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# **Optimal Multiproduct Nonlinear Pricing**

## **with Correlated Consumer Types<sup>\*</sup>**

By

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*Abstract:* We examine the design of nonlinear prices by a multiproduct monopolist who serves customers with multidimensional but correlated types. We show that the monopoly can exploit the correlations between consumers' types to design pricing mechanisms that fully extract the surplus from each consumer. Our main insight is that regardless of the dimensionality of the consumers types and the number of goods that the monopoly produces, the surplus that each consumer gets from buying is a scalar. Hence, it is possible to design a two step mechanism where in the first step the monopoly induces the consumers to make efficient purchasing decisions (given their private information), and in the second step the monopoly extracts the surplus from each consumer via a (random) fixed fee.

*Keywords:* nonlinear prices, multidimensional types, correlated types, incremental cost, Clarke-Groves mechanisms

*JEL classification numbers:* D42, D82

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## 1. Introduction

The mechanism design literature has traditionally focused on cases where the private information of agents can be captured by a single parameter. Yet, in many applications of mechanism design (e.g., nonlinear pricing, the design of product lines, optimal regulation, optimal taxation, etc.), agents have multiple characteristics and cannot be sorted out in a satisfactory manner according to only one of these characteristics. Indeed, in many applications, the data on agents is only imperfectly correlated with their types so the ability to use as many characteristics as possible is highly advantageous. The fact that agents may have multidimensional types is troubling because, as Sibley and Srinagesh (1997), Armstrong (1996), Rochet and Chone (1998), and Rochet and Stole (1999) demonstrate, some of the insights obtained in the unidimensional case do not generalize to the multidimensional case. Moreover, apart from highly specific cases, the standard methodology for characterizing optimal mechanisms in the unidimensional case is not directly applicable in the multidimensional case.

Recently, some progress was made in relaxing the restrictive assumption that agents are characterized by a single parameter. Wilson (1993) develops an innovative and powerful technique for characterizing optimal nonlinear prices in the unidimensional case and shows that this technique can also be used to solve specific examples in the multidimensional case. Sibley and Srinagesh (1997) analyze optimal multiproduct nonlinear pricing and show that if the demand curves are uniformly ordered (i.e., the ordering of the demand functions in any market gives rise to an ordering of the utility functions that is independent of prices), the monopoly's problem amounts to designing optimal prices separately for each market. Armstrong (1996) examines a similar problem and shows that generically, the monopoly will exclude some low value consumers from all markets. He then goes on to show that in a special class of cases, it is possible to extend the standard approach, initiated by Mirrlees (1971), to obtain closed form solutions for the monopoly's problem. Armstrong (1999) shows that if the number of products offered by the monopolist is large, then it is possible to design almost optimal cost-based two part tariffs.<sup>1</sup> Rochet and Chone (1998) design a new technique, called the "sweeping" procedure,

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<sup>1</sup> Bakos and Brynjolfsson (1999) use similar techniques and show that a monopoly that provides a large number of products that can each be produced at a small marginal cost (e.g., information goods) may wish to bundle these products rather than sell them separately.

to deal with the bunching problem that was pointed out by Armstrong (1996). Using this technique, they extend the Mussa and Rosen (1978) model of monopoly quality discrimination to the case where each consumer is parametrized by a multidimensional vector of characteristics. Rochet and Stole (1999) extend the Mussa and Rosen model by considering the case in which consumers' types are two-dimensional and specify not only the marginal willingness to pay for quality but also the reservation utility of the consumer (i.e., the utility from not buying). They develop a methodology for addressing this class of problems and show that although many properties of the optimal solution in the single dimensional case re-emerge, at the bottom of the distribution it may no longer be optimal to distort the level of quality or ensure complete separation of types.

While these papers make an important progress in analyzing a very difficult and empirically relevant problem, they retain the traditional assumption (e.g., of Mussa and Rosen 1978, Myerson 1981, and Maskin and Riley 1984) that consumers' types are independently drawn from some commonly known distribution. In practice though, the independence assumption appears to be quite strong since it suggests that information about one agent is completely uninformative about the types of other agents. In the context of nonlinear prices for instance, this assumption suggests that data about the demands of existing customers cannot be used in predicting the pattern of demands of new customers.

In this paper we relax the independence assumption and examine the optimal design of nonlinear prices by a multiproduct monopolist who serves consumers with multidimensional but correlated types. We show that the plausible assumption that consumers' types are correlated has a dramatic impact on the results. First, despite the fact that the monopoly produces many products and consumers' types are multidimensional, the characterization of the optimal pricing mechanism is surprisingly straightforward and only requires us to solve a system of linear inequalities (albeit quite large). Second, if the correlations between consumers are sufficiently strong, the monopoly can design a pricing mechanism that fully captures, in expectation, the entire social surplus.<sup>2</sup> Third, our approach does not require us to impose many of restrictions

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<sup>2</sup> An important caveat is that we assume that consumers are risk neutral and not subject to liquidity constraints, and the correlations between them are common knowledge. Robert (1991)

that Wilson (1993), Armstrong (1996, 1999), Armstrong and Rochet (1999), Rochet and Chone (1998), Rochet and Stole (1999), and Sibley and Srinagesh (1997) had to impose on the preferences of consumers, the distribution of types, and the monopoly's cost functions, in order to analyze the independent types case. In particular, we do not need to assume that consumers' preferences satisfy any type of a single crossing property, or assume that the monopoly has a constant returns to scale technology.

A main insight in this paper is that although the monopoly produces multiple products and although consumers have multidimensional types, it is nonetheless possible to construct simple price mechanisms (e.g., incremental cost pricing or a Clarke-Groves mechanism) that induce each consumer to make an efficient buying decision. While these price mechanisms induce efficient outcomes, they do a poor job in extracting consumers' surplus. In fact, under incremental cost pricing, the monopoly just breaks even and consumers capture the entire surplus. But since at this point the surplus of each consumers is a scalar, we are back in the unidimensional case.<sup>3</sup> Assuming that consumers' types are correlated, we can augment the price mechanism with a (random) fixed fee that will extract, in expectation, the full surplus from each consumer, using the techniques of Crémer and McLean (1985, 1988) when the set of consumers' types is finite and McAfee and Reny (1992) when it is infinite. The fixed fee that each consumer pays is random because it depends on the purchasing decisions (i.e., reports) of other consumers, whose exact types the consumer still does not know when making his/her own report.

Our approach can be viewed as a two step mechanism. In the first step, the monopoly

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shows in the context of a single good auction problem, that full extraction of the surplus is impossible if buyers are risk averse or subject to limited liability constraints. Neeman (1999) shows that if the correlations between agents' types are not common knowledge, it may be impossible to design a mechanism that fully extracts the surplus. The characterization of optimal nonlinear prices when consumers are risk averse or subject to liquidity constraints is of course a much harder problem. Indeed, so far the literature on multidimensional screening (e.g., Armstrong 1996 and 1999, Armstrong and Rochet 1999, Rochet and Chone 1998, Rochet and Stole 1999, and Sibley and Srinagesh, 1997) has only considered risk neutral consumers.

