each point. The display of these two choices is independent on participant's choices and is expected to make the inference of the whole algorithm more difficult.

The procedure may be best understood with a numerical example. In the experiment we started the elicitation with the following pair of prospects: $A = (20, 0.5; 10) \sim B = (x_1, 0.5; 0)$. The potential range of x_1 is $[15, 45]$. Participants will then face the following sequence of choices.

No.	Alternatives	Choice	Inference
1	$A = (20, 0.5; 10)$ vs $B = (30, 0.5; 0)$	A	$x_1 \in [30, 45]$
2	$A = (20, 0.5; 10)$ vs $B = (24, 0.5; 0)$	A	$x_1 \in [30, 45]$
3	$A = (20, 0.5; 10)$ vs $B = (38, 0.5; 0)$	A	$x_1 \in [38, 45]$
4	$A = (20, 0.5; 10)$ vs $B = (34, 0.5; 0)$	A	$x_1 \in [38, 45]$
5	$A = (20, 0.5; 10)$ vs $B = (41, 0.5; 0)$	B	$x_1 \in [38, 41]$
6	$A = (20, 0.5; 10)$ vs $B = (39, 0.5; 0)$	A	$x_1 \in [39, 41]$
7	$A = (20, 0.5; 10)$ vs $B = (40, 0.5; 0)$	A	$x_1 \in [40, 41]$
8	$A = (20, 0.5; 10)$ vs $B = (41, 0.5; 0)$	B	$x_1 \in [40, 41]^1$

Based these choices, x_1 is set to equal to the middle point of the final range $[40, 41]$, that is, 40.5. If subjects choose A all the way, we simply set x_1 equal to the upper bound of the initial range, which is 45.

After the elicitation of the six outcomes $x_1, x_2, \ldots, x_6$, we presented subjects an additional pair of choices for each x_i to check for the consistency of choices. This additional pair of choices was taken to be the 7^{th} iteration for each x_i. Note, that although this additional pair of choices has not reached the final indifference relationship, the remaining interval is quite small. This makes the consistency check a rather tough test that would strongly support stable preferences. Given the number of choices a memorization of choices seems rather unlikely.

3.2 Eliciting the probability weighting function: PW experiment

Having elicited a sequence of x_i, we proceeded to the sequence of probabilities. $, p_1, ..., p_5$. Subjects were presented with pairs of prospects of structure (6), $(x_0, p_i; x_6)$ and (x_i). Here p_i was varied to establish a indifference relationship. Again the indifference relationships were established via a (modified) algorithm.

1. For each p_i subjects were first presented with a sequence of five pairs of prospects of structure (6), where p_i is successively set to $.1, .9, .3, .7, .5$. After this was done for

all $x_i, i = 1, \ldots, 5$, it proceeded to the normal bisection procedure.

2. If there was only one switching point, two further iterations would be employed to find the point of indifference. For instance, if for a given x_i a subject preferred B over A for $p_i = 0.3$ and A over B for $p_i = .5$, then it could be inferred that her indifference probability must lie within the interval $[.3, .5]$. The bisection procedure (proceeding with $p_i = .4$) would be applied two times to elicit the indifference probability for this x_i.

3. If there were two or more switching points, a interval encompassing all switching points would be determined and a maximum of 4 iterations of the bisection procedure would be employed to find out the indifference probability.

With a standard sequence of $x_1, x_2, \ldots, x_6$, we got 5 probabilities $p_1, p_2, \ldots, p_5$. We also performed a consistency check, asking participants to choose between:(x_3) and $(x_4, p_6; x_2, 1-p_2)$. According to CPT choices are consistent it $p_6 = p_3$.

4 Results

We report the results in two steps, starting with some general results for utility and probability weighting, proceed with the classification of them in terms of risk attitudes and then turn to our main result: the relationship between these two dimensions of risk attitudes.

4.1 General result

Subjects' average earning was 16 Euro. 124 students participated in the experiment. For the analysis we discarded 5 subjects, partly due to computer problems and partly due to insufficient sensitivity towards the stimulus.

