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# Dynamic Trading and Asset Prices: Keynes vs. Hayek

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# Dynamic Trading and Asset Prices: Keynes vs. Hayek

## Abstract

We investigate the dynamics of prices, information and expectations in a competitive, noisy, dynamic asset pricing equilibrium model. We show that prices are farther away from (closer to) fundamentals compared with average expectations if and only if traders over- (under-) rely on public information with respect to optimal statistical weights. Both phenomena, in turn, occur whenever traders speculate on short-run price movements. For a given, positive level of residual payoff uncertainty, over-reliance on public information obtains if noise trade displays low persistence. This defines a “Keynesian” region; the complementary region is “Hayekian” in that prices are systematically closer to fundamentals than average expectations. The standard case of no residual uncertainty and noise trading following a random walk is on the frontier of the two regions and identifies the set of deep parameters for which traders abide by Keynes’ dictum of concentrating on an asset “long term prospects and those only.” The analysis explains accommodation and trend chasing strategies as well as momentum and reversal.

JEL Code: G10, G12, G14.

Keywords: efficient market hypothesis, long and short-term trading, average expectations, higher order beliefs, over-reliance on public information, opaqueness, momentum, reversal.

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# 1 Introduction

Do investors excessively focus their attention on market aggregate behavior and public information, disregarding their private judgement? Are asset prices aligned with the consensus opinion (average expectations) on the fundamentals in the market? Undeniably, the issues above have generated much debate among economists. In his *General Theory*, Keynes pioneered the vision of stock markets as beauty contests where investors try to guess not the fundamental value of an asset but the average opinion of other investors, and end up chasing the crowd.<sup>1</sup> This view tends to portray a stock market dominated by herding, behavioral biases, fads, booms and crashes (see, for example, Shiller (2000)), and goes against the tradition of considering market prices as aggregators of the dispersed information in the economy advocated by Hayek (1945). According to the latter view prices reflect, perhaps noisily, the collective information that each trader has about the fundamental value of the asset (see, for example, Grossman (1989)), and provide a reliable signal about assets' liquidation values.

Keynes distinguished between enterprise, or the activity of forecasting the prospective yield of assets over their whole life, and speculation, or the activity of forecasting the psychology of the market. In the former the investor focuses on the “long-term prospects and those only” while in the latter he tries to anticipate a change in the convention that guides the stock market valuation of actual investments. Keynes thought that in modern stock markets speculation would be king. Recurrent episodes of bubbles or departures of asset prices from fundamental values have the flavor of Keynes' speculation with traders trying to guess what others will do while prices seem far away from average expectations of fundamentals in the market. In fact, a (somewhat simplistic) version of the Efficient Market Hypothesis (EMH) would say that competition among rational investors will drive prices to be centered around the consensus estimate of underlying value given available information. In other words, prices should equal average expectations of value plus noise.<sup>2</sup>

In this paper, we address the tension between the Keynesian and the Hayekian visions in a dynamic finite horizon market where investors, except for noise traders, have no behavioral bias and hold a common prior on the liquidation value of the risky asset. We therefore allow for the possibility that traders concentrate on “long-term prospects and those only” in a rich noisy dynamic rational expectations environment where there is residual uncertainty on the

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<sup>1</sup>Keynes' vision of the stock market as a beauty contest – i.e., the situation in which judges are more concerned about the opinion of other judges than of the intrinsic merits of the participants in the contest – is vividly expressed in the twelfth chapter of the *General Theory*: “. . . professional investment may be likened to those newspaper competitions in which the competitors have to pick out the six prettiest faces from a hundred photographs, the prize being awarded to the Competitor whose choice most nearly corresponds to the average preferences of the competitor as a whole; so that each competitor has to pick, not those faces which he himself finds prettiest, but those which he thinks likeliest to catch the fancy of the other competitors, all of whom are looking at the problem from the same point of view.” (Keynes, Ch. 12, *General Theory*, 1936).

<sup>2</sup>Professional investors attribute considerable importance to the consensus estimate as a guide to selecting stocks. Bernstein (1996) reports how in 1995 Neil Wrigth, chief investment officer of ANB Investment Management & Trust, introduced a strategy “explicitly designed to avoid the Winner's Curse.” Such a strategy was based on the composition of a portfolio from stocks with a narrow trading range, “an indication that [these stocks] are priced around consensus views, with sellers and buyers more or less evenly matched. The assumption is that such stocks can be bought for little more than their consensus valuation.”

liquidation value of the asset and where noise trading follows a general process.

We find that the simplistic EMH does not hold in our model except in very particular circumstances.<sup>3</sup> In our model asset prices are systematically closer or farther away from the liquidation value compared with the consensus opinion about the fundamentals and display over- or under-reliance on public information. Both phenomena, in turn, are driven by traders' endogenous short-term speculative behavior.<sup>4</sup> In a *static* market agents speculate on the difference between the price and the liquidation value, prices are aligned with investors' average expectations about this value, and traders put the optimal statistical weight on public information. In a *dynamic* market traders speculate *also* on short-run price differences. When traders have heterogeneous information, this may misalign prices and investors' average expectations, potentially leading prices either closer or farther away from the fundamentals compared to traders' average opinions. Two key deep parameters, the level of residual payoff uncertainty and the degree of persistence of noise trades, determine whether either over- or under-reliance on public information and systematic departures of prices from average expectations occur. When there is no residual uncertainty on the asset liquidation value and noise trading follows a random walk then prices are aligned with consensus expectations like in a static market. This is one of the boundary cases where rational traders do not have incentives to speculate on short run price movements. For a given, positive level of residual uncertainty, low persistence generates over-reliance; conversely, high noise trades' persistence tends to generate under-reliance on public information. This partitions the parameter space into a Keynesian region, where prices are farther away from fundamentals than average expectations, and a Hayekian region where the opposite occurs. The boundary of these regions reflects Keynes' situation where traders concentrate on "long-term prospects and those only" and where the (simplistic) EMH holds. In the Keynesian region short run price speculation based on market making motives (reversion of the noise trades process) predominates, while in the Hayekian region short run price speculation based on information (trend chasing) predominates. As a consequence of our analysis we can explain also accommodation and trend chasing strategies in a model with rational traders and study how do they map to momentum (recent performance tends to persist in the near future) and reversal (a longer history of performance tends to revert).

The intuition for our results is as follows. In a dynamic market, the relationship between price and fundamentals depends both on the quality of traders' information *and* on their reaction to order imbalances. Suppose a trader observes a positive signal and faces a positive order-flow. Upon the receipt of good news he increases his long position in the asset. On the other hand, his reaction to the order imbalance is either to accommodate it, counting on a future price reversal (and thus acting as a "market-maker"), or to follow the market and further

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<sup>3</sup>It should be no surprise that in a *noisy* rational expectations equilibrium prices may be systematically closer or farther away from the fundamentals compared with investors' average expectations. This result depends on the relative weights that in equilibrium traders put on private and public information and, obviously, could not arise in a *fully revealing* equilibrium where the price coincides with the liquidation value.

<sup>4</sup>Over-reliance on public information may have deleterious welfare consequences (see, e.g., Vives (1997), Morris and Shin (2002), and Angeletos and Pavan (2007)). In this paper we stay within the bounds of a positive analysis.

increase his long position anticipating an additional price rise (in this way “chasing” the trend). The more likely it is that the order imbalance reverts over time, e.g., due to liquidity traders’ transient demand, the more actively the trader will want to accommodate it. Conversely, the more likely it is that the imbalance proxies for a stable trend, e.g., because of intense informed speculators’ activity, the more the trader will want to follow the market.<sup>5</sup> In the former (latter) case, the trader’s speculative position is partially offset (reinforced) by his market making (trend chasing) position. Thus, the impact of private information on the price is partially sterilized (enhanced) by traders’ market making (trend chasing) activity. This, in turn, loosens (tightens) the price from (to) the fundamentals in relation to average expectations, yielding over- (under-) reliance on public information.<sup>6</sup>

Low noise trades’ persistence strengthens the mean reversion in aggregate demand, and tilts traders towards accommodating order imbalances. This effect is extreme when the stock of noise traders’ demand is independent across periods.<sup>7</sup> The impact of residual uncertainty over the liquidation value, on the other hand, enhances the hedging properties of future positions, boosting traders’ signal responsiveness and leading them to speculate more aggressively on short-run price differences. Thus, depending on the persistence of noise traders’ demand, over- or under-reliance on public information occurs, respectively yielding the Keynesian and the Hayekian regions. Conversely, when noise traders’ demand is very persistent (i.e., when noise trades increments are i.i.d.) and absent residual uncertainty, traders act as in a static market, and the price assigns the optimal statistical weight to public information. This, together with the boundary between the Keynesian and the Hayekian regions, identifies the set of parameter values for which traders concentrate on the asset long term prospects, shying away from short term speculation.

Interestingly, the Keynesian and Hayekian regions can be characterized in terms of traders’ consensus opinion about the systematic behavior of future price changes. Indeed, in the Hayekian (Keynesian) region, traders chase the market (accommodate the order flow) because the consensus opinion is that prices will systematically continue a given trend (revert) in the upcoming trading period. We illustrate how expected price behavior under the latter metric *does not* always coincide with a prediction based on the unconditional correlation of returns. This is due to the usual signal extraction problem traders face in the presence of heterogeneous information. Thus, in our setup, depending on the patterns of information arrival, returns can display both reversal and momentum. However, these phenomena are compatible with both the Hayekian and Keynesian equilibrium.

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<sup>5</sup>In this case, indeed, the order imbalance is likely to proxy for upcoming good news that are not yet completely incorporated in the price. There is a vast empirical literature that documents the transient impact of liquidity trades on asset prices as opposed to the permanent effect due to information-driven trades. See e.g. Wang (1994), and Llorente et al. (2002).

<sup>6</sup>Other authors have emphasized the consequences of traders’ reaction to the aggregate flow of orders. For example, Genotte and Leland (1990) argue that investors may exacerbate the price impact of trades, yielding potentially destabilizing outcomes, by extracting information from the order flow.

<sup>7</sup>Indeed, assuming that the stock of noise trade is i.i.d. implies that the gross position noise traders hold in a given period  $n$  completely reverts in period  $n + 1$ . This lowers the risk of accommodating order imbalances in any period, as speculators can always count on the possibility of unwinding their inventory of the risky asset to liquidity traders in the coming round of trade.

This paper contributes to the recent literature that analyzes the effect of higher order expectations in asset pricing models where traders have differential information, but agree on a common prior over the liquidation value. In a dynamic market with risk averse short-term traders, differential information, and an independent stock of noisy supply across periods Allen, Morris, and Shin (2006) argue that prices are always farther away from fundamentals than traders' average expectations and display over-reliance on public information. We show how Keynesian dynamics can arise with long-term traders and how the properties of the noise trading process affect them. Indeed, in our market traders have endogenously short horizons. Bacchetta and van Wincoop (2006) study the role of higher order beliefs in asset prices in an infinite horizon model showing that higher order expectations add an additional term to the traditional asset pricing equation, the higher order "wedge," which captures the discrepancy between the price of the asset and the average expectations of the fundamentals. According to our results, higher order beliefs do not necessarily enter the pricing equation. In other words, for the higher order wedge to play a role in the asset price we need residual uncertainty to affect the liquidation value or noise trade increments predictability when traders have long horizons; Nimark (2007), in the context of Singleton (1987)'s model, shows that under some conditions both the variance and the impact that expectations have on the price decrease as the order of expectations increases.

Other authors have analyzed the role of higher order expectations in models where traders hold different initial beliefs about the liquidation value. Biais and Bossaerts (1998) show that departures from the common prior assumption rationalize peculiar trading patterns whereby traders with low private valuations may decide to buy an asset from traders with higher private valuations in the hope to resell it later on during the trading day at an even higher price. Cao and Ou-Yang (2005) study conditions for the existence of bubbles and panics in a model where traders' opinions about the liquidation value differ.<sup>8</sup> Banerjee et al. (2006) show that in a model with heterogeneous priors differences in higher order beliefs may induce price drift.

The paper also contributes to the literature analyzing asset pricing anomalies within the rational expectations equilibrium paradigm. Biais, Bossaerts and Spatt (2008), in a multi-asset, noisy, dynamic model with overlapping generations show that momentum can arise in equilibrium. Vayanos and Woolley (2008) present a theory of momentum and reversal based on delegated portfolio considerations. We add to this literature by showing how momentum and reversal relate to price over- and under-reliance on public information.

Finally, our paper is related to the literature emphasizing the existence of "limits to arbitrage." De Long et. al (1990) show how the risk posed by the existence of an unpredictable component in the aggregate demand for an asset can crowd-out rational investors, thereby limiting their arbitrage capabilities.<sup>9</sup> In our setup, it is precisely the risk of facing a reversal in noise traders' positions that tilts informed traders towards accommodating more eagerly order imbalances. In turn, this effect is responsible for the over-reliance that asset prices place on

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<sup>8</sup>Kandel and Pearson (1995) provide empirical evidence supporting the non-common prior assumption.

<sup>9</sup>Kondor (2004) shows that limits to arbitrage also occur in a 2-period model where informed traders have market power.

public information.

The paper is organized as follows: in the next section we present the static benchmark, showing that in this framework the asset price places the optimal statistical weight on public information and is just a noisy version of investors' average expectations. In section 3 we analyze the dynamic model and argue that prices display over- or under-reliance on public information whenever, in the presence of heterogeneous information, traders speculate on short term returns. Section 4 analyzes the implications of our model for return regularities. The final section provides concluding remarks.

## 2 A Static Benchmark

Consider a one-period stock market where a single risky asset with liquidation value  $v + \delta$ , and a riskless asset with unitary return are traded by a continuum of risk-averse speculators in the interval  $[0, 1]$  together with noise traders. We assume that  $v \sim N(\bar{v}, \tau_v^{-1})$ ,  $\delta \sim N(0, \tau_\delta^{-1})$ , and  $\delta$  orthogonal to  $v$ . Speculators have CARA preferences (denote with  $\gamma$  the risk-tolerance coefficient) and maximize the expected utility of their wealth:  $W_i = (v - p)x_i$ .<sup>10</sup> Prior to the opening of the market every informed trader  $i$  obtains private information on  $v$ , receiving a signal  $s_i = v + \epsilon_i$ ,  $\epsilon_i \sim N(0, \tau_\epsilon^{-1})$ , and submits a demand schedule (generalized limit order) to the market  $X(s_i, p)$  indicating the desired position in the risky asset for each realization of the equilibrium price.<sup>11</sup> Assume that  $v$  and  $\epsilon_i$  are independent for all  $i$ , and that error terms are also independent across traders. Noise traders submit a random demand  $u$  (independent of all other random variables in the model), where  $u \sim N(0, \tau_u^{-1})$ . Finally, we make the convention that, given  $v$ , the average signal  $\int_0^1 s_i di$  equals  $v$  almost surely (i.e. errors cancel out in the aggregate:  $\int_0^1 \epsilon_i di = 0$ ).<sup>12</sup> The random term  $\delta$  in the liquidation value thus denotes the residual uncertainty affecting the final pay off about which no trader possesses information, and can be used as a proxy for the level of market opaqueness.<sup>13</sup>

We denote by  $E_i[Y]$ ,  $\text{Var}_i[Y]$  the expectation and the variance of the random variable  $Y$  formed by a trader  $i$ , conditioning on the private and public information he has:  $E_i[Y] = E[Y|s_i, p]$ ,  $\text{Var}_i[Y] = \text{Var}[Y|s_i, p]$ . Finally, let  $\alpha_E = \tau_\epsilon/\tau_i$ , where  $\tau_i \equiv (\text{Var}_i[v])^{-1}$ , and  $\bar{E}[v] = \int_0^1 E_i[v] di$ .

We will use the above CARA-normal framework to investigate conditions under which the equilibrium price is systematically farther away from the fundamentals compared to investors' average expectations. Similarly as in Allen et al. (2006) this occurs whenever for all  $v$ ,

$$|E[p - v|v]| > |E[\bar{E}[v] - v|v]|. \quad (1)$$

<sup>10</sup>We assume, without loss of generality with CARA preferences, that the non-random endowment of traders is zero.

<sup>11</sup>The unique equilibrium in linear strategies of this model is symmetric.

<sup>12</sup>See Section 3.1 in the Technical Appendix of Vives (2008) for a justification of the convention.

<sup>13</sup>One can think that the actual liquidation value of the asset results from the sum of two, orthogonal, random components:  $v$  and  $\delta$ . The former relates to the "traditional" business of the firm, so that an analyst or an expert can obtain information about it. The latter component, instead, originates from decisions and actions that insiders make and regarding which the market is totally clueless.



In the market, two estimators of the fundamentals are available: the equilibrium price,  $p$ , and the average expectation traders hold about  $v$  (the “consensus opinion”),  $\bar{E}[v]$ . The above condition then holds if, for any liquidation value, averaging out the impact of noise trades, the discrepancy between the price and the fundamentals is always larger than that between investors’ consensus opinion and the fundamentals.

Interestingly, condition (1) turns out to be satisfied whenever traders assign extra weight to public information compared to the optimal statistical weight in the estimation of  $v$ . Indeed, at a linear symmetric equilibrium for a given private signal responsiveness  $a > 0$  the price can be expressed as

$$p = \alpha_P \left( v + \frac{1}{a}u \right) + (1 - \alpha_P)E[v|p],$$

where  $\alpha_P = a(1 + \kappa)/\gamma\tau_i$ , and  $\kappa \equiv \tau_\delta^{-1}\tau_i$ . It thus follows that

$$p - v = (1 - \alpha_P)(E[v|p] - v) + \alpha_P \frac{1}{a}u.$$

Owing to normality, on the other hand, one can immediately verify that

$$E_i[v] = \alpha_E s_i + (1 - \alpha_E)E[v|p],$$

where  $\alpha_E$  denotes the optimal statistical weight to private information. Because of our convention, we have  $\bar{E}[v] = \alpha_E v + (1 - \alpha_E)E[v]$ . Since

$$E[\bar{E}[v] - v|v] = (1 - \alpha_E)(E[E[v|p]|v] - v),$$

and

$$E[p - v|v] = (1 - \alpha_P)(E[E[v|p]|v] - v),$$

condition (1) holds if and only if the equilibrium price displays over-reliance on public information in relation to the optimal statistical weight:

$$\alpha_P < \alpha_E \Leftrightarrow a < \frac{\gamma\tau_\epsilon}{1 + \kappa}, \quad (2)$$

where the latter equivalence follows immediately from the definitions of  $\alpha_P$  and  $\alpha_E$ .