<sup>3</sup> Sharkey and Sibley (1993) and Armstrong and Vickers (1999) also exploit the fact that consumer surplus is a scalar and study competition with nonlinear prices in terms of the surplus induced by potentially complex and multidimensional pricing schemes.

ensures efficiency by using a pricing mechanism that induces consumers to maximize their surplus. In the second step, the monopolist extracts the surplus from each consumer through an appropriate fixed fee.<sup>4</sup> When the set of consumers' types is finite, the fixed fees fully extract the expected surplus from each consumer; in the infinite case, the surplus extraction is arbitrarily close to being full.<sup>5</sup>

The results in this paper generalize the results in Crémer and McLean (1985, 1988) in two ways. First, Crémer and McLean (1985) also consider a monopoly price discrimination problem and show that under certain conditions the monopoly can fully extract the expected surplus from every consumer. However, unlike in our paper, consumers in their model have essentially unidimensional types because Assumption 2 in their paper states that consumers' types can be ranked according to the consumers' marginal willingness to pay. In the current paper in contrast, types are multidimensional and cannot be ranked from "high" to "low." In addition, we consider a multiproduct monopoly whereas they consider a single product monopoly. Second, Crémer and McLean (1988) consider a single-unit auction, where the reservation value of the seller is 0, and the expected payoff of agent  $i$  (using the notations of the current paper) is  $q_i t_i - p_i$ , where  $q_i$  is the probability that agent  $i$  gets the object,  $t_i$  is the agent's (possibly multidimensional) type, and  $p_i$  is the agent's payment. Their model can be reinterpreted as a single-product monopoly setting in which the monopoly can produce a fixed quantity at no cost, and the buyers have Mussa-Rosen (1978) utility functions, with  $q_i$  representing a quantity rather than the probability of winning. Viewed in this way, their setting is obviously much more restrictive than ours, as in our setting

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<sup>4</sup> Shinotsuka and Wilkie (1999) have used a similar two step approach in the context of optimal multi-object auctions. Our two step approach is also reminiscent of Matthews (1997) who shows that it is optimal for a principal who needs to induce an agent to exert effort while insuring the agent against risk to break the contracting process into two steps. First, before the agent exerts effort, the principal should offer the agent a menu of output levels that the agents needs to deliver and associated wages. Then, after the agent has already exerted effort, but before uncertainty is realized, the principal should sell the agent insurance. Unlike in Matthews where the problem is one of moral hazard, we do not need to worry about the timing of the two steps as the problem that we consider is one of adverse selection.

<sup>5</sup> Our approach can also be easily extended to the case where the monopoly is regulated and is allowed to capture only a fraction of the consumer surplus. In that case, the first step remains unchanged while in the second step, the fixed fees should be multiplied by the required fraction.

the monopoly produces multiple products (i.e.,  $q_i$  is a vector rather than a scalar), the expected payoff of buyer  $i$  is  $V(q_i, t_i) - p_i$ , where  $V(\cdot)$  is any function that is continuous and increasing in  $q_i$  with  $V(0, t_i) = 0$  (but not necessarily multiplicative), and the monopoly has a general cost function.

It should be emphasized that although the fixed fee of each consumer depends in our model on the purchasing decisions (i.e., reports) of all other consumers, it is equally possible to condition it instead on the realization of a public signal that is correlated with the consumers' types (e.g., the state of the economy, consumption of related products, or consumption of similar products in other markets). This alternative approach was first used by Riordan and Sappington (1988) in the context of procurement contracts; they show that if the public signal is "sufficiently rich," then a buyer can design a mechanism that fully extracts the expected surplus from the seller.<sup>6</sup> Although it is straightforward to rewrite our model using the Riordan and Sappington approach (this requires only minor adjustments), we chose to condition the fixed fees on the reports of other consumers in order to keep our analysis as close as possible to that in Crémer and McLean (1985, 1988).

The rest of the paper is organized as follows. In Section 2 we present the model. In Section 3 we characterize the optimal pricing mechanism when the monopoly's cost function is decomposable in the sense that the cost of serving one consumer is independent of the quantity produced for other consumers. In Section 4 we illustrate our approach by means of an example. Then we extend the results in Section 5 to the case where the monopoly's cost is not decomposable, and in Section 6 to the case where consumers have continuous type spaces. We conclude in Section 7.

## 2. The model

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<sup>6</sup> In general, the public signal may need to have at least as many possible realizations as there are consumers' types in order to be "sufficiently rich" (otherwise, the equivalent of condition CM1 that appears in Section 3 below may fail). However, Riordan and Sappington (1988, Corollary 4) and Bose and Zhao (2000) show that under certain conditions, full extraction of the surplus may be still possible even when there are fewer signals than types.

A monopoly provides  $k$  products or services to a set of consumers  $N = \{1, \dots, n\}$ . The utility function of consumer  $i \in N$  when he buys a bundle of products,  $q_i \in \mathfrak{R}_+^k$ , at a price  $p_i$ , is quasi-linear and given by  $V(q_i, t_i) - p_i$ , where  $t_i$  is an  $m_i$  dimensional vector of characteristics that describes the consumer's (multidimensional) type and is drawn from a compact non-empty metric space  $T_i \subset \mathfrak{R}^{m_i}$ . We assume that the gross utility function,  $V(q_i, t_i)$ , is continuous and increasing in  $q_i$ , and if a consumer does not buy anything, his/her gross utility is  $V(0, t_i) = 0$ . Let  $T = T_1 \times \dots \times T_n$  be the consumers type space and let  $T_{-i} = T \setminus T_i$  be the type space of all consumers but consumer  $i$ . We will call elements of  $T$  **consumer profiles** and denote them by  $t$ . There is a density function,  $f$ , defined over  $T$ . We denote by  $f_i(t_i, t_{-i})$  the marginal density of  $f$ , and use  $f(t_{-i} | t_i) = f(t_i, t_{-i}) / f_i(t_i, t_{-i})$  to denote the probability distribution on  $T_{-i}$ , conditional on consumer  $i$ 's type being  $t_i$ . Given a vector of bundles  $q \equiv (q_1, \dots, q_n)$ , the monopoly's cost function is  $C(q)$ . We assume that all consumers and the monopolist are risk neutral.

In most of the paper we shall assume that  $T_i$  is a finite set for all  $i \in N$  and use  $\ell_i$  to denote its cardinality (i.e., the number of possible types that consumer  $i$  can assume). Using analogous notation, we denote the cardinality of the set  $T_{-i}$  by  $\ell_{-i} = \sum_{j \neq i} \ell_j$ . In Section 6 below, we shall relax the assumption that the consumers type space is finite and allow  $T_i$  to be infinite for all  $i \in N$ .

As usual we invoke the revelation principle and restrict attention to direct revelation mechanisms. Under a direct revelation pricing mechanism, each consumer reports his/her type to the monopoly and given a vector of reported types,  $\hat{t} \in T$ , the monopoly chooses an allocation of bundles  $q(\hat{t}) \equiv (q_1(\hat{t}), \dots, q_n(\hat{t}))$  and a corresponding vector of prices  $p(\hat{t}) \equiv (p_1(\hat{t}), \dots, p_n(\hat{t}))$ . We will often refer to pairs of bundles and prices as **offers**. Using this terminology, a pricing mechanism simply specifies a list of offers, one for each reported type. We now introduce three useful definitions:

**Definition 1:** *A pricing mechanism is ex post efficient at consumer profile  $t$  if*



$$q^*(t) \in \underset{q}{\operatorname{Argmax}} \sum_{i \in N} V(q_i, t_i) - C(q). \quad (1)$$

That is, a pricing mechanism is ex post efficient if it selects an allocation  $q(t)$  that maximizes the social surplus.

**Definition 2:** A pricing mechanism is incentive compatible if for all  $i \in N$ , all  $t_i, \hat{t}_i \in T_i$  and all  $t_{-i} \in T_{-i}$

$$V(q_i(t), t_i) - p_i(t) \geq V(q_i(t_{-i}, \hat{t}_i), t_i) - p_i(t_{-i}, \hat{t}_i). \quad (2)$$

When a pricing mechanism is incentive compatible, it is a dominant strategy for each consumer to make a truthful report so  $\hat{t}_i = t_i$  for all  $i \in N$ .

