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Economics Working Paper  
No 2009-11

# Firms' heterogeneity, endogenous entry, and exit decisions

by Alexander Totzek



# Firms' Heterogeneity, Endogenous Entry, and Exit Decisions\*

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## Abstract

As GDP is highly correlated with both entering and exiting firms, we develop a totally microfounded DSGE model with endogenous firms entry as well as exit decisions. We show that the simplifying assumption of a constant firms' death rate made by the recent literature on DSGE modelling can lead to counterfactual implications of the resulting dynamics. We further demonstrate that the feature of endogenous exits significantly improves the performance of the resulting model when comparing the generated second moments with those of existing models assuming exogenous exits and with the data. Moreover, we estimate the resulting Phillips curve which turns out to be also a function of the change in the mass of producers using the generalized method of moments.

*JEL classification:* E32; E31

*Keywords:* Heterogeneity; Producer entry and exit; Business cycles; GMM

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# 1 Introduction

The number of producing firms varies over time and significantly co-moves with GDP [see amongst others Devereux, Head, and Lapham (1996) or Bergin and Corsetti (2008)]. Moreover, the empirical study of Campbell (1998) shows that, although the entry rate of new firms is significantly correlated with GDP, the co-movement between the business cycle and firms' failures is even larger.<sup>1</sup> This result is confirmed by Jaimovich and Floetotto (2008) who also find negative and highly significant correlations between GDP and firms' failures based on industry level data. However, in the recent theoretical literature an endogenous tendency for firms to leave the market has been totally neglected, yet. The substantial cyclical behavior of firms' failures implies that a closer examination of this topic may help to explain how shocks to the economy generate large and persistent business cycle fluctuations.

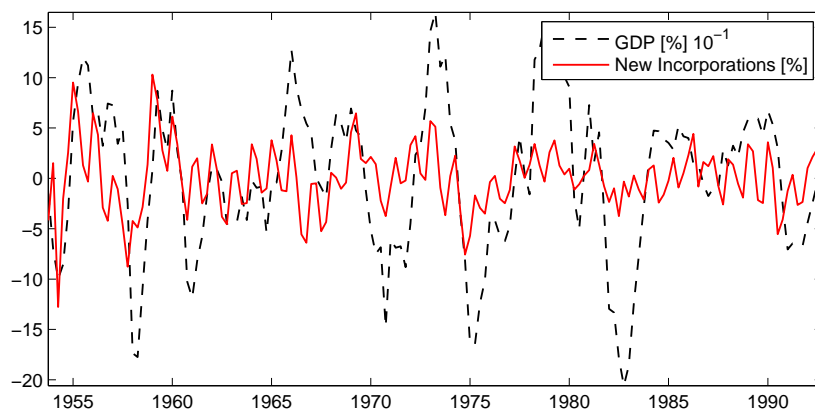


Figure 1: Firms birth rate vs GDP in the US (Hodrick-Prescott filtered data in logs)

Figure 1 and Figure 2 depict new incorporations and firms' failures in comparison with GDP for the US economy based on quarterly data (1953Q1:1992Q4 and 1953Q1:1998Q4), respectively.<sup>2</sup> The data is represented in logs and detrended by application of the Hodrick-Prescott filter. Figure 1 and Figure

<sup>1</sup>Campbell (1998) finds a correlation between the exit (entry) rate and GDP growth of 0.51 (0.28) for the US economy (1972Q2-1988Q4).

<sup>2</sup>The data of new incorporations and firms' failures is provided by the "Survey of Current Business" and the "Economic Report of the President" by the Council of Economic Advisors.

2 show that there exists a positive (negative) co-movement of firms' creation (failures) with GDP.

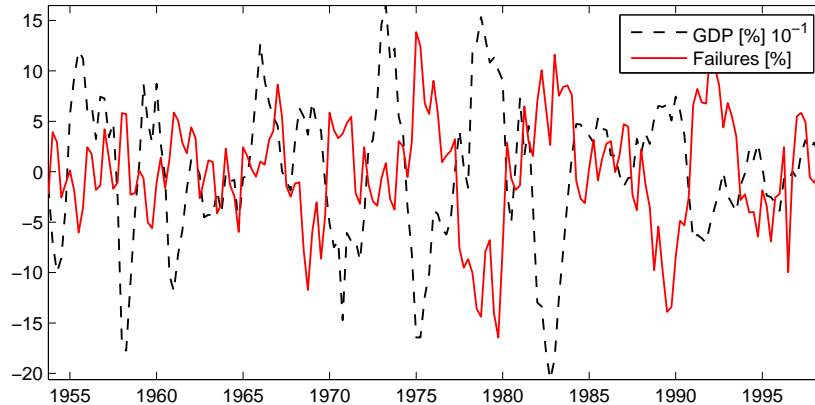


Figure 2: Firms failures vs GDP in US (Hodrick-Prescott filtered data in logs)

Figure 3 depicts the corresponding cross-correlations between GDP and firms' failures as well as between GDP and new incorporations for different leads and lags. It shows that although firms' creation is strongly correlated with GDP (0.41), an even stronger (negative) correlation exists between GDP and firms' failures (-0.57) which is consistent with the findings of Campbell (1998). This result is analogous to labor market data where job destruction turns out to be more cyclical than job creation [see Blanchard and Diamond (1990) or Davis and Haltiwanger (1996)]. Moreover, it is worthwhile to mention that in line with the results of Devereux, Head, and Lapham (1996), entries takes place slightly prior to an increase in GDP while exits takes place contemporaneously.

The disregard of the correlation between GDP and firms' failures would thus be an empirical shortcoming. However, the implementation of an endogenous counter-cyclical tendency of firms to leave the market has totally been neglected in the recent theoretical literature. There just exists a small strand of literature dealing with endogenous firms' entries initiated by Ghironi and Melitz (2005) to which Lewis (2009a), Jaimovich and Floetotto (2008), Bilbiie, Ghironi, and Melitz (2007a) [henceforth: BGMa], Corsetti, Martin, and Pesenti (2007), and Beaudry, Collard, and Portier (2006) have contributed some interesting aspects,

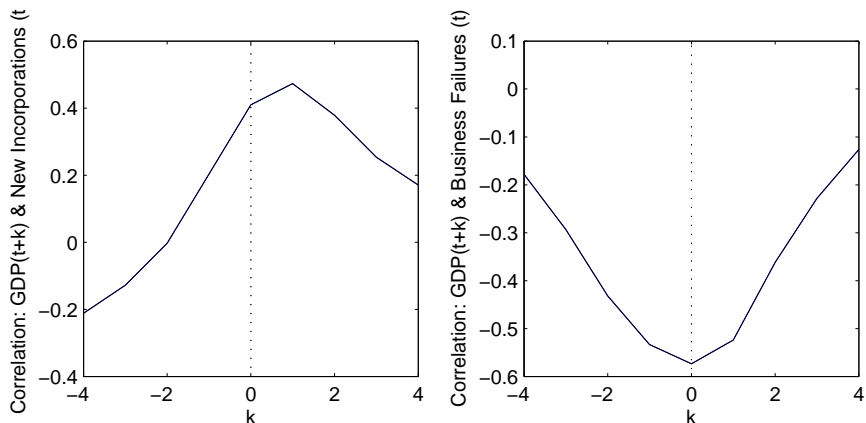


Figure 3: Cyclical properties of entry and exit

on the counter-cyclical nature of mark-ups and business cycle movements. By introducing nominal rigidities to the framework of BGMa, the study of Bilbiie, Ghironi, and Melitz (2007b) [henceforth: BGMb] has become the workhorse New Keynesian model for analyzing monetary policy issues in a framework with endogenous firms' entries [see e.g. Faia (2009), Bergin and Corsetti (2008), Lewis (2009b), and Elkhoury and Mancini-Griffoli (2006)]. However, the authors mentioned above assume that the firms death rate is constant over time. More precisely, they assume that with a given (constant) probability firms are hit with a death shock at the very end of each period. A main problem of these models is however that they do not perform better than standard RBC models when having a look at the generated second moments [see BGMa and BGMb].

The purpose of our paper is therefore to develop a totally microfounded New Keynesian model with heterogeneous firms and endogenous firms' entries as well as exits. We assume the firms to be heterogeneous in their individual productivity. They thus produce with different technologies. Thereby, both the entry as well as the exit decision of firms are based on present value criteria which are defined as the respective discounted sum of current and expected future profits. More precisely, if an existing firm expects a non-positive present value of production, it will consequently leave the market. On the other hand, a new firm will enter, if its entry is profitable, i.e. if the present value of

production exceeds the entry costs. Of course, the entry and exit decisions crucially depend on the respective individual productivity level in our model. As a result, good (i.e. productive) firms will thus stay in the market or will enter it, while bad firms will leave.

In comparison to BGMa, the introduction of an endogenous tendency of firms to leave the market does not only enhance the performance of the resulting model with respect to the generated second moments but also solves a general difficulty of Real Business Cycle (RBC) and New Keynesian (NK) models concerning the pro-cyclicality of real variables. More precisely, the generated absolute and relative standard deviations of output, consumption, investment, and total hours worked are unambiguously closer to the data than those obtained by the exogenous exit model of BGMa. By the introduction endogenous exits – which react counter-cyclical – we moreover do not obtain the common DSGE problem that all variables react too pro-cyclical in comparison to the data. In addition, when having a look at the impulse responses to aggregate productivity, to government spending, and to interest rate shocks, our model generates more plausible reactions in some important aspects as the workhorse model of BGMb.

More precisely, in the case of an expansionary overall technology shock the impact reaction of inflation generated by BGMb is counterfactual, i.e. positive, for high but commonly estimated degrees of shock persistence [see Lewis (2009b)]. This is actually not in line with empirical findings of amongst others Dedola and Neri (2007), Galí and Rabanal (2004), or Smets and Wouters (2003, 2007). In our model, on the other hand, the resulting inflation dynamics are completely negative as the data suggests. This result holds independently of the assumed degree of the shock persistence. When modifying our model to a comparable framework, i.e. when we set the firms' death rate equal to the respective constant steady state value, we can generate the same empirically counterfactual dynamics of inflation, if the shock's autocorrelation coefficient

exceeds 0.9.<sup>3</sup>

Thereby and in line with the empirical findings of Galí, Gertler, and López-Salido (2007), Martins, Scapetta, and Pilat (1996), and Rotemberg and Woodford (1991, 1999), our model is also able to generate counter-cyclical mark-up movements without implying counter-cyclical profits.<sup>4</sup> The underlying intuition for generating counter-cyclical mark-ups is straightforward. When an expansionary shock occurs, the incentive for potential producers increases to enter the market. Contemporaneously, less firms leave. The rising number of firms then leads to a decreasing market share of the single producer. The monopoly power and thus the mark-up will consequently decline.

In addition, our model can contribute to the debate in the RBC literature initiated by Galí (1999), whether an overall productivity shock leads to an expansionary or contractionary reaction of aggregate labor. In the empirical literature, there is a widespread agreement that there exists a negative correlation between total hours worked and GDP [see amongst others Francis and Ramey (2004, 2005), Galí and Rabanal (2004), and Galí (1999)]. However, standard RBC models generate a positive co-movement. By making prices totally flexible, the resulting RBC core of our model, on the other hand, can depict both possibilities as the intertemporal elasticity of substitution is slightly varied within the plausible range between 0.5 and 2. The underlying driving force is the development of the extensive margin, i.e. the mass of producing firms.

Another important advantage of our framework is that in the case of an expansionary shock to monetary policy our model delivers an expansionary reaction of both variables as the data suggests. BGMB, on the other hand, generate a contractionary reaction of firms' creation and the total number of

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<sup>3</sup>Note that as estimated by amongst others Smets and Wouters (2003, 2007), the autocorrelation coefficient of a technology shock is however at least 0.95.