In the static model it is easy to verify that a unique equilibrium in linear strategies exists in the class of equilibria with a price functional of the form  $P(v, u)$  (see e.g. Admati (1985), Vives (2008)). The equilibrium strategy of a trader  $i$  is given by

$$X(s_i, p) = \frac{a}{\alpha_E}(E_i[v] - p),$$

where  $a$  is given by the unique solution to the cubic equation  $\phi(a) \equiv a(1 + \kappa) - \gamma\tau_\epsilon = 0$ .<sup>14</sup> Therefore, since in equilibrium

$$a = \frac{\gamma\tau_\epsilon}{1 + \kappa}, \quad (3)$$

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<sup>14</sup>It is easy to verify that  $\phi(a) = a^3\tau_u + a(\tau_v + \tau_\epsilon + \tau_\delta) - \gamma\tau_\delta\tau_\epsilon = 0$  possesses a unique real solution. Indeed,  $\phi(0) = -\gamma\tau_\delta\tau_\epsilon < 0$ ,  $\phi(\gamma\tau_\epsilon) = a(a^2\tau_u + \tau_v + \tau_\epsilon) > 0$ , implying that a real solution  $a^*$  exists in the interval  $(0, \gamma\tau_\epsilon)$ . Finally, since  $\phi'(a)|_{a=a^*} > 0$ , the result follows.

we can conclude that in a static setup, condition (2) can never be satisfied, and the equilibrium price always assigns the optimal statistical weight to public information.<sup>15</sup>

**Remark 1.** There is an alternative, more direct way to verify whether condition (1) is satisfied. Indeed, as traders' aggregate demand is proportional to  $\int_0^1 (E_i[v] - p)di$ , imposing market clearing in the above model yields

$$\int_0^1 x_i di + u = \int_0^1 \frac{a}{\alpha_E} (E_i[v] - p) di + u = 0,$$

and solving for the equilibrium price we obtain

$$p = \bar{E}[v] + \frac{\alpha_E}{a} u. \quad (4)$$

In other words, in equilibrium the price is given by the sum of traders' average expectations and noise (times a constant). As  $u$  and  $v$  are by assumption orthogonal, we can therefore conclude that in a static setup the price assigns the optimal statistical weight to public information. To obtain over-reliance on public information, we thus need to find conditions under which traders' aggregate demand is no longer proportional to  $\bar{E}[v] - p$  and this, in a static context with CARA preferences can never happen.  $\square$

In the following sections we will argue that price over-reliance on public information can be traced to traders' speculative activity on short-run price movements that makes strategies depart from the solution of the static setup.

### 3 A 3-Period Model

Consider now a 3-period extension of the market considered in the previous section. We assume that any speculator  $i \in [0, 1]$  has CARA preferences and maximizes the expected utility of his final wealth  $W_{i3} = (v - p_3)x_{i3} + \sum_{n=1}^2 (p_{n+1} - p_n)x_{in}$ .<sup>16</sup> In period  $n$  an informed trader  $i$  receives a signal  $s_{in} = v + \epsilon_{in}$ , where  $\epsilon_{in} \sim N(0, \tau_{\epsilon_n}^{-1})$ ,  $v$  and  $\epsilon_{in}$  are independent for all  $i, n$  and error terms are also independent both across time periods and traders. Denote with  $s_i^n \equiv \{s_{it}\}_{t=1}^n$  and  $p^n \equiv \{p_t\}_{t=1}^n$ , respectively, the sequence of private signals and prices a trader observes at time  $n$ . Informed traders submit a demand schedule (generalized limit order) to the market  $X_n(s_i^n, p^{n-1}, p_n)$  indicating the desired position in the risky asset for each realization of the equilibrium price.

The stock of noise trades is assumed to follow an AR(1) process:  $\theta_n = \beta\theta_{n-1} + u_n$ , where  $u_n \sim N(0, \tau_u^{-1})$  is orthogonal to  $\theta_{n-1}$ , and  $\beta \in [0, 1]$ . To interpret, suppose  $\beta < 1$ , then at any period  $n > 1$  market clearing involves the  $n - 1$ -th and  $n$ -th period aggregate demands of informed traders (respectively,  $x_{n-1} \equiv \int_0^1 x_{in-1} di$ , and  $x_n \equiv \int_0^1 x_{in} di$ ), a fraction  $1 - \beta$  of the demand coming from the  $n - 1$ -th generation of noise traders' who revert their positions, and

<sup>15</sup>If  $E[u]$  is non null, e.g. if  $E[u] = \bar{u} > 0$ , we have to replace the price  $p$  by the price net of the expected noise component  $\hat{p} = p - \bar{u}\text{Var}_i[v + \delta]/\gamma$ . Using this definition it is immediate to verify that also when  $\bar{u} > 0$ , in a static market the equilibrium price assigns the optimal statistical weight to public information.

<sup>16</sup>We assume, as before without loss of generality, that the non-random endowment of traders is zero.

the demand of the new generation of noise traders. Considering the equilibrium conditions for the first two trading dates, and letting  $\Delta x_2 \equiv x_2 - x_1$ ,  $\Delta \theta_2 \equiv \theta_2 - \theta_1 = u_2 + (\beta - 1)\theta_1$ , this implies

$$\begin{aligned} x_1 + \theta_1 &= 0 \\ \Delta x_2 + \Delta \theta_2 &= 0 \Leftrightarrow x_2 + \beta \theta_1 + u_2 = 0. \end{aligned}$$

Thus, assuming that noise trading follows an AR(1) process allows to take into account the possibility that only part of the trades initiated by noise traders at time  $n$  actually reverts at time  $n + 1$ . The lower (higher) is  $\beta$ , the higher (lower) is the fraction of period  $n$  noise traders who will (will not) revert their positions at time  $n + 1$ , and thus won't (will) be in the market at time  $n + 1$ . Equivalently, for  $0 \leq \beta < 1$ , a high, positive demand from noise traders at time  $n$  is unlikely to show up with the same intensity at time  $n + 1$ , implying that  $\text{Cov}[\Delta \theta_n, \Delta \theta_{n+1}] < 0$ . Intuitively, a low  $\beta$  is likely to occur when the time between two consecutive trades is large. Conversely, a high  $\beta$  depicts a situation in which the time between two consecutive transactions is small, so that investors make repeated use of the market to satisfy their trading needs.

Extending the notation adopted in the previous section, we denote by  $E_{in}[Y] = E[Y|s_i^n, p^n]$ ,  $E_n[Y] = E[Y|p^n]$  ( $\text{Var}_{in}[Y] = \text{Var}[Y|s_i^n, p^n]$ ,  $\text{Var}_n[Y] = \text{Var}[Y|p^n]$ ), respectively the expectation (variance) of the random variable  $Y$  formed by a trader conditioning on the private and public information he has at time  $n$ , and that obtained conditioning on public information only. Finally, we let  $\alpha_{E_n} = \sum_{t=1}^n \tau_{\epsilon t} / \tau_{in}$ , where  $\tau_{in} \equiv (\text{Var}_{in}[v])^{-1}$  and make the convention that, given  $v$ , at any time  $n$  the average signal  $\int_0^1 s_{in} di$  equals  $v$  almost surely (i.e. errors cancel out in the aggregate:  $\int_0^1 \epsilon_{in} di = 0$ ).

### 3.1 The Equilibrium

In period  $1 \leq n \leq 3$  each informed trader has the vector of private signals  $s_i^n$  available. It follows from Gaussian theory that the statistic  $\tilde{s}_{in} = (\sum_{t=1}^n \tau_{\epsilon t})^{-1} (\sum_{t=1}^n \tau_{\epsilon t} s_{it})$  is sufficient for the sequence  $s_i^n$  in the estimation of  $v$ . An informed trader  $i$  in period  $n$  submits a limit order  $X_n(\tilde{s}_{in}, p^{n-1}, \cdot)$ , indicating the position desired at every price  $p_n$ , contingent on his available information. We will restrict attention to linear equilibria where in period  $n$  a speculator trades according to  $X_n(\tilde{s}_{in}, p^n) = a_n \tilde{s}_{in} - \varphi_n(p^n)$ , where  $\varphi_n(\cdot)$  is a linear function of the price sequence  $p^n$ . Let us denote with  $z_n$  the intercept of the  $n$ -th period net aggregate demand  $\int_0^1 \Delta x_{in} di + u_n$ , where  $\Delta x_{in} = x_{in} - x_{in-1}$ . The random variable  $z_n \equiv \Delta a_n v + u_n$  represents the informational addition brought about by the  $n$ -th period trading round, and can thus be interpreted as the informational content of the  $n$ -th period order-flow (where  $\Delta a_n \equiv a_n - \beta a_{n-1}$ ). The following proposition characterizes equilibrium prices and strategies:

**Proposition 1.** *At any linear equilibrium of the 3-period market the equilibrium price is given by*

$$p_n = \alpha_{P_n} \left( v + \frac{\theta_n}{a_n} \right) + (1 - \alpha_{P_n}) E_n[v], \quad n = 1, 2, 3, \quad (5)$$

where  $\theta_n = u_n + \beta\theta_{n-1}$ . For  $n = 1, 2$ , a trader's strategy is given by:

$$X_n(\tilde{s}_{in}, z^n) = \frac{a_n}{\alpha_{E_n}}(E_{in}[v] - p_n) + \frac{\alpha_{P_n} - \alpha_{E_n}}{\alpha_{E_n}} \frac{a_n}{\alpha_{P_n}}(p_n - E_n[v]), \quad (6)$$

while at time 3:

$$X_3(\tilde{s}_{i3}, z^3) = \frac{a_3}{\alpha_{E_3}}(E_{i3}[v] - p_3), \quad (7)$$

where  $\alpha_{E_n} = \sum_{t=1}^n \tau_{\epsilon_t} / \tau_{in}$ , and expressions for  $\alpha_{P_n}$  and  $a_n$  are provided in the appendix (see equations (32), (50), (68), and (31), (46), (71), respectively). The parameters  $\alpha_{P_n}$  and  $a_n$  are positive for  $n = 2, 3$ . Numerical simulations show that  $\alpha_{P_1} > 0$  and  $a_1 > 0$ .

*Proof.* See the appendix. □

Proposition 1 extends Vives (1995), restating a result due to He and Wang (1995), providing an alternative, constructive proof. According to (5), at any period  $n$  the equilibrium price is a weighted average of the market expectation about the fundamentals  $v$ , and a monotone transformation of the  $n$ -th period aggregate demand intercept.<sup>17</sup> A straightforward rearrangement of (5) yields

$$\begin{aligned} p_n - E_n[v] &= \frac{\alpha_{P_n}}{a_n} E_n[\theta_n] \\ &= \Lambda_n (a_n (v - E_n[v]) + \theta_n). \end{aligned} \quad (8)$$

According to (8), the discrepancy between  $p_n$  and  $E_n[v]$  is due to the contribution that noise traders are expected to give to the  $n$ -th period aggregate demand. The parameter  $\Lambda_n \equiv \alpha_{P_n} / a_n$  is a measure of market depth. The smaller is  $\Lambda_n$  and the smaller is the anticipated (and realized) contribution that the stock of noise gives to the aggregate demand and to the price.

At any period  $n < 3$ , a trader's strategy is the sum of two components. The first component captures the trader's activity based on his *private* estimation of the difference between the fundamentals and the  $n$ -th period equilibrium price. This can be considered as "long-term" speculative trading, aimed at profiting from the liquidation value of the asset. The second component captures the trader's activity based on the extraction of order flow, i.e. *public*, information. This trading is instead aimed at exploiting short-run movements in the asset price determined by the evolution of the future aggregate demand. Upon observing this information, and depending on the sign of the difference  $\alpha_{P_n} - \alpha_{E_n}$ , traders engage either in "market making" (when  $\alpha_{P_n} - \alpha_{E_n} < 0$ , thereby accommodating the aggregate demand) or in "trend chasing" (when  $\alpha_{P_n} - \alpha_{E_n} > 0$ , thus following the market).

To fix ideas, consider the following example. Suppose that  $p_n - E_n[v] > 0$ . According to (8), we know that the market attributes the discrepancy between the price and the public expectation to the presence of a positive expected stock of demand coming from noise traders:  $E_n[\theta_n] > 0$ . A trader's reaction to this observation depends on whether he believes it to be driven by noise or information. In the former (latter) case, the forward looking attitude implied by rational behavior, would advise the trader to accommodate (join) the aggregate demand in

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<sup>17</sup>This is immediate since in any linear equilibrium  $\int_0^1 x_{in} di + \theta_n = a_n v + \theta_n - \varphi_n(p^n)$ .

the expectation of a future price reversion (further increase).<sup>18</sup> Suppose  $\alpha_{P_n} < \alpha_{E_n}$ , then informed traders count on the reversal of noise traders' demand in the next period(s) and take the other side of the market, acting as market makers. They thus short the asset expecting to buy it back in the future at a lower price.<sup>19</sup> If, on the other hand,  $\alpha_{P_n} > \alpha_{E_n}$ , informed traders anticipate that the role of "positive" fundamental information looms large in the  $n$ -th period aggregate demand and that this is most likely affecting the sign of  $E_n[\theta_n]$ . As a consequence, they buy the asset, expecting to re sell it once its price has incorporated the positive news, effectively chasing the trend.<sup>20</sup>

Finally, note that according to (7), in the third period agents concentrate in "long term speculation." Indeed, at  $n = 3$ , agents anticipate that the asset will be liquidated in the next period and thus that its value will *not* depend on the information contained in that period's aggregate demand. As a consequence, they choose their position only taking into account their information on the fundamentals, acting like in a static market.

**Remark 2.** While for  $N = 3$  existence is daunting to show, assuming  $N = 2$  we are able to prove that an equilibrium in linear strategies always exists.<sup>21</sup> In this latter case, multiple equilibria may in principle arise. For some parameter values, it is easy to find equilibria. For instance, if noise increments are i.i.d., and traders only receive private information in the first period (i.e., if  $\beta = 1$  and  $\tau_{\epsilon_2} = 0$ ), there always exists an equilibrium where  $a_1 = a_2 = (1 + \kappa)^{-1} \gamma \tau_{\epsilon_1}$ , whereas for large values of  $\tau_\delta$  another equilibrium where  $a_1 = (\gamma \tau_u)^{-1} (1 + \kappa + \gamma^2 \tau_{\epsilon_1} \tau_u) > a_2 = (1 + \kappa)^{-1} \gamma \tau_{\epsilon_1}$  may also arise (Grundy and McNichols (1989)). The first equilibrium disappears when  $\beta < 1$ . In the absence of residual uncertainty (i.e., if  $\tau_\delta^{-1} = 0$ ),  $\kappa = 0$ , and the equilibrium with  $a_1 = a_2 = \gamma \tau_{\epsilon_1}$  is unique (see Section 3.3).  $\square$

As argued above, the difference  $\alpha_{P_n} - \alpha_{E_n}$  plays a crucial role in shaping traders' reactions to public information and thus their trading behavior. In our static benchmark, on the other hand, the same difference also determines how "close" the price is to the fundamentals compared to the average expectations traders hold about it. This fact is also true in a dynamic market. Indeed, since

$$\bar{E}_n[v] \equiv \int_0^1 E_{in}[v] di = \alpha_{E_n} v + (1 - \alpha_{E_n}) E_n[v],$$

and using (5), a straightforward extension of the argument used in section 2 allows to obtain the following

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<sup>18</sup>In other words, owing to the traditional signal extraction problem, it is entirely possible that the sign of  $E_n[\theta_n]$  is due to the presence of a positive demand coming from informed traders.

<sup>19</sup>When  $\alpha_{P_n} - \alpha_{E_n} < 0$ , the reaction to the aggregate demand traders display in the above example may seem akin to a "contrarian" strategy. However, while value investors tend to buy at low prices in the expectation that the intrinsic value of an asset will eventually show up, our traders take the other side of the market just to exploit the regularity in the pattern of noise traders' demand.

<sup>20</sup>Note that the intensity of the trading based on order flow information is positively related to the depth of the period  $n$  market. Indeed, in a deeper market both a market maker and a market chaser face smaller adverse price movements, and are thus willing to trade more aggressively.

<sup>21</sup>The proof is available from the authors upon request.

**Corollary 1.** *At any linear equilibrium of the 3-period market*

$$|E[p_n - v|v]| > \left| E \left[ \int_0^1 E_{in}[v] di - v|v \right] \right|, \quad (9)$$

if and only if  $\alpha_{P_n} < \alpha_{E_n}$ , for  $n \leq 3$ .

*Proof.* We use here the direct proof, based on the analysis of the market clearing equation, adopted in Section 2. Using the expression for strategies in Proposition 1, at any period  $n < 3$  at equilibrium we have

$$\int_0^1 x_{in} di + \theta_n = 0 \Leftrightarrow \frac{a_n}{\alpha_{E_n}} (\bar{E}_n[v] - p_n) + \frac{\alpha_{P_n} - \alpha_{E_n}}{\alpha_{E_n}} \frac{a_n}{\alpha_{P_n}} (p_n - E_n[v]) + \theta_n = 0.$$

Solving for the price and rearranging yields

$$p_n = \bar{E}_n[v] + \frac{\alpha_{P_n} - \alpha_{E_n}}{a_n} E_n[\theta_n] + \frac{\alpha_{E_n}}{a_n} \theta_n,$$

where  $E_n[\theta_n] = a_v(v - E_n[v]) + \theta_n$ . This, in turn, implies that

$$p_n - v = \bar{E}_n[v] - v + \frac{\alpha_{P_n} - \alpha_{E_n}}{a_n} E_n[\theta_n] + \frac{\alpha_{E_n}}{a_n} \theta_n.$$

Thus, if  $\alpha_{P_n} < \alpha_{E_n}$  the price is closer to the fundamentals compared the consensus opinion, while the opposite occurs whenever  $\alpha_{P_n} > \alpha_{E_n}$ .  $\square$

We can now put together the results obtained in proposition 1 and corollary 1: if upon observing the  $n$ -th period aggregate demand traders expect it to be mostly driven by noise trades, they accommodate the order flow. As a consequence, their behavior drives the price away from the fundamentals compared to the average market opinion. If, instead, they deem the aggregate demand to be mostly information driven, they align their short term positions to those of the market. This, in turn, drives the price closer to the fundamentals, compared to traders' average expectations.

Alternatively, when traders speculate on short term returns the equilibrium price and the consensus opinion have different dynamics:

$$p_n = \bar{E}_n[v] + \frac{\alpha_{P_n} - \alpha_{E_n}}{a_n} E_n[\theta_n] + \frac{\alpha_{E_n}}{a_n} \theta_n. \quad (10)$$

Indeed, as the price originates from market clearing, it reflects *both* determinants of traders' demand, i.e. their long term forecast *and* their short term speculative activity. Conversely, as the consensus opinion is *only* based on traders' long term expectations, it *does not* reflect the impact of short term speculation.

To establish the direction of inequality (9) we thus need to determine what is the force that drives a trader's reaction to the information contained in the order flow. Prior to that we consider a special case of our model in which traders do not receive private signals at any period  $n$ . In this case short term speculation is disconnected from the existence of over- or under-reliance of prices on public information, as we show in the following section.

### 3.2 Homogeneous Information and Short Term Speculation

In this section we assume away heterogeneous information, setting  $\tau_{\epsilon_n} = 0$ , for all  $n$ . This considerably simplifies the analysis and allows us to show that in the absence of heterogeneous information short term speculation *does not* lead prices to be systematically closer or farther away from the fundamentals compared to traders' average expectations. We start by characterizing the equilibrium in this setup, and then analyze its properties.