**Definition 3:** Given a pricing mechanism and given his/her report, consumer  $i$ 's expected surplus is

$$S_i(\hat{t}_i | t_i) \equiv \sum_{t_{-i} \in T_{-i}} [V(q_i(t_{-i}, \hat{t}_i), t_i) - p_i(t_{-i}, \hat{t}_i)] f(t_{-i} | t_i). \quad (3)$$

A pricing mechanism is Bayesian incentive compatible if, for all  $i \in N$  and all  $t_i, \hat{t}_i \in T_i$

$$S_i(t_i) \equiv S_i(t_i | t_i) \geq S_i(\hat{t}_i | t_i), \quad (4)$$

and it is interim individually rational if for all  $i \in N$  and all  $t_i \in T_i$

$$S_i(t_i) \geq 0. \quad (5)$$

Bayesian incentive compatibility is a weaker concept than incentive compatibility because it only requires that each consumer will find it optimal to make a truthful report, given his/her beliefs about other agents' types.

Given an incentive compatible and interim individually rational pricing mechanism  $M$ ,

the monopoly's expected profit is given by

$$\pi(M) = \sum_{t \in T} \left[ \sum_{i \in N} p_i(t) - C(q(t)) \right] f(t). \quad (6)$$

Note that in writing  $\pi(M)$  we use the fact that  $M$  is incentive compatible to evaluate the allocation  $q$  at the true consumer profile  $t$  (rather than an arbitrary vector of reports,  $\hat{t}$ ) and use the fact that  $M$  is interim individually rational to integrate profits over all possible realizations of consumer profiles (i.e., no type of consumer is ever excluded).

If the monopoly knew the true consumer profile,  $t \in T$ , it could have engaged in first degree price discrimination and capture the maximal social surplus given  $t$ . Thus the maximal social surplus places an upper bound on the expected profit that the monopoly can attain. We will say that a pricing mechanism **fully extracts the surplus** if it enables the monopoly to reach this upper bound in expectation.<sup>7</sup> Formally, we introduce the following definition:

**Definition 4:** *A pricing mechanism  $M$  is said to fully extract the surplus if*

$$\pi(M) = \sum_{t \in T} \left[ \sum_{i \in N} p_i^*(t) - C(q^*(t)) \right] f(t), \quad (7)$$

where  $(p_1^*(t), \dots, p_n^*(t))$  and  $q^*(t) \equiv (q_1^*(t), \dots, q_n^*(t))$  are the prices and bundles offered under first degree price discrimination.

When a pricing mechanism fully extracts the surplus, the monopoly completely overcomes the problem of informational asymmetry in the sense that, in expectation, it does as well as under first degree price discrimination. The following lemma provides a straightforward but useful characterization of the conditions under which full extraction of the surplus is possible. The lemma is closely related to Lemma 1 in Crémer and McLean (1988).

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<sup>7</sup> Since for some realizations of  $t$ , the monopoly's profit may exceed the social surplus (in which case at least some consumers will end up with a negative utility), while for other realizations it may fall short of the social surplus (in which case at least some consumers will have a positive utility); yet in expectation, each consumer breaks even so the expected profit of the monopoly is maximal.

**Lemma 1:** *A pricing mechanism  $M$  fully extracts the surplus if and only if it is ex-post efficient at the true consumer profile  $t$  and if  $S_i(t_i) = 0$  for all  $i \in N$ .*

**Proof:** First, we prove that ex-post efficiency and  $S_i(t_i) = 0$  for all  $i \in N$  imply that  $M$  fully extracts the surplus. To this end, note that by Definition 1, ex post efficiency implies that the surplus is maximal at every consumer profile. Since  $S_i(t_i) = 0$  for all  $i \in N$ , the monopoly fully captures the expected surplus. Next we prove that if  $M$  fully extracts the surplus then it must be that  $M$  is ex post efficient and  $S_i(t_i) = 0$  for all  $i \in N$ . By definition 4,  $M$  fully extracts the surplus if, on average, it generates a surplus that equals the maximal level of social surplus. But since by definition,  $M$  cannot achieve more than the maximal level of social surplus at any consumer profile, it can generate on average the maximal level of social surplus only if it does so at every consumer profile (i.e., it is ex post efficient). Clearly then, if  $M$  fully extracts the social surplus, each consumer must end up, on average, with a 0 level of utility. ■

### 3. Decomposable cost functions

In this section we consider the case where the cost of serving one customer is independent of the quantity produced for other customers. To this end we introduce the following definition:

**Definition 5:** *The cost function is said to be decomposable if  $C(q) = \sum_{i \in N} C_i(q_i)$ , where  $C_i(q_i)$  is the cost of producing for consumer  $i$ .*

When  $C(q)$  is decomposable, each of its components can be directly attributed to a specific consumer. Therefore,  $C(q)$  is simply the sum of the  $n$  consumer-specific cost functions, each of which is computed as if the consumer was the only one served by the monopoly. It is important to note that decomposability does not place any restrictions on the consumer-specific cost functions. In particular, the cost of serving consumer  $i$  can have a consumer-specific fixed cost component and the variable cost of serving consumer  $i$  need not exhibit constant returns to scale or to scope. The case of decomposable cost functions has also been considered by Armstrong (1996). It includes as a special case constant marginal costs which are typically

assumed in the literature (e.g., Mussa and Rosen 1978, Maskin and Riley 1984, Sibley and Srinagesh, 1997, and Rochet and Chone, 1998), but is much more general because it allows the customer specific costs to be nonlinear. In the next lemma we prove an important implication of decomposability that will be useful in what follows.

**Lemma 2:** *If  $C(q)$  is decomposable, the efficient allocation at a consumer profile  $t$  is such that each consumer gets an offer that depends only on the consumer's own report.*

**Proof:** When  $C(q)$  is decomposable, the social surplus can be written as

$$\sum_{i \in N} [V(q_i, t_i) - C_i(q_i)]. \quad (8)$$

Since this expression is separable across consumers, the efficient allocation at consumer profile  $t$  is such that  $(q_1^*(t_1), \dots, q_n^*(t_n))$ . That is, the bundle of each consumer depends only on the consumer's own report but not on the reports of other consumers. ■

Next, we introduce the augmented incremental cost (AIC) mechanism.

**Definition 6:** *Given a vector of reported types  $\hat{t}$ , the augmented incremental cost (AIC) mechanism specifies for each consumer  $i \in N$ ,*

- (i) *a bundle  $q_i^*(\hat{t})$  that corresponds to the efficient allocation at  $\hat{t}$ ,*
- (ii) *a usage fee,  $r_i$ , such that  $r_i(q(\hat{t})) = C(q(\hat{t})) - C(q_{-i}(\hat{t}), 0)$ .*
- (iii) *a (random) fixed fee,  $z_i$ .*

Under the AIC mechanism, each consumer reports his/her type to the monopoly. Given a vector of reports,  $\hat{t}$ , each consumer gets an offer that specifies a bundle of products dictated by the efficient allocation at  $\hat{t}$ , and a price that consists of a fixed fee plus a usage fee. The latter is equal to the incremental cost of producing the consumer's bundle, assuming that all other consumers are already served. In the case where  $C(q)$  is decomposable, the offers under the AIC mechanism are in fact simpler since by Lemma 2, the allocation is such that each consumer gets

a bundle that depends only on his/her report. Consequently, given a vector of reports,  $\hat{t}$ , the allocation of bundles under the AIC mechanism is  $(q_1^*(\hat{t}_1), \dots, q_n^*(\hat{t}_n))$  and the usage fees are such that  $r_i(q_i^*(\hat{t}_i)) = C_i(q_i^*(\hat{t}_i)) - C_i(0)$  for all  $i \in N$ . The fixed fees dictated by the AIC mechanism will be characterized in Theorem 1 below.