4.2 Classification of utility functions

To check for consistency in participant's choices the $7^t h$ choice pair of each x_i was repeated. Preference reversal occurred in 30% of the cases. Though this number may seem large, note that the remaining interval for the inference of x_i at the 7th choice is already quite small. Thus a consistent choice suggests strongly data reliability whereas a inconsistent choice does not necessarily imply a poor decision. The value is also comparable to the findings in Starmer and Sugden (1989) (26.5%) and Camerer (1989) (31.6%), which suggests that the elicited x_i are rather reliable.[2]

We classified the participants' utility or value function using $u(x) = x^\alpha$, which is often used in the literature. The sequence of values, $x_1, x_2, ..., x_6$ enables us to estimate α for each subject. An $\alpha < 1$ implies a concave utility function, while $\alpha \approx 1$ implies a linear utility function, and $\alpha > 1$ implies convex utility function. For a linear utility we set a tolerance level $0.9 < \alpha < 1.1$. According to our classification 67 subjects to have concave ($\alpha < 0.9$) , 24 subjects to have linear ($0.9 < \alpha < 1.1$), and 28 subjects to have convex utility ($\alpha > 1.1$). We varied this tolerance level slightly and found results to be robust.

Figure displays the distribution of α.

Since a wrong choice of parametric specification may bias results, we additionally used the non-parametric difference method. We calculated the first order difference $\Delta'_i = |x_i - x_{i-1}|$ for $i = 1, ..., 6$ and the second order difference $\Delta''_j = \Delta'_{j+1} - \Delta'_j$ for $j = 1, ..., 5$. Similar to Abdellaoui (2000), we classify

- a utility function to be concave if $\Delta''_j > 0$ for more than 3 out of 5 times,
- a utility function to be convex if $\Delta''_j < 0$ for more than 3 out of 5 times, ,
- a utility function to be linear.if $\Delta''_j \approx 0$ for more than 3 out of 5 times,

[2] Note that for x_1, when the interval is rather small preference reversal occurs in 39% of the cases, while it lowers to 23% for x_6. This further suggests that preference reversal was a result of the rather small choice interval.

	α	Difference	γ	Difference
Risk averse	67	68	95	81
Risk neutral	24	16	1	34(15+19)
Risk seeking	28	15	23	4

Table 1: Classification of the two functions

With these criteria, we filed 68 subjects with concave, 16 with linear, and 15 subjects with convex utility. The remaining 20 subjects could not be classified via this method. As shown in table (4.2), the two classification methods yield similar results. Hence, α reasonably captures the shape of the utility function.

4.3 Classification of probability weighting functions

A similar check of reliability was included in the PW experiment. Here consistency is checked by comparing $(x_6, p_3; x_0) \sim (x_3)$ and $(x_4, p_3'; x_2) \sim (x_3)$. According to CPT, the two probabilities should be the equal ($p_3 = p_3'$). Both median values of (p_3) and of (p_3') is equal to 0.5, and they are not significantly different (the mean difference $p_3 - p_3' = -0.015$, $p > 0.10$).

To confirm the non-linearity of probability functions, we performed a Friedman-test. The hypothesis that the probability weighting function is linear can be rejected at the 5% of level ($\chi^2 = 15.9137$ with $p < .0031$).

Classifying the probability weighting function is more difficult. Tversky and Kahneman (1992) advocate an inverse S shape. However, previous experiments find S, inverse S, linear, S, convex, as well as concave shaped probability weighting functions. The shape of the probability weighting has an important impact on risk attitude. For example, an inverse S shaped probability weighting function implies risk aversion for large probability gains and risk seeking behavior for small probability gains, while an S shaped probability weighting function implies the opposite. A convex probability weighting function implies risk aversion for gains, while a concave probability weighting function implies risk seeking for gains.