**Proposition 2.** *In the 3-period market with homogeneous information, there exists a unique equilibrium in linear strategies, where prices are given by*

$$p_n = \bar{v} + \Lambda_n \theta_n, \quad (11)$$

where

$$\Lambda_3 = \frac{1 + \kappa}{\gamma \tau_v} \quad (12)$$

$$\Lambda_2 = \Lambda_3 \left( 1 + \frac{(\beta - 1) \gamma^2 \tau_u \tau_v}{1 + \kappa + \gamma^2 \tau_u \tau_v} \right) \quad (13)$$

$$\Lambda_1 = \Lambda_2 \left( 1 + \frac{(\beta - 1) \gamma^2 \tau_u \tau_v ((1 + \kappa)(1 - \beta) + \gamma^2 \tau_u \tau_v)}{(1 + \kappa + \gamma^2 \beta^2 \tau_u \tau_v)(1 + \kappa) + \gamma^2 \tau_u \tau_v (1 + \kappa + \gamma^2 \tau_u \tau_v)} \right), \quad (14)$$

and  $\kappa = \tau_v / \tau_\delta$ . Risk averse speculators trade according to

$$X_n(p^n) = -\Lambda_n^{-1}(p_n - \bar{v}), \quad n = 1, 2, 3. \quad (15)$$

*Proof.* See the appendix □

In a market with homogeneous information, at any period  $n$  traders have no private signal to use when forming their position. As a consequence, the aggregate demand only reflects the stock of noise trades. According to (15), this implies that speculators always take the other side of the market, buying the asset when  $p_n < \bar{v} \Leftrightarrow \theta_n = \Lambda_n^{-1}(p_n - \bar{v}) < 0$ , and selling it otherwise. Indeed, in the absence of private information, risk averse traders face no adverse selection problem when they clear the market. The discrepancy between the equilibrium price and the unconditional expected value reflects the risk premium traders demand in order to accommodate the liquidity needs of noise traders. Even in the absence of adverse selection risk, in fact, traders anticipate the possibility that the liquidation value  $v$  may be lower (higher) than the price they pay for (at which they sell) the asset.

If  $\beta < 1$ , risk averse traders also speculate on short term asset price movements providing additional order flow accommodation at any time  $n = 1, 2$ . This can be seen rearranging (15) in the following way:

$$X_n(p^n) = \Lambda_3^{-1}(\bar{v} - p_n) - (\Lambda_n^{-1} - \Lambda_3^{-1})(p_n - \bar{v}).$$

As a result, for  $\beta \in (0, 1)$ , market depth decreases across trading periods:

$$0 < \Lambda_1 < \Lambda_2 < \Lambda_3,$$

and *within* each period it decreases in  $\beta$ :

$$\frac{\partial \Lambda_n}{\partial \beta} > 0,$$

as one can immediately see from (12), (13), and (14). The intuition for these results is that if  $\beta < 1$ , as noise trades increments are negatively correlated, prior to the last trading round traders have more opportunities to unload their risky position. This reduces the risk they bear, and lowers the impact that the noise shock has on the price. If  $\beta = 1$  noise trades increments are i.i.d.. Therefore, speculators cannot count on the future reversion in the demand of noise traders and their extra order flow accommodation disappears. As a consequence, depth is constant across periods:  $\Lambda_1 = \Lambda_2 = \Lambda_3 = (\gamma\tau_v)^{-1}(1 + \kappa)$ .<sup>22</sup>

As one would intuitively expect, short term speculation arises insofar as traders can map the partial predictability of noise trades' increments into the anticipation of short term returns. The following proposition formalizes this intuition:

**Corollary 2.** *At  $n = 1, 2$  traders speculate on short term asset price movements if and only if, provided  $\theta_n > 0$  ( $\theta_n < 0$ ), they expect the next period return to revert:  $E_n[p_{n+1} - p_n] < 0$  ( $E_n[p_{n+1} - p_n] > 0$ ).*

*Proof.* Using (11) we can easily obtain

$$E_n[p_{n+1} - p_n] = (\beta\Lambda_{n+1} - \Lambda_n) \theta_n.$$

Fixing  $n = 2$ , and using (13) we then obtain

$$\begin{aligned} (\beta\Lambda_3 - \Lambda_2) \theta_2 &= \Lambda_3(\beta - 1) \frac{1 + \kappa}{1 + \kappa + \gamma^2\tau_u\tau_v} \theta_2 \\ &= \Lambda_3\Lambda_2^{-1}(\beta - 1) \frac{1 + \kappa}{1 + \kappa + \gamma^2\tau_u\tau_v} (p_2 - \bar{v}). \end{aligned} \quad (16)$$

In a similar way, fixing  $n = 1$ , and using (14) yields

$$\begin{aligned} (\beta\Lambda_2 - \Lambda_1) \theta_1 &= \Lambda_2(\beta - 1) \frac{(1 + \kappa + \gamma^2\beta^2\tau_u\tau_v)(1 + \kappa) + \gamma^2\beta(1 + \kappa)\tau_u\tau_v}{(1 + \kappa + \gamma^2\tau_u\tau_v)(1 + \kappa) + \gamma^2\tau_u\tau_v(1 + \kappa + \gamma^2\tau_u\tau_v)} \theta_1 \\ &= \Lambda_2\Lambda_1^{-1} \frac{(\beta - 1)(1 + \kappa + \gamma^2\beta^2\tau_u\tau_v)(1 + \kappa) + \gamma^2\beta(1 + \kappa)\tau_u\tau_v}{(1 + \kappa + \gamma^2\tau_u\tau_v)(1 + \kappa) + \gamma^2\tau_u\tau_v(1 + \kappa + \gamma^2\tau_u\tau_v)} (p_1 - \bar{v}). \end{aligned} \quad (17)$$

Since for  $\beta \in [0, 1)$ , the terms multiplying  $\theta_n$  in (16) and (17) are both negative,  $E_n[p_{n+1} - p_n] < 0 \Leftrightarrow \theta_n > 0$ . If  $\beta = 1$  traders do not speculate on short term returns, and  $\Lambda_1 = \Lambda_2 = \Lambda_3 = (\gamma\tau_v)^{-1}(1 + \kappa)$ . This, in turn, implies that  $E_n[p_{n+1} - p_n] = 0$ , for  $n = 1, 2$ , proving our claim.  $\square$

Both in the market with homogeneous information and in the one with heterogeneous information traders speculate on short term returns. However, while in the latter market this possibly leads to the fact that prices over-rely on public information, in the presence of symmetric information this never happens:

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<sup>22</sup>This matches the result that He and Wang obtain when looking at the case of homogeneous information when signal are fully informative on  $v$ , i.e. with  $\tau_{\epsilon_n} \rightarrow \infty$ .



**Corollary 3.** *With homogeneous information at  $n = 1, 2, 3$ , the price is as far away from the fundamentals as traders' average expectations.*

*Proof.* According to (11), the equilibrium price can be expressed as the sum of traders' average expectations and a noise term  $\theta_n$  which is by assumption orthogonal to  $v$ . Hence,

$$E[p_n - v|v] = E[\bar{v} + \Lambda_n \theta_n - v|v] = \bar{v} - v.$$

Given that traders do not have private information, the price only reflects the noise term  $\theta_n$ , and  $E_{in}[v] = E[v] = \bar{v}$ . Hence,

$$E[\bar{E}_n[v] - v|v] = \bar{v} - v.$$

Thus  $E[\bar{E}_n[v] - v|v] = E[p_n - v|v]$ , which proves our result.  $\square$

As risk-averse traders have no private information to trade with, their orders do not impound fundamental information in the price. As a consequence, as shown in Proposition 2, at any period  $n$  traders are able to extract the realization of the noise stock  $\theta_n$  from the observation of the aggregate demand, implying that the price perfectly reflects  $\theta_n$ . As the latter is orthogonal to  $\bar{v}$ , and in the absence of heterogeneous signals  $\bar{E}_n[v] = \bar{v}$ , both prices and speculators' consensus opinion about fundamentals stand at the same "distance" from  $v$ .

The last result of this section draws an implication of our analysis for the time series behavior of returns, showing that second and third period returns display reversal if noise trade increments are correlated:

**Corollary 4.** *At  $n = 1, 2, 3$  returns exhibit reversal if and only if  $\beta < 1$ .*

*Proof.* To see this, first we compute the covariance between second and third period returns:

$$\begin{aligned} \text{Cov}[p_3 - p_2, p_2 - p_1] &= (\Lambda_2 (\beta\Lambda_3 - \Lambda_2) + \beta (\beta\Lambda_3 - \Lambda_2) (\beta\Lambda_2 - \Lambda_1)) \tau_u^{-1} \\ &= \left( \frac{\beta\Lambda_3 - \Lambda_2}{\tau_u} \right) (\Lambda_2 (1 + \beta^2) - \beta\Lambda_1). \end{aligned}$$

Given that as argued above  $\Lambda_1 < \Lambda_2$ , a necessary and sufficient condition for  $\text{Cov}[p_3 - p_2, p_2 - p_1] < 0$  is that  $(\beta\Lambda_3 - \Lambda_2) < 0$ . We know from (16) that

$$\beta\Lambda_3 - \Lambda_2 = (\beta - 1)(1 + \kappa) \left( \frac{1}{\gamma\tau_v} + \frac{\gamma\tau_v}{1 + \kappa + \gamma^2\tau_u\tau_v} \right) < 0,$$

for all  $\beta \in [0, 1)$ . Similarly, from (17),  $\text{Cov}[p_2 - p_1, p_1 - \bar{v}] = \Lambda_1(\beta\Lambda_2 - \Lambda_1)\tau_u^{-1} < 0$  for  $\beta \in [0, 1)$ . Finally,  $\text{Cov}[v - p_3, p_3 - p_2] = -\Lambda_3(\Lambda_3 + \beta(\beta\Lambda_3 - \Lambda_2)(1 + \beta^2))\tau_u^{-1} < 0$  for  $\beta \in [0, 1)$ .  $\square$

With homogeneous information, reversal occurs because with  $\beta < 1$ , the impact of liquidity shocks "evaporates" across trading periods. Thus, a given liquidity shock  $u_n$  has a stronger impact on the  $n$ -th period price compared to the  $(n + 1)$ -th price. As a consequence, the price change spurred by  $u_n$  across times  $n$  and  $n + 1$  is negative, and more than compensates for any effect generated by the former periods' liquidity shocks, implying that  $\text{Cov}[p_{n+1} - p_n, p_n - p_{n-1}] < 0$ . To be sure, consider the following example. Suppose that  $u_1, u_2 > 0$ . Then first period noise

traders' demand has a positive impact on the first period price which is larger than the one it has on the second and third period prices. In turn, the second period noise traders' demand has a stronger positive impact on the second period price than on the third period price. Formally:  $p_3 - p_2 = \Lambda_3 u_3 + \beta(\beta\Lambda_3 - \Lambda_2)u_2 + \beta u_1(\beta\Lambda_3 - \Lambda_2)$ , and  $p_2 - p_1 = \Lambda_2 u_2 + (\beta\Lambda_2 - \Lambda_1)u_1$ , with  $\beta\Lambda_n - \Lambda_{n-1} < 0$ . Thus, both  $u_1$  and  $u_2$  have an impact on  $\text{Cov}[p_3 - p_2, p_2 - p_1]$ , the former is positive while the latter is negative. At equilibrium the latter effect is always stronger than the former.

Summarizing, in the model with homogeneous information traders speculate on short term asset price movements if and only if they can exploit the predictability of future noise trades' increments. However, this is *not* enough to induce over- or under-reliance of prices on public information. Indeed, in the absence of heterogeneous information, prices are as far away from fundamentals as the consensus opinion. Furthermore, corollaries 2 and 4 imply that at any time  $n = 1, 2$ , and for all  $(\beta, 1/\tau_\delta) \in [0, 1) \times \mathbb{R}_+$  the short term, contrarian strategy based on the realization of  $\theta_n$  univocally maps into return reversal.

### 3.3 The Effect of Heterogeneous Information

As explained in Section 3.1, the assumption  $\beta < 1$  implies that noise trades' increments are negatively correlated, and introduces a mean reverting component in the evolution of the aggregate demand. In the market with homogeneous information analyzed in Section 3.2, as the noise stock is perfectly observable, this leads traders to speculate on short term returns, providing additional order flow accommodation. When speculators have private signals, the aggregate demand features also a component that reflects fundamental information. As a consequence, the noise stock *cannot* be perfectly retrieved, and informed traders face an adverse selection problem when clearing the market. Thus, when faced with the aggregate demand, they *estimate* the noise stock and choose the side of the market on which to stand, based on which component (noise or information) they trust will influence the evolution of the future aggregate demand. Mean reversion in noise increments pushes traders to take the other side of the market (see Section 3.2). In this section we will argue that with heterogeneous information, if  $\tau_\delta^{-1} > 0$  traders scale up their signal responsiveness prior to the last trading round. This, in turn, implies that prior to the last trading round informed traders are more inclined to attribute a given aggregate demand realization to the impounding of fundamental information, and are pushed to follow the market. Both effects eventually bear on the magnitude of the weight the price assigns to the fundamentals:

**Proposition 3.** *In the presence of residual uncertainty, at any linear equilibrium the weight the price assigns to the fundamentals at time  $n = 1, 2$  is given by*

$$\alpha_{P_1} = \alpha_{E_1} \left( 1 + (\beta\rho_1 - \rho_2)\Upsilon_1^1 + (\beta\rho_2 - 1)\Upsilon_1^2 \right) \quad (18)$$

$$\alpha_{P_2} = \alpha_{E_2} \left( 1 + (\beta\rho_2 - 1)\Upsilon_2^1 \right), \quad (19)$$

where

$$\rho_n = \frac{a_n(1 + \kappa)}{\gamma \sum_{t=1}^n \tau_{\epsilon_t}}, \quad (20)$$

$\kappa = \tau_\delta^{-1} \tau_{i3}$ , and the expressions for  $\Upsilon_n^k$ ,  $a_n$  are provided in the appendix for  $k, n \in \{1, 2\}$  (see equations (51), (69), (70), and (31), (46), (71), respectively). The parameter  $\Upsilon_2^1$  is positive. Numerical simulations show that  $\Upsilon_1^1 > 0$  and  $\Upsilon_2^2 > 0$ , and that  $\rho_1 \geq \rho_2 \geq 1$ .<sup>23</sup>

*Proof.* See the appendix. □

According to the above result, at any linear equilibrium the magnitude of  $\alpha_{P_n}$  depends on the sign of the differences  $\beta\rho_1 - \rho_2$  and  $\beta\rho_2 - 1$ . While  $\beta < 1$  implies that noise traders' demand increments are negatively correlated,  $\rho_n$  captures the deviation that residual uncertainty induces in traders' signal responsiveness with respect to the "long term" solution.<sup>24</sup>

To better separate the impact that noise traders' mean reversion and the residual uncertainty affecting fundamentals have on  $\alpha_{P_n}$ , we start by considering the case in which  $\tau_\delta^{-1} = 0$ . In this case  $\kappa = 0$ , and there exists a unique equilibrium in linear strategies in the market (He and Wang (1995) and Vives (1995)). Furthermore,  $\rho_n = 1$  for all  $n$ , and a closed form solution is available which partially simplifies the analysis and allows to show

**Corollary 5.** *In the absence of residual uncertainty, at any period  $n = 1, 2$ , (a)  $a_n = \gamma \sum_{t=1}^n \tau_{\epsilon_t}$ , and (b) the  $n$ -th period price displays over reliance on public information if and only if  $\beta < 1$ .*

*Proof.* See the appendix. □

According to the above result, if  $\tau_\delta^{-1} = 0$ , traders' responsiveness to private information matches the static solution. Hence,  $\rho_n = 1$  and (18)–(19) become

$$\alpha_{P_1} = \alpha_{E_1} (1 + (\beta - 1) (\Upsilon_1^1 + \Upsilon_1^2)) \quad (21)$$

$$\alpha_{P_2} = \alpha_{E_2} (1 + (\beta - 1) \Upsilon_2^1). \quad (22)$$

We know that  $\Upsilon_2^1 > 0$  from proposition 3. In the appendix we show that  $\tau_\delta^{-1} = 0$  implies  $\Upsilon_1^1 + \Upsilon_1^2 > 0$ , lending support to our conclusion. Intuitively, if  $\tau_\delta^{-1} = 0$ , when  $\beta < 1$  at any time  $n = 1, 2$  the only source of predictability in the future aggregate demand comes from the mean reverting nature of the noise trading process, and traders' short term behavior is akin to the one they display in the market with homogeneous information. Thus, upon observing  $p_n > E_n[v] \Leftrightarrow E_n[\theta_n] > 0$  ( $p_n < E_n[v] \Leftrightarrow E_n[\theta_n] < 0$ ), traders accommodate the expected positive noise traders' demand (supply), selling (buying) the asset in the anticipation of a future price reversion. As these price movements do not reflect fundamental information, this drives the price away from the terminal pay off.

Corollary 5 argues that, absent residual uncertainty, traders' sole motive to speculate on price differences is the possibility to profit from the mean reversion of noise trades. This suggests that shutting down this prediction channel should eliminate any short term speculative activity:

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<sup>23</sup>Simulations have been run assuming that either private information flows at a constant rate in the three trading periods ( $\tau_{\epsilon_n} = \tau_{\epsilon_1}$ , for  $n = 2, 3$ ) or that it arrives in the first period only ( $\tau_{\epsilon_n} = 0$ , for  $n = 2, 3$ ) with the following parameter values:  $\tau_v, \tau_u, \tau_{\epsilon_1} \in \{.1, .2, \dots, 2\}$ ,  $\beta \in \{0, .1, \dots, 1\}$  and  $\gamma \in \{1, 3\}$ ,  $\tau_\delta \in \{1, 10\}$ .

<sup>24</sup>If at time  $n = 1, 2$  traders were to neglect short run price movements and be forced to focus on long term speculation only, they would respond to their private information according to  $(1 + \kappa)^{-1} \gamma \sum_{t=1}^n \tau_{\epsilon_t}$ .

**Corollary 6.** *In the absence of residual uncertainty and assuming  $\beta = 1$ ,  $\alpha_{P_n} = \alpha_{E_n}$  for  $n = 1, 2$ .*

*Proof.* This follows immediately by replacing  $\beta = 1$  in (21) and (22).  $\square$

If  $\tau_\delta^{-1} = 0$ , and  $\beta = 1$ , noise trades increments are i.i.d. and at any period  $n < 3$  traders have no way to exploit the predictability of future periods' aggregate demand. As a consequence, they concentrate their trading activity on long term speculation, and  $\alpha_{P_n} = \alpha_{E_n}$ .

We can now bring back the effect of residual uncertainty. As argued in section 3.1 in the last trading round agents concentrate on the long term value of the asset, speculating as in a static market without exploiting any pattern in the evolution of the aggregate demand. This implies that their responsiveness to private information is given by

$$a_3 = \frac{\gamma \sum_{t=1}^3 \tau_{\epsilon_t}}{1 + \kappa}.$$

The above expression generalizes (3) and shows that in a *static* market with residual uncertainty, the weight traders assign to private information is the risk-tolerance weighted sum of their private signal precisions, scaled down by a factor  $1 + \kappa$ , which is larger, the larger is  $\tau_\delta^{-1}$ . Indeed, the larger is  $\tau_\delta^{-1}$ , the larger is the impact of residual uncertainty on the fundamentals, and the less informative are traders' private signals about the liquidation value. Thus, traders feel less confident about their information and scale down their signal responsiveness.