By construction, the AIC mechanism is ex post efficient. In the next lemma we prove that absent fixed fees, the AIC mechanism is also incentive compatible and individually rational.

**Lemma 3:** *If  $C(q)$  is decomposable, then absent fixed fees, the AIC mechanism is incentive compatible and interim individually rational.*

**Proof:** Recalling that when  $C(q)$  is decomposable, that  $q_i^*$  depends only on consumer  $i$ 's own report, and that the usage fee that consumer  $i$  pays is  $r_i(q_i^*(\hat{t}_i)) = C_i(q_i^*(\hat{t}_i)) - C_i(0)$ , it follows that absent fixed fees, the problem that consumer  $i$  faces when deciding what to report is given by

$$\text{Max}_{\hat{t}_i \in T_i} V_i(q_i^*(\hat{t}_i), t_i) - (C_i(q_i^*(\hat{t}_i)) - C_i(0)). \quad (9)$$

Since  $q_i^*(\hat{t})$  is ex post efficient at  $\hat{t}$ , it is by definition optimal for consumer  $i$  to make a truthful report no matter what other consumers report. Therefore the AIC mechanism without fixed fees is incentive compatible.

Since the AIC mechanism is ex post efficient and incentive compatible, it selects the efficient allocation at the truthful vector of reports  $t$ . Hence, the social surplus at  $q^*(t)$  must be at least as high as the social surplus at any other allocation, including the allocation  $(q_{-i}^*, 0)$  in which every consumer but  $i$  gets the same bundle as in  $q^*(t)$  while consumer  $i$  gets 0. Hence,

$$\sum_{i \in N} V(q_i^*(t), t_i) - C(q^*(t)) \geq \sum_{j \in N \setminus i} V(q_j^*(t), t_j) + V(0, t_i) - C(q_{-i}^*(t), 0). \quad (10)$$

Using Lemma 2 and the assumption that  $C(q)$  is decomposable, recalling that  $V(0, t_i) = 0$ , and rearranging terms, the inequality becomes:

$$V(q_i^*(t_i), t_i) - (C_i(q_i^*(t_i)) - C_i(0)) \geq 0. \quad (11)$$

Noting that the expression inside the brackets is just the usage fee of consumer  $i$ , equation (11) implies that absent a fixed fee, each consumer gets a nonnegative expected surplus. By Definition 3, the mechanism is therefore interim individually rational. ■

Lemma 3 shows that when  $C(q)$  is decomposable, then under an AIC mechanism with no fixed fee, it is a dominant strategy for each consumer to make a truthful report. The reason for this is that when  $C(q)$  is decomposable, the social surplus is separable across consumers, so the bundle that maximizes the utility of each consumer is also the one that maximizes social surplus. Therefore, it is optimal for each consumer to report truthfully and thereby enable the monopoly to select the bundle that maximizes the consumer's utility.

Since the usage fees only cover the monopoly's variable costs, consumers are left with the entire surplus. Absent fixed fees, the expected surplus of consumer  $i$  under the AIC mechanism, given the consumer's type, is given by

$$S_i^{AIC}(t_i) \equiv S_i^{AIC}(t_i | t_i) = \sum_{t_{-i} \in T_{-i}} [V_i(q_i^*(t_i), t_i) - r_i(q_i^*(t_i))] f(t_{-i} | t_i). \quad (12)$$

The monopoly's problem now is to design the fixed fees so as to extract the surplus from each consumer without violating incentive compatibility and interim individual rationality. In order to ensure that it is possible to design such fixed fees, we introduce the following condition which is based on Crémer and McLean (1988):

**Condition CM1 (Crémer and McLean 1988):** For all  $i \in N$ , there does not exist a type  $t_i \in T_i$  and a list of  $\ell-1$  numbers,  $(\rho_1, \dots, \rho_{\ell-1})$ , not all equal to 0, such that:

$$f(t_{-i} | t_i) = \sum_{j \in N \setminus i} \rho_j f(t_{-i} | t_j), \quad \text{for all } t_{-i} \in T_{-i}. \quad (13)$$

When Condition CM1 holds, the  $\ell_i \times \prod_{j \in N \setminus i} \ell_j$  matrix of conditional probabilities,  $\Gamma_i$ , whose

rows are indexed by the elements of  $T_i$  and whose columns are indexed by the elements of  $T_{-i}$  is of rank  $\ell_i$ . In Section 4 below we provide an example that illustrates the matrix  $\Gamma_i$  and the full rank condition. Roughly speaking, this means that every realization of  $t_i$  gives consumer  $i$  some information about the types of other consumers,  $t_{-i}$ , in the sense that it induces a different belief about the likelihood of different realizations of  $t_{-i}$ . The implication of condition CM1 is that since  $\text{rank } \Gamma_i = \ell_i$ , then (see e.g., the Corollary on p. 37 in Gale, 1960) there exists for any vector  $x_i \in \mathfrak{R}^{\ell_i}$ , another vector  $z_i \in \mathfrak{R}^{\ell_{-i}}$ , such that for all  $i \in N$ ,

$$x_i = \sum_{t_{-i} \in T_{-i}} z_i f(t_{-i} | t_i), \quad \text{for all } t_i \in T_i. \quad (14)$$

Note from (14) that for all  $i \in N$ , the vector  $z_i$  depends only on  $t_{-i}$ . Hence we can write a vector  $z_i$  that satisfies equation (14) as  $z_i(t_{-i})$ . Now, if we set  $x_i = S_i^{\text{AIC}}(t_i)$ , where  $S_i^{\text{AIC}}(t_i)$  is given by equation (12), then a fixed fee  $z_i(t_{-i})$  defined implicitly by equation (14), ensures full extraction of the surplus from consumer  $i$ . We can now state the following theorem:

**Theorem 1:** *Suppose that  $C(q)$  is decomposable and Condition CM1 holds. Then, an AIC mechanism in which  $z_i$  is defined implicitly by equation (14), with  $x_i = S_i^{\text{AIC}}(t_i)$ , is incentive compatible, interim individually rational, and fully extracts the surplus.*

**Proof:** Lemma 3 shows that when  $C(q)$  is decomposable, the AIC mechanism without fixed fees is incentive compatible. Since  $z_i(t_{-i})$  does not depend on consumer  $i$ 's report, the AIC mechanism with fixed fees  $(z_1(t_{-1}), \dots, z_n(t_{-n}))$  is still incentive compatible. Moreover, by construction,  $z_i(t_{-i})$  extracts the full surplus from each consumer, so  $S_i(t_i) = 0$  for all  $i \in N$  and all  $t_i \in T_i$ . Hence, the mechanism is interim individually rational. Finally, since the mechanism selects the efficient allocation at the true consumer profile and since  $S_i(t_i) = 0$  for all  $i \in N$  and all  $t_i \in T_i$ , it follows from Lemma 1 that the mechanism fully extracts the surplus. ■

Theorem 1 shows that when  $C(q)$  is decomposable and Condition CM1 holds, the AIC mechanism allows the monopoly to obtain, in expectation, the same profit it would get under first

degree price discrimination. In other words, the monopoly completely overcomes the informational asymmetry problem. Moreover, following Theorem 1 in Crémer and McLean (1988), one can construct an example that shows that if condition CM1 fails, then it is impossible to find an incentive compatible and individually rational pricing mechanism that will guarantee full extraction of the surplus for any arbitrary choice of the gross utility functions.<sup>8</sup>