<sup>4</sup>This result is also obtained by Jaimovich and Floetotto (2008) and BGMA in a framework with endogenous entry. See Ravn, Schmitt-Grohé, and Uribe (2006, 2008) for an alternative theoretical approach which also generates counter-cyclical mark-up movements by introducing 'deep habits'. Standard DSGE models, on the other hand, predict pro-cyclical mark-up movements.



producers which however conflicts with empirical evidence of Lewis (2009b) and Bergin and Corsetti (2008). The underlying driving force for our result is an endogenous cost push shock which results from endogenizing firms' exits and which has an expansionary impact in this case.

As PPI and CPI inflation do not have to coincide in our approach, we derive two specifications of the Phillips curve. We show that PPI inflation is only affected by the expected future inflation and the labor share as the baseline Phillips curve.<sup>5</sup> This result is moreover supported by US economy data as there does not exist a significant correlation between PPI inflation and the extensive margin. In the case of CPI inflation, there however exists a variety effect in our theoretical framework as the CPI Phillips curve is also a function of the change in the number of producers. This is moreover supported by US economy data which shows that CPI inflation is significantly correlated with the change in the number of producers.<sup>6</sup> We estimate the latter specification of the Phillips curve using the generalized method of moments. We show that the impact of the change in the extensive margin on CPI inflation is highly significant in the reduced form as well as in the structural estimation. In comparison to the baseline New Keynesian Phillips curve our CPI Phillips curve becomes flatter in an inflation/labor share-space which implies that the introduction of an endogenous number of producers causes the impact of the marginal costs on inflation to decrease as there occur additional effects from changes in product variety on CPI inflation.

The remaining paper is organized as follows. In section 2, we develop the totally microfounded New Keynesian model with endogenous firms' entries and exits. Section 3 provides our baseline calibration and discusses the impulse responses to persistent shocks to aggregate technology, to the interest rate, and to government spending. In section 4, we compare the generated second moments of our model to those of BGMA and to the data. The GMM estimations of the

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<sup>5</sup>We assume labor to be the only input factor for this exercise.

<sup>6</sup>The corresponding correlation is  $-0.13$  at a 95% significance level.

resulting Phillips curve is done in section 5. The last section concludes.

## 2 The Model

### 2.1 Producers

Following amongst others Christoffel, Kuester, and Linzert (2009), Faia, Lechthaler, and Merkl (2009), and Lechthaler and Snower (2008), we assume three sectors of production in order to separate the mark-up pricing decision from the input factor demand. We will distinguish between intermediate good producers, firms in the wholesale sector, and final good producers. The model structure is depicted in Figure 4.

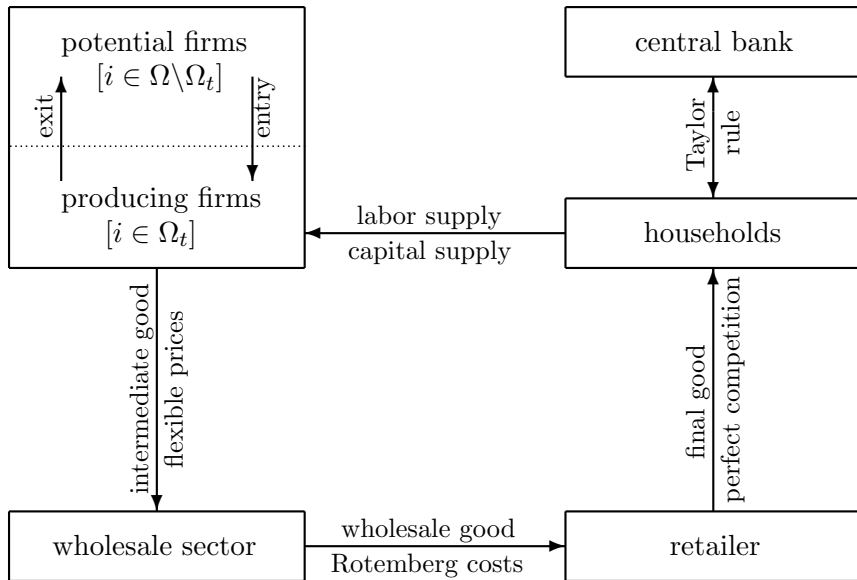


Figure 4: Model structure

The intermediate good producers (or: firms) differ in their individual productivity level. They are thus heterogeneous. Firms are indexed by  $i \in \Omega$  where  $\Omega$  is the bounded set of existing and potential firms. Due to firms' entries and exits only a subset of intermediate goods  $\Omega_t \subset \Omega$  is always available in the market.<sup>7</sup> The intermediate good is sold under totally flexible prices to

<sup>7</sup>For the sake of simplicity, we assume a one-to-one identification between a product and a firm.

the wholesale sector.

Firms in the wholesale sector, on the other hand, differentiate the intermediate goods by using a CES technology and sell them to the retail sector under monopolistic competition. They are moreover faced with quadratic adjustment costs in the spirit of Rotemberg (1982). Eventually, the final good producers (or: retailers) bundle the differentiated wholesale goods and sell them under perfect competition to the households.

### 2.1.1 Retail Sector

The retailer bundles the wholesale goods,  $y_{j,t}$ , according to the CES technology function given by

$$Y_t \equiv \left( \int_0^1 y_{j,t}^{\frac{\zeta-1}{\zeta}} dj \right)^{\frac{\zeta}{\zeta-1}} \quad (1)$$

where  $\zeta > 1$  denotes the elasticity between the wholesale goods. By cost minimization, we obtain the standard price index,  $P_t$ :

$$P_t = \left( \int_0^1 P_{j,t}^{1-\zeta} dj \right)^{\frac{1}{1-\zeta}} \quad (2)$$

The retailer acts under perfect competition.

### 2.1.2 Wholesale Sector

For the sake of simplicity, we assume the same elasticity between the intermediate and wholesale goods as well as between the wholesale and final goods.

The firms in the wholesale sector – indexed with  $j \in (0, 1)$  – differentiate the intermediate goods,  $y_{i,t}$ , according to the CES technology

$$y_{j,t} \equiv \left[ \int_{\Omega} y_{i,t}^{\frac{\zeta-1}{\zeta}} di \right]^{\frac{\zeta}{\zeta-1}} \quad (3)$$

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As standard in the macroeconomic theory, we thus do not model multi-product firms.

which implies a price level given by

$$P_{j,t} = \left[ \int_{\Omega} P_{i,t}^{1-\zeta} di \right]^{\frac{1}{1-\zeta}} \quad (4)$$

Being faced with quadratic adjustment costs in the spirit of Rotemberg (1982), the wholesale goods are sold under monopolistic competition to the retailers. The real profit of a wholesale firm  $j$  is then given by

$$E_0 \sum_{t=0}^{\infty} \Delta_{0,t} \left\{ \frac{P_{j,t}}{P_t} y_{j,t} - mc_{j,t} y_{j,t} - \frac{\theta}{2} \left( \frac{P_{j,t}}{P_{j,t-1}} - \bar{\pi}_j \right)^2 y_{j,t} \right\} \quad (5)$$

where  $\Delta_{0,t}$  and  $\bar{\pi}_{j,t}$  represent the stochastic real discount factor and the steady state value of producer price changes, respectively.  $mc_{j,t}$  are the marginal costs.  $\theta$  is interpreted as the menu costs resulting from relative price changes.

Optimizing (5) subjected to the demand for the wholesale good,  $y_{j,t} = \left( \frac{P_{j,t}}{P_t} \right)^{-\zeta} Y_t$ , yields the Phillips curve in non-aggregated terms

$$\begin{aligned} \rho_{j,t} = & \frac{\zeta}{\zeta-1} mc_{j,t} - \frac{\theta}{\zeta-1} \left[ (\pi_{j,t} - \bar{\pi}_j) \pi_{j,t} - E_t \left\{ \frac{\Delta_{0,t+1}}{\Delta_{0,t}} (\pi_{j,t+1} - \bar{\pi}_j) \pi_{j,t+1} \frac{y_{j,t+1}}{y_{j,t}} \right\} \right] \\ & + \frac{\zeta}{\zeta-1} \frac{\theta}{2} \left[ (\pi_{j,t} - \bar{\pi}_j)^2 - E_t \left\{ \frac{\Delta_{0,t+1}}{\Delta_{0,t}} (\pi_{j,t+1} - \bar{\pi}_j)^2 \frac{y_{j,t+1}}{y_{j,t}} \right\} \right] \end{aligned} \quad (6)$$

where  $\rho_{j,t} \equiv \frac{P_{j,t}}{P_t}$  and  $\pi_{j,t} \equiv \frac{P_{j,t}}{P_{j,t-1}}$ . The wholesale firm  $j$  thus sets its optimal price level as a mark-up,  $\mu_{j,t}$ , over the nominal marginal costs which implies

$$\rho_{j,t} = \mu_{j,t} mc_{j,t} \quad (7)$$

where

$$\mu_{j,t} \equiv \frac{\zeta}{(\zeta-1) + \frac{\theta}{\rho_{j,t}} \left[ \Upsilon_t - E_t \left\{ \frac{\Delta_{0,t+1}}{\Delta_{0,t}} \Upsilon_{t+1} \frac{y_{j,t+1}}{y_{j,t}} \right\} \right]} \quad (8)$$

with  $\Upsilon_t \equiv (\pi_{j,t} - \bar{\pi}_j) \left[ \pi_{j,t} - \frac{\zeta}{2} (\pi_{j,t} - \bar{\pi}_j) \right]$ .

### 2.1.3 Intermediate good producers

The intermediate good producers are heterogeneous. Following Ghironi and Melitz (2005), we assume that these firms differ in their individual productivity denoted with  $z_i$ . They thus produce with different technologies.

For production firms need capital and labor. The production function of a firm  $i$  is given by

$$y_{i,t} = A_t z_i l_{i,t}^\alpha k_{i,t}^{1-\alpha} \quad (9)$$

where  $l_{i,t}$  and  $k_{i,t}$  represent the labor and capital demand of firm  $i$ , respectively.  $A_t$  is an overall productivity shock. The individual productivity level,  $z_i$ , is assumed to be Pareto distributed across firms. This assumption implies that the firms' size distribution is also Pareto distributed which fits firm level data quite well [see Ghironi and Melitz (2005)]. The probability distribution function (PDF) and the cumulative distribution function (CDF) of  $z_i$  then follow  $k \frac{z_i^{-k-1}}{z_{\min}^{-k}}$  and  $1 - \left(\frac{z_i}{z_{\min}}\right)^{-k}$ , respectively, where  $k$  and  $z_{\min}$  are scaling parameters.

The optimal factor demand relation is given by

$$\frac{r_t^K}{w_t} = \frac{\alpha}{1-\alpha} \frac{l_{i,t}}{k_{i,t}} \quad (10)$$

where  $w_t$  and  $r_t^K$  are the real wage and the rental rate on capital, respectively.

Productivity differences across firms translate into differences in real marginal costs. Expressed in terms of the real wage, they follow

$$mc_{i,t} = \frac{w_t}{A_t z_i (1-\alpha) \left(\frac{k_{i,t}}{l_{i,t}}\right)^\alpha} \quad (11)$$

The intermediate good is sold under completely flexible prices to the wholesale sector.

After observing the shocks, an intermediate good producer which is cur-

rently existent in the market, i.e.  $i \in \Omega_t$ , decides, whether to stay or to leave the market before he actually starts producing. The firm will exit, if its generated present value of production – defined as the discounted sum of current and future profits – is non-positive, i.e.

$$\Psi_{i,t} = (\rho_{i,t} - mc_{i,t}) A_t z_i l_{i,t}^{1-\alpha} k_{i,t}^\alpha + E_t \left\{ \frac{\Delta_{0,t+1}}{\Delta_{0,t}} \Psi_{i,t+1} \right\} \leq 0 \quad (12)$$

The expected next period profit,  $E_t \Psi_{i,t+1}$ , is given by

$$E_t \Psi_{i,t+1} = E_t \left\{ (1 - \gamma_{t+1}) \left[ (\rho_{i,t+1} - mc_{i,t+1}) A_{t+1} z_i l_{i,t+1}^{1-\alpha} k_{i,t+1}^\alpha + \frac{\Delta_{0,t+2}}{\Delta_{0,t+1}} \Psi_{i,t+2} \right] \right\} \quad (13)$$

where  $\gamma_t$  denotes the time dependent probability of exiting which will be specified below.