Residual uncertainty also affects a trader's signal responsiveness at any time  $n < 3$ , and this is reflected by the parameter

$$\rho_n = \frac{a_n(1 + \kappa)}{\gamma \sum_{t=1}^n \tau_{\epsilon_t}}. \quad (23)$$

Expression (23), captures the deviation from the long term private signal responsiveness due to the presence of residual uncertainty. As stated in proposition 3 our numerical simulations show that in the presence of residual uncertainty  $\rho_1 \geq \rho_2 \geq 1$ . Thus, prior to the last trading round, traders react to their private signals *more* aggressively than if they were just about to observe the liquidation value:

$$a_n \geq \frac{\gamma \sum_{t=1}^n \tau_{\epsilon_t}}{1 + \kappa}, \quad n = 1, 2.$$

Indeed, while residual uncertainty makes traders less confident about their signals, the presence of additional trading rounds increases the opportunities to adjust suboptimal positions prior to liquidation. This, in turn, boosts traders' reaction to private information, the more, the longer is the amount of time prior to liquidation, as  $\gamma$  more trading opportunities are available to revise traders' positions. Furthermore, this also implies that a given aggregate demand realization may be driven by informed traders, contributing to explain the component capturing trading based on order flow information in traders' strategies:

**Corollary 7.** *In the presence of residual uncertainty, at any linear equilibrium the second period price displays over reliance on public information if and only if  $\beta\rho_2 < 1$ .*

*Proof.* See the appendix. □

To fix ideas, suppose  $\beta = 1$  and assume that at time 2 traders observe  $p_2 > E_2[v]$  (i.e.,  $E_2[\theta_2] > 0$ ). Given that the demand of noise traders displays no predictable pattern, a short term position based on shorting the asset in the anticipation of buying it back at a lower price one period ahead is suboptimal. At the same time, the fact that  $\rho_2 \geq 1$  implies that informed traders react more aggressively to their private signal than in a static market. This generates additional informed trading which may be responsible for the observed price realization. Informed traders thus go long in the asset in the anticipation of a further price increase in the coming period. If  $\beta < 1$ , the mean reversion effect of noise trades kicks in and traders' decisions as to the side of the market in which to position themselves needs to trade off this latter pattern against the one driven by fundamental information.

For trend chasing to be optimal in the first period, the impact of the mean reverting component due to noise trades on future prices must be weaker than the effect of informed traders' overreaction to private information in *both* periods. As  $\Upsilon_1^1 > 0$  and  $\Upsilon_1^2 > 0$ , inspection of (18) suggests that this depends on the sign of both  $\beta\rho_1 - \rho_2$  and  $\beta\rho_2 - 1$ . Indeed, we have the following numerical result:

**Numerical Result.** *In the presence of residual uncertainty, at any linear equilibrium of the market, if  $\beta\rho_2 > 1$ , a sufficient condition for  $\alpha_{P_1} > \alpha_{E_1}$  is that  $\beta\rho_1 > \rho_2$ .*

Thus, in the first period having  $\beta\rho_1 > \rho_2$  is *not enough* to ensure that traders are willing to chase the market if  $E_1[\theta_1] > 0$ . The intuition is as follows. In the appendix we show that a trader's first period optimal strategy can also be expressed as follows,

$$X_1(s_{i1}, p_1) = \Gamma_1^1 E_{i1}[p_2 - p_1] + \Gamma_1^2 E_{i1}[x_{i2}] + \Gamma_1^3 E_{i1}[x_{i3}],$$

where expressions for  $\Gamma_1^1$ ,  $\Gamma_1^2$ , and  $\Gamma_1^3$  are provided in the appendix. Thus, in the first period both the second and third period expected positions impinge on a trader's decision. Suppose  $p_1 > E_1[v]$  (i.e.,  $E_1[\theta_1] > 0$ ), and  $\beta\rho_1 > \rho_2$ , but  $\beta\rho_2 < 1$ . Upon observing a high first period price, given that  $\beta\rho_1 > \rho_2 > 1$ , a trader may think to side with the market in the expectation of selling in period 2 once the anticipated further appreciation has realized. This, however, is not enough. Indeed, given that  $\beta\rho_2 < 1$ , upon observing  $E_2[\theta_2] > 0$ , (i.e.,  $p_2 > E_2[v]$ ) traders in the second period, anticipating their third period position, will take the other side of the market. This, in turn, may depress  $p_2$  and compromise the trend chasing strategy set up in the first period. Thus, if the mean reverting effect of noise trades leading to extra order flow accommodation in the second period is strong enough, even if  $\beta\rho_1 > \rho_2$ , in the first period traders will take the other side of the market. This, in turn, implies that  $\alpha_{P_1} < \alpha_{E_1}$ .<sup>25</sup>

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<sup>25</sup>Notice that in the absence of residual uncertainty, this could not happen. In that case, the only source of predictability comes from noise trades mean reversion. Thus, given that  $\beta$  is constant across time, provided  $\beta < 1$ , the condition for price over- or under-reliance on public information does not change in the two trading periods. To be sure, suppose that  $\kappa = 0$  and that at time 1  $E_1[\theta_1] > 0$ . Traders short the asset expecting to buy it back either in period 2 or 3. If at time 2  $E_2[\theta_2] > 0$ , they keep shorting, coherently with what they decided in period 1.

### 3.4 Public Information Reliance and Consensus Opinion: Keynes vs. Hayek

Summarizing the results we obtained in the previous section, the systematic discrepancy between prices and the consensus opinion in the estimation of the fundamentals, depends on the joint impact that noise trades' mean reversion and informed traders' overreaction to private information have on short term speculative activity. According to corollary 5, lacking residual uncertainty, noise trades' mean reversion pushes informed traders to act as market makers. This pulls the price away from the fundamentals compared to the average market opinion. When residual uncertainty is introduced, corollary 7 together with our numerical results imply that the decision to “make” the market or “chase” the trend arises as a solution to the trade off between the strength of noise trades' mean reversion and that of informed traders' overreaction to private information. Finally, when noise trades' increments are i.i.d., corollary 6 and proposition 3 respectively imply that lacking residual uncertainty traders concentrate on long term speculation only, while introducing residual uncertainty they tend to chase the market. This, in turn, leads to a price that is either as far away from, or closer to the fundamentals compared to traders' average opinion. Table 1 summarizes this discussion.

		Noise trades' persistence		
		$\beta = 0$	$0 < \beta < 1$	$\beta = 1$
Residual uncertainty	$\tau_\delta^{-1} = 0$	$\alpha_{P_n} < \alpha_{E_n}$	$\alpha_{P_n} < \alpha_{E_n}$	$\alpha_{P_n} = \alpha_{E_n}$
	$\tau_\delta^{-1} > 0$	$\alpha_{P_n} < \alpha_{E_n}$	$\alpha_{P_n} \lesseqgtr \alpha_{E_n}$	$\alpha_{P_n} > \alpha_{E_n}$

Table 1: A summary of the results for  $n = 1, 2$ .

Our summary suggests that in both periods and for  $\tau_\delta^{-1} \geq 0$ , there must exist a  $\beta$  such that  $\alpha_{P_n} = \alpha_{E_n}$ , and traders are willing to forgo short term speculation. Numerical simulations confirm this insight as shown in figures 1 and 2. The figures plot the locus  $\Omega_n \equiv \{(\beta, 1/\tau_\delta) \in [0, 1] \times \mathbb{R}_+ | \alpha_{P_n} = \alpha_{E_n}\}$ ,  $n = 1, 2$ , assuming that traders receive a private signal in every trading period of the same precision. At any period  $n$ , the set  $\Omega_n$  divides the parameter space  $(\beta, 1/\tau_\delta)$  into a Keynesian region (to the left of the locus) with over-reliance on public information and a Hayekian region (the rest) where the opposite occurs. With no residual uncertainty ( $\tau_\delta^{-1} = 0$ ) and i.i.d. noise trade increments ( $\beta = 1$ ),  $\Omega_n = (1, 0)$  (corollary 6). The introduction of residual uncertainty, on the other hand, may have a non-monotone effect on  $\Omega_n$ . Observing the figures for small (large) values of  $\tau_\delta^{-1}$  the Hayekian region widens (shrinks). This is especially true for high levels of risk tolerance. The intuition is as follows. For small levels of residual uncertainty, the fact that speculators can re trade in a dynamic market has a first order impact on  $\rho_n$  as the possibility to readjust one's position more than compensates for the increase in risk due to the augmented residual uncertainty over the liquidation value. As  $\tau_\delta^{-1}$  grows larger, the possibility

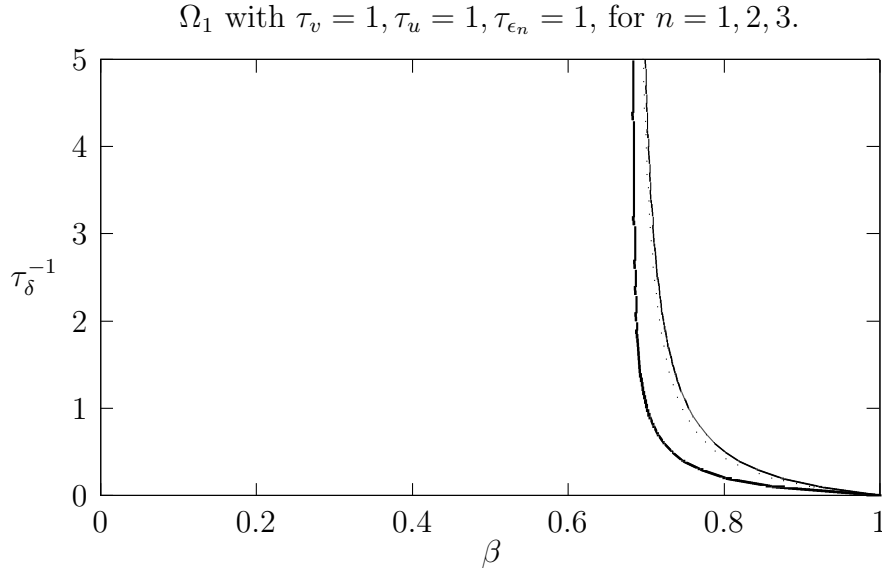


Figure 1: The Keynesian and Hayekian regions for  $n = 1$  with “constant” arrival of information:  $\tau_{\epsilon_n} = \tau_\epsilon$  for  $n = 1, 2, 3$ . The bold, dotted, and thin curves are associated respectively to  $\gamma = 1$ ,  $\gamma = 1/2$ , and  $\gamma = 1/4$ .

to retrade has an increasingly weaker effect on a trader’s dynamic responsiveness, as private signals become less and less relevant to forecast the fundamentals. Traders thus scale back their responsiveness and more noise trades persistence is needed to make traders forgo short term speculation.<sup>26</sup>

According to our simulations, at any trading period the Hayekian (Keynesian) region widens (shrinks) whenever the impact of traders’ overreaction to private information on aggregate demand realizations is strong. This occurs for large values of  $\gamma$ ,  $\tau_\epsilon$ , and  $\tau_u$ . When, on the other hand,  $\tau_v$  is large, traders enter the market with sufficiently good prior information, and the trading process is unlikely to have a strong informational impact on the price. In this case, the Hayekian (Keynesian) region shrinks (widens). Interestingly, when traders only receive information in the first and second period we find that  $\alpha_{P_2} < \alpha_{E_2}$ . Similarly, our numerical simulations show that if  $\tau_{\epsilon_2} = 0$ , the same happens in the first period as well, implying that the Hayekian region disappears in both period 1 and 2, and  $\Omega_n = \{(1, \tau_\delta^{-1}), \text{ for } \tau_\delta^{-1} > 0\}$ . The intuition is as follows: from our previous analysis the reason why informed traders may want to side with the market is that they believe that fundamental information drives the aggregate demand realization. However, with this pattern of information arrival, traders do not receive any new signal after the first (or second) trading round. As a consequence, in the presence of a mean reverting demand from noise traders, siding with the market exposes informed traders to a considerable risk of trading in the expectation of a price increase (decrease) in the second

<sup>26</sup>According to the figures above as  $\tau_\delta^{-1}$  grows unboundedly traders’ private signal responsiveness shrinks but the Hayekian region *does not* disappear. In the 2-period model it is easy to see that when  $\tau_\delta^{-1} \rightarrow \infty$ ,  $\Omega_1$  becomes a constant. Indeed, in this case  $\Omega_1 = \{(\beta, 1/\tau_\delta) \in [0, 1] \times \mathbb{R} | \beta \rho_1 = 1\}$ , and  $\lim_{\tau_\delta^{-1} \rightarrow \infty} \rho_1 = (\tau_v + \tau_{\epsilon_1})^{-1}(\tau_v + \tau_{\epsilon_1} + \tau_{\epsilon_2}) > 1$  is a constant that only depends on deep parameters. Therefore,  $\beta \rho_1 = 1$  can be explicitly solved, yielding  $\beta = (\tau_v + \tau_{\epsilon_1} + \tau_{\epsilon_2})^{-1}(\tau_v + \tau_{\epsilon_1}) < 1$ . In the three-period model our numerical simulations show that a similar effect is at work.

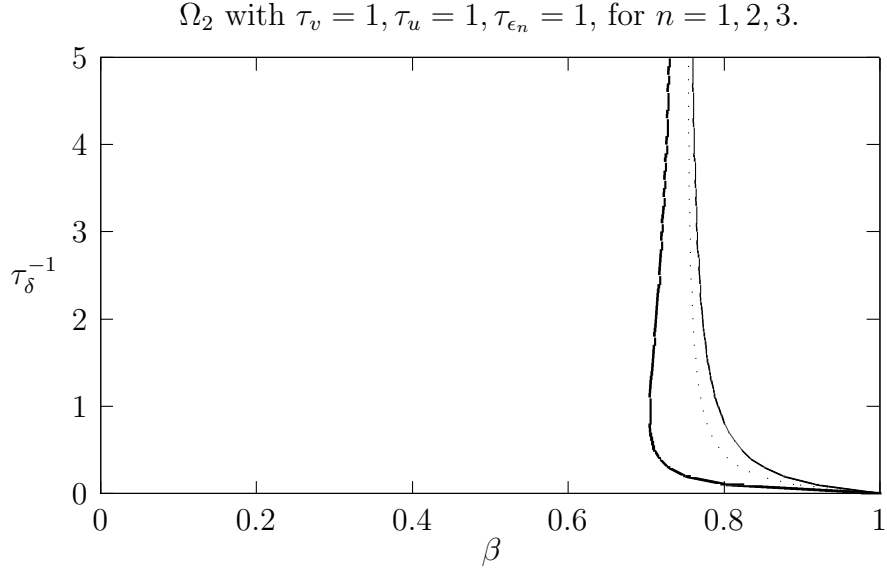


Figure 2: The Keynesian and Hayekian regions for  $n = 2$  with “constant” arrival of information:  $\tau_{\epsilon_n} = \tau_\epsilon$  for  $n = 1, 2, 3$ . The bold, dotted, and thin curves are associated respectively to  $\gamma = 1$ ,  $\gamma = 1/2$ , and  $\gamma = 1/4$ .

and third period and instead being faced with a price decrease (increase).<sup>27</sup>

In the light of our discussion in section 4.2,  $\Omega_n$  captures the set of deep parameter values granting the existence of an equilibrium in which traders only focus on an asset “long-term prospects and those only.” This is the attitude towards investment that Keynes contrasted to the Beauty Contest (General Theory, Ch. 12). The exclusive focus on an asset long term prospects arises either in the absence of any systematic pattern in the evolution of the aggregate demand (as argued in corollary 6) or when the forces backing trend chasing are exactly offset by those supporting market making (as shown in figures 1 and 2). In both cases, along the region  $\Omega_n$ , long term traders can only devote their attention to forecasting the fundamentals, shying away from the exploitation of the profits generated by short-term price movements. As a consequence, the price ends up being as close to the fundamentals as the market average opinion.

Corollary 2 argues that in the presence of symmetric information it is possible to map observed price departures from the public expectation at a given period  $n$  (i.e.,  $p_n - E_n[v]$ ), into a position which is coherent with traders’ expectations about the future evolution of the market price. The following corollary shows that an equivalent result also holds in the market with heterogeneous information, characterizing the consensus opinion about the evolution of future prices in the Hayekian and Keynesian regions:

**Corollary 8.** *In the presence of residual uncertainty, at any linear equilibrium*

$$E[p_2 - E_2[v]|v] > 0 \Leftrightarrow E[\bar{E}_2[p_3 - p_2]|v] > 0,$$

<sup>27</sup>The figures in the text refer to a set of numerical simulations that were conducted assuming  $\tau_v, \tau_u, \tau_{\epsilon_n} \in \{1, 4\}$ ,  $\gamma \in \{1/4, 1/2, 1\}$ , and  $\beta \in \{0, 0.001, 0.002, \dots, 1\}$ ,  $\tau_\delta^{-1} \in \{0.1, 0.2, \dots, 5\}$ , for each pattern of private information arrival.



if and only if  $\alpha_{P_2} > \alpha_{E_2}$ . If  $\tau_\delta^{-1} = 0$

$$E[p_n - E_n[v]|v] > 0 \Leftrightarrow E[\bar{E}_n[p_{n+1} - p_n]|v] < 0.$$

*Proof.* See the appendix. □

Thus, in the Hayekian (Keynesian) region, a systematic positive price departure from the public expectation about the fundamentals at time 2 “generates” the consensus opinion that prices will systematically further rise (decrease) in the third period. In the first period numerical simulations confirm that a similar result holds:  $E[p_1 - E_1[v]|v] > 0 \Leftrightarrow E[\bar{E}_1[p_2 - p_1]|v] > 0$ . If  $\tau_\delta^{-1} = 0$  informed traders never overreact to their private information. Hence, provided  $\beta < 1$ , only the Keynesian equilibrium can arise and a systematic positive discrepancy between prices and public expectations creates the consensus opinion that prices will systematically revert. Finally, along the region  $\Omega_n$ , the market consensus opinion is that the next period price won’t change in any systematic way. As a consequence,  $E[\bar{E}_n[p_{n+1} - p_n]|v] = 0$ , and traders concentrate on the asset long term prospects.

## 4 Discussion

### 4.1 Reversal and Momentum

A vast empirical literature has evidenced the existence of return predictability based on a stock’s past performance. DeBondt and Thaler (1986) document a “reversal” effect, whereby stocks with low past returns (*losers*) tend to outperform stocks with high past returns (*winners*) over medium/long future horizons. Jegadeesh and Titman (1993), instead, document a “momentum” effect, showing that recent past winners tend to outperform recent past losers in the following near future. In our framework, as we argued in Section 3.2, when traders have homogeneous information, noise trades’ low persistence implies that returns are negatively correlated, and thus exhibit reversal.<sup>28</sup>

In this section we turn to the model with heterogeneous information, and analyze its implications for returns’ correlation. The introduction of a strongly persistent factor affecting asset prices (i.e., fundamental information) contrasts the impact of the transient component represented by the noise stock. As a consequence, and except for the case in which  $\beta = 0$ , momentum and reversal can arise in both the Keynesian and the Hayekian equilibrium.