We now turn to Bayesian incentive compatible pricing mechanisms. Such mechanisms will allow the monopoly to extract the full expected surplus in a broader class of problems than those considered so far. In particular they will allow the monopoly to extract the full expected surplus in some cases where condition CM1 fails. The difference though is that now, the fixed fee of consumer  $i$ ,  $z_i$ , will depend in general on consumer  $i$ 's report, so truthful reporting will no longer be a dominant strategy for consumer  $i$ . Rather, assuming that all other consumers report truthfully, each consumer will find it worthwhile to report truthfully given the consumer's beliefs about the types of other consumers. We now introduce the following condition:

**Condition CM2 (Crémer and McLean 1988):** *For all  $i \in N$ , there does not exist a type  $t_i \in T_i$  and a list of  $\ell_i - 1$  nonnegative numbers,  $(\rho_1, \dots, \rho_{\ell_i - 1})$ , such that:*

$$f(t_{-i} | t_i) = \sum_{j \in N \setminus i} \rho_j f(t_{-i} | t_j), \quad \text{for all } t_{-i} \in T_{-i}. \quad (15)$$

Condition CM2 requires that the  $\ell_i \times \prod_{j \in N \setminus i} \ell_j$  matrix of conditional probabilities,  $\Gamma_i$ , will be such that no row in  $\Gamma_i$  will be in the finite cone generated by all other rows in the matrix. The condition is weaker than condition CM1 because it only requires the non existence of a list of nonnegative numbers  $(\rho_1, \dots, \rho_{\ell_i - 1})$  such that equation (15) holds, whereas condition CM1 requires the non existence of any list of numbers  $(\rho_1, \dots, \rho_{\ell_i - 1})$  (negative or positive), such that equation (15) holds.

When Condition CM2 holds, it is possible to construct an AIC mechanism that will fully

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<sup>8</sup> In specific examples however, full extraction of the surplus may still be possible even when the full rank condition fails. For instance, Riordan and Sappington (1988, Corollary 4) provide conditions under which that is the case in the context of a procurement model.



extract the surplus in a larger class of cases than those permitted by Condition CM1. As before, the usage fees will be as in Definition 6. To construct the fixed fees, note that when Condition CM2 holds, Farkas' Lemma (see e.g., Theorem. 2.6, Gale, 1960, p. 44) implies that for all  $i \in N$  and all  $t_i \in T_i$ , there exists a vector  $g_{t_i} \in \mathfrak{R}^{\ell_{-i}}$ , that attaches a number  $g_{t_i}(t_{-i})$  to each vector of reports by consumers other than  $i$ , such that

$$a_i(t_i) \equiv \sum_{t_{-i} \in T_{-i}} g_{t_i}(t_{-i}) f(t_{-i} | t_i) > 0, \quad (16)$$

and

$$\sum_{t_{-i} \in T_{-i}} g_{t'_i}(t_{-i}) f(t_{-i} | t'_i) \leq 0, \quad \text{for all } t'_i \in T_i \setminus t_i. \quad (17)$$

Now, given a vector of reported types  $\hat{t}$ , let the fixed fee paid by consumer  $i \in N$  be

$$z_i^{AIC}(\hat{t}) \equiv S_i^{AIC}(\hat{t}_i | t_i) + \lambda_i (a_i(\hat{t}_i) - g_{\hat{t}_i}(t_{-i})), \quad (18)$$

where  $S_i^{AIC}(\hat{t}_i | t_i)$  is defined by equation (12),  $\lambda_i$  is a sufficiently large positive scalar, and  $g_{\hat{t}_i}(t_{-i})$  satisfies inequalities (16) and (17). Notice that by construction, if the reports are truthful, then

$$\begin{aligned} E_{t_{-i} \in T_{-i}} [z_i^{AIC}(t) | t_i] &= S_i^{AIC}(t_i) + \lambda_i \sum_{t_{-i} \in T_{-i}} (a_i(t_i) - g_{t_i}(t_{-i})) f(t_{-i} | t_i) \\ &= S_i^{AIC}(t_i) + \lambda_i \left( a_i(t_i) - \sum_{t_{-i} \in T_{-i}} g_{t_i}(t_{-i}) f(t_{-i} | t_i) \right) = S_i^{AIC}(t_i). \end{aligned} \quad (19)$$

This implies that if all consumers report their types truthfully (i.e.,  $\hat{t} = t$ ), the fixed fees fully extract the surplus from each consumer in expectation.

**Theorem 2:** *Suppose that  $C(q)$  is decomposable and Condition CM2 holds. Then, an AIC mechanism in which  $z_i = z_i^{AIC}(\hat{t}_i)$ , is Bayesian incentive compatible, interim individually rational, and fully extracts the surplus.*

**Proof:** By Lemma 3 and the assumption that  $C(q)$  is decomposable, the AIC mechanism without

fixed fees is incentive compatible. Hence it is also Bayesian incentive compatible. We now need to show that adding the fixed fees does not change this property. To this end, note that if consumer  $i$  believes that others will report their types truthfully, the expected value of the consumer's fixed fee when reporting  $\hat{t}_i$ , conditional on the consumer's type  $t_i$ , is

$$\begin{aligned} E_{t_{-i} \in T_{-i}} \left[ z_i^{AIC}(t_{-i}, \hat{t}_i) | t_i \right] &= S_i^{AIC}(\hat{t}_i | t_i) + \lambda_i \sum_{t_{-i} \in T_{-i}} (a_i(\hat{t}_i) - g_{\hat{t}_i}(t_{-i})) f(t_{-i} | t_i) \\ &= S_i^{AIC}(\hat{t}_i | t_i) + \lambda_i \left( a_i(\hat{t}_i) - \sum_{t_{-i} \in T_{-i}} g_{\hat{t}_i}(t_{-i}) f(t_{-i} | t_i) \right). \end{aligned} \quad (20)$$

Since inequalities (16) and (17) imply that the expression in brackets vanishes if  $\hat{t}_i = t_i$  (i.e.,  $i$  makes a truthful report) and is positive otherwise, we can ensure that for all  $t_i \in T_i$ , the conditional expectation of the fixed fee exceeds  $S_i^{AIC}(t_i)$  by making  $\lambda_i$  sufficiently large. Recalling that under truthful report, consumer  $i$ 's expected fixed fee is  $S_i^{AIC}(t_i)$ , truthful reporting is Bayesian incentive compatible. Moreover, since by construction the fixed fees extract the full surplus from each consumer,  $S_i(t_i) = 0$  for all  $i \in N$  and all  $t_i \in T_i$ . Hence the mechanism is interim individually rational. Finally, since the mechanism selects the efficient allocation at the true consumer profile and since  $S_i(t_i) = 0$  for all  $i \in N$ , it follows from Lemma 1 that the mechanism fully extracts the surplus. ■

#### 4. An example

In this section we illustrate our two-step approach with an example that is based on the example in Section 3 in Armstrong and Rochet (1999). A monopoly produces goods A and B at constant marginal costs,  $c^A$  and  $c^B$ , and serves two consumers, 1 and 2. The preferences of the two consumers are additively separable in the two goods and quasi-linear in income. Using  $P_i$  to denote the total payment of consumer  $i$ , the net utility of consumer  $i$  is:

$$U(q^A, q^B, t_i^A, t_i^B) = t_i^A \sqrt{q^A} + t_i^B \sqrt{q^B} - P_i, \quad t_i^A, t_i^B \in \{H, L\}. \quad (21)$$

Since  $t_i^A$  and  $t_i^B$  are either H or L, each consumer has four possible types:  $\{(L,L), (L,H), (H,L), (H,H)\}$ . To simplify the notations, we shall call these types 1, 2, 3, and 4, so that (L,L) is type 1, (L,H) is type 2, (H,L) is type 3, and (H,H) is type 4.