In contrast to the exiting decision, we assume entering not to be costless. A non-producing firm  $i \in \Omega \setminus \Omega_t$  will thus enter, if the market entry costs do not exceed the expected generated present value of entering the market, i.e. if its net present value is non-negative

$$\Psi_{i,t} = (\rho_{i,t} - mc_{i,t}) A_t z_i l_{i,t}^{1-\alpha} k_{i,t}^\alpha + E_t \left\{ \frac{\Delta_{0,t+1}}{\Delta_{0,t}} \Psi_{i,t+1} \right\} \geq f_{E,t} A_t z_i l_{i,t}^{1-\alpha} k_{i,t}^\alpha \quad (14)$$

where  $f_{E,t}$  denotes the market entry costs proportional to the production volume of the intermediate good.<sup>8</sup>

The change in the mass of producing firms in the market,  $\Delta N_t$ , is defined as the number of new firms,  $N_{E,t}$ , minus the firms which leave the market,  $N_{X,t}$ . When denoting the time-dependent probability of leaving the market with  $\gamma_t$ , the number of exiting firms is given by  $N_{X,t} = \gamma_t N_{t-1}$ . By assumption, there exists a constant set of potential entrants  $N^{\max}$  which want to enter the market,

<sup>8</sup>This assumption is in line with Bergin and Corsetti (2008) and Blanchard and Giavazzi (2003) but contrasts with Bilbiie, Ghironi, and Melitz (2007a, 2007b) or Ghironi and Melitz (2005) who assume that entry costs are paid proportional to marginal costs.

if their respective entry is profitable, such that  $N_{E,t} = \phi_{t-1}(N^{\max} - N_{t-1})$  where  $\phi_t$  denotes the probability of entering. We thus implement a time-to-build lag which captures the empirical finding of the lagged firm creation [see Figure 3].<sup>9</sup>

The number of producing firms in the market then follows

$$N_t = \phi_{t-1}N^{\max} + N_{t-1}(1 - \phi_{t-1} - \gamma_t) \quad (15)$$

More precisely, the aggregate probabilities of entering and exiting are given by

$$\phi_t = \Gamma(\delta_t^{in}), \quad \gamma_t = 1 - \Gamma(\delta_t^{out}) \quad (16)$$

where  $\Gamma$  is the CDF of the idiosyncratic productivity level, i.e. all firms which are actually not existent in the market and have an idiosyncratic productivity which is above the threshold,  $\delta_t^{in}$ , will enter the market since their net present value of entering (14) is at least non-negative. On the other hand, all existing firms which have a productivity below the threshold,  $\delta_t^{out}$ , will consequently leave since their present value of production (12) is non-positive.

## 2.2 Aggregation

The price index, defined in (2), can be expressed in terms of the average producer price<sup>10</sup>

$$P_t = N_t^{\frac{1}{1-\zeta}} \tilde{P}_t \quad \Leftrightarrow \quad \tilde{\rho}_t = N_t^{\frac{1}{\zeta-1}} \quad (17)$$

where  $\tilde{P}_t \equiv P_{i,t}(\tilde{z})$ . As in Melitz (2003), the average idiosyncratic productivity level,  $\tilde{z}$ , is based on a weight which is proportional to the relative output share

<sup>9</sup>Although, the authors implement the time-to-build lag in a different manner, an equivalent assumption can also be found in Bergin and Corsetti (2008), Bilbiie, Ghironi, and Melitz (2007a, 2007b), and Ghironi and Melitz (2005).

<sup>10</sup>See the Appendix for a proof.

of firms

$$\tilde{z} \equiv \left[ \int_{z_{min}}^{\infty} z_i^{\zeta-1} g(z_i) dz_i \right]^{\frac{1}{\zeta-1}} = \left[ \frac{k}{k - (\zeta - 1)} \right]^{\frac{1}{\zeta-1}} z_{min} \quad (18)$$

where  $g(\cdot)$  is the PDF of the Pareto distribution.

Hence, the real price of the average intermediate good,  $\tilde{\rho}_t$ , is a function of product variety, as it increases in the number of firms since  $\zeta > 1$ . This also holds for aggregate production which is given by<sup>11</sup>

$$Y_t = N_t^{\frac{\zeta}{\zeta-1}} \tilde{y}_t \quad (19)$$

As in Bergin and Corsetti (2008), a rising number of firms thus causes the aggregate level of output to increase.

The aggregated production function consequently follows

$$Y_t = A_t \tilde{z} N_t^{\frac{1}{\zeta-1}} L_t^{1-\alpha} K_t^\alpha \quad (20)$$

where aggregate labor,  $L_t$ , and aggregate capital,  $K_t$ , are respectively given by  $N_t \tilde{l}_t$  and  $N_t \tilde{k}_t$  with  $\tilde{l}_t \equiv l_{i,t}(\tilde{z})$  and  $\tilde{k}_t \equiv k_{i,t}(\tilde{z})$ . The total factor productivity, TFP, defined as  $Y_t / (L_t^{1-\alpha} K_t^\alpha)$ , is thus not only a function of productivity but also of the number of producers

$$TFP_t = A_t \tilde{z} N_t^{\frac{1}{\zeta-1}} \quad (21)$$

According to (17), the change in the average individual firms' price level,  $\tilde{\pi}_t$ , i.e. producer price index (PPI) inflation, can be expressed as

$$\tilde{\pi}_t = \pi_t \left( \frac{N_t}{N_{t-1}} \right)^{\frac{1}{\zeta-1}} \quad (22)$$

Note that the two inflation rates coincide in the steady state.

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<sup>11</sup>See the Appendix for a proof.



Inserting (17) as well as the aggregated version of (10) and (11) in (7) yields the mark-up in aggregated terms

$$\mu_t = \frac{\tilde{z}A_t(1-\alpha)^{1-\alpha}\alpha^\alpha}{N_t^{\frac{\zeta-2}{\zeta-1}}(r_t^K)^\alpha w_t^{1-\alpha}} \quad (23)$$

The aggregated mark-up,  $\mu_t$ , is thus a decreasing function in the number of producers since the elasticity between the goods,  $\zeta$ , is typically assumed to be larger than two.<sup>12</sup> With an exogenous (constant) number of homogeneous producers, equation (23) simplifies to the common negative relation between the mark-up and the real marginal costs.

### 2.3 Households

In opposition to firms, households are homogeneous. They supply their labor force and capital to all kinds of producing firms.

Since firms decide to leave the market before they start producing, i.e. before they have a need for input factors, households do not supply capital and labor to exiting firms. Instead, they just supply input factors to producing firms. The probability of exiting the market can thus be neglected in the decision process of the household. Without loss of generality, we moreover assume the representative household to be faced with a mutual fund that pays dividends equal to total average profits,  $\Psi_t$ , instead of being faced with the heterogeneous single firms. Writing the problem in terms of share holdings in individual firms would complicate the notation and ultimately result in identical equilibrium conditions [see Ghironi and Melitz (2005)].

The representative household maximizes its expected utility life-time value

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<sup>12</sup>The commonly applied range of parameter values for this elasticity is between 2 as in Christiano, Eichenbaum, and Evans (2005) and 17 as in Uusküla (2008).

given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{1}{1+\eta} L_t^{1+\eta} \right) \quad (24)$$

subjected to its budget constraint which in real terms follows

$$C_t + \frac{B_{t+1}}{P_t} + K_{t+1} - (1-\chi)K_t = \frac{R_{t-1}}{P_t} B_t + r_{t-1}^K K_t + w_t L_t + \Psi_t + T_t \quad (25)$$

where  $C_t$ ,  $L_t$ ,  $K_t$ ,  $B_t$ , and  $T_t$  respectively represent the household's real consumption expenditure, the labor supply, the physical capital supply, bonds holdings, and transfers in period  $t$ . The inverse of the intertemporal elasticity of substitution and the inverse of the Frisch elasticity of labor supply are represented by  $\sigma$  and  $\eta$ , respectively.  $R_t$  denotes the gross nominal interest rate.  $\beta \in (0, 1)$  is a discount factor and  $\chi$  is the depreciation rate.

The following optimality conditions hold

$$w_t = L_t^\eta C_t^\sigma \quad (26)$$

$$C_t^{-\sigma} = \beta E_t \left\{ C_{t+1}^{-\sigma} \frac{R_t}{\pi_{t+1}} \right\} \quad (27)$$

$$C_t^{-\sigma} = \beta E_t \left\{ C_{t+1}^{-\sigma} [r_t^K - (1-\chi)] \right\} \quad (28)$$

## 2.4 Overall Resource Constraint

Inserting aggregated profits into the aggregated budget constraint of the households considering  $B_{t+1} - R_{t-1}B_t - P_t T_t = P_t G_t$ , yields the overall resource constraint

$$C_t = Y_t - I_t - G_t - \frac{\theta}{2} (\pi_t - \bar{\pi})^2 Y_t - \phi_{t-1} N_t^{-\frac{1}{\zeta-1}} f_{E,t} Y_t \quad (29)$$

where  $G_t$  is government expenditure and investments,  $I_t$ , are defined as

$$I_t \equiv K_{t+1} - (1 - \chi)K_t \quad (30)$$

## 2.5 Monetary Policy and Endogenous Trade-Off

By endogenizing firms' exits, our model generates an endogenous trade-off for monetary policy. To show that, we aggregate and log-linearize the Phillips curve (6) and insert equation (17).<sup>13</sup>

$$\widehat{\pi}_t = \beta \widehat{\pi}_{t+1} + \frac{\zeta - 1}{\theta} \widehat{mc}_t - \frac{1}{\theta} \widehat{N}_t \quad (31)$$

Log-linearizing the aggregate version of (12) yields

$$\widehat{mc}_t = \frac{\rho - mc}{mc} \left[ \widehat{Y}_t + \widehat{\delta}_t^{out} + \frac{\rho}{\rho - mc} \widehat{\rho}_t \right] + \frac{\beta \Psi}{mc \delta^{out} Y} \left[ E_t \widehat{\Psi}_{t+1} + \widehat{N}_t - E_t \widehat{N}_{t+1} \right] \quad (32)$$

It follows from (31) and (32) that the threshold for the exiting decision,  $\widehat{\delta}_t^{out}$ , acts as an endogenous cost push shock and thus generates an endogenous trade-off for monetary policy between the stabilization of output and inflation. If the threshold for exiting increases, the endogenous cost push shock leads to an increase in the marginal costs since  $\frac{\rho - mc}{mc} > 0$ . This result is analogous to models concerning labor turnover costs, where the introduction of firing costs generates an equivalent trade-off [see Faia, Lechthaler, and Merkl (2009)].

There is however no attempt to derive the optimal monetary policy in this paper. We leave that for future research. Instead, the monetary authority simply follows a standard Taylor rule given by

$$\frac{R_t}{R} = \left( \frac{\pi_t}{\pi} \right)^{\lambda_\pi} \left( \frac{Y_t}{Y} \right)^{\lambda_y} \exp(\kappa_t) \quad (33)$$

where  $\kappa$  represents an interest rate shock.

<sup>13</sup>In the following, a hat denotes a variable which is log-linearized around a zero inflation steady state. Variables without time index are steady state values.

### 3 Parameterizations and Impulse Responses

In this section, we will show up our baseline calibration and analyze the impulse responses to an aggregate productivity shock, a government spending shock, and a shock to monetary policy.<sup>14</sup>

In the baseline calibration, we set the discount factor,  $\beta$ , to 0.99 which implies a steady state value of the annual interest rate of about 4%. As widely applied in the literature, the inverse of the elasticity of intertemporal substitution,  $\sigma$ , and the Frisch elasticity of labor supply,  $\eta$ , are respectively set to 1 and 2. The price elasticity of the demand for the intermediate good,  $\zeta$ , is assumed to be equal to 8, implying a steady-state mark-up over the nominal costs of about 14%. As standard in the literature, we set the depreciation rate,  $\chi$ , to 0.025. Moreover, we calibrate  $\theta$  to 83 in order to obtain a slope of the Phillips curve equal to the baseline New Keynesian model corresponding with an average price duration of three quarters, i.e. a Calvo parameter of  $2/3$ .

Following Ghironi and Melitz (2005), we arbitrarily set the entry costs,  $f_E$ , equal to 1. For calibrating the first scaling parameter of the Pareto distribution,  $k$ , it is important that the condition  $k > \zeta - 1$  holds in order to assure the standard deviation of the idiosyncratic shock to be finite and positive.<sup>15</sup> Moreover,  $k$  is calibrated to obtain the same  $k/\zeta$  ratio as in Ghironi and Melitz (2005). To be able to compare our results with models without heterogeneity, the second scaling parameter,  $z_{\min}$ , is chosen to obtain an average individual productivity level equal to one. As standard in the literature, we set  $\alpha$  to 0.2 implying that 80 percent of total costs are represented by wages. The exogenous government spending/GDP ratio in steady state,  $G/Y$ , is calibrated at a 25% level as in Faia (2009). Moreover, we set the steady state values of the probability of exiting and entering,  $\phi$  and  $\gamma$ , both equal to 0.025 implying an

<sup>14</sup>We simulate the model in log-linear form using Dynare V. 4.01 [see Juillard (2001)]. The complete log-linear equation system can be found in the Appendix.

<sup>15</sup>Remark: The standard deviation of the Pareto distributed individual productivity level is given by  $(k - \zeta + 1)^{-1}$ .

average annual firms' birth and death rate of 10% which is consistent with US economy data.

All analyzed shocks – to aggregate productivity, to government spending, and to the interest rate – follow an AR(1) process:  $x_t = \rho_x x_{t-1} + \varepsilon_t^x$  with  $x = \{a, g, \kappa\}$ . We calibrate these processes to the estimated values of Smets and Wouters (2007). Hence, the corresponding autocorrelation coefficients  $(\rho_a, \rho_g, \rho_\kappa)$  are respectively 0.95, 0.97, and 0.15 whereas the standard errors are 0.45, 0.53, and 0.24. Finally, the Taylor rule is calibrated in the standard fashion, i.e.  $\lambda_\pi$  and  $\lambda_y$  are set equal to 1.5 and 0.5, respectively.

### 3.1 Overall productivity shock

Figure 5 shows the impulse responses to the persistent overall productivity shock.<sup>16</sup> As expected, the shock causes aggregate output and consumption to increase while aggregate inflation decreases.

However, the production of the average firm just increases on impact and then turns negative. This is in line with the results of BGMB. The economic interpretation is that although the aggregate productivity shock has a positive effect on the production level of the individual firm, this effect is compensated by a decreasing market share. As higher productivity leads to higher profit opportunities – which causes the thresholds for entering and exiting the market to decrease – the probability of entering (exiting) increases (decreases). As a result, the rising number of producing firms causes the market share of the individual firm – defined as  $\tilde{y}_t/Y_t$  – per se to decrease.<sup>17</sup> Hence, our model depicts the empirically observed pro-cyclical movement of entries as well as the counter-cyclical adjustment process of exits. In line with the findings of BGMB, the adjustment time-path of the intensive margin, i.e. aggregate production, is humped-shaped whereas this is not the case for the extensive margin, i.e. the

<sup>16</sup>The number of years are on the abscissa. However, we interpret periods as quarters.

<sup>17</sup>Note that in log-linear representation the market share follows  $-\frac{\zeta}{\zeta-1}\hat{N}_t$  such that the resulting dynamics look qualitative equivalent to those of the number of firms with inverse sign. They are thus humped shaped and negative over the total adjustment path.

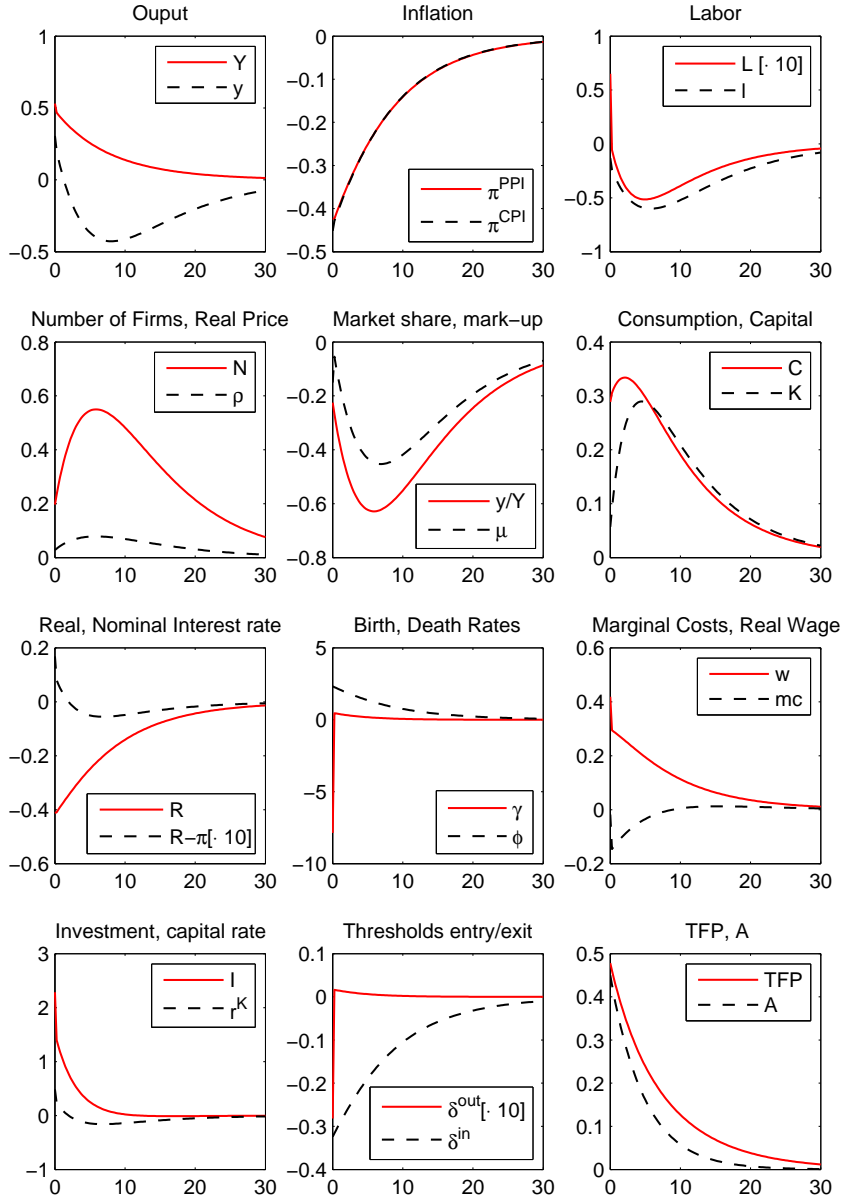


Figure 5: Impulse responses to a persistent overall productivity shock

number of producing firms.

Moreover, by assuming an endogenous mass of firms, the shock impact is amplified since TFP increases more than aggregate productivity which directly follows from equation (21) as the number of producers rises. This result is in line with the findings of Jaimovich and Floetotto (2008) who show that TFP is

a decreasing function of the firms' mark-up.<sup>18</sup>

In line with the empirical findings of Galí, Gertler, and López-Salido (2007), Martins, Scapetta, and Pilat (1996), and Rotemberg and Woodford (1991, 1999), our model is also able to generate counter-cyclical mark-up movements without implying counter-cyclical profits.<sup>19</sup> Technically, the counter-cyclical reaction of the mark-up results from equation (23) as the number of producers increases. The underlying economic intuition for generating counter-cyclical mark-ups is straightforward. When an expansionary shock occurs, the incentive for potential producers to enter the market increases. Contemporaneously, less firms leave. The rising extensive margin then leads to a decreasing market share of the single producer. The monopoly power and thus the mark-up of the average firm will consequently decline.

In the baseline calibration, we obtain a positive impact reaction of aggregate labor. Thereafter labor reacts contractionary. This is in line with the widespread agreement in the empirical literature that there exists a negative correlation between productivity shocks and total hours worked [see amongst others Francis and Ramey (2004, 2005), Galí and Rabanal (2004), and Galí (1999)].<sup>20</sup> Individual labor behaves qualitatively equivalent. However, the reaction of total hours worked is very sensitive to the calibration of the households' utility function. By decreasing the inverse of the intertemporal elasticity of substitution,  $\sigma$ , to 0.5, our model generates a totally expansionary reaction of total hours worked leaving the remaining variables qualitatively unchanged. Note that also a totally contractionary reaction of aggregate labor can be generated when setting  $\sigma = 2$ . The corresponding time paths of total hours worked for

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<sup>18</sup>The mark-up then turns out to be a declining function of the number of producers.

<sup>19</sup>This result is also obtained by Jaimovich and Floetotto (2008) and BGMa in a framework with endogenous entry. See Ravn, Schmitt-Grohé, and Uribe (2006, 2008) for an alternative theoretical approach which also generates counter-cyclical mark-up movements by introducing 'deep habits'. Standard DSGE models, on the other hand, predict pro-cyclical mark-up movements.

<sup>20</sup>Note however that there is also empirical support for the increase in aggregate labor, e.g. by Dedola and Neri (2007) who emphasizes a positive correlation between total hours worked and productivity in the US economy.

different values of  $\sigma$  are depicted in Figure 6 (a).

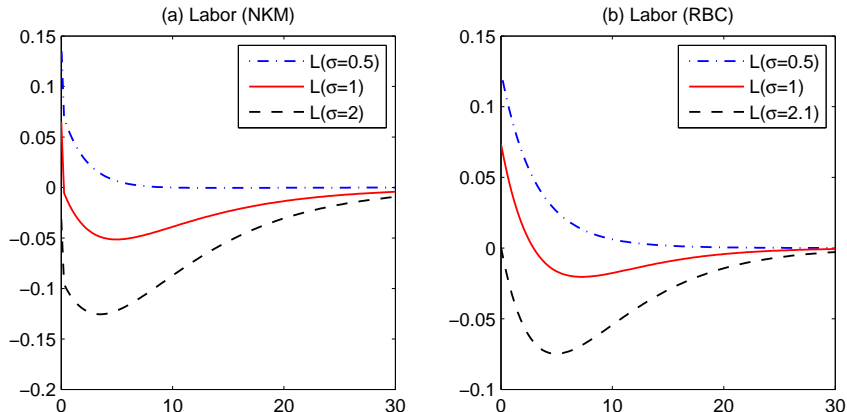


Figure 6: Labor market adjustment in a RBC and NKM world under different  $\sigma$

It is worthwhile to mention that this result can also be obtained in the RBC core of our model. We can thus contribute to the debate in the RBC literature initiated by Galí (1999), whether an overall technology shock leads to an expansionary or contractionary reaction of aggregate labor. By setting  $\theta = 0$ , the RBC core version of our model can depict both possibilities when varying the inverse of the intertemporal elasticity of consumption leaving other variables qualitatively unchanged. Normal RBC models generate a positive comovement. The corresponding impulse responses of total hours worked under flexible prices are depicted in Figure 6 (b). We obtain an expansionary reaction of aggregate labor in the case of  $\sigma = 0.5$ , while our model generates a completely contractionary reaction when the inverse of the intertemporal elasticity of substitution slightly exceeds 2.

The economic intuition for the change in the reaction of total hours worked is that by increasing the inverse of the intertemporal elasticity of substitution, future consumption is shifted to the present. The impact reaction of consumption and output are thus amplified. Due to an increasing goods demand, the thresholds for entering and exiting both decrease. Consequently, the number of firms and thus aggregate labor increase in the inverse of the intertemporal elasticity of substitution.



However, the adjustment time paths of the second input factor, capital, are unambiguous. Capital and investments both increase in a humped-shaped manner. Due to the increasing demand, the rental rate on capital also reacts expansionary.

When comparing our impulse responses to those of BGMB, it turns out that beside the dynamics of inflation our model behaves qualitatively equivalent. However, BGMB generate a positive initial reaction of inflation after an expansionary technology shock for high but commonly estimated degrees of shock persistence [see Lewis (2009b)]. Our model, on the other hand, generates a totally negative reaction of inflation independently of the persistence coefficient which is in line with the empirical studies of e.g. Dedola and Neri (2007), Galí and Rabanal (2004), and Smets and Wouters (2003, 2007).

In order to find the crucial factor for this counterfactual behavior in the model of BGMB, we have to modify our model to obtain a comparable framework. For this exercise, we thus set the time dependent exiting rate,  $\gamma_t$ , equal to the constant steady state level. In addition, BGMB assume labor to be the only input factor implying  $\alpha = 0$  in our framework. Following BGMB, we moreover assume the elasticity between the intermediate goods,  $\zeta$ , to be 3.8 and the Taylor rule to follow:  $\widehat{R}_t = 1.5E_t\widehat{\pi}_{t+1}$  for this exercise.<sup>21</sup> Figure 7 shows the resulting impulse responses.

The most remarkable qualitative change is the impact reactions of aggregate inflation (or: CPI) and PPI inflation which now turn positive.<sup>22</sup> Another interesting result is that even though the marginal costs now increase due to the overall productivity shock, inflation still decreases. The reason for that is the adjustment path of the real price of the intermediate good,  $\widetilde{\rho}_t$ , which reacts expansionary due to the variety effect [see equation (17)]. By having a look at the Phillips curve (6), it turns out that although inflation is an increasing

<sup>21</sup>Furthermore, we have to re-calibrate the second scaling parameter in order to ensure the condition  $k > \zeta - 1$  to hold. We thus set  $k$  to 3.4 as in Ghironi and Melitz (2005).

<sup>22</sup>Of course, the generated degree of persistence declines in the absence of capital since  $K_t$  is predetermined.

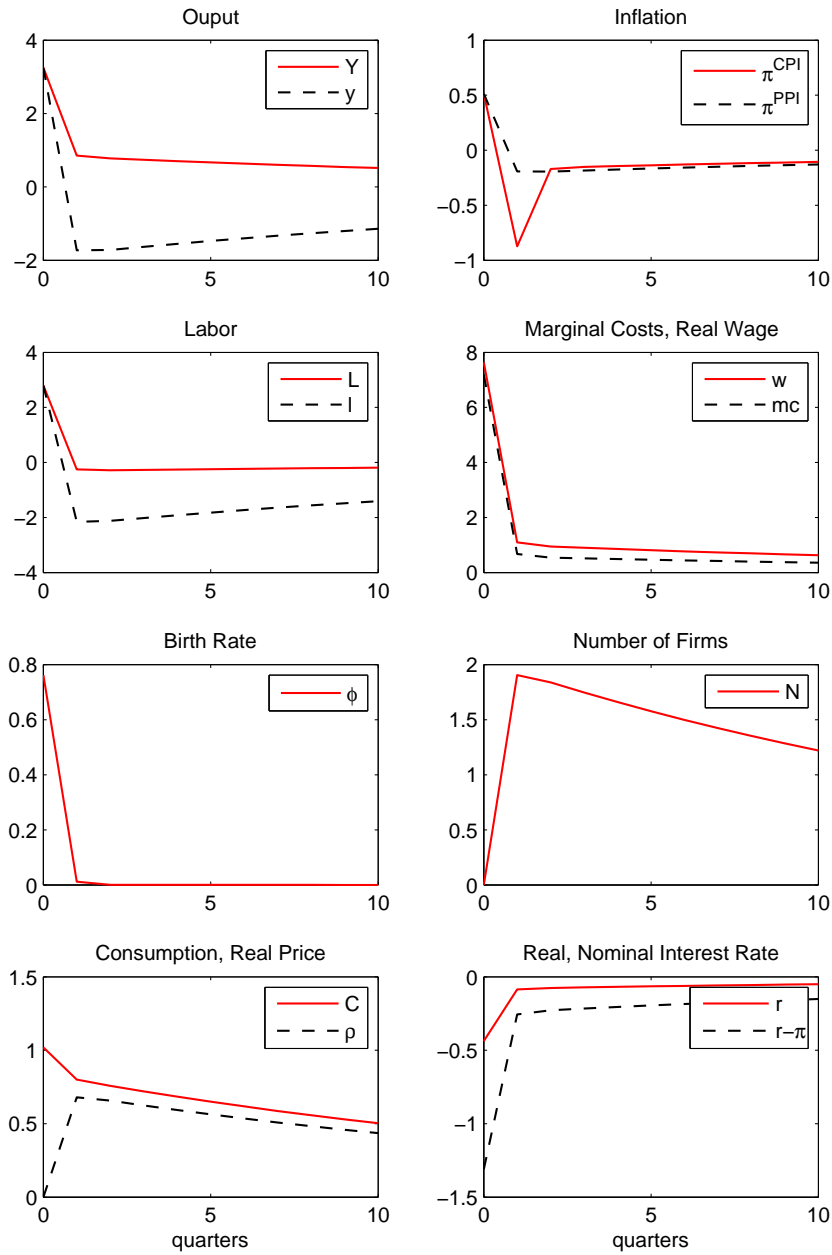


Figure 7: Impulse responses to persistent overall productivity shock with a constant death rate

function of marginal costs, it is also negatively correlated with the real price. In the current framework, the latter effect dominates the first one.

As Figure 8 (a) depicts, we only obtain the empirically suggested contrac-

tionary response of inflation for a shock persistence below 0.9 when exits are exogenous. However, empirical studies have shown that the autocorrelation coefficient of a technology shock is rather above 0.95 [see amongst others Smets and Wouters (2003, 2007)]. On the other hand, we never obtain an expansionary inflation reaction in the case of endogenous exits. The corresponding impulse responses are depicted in Figure 8 (b).

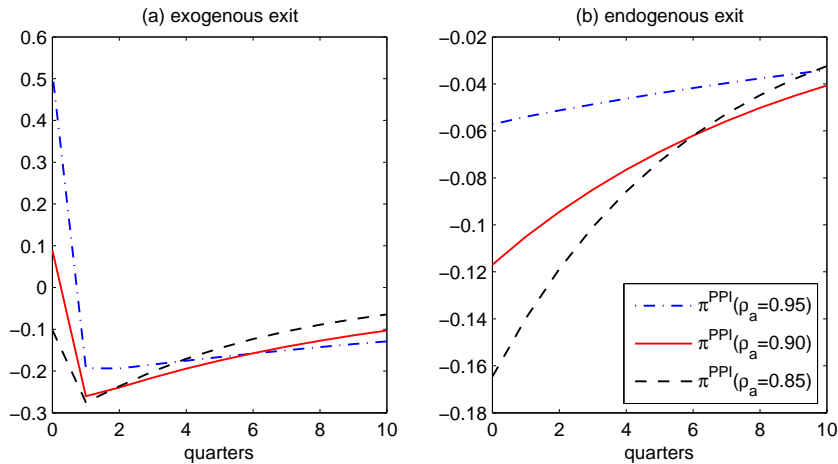


Figure 8: Inflation adjustment with exogenous vs. endogenous firms' exit and  $\alpha = 0$

The counterfactual adjustment pattern of inflation in BGMB thus results from the assumption of an exogenous (constant) exit rate. The underlying intuition is that by endogenizing the firms' failure rate, the extensive margin reacts more expansionary than in the case of exogenous exits. As shown in Figure 5, the exit rate reacts contractionary in our model and thus amplifies the positive effect on the extensive margin resulting from an increasing number of new firms. Under a constant exit rate, the mark-up of producers consequently increases less. As a result, firms set a lower (real) price level leading to an upward pressure on inflation according to the Phillips curve (6).

### 3.2 Interest rate shock

Figure 9 shows the impulse responses to an expansionary interest rate shock. Following, BGMB we assume the monetary authority to follow a smoothed

Taylor rule with  $\rho_\lambda = 0.8$  for this exercise.

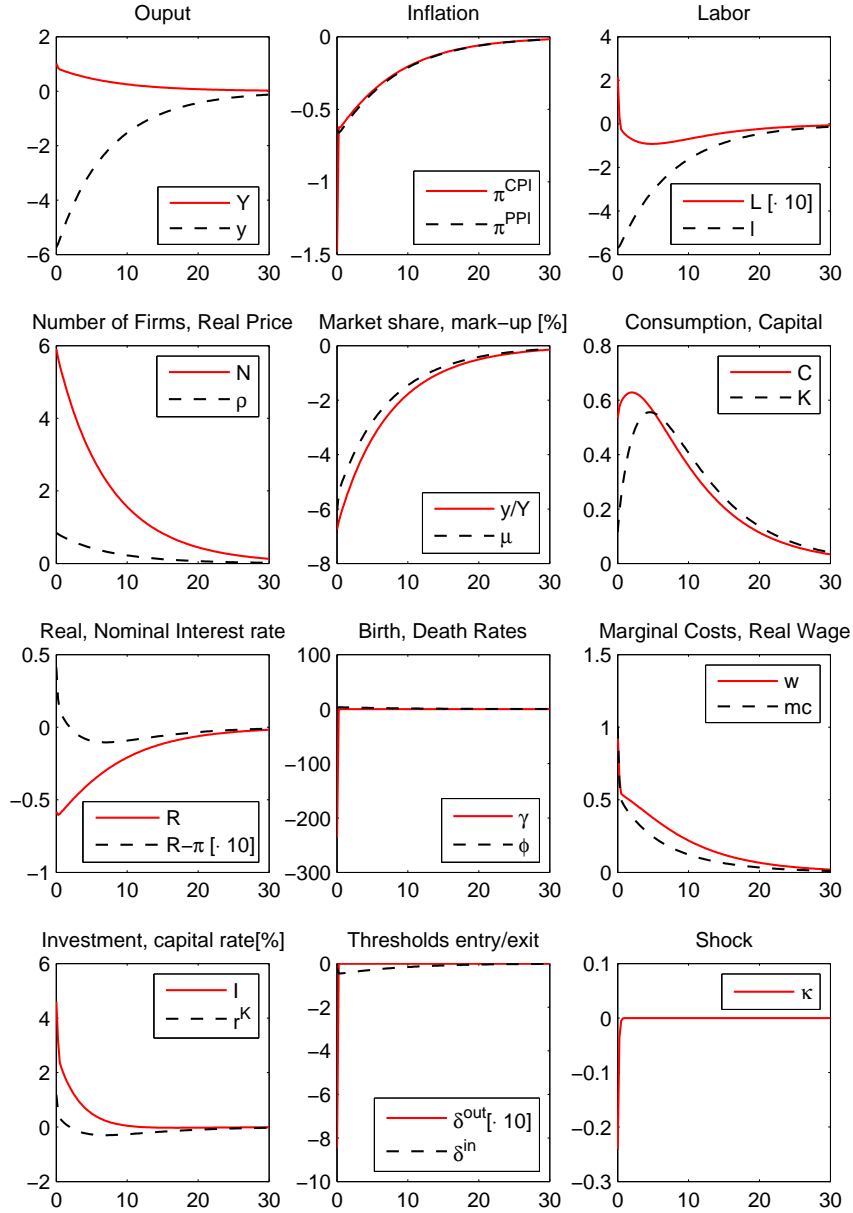


Figure 9: Impulse responses to an interest rate shock

As expected and in line with various VAR estimates of amongst others Lewis (2009b) or Smets and Wouters (2003, 2007), we obtain an increase in aggregate output, consumption, and investment. Moreover, the real wage, the rental rate on capital, and investment rise, too.

As the demand for the intermediate good increases due to the expansionary reaction of consumption and investment, the threshold for exiting the market decreases. According to (32), this causes the marginal costs to decrease leading to a downward-pressure on inflation [see the Phillips curve (31)]. As a result, both CPI and PPI inflation decrease caused by the endogenous cost-push shock which has an expansionary impact in this case.

In comparison to BGMB, we moreover do not obtain the counter-intuitive result that the extensive margin reacts contractionary to an expansionary monetary policy shock which conflicts with empirical evidence of Lewis (2009b) and Bergin and Corsetti (2008). In our framework, the extensive margin clearly reacts expansionary as the birth rate increases while the death rate decreases. This is another advantage of our framework over models with exogenous exits. As shown by Uusküla (2009), these sticky price models cannot depict an expansionary reaction of the number of producers, at all.<sup>23</sup>

The underlying intuition is as follows. There exist two opposing effects which determine the reaction of the extensive margin. (i) According to the Euler consumption equation, future consumption is shifted to present when the interest rate falls. As a result, consumption reacts expansionary leading to higher profit opportunities for producing and potential firms. (ii) The increase in marginal costs caused by a higher demand for both capital and labor results in an opposing negative effect on the number of firms. In our framework, the first effect dominates the second one. However, without the endogenous expansionary cost push shock – which results from endogenizing firms’ exits – the second effect would not be dampened leading to an overall contractionary reaction on the extensive margin.

In the baseline calibration, aggregate labor reacts expansionary on impact and then turns negative. However, the reaction of total hours worked is again very sensitive to the calibration of the inverse of the intertemporal elasticity of

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<sup>23</sup>Uusküla (2009) moreover suggest the application of a limited participation model to get rid of this problem.

substitution.

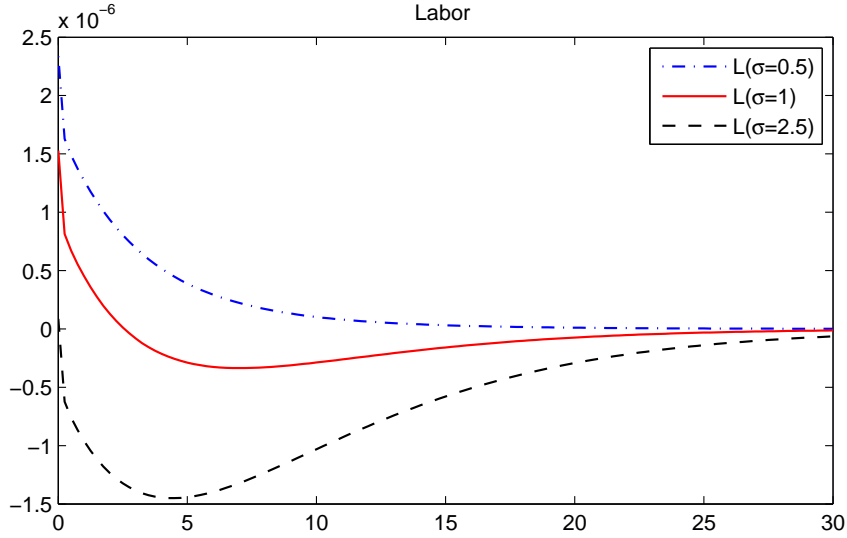


Figure 10: Labor market adjustment under different  $\sigma$

As shown in Figure 10, we obtain the empirically observed positive co-movement of total hours worked with GDP to an expansionary monetary shock for  $\sigma < 1$ . However, also a completely contractionary reaction of aggregate labor is possible in our framework when  $\sigma > 2.5$  leaving the adjustment time paths of the remaining variables qualitatively unchanged. The underlying intuition is the same as in 3.1.

### 3.3 Government spending shock

Figure 11 depicts the impulse responses to an expansionary shock to government spending.<sup>24</sup> It is a common result that government spending causes a crowding-out in consumption as the real interest rate rises. However, the contractionary reaction of private consumption is dominated by the expansionary effect on government spending such that the aggregate goods demand increases. Due to that, the thresholds for entry and exit decrease on impact. All in all, the extensive margin reacts expansionary in a humped-shaped manner and amplifies

<sup>24</sup>In this case, we cannot compare our impulse responses to BGMB since they do not consider government spending shocks.

the rise in (individual) output. During the remaining adjustment process, the reaction of individual output turns negative due to the decreasing market share leading to counter-cyclical mark-up movements.

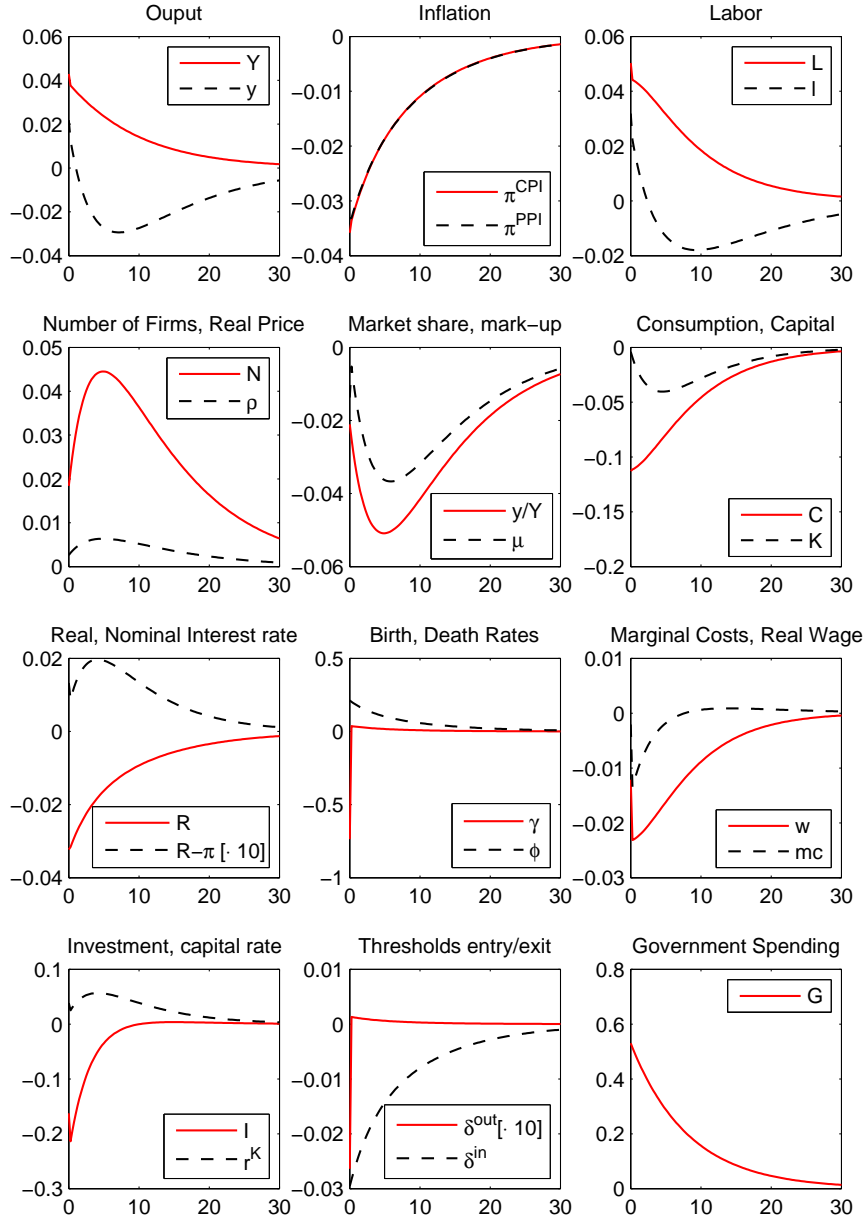


Figure 11: Impulse responses to a government spending shock

Hence, beside inflation all variable behave in line with the empirical findings

of Smets and Wouters (2003, 2007) and Lewis (2009b). As shown in Linnemann and Schabert (2003), the qualitative reaction of inflation, however, crucially depends on the design of monetary policy. More precisely, the coefficient on output,  $\lambda_y$ , is the decisive factor. Figure 12 shows that the reaction of inflation turns from totally negative to completely expansionary when decreasing the coefficient on output in the Taylor rule.

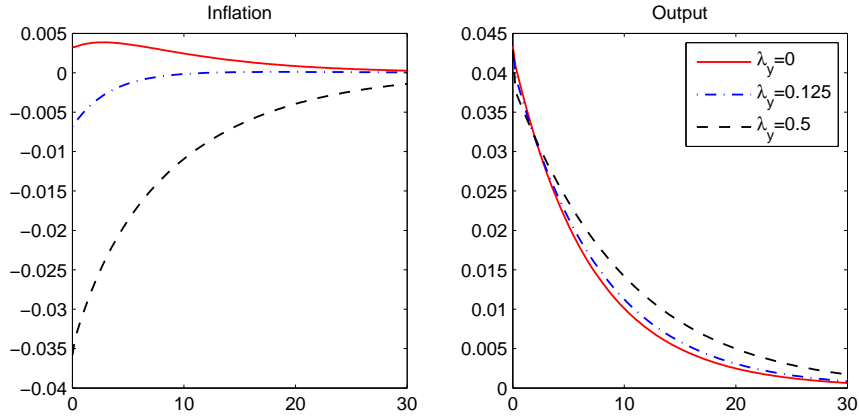


Figure 12: Impulse responses of inflation and output to a government spending shock under varied  $\lambda_y$

The economic intuition is that by decreasing the weight on output in the Taylor rule, the decline in the nominal interest rate will be larger leading to a higher goods demand. The increasing marginal costs consequently result in an upward-pressure on inflation. Figure 12 moreover shows that in the case of an expansionary shock to government spending output always reacts expansionary. The effects resulting from the endogenous cost-push shock are thus dominated in this case.

## 4 Second Moments

We will now have a look at the generated second moments of the developed model and compare them to the empirical ones provided by King and Rebelo (1999). As the model of BGMa represents an RBC framework and BGMb do



not consider capital in their analysis, we will first compare our results under flexible prices to those of BGMa and second show that the introduction of sticky prices does not yield any significant differences for this exercise.

For this exercise, we simulate the reaction of our model to an aggregate productivity shock 500 times for 200 periods. We use the Hodrick-Prescott filter with a smoothing parameter of  $10^5$ . In order to deliver comparable results with the data and BGMa, we calibrate the productivity shock process according to the empirically observed values of King and Rebelo (1999). We thus set the standard deviation,  $\sigma_a$ , and the autocorrelation coefficient,  $\rho_a$ , of the shock respectively to 0.0072 and 0.979. All remaining parameters follow the baseline calibration.

Table 1 shows the simulated second moments of our model under flexible prices as log-deviations from the HP-trend in comparison to the empirically reported values (bold values) and the values obtained by BGMa (in parenthesis).<sup>25</sup> Following BGMa, we measure total investment,  $I^T$ , with investment in physical capital,  $I_t$ , and the real value of new firms creation,  $N_{E,t}/N_t\Psi_t$ .

$X$	$\sigma_X$		$\sigma_X/\sigma_Y$		$E(X_t X_{t-1})$		$corr(X, Y)$	
$Y$	<b>1.81</b>	1.40 (1.40)	<b>1.00</b>		<b>0.84</b>	0.70 (0.70)	<b>1.00</b>	
$C$	<b>1.35</b>	1.16 (0.57)	<b>0.74</b>	0.83 (0.41)	<b>0.80</b>	0.74 (0.65)	<b>0.88</b>	0.71 (0.99)
$I^T$	<b>5.30</b>	4.30 (3.33)	<b>2.93</b>	3.07 (2.38)	<b>0.87</b>	0.66 (0.73)	<b>0.80</b>	0.56 (1.00)
$L$	<b>1.79</b>	1.46 (0.97)	<b>0.99</b>	1.04 (0.69)	<b>0.88</b>	0.70 (0.71)	<b>0.88</b>	0.65 (0.99)

Table 1: Second moments to an aggregate productivity shock [**data**, model, *BGMb* (2007a)]

Table 1 shows that our model performs unambiguously better than BGMa concerning all absolute and relative standard deviations in comparison with the data. With respect to the generated autocorrelations, both models however perform equivalently. Hence, both models do not generate enough endogenous persistence.<sup>26</sup>

<sup>25</sup>More precisely, we compare our results with those of BGMa under a CES technology and capital in production.

<sup>26</sup>This is however a general problem of NK and RBC models.

By introducing endogenous exits – which react counter-cyclical – we however do not obtain the standard difficulty of DSGE models – including BGMa and BGMb – that all variables react too pro-cyclical in comparison to the data. BGMa obtain cross-correlations between the depicted variables and GDP which are very close or even equal to one. However, this does not hold for our entry and exit model as the corresponding simulated co-movements of consumption, investment, and total hours worked with GDP do not exceed the empirical ones.

For the next exercise, we assume sticky prices again. Following BGMb, we moreover assume the monetary authority to follow a smoothed Taylor rule with a smoothing parameter of 0.8, a higher weight on inflation ( $\lambda_\pi = 3.5$ ), and a zero-weight on output ( $\lambda_y = 0$ ). The generated second moments of our model under sticky prices are shown in Table 2 in comparison with those under flexible prices.

$X$	$\sigma_X$			$\sigma_X/\sigma_Y$			$E(X_t X_{t-1})$			$corr(X, Y)$		
$Y$	<b>1.81</b>	1.42	(1.40)	<b>1.00</b>			<b>0.84</b>	0.70	(0.70)	<b>1.00</b>		
$C$	<b>1.35</b>	1.16	(1.16)	<b>0.74</b>	0.82	(0.83)	<b>0.80</b>	0.74	(0.74)	<b>0.88</b>	0.73	(0.71)
$I^T$	<b>5.30</b>	4.34	(4.30)	<b>2.93</b>	3.06	(3.07)	<b>0.87</b>	0.66	(0.66)	<b>0.80</b>	0.63	(0.56)
$L$	<b>1.79</b>	1.59	(1.46)	<b>0.99</b>	1.12	(1.04)	<b>0.88</b>	0.65	(0.70)	<b>0.88</b>	0.60	(0.65)

Table 2: Second moments to an aggregate productivity shock [**data**, model with sticky prices, (*model with flexible prices*)]

In line with the findings of BGMb, the introduction of sticky prices does not deliver an important enhancement for this exercise as the generated second moments do not change significantly.

## 5 An empirical exercise

In this section, we will estimate the log-linearized Phillips curve with the generalized method of moments (GMM) as in Galí and Gertler (1999).

For the sake of simplicity, we assume for this exercise  $\alpha = 0$ , i.e. there is only one input factor, labor. Inserting the aggregate expression for the marginal costs as well as the aggregated production function (20) in the Phillips curve

(6) and log-linearizing yields

$$\widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} + \underbrace{\frac{\zeta - 1}{\theta}}_{\omega} \widehat{S}_t \quad (34)$$

where  $\widehat{S}_t \equiv \widehat{w}_t + \widehat{N}_t - \widehat{Y}_t$  is the labor share.<sup>27</sup>  $\omega$  represents the reduced form slope coefficient of the Phillips curve. Equation (34) represents a standard Phillips curve just depending on expected future inflation and the labor share. Hence, the number of producers does not effect PPI inflation in our approach. This result is moreover supported by US economy data as there is no significant correlation between PPI inflation and the extensive margin.<sup>28</sup>

However, by inserting the log-linearized version of (22) in (34), we obtain

$$\widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} + \frac{\zeta - 1}{\theta} \widehat{S}_t + \frac{1}{\zeta - 1} \left[ \beta E_t \Delta \widehat{N}_{t+1} - \Delta \widehat{N}_t \right] \quad (35)$$

where  $\Delta \widehat{N}_t \equiv \widehat{N}_t - \widehat{N}_{t-1}$ . Hence, the change in the number of producers occurs additionally to the labor share in the CPI Phillips curve (35). When having a look at US economy data, it moreover turns out that the cross-correlation between CPI inflation and the change in the number of firms is  $-0.13$  which is significant at a 95% level. This finding indicates that there seems to exist a variety effect on CPI inflation.

For our estimations, we follow Galí and Gertler (1999) by using quarterly data for the US economy over the period 1960Q1:1997Q4.<sup>29</sup> The instrument set includes four lags of the output gap, the long-short interest rate spread, wage inflation, commodity price inflation, the non-farm labor's share, and overall GDP deflator inflation. Additionally, we take the data for the extensive margin seasonally adjusted and de-trended by application of the HP-filter. The data is constructed from new incorporations and firms' failures which are provided by

<sup>27</sup>Note however that  $\widehat{m}c_t \neq \widehat{S}_t$ .

<sup>28</sup>The applied data set is described below.

<sup>29</sup>The data of the extensive margin is just capable for 1959Q3:1998Q3. We thus do not loose many observations by applying the data range of Galí and Gertler (1999).

the "Survey of Current Business" and the "Economic Report of the President" by the Council of Economic Advisors. Following Galí and Gertler (1999), we use a 12-lag Newey-West estimate of the covariance matrix.

In order to generate a benchmark, we first estimate the standard Phillips curve with Calvo pricing where the marginal costs can be approximated by the labor share [see Galí and Gertler (1999)].<sup>30</sup> It is given by

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \underbrace{\frac{(1-\vartheta)(1-\beta\vartheta)}{\vartheta}}_{\omega'} \hat{S}_t \quad (36)$$

where  $\vartheta \in (0, 1)$  represents the Calvo parameter.  $\omega'$  is the resulting reduced form slope parameter. Note that in contrast to Galí and Gertler (1999) but for the sake of comparability, we take the CPI for generating inflation for this exercise.<sup>31</sup> Under rational expectations the corresponding orthogonality condition is given by

$$E_t \left\{ (\hat{\pi}_t - \beta \hat{\pi}_{t+1} - \omega' \hat{S}_t) z_t \right\} = 0 \quad (37)$$

where  $z_t$  is the vector of instruments. All instruments are observable at time  $t$ .

In the following an asterisk indicates significance at a 99% level. The estimation of (37) in reduced form yields

$$\hat{\pi}_t = 0.9814^* E_t \hat{\pi}_{t+1} + 0.2062^* \hat{S}_t \quad (38)$$

Both estimates are significant and reasonable. As already shown by Galí and Gertler (1999), inflation is significantly affected by the labor share beside future inflation as the theoretical literature suggests.

The corresponding structural estimation of (37) using a nonlinear instru-

<sup>30</sup>Remark: As we also want to estimate the baseline Phillips curve in structural form we cannot use the standard Phillips curve with Rotemberg adjustment costs as a benchmark because it has two reduced-form parameters but three structural parameters. There would thus exist an identification problem.

<sup>31</sup>Galí and Gertler (1999) take the GDP deflator for generating inflation.

mental variables estimator yields a discount factor,  $\beta$ , equal to 0.98 and a Calvo parameter of 0.63 implying an average price duration of 2.75 quarters. Both parameter estimates are significant at a 99% level and very close to commonly assumed and estimated values.<sup>32</sup> The resulting reduced form slope coefficient,  $\omega'$ , is 0.20.

After generating a benchmark, we will now estimate our CPI Phillips curve (35) which additionally depends on changes in the number of producers. The corresponding orthogonality condition is given by

$$E_t \left\{ \left( \hat{\pi}_t - \beta \hat{\pi}_{t+1} - \frac{\zeta - 1}{\theta} \hat{S}_t - \frac{1}{\zeta - 1} \left[ \beta \Delta \hat{N}_{t+1} - \Delta \hat{N}_t \right] \right) z_t \right\} = 0 \quad (39)$$

In order to show that the impact of the extensive margin has a significant effect additionally to that of the labor share on current inflation, we first estimate (39) in reduced form. The resulting estimated equation is given by

$$\hat{\pi}_t = 0.9895^* E_t \hat{\pi}_{t+1} + 0.1599^* \hat{S}_t + 0.3684^* E_t \Delta \hat{N}_{t+1} - 0.1351^* \Delta \hat{N}_t \quad (40)$$

Hence, both the future as well as the present change in the number of producers have a significant impact on CPI inflation which is consistent with our theoretical approach. When comparing the reduced form estimation in (40) with that of our benchmark (38), it turns out that the slope of the Phillips curve becomes flatter in a  $\hat{\pi}_t / \widehat{m}c_t$ -space by introducing the extensive margin. This implies that the introduction of an endogenous number of producers causes the impact of the marginal costs on CPI inflation to decrease as there occur additional effects from changes in product variety.

Finally, we estimate (35) in structural form. Also in this case, the GMM estimation delivers very plausible and highly significant parameter values for CPI data. They are shown in Table 3 in comparison to the benchmark estimation.<sup>33</sup>

<sup>32</sup>For example Galí (2008) assumes  $\beta = 0.99$  and  $\vartheta = 2/3$ .

<sup>33</sup>Note that the estimation of the CPI Phillips curve (37) using PPI data delivers completely insignificant and implausible estimates which again indicates the absence of a significant cor-

Phillips curve	$\beta$	$\zeta$	$\theta$	$\vartheta$	$\omega$ ( $\omega'$ )
(37)	0.9797*			0.6325*	0.2210*
(39)	0.9861*	14.6454*	73.1282*		0.2003*

Table 3: Structural parameter estimates [\*: 99% significance level]

The elasticity of substitution between the goods,  $\zeta$ , is estimated to be 14.65 which is within the commonly applied/estimated range between 6 [Christiano, Eichenbaum, and Evans (2005)] and 17 [Uusküla (2008)]. This value thus seems to be plausible. The absolute value of the Rotemberg parameter,  $\theta$ , is hard to interpret as it is commonly set just to obtain an appropriate slope of the Phillips curve. The resulting slope coefficient,  $\omega$ , on the other hand becomes slightly lower as in our benchmark estimation which is in line with the reduced form estimations [cf. (38) and (40)].

The histograms of the estimation errors which respectively result from the structural estimations of (37) and (39) are depicted in Figure 13. The figure indicates that both errors are normally distributed which is supported by the Jarque-Bera test at a 99% significance level.

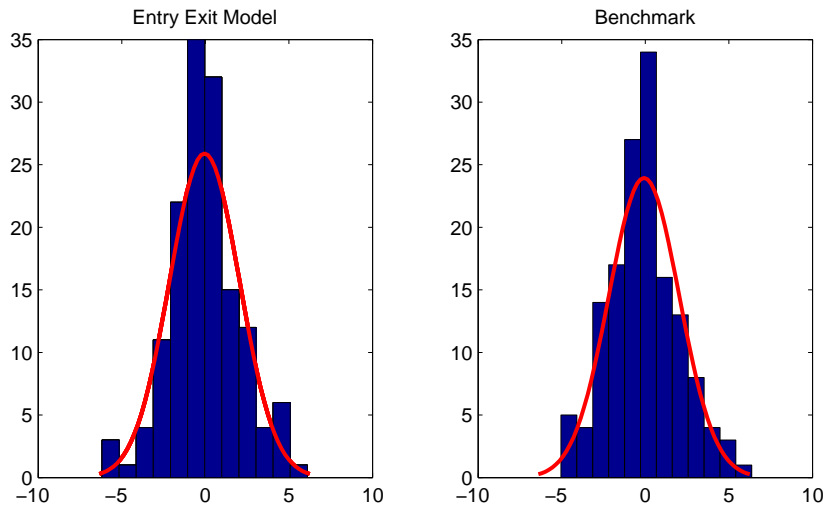


Figure 13: Histograms

The properties of the estimation errors are finally shown in Table 4. It  


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relation between the extensive margin and PPI inflation.

shows that the estimation error resulting from (39) has a mean which is closer to zero, has a lower standard deviation and skewness, and is less autocorrelated in comparison to that resulting from (37). Beside the skewness however the differences are only marginally.

Phillips curve	expected value	standard deviation	skewness	auto-correlation
(37)	-0.075	2.107	0.264	-0.240
(39)	-0.048	2.096	0.152	-0.216

Table 4: Moments of estimation errors

## 6 Conclusion

As GDP is even higher correlated with firms' failures than with firms' creations and since an endogenous tendency of firms to leave the market has been totally neglected in recent theoretic literature, yet, we build up a totally microfounded New Keynesian model with endogenous firms' entries as well as exits.

It turns out that the resulting model can solve some empirical problems of existing theoretical models that results in counterfactual developments of important economic variables. More precisely, we show that disregarding endogenous exits can lead to empirically implausible responses of inflation in the case of a technology shock as well as of firms' creation and the total number of producers in the case of a shock to monetary policy. Moreover, we can contribute to the debate in RBC literature initiated by Galí (1999) such that the RBC core of our model can depict both an expansionary and a contractionary reaction of total hours worked in the case of an aggregate productivity shock.

Additionally, the introduction of an endogenous tendency of firms to leave the market does not only enhance the performance of the resulting model with respect to the generated second moments but also solves a general difficulty of RBC and NK models. More precisely, the generated absolute and relative standard deviations are unambiguously closer to the data than those obtained

by the exogenous exit model of BGMa. Moreover, the problem that all variables react too pro-cyclical in DSGE models vanishes by the introduction of endogenous exits which react counter-cyclical. All in all, firms exits should not be neglected.

Furthermore, we show that the resulting CPI Phillips curve turns out to be dependent on the extensive margin while PPI inflation – like the baseline NK Phillips curve – is only affected by expected future inflation and the labor share. The GMM estimation of the CPI Phillips curve shows that the impact of the change in the number of producers on CPI inflation is highly significant and in line with our theoretical findings. Moreover, it turns out that the Phillips curve becomes flatter in a  $\hat{\pi}_t/\widehat{mc}_t$ -space in comparison to the standard NK Phillips curve which implies that the introduction of an endogenous number of producers causes the impact of the marginal costs on inflation to decrease as there occur additional effects from changes in product variety on CPI inflation. However, the resulting estimation errors are only marginally better than those of the baseline NK Phillips curve.

By endogenizing firms' exits, our model generates an endogenous trade-off between stabilizing output and inflation for monetary policy. Future research should thus concern about the optimal monetary policy.

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## A Proofs

### A.1 Proof of (17)

In equilibrium, there exist  $N_t$  firms which are Pareto distributed according to  $g(z_i)$  where  $g(\cdot)$  is the PDF of the Pareto distribution. The price level of a wholesale firm (4) then follows

$$P_{j,t} = \left( \int_{z_{\min}}^{\infty} N_t P_{i,t}(z_i)^{1-\zeta} g(z_i) dz_i \right)^{\frac{1}{1-\zeta}} \quad (\text{A1})$$

Inserting the nominal marginal costs (11) and the capital/labor input relation (10) yields

$$\begin{aligned} P_{j,t} &= \left( \int_{z_{\min}}^{\infty} N_t \left[ \frac{w_t^{\text{nominal}}}{A_t z_i (1-\alpha) \left( \frac{k_{i,t}}{l_{i,t}} \right)^\alpha} \right]^{1-\zeta} g(z_i) dz_i \right)^{\frac{1}{1-\zeta}} \\ &= \left( \int_{z_{\min}}^{\infty} N_t \left[ \frac{w_t^{\text{nominal}}}{A_t z_i (1-\alpha) \left( \frac{\alpha}{1-\alpha} \frac{r_t^K}{w_t} \right)^\alpha} \right]^{1-\zeta} g(z_i) dz_i \right)^{\frac{1}{1-\zeta}} P_{j,t} \\ &= N_t^{\frac{1}{1-\zeta}} \left[ \frac{w_t^{\text{nominal}}}{A_t (1-\alpha) \left( \frac{\alpha}{1-\alpha} \frac{r_t^K}{w_t} \right)^\alpha} \right] \left( \int_{z_{\min}}^{\infty} z_i^{\zeta-1} g(z_i) dz_i \right)^{\frac{1}{1-\zeta}} \end{aligned} \quad (\text{A2})$$

When defining

$$\tilde{z} \equiv \left( \int_{z_{\min}}^{\infty} z_i^{1-\zeta} g(z_i) dz_i \right)^{\frac{1}{1-\zeta}} \quad (\text{A3})$$

it follows that

$$P_{j,t} = N_t^{\frac{1}{1-\zeta}} \tilde{P}_t \quad (\text{A4})$$

where  $\tilde{P}_t \equiv P_{i,t}(\tilde{z})$ .

Due to symmetry across wholesale sector firms, equation (17) holds.

## A.2 Proof of (19)

From equation (A4) follows

$$1 = N_t^{\frac{1}{1-\zeta}} \frac{\tilde{P}_t}{P_{j,t}} \quad (\text{A5})$$

Raising this expression to the power of  $-\zeta$  and expanding the resulting expression by  $y_{j,t}$ , yields

$$y_{j,t} = N_t^{\frac{\zeta}{\zeta-1}} \left( \frac{\tilde{P}_t}{P_{j,t}} \right)^{-\zeta} y_{j,t} \quad (\text{A6})$$

When defining  $\tilde{y}_t \equiv y_{i,t}(\tilde{z}) = \left( \frac{\tilde{P}_t}{P_{j,t}} \right)^{-\zeta} y_{j,t}$ , we obtain

$$y_{j,t} = N_t^{\frac{\zeta}{\zeta-1}} \tilde{y}_t \quad (\text{A7})$$

According to symmetry across wholesale firms equation (19) holds.

## B The Set of Equations

In the following, hats denote log-linearized variables, while variables without time indexes represent steady state values.

The complete model in log-linear form then follows

$$\begin{aligned}
\widehat{Y}_t &= \frac{C}{Y}\widehat{C}_t + \frac{G}{Y}\widehat{G}_t + \frac{I}{S}\widehat{I}_t + \phi N^{-\frac{1}{\zeta-1}} f_E \left[ \widehat{Y}_t + \widehat{\phi}_t - \frac{1}{\zeta-1}\widehat{N}_t \right] \\
\widehat{C}_t &= E_t \widehat{C}_{t+1} - \frac{1-(1-\chi)\beta}{\sigma} \widehat{R}_t^K \\
\widehat{C}_t &= E_t \widehat{C}_{t+1} - \frac{1}{\sigma} (\widehat{R}_t - E_t \widehat{\pi}_{t+1}) \\
\widehat{I}_t &= \frac{1}{\chi} E_t \widehat{K}_{t+1} - \frac{1-\chi}{\chi} \widehat{K}_t \\
\widehat{w}_t &= \eta \widehat{L}_t + \sigma \widehat{C}_t \\
Y_t &= \widehat{A}_t + \frac{1}{\zeta-1} \widehat{N}_t + (1-\alpha) \widehat{L}_t + \alpha \widehat{K}_t \\
\widehat{w}_t &= \widehat{A}_t + \widehat{m}c_t + \alpha (\widehat{K}_t - \widehat{L}_t) \\
\widehat{R}_t^K &= \widehat{A}_t + \widehat{m}c_t + (1-\alpha) (\widehat{L}_t - \widehat{K}_t) \\
\widehat{\phi}_t &= -k \widehat{\delta}_t^{in} \\
\widehat{\gamma}_t &= k \frac{(z_{\min})^k}{(\delta^{out})^k - (z_{\min}^k)} \widehat{\delta}_t^{out} \\
\widehat{N}_t &= (1-\phi-\gamma) \widehat{N}_{t-1} + \phi \frac{N^{\max} - N}{N} \widehat{\phi}_{t-1} - \gamma \widehat{\gamma}_t \\
0 &= \frac{1}{\Psi} (\rho - mc) \delta^{out} (L^{1-\alpha} K^\alpha) \left[ (1-\alpha) \widehat{L}_t + \alpha \widehat{K}_t + \widehat{\delta}_t^{out} + \widehat{A}_t + \frac{\rho}{\rho - mc} \widehat{\rho}_t - \frac{mc}{\rho - mc} \widehat{m}c_t \right] \\
&\quad + \beta E_t \widehat{\Psi}_{t+1} + \beta (\widehat{N}_t - E_t \widehat{N}_{t+1}) \\
E_t \widehat{\Psi}_{t+1} &= \frac{\beta}{1-(1-\gamma)\beta} E_t \widehat{\Psi}_{t+2} + \frac{1}{1-(1-\gamma)\beta} \left[ (1-\alpha) E_t \widehat{L}_{t+1} + \alpha E_t \widehat{K}_{t+1} + E_t \widehat{A}_{t+1} - E_t \widehat{\gamma}_{t+1} \right] \\
&\quad + \beta (E_t \widehat{N}_{t+1} - E_t \widehat{N}_{t+2}) + \beta \gamma E_t \widehat{\gamma}_{t+1} \\
0 &= \frac{1}{\Psi} (\rho - mc) \delta^{in} (L^{1-\alpha} K^\alpha) \left[ (1-\alpha) \widehat{L}_t + \alpha \widehat{K}_t + \widehat{\delta}_t^{in} + \widehat{A}_t + \frac{\rho}{\rho - mc} \widehat{\rho}_t - \frac{mc}{\rho - mc} \widehat{m}c_t \right] \\
&\quad + \beta E_t \widehat{\Psi}_{t+1} + \beta (\widehat{N}_t - E_t \widehat{N}_{t+1}) - \phi f_E N^{\frac{1}{\zeta-1}} \frac{\delta^{in} Y}{\Psi} \left[ \widehat{\delta}_t^{in} + \widehat{Y}_t + \frac{1}{\zeta-1} \widehat{N}_t + \widehat{\phi}_t \right] \\
\widehat{\rho}_t &= \widehat{m}c_t - \frac{\theta}{\zeta-1} \left[ \widehat{\pi}_t - \beta E_t \widehat{\pi}_{t+1} \right] \\
\widehat{R}_t &= \lambda_\pi \widehat{\pi}_t + \lambda_y \widehat{Y}_t + \kappa_t
\end{aligned}$$

$$\begin{aligned}
\widehat{\pi}_t &= \widehat{\pi}_t + \frac{1}{\zeta - 1} [\widehat{N}_t - \widehat{N}_{t-1}] \\
\widehat{\rho}_t &= \frac{1}{\zeta - 1} \widehat{N}_t \\
\frac{\widehat{y}_t}{\widehat{Y}_t} &= -\frac{\zeta}{\zeta - 1} \widehat{N}_t \\
\widehat{l}_t &= \widehat{L}_t - \widehat{N}_t \\
\widehat{\mu}_t &= \widehat{\rho}_t - \widehat{N}_t - \widehat{m}c_t \\
\widehat{TFP}_t &= \widehat{Y}_t - (1 - \alpha)\widehat{L}_t - \alpha\widehat{K}_t
\end{aligned}$$