Using (8), we concentrate on the covariance between second and third period returns, as

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<sup>28</sup>More in detail, DeBondt and Thaler (1986) classify all the NYSE-traded stocks according to their past three-year return in relation to the corresponding market average in the period spanning January 1926 to December 1982 in stocks that outperform the market (“winners”) and stocks that underperform it (“losers”). According to their results, in the following three years, portfolios of losers outperform the market by 19.6% on average while portfolios of winners underperform the market by 5% on average. Jegadeesh and Titman (1993), classify NYSE stocks over the period from January 1963 to December 1989 according to their past six-month returns. Their results show that the top prior winners tend to outperform the worst prior losers by an average of 10% on an annual basis. Research on momentum and reversal is extensive (see Vayanos and Woolley (2008) and Asness, Moskowitz and Pedersen (2008) for a survey of recent contributions).

this fully depends on endogenous prices:

$$\begin{aligned} \text{Cov}[p_3 - p_2, p_2 - p_1] &= \text{Cov}[E_3[v] - E_1[v], \Lambda_3 E_3[\theta_3] - \Lambda_1 E_1[\theta_1]] \\ &\quad + \text{Cov}[\Lambda_2 E_2[\theta_2] - \Lambda_1 E_1[\theta_1], \Lambda_3 E_3[\theta_3] - \Lambda_2 E_2[\theta_2]]. \end{aligned} \quad (24)$$

Explicitly computing the covariances in (24) and rearranging yields:

$$\begin{aligned} \text{Cov}[p_3 - p_2, p_2 - p_1] &= \left( \frac{\beta\Lambda_3 - \Lambda_2}{\tau_u} \right) \times \\ &\quad \left( \Lambda_2 (1 + \beta^2) - \beta\Lambda_1 + \frac{a_2\tau_u(1 - \alpha_{P_2})}{\tau_2} - \frac{\beta a_1\tau_u(1 - \alpha_{P_1})}{\tau_1} \right). \end{aligned} \quad (25)$$

The latter expression shows that in a market with heterogeneous information the covariance of returns is generated by two effects. The first one is captured by

$$\left( \frac{\beta\Lambda_3 - \Lambda_2}{\tau_u} \right) (\Lambda_2 (1 + \beta^2) - \beta\Lambda_1),$$

which coincides with the expression given for the third period returns' covariance in the model with homogeneous information. As we argued in Section 3.2, this component reflects the impact of the noise shocks affecting the first and second period aggregate demand. The second component is given by

$$\left( \frac{\beta\Lambda_3 - \Lambda_2}{\tau_u} \right) \left( \frac{a_2\tau_u(1 - \alpha_{P_2})}{\tau_2} - \frac{\beta a_1\tau_u(1 - \alpha_{P_1})}{\tau_1} \right),$$

and captures the impact of the fundamental information shocks affecting the first and second period aggregate demand.

Inspection of (25) shows that if  $\beta = 0$ , then  $\text{Cov}[p_3 - p_2, p_2 - p_1] < 0$ , implying that if noise trades' increments are strongly negatively correlated (i.e., the stock of noise trades is transient, and i.i.d), returns can only exhibit reversal. Hence, when  $\beta = 0$  equilibria are Keynesian (in that the price over relies on public information) and display negative returns' autocorrelation.

As  $\beta$  increases away from zero, depending on the patterns of private information arrival, momentum can arise. To see this, we start by assuming away residual uncertainty and set  $\beta = 1$ , so that any pattern in the correlation of returns must depend on the time distribution of private information. In this situation, as argued in Corollary 6, the equilibrium is unique and we have  $\alpha_{P_n} = \alpha_{E_n} = \tau_{in}^{-1} \sum_{t=1}^n \tau_{\epsilon_t}$ ,  $a_n = \gamma \sum_{t=1}^n \tau_{\epsilon_t}$ , and

$$\Lambda_n = \frac{1}{\gamma\tau_{in}},$$

implying that, provided traders receive information at all trading dates, and differently from what happens in the market with homogeneous information, market depth *improves* over time.<sup>29</sup> As a consequence,  $\Lambda_3 < \Lambda_2$  and, similarly to the case with homogeneous information, the impact of a given liquidity shock “evaporates” across trading periods. Note, however, that as now market depth depends on the patterns of information arrival, the presence of heterogeneous

<sup>29</sup>In the market with homogeneous information if  $\beta = 1$ ,  $\Lambda_n = (\gamma\tau_v)^{-1}(1 + \kappa)$ , for  $n = 1, 2, 3$ .

information makes it possible for the impact of the first period liquidity shock to overpower that of the liquidity shock arriving in the second period. Indeed, as one can verify:

$$\begin{aligned} \text{Cov}[p_3 - p_2, p_2 - p_1] > 0 &\Leftrightarrow 2\Lambda_2 - \Lambda_1 + \frac{a_2\tau_u(1 - \alpha_{P_2})}{\tau_2} - \frac{a_1\tau_u(1 - \alpha_{P_1})}{\tau_1} < 0 \\ &\Leftrightarrow \tau_{\epsilon_2} > \frac{\tau_{i1}}{1 + \gamma\tau_u a_1}, \end{aligned}$$

and given that  $(1 + \gamma\tau_u a_1)^{-1}\tau_{i1} > \tau_{\epsilon_1}$ , we can conclude that with no residual uncertainty and i.i.d. noise increments, returns are positively correlated provided that traders receive private information at all trading dates (i.e.,  $\tau_{\epsilon_n} > 0$ , for all  $n$ ), and the quality of such information shows sufficient improvement across periods 1 and 2. In this situation, market depth considerably increases between the first and second period. This implies that the impact of the first period liquidity shock is always stronger than the one coming from  $u_2$ , building a positive trend in returns.<sup>30</sup> Furthermore, a large second period private precision strengthens the impact of fundamental information, eventually yielding  $\text{Cov}[p_3 - p_2, p_2 - p_1] > 0$ .

When  $\beta < 1$  (keeping  $\tau_\delta^{-1} = 0$ ), noise trades' persistence is lower and this helps to generate a negative covariance. As a result, the value of  $\tau_{\epsilon_2}$  which is needed for the model to display momentum, increases. Adding residual uncertainty, lowers traders' responsiveness to private information. This, in turn, implies that for any  $\beta$ , the value of  $\tau_{\epsilon_2}$  that triggers momentum further increases (see Figure 3).

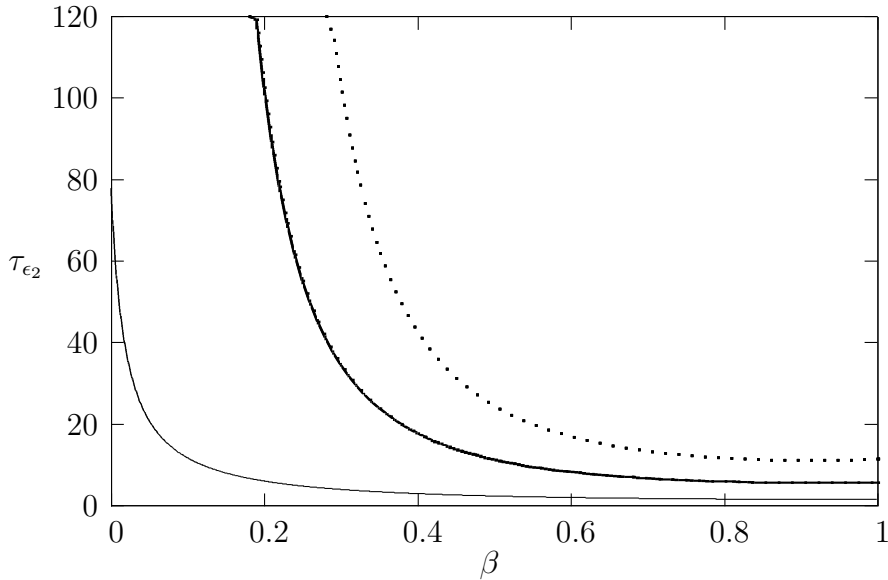


Figure 3: The figure displays the set  $\{(\beta, \tau_{\epsilon_2}) \in [0, 1] \times \mathbb{R}_+ | \text{Cov}[p_3 - p_2, p_2 - p_1] = 0\}$ , partitioning the parameter space  $[0, 1] \times \mathbb{R}_+$  into two regions: points above the plot identify the values of  $(\beta, \tau_{\epsilon_2})$  such that there is momentum. Points below the plot identify the values of  $(\beta, \tau_{\epsilon_2})$  such that there is reversal. Parameters' values are  $\tau_v = \tau_u = \tau_{\epsilon_1} = \tau_{\epsilon_3} = 1$ . The thin, thick and dotted line respectively correspond to  $\tau_\delta^{-1} = 0$ ,  $\tau_\delta^{-1} = .2$ , and  $\tau_\delta^{-1} = .3$ .

Summarizing, when  $\beta = 0$  as argued in section 3.4 the Keynesian equilibrium realizes. There we obtain excessive reliance on public information, and prices that are farther away from the

<sup>30</sup>Formally,  $2\Lambda_2 - \Lambda_1 |_{\tau_{\epsilon_2}=(1+\gamma a_1 \tau_u)^{-1}\tau_{i1}} < 0$ .

fundamentals compared to the consensus opinion. Traders accommodate a positive expected liquidity demand, as the consensus opinion is that prices systematically revert. Furthermore, returns are negatively correlated. As  $\beta$  grows larger, for intermediate values of the residual uncertainty parameter the Hayekian equilibrium may occur, with insufficient reliance on public information, and prices that are closer to the fundamentals compared to the consensus opinion. Upon observing a positive realization of the expected liquidity demand, traders chase the trend, as in this case the consensus opinion is that prices will systematically increase. In this equilibrium, momentum obtains provided that the quality of traders' private information improves sufficiently across trading dates. Momentum and reversal are therefore compatible with both types of equilibria.<sup>31</sup>

Inspection of figure 3 suggests that for a given  $\tau_{e_2}$ , higher values of  $1/\tau_\delta$  require a larger noise trades' persistence for  $\text{Cov}[p_3 - p_2, p_2 - p_1] = 0$ . Numerical simulations confirm this insight, showing that the set of parameter values  $(\beta, 1/\tau_\delta)$  for which  $\text{Cov}[p_3 - p_2, p_2 - p_1]$  is null has the shape displayed by the thick line in figure 4. Points above (below) the thick line represent combinations of  $(\beta, 1/\tau_\delta)$  such that the third period returns display reversal (momentum), so that  $\text{Cov}[p_3 - p_2, p_2 - p_1] < 0$  ( $\text{Cov}[p_3 - p_2, p_2 - p_1] > 0$ ). It is useful to also draw the set  $\Omega_2 = \{(\beta, 1/\tau_\delta) \in [0, 1] \times \mathbb{R}_+ | \alpha_{P_2} = \alpha_{E_2}\}$  for the chosen parameter configuration. This partitions the parameter space  $[0, 1] \times \mathbb{R}_+$  into four regions. Starting from the region  $HR$  in which there is under reliance on public information and reversal and moving clockwise, we have the region  $HM$  with under reliance on public information and momentum; the region  $KM$  with over reliance on public information and momentum; the region  $KR$  with over reliance on public information and reversal.<sup>32</sup>

According to Corollary 8, in the Hayekian (Keynesian) region traders' short term strategies reflect the consensus opinion about the systematic behavior of future prices. For instance, in the region to the right of  $\Omega_2$  (i.e., the region  $H$ ), a systematic positive discrepancy between  $p_2$  and  $E_2[v]$  creates the consensus opinion that the third period price will increase above  $p_2$ . This rationalizes informed traders' decisions to ride the market upon observing  $p_2 - E_2[v] > 0$ . As figure 4 clarifies, in this region the consensus opinion about the systematic future price behavior *does not* always coincide with the forecast based on unconditional correlation. Indeed, suppose that at time 2 traders observe  $p_2 > p_1 > E_2[v]$ . For  $(\beta, 1/\tau_\delta) \in HR$ , unconditional correlation predicts that the short term increase in prices across the first two periods will be followed by a reversal, in stark contrast with the prediction based on the consensus opinion. To understand the reason for this difference, it is useful to refer to the case with homogeneous information. In that case, upon observing the *realization* of a positive noise stock  $\theta_2 > 0$ , traders speculate on short run price differences by taking the other side of the market. Furthermore, unconditional correlation predicts a price reversal. Indeed, with homogeneous information the only factor moving prices is represented by noise traders' demand which is transient. Therefore, both a

<sup>31</sup>Therefore, as momentum can arise also in the Keynesian region, a price runup is entirely compatible with a situation in which prices are farther away from the fundamentals compared with the consensus opinion.

<sup>32</sup>In the figure we use parameters' values in line with Cho and Krishnan (2000)'s estimates based on S&P500 data. Thus, we set  $\tau_v = 1/25$ ,  $\tau_u = 1/0.011^2$ ,  $\gamma = 1/2$  and  $\tau_{e_1} = 1/144$ ,  $\tau_{e_2} = \tau_{e_3} = 4/144$ .

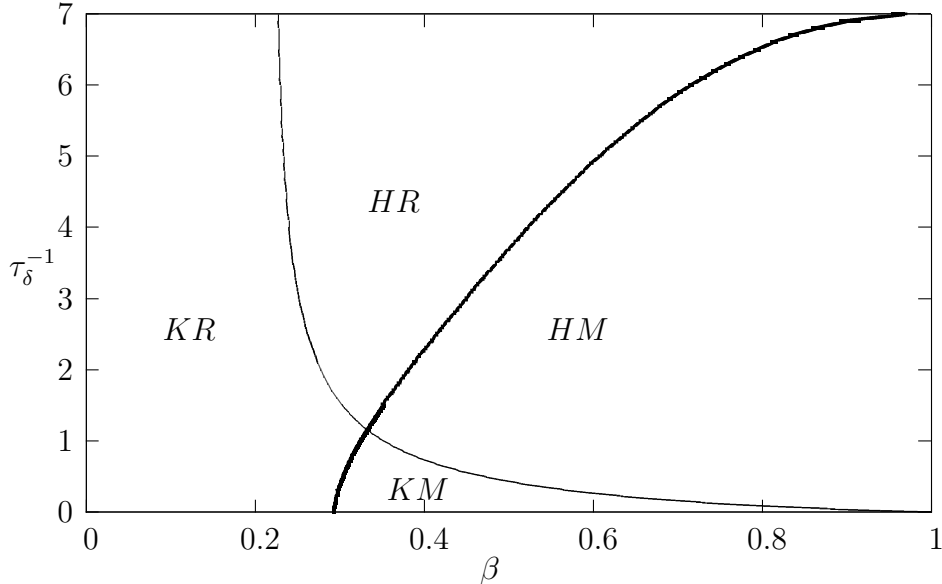


Figure 4: The figure displays the set  $\Omega_2 = \{(\beta, 1/\tau_\delta) \in [0, 1] \times \mathbb{R}_+ | \alpha_{P_2} = \alpha_{E_2}\}$  (thin line) and the set  $\{(\beta, 1/\tau_\delta) \in [0, 1] \times \mathbb{R}_+ | \text{Cov}[p_3 - p_2, p_2 - p_1] = 0\}$  (thick line). Parameters' values are  $\tau_v = 1/25$ ,  $\tau_u = 1/0.011^2$ ,  $\gamma = 1/2$  and  $\tau_{\epsilon_1} = 1/144$ ,  $\tau_{\epsilon_2} = \tau_{\epsilon_3} = 4/144$ .

positive liquidity stock and a price increase are deemed to be temporary. In the presence of heterogeneous information, on the other hand, fundamental information, which is persistent, also affects prices. This creates a signal extraction problem, implying that traders base their short term strategies on the realization of the *expected* noise stock,  $E_2[\theta_2]$ , filtered out of the observed aggregate demand. In this situation, it is natural that the anticipation of future price behavior crucially depends on the information set on which such a forecast is based.

The latter result is reminiscent of Biais, Bossaerts, and Spatt (2008) who study the empirical implications that a multi-asset, dynamic, noisy rational expectations equilibrium model has for optimal trading behavior. One of their findings points to the existence of a discrepancy between momentum strategies based on unconditional correlation and the optimal, price contingent strategies that traders adopt in their model.<sup>33</sup>

## 4.2 Higher Order Expectations, Short term Horizons, and Price Over-reliance on Public Information

Allen, Morris and Shin (2006), in a model with short term traders where  $\beta = \tau_\delta^{-1} = 0$ , relate the over-reliance of prices on public information to the well-known Keynesian “Beauty Contest” metaphor. They argue that if traders have a short-term speculative horizon, they engage in the activity of guessing the average behavior of the market. To use Keynes’ words, agents trade with a view to anticipate a change in the “convention” that guides the stock market valuation of actual investments. As a result, the price ends up assigning too high a weight to public information compared to the optimal statistical weight, rendering condition (9) always satisfied.

<sup>33</sup>Biais, Bossaerts, and Spatt (2008) also find that price contingent strategies are empirically superior to momentum strategies.

We can easily reproduce this argument with our notation under Allen et al. (2006) assumptions. Suppose thus that in the dynamic market analyzed in the previous section traders have short-term horizons (i.e. they take a position in period  $n$  and unwind it in period  $n + 1$ ) and that the private information each trader  $i$  receives in every period  $n$  is transmitted to the corresponding trader in period  $n + 1$ . Thus, in any period  $n$  every trader  $i$  maximizes the expected utility of his short-term profits  $\pi_{in} = (p_{n+1} - p_n)x_{in}$ ,  $E[U(\pi_{in})|\tilde{s}_{in}, p^n] = -E[\exp\{-\pi_{in}/\gamma\}|\tilde{s}_{in}, p^n]$ . Under these assumptions, at any time  $n$ , a trader's strategy is given by  $X_n(\tilde{s}_{in}, p^n) = \gamma \text{Var}_{in}[p_{n+1}]^{-1} E_{in}[p_{n+1} - p_n]$ . Assuming  $N = 3$ , in the third trading period imposing market clearing yields

$$\int_0^1 X_3(\tilde{s}_{i3}, p^3) di + \theta_3 = 0,$$

and since  $p_4 = v$ , solving for the equilibrium price we obtain

$$p_3 = \bar{E}_3[v] + \frac{\text{Var}_{i3}[v]}{\gamma} \theta_3.$$

Similarly, in the second period, imposing market clearing

$$\int_0^1 X_2(\tilde{s}_{i2}, p^2) di + \theta_2 = 0,$$

and solving for the equilibrium price we obtain

$$p_2 = \bar{E}_2[p_3] + \frac{\text{Var}_{i2}[p_3]}{\gamma} \theta_2. \quad (26)$$

Substituting the above obtained expression for  $p_3$  in (26) yields

$$p_2 = \bar{E}_2[\bar{E}_3[v]] + \frac{\text{Var}_{i2}[p_3]}{\gamma} \theta_2. \quad (27)$$

Finally, in the first period, a similar argument yields

$$p_1 = \bar{E}_1[\bar{E}_2[\bar{E}_3[v]]] + \frac{\text{Var}_{i1}[p_2]}{\gamma} \theta_1. \quad (28)$$

Equations (27) and (28) show that in this case the equilibrium price at period  $n$  is the average expectation at  $n$  of the average expectation at  $n + 1$  of the average expectation at  $n + 2$  of... the liquidation value in period  $N + 1$ , plus the corresponding period, risk-adjusted noise shock. This is reminiscent of Keynes' vision of the stock market as a beauty contest.