Unlike in Armstrong and Rochet's original example where consumers' types are independent, here consumers types are correlated. Let the matrix of conditional probabilities be given by

$$\Gamma = \begin{bmatrix} r & 0 & 2/3 - r & 1/3 \\ 1/3 & 1/3 & 1/3 & 0 \\ 0 & 1/3 & 1/3 & 1/3 \\ 1/2 & 0 & 1/4 & 1/4 \end{bmatrix}, \quad (22)$$

where the entry in the  $l$ 'th row and the  $m$ 'th column represents the probability that consumer  $i$  assigns to the event that consumer  $j$ 's type is  $m$  when consumer  $i$ 's type is  $l$ ; for instance,  $1/2$  is the probability that consumer  $i$  assigns to consumer  $j$  being of type 1, when consumer  $i$ 's type is 4. Note that each row in  $\Gamma$  sums up to 1. To ensure that all entries in the first row of  $\Gamma$  are nonnegative, let  $0 \leq r \leq 2/3$ . Since in this example the two consumers are identical ex ante (i.e., before their types are realized), we do not need to index  $\Gamma$  by  $i$ .

Now, suppose that the monopoly uses the AIC mechanism. Since the monopoly has a linear cost function, the incremental cost of each unit of A is  $c^A$  and the incremental cost of each unit of B is  $c^B$ . Therefore, if consumer  $i$ 's type is  $t_i = 1, 2, 3, 4$ , and if the consumer buys the bundle  $(q^A(t_i), q^B(t_i))$ , then the total payment of the consumer is

$$p_i(t_i) = c^A q^A(t_i) + c^B q^B(t_i) + z_i, \quad (23)$$

where  $z_i$  is a fixed fee. Given  $p_i(t_i)$ , the optimal bundle for consumer  $i$ , given  $t_i$ , is

$$q^A(t_i) = \left( \frac{t_i}{2c^A} \right)^2, \quad q^B(t_i) = \left( \frac{t_i}{2c^B} \right)^2. \quad (24)$$

Substituting  $q^A(t_i)$  and  $q^B(t_i)$  into equation (21) and rearranging, the surplus of each type of consumer is

$$\begin{aligned}
S^{AIC}(1) &= \frac{L^2}{4c^A} + \frac{L^2}{4c^B}, & S^{AIC}(2) &= \frac{L^2}{4c^A} + \frac{H^2}{4c^B}, \\
S^{AIC}(3) &= \frac{H^2}{4c^A} + \frac{L^2}{4c^B}, & S^{AIC}(4) &= \frac{H^2}{4c^A} + \frac{H^2}{4c^B}.
\end{aligned} \tag{25}$$

The remaining task is to construct the fixed fees,  $z_1$  and  $z_2$ , to extract the full surplus from each consumer. We will consider two cases: one in which condition CM1 holds, and one in which CM1 fails but condition CM2 holds.

### Case 1: condition CM1 holds

Condition CM1 holds if  $\Gamma$  has full rank. Noting that the determinant of  $\Gamma$  is  $(5-12r)/108$ , it follows that  $\Gamma$  has a full rank if and only if  $r \neq 5/12$ . As we saw in Section 3, when  $\Gamma$  has a full rank, the fixed fee of each consumer depends only on the reports of other consumers but not on the consumer's own report. Given that the other consumer can make four possible reports (corresponding to each of the four possible types), we need to construct a vector  $(z_1, z_2, z_3, z_4)$  such that the following four individual rationality constraints hold:

$$S^{AIC}(l) = f_{l1}z_1 + f_{l2}z_2 + f_{l3}z_3 + f_{l4}z_4, \quad l = 1, 2, 3, 4, \tag{26}$$

where  $f_{lm}$  is the entry in the  $l$ 'th row and the  $m$ 'th column in  $\Gamma$  (i.e.,  $f_{lm}$  is the probability that consumer  $j$ 's type is  $m$  when consumer  $i$ 's type is  $l$ ). Solving the system yields

$$\begin{aligned}
z_1 &= \frac{-3S^{AIC}(1) + 3(1-3r)(S^{AIC}(2) - S^{AIC}(3)) + 4(2-3r)S^{AIC}(4)}{5-12r}, \\
z_2 &= \frac{-6S^{AIC}(1) + 6(1-3r)S^{AIC}(2) + 9(1-2r)S^{AIC}(3) - 4(1-6r)S^{AIC}(4)}{5-12r}, \\
z_3 &= \frac{9S^{AIC}(1) + 3(2-3r)(S^{AIC}(2) - S^{AIC}(3)) - 4(1+3r)S^{AIC}(4)}{5-12r}, \\
z_4 &= \frac{-3S^{AIC}(1) - 4(3-7r)(S^{AIC}(2) - S^{AIC}(3)) + 4(2-3r)S^{AIC}(4)}{5-12r}.
\end{aligned} \tag{27}$$

Note that as  $r \rightarrow 5/12$ , the correlation between the consumers' types vanishes, and as a result, the fixed fees blow up.

### Case 2: condition CM1 fails but condition CM2 holds

Now suppose that  $r = 5/12$ . Then the determinant of  $\Gamma$  vanishes, so condition CM1 fails; hence, it is now impossible to construct fixed fees that will fully extract the surplus from each consumer and will induce truthful revelation as a dominant strategy. Condition CM2 however may still hold provided that we cannot find 3 nonnegative numbers,  $(\rho_1, \rho_2, \rho_3)$ , such that

$$f_{ll} = \rho_1 f_{l2} + \rho_2 f_{l3} + \rho_3 f_{l4} \quad l = 1, 2, 3, 4. \tag{28}$$

To prove the nonexistence of such 3 numbers, consider the following expression:

$$W \equiv \min_{\rho_1 \geq 0, \rho_2 \geq 0, \rho_3 \geq 0} \left\{ \sum_{l=1}^4 (f_{ll} - \rho_{l2} f_{l2} - \rho_{l3} f_{l3} - \rho_{l4} f_{l4})^2 \right\}. \tag{29}$$

Condition CM2 holds if  $W \neq 0$ , because then there do not exist 3 nonnegative numbers,  $(\rho_1, \rho_2, \rho_3)$ , that satisfy equation (28). Given that  $r = 5/12$ , we have  $W = 1/25$ , so condition CM2 holds. Hence, it is possible to construct one fixed fee for each combination of reports such that the monopoly will extract the full expected surplus from each consumer. That is, it is possible to construct a vector  $(z_{11}, \dots, z_{14}, z_{21}, \dots, z_{24}, z_{31}, \dots, z_{34}, z_{41}, \dots, z_{44})$  such that each consumer will get a 0 expected payoff by making a truthful report, and a nonpositive payoff by making a false report. Hence the following 16 constraints need to hold:

$$S^{AIC}(l) - f_{l1}z_{l1} + f_{l2}z_{l2} + f_{l3}z_{l3} + f_{l4}z_{l4} = 0, \quad l = 1, 2, 3, 4, \quad (30)$$

$$S^{AIC}(l) - (f_{11}z_{l1} + f_{12}z_{l2} + f_{13}z_{l3} + f_{14}z_{l4}) \leq 0, \quad l = 2, 3, 4, \quad (31)$$

$$S^{AIC}(l) - (f_{21}z_{l1} + f_{22}z_{l2} + f_{23}z_{l3} + f_{24}z_{l4}) \leq 0, \quad l = 1, 3, 4, \quad (32)$$

$$S^{AIC}(l) - (f_{31}z_{l1} + f_{32}z_{l2} + f_{33}z_{l3} + f_{34}z_{l4}) \leq 0, \quad l = 1, 2, 4, \quad (33)$$

and

$$S^{AIC}(l) - (f_{41}z_{l1} + f_{42}z_{l2} + f_{43}z_{l3} + f_{44}z_{l4}) \leq 0, \quad l = 1, 2, 3. \quad (34)$$

