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Working Paper — Digitized Version

Reference-price-independent welfare prescriptions

ZEW Discussion Papers, No. 91-04

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Suggested Citation: Blackorby, Charles; Laisney, François; Schmachtenberg, Rolf (1991) : Reference-price-independent welfare prescriptions, ZEW Discussion Papers, No. 91-04, Zentrum für Europäische Wirtschaftsforschung (ZEW), Mannheim

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Discussion Paper

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Reference-Price-Independent Welfare Prescriptions

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ZEW

Zentrum für Europäische
Wirtschaftsforschung GmbH

Public Finance and
Company Taxation Series

19. DEZ. 1991

Weltwirtschaft
Kiel

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636-91,4 JK

Reference-Price-Independent Welfare Prescriptions

by

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Abstract

A number of studies have evaluated the social welfare impact of price and income changes using equivalent incomes that are computed at some reference price vector, and an aggregator with these equivalent incomes as arguments in place of a social welfare function. This paper investigates the impact of the choice of the reference price vector on the results of such exercises, distinguishing the case of individualized prices from the constrained case where all individuals face the same price vector. We characterize preferences and aggregators leading to reference-price-independent welfare prescriptions.

Acknowledgements

This paper is a revision of Laisney and Schmachtenberg (1989). The second author wishes to thank the Deutsche Forschungsgemeinschaft for support. The benefits of discussions with Rafiq Baccouche, Salvador Barbera, Walter Bossert, François Bourguignon, Russell Davidson, David Donaldson, Mervin King, and Ian Preston are gratefully acknowledged. The comments of four anonymous referees greatly helped revising the paper.

1 Introduction

Following the work of King (1983a), a number of studies have evaluated the social welfare impact of price and income changes using equivalent incomes that are computed at some reference price vector, and a *social aggregator function* with these equivalent incomes as arguments in place of a social welfare function (e.g. Baccouche and Laisney (1990), Colombino and Del Boca (1989), King (1983a, 1983b), Nichèle (1990), Patrizi and Rossi (1989). However, typically, the impact of the choice of the reference price vector on the results of such exercises has not been clearly delineated.

King (1983a) devotes limited attention to the characterization of situations (in terms of individual or household preferences and a social welfare function) admitting *Reference Price Independent Welfare Prescriptions* (RPIWP), referring loosely to the work of Muellbauer (1974) and Roberts (1980). Roberts studies the robustness of welfare changes using incomes only, and gives partial characterizations of the situations admitting *Price-Independent Welfare Prescriptions* (PIWP). Slivinski (1983) gives a complete characterization of PIWP if households face individualized prices.

In Section 3, we show that RPIWP implies PIWP on all price domains. In Section 4 we provide a complete characterization of RPIWP if there are household-specific prices, and in Section 5 there is a partial characterization of RPIWP when households face common prices.

2 Notation

We consider H households with preferences over n goods. The preferences are described by an expenditure function whose image is $e^h(u_h, p_h)$ where u_h is the utility level and p_h is the vector of prices faced by household h .¹

From the expenditure function we can derive the indirect utility function, v^h , of household h

$$y_h = e^h(u_h, p_h) \leftrightarrow u_h = v^h(y_h, p_h) \quad (2.1)$$

where y_h is the nominal income of household h .

¹ We assume that e^h is a continuous and increasing function of its arguments and that it is linearly homogeneous and concave in prices. In Section 3 we permit prices to be household-specific whereas in section 4 we require all households to face the same price vector.

Given a reference price vector² q_h for household h , the equivalent income of household h , ξ_h , is given by

$$\xi_h := e^h(u_h, q_h) = e^h(v^h(y_h, p_h), q_h). \quad (2.2)$$

The equivalent income function is a particular representation of the household's preferences for each reference price vector q_h .³

Let $W(u)$, $u = (u_1, \dots, u_H)$ be a Bergson-Samuelson social welfare function. We define a Bergson-Samuelson indirect social welfare function by

$$V(y, p) := W(\{v^h(y_h, p_h)\}) \quad (2.3)$$

where $y = (y_1, \dots, y_H)$, $p = (p_1, \dots, p_H)$ and $\{v^h(y_h, p_h)\} = (v^1(y_1, p_1), \dots, v^H(y_H, p_H))$, and W is continuous, increasing, and quasi-concave.