An interesting observation by Allen et al. (2006) is that, when averaging over the realizations of noise trading, the price at date  $n$  – the average expectation of the average expectation of the... – will in general not coincide with the period  $n$  average expectation of the fundamental value (the price at  $N + 1$ ). In this sense the consensus value of the fundamentals  $\bar{E}_n[v]$  does not coincide with the price  $p_n$ , with the exception of the last period  $n = N$ . The mean price path  $p_n$  gives a higher weight to history – relies more on public information – than the mean consensus path  $\bar{E}_n[v]$ . This is because of the bias towards public information when a Bayesian agent has to forecast the average market opinion knowing that it is based also on the public

information observed by other agents. This also implies that the current price will be always farther away from fundamentals than the average of investors' expectations and that it will be more sluggish to adjust. To see this within our example, let us compute the higher order expectation component of (27) and (28):

$$\bar{E}_2 [\bar{E}_3[v]] = \bar{\alpha}_{E_2} v + (1 - \bar{\alpha}_{E_2}) E_2[v], \quad \bar{E}_1 [\bar{E}_2 [\bar{E}_3[v]]] = \bar{\alpha}_{E_1} v + (1 - \bar{\alpha}_{E_1}) E_1[v],$$

where

$$\bar{\alpha}_{E_1} = \alpha_{E_1} \left( 1 - \frac{\tau_1}{\tau_2} (1 - \bar{\alpha}_{E_2}) \right), \quad \bar{\alpha}_{E_2} = \alpha_{E_2} \left( 1 - \frac{\tau_2}{\tau_3} (1 - \alpha_{E_3}) \right).$$

Given (27) and (28), it should be clear that in this case the weight the price assigns to the fundamentals (i.e.,  $\alpha_{P_n}$ ) is given by  $\bar{\alpha}_{E_n}$ :  $\alpha_{P_n} \equiv \bar{\alpha}_{E_n}$ . Now, as one can easily verify, at any linear equilibrium  $\alpha_{P_n} < \alpha_{E_n}$ . Hence, under Allen et al. (2006) assumptions, in the presence of short term traders (and assuming  $\beta = \tau_\delta^{-1} = 0$ ), the price is farther away from the fundamentals compared to traders' average opinion. If  $\beta \in (0, 1)$ , and keeping the assumption of short term horizons, expressions (27) and (28) are no longer correct, as the stock of noise trades at any period  $n$ , partially depends on the previous period one:  $\theta_n = u_n + \beta\theta_{n-1}$ . This in turn, implies that  $p_n$  depends, among other things, on higher order expectations about noise trades (see Cespa and Vives (2009)).

As we have argued in the previous section, in the present setup lack of residual uncertainty *coupled* with the negative correlation of noise increments ( $\beta \in [0, 1)$ ) induces even long term traders to exploit short term patterns in returns. This, in turn, is responsible for the occurrence of Keynesian equilibria in the market with long term traders. Our analysis, however, has shown that short term speculation can also lead to a markedly different outcome. Indeed, the introduction of an additional source of risk in the liquidation value, by boosting traders' signal responsiveness prior to the last trading date, opens the possibility for Hayekian equilibria with very distinct properties.

## 5 Conclusions

In this paper we have investigated the relationship between prices and consensus opinion as estimators of the fundamentals. We have shown that whenever heterogeneously informed, long term traders find it optimal to exploit short term price movements, prices can either be systematically farther away or closer to the fundamentals compared to the consensus opinion. This gives rise to a Keynesian and a Hayekian region in the space of our deep parameters (i.e., the persistence of noise trades and the dispersion of residual uncertainty affecting the asset liquidation value). In the Hayekian (Keynesian) region a systematic positive price departure from the public expectation about the fundamentals "generates" the consensus opinion that prices will systematically further rise (decrease) in the upcoming period. On the boundary between the two regions, on the other hand, the market consensus opinion is that the next period price won't change in any systematic way. As a consequence, traders concentrate on "the asset long term prospects and those only," abiding by Keynes's dictum.

Our paper provides a number of empirical implications. According to our results, for a given level of residual uncertainty, traders tend to use accommodating strategies when noise trading is strongly mean reverting. Conversely, they are trend chasers when noise trading is close to random walk and there is a continuous flow of private information. The latter parameter region widens when traders are more risk tolerant, receive better private information and a lower level of noise affects prices.

Furthermore, as in our setup the evolution of prices is governed by a transient and a persistent component, depending on the quality of private information, our model can generate empirically documented return regularities. Interacting the space of parameter values yielding momentum and reversal with the Keynesian and Hayekian regions, we have illustrated that the set of deep parameters yielding the two phenomena are different. As we argued, the consensus opinion can be taken as a measure of the market view of an asset fundamentals which, differently from the market price, is free from the influence of short term speculation dynamics. Therefore, our theory gives indications as to when a price runup (momentum) should be associated with a situation in which prices get closer or farther away from the fundamentals compared to the investors' average expectations. Low residual uncertainty in the liquidation value together with a high noise trades' persistence are likely to characterize situations of the first type. On the other hand, low noise trades' persistence (again coupled with low residual uncertainty) can be responsible of prices growing increasingly apart from fundamentals compared to the market consensus opinion.

Overall, our analysis points to the fact a low persistence of liquidity trades and an environment in which fundamentals are very opaque may be responsible for a failure of the simplistic EMH. Indeed, as we have shown, these conditions make the evolution of the aggregate demand, and thus of the asset returns, predictable. This lures traders towards the exploitation of these regularities, partially diverting them from the activity of evaluating the fundamentals. As a result, the equilibrium price ends up reflecting both components of traders' strategies (long and short term speculation), decoupling its dynamic from that of the consensus opinion. In these conditions, we have also argued that reversal occurs, and prices display over-reliance on public information. Momentum, instead, needs high noise trading persistence, and a transparent environment to arise. Hence, insofar as a high  $\beta$  proxies for a high trading frequency, we can conclude that any technological arrangement conducive to an increase in trading frequency together with improved disclosure is likely to promote positive return correlation and price under-reliance on public information.

A number of issues are left for future research. Our analysis has concentrated on the case in which traders have long horizons. Indeed, short term speculation in our setup arises endogenously whenever traders find it optimal to exploit regularities in the evolution of future returns. In a companion paper we analyze the implications of forcing on traders a short term horizon and show that in our general framework this is conducive to multiple equilibria with either Keynesian or Hayekian features (Cespa and Vives (2009)). Furthermore, while our paper gives a very detailed characterization of the conditions leading to traders' over-reliance



on public information, it does not assess the welfare consequences that this may have for market participants. In particular, in the Keynesian equilibrium informed traders explicitly take advantage of noise traders, exploiting the low persistence of their demand shocks. A model in which the noise in the price is due to rational traders entering the market to hedge a shock to their endowment would allow to analyze the welfare properties of this equilibrium. Furthermore, it would also allow to see whether in response to informed traders' activity liquidity patterns can change over time, thereby inducing a time-varying degree of noise trades' persistence, and ultimately affecting the sign and magnitude of the discrepancy between prices and average expectations in the estimation of fundamentals.<sup>34</sup>

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<sup>34</sup>Several authors have made a foray into the welfare analysis of noisy, dynamic rational expectations equilibrium models (see, e.g., Brennan and Cao (1996), and Cespa and Foucault (2008)).

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# Appendix

The following lemma establishes that working with the sequence  $z^n \equiv \{z_t\}_{t=1}^n$  is equivalent to working with  $p^n \equiv \{p_t\}_{t=1}^n$ :

**Lemma 1.** *In any linear equilibrium the sequence of informational additions  $z^n$  is observationally equivalent to  $p^n$ .*

*Proof.* Consider a candidate equilibrium in linear strategies  $x_{in} = a_n \tilde{s}_{in} - \varphi_n(p^n)$ . In the first period imposing market clearing yields  $\int_0^1 a_1 s_{i1} - \varphi_1(p_1) di + \theta_1 = a_1 v - \varphi_1(p_1) + \theta_1 = 0$  or, denoting with  $z_1 = a_1 v + \theta_1$  the informational content of the first period order-flow,  $z_1 = \varphi_1(p_1)$ , where  $\varphi_1(\cdot)$  is a linear function. Hence,  $z_1$  and  $p_1$  are observationally equivalent. Suppose now that  $z^{n-1} = \{z_1, z_2, \dots, z_{n-1}\}$  and  $p^{n-1} = \{p_1, p_2, \dots, p_{n-1}\}$  are observationally equivalent and consider the  $n$ -th period market clearing condition:  $\int_0^1 X_n(\tilde{s}_{in}, p^{n-1}, p_n) di + \theta_n = 0$ . Adding and subtracting  $\sum_{t=1}^{n-1} \beta^{n-t+1} a_t v$ , the latter condition can be rewritten as follows:

$$\sum_{t=1}^n z_t - \varphi_n(p^n) = 0,$$

where  $\varphi_n(\cdot)$  is a linear function,  $z_t = \Delta a_t v + u_t$  denotes the informational content of the  $t$ -th period order-flow, and  $\Delta a_t = a_t - \beta a_{t-1}$ . As by assumption  $p^{n-1}$  and  $z^{n-1}$  are observationally equivalent, it follows that observing  $p_n$  is equivalent to observing  $z_n$ .  $\square$

## PROOF OF PROPOSITION 1

To prove our argument, we proceed backwards. In the last trading period traders act as in a static model and owing to CARA and normality we have

$$X_3(\tilde{s}_{i3}, z^3) = \gamma \frac{E_{i3}[v] - p_3}{\text{Var}_{i3}[v + \delta]}, \quad (29)$$

and

$$p_3 = \alpha_{P_3} \left( v + \frac{\theta_3}{a_3} \right) + (1 - \alpha_{P_3}) E_3[v], \quad (30)$$

where

$$a_3 = \frac{\gamma \sum_{t=1}^3 \tau_{\epsilon_t}}{1 + \kappa}, \quad (31)$$

$$\alpha_{P_3} = \frac{\sum_{t=1}^3 \tau_{\epsilon_t}}{\tau_{i3}}, \quad (32)$$

$\kappa = \tau_\delta^{-1} \tau_{i3}$ . An alternative way of writing the third period equilibrium price is

$$p_3 = \lambda_3 z_3 + (1 - \lambda_3 \Delta a_3) \hat{p}_2, \quad (33)$$

where

$$\lambda_3 = \alpha_{P_3} \frac{1}{a_3} + (1 - \alpha_{P_3}) \frac{\Delta a_3 \tau_u}{\tau_3}, \quad (34)$$

captures the price impact of the net informational addition contained in the 3rd period aggregate demand, while

$$\begin{aligned}\hat{p}_2 &= \frac{\alpha_{P_3}\tau_3\beta(\sum_{t=1}^2\beta^{2-t}z_t) + (1 - \alpha_{P_3})a_3\tau_2E_2[v]}{\alpha_{P_3}\tau_3\beta a_2 + (1 - \alpha_{P_3})a_3\tau_2} \\ &= \frac{\gamma\tau_2E_2[v] + \beta(1 + \kappa)(z_2 + \beta z_1)}{\gamma\tau_2 + \beta a_2(1 + \kappa)},\end{aligned}\quad (35)$$

$z_n = \Delta a_n v + u_n$ , and  $\Delta a_n = a_n - \beta a_{n-1}$ .

## SECOND PERIOD

Substituting (29) in the second period objective function, a trader in the second period maximizes

$$E_{i2}[U(\pi_{i2} + \pi_{i3})] = -E_{i2}\left[\exp\left\{-\frac{1}{\gamma}\left((p_3 - p_2)x_{i2} + \frac{x_{i3}^2 \text{Var}_{i3}[v + \delta]}{2\gamma}\right)\right\}\right]. \quad (36)$$

Let  $\phi_{i2} = (p_3 - p_2)x_{i2} + x_{i3}^2 \text{Var}_{i3}[v + \delta]/(2\gamma)$ . The term  $\phi_{i2}$  is a quadratic form of the random vector  $Z_2 = (x_{i3} - \mu_1, p_3 - \mu_2)'$ , which is normally distributed (conditionally on  $\{\tilde{s}_{i2}, z^2\}$ ) with zero mean and variance covariance matrix

$$\Sigma_2 = \begin{pmatrix} \text{Var}_{i2}[x_{i3}] & \text{Cov}_{i2}[x_{i3}, p_3] \\ \text{Cov}_{i2}[x_{i3}, p_3] & \text{Var}_{i2}[p_3] \end{pmatrix}, \quad (37)$$

where

$$\begin{aligned}\text{Var}_{i2}[x_{i3}] &= \frac{(\Delta a_3(1 + \kappa) - \gamma\tau_{\epsilon_3})^2\tau_u + \tau_{i2}((1 + \kappa)^2 + \gamma^2\tau_u\tau_{\epsilon_3})}{\tau_{i2}\tau_u(1 + \kappa)^2}, \\ \text{Cov}_{i2}[x_{i3}, p_3] &= \lambda_3 \left( \frac{\gamma\tau_{\epsilon_3}\Delta a_3\tau_u - (1 + \kappa)(\tau_3 + \sum_{t=1}^2\tau_{\epsilon_t})}{\tau_{i2}\tau_u(1 + \kappa)} \right), \\ \text{Var}_{i2}[p_3] &= \lambda_3^2 \left( \frac{\tau_3 + \sum_{t=1}^2\tau_{\epsilon_t}}{\tau_{i2}\tau_u} \right),\end{aligned}$$

and

$$\mu_1 \equiv E_{i2}[x_{i3}] = \frac{a_3(1 - \lambda_3\Delta a_3)}{\alpha_{E_3}}(E_{i2}[v] - \hat{p}_2) \quad (38)$$

$$\mu_2 \equiv E_{i2}[p_3] = \lambda_3\Delta a_3E_{i2}[v] + (1 - \lambda_3\Delta a_3)\hat{p}_2. \quad (39)$$

Writing in matrix form:

$$\phi_{i2} = c_2 + b_2'Z_2 + Z_2'A_2Z_2,$$

where  $c_2 = (\mu_2 - p_2)x_{i2} + \mu_1^2 \text{Var}_{i3}[v + \delta]/(2\gamma)$ ,  $b_2 = (\mu_1 \text{Var}_{i3}[v + \delta]/\gamma, x_{i2})'$ , and  $A_2$  is a  $2 \times 2$  matrix with  $a_{11} = \text{Var}_{i3}[v + \delta]/(2\gamma)$  and the rest zeroes. Using a well-known result from normal theory we can now rewrite the objective function (36) as

$$\begin{aligned}E_{i2}[U(\pi_{i2} + \pi_{i3})] &= \\ &= -|\Sigma_2|^{-1/2} |\Sigma_2^{-1} + 2/\gamma A_2|^{-1/2} \times \exp\left\{-1/\gamma \left(c_2 - \frac{1}{2\gamma} b_2' (\Sigma_2^{-1} + 2/\gamma A_2)^{-1} b_2\right)\right\}.\end{aligned}\quad (40)$$

Maximizing the above function with respect to  $x_{i2}$  yields

$$x_{i2} = \Gamma_2^1(\mu_2 - p_2) + \Gamma_2^2\mu_1, \quad (41)$$

where

$$\Gamma_2^1 = \frac{\gamma}{h_{2,22}}, \quad \Gamma_2^2 = -\frac{h_{2,21}\text{Var}_{i3}[v + \delta]}{\gamma h_{2,22}},$$

and  $h_{2,ij}$  denotes the  $ij$ -th term of the symmetric matrix  $H_2 = (\Sigma_2^{-1} + 2/\gamma A_2)^{-1}$ :

$$h_{2,12} = -\frac{\lambda_3\tau_{i3}^2(1+\kappa)(1-\lambda_3\gamma\tau_{\epsilon_3}/(1+\kappa))}{D_2/\gamma^2} \quad (42)$$

$$h_{2,22} = \frac{\lambda_3^2\tau_{i3}((1+\kappa)(\tau_3 + \sum_{t=1}^2\tau_{\epsilon_t}) + \tau_{\epsilon_3})}{D_2/\gamma^2}, \quad (43)$$

and

$$\frac{D_2}{\gamma^2} = \tau_{i3} \left( \tau_{i3} (\lambda_3^2\tau_{i2} + (1 - \lambda_3\Delta a_3)^2\tau_u) + \tau_{i2}\tau_u\kappa \right). \quad (44)$$

Substituting (38) and (39) into (41) and rearranging yields

$$X_2(\tilde{s}_{i2}, z^2) = \frac{a_2}{\alpha_{E_2}} (E_{i2}[v] - \hat{p}_2) - \frac{\gamma}{h_{2,22}} (p_2 - \hat{p}_2), \quad (45)$$

where  $a_2$  denotes the 2nd period trading aggressiveness:

$$a_2 = \frac{\gamma(\sum_{t=1}^2\tau_{\epsilon_t})\tau_{i3}(1+\kappa)(1+\gamma\tau_u\Delta a_3)}{(1+\kappa+\gamma\tau_u\Delta a_3)(\tau_{\epsilon_3} + (\tau_3 + \sum_{t=1}^2\tau_{\epsilon_t})(1+\kappa))}. \quad (46)$$

Imposing market clearing yields

$$\int_0^1 \frac{a_2}{\alpha_{E_2}} (E_{i2}[v] - \hat{p}_2) di - \frac{\gamma}{h_{2,22}} (p_2 - \hat{p}_2) + \theta_2 = 0,$$

which after rearranging implies

$$\frac{\gamma\tau_2(\beta\rho_2 - 1)}{\gamma\tau_2 + \beta a_2(1+\kappa)} E_2[\theta_2] = \frac{\gamma}{h_{2,22}} (\hat{p}_2 - p_2), \quad (47)$$

where  $\rho_2 \equiv a_2(1+\kappa)/(\gamma\sum_{t=1}^2\tau_{\epsilon_t})$ . As a consequence, a trader  $i$ 's second period strategy can be written as follows:

$$X_2(\tilde{s}_{i2}, z^2) = \frac{a_2}{\alpha_{E_2}} (E_{i2}[v] - p_2) + \frac{(\gamma + h_{2,21})(\beta\rho_2 - 1)\tau_2}{\gamma\tau_{i3}} E_2[\theta_2]. \quad (48)$$

Using (45) we can obtain an expression for the second period equilibrium price that clarifies the role of the impact of expected noise traders' demand. Indeed, imposing market clearing yields

$$\frac{a_2}{\alpha_{E_2}} (\bar{E}_2[v] - p_2) + \frac{(\gamma + h_{2,21})(\beta\rho_2 - 1)\tau_2}{\gamma\tau_{i3}} E_2[\theta_2] + \theta_2 = 0,$$

where  $\bar{E}_2[v] \equiv \int_0^1 E_{i2}[v] di$ . Isolating  $p_2$  and rearranging we obtain

$$p_2 = \alpha_{P_2} \left( v + \frac{\theta_2}{a_2} \right) + (1 - \alpha_{P_2}) E_2[v], \quad (49)$$

where

$$\alpha_{P_2} = \alpha_{E_2} (1 + (\beta\rho_2 - 1)\Upsilon_2^1) \quad (50)$$

denotes the weight that the second period price assigns to  $v$ , and

$$\Upsilon_2^1 = \frac{\gamma\tau_2\tau_u(\gamma\tau_2 + \beta a_2(1 + \kappa) + \gamma\tau_{i2}\kappa)}{D_2} > 0. \quad (51)$$

Using (49) and (50) in (48) yields:

$$X_2(\tilde{s}_{i2}, z^2) = \frac{a_2}{\alpha_{E_2}}(E_{i2}[v] - p_2) + \frac{\alpha_{P_2} - \alpha_{E_2}}{\alpha_{E_2}} \frac{a_2}{\alpha_{P_2}}(p_2 - E_2[v]). \quad (52)$$

Finally, note that in period 2 as well we can obtain a recursive expression for the price that confirms the formula obtained in (33). Indeed, rearranging (49) we obtain

$$p_2 = \lambda_2 z_2 + (1 - \lambda_2 \Delta a_2) \hat{p}_1, \quad (53)$$

where

$$\lambda_2 = \alpha_{P_2} \frac{1}{a_2} + (1 - \alpha_{P_2}) \frac{\Delta a_2 \tau_u}{\tau_2}, \quad (54)$$

measures the price impact of the new information contained in the second period aggregate demand (since  $\int_0^1 x_{i2} di + \theta_2 = a_2 v + \theta_2 - \varphi_2(p_1, p_2) = z_2 + \beta z_1 - \varphi_2(p_1, p_2)$ ), and

$$\hat{p}_1 = \frac{\alpha_{P_2} \tau_2 \beta z_1 + (1 - \alpha_{P_2}) a_2 \tau_1 E_1[v]}{\alpha_{P_2} \tau_2 \beta a_1 + (1 - \alpha_{P_2}) a_2 \tau_1}. \quad (55)$$

An alternative expression for  $\lambda_2$  is as follows:

$$\lambda_2 = \underbrace{\frac{1 + \kappa + \gamma\tau_u\rho_2\Delta a_2}{\gamma\rho_2\tau_{i2}}}_{\lambda_2^S} + \frac{(\beta\rho_2 - 1)(1 + \kappa)\tau_u(\gamma\tau_2 + \beta a_2(1 + \kappa) + \gamma\tau_{i2}\kappa)(\tau_2 - a_2\Delta a_2\tau_u)}{\rho_2\tau_{i2}D_2}, \quad (56)$$

where  $\lambda_2^S$  denotes the “static” measure of the price impact of trade. The above expression thus highlights how noise trade predictability and the presence of residual uncertainty affect the static measure of the price impact of trade.