The problem of finding a vector of fixed fees that satisfies the 4 equalities and 12 inequalities is a straightforward linear programming problem and can be solved using standard techniques. It turns out that the problem has multiple solutions. For instance, it can be verified that one family of solutions to this problem is

$$z_{l3} \in \mathfrak{R}, \quad z_{l1} = z_{l4} = \frac{4 S^{AIC}(l) - z_{l3}}{3}, \quad z_{l2} = \frac{5 S^{AIC}(l) - 2 z_{l3}}{3}, \quad l = 1, 2, 3, 4. \quad (35)$$

For this family of solutions, all the constraints are holding with equality.

## 5. General cost functions

Having considered decomposable cost functions, we next turn to the harder case where the cost of serving one customer depends on the quantity produced for other customers. In other words, the cost of serving consumer  $i$  depends now on the entire allocation of bundles,  $q = (q_1, \dots, q_n)$ , rather than just on  $q_i$ . In order to characterize the solution to the monopoly's problem in this case we introduce the augmented Clarke-Groves (ACG) mechanism.

**Definition 7:** *Given a vector of reported types  $\hat{t}$ , the augmented Clarke-Groves (ACG) mechanism specifies for each consumer  $i \in N$ ,*

- (i) *a bundle  $q_i^*(\hat{t})$  that corresponds to the efficient allocation at  $\hat{t}$ ,*

(ii) a usage fee,  $w_i(\hat{t})$ , such that

$$w_i(\hat{t}) = \left[ C(q^*(\hat{t})) - \sum_{j \in N \setminus i} V_j(q_j^*(\hat{t}), t_j) \right] - \left[ C(q^*(\hat{t}_{-i})) - \sum_{j \in N \setminus i} V_j(q_j^*(\hat{t}_{-i}), t_j) \right],$$

where  $q^*(\hat{t}_{-i})$  is an  $n-1$  dimensional vector that specifies the optimal allocation of bundles when consumer  $i$  is not served.

(iii) a (random) fixed fee,  $z_i$ .

The ACG mechanism differs from the AIC mechanism in that the usage fees are no longer independent of the reports of other consumers. Specifically, the usage fee of each consumer is equal to the difference between (i) the difference between the monopoly's cost and the gross utilities of all consumers but  $i$ , when all consumers are served, and (ii) the difference between the monopoly's cost and the gross utilities of all consumers but  $i$  when the consumer  $i$  is excluded. In other words, the usage fees correspond to the payments that will result from applying the Clarke-Groves mechanism to the monopoly's problem. Since the ACG mechanism is by construction ex post efficient, it maximizes the social surplus. The monopoly's problem then is to construct the fixed fees that fully extract the surplus. Before turning to the fixed fees, we first prove the following result.

**Lemma 4:** *Absent fixed fees, the ACG mechanism is ex post efficient, incentive compatible, and interim individually rational.*

**Proof:** Note that the second bracketed term in  $w_i(\hat{t})$  does not depend on consumer  $i$ 's report. Hence, when the consumer decides what to report, his/her maximization problem could be written as follows:

$$\underset{\hat{t}_i \in T_i}{\text{Max}} \ V_i(q_i^*(\hat{t}_{-i}, \hat{t}_i), t_i) - C(q^*(\hat{t}_{-i}, \hat{t}_i)) - \sum_{j \in N \setminus i} V_j(q_j^*(\hat{t}_{-i}, \hat{t}_i), t_j). \quad (36)$$

Since  $q^*(\hat{t})$  is efficient, it is by definition optimal for consumer  $i$  to make a truthful report. Therefore the mechanism is incentive compatible. Moreover, given truthful reports, the efficiency of  $q^*(t)$  implies that

$$\sum_{i \in N} V_i(q_i^*(t), t_i) - C(q^*(t)) \geq \sum_{j \in N \setminus i} V_j(q_j^*(t_{-i}), t_j) - C(q^*(t_{-i})). \quad (37)$$

Using the definition of  $w_i(t)$ , this implies in turn that for all  $i \in N$ :

$$V_i(q_i^*(t), t_i) - w_i(t) \geq 0, \quad \text{for all } t_i \in T_i. \quad (38)$$

Hence absent fixed fees, the ACG mechanism is interim individually rational. Since all consumers participate and make truthful reports, the monopoly is able to chose the ex post efficient allocation. ■

With Lemma 4 in place, we are now ready to characterize the fixed fees. After paying a usage fee, the expected surplus of consumer  $i$ , conditional on the consumer's type,  $t_i$ , is

$$S^{ACG}(t_i) \equiv S^{ACG}(t_i | t_i) = \sum_{t_{-i} \in T_{-i}} [V_i(q_i^*(t), t_i) - w_i(t)] f(t_{-i} | t_i). \quad (39)$$

Given  $S_i^{ACG}(t_i)$ , we can now use exactly the same proofs as in Theorems 1 and 2 (the only difference is that now  $S_i^{ACG}(t_i)$  replaces  $S_i^{AC}(t_i)$ ), to establish the following result:

**Theorem 3:** (i) Suppose that Condition CM1 holds and let the fixed fees  $(z_1^{ACG}(t_1), \dots, z_n^{ACG}(t_n))$  be implicitly defined by

$$S_i^{ACG}(t_i) = \sum_{t_{-i} \in T_{-i}} z_i(t_{-i}) f(t_{-i} | t_i), \quad \text{for all } t_i \in T_i. \quad (40)$$

Then the ACG mechanism is incentive compatible, interim individually rational, and it fully extracts the surplus.

(ii) Suppose that condition CM1 fails but condition CM2 holds, and let the fixed fees be defined by

$$z_i^{ACG}(t) = S_i^{ACG}(\hat{t}_i | t_i) + \lambda_i^{ACG} (a_i(\hat{t}_i) - g_i(t_{-i})), \quad (41)$$

where  $a_i(t_i)$  and  $g_i(t_{-i})$  are as in equation (18). Then, for sufficiently large  $\lambda_i^{ACG}$ , the ACG mechanism with fixed fees  $(z_1^{ACG}(t), \dots, z_n^{ACG}(t))$  is Bayesian incentive compatible, interim



*individually rational, and fully extracts the surplus.*

## 6. The case of infinite types space

In this section we extend our results to the case where the consumers' type space is infinite. Specifically, we consider the case where for each  $i \in N$ , the type space,  $T_i$ , is a non-empty compact convex subset of  $\mathfrak{R}^n$ . In addition we make the following assumptions: (i) the density function  $f(t_i, t_{-i})$  is continuous on  $T$ , (ii) there are no degenerate types in the sense that  $f_i(t_i, t_{-i}) > 0$  for all  $i \in N$ , all  $t_i \in T_i$ , and all  $t_{-i} \in T_{-i}$ ; (iii) the gross utility function  $V(q_i, t_i)$  is continuous in  $q_i$  and  $t_i$  for all  $i \in N$ , and the monopoly's cost function is decomposable and continuous in each  $q_i$ ; and (iv) for each  $t_i$ , the surplus maximization problem of each consumer has a bounded solution. The latter assumption implies that we restrict attention to cases where the optimal bundles lie in some compact subset of  $K \subset \mathfrak{R}_+^k$ . To simplify matter we retain the assumption that the monopoly's cost are decomposable. We now show the following result:

**Lemma 5:** *Under the AIC mechanism, the expected surplus of each customer before paying the fixed fee, is continuous in the consumer's type.*

**Proof:**<sup>9</sup> First note that since the monopoly's cost is decomposable, the surplus of each consumer,  $S(t_i) = V_i(q_i^*) - c(q_i^*)$ , depends only on  $t_i$  but not on  $t_{-i}$ . Second, since the gross utility function is continuous in  $q_i$  and  $t_i$  and the cost function is continuous in  $q_i$  on  $K$ , then by the Theorem of the Maximum,  $S(t_i)$  is continuous in  $t_i$ . As by assumption  $f(t_i, t_{-i})$  is continuous on  $T$ , it is also uniformly continuous on  $T$  since  $T$  is compact. Uniform continuity means, in particular, that for every  $\varepsilon > 0$  there is a  $\delta > 0$  such that if  $|t_i, t_i'| < \delta$ , then  $|f(t_i, t_{-i}) - f(t_i', t_{-i})| < \varepsilon$ . Hence

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<sup>9</sup> We thank Aviad Heifetz for his help with the proof.

$$\begin{aligned}
\left| f_i(t_i, t_{-i}) - f_i(t'_i, t_{-i}) \right| &= \left| \int_{T_{-i}} f(t_i, t_{-i}) dt_{-i} - \int_{T_{-i}} f(t'_i, t_{-i}) dt_{-i} \right| \\
&< \int_{T_{-i}} |f(t_i, t_{-i}) - f(t'_i, t_{-i})| dt_{-i} < \varepsilon \phi(T),
\end{aligned} \tag{42}$$

where  $\phi(T)$  is the finite volume of compact set  $T$ . Therefore  $f_i(t_i, t_{-i})$  is continuous. By the assumption that  $f_i(t_i, t_{-i}) > 0$ , we have that  $f(t_{-i} | t_i)$  is continuous in  $t_i$ . Noting that  $T$  and  $K$  are compact and using the Dominated Convergence Theorem, this implies that if the sequence  $\{t_i^k\}$  converges to  $t_i$  then

$$\int_{T_{-i}} S_i(t_i^k) f(t_{-i} | t_i^k) dt_{-i} \rightarrow \int_{T_{-i}} S_i(t_i) f(t_{-i} | t_i) dt_{-i}, \tag{43}$$

as required. ■

Having established that the expected surplus of each consumer is continuous in the consumer's type and recalling that the AIC Mechanism is Bayesian Incentive Compatible for any finite subset of type profiles, we can invoke Theorem 2 in McAfee and Reny (1992). This theorem implies that under the continuous analogue of condition CM2 (which we define below), we can define for any  $\varepsilon > 0$ , a fine enough grid over the type space  $T$ , such that after applying the AIC mechanism with fixed fees as in Theorem 2 above, each consumer will be left with an expected net surplus  $\varepsilon$ . Given that the AIC mechanism leaves each consumer with a unidimensional surplus, the application of McAfee and Reny (1992) is straightforward.

**Condition MR2 (McAfee and Reny 1992):** *For all  $i \in N$ , there does not exist a type  $t_i \in T_i$  and a positive measure,  $\mu \in \Delta(T_i)$ , such that:*

$$\mu(t_i) \neq 1 \text{ and } f(t_{-i} | t_i) = \int_{T_i} f(t_{-i} | \tilde{t}_i) \mu(d\tilde{t}_i). \tag{44}$$

For any finite grid  $T'$  over the type space  $T$ , condition MR2 implies condition CM2. Thus, we can invoke Theorem 2 above that states that the AIC mechanism is Bayesian Incentive Compatible and fully extracts the surplus for the type space  $T'$ . As the surplus of each consumer

is continuous in the consumer's type, we can apply Theorem 2 in McAfee and Reny (1992) directly to establish the following result:

**Theorem 4:** *Suppose that Condition MR2 holds. Then for any  $\varepsilon > 0$ , we can define a fine enough grid over the type space  $T$ , such that the AIC mechanism with fixed fees as in Theorem 2 above, is incentive compatible, interim individually rational, and extracts all but  $\varepsilon$  of each consumer's surplus.*

Theorem 4 indicates that when the consumers' type space is infinite, the monopoly can come arbitrarily close to full surplus extraction from every consumer. Hence, once again, the correlation between consumers' types can be used to construct relatively simple pricing mechanisms that can completely overcome the informational asymmetry problem.

## 7. Conclusion

In this paper we examined the design of nonlinear prices by a multiproduct monopolist who serves customers with multidimensional types. In general, this problem is very hard because one cannot use standard techniques to solve it. Recently, some important progress was made by Wilson (1993), Armstrong (1996, 1999), Sibley and Srinagesh (1997), Armstrong and Rochet (1999), Rochet and Chone (1998), and Rochet and Stole (1999). These papers maintain the traditional assumption in the literature that consumers types are independently drawn from some commonly known distribution. In reality however, consumer types are often correlated because the demands of different consumers are affected by the same macroeconomic or technological factors. Our paper shows that if the correlations between different types of consumers are sufficiently strong, the monopoly can exploit them and design surprisingly simple pricing mechanisms that fully extract the surplus from each consumer.

The main insight is that the monopolist does not need to achieve efficiency and rent extraction all at one step. Instead, it is possible to use a two step approach where in the first step the monopoly offers a simple usage fee to induce consumers to make efficient purchasing

decisions (given their private information), and in the second step, it extracts the surplus from each consumer via a random fixed fee. The advantage of this two step approach is that regardless of the dimensionality of the consumers' types and the number of products that each consumer buys from the monopoly, the surplus that each consumer gets after buying is a scalar. Hence, we can design the fixed fees by adopting the techniques developed by Crémer and McLean (1985, 1988) for the finite type space and McAfee and Reny (1992) for the infinite case. This implies that at least theoretically, the monopoly can completely overcome the informational asymmetry problem even though it produces many products and even if its customers have multidimensional types.

Although the pricing mechanisms that we propose can completely overcome the informational asymmetry problem, they have at least two practical limitations that are common in the literature. First, the mechanisms rely on the assumption that it is commonly known that two different types of an agent with different valuations cannot hold the same beliefs. As Neeman (2000) shows, when this assumption fails, it may be impossible to design a mechanism that fully extracts the surplus. Second, the mechanisms rely heavily on the assumption that consumers are risk neutral and are not liquidity constrained (i.e., can always pay the fixed fees).

These limitations suggest that our results should be mainly viewed as a benchmark. This benchmark shows that unlike in the independent types' case where the multidimensionality of the problem makes it very hard to characterize the optimal pricing mechanism, under the plausible assumption that consumers' types are correlated, it is surprisingly easy to characterize the optimal mechanism in a broad class of environments. Depending on the monopoly's cost function, this mechanism consists of either incremental or Clarke-Groves prices plus a fixed fee. This suggests in turn that the multidimensionality of the problem (consumers' types and the number of products that the monopoly sells) should not, in of itself, be problematic unless it is coupled with additional factors like imperfect knowledge of the distribution of types, risk aversion, and limited liability constraints. Therefore, it seems that in future research on multidimensional, multiproduct nonlinear prices, these additional factors should be explicitly considered.

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