To properly classify probability weighting functions, we first check each subject's array of p_i for patterns. The vast majority of subjects has a convex probability weighting pattern. The non-parametric difference method confirms this. Note that the pattern of probability weighting is best discovered when p is close to 0 or 1. Here probability weighting is suspected to be most severe, while the middle range, i.e., when p is close to 0.5, patterns may be less obvious. Thus a crude but simple way to check for the shape of probability weighting functions is to compare the pairs $(w_1 \sim p_1)$ and $(w_5 \sim p_5)$. A convex probability weighting function implies $w_1 < p_1$ and $w_5 < p_5$, while a concave probability weighting function implies $w_1 > p_1$ and $w_5 > p_5$, an inverse S-shaped probability weighting function implies $w_1 > p_1$ and $w_5 < p_5$, and finally an S-shaped probability weighting function implies $w_1 < p_1$ and $w_5 > p_5$. Based on these criteria, we classified 81 subjects as pessimistic, 4 subjects as optimistic, 19 subject as inverse S-shaped, and 15 subjects as S-shaped.

This finding is not unusual comparing to previous literature. van de Kuilen (2008) and van de Kuilen et al. (2007) also found that majority of subjects possess a convex probability weighting function and there was little evidence for inverse S shaped probability weighting functions. Recall that in both Hong et al. (1987) and Schmidt and Zank (2008), a convex, concave or linear probability weighting function corresponds to risk aversion, risk seeking, or risk neutrality in this dimension. Also as discussed above, subjects with an inverse S or S shaped probability weighting are risk averse with some probabilities but risk seeking with other probabilities, which makes a simple classification of subjects in terms of risk attitudes difficult. To discuss the relationship between the two dimensions of risk attitudes, we need to obtain a better classification of probability weighting function. This measure should consistently classify subjects' risk attitudes in the probability dimension. Taking the data pattern and the practical classification problem into account, we assume a probability weighting function with the shape $w(p) = p^{\gamma}$. Here the precise value of γ is not important, what we need is only a order of subjects' risk attitudes. With five data points $p_i, i = 1, 2, 3, 4, 5$, we estimated a γ for each subject.

	concave α	linear α	convex α	sum
pessimistic γ	53	19	23	95
neutral γ	0	1	0	1
optimistic γ	14	4	5	23
sum	67	24	28	119

Table 2: The two dimensions of risk attitudes

In order to highlight the different dimensions of risk attitude, we classify the probability weighting function as follows:

- **Optimistic**: a subject is optimistic if her probability weighting function is concave ($\gamma < 1$),
- **Neutral**: a subject is neutral if her probability weighting function is linear ($\gamma \approx 1$), and
- **Pessimistic**: a subject is pessimistic if her probability weighting function is convex ($\gamma > 1$).

Again, we varied the tolerance level for γ, and the classification result was robust. We fixed the range for linear probability weighting to $0.98 < \gamma < 1.02$. According these criteria, we classified 95 subjects as pessimistic, 23 subjects as optimistic, and 1 subject as linear. Note, that these results are similar comparable to the non-parametric difference method.

4.4 Central result

Last we turn to our main hypothesis: what's the relationship between the two dimensions of risk? This result is reported in Table (4.4).

The largest group in Table (4.4) are the subjects with concave utility and pessimism in the probability weighting dimension (53 subjects). This finding is amiable to economists, since most theoretical models rely on the assumption that agents are risk averse. Our result suggests that the majority of the population may indeed be risk averse in both dimensions

There are further interesting patterns in the data. The third cell in the first row denotes the convex/pessimistic subjects. They are the second largest group in our classification (23 subjects). Mirroring this is the first cell in the third row. This cell denotes the concave/optimistic subjects. Here we have 14 subjects. These subjects are risk averse in one dimension but risk seeking in the other. This is interesting since although both utility functions and probability weighting functions captures information about risk attitudes, they seem to have different foundations.

The subjects who are risk averse in both dimension represent the largest proportion. Among these subjects, one natural question to ask will be: is a subject who is more risk averse in one dimension is also more likely to be risk averse in the other? If this is so, these two dimensions of risk attitudes are well correlated. As a result it might not be that problematic to use the curvature of utility function as the single proxy for risk attitudes. To test this hypothesis, we ran a Spearman's ρ rank correlation test between α and γ for these 53 subjects. The correlation is insignificant (Spearman's ρ, $p > 0.10$). This finding suggests that these two dimensions of risk are different and, therefore, necessary to consider. Each captures a different characteristic of an individuals' attitude towards.

These results raises a question: what is the nature of risk attitudes? Schunk and Betsch (2006) argue that risk attitudes result from the cognitive limitations. They found that people who have large bias also exhibit higher risk aversion. To look into the nature of risk attitudes, we examine the *decision time* in comparison to risk attitude. This relationship is not straightforward. One can argue in both directions: taking more time for decision making can imply a more thought through decision and therefore *use* more time to make a decision is more likely to be less biased and consequently less risk averse. The reverse - a negative relation - can also be argued: the more time a subject needs, the less cognitive capacity she has and the more risk averse she is. These two arguments suggest that a clean relation between decision time and risk attitudes is best found on a homogenous pool. The analysis done on the whole subject pool confirms the above intuition. We found no correlation between decision time and risk attitudes in either dimension.

Looking at table (4.4): subjects in the same cell may have more similar characteristics than the group of all subjects and are thus more homogeneous. To test this, we performed a correlation analysis on the three above mentioned cells: Subjects who are risk averse in both dimensions, subjects who are risk averse in one dimension but risk seeking in the other. We found that the decision time were positively correlated with both dimensions of risk attitudes (Spearmann correlation test, $p < 0.05$). This suggests that risk attitudes are indeed due to cognitive limitations. Subjects who needed more time to make a decision are more likely to have high cognitive limitation and are therefore more likely to be risk averse, in either dimensions.

5 Discussion

It is now probably less controversial to argue that risk attitudes have two dimensions. Yet, to the best of our knowledge no study so far looked at the relation between these two dimensions of risk. This paper serves to answer this question. Our result suggests that these two dimensions of risk attitudes seem to capture different characteristics of individuals' risk attitudes. Although individuals who are risk averse in one dimension are also likely to be risk averse in the other dimension, the two dimensions show no significant correlation. Therefore an accurate measurement of risk attitudes requires the measurement of both. Predictions based risk attitudes captured only by the curvature of utility functions can be quite far way from real behaviors, as showed by the findings in numerous literature.

A deeper understanding of the two dimensions of risk attitudes and their interplay would require further research on the nature of risk attitudes. One way to examine the nature of risk attitudes might be to use brain scanning. Possibly comparing decision making under certainty, where only the utility function is involved activates different parts of brain, to decision making under risk, where both the utility function and probability weighting function are needed.

6 Appendix: Experimental Instructions

6.1 General Information

Thank you for participating in our experiment. Please end all conversations now and switch off your cell phone. Please read the instruction carefully. The money you earn will depend on the choice you make. The money will be paid to you in cash at the end of the experiment. Throughout the experiment, we shall speak of ECU (experimental currency units) rather than Euro. The exchange rate between ECU and Euro is fixed to

20 ECU= 1 Euro Please do not communicate during the experiment, and raise your hand if you have questions. We will answer your questions individually. It is very important that you obey these rules, since we would otherwise be forced to exclude you from the experiment and hence from payment.

The Experiments consists of four parts. Each part consists of several rounds. In each round you have to make a decision. At the end of the experiment one round of each part is selected for payment. In all four rounds will be relevant for your payment.

6.2 Instructions for the TO experiment

The first part of the experiment comprises 42 rounds. In each round, you will be presented with a pair of risky alternatives. Your task is to pick your preferred alternative. To make the comparisons easier, the payoffs are also presented in the upper right corner of the screen. The pairs of risky alternatives will have the following format:

The alternatives shown above can be better understood by using the following thinking. Imagine a big watch with one arm. In above figure, 40% of the panel is covered by white and 60% of the panel is covered by black. The arm of the watch stops equally likely at each position of the watch. Suppose now you have chosen alternative A from the above pair. Then, if the arm stops in the white area, you are paid 300 ECU, if the arm stops at the black area, you are paid 100 ECU. (Equivalent, had you chosen B you would be paid 200 in case of black and 50 in case of white)

At the end of this part of the experiment, one of your choices will be randomly selected and played, and the resulting outcome will be your experimental earning in this part.

6.3 Instructions for the PW experiment

This part is similar to the first part. Again you will be asked for your preference between two lotteries, the difference being that lottery B always gives a fixed payoff. Another difference is that the probabilities in lottery A change for each decision. Using the picture of the first part: the division of the circle between black and white changes for each decision. Please think carefully before each decision, since a confirmed choice cannot be changed.

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