## FIRST PERIOD

To solve for the first period strategy, we now plug (41) into the argument of the exponential in (40):

$$c_2 - \frac{1}{2\gamma} b_2' (\Sigma_2^{-1} + 2/\gamma A_2)^{-1} b_2 = (E_{i2}[p_3] - p_2) x_{i2} + \frac{\text{Var}_{i3}[v + \delta]}{2\gamma} (E_{i2}[x_{i3}])^2 - \frac{1}{2\gamma} \left( \begin{array}{c} \frac{\text{Var}_{i3}[v + \delta]}{\gamma} E_{i2}[x_{i3}] \\ x_{i2} \end{array} \right) \left( \begin{array}{cc} h_{2,11} & h_{2,12} \\ h_{2,21} & h_{2,22} \end{array} \right) \left( \begin{array}{c} \frac{\text{Var}_{i3}[v + \delta]}{\gamma} E_{i2}[x_{i3}] \\ x_{i2} \end{array} \right).$$



Carrying out the matrix multiplication and simplifying yields

$$c_2 - \frac{1}{2\gamma} b_2' (\Sigma_2^{-1} + 2/\gamma A_2)^{-1} b_2 = \frac{1}{2\gamma} \left( h_{2,22} x_{i2}^2 + \frac{\gamma^2 (1 + \kappa)^2 \tau_{i2} \tau_u}{D_2} (E_{i2}[x_{i3}])^2 \right),$$

implying that

$$E_{i2}[U(\pi_{i2} + \pi_{i3})] = -|\Sigma_2|^{-1/2} |\Sigma_2^{-1} + (2/\gamma)A_2|^{-1/2} \times \exp \left\{ -\frac{1}{2\gamma^2} \left( h_{2,22} x_{i2}^2 + \frac{\gamma^2 (1 + \kappa)^2 \tau_{i2} \tau_u}{D_2} (E_{i2}[x_{i3}])^2 \right) \right\}.$$

The first period objective function now reads as follows:

$$E_{i1}[U(\pi_{i1} + \pi_{i2} + \pi_{i3})] = -E_{i1} \left[ \exp \left\{ -\frac{1}{\gamma} \left( (p_2 - p_1)x_{i1} \right. \right. \right. \quad (57) \\ \left. \left. \left. + \frac{1}{2\gamma} \left( h_{2,22} x_{i2}^2 + \frac{\gamma^2 (1 + \kappa)^2 \tau_{i2} \tau_u}{D_2} (E_{i2}[x_{i3}])^2 \right) \right) \right\} \right].$$

Note that since

$$E_{i2}[x_{i3}] = \frac{\gamma \tau_2}{1 + \kappa} (E_{i2}[v] - E_2[v]) - \beta E_{i2}[\theta_2],$$

we have

$$E_{i2}[v] - E_2[v] = \frac{(1 + \kappa)(E_{i2}[x_{i3}] + \beta E_{i2}[\theta_2])}{\gamma \tau_2},$$

and replacing the latter in the expression for  $x_{i2}$  yields

$$E_{i2}[x_{i3}] = \frac{x_{i2} + (1 - \beta \rho_2) E_{i2}[\theta_2]}{\rho_2}. \quad (58)$$

Thus, denoting by  $\phi_{i1}$  the argument of the exponential in (57) we obtain:

$$\phi_{i1} = (p_2 - p_1)x_{i1} + \frac{1}{2\gamma} \left( h_{2,22} x_{i2}^2 + \frac{\gamma^2 (1 + \kappa)^2 \tau_{i2} \tau_u}{D_2} \left( \frac{x_{i2} + (1 - \beta \rho_2) E_{i2}[\theta_2]}{\rho_2} \right)^2 \right).$$

Finally, as one can verify, letting  $\nu_1 = \alpha_{E_2}$ ,  $\nu_2 = -(\lambda_2 \tau_{i2})^{-1}(\tau_2 - a_2 \Delta a_2 \tau_u)$ , and  $\nu_3 = 1$ , we have

$$\nu_1 x_{i2} + \nu_2 p_2 + \nu_3 E_{i2}[\theta_2] = \frac{1}{\lambda_2 \tau_{i2}} (\Delta a_2 \tau_u \beta z_1 - \tau_1 E_1[v]) \equiv c(z_1), \quad (59)$$

implying that

$$E_{i2}[\theta_2] = c(z_1) - \alpha_{E_2} x_{i2} + \frac{\tau_2 - a_2 \Delta a_2 \tau_u}{\lambda_2 \tau_{i2}} p_2.$$

Given a trader's information set at time 1,  $c(z_1)$  is a constant. Hence, the uncertainty that a trader  $i$  faces at time 1 is reflected in  $\phi_{i1}$  through  $p_2$  and  $x_{i2}$  only:

$$\phi_{i1} = (p_2 - p_1)x_{i1} + \frac{1}{2\gamma} \left( h_{2,22} x_{i2}^2 + \frac{\gamma^2 (1 + \kappa)^2 \tau_{i2} \tau_u}{\rho_2^2 D_2} \times \right. \quad (60) \\ \left. \left( (1 - (1 - \beta \rho_2) \alpha_{E_2}) x_{i2} + c(z_1) (1 - \beta \rho_2) + \frac{(\tau_2 - a_2 \Delta a_2 \tau_u) (1 - \beta \rho_2)}{\lambda_2 \tau_{i2}} p_2 \right)^2 \right).$$

The term  $\phi_{i1}$  is a quadratic form of the random vector  $Z_1 \equiv (x_{i2} - \mu_1, p_2 - \mu_2)$ , which is normally distributed conditionally on  $\{s_{i1}, z_1\}$  with mean zero and variance-covariance matrix

$$\Sigma_1 = \begin{pmatrix} \text{Var}_{i1}[x_{i2}] & \text{Cov}_{i1}[x_{i2}, p_2] \\ \text{Cov}_{i1}[x_{i2}, p_2] & \text{Var}_{i1}[p_2] \end{pmatrix},$$

where  $\mu_1 \equiv E_{i1}[x_{i2}]$ ,

$$\mu_1 = \frac{(1 - \lambda_2^S \Delta a_2) a_2}{\alpha_{E_2}} (E_{i1}[v] - \hat{p}_1) + \frac{a_2 \tau_1 (\alpha_{P_2} - \alpha_{E_2})}{\alpha_{P_2} \alpha_{E_2} \tau_2} (\hat{p}_1 - E_1[v]), \quad (61)$$

and  $\mu_2 \equiv E_{i1}[p_2]$ ,

$$\mu_2 = \lambda_2 \Delta a_2 E_{i1}[v] + (1 - \lambda_2 \Delta a_2) \hat{p}_1, \quad (62)$$

while

$$\begin{aligned} \text{Var}_{i1}[x_{i2}] &= \frac{(\Delta a_2 \sum_{t=1}^2 \tau_{\epsilon_t} - a_2 \tau_{\epsilon_2})^2 \tau_u + \tau_{i1} ((\sum_{t=1}^2 \tau_{\epsilon_t})^2 + a_2^2 \tau_u \tau_{\epsilon_2})}{(\sum_{t=1}^2 \tau_{\epsilon_t})^2 \tau_{i1} \tau_u} \\ \text{Cov}_{i1}[x_{i2}, p_2] &= \lambda_2 \left( \frac{a_2 \Delta a_2 \tau_u \tau_{\epsilon_2} - (\tau_2 + \tau_{\epsilon_1}) (\sum_{t=1}^2 \tau_{\epsilon_t})}{(\sum_{t=1}^2 \tau_{\epsilon_t}) \tau_{i1} \tau_u} \right) \\ \text{Var}_{i1}[p_2] &= \lambda_2^2 \left( \frac{\tau_2 + \tau_{\epsilon_1}}{\tau_{i1} \tau_u} \right). \end{aligned}$$

Writing in matrix form:

$$\phi_{i1} = c_1 + b_1' Z_1 + Z_1' A_1 Z_1,$$

where

$$c_1 = (\mu_2 - p_1) x_{i1} + a_{11} \mu_1^2 + a_{22} \mu_2^2 + m_3 c(z_1)^2 + 2(m_1 \mu_1 c(z_1) + m_2 \mu_2 c(z_1) + a_{12} \mu_1 \mu_2),$$

$b_1 = (2(a_{11} \mu_1 + a_{12} \mu_2 + m_1 c(z_1)), 2(a_{22} \mu_2 + a_{12} \mu_1 + m_2 c(z_1)) + x_{i1})'$ , and

$$A_1 = \begin{pmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{pmatrix},$$

with

$$\begin{aligned} a_{11} &= \frac{h_{2,22}}{2\gamma} + \frac{a_{22}}{\nu_2^2} \left( \frac{1 - (1 - \beta \rho_2) \alpha_{E_2}}{1 - \beta \rho_2} \right)^2 & a_{12} &= -\frac{a_{22}}{\nu_2} \left( \frac{1 - (1 - \beta \rho_2) \alpha_{E_2}}{1 - \beta \rho_2} \right) \\ a_{22} &= \frac{(\nu_2 (1 - \beta \rho_2))^2}{2\gamma} \left( \frac{\gamma^2 (1 + \kappa)^2 \tau_{i2} \tau_u}{\rho_2^2 D_2} \right), \end{aligned}$$

and

$$m_1 = \frac{a_{22}}{\nu_2^2} \frac{1 - (1 - \beta \rho_2) \alpha_{E_2}}{1 - \beta \rho_2}, \quad m_2 = -\frac{a_{22}}{\nu_2}, \quad m_3 = \frac{a_{22}}{\nu_2^2}.$$

Along the lines of the second period maximization problem we then obtain

$$\begin{aligned} E_{i1}[U(\pi_{i1} + \pi_{i2} + \pi_{i3})] &= \\ &= -|\Sigma_1|^{-1/2} |\Sigma_1^{-1} + 2/\gamma A_1|^{-1/2} \exp \left\{ -1/\gamma \left( c_1 - \frac{1}{2\gamma} b_1' (\Sigma_1^{-1} + 2/\gamma A_1)^{-1} b_1 \right) \right\}. \end{aligned} \quad (63)$$

Maximizing (63) with respect to  $x_{i1}$ , solving for  $x_{i1}$  and rearranging yields

$$X_1(s_{i1}, p_1) = \Gamma_1^1 E_{i1}[p_2 - p_1] + \Gamma_1^2 E_{i1}[x_{i2}] + \Gamma_1^3 E_{i1}[x_{i3}], \quad (64)$$

where

$$\Gamma_1^1 = \frac{\gamma}{h_{1,22}}, \quad \Gamma_1^2 = -\frac{h_{1,12}h_{2,22}}{\gamma h_{1,22}}$$

$$\Gamma_1^3 = -\frac{\gamma^2(1+\kappa)^2\tau_u((\beta\rho_2-1)\tau_{i2}\nu_2h_{1,22} + \tau_{i3}(1-\lambda_3\Delta a_3)h_{1,12})}{D_2h_{1,22}}.$$

and the terms  $h_{1,ij}$  denote the  $ij$ -th elements of the symmetric matrix  $H_1 = (\Sigma_1^{-1} + 2/\gamma A_1)^{-1}$ :

$$h_{1,22} = \frac{\tau_{i3}}{\Phi_1 D_2 \rho_2^2 \tau_{\epsilon_2}} \left( (1+\kappa)^2((\tau_2 + \tau_{\epsilon_1})(\kappa\tau_u + \lambda_3^2\tau_{i3}) + (1-\lambda_3\Delta a_3)^2\tau_{i3}\tau_u) \right. \quad (65)$$

$$\left. + \lambda_3^2\rho_2^2\tau_{\epsilon_2} \left( (1+\kappa) \left( \tau_3 + \sum_{t=1}^2 \tau_{\epsilon_t} \right) + \tau_{\epsilon_3} \right) \right),$$

$$h_{1,12} = \frac{(1+\kappa)^2}{\Phi_1 D_2 \rho_2^2 \tau_{\epsilon_2} \gamma^2 \tau_{i2} \lambda_2 (\sum_{t=1}^2 \tau_{\epsilon_t})} \left( \left( a_2 \Delta a_2 \tau_u \tau_{\epsilon_2} - (\tau_2 + \tau_{\epsilon_1}) \left( \sum_{t=1}^2 \tau_{\epsilon_t} \right) \right) D_2 \right. \quad (66)$$

$$\left. + (\tau_2 - a_2 \Delta a_2 \tau_u) (\beta\rho_2 - 1) \tau_u \tau_{i3} (1 - \lambda_3 \Delta a_3) \gamma^2 \tau_{\epsilon_2} \sum_{t=1}^2 \tau_{\epsilon_t} \right),$$

and

$$\Phi_1 = |\Sigma_1|^{-1} \left( 1 + \frac{2a_{22}}{\gamma} \left( \text{Var}_{i1}[p_2] + \text{Var}_{i1}[x_{i2}] \frac{\tau_{i3}^2(1-\lambda_3\Delta a_3)^2}{\nu_2^2\tau_{i2}^2(1-\beta\rho_2)^2} \right. \right.$$

$$\left. \left. - 2\frac{\tau_{i3}(1-\lambda_3\Delta a_3)}{\nu_2\tau_{i2}(1-\beta\rho_2)} \text{Cov}_{i1}[p_2, x_{i2}] \right) + \frac{h_{2,22}\text{Var}_{i1}[x_{i2}]}{\gamma^2} \right) + 2\frac{a_{22}h_{2,22}}{\gamma^3}.$$

Substituting (61) and (62) into (64) and imposing market clearing, yields

$$p_1 = \alpha_{P_1} \left( v + \frac{\theta_1}{a_1} \right) + (1 - \alpha_{P_1}) E_1[v], \quad (67)$$

where

$$\alpha_{P_1} = \alpha_{E_1} \left( 1 + (\beta\rho_1 - \rho_2)\Upsilon_1^1 + (\beta\rho_2 - 1)\Upsilon_1^2 \right), \quad (68)$$

denotes the weight that the first period price assigns to the fundamentals and

$$\Upsilon_1^1 = \left( \frac{\gamma}{h_{1,22}} - \frac{a_1}{\alpha_{E_1}} \right) \frac{h_{1,22}\rho_1\tau_1\alpha_{E_2}}{\gamma a_2(1-\lambda_2\Delta a_2)}, \quad (69)$$

$$\Upsilon_1^2 = \left( 1 - \frac{h_{1,22}}{\gamma} \left( \frac{\gamma}{h_{1,22}} - \frac{a_1}{\alpha_{E_1}} \right) \right) \frac{\gamma\beta\tau_1\tau_u}{h_{1,22}(1-\lambda_2\Delta a_2)D_2} \times$$

$$\left( \frac{\gamma\tau_{i3}\lambda_3(1+\kappa)^3(1+\gamma\tau_u\Delta a_3)\Phi_2}{\rho_2^2 D_2 \lambda_2} - h_{1,12}h_{2,22}(\tau_{i3}(1-\lambda_3\Delta a_3) + \tau_{i2}\kappa) \right) \quad (70)$$

$$- \left( \frac{\gamma}{h_{1,22}} - \frac{a_1}{\alpha_{E_1}} \right) \frac{h_{1,22}\rho_1\tau_1 \sum_{t=1}^2 \tau_{\epsilon_t} \tau_2\tau_u(a_2(1+\kappa) + \gamma\tau_2\beta\rho_1)(\tau_{i3}(1-\lambda_3\Delta a_3) + \tau_{i2}\kappa)}{\gamma a_2\tau_2(1-\lambda_2\Delta a_2) \tau_{i2}D_2},$$

and  $a_1$  denotes a trader  $i$ 's first period private signal responsiveness:

$$a_1 = \frac{\alpha_{E_1}}{h_{1,22}} \left( \lambda_2 \Delta a_2 (\gamma - 2(h_{1,22} a_{22} + h_{1,12} a_{12})) + \right. \\ \left. - 2(h_{1,22} a_{12} + h_{1,12} a_{11}) \left( \frac{a_2 \tau_1}{\sum_{t=1}^2 \tau_{\epsilon_t}} + \beta a_1 \right) \right). \quad (71)$$

Using (67) in (64) and rearranging yields

$$X_1(s_{i1}, z_1) = \frac{a_1}{\alpha_{E_1}} (E_{i1}[v] - p_1) + \frac{\alpha_{P_1} - \alpha_{E_1}}{\alpha_{E_1}} \frac{a_1}{\alpha_{P_1}} (p_1 - E_1[v]). \quad (72)$$

Note that (72), together with (67) show that the expressions for equilibrium prices and traders' strategies have a recursive structure. Finally, note that as obtained in periods 2 and 3, we can express the first period equilibrium price as follows

$$p_1 = \lambda_1 z_1 + (1 - \lambda_1 a_1) \bar{v},$$

where

$$\lambda_1 = \alpha_{P_1} \frac{1}{a_1} + (1 - \alpha_{P_1}) \frac{a_1 \tau_u}{\tau_1}.$$

Finally, to obtain the expressions for the equilibrium prices in section 4.2, we use (29), (41), and (64) and via recursive substitution obtain

$$\Psi_2^1 = \frac{\text{Var}_{i3}[v + \delta]}{\gamma}, \quad \Psi_2^2 = -\frac{h_{2,12} \text{Var}_{i3}[v + \delta]}{\gamma^2}, \quad \Psi_2^3 = \frac{h_{2,22}}{\gamma},$$

and

$$\Psi_1^1 = \Psi_2^1, \quad \Psi_1^2 = \Psi_2^2, \quad \Psi_1^3 = \Psi_2^3, \\ \Psi_1^4 = -\frac{\gamma(1 + \kappa)^2 \tau_u (\tau_{i2} (\beta \rho_2 - 1) \nu_2 h_{1,22} + \tau_{i3} (1 - \lambda_3 \Delta a_3) h_{1,12})}{D_2} \\ \Psi_1^5 = \frac{h_{1,12} h_{2,22}}{\gamma^2}, \quad \Psi_1^6 = -\frac{h_{1,22}}{\gamma}.$$

This completes our proof.