A tax reform which moves society from an income-price vector, (y^b, p^b) (b as 'before') to an 'after' income-price vector (y^a, p^a) is a social improvement if and only if

$$W(u^a) \geq W(u^b). \quad (2.4)$$

This can also be written in terms of the equivalent income functions $\{\xi_h\}$ by using (2.1) and (2.2) to obtain

$$u_h = v^h(y_h, p_h) = v^h(\xi_h, q_h); \quad (2.5)$$

substituting (2.5) into W yields

$$U(\xi, q) := W(\{v^h(\xi_h, q_h)\}) \quad (2.6)$$

where $\xi = (\xi_1, \dots, \xi_H)$ and $q = (q_1, \dots, q_H)$.

U is an attractive form of the social welfare function because monetary measures of social welfare changes due to a tax reform can be defined. Following page 198 in King(1983) the social gain SG from a tax reform at reference prices SG can be defined implicitly by

$$U(\xi^b + SG, q) = U(\xi^a, q). \quad (2.7)$$

² When households face different prices we allow the reference prices to be household-specific as well. In Section 4 where we require households to face a common price vector we require the reference price vector to be common as well. The equivalent income is also known as the indirect money-metric. See Weymark (1983) or Blackorby and Donaldson (1988).

³ Is it possible to pick all ordinal representations of a preference ordering by changing the reference price vector? (This question was posed by one of the referees.) The following example shows that this is not possible. Let a household consume only good one so that $v(y, p) = \phi(y/p)$. Then $\xi = q\psi(u)$ where ψ is the inverse of ϕ . Hence changing reference prices can only pick up all linear transforms of $x = \psi(u)$ and no others.

As a practical problem it can be difficult to find a tractable functional form for U given a social welfare function W and a set of preferences $\{v^h\}$ and one may be tempted to construct a functional form directly on the equivalent incomes themselves, ignoring the primitives from which it must be derived.⁴

3 Ethical Consistency and Reference-Price-Independent Welfare Prescriptions

We noted above that the practical difficulties involved in deriving a tractable U might lead researchers to specify a functional form directly on the equivalent incomes. This is the path followed by King (1983, p. 196) who posits the existence of an *equivalent-income aggregator* whose image is $G(\xi)$ which he then uses to evaluate social changes.⁵ Under what conditions is this equivalent-income aggregator ethically consistent?

Ethical Consistency: G and $\{v^h\}$ are ethically consistent if and only if there exists some Bergson-Samuelson social welfare function W such that

$$G(\xi^a) \geq G(\xi^b) \iff W(u^a) \geq W(u^b). \tag{3.1}$$

Another way to look at the potential problems associated with using the equivalent-income aggregator is to note that it depends upon the reference price vector q whereas the indirect Bergson-Samuelson social welfare function does not. Rewriting the equivalent-income aggregator as

$$F(y, p, q) = G(\{e^h(v^h(y_h, p_h), q_h)\}) \tag{3.2}$$

we are interested in the circumstances in which the social changes that it recommends are independent of the reference price vector q .

Reference-Price-Independent Welfare Prescriptions (RPIWP): G and $\{v^h\}$ admit RPIWP if and only if for all (y, p) , (y', p') , and (q, q') we have

$$F(y, p, q) \geq F(y', p', q) \iff F(y, p, q') \geq F(y', p', q'). \tag{3.3}$$

From Corollary 3.2.1 in Blackorby, Primont, and Russell (1978) this is equivalent to the following functional representation.

⁴ Even this exaggerates somewhat the value of U . In general, the equivalent income functions are not concave representations of preferences; see Blackorby and Donaldson (1988). Hence the U defined above may not be able to pick up all possible allocations of resources.

⁵ For example, suppose that households 1 and 2 consume only goods 1 and 2 respectively. If $u^h(x) = x$ for $h = 1, 2$, the sum of the equivalent income functions is $q_1 u^1 + q_2 u^2$. That is, the utilities are weighted in an arbitrary and capricious manner. Of course this can easily be undone in this simple example by dividing the first argument by q_1 and the second by q_2 . In general this is difficult to do and to the best of our knowledge no one has ever bothered to do it.

Proposition 3.1 G and $\{v^h\}$ admit RPIWP if and only if (y, p) is separable from q in F , that is,

$$F(y, p, q) = \hat{F}(E(y, p), q) \quad (3.4)$$

where \hat{F} is increasing in its first argument.

An immediate consequence of (3.1) and (3.2) is

Theorem 3.1: G and $\{v^h\}$ are ethically consistent if and only if they satisfy RPIWP.

Proof: If G and $\{v^h\}$ are ethically consistent, then, using (2.3) and (3.2), we obtain

$$F(y^a, p^a, q) \geq F(y^b, p^b, q) \leftrightarrow V(y^a, p^a) \geq V(y^b, p^b) \quad (3.5)$$

which implies (3.4). On the other hand, if G and $\{v^h\}$ admit RPIWP then

$$F(y^a, p^a, q) \geq F(y^b, p^b, q) \leftrightarrow E(y^a, p^a) \geq E(y^b, p^b). \quad (3.6)$$

Hence, we can set $q_h = 1_n$ and define, using (3.2),

$$W(u) := G(\{e^h(u_h, 1_n)\}) \quad (3.7)$$

which satisfies ethical consistency. •

On the way we have shown that if G and $\{v^h\}$ satisfy RPIWP, that is if (3.4) holds, then E is a Bergson-Samuelson indirect social welfare function.

The phrase *Price-Independent Welfare Prescriptions* was introduced by Roberts (1980) to describe those situations where policy prescriptions could be made on the basis of incomes alone even though prices had changed. It is defined as follows:

Price-Independent-Welfare Prescriptions (PIWP): W and $\{v^h\}$ admit PIWP if and only if the Bergson-Samuelson indirect social welfare function, (2.3), can be written as

$$V(y, p) = \bar{V}(\bar{V}(y), p) \quad (3.8)$$

where \bar{V} is increasing in its first argument. (In this case \bar{V} can be picked to be positively linearly homogeneous without loss of generality.)⁶

Roberts' notion of PWIP for a Bergson-Samuelson social welfare function leads to two notions of price-independent welfare prescriptions for the equivalent-income aggregator,

Local Price-Independent Welfare Prescriptions (LPIWP): G and $\{v^h\}$ admit LPIWP if and only if for all (y, p) , (y', p') , and q

$$F(y, p, q) \geq F(y', p, q) \leftrightarrow F(y, p', q) \geq F(y', p', q), \quad (3.9)$$

⁶ See Roberts (1980) or Blackorby and Donaldson (1985).

and

Global Price-Independent Welfare Prescriptions (GPIWP): G and $\{v^h\}$ admit GPIWP if and only if for all (y, p, q) and (y', p', q')

$$F(y, p, q) \geq F(y', p, q) \leftrightarrow F(y, p', q') \geq F(y', p', q'). \quad (3.10)$$

Similar to the representation result for RPIWP we have two representation results given by

Proposition 3.2: G and $\{v^h\}$ admit LPIWP if and only if

$$F(y, p, q) = \bar{F}(D(y, q), p, q) \quad (3.11)$$

where \bar{F} is increasing in its first argument

and

Proposition 3.3: G and $\{v^h\}$ admit GPIWP if and only if

$$F(y, p, q) = \bar{F}(C(y), p, q) \quad (3.12)$$

where \bar{F} is increasing in its first argument.

LPIWP guarantees price independence for a given reference price while GPIWP requires consistency of these welfare prescriptions across different reference prices.

The relationships between the several definitions of price-independent welfare prescriptions which hold on both of the price domains considered are given in the following two propositions.

Theorem 3.2: Given RPIWP, LPIWP implies GPIWP.

Proof: From Proposition 3.1, RPIWP implies that y is separable from q conditional on p whereas from Proposition 3.2, LPIWP implies y is separable from p conditional on q . Together these imply GPIWP. •

Theorem 3.3: Assume that prices and reference prices have the same domain. There exists a G such that G and $\{v^h\}$ admit RPIWP if and only if there exists a W such that W and $\{v^h\}$ admit PIWP.

Proof:⁷ Under RPIWP we have

$$G(\xi^a) \geq G(\xi^b) \leftrightarrow F(E(y^a, p^a), q) \geq F(E(y^b, p^b), q). \quad (3.13)$$

Setting $p^a = p^b = q$ and using (2.2) yields

$$\xi_h = e^h(v^h(y_h, p_h), p_h) = y_h, \quad (3.14)$$

and hence we obtain

⁷ This was discovered independently and given as Proposition 1 in Laisney and Schmachtenberg (1989) and Theorem 1b in Donaldson (1990). We present the proof given by Donaldson because of its simplicity.

$$G(y^a) \geq G(y^b) \leftrightarrow E(y^a, p) \geq E(y^b, p). \quad (3.15)$$

Setting $V = E$ yields an indirect social welfare function which satisfies PIWP.

On the other hand, if W and $\{v^h\}$ admit PIWP, then from (3.8) we have

$$W(u) = \bar{V}(\bar{V}(y), p) = \bar{V}(\bar{V}(\{e^h(u_h, p_h)\}), p). \quad (3.16)$$

Therefore,

$$\begin{aligned} \bar{V}(\{e^h(u_h^a, q_h)\}) &\geq \bar{V}(\{e^h(u_h^b, q_h)\}) \\ \leftrightarrow \bar{V}(\bar{V}(\{e^h(u_h^a, q_h)\}), q) &\geq \bar{V}(\bar{V}(\{e^h(u_h^b, q_h)\}), q) \\ \leftrightarrow W(u^a) &\geq W(u^b). \end{aligned} \quad (3.17)$$

Setting $G = \bar{V}$ yields RPIWP.

4 The Equivalence of RPIWP, LPIWP, and GPIWP on Household-Specific Price Domains

In this section we prove that if all households face different prices, then all three notions of price-independent welfare prescriptions are equivalent and require that the Bergson-Samuelson social welfare function and the equivalent-income aggregator must be Cobb-Douglas and that individual preferences must be homothetic (but not necessarily identical).

To do this we first use a result of Slivinski (1983) who proves that PIWP on household-specific price domains yields a Cobb-Douglas representation of the Bergson-Samuelson social welfare function and individual homotheticity. Therefore, the equivalent-income aggregator must be Cobb-Douglas as well for RPIWP to hold. Second, we show directly that LPIWP implies that the equivalent-income aggregator must be Cobb-Douglas and that individual preferences must be homothetic which implies RPIWP. Theorem 3.2 implies therefore GPIWP. For simplicity we break this argument into a series of Lemmata and assemble the main result at the end. We frequently use the symbol " $\overset{\circ}{=}$ " to mean "ordinally equivalent to" in what follows.

Lemma 4.1: *G and $\{v^h\}$ admit RPIWP, on household specific price domains, if and only if the equivalent-income aggregator is Cobb-Douglas and individual preferences are homothetic, that is,*

$$G(\xi) \overset{\circ}{=} \sum_{h=1}^H b_h \ln \xi_h \quad (4.1)$$

and

$$v^h(y_h, p_h) = y_h \alpha^h(p_h) \quad (4.2)$$

where α^h is homogeneous of degree minus one.

Proof: From Theorem 3.3 RPIWP implies PIWP and on household-specific price domains this yields individual homotheticity, (4.2), and a social welfare function that is ordinally equivalent to a Cobb-Douglas, that is,

$$W(u) = \sum_{h=1}^H b_h \ln u_h \quad (4.3)$$

by Proposition 4 in Slivinski (1983). From (3.7) we must have

$$G(\{e^h(u_h, 1_n)\}) = W(u) \quad (4.4)$$

for RPIWP to hold, and thus the equivalent-income aggregator must be ordinally equivalent to a Cobb-Douglas.

Conversely, given (4.1) and (4.2) we can first rewrite (4.2) as

$$v^h(y_h, p_h) = \Psi^h(y_h \alpha^h(p_h)) \quad (4.5)$$

and hence the expenditure function as

$$e^h(u_h, p_h) = \frac{\Psi^h(u_h)}{\alpha^h(p_h)} \quad (4.6)$$

where Ψ^h is the inverse of ϕ^h . Thus, the equivalent income can be written as

$$\xi^h = \frac{