QED

### PROOF OF PROPOSITION 3.2

Suppose that  $\tau_{\epsilon_1} = \tau_{\epsilon_2} = \tau_{\epsilon_3} = 0$ . Then, since in equilibrium

$$a_3 = \frac{\gamma \sum_{t=1}^3 \tau_{\epsilon_t}}{1 + \kappa} \\ a_2 = \frac{\gamma (\sum_{t=1}^2 \tau_{\epsilon_t}) \tau_{i3} (1 + \kappa) (1 + \gamma \tau_u \Delta a_3)}{(1 + \kappa + \gamma \tau_u \Delta a_3) (\tau_{\epsilon_3} + (\tau_3 + \sum_{t=1}^2 \tau_{\epsilon_t}) (1 + \kappa))},$$

we immediately obtain  $a_2 = a_3 = 0$ . Note that this is in line with what one should assume in a linear equilibrium where traders possess no private information. Indeed, at any candidate

linear equilibrium a trader's strategy at time  $n$  is given by  $X_n(p^n) = \varphi(p^n)$ , where  $\varphi(\cdot)$  is a linear function. Imposing market clearing, in turn implies that  $\varphi(p^n) = \theta_n$ , so that at any linear equilibrium the price only incorporates the supply shock ( $a_n = 0$ ) which is therefore perfectly revealed to risk averse speculators.

This, in turn, implies that  $\tau_n = \tau_{in} = \tau_v$ ,

$$E_n[v] = E_{in}[v] = \bar{v},$$

and that  $\alpha_{P_n} = \alpha_{E_n} = 0$ . Now, we can go on and characterize the strategies that traders adopt, using the expressions that appear in proposition 1 in the paper:

$$X_3(p^3) = \frac{\gamma\tau_v}{1+\kappa}(\bar{v} - p_3) \quad (73)$$

$$X_2(p^2) = \frac{\gamma\tau_v}{1+\kappa}(\bar{v} - p_2) + \frac{(\beta-1)\gamma^3\tau_v^2\tau_u}{(1+\kappa)(1+\kappa+\beta\gamma^2\tau_u\tau_v)}(p_2 - \bar{v}). \quad (74)$$

The second component of the latter expression, in particular, comes from the fact that

$$\lim_{\tau_{\epsilon_n} \rightarrow 0} \frac{\alpha_{P_1} - \alpha_{E_1}}{\alpha_{E_1}} \frac{a_1}{\alpha_{P_1}} = \frac{(\beta-1)\gamma^3\tau_v^2\tau_u}{(1+\kappa)(1+\kappa+\beta\gamma^2\tau_u\tau_v)}.$$

Imposing market clearing we obtain:

$$p_2 = \bar{v} + \frac{(\beta-1)(1+\kappa)\gamma\tau_u}{1+\kappa+\gamma^2\tau_v\tau_u}E_2[\theta_2] + \frac{1+\kappa}{\gamma\tau_v}\theta_2. \quad (75)$$

Given that  $a_2 = 0$ ,  $z_2 = u_2$ , and since traders at time 2 have also observed  $z_1 = \theta_1$ , the second period stock of noise  $\theta_2 = \beta\theta_1 + u_2$  can be exactly determined, and

$$E_2[\theta_2] = \theta_2.$$

Hence, as argued above, traders perfectly anticipate the noise shock and accommodate it, and the price only reflects noise. But then this implies that

$$p_2 = \bar{v} + \frac{(\beta-1)(1+\kappa)\gamma\tau_u}{1+\kappa+\gamma^2\tau_v\tau_u}\theta_2 + \frac{1+\kappa}{\gamma\tau_v}\theta_2. \quad (76)$$

As a last step we need to characterize the first period equilibrium. Substituting the second period optimal strategy in the corresponding objective function and rearranging, at time 1 a trader chooses  $x_{i1}$  to maximize

$$-E_{i1} \left[ \exp \left\{ -\frac{1}{\gamma} \left( (p_2 - p_1)x_{i1} + \frac{(1+\kappa)(1+\kappa+\gamma\beta^2\tau_v\tau_u)}{2\gamma\tau_v(1+\kappa+\gamma^2\tau_v\tau_u)}\theta_2^2 \right) \right\} \right].$$

According to (75)  $p_2$  only depends on  $\theta_2$ . Hence, in the first period the argument of the trader's objective function is a quadratic form of the random variable  $\theta_2$  which is normally distributed:

$$\theta_2|\theta_1 \sim N(\beta\theta_1, \tau_u^{-1}) \Rightarrow (\theta_2 - \beta\theta_1)|\theta_1 \sim N(0, \tau_u^{-1}),$$

and we can apply the usual transformation to compute the above expectation, obtaining that the function maximized by the trader is given by

$$(\bar{v} - p_1)x_{i1} + \beta\theta_1(m_1x_{i1} + m_2\beta\theta_1) - \frac{1}{2\gamma(\tau_u + (2/\gamma)m_2)}(m_1x_{i1} + 2m_2\beta\theta_1)^2,$$

where

$$m_1 = \frac{(1 + \kappa + \gamma^2 \beta \tau_u \tau_v)(1 + \kappa)}{(1 + \kappa + \gamma^2 \tau_u \tau_v) \gamma \tau_v}, \quad m_2 = \frac{(1 + \kappa + \gamma^2 \beta^2 \tau_u \tau_v)(1 + \kappa)}{(1 + \kappa + \gamma^2 \tau_u \tau_v) 2 \gamma \tau_v}.$$

Computing the first order condition and solving for  $x_{i1}$  yields

$$X_1(p_1) = \frac{\gamma \tau_v}{1 + \kappa} (\bar{v} - p_1) + \frac{(\beta - 1)((1 + \kappa + \gamma^2 \tau_u \tau_v)(1 + \kappa + \gamma^2 \tau_u \tau_v (1 + \beta)) + (1 + \kappa)^2 (1 - \beta))}{(1 + \kappa)(1 + \kappa + \beta \gamma^2 \tau_u \tau_v)((1 + \kappa + \gamma^2 \beta \tau_u \tau_v)^2 + \beta \gamma^4 \tau_u^2 \tau_v^2 (1 - \beta))} (p_1 - \bar{v}). \quad (77)$$

Imposing market clearing and explicitly solving for the price

$$p_1 = \bar{v} + \Lambda_1 \theta_1, \quad (78)$$

where

$$\Lambda_1 = \left( \frac{\gamma \tau_v}{1 + \kappa} - \frac{(\beta - 1)((1 + \kappa + \gamma^2 \tau_u \tau_v)(1 + \kappa + \gamma^2 \tau_u \tau_v (1 + \beta)) + (1 + \kappa)^2 (1 - \beta))}{(1 + \kappa)(1 + \kappa + \beta \gamma^2 \tau_u \tau_v)((1 + \kappa + \gamma^2 \beta \tau_u \tau_v)^2 + \beta \gamma^4 \tau_u^2 \tau_v^2 (1 - \beta))} \right)^{-1},$$

which can be rearranged to obtain (14). QED

### PROOF OF PROPOSITION 3

Follows immediately from the definition of equations (50), and (68). QED

### PROOF OF COROLLARY 5

Note that for  $\kappa = 0$  (31) and (46) imply  $a_n = \gamma(\sum_{t=1}^n \tau_{\epsilon_t})$ , for  $n = 2, 3$ . Hence,  $\rho_2 = 1$  and (56), (50) respectively become:

$$\lambda_2 = \frac{1 + \gamma \tau_u \Delta a_2}{\gamma \tau_2 + a_2} + \frac{(\beta - 1) \tau_u (\gamma \tau_2 + \beta a_2) (\tau_2 - a_2 \Delta a_2 \tau_u)}{\tau_{i2} D_2} \quad (79)$$

$$\alpha_{P_2} = \alpha_{E_2} \left( 1 + \frac{(\beta - 1) \gamma \tau_2 \tau_u (\gamma \tau_2 + \beta a_2)}{D_2} \right), \quad (80)$$

so that

$$\Upsilon_2^1 = \frac{\gamma \tau_2 \tau_u (\gamma \tau_2 + \beta a_2)}{D_2} > 0.$$

In the first period tedious algebra allows to show that

$$h_{1,12} = -\frac{\lambda_2 \tau_{i2}^2 (1 - \lambda_2 \gamma \tau_{\epsilon_2})}{D_1}, \quad h_{1,22} = \frac{(\lambda_2 \tau_{i2})^2}{D_1}, \quad (81)$$

where

$$D_1 = \tau_{i2}^2 \left( \lambda_2^2 \tau_{i1} + (1 - \lambda_2 \Delta a_2)^2 \tau_u + \frac{(\beta - 1)^2 (\tau_2 - a_2 \Delta a_2 \tau_u)^2 \tau_u h_{2,22}}{D_2} \right). \quad (82)$$

Substituting (61), (62), and (81) in (64) and rearranging yields:

$$X_1(s_{i1}, p_1) = \frac{a_1}{\alpha_{E_1}} (E_{i1}[v] - p_1) + \frac{\gamma}{h_{1,22}} (1 - \tau_{i1} h_{1,22}) (\hat{p}_1 - p_1) - \frac{\gamma \tau_u \tau_1 \beta (\beta - 1)^2 (\tau_2 - a_2 \Delta a_2 \tau_u) (\gamma \tau_{i3} \lambda_3)^2}{\lambda_2 (1 - \lambda_2 \Delta a_2) D_2^2} E_1[\theta_1]. \quad (83)$$

Using (81), we can now simplify (71) to obtain

$$a_1 = \frac{\tau_{\epsilon_1}}{\lambda_2 \tau_{i1} \tau_{i2}} \left( \frac{D_1 \Delta a_2}{\tau_{i2}} - (\gamma \tau_1 + \beta a_1) (\Delta a_2 \tau_u (1 - \lambda_2 \Delta a_2) - \lambda_2 \tau_{i1}) + \right. \\ \left. - \frac{(1 - \beta) (\tau_2 - a_2 \Delta a_2 \tau_u) \Delta a_2 \tau_u (\gamma (1 - \beta) (\tau_2 - a_2 \Delta a_2 \tau_u) - (\gamma \tau_2 + \beta a_2) (1 - \lambda_2 \gamma \tau_{\epsilon_2}))}{D_2} \right) \\ = \gamma \tau_{\epsilon_1}, \quad (84)$$

since, as one can verify,

$$\frac{D_1}{\tau_{i2}} = \lambda_2 \tau_{i1} (1 + \gamma \Delta a_2 \tau_u) + (1 - \lambda_2 \Delta a_2) \tau_u (\gamma \tau_1 + \beta a_1) + \\ + \frac{(1 - \beta) (\tau_2 - a_2 \Delta a_2 \tau_u) \Delta a_2 \tau_u (\gamma (1 - \beta) (\tau_2 - a_2 \Delta a_2 \tau_u) - (\gamma \tau_2 + \beta a_2) (1 - \lambda_2 \gamma \tau_{\epsilon_2}))}{D_2}.$$

Finally, imposing market clearing yields

$$\frac{(\beta - 1) \tau_1}{1 - \lambda_2 \Delta a_2} \left( \frac{\alpha_{P_2} (\beta - 1) (1 - \alpha_{E_2}) + \alpha_{P_2} - \alpha_{E_2}}{\alpha_{E_2} \tau_2 (\beta - 1)} + \frac{\beta (\beta - 1) \gamma \tau_u (\tau_2 - a_2 \Delta a_2 \tau_u) (\gamma \tau_{i3} \lambda_3)^2}{\lambda_2 D_2^2} \right) E_1[\theta_1] = \frac{\gamma}{h_{1,22}} (\hat{p}_1 - p_1). \quad (85)$$

We can now substitute (85) in (83). Imposing market clearing and rearranging allows to obtain an expression for the first period price as (67), where

$$\alpha_{P_1} = \alpha_{E_1} \left\{ 1 + \frac{(\beta - 1) \gamma \tau_1 (1 - h_{1,22} \tau_{i1})}{1 - \lambda_2 \Delta a_2} \frac{\alpha_{P_2}}{a_2} + \frac{(\beta - 1) \tau_1}{1 - \lambda_2 \Delta a_2} \times \right. \\ \left. \left( h_{1,22} \tau_{i1} \frac{\beta (1 - \beta) \gamma \tau_u (\tau_2 - a_2 \Delta a_2 \tau_u) (\gamma \tau_{i3} \lambda_3)^2}{\lambda_2 D_2^2} + (1 - h_{1,22} \tau_{i1}) \frac{\alpha_{P_2} - \alpha_{E_2}}{\alpha_{E_2} \tau_2 (\beta - 1)} \right) \right\}. \quad (86)$$

Finally, for  $\alpha_{P_2}$ , using (80), the result stated in the corollary is immediate. For  $\alpha_{P_1}$ , inspection of (86) shows that  $\alpha_{P_1} < \alpha_{E_1}$  if and only if  $\beta < 1$  since the sum of the terms multiplying  $\beta - 1$ :

$$\Upsilon_1^1 + \Upsilon_1^2 = \frac{\gamma \tau_1 (1 - h_{1,22} \tau_{i1})}{1 - \lambda_2 \Delta a_2} \frac{\alpha_{P_2}}{a_2} + \frac{\tau_1}{1 - \lambda_2 \Delta a_2} \times \\ \left( h_{1,22} \tau_{i1} \frac{\beta (1 - \beta) \gamma \tau_u (\tau_2 - a_2 \Delta a_2 \tau_u) (\gamma \tau_{i3} \lambda_3)^2}{\lambda_2 D_2^2} + (1 - h_{1,22} \tau_{i1}) \frac{\alpha_{P_2} - \alpha_{E_2}}{\alpha_{E_2} \tau_2 (\beta - 1)} \right),$$

can be verified to be always positive.

QED

### PROOF OF COROLLARY 8

For the first part of the corollary, consider the following argument. From the first order condition of the trader's problem in the second period

$$x_{i2} = \gamma \frac{E_{i2}[p_3 - p_2]}{h_{2,22}} - \frac{h_{2,21} (1 + \kappa)}{\gamma h_{2,22}} E_{i2}[x_{i3}].$$

Imposing market clearing, using (38) and (39), and rearranging yields

$$\frac{\tau_2 (\beta \rho_2 - 1)}{h_{2,22} \tau_{i3} (1 - \lambda_3 \Delta a_3)} \left( h_{2,22} - \frac{\lambda_3 \Delta a_3 (1 + \kappa)}{\rho_2 \tau_{i2}} \right) E_2[\theta_2] - \frac{h_{2,21} (1 + \kappa) (1 - \alpha_{E_2}) (1 - \beta \rho_2)}{\gamma h_{2,22} \rho_2 \tau_{i2} \tau_{i3}} E_2[\theta_2] \\ + \left( 1 + \frac{\alpha_{E_2}}{a_2} \left( \frac{h_{2,21} (1 + \kappa) a_3 (1 - \lambda_3 \Delta a_3)}{\gamma h_{2,22} \tau_{i3} \alpha_{E_3}} - \frac{\gamma \lambda_3 \Delta a_3}{h_{2,22}} \right) \right) \theta_2 = 0.$$

The first line in the above equation respectively captures the impact that the expected change in price and the expected third period position have on traders' aggregate second period strategy. Rearranging the term multiplying  $\theta_2$  in the second line yields

$$1 + \frac{\alpha_{E_2}}{a_2} \left( \frac{h_{2,21}(1 + \kappa)a_3(1 - \lambda_3\Delta a_3)}{\gamma h_{2,22}\tau_{i3}\alpha_{E_3}} - \frac{\gamma\lambda_3\Delta a_3}{h_{2,22}} \right) = 1 + \frac{\alpha_{E_2}}{a_2} \left( -\frac{\alpha_{E_2}}{a_2} \right) = 0.$$

The above result implies that for any realization of  $E[E_2[\theta_2]|v] = (a_2/\alpha_{P_2})E[p_2 - E_2[v]|v]$ ,

$$\frac{\tau_2(\beta\rho_2 - 1)}{h_{2,22}\tau_{i3}(1 - \lambda_3\Delta a_3)} \left( h_{2,22} - \frac{\lambda_3\Delta a_3(1 + \kappa)}{\rho_2\tau_{i2}} \right) E[E_2[\theta_2]|v]$$

and

$$-\frac{h_{2,21}(1 + \kappa)(1 - \alpha_{E_2})(1 - \beta\rho_2)}{\gamma h_{2,22}\rho_2\tau_{i2}\tau_{i3}} E[E_2[\theta_2]|v],$$

must have opposite sign. Given that  $h_{2,21}$  can be verified to be negative, this implies that if  $\beta\rho_2 > 1$ ,  $E[\bar{E}_2[p_3 - p_2]|v]$  is positive. If  $\kappa = 0$ , then a similar argument shows that at time 2  $E[p_2 - E_2[v]|v] < 0 \Leftrightarrow E[\bar{E}_2[p_3 - p_2]|v] > 0$  for  $\beta < 1$ .

In the absence of residual uncertainty, at time  $n = 1$ , using (85) and rearranging the market clearing equation yields

$$\frac{h_{1,22}}{\gamma} \frac{(\beta - 1)\tau_1}{1 - \lambda_2\Delta a_2} \left( \frac{\alpha_{P_2}(\beta - 1)(1 - \alpha_{E_2}) + \alpha_{P_2} - \alpha_{E_2}}{\alpha_{E_2}\tau_2(\beta - 1)} + \frac{\beta(\beta - 1)\gamma\tau_u(\tau_2 - a_2\Delta a_2\tau_u)(\gamma\tau_{i3}\lambda_3)^2}{\lambda_2 D_2^2} \right) E_1[\theta_1] = \hat{p}_1 - p_1. \quad (87)$$

Averaging out noise in the above expression, in this case the sign of  $E[\bar{E}_1[p_2 - p_1]|v]$  depends on the sign of the sum of the term multiplying  $E[E_1[\theta_1]|v]$  in the above expression and

$$\lambda_2\Delta a_2 \left( \frac{\alpha_{E_1}}{a_1} - \frac{\beta\alpha_{P_2}}{a_2(1 - \lambda_2\Delta a_2)} \right), \quad (88)$$

which after rearranging can be shown to be always negative provided  $\beta < 1$ .

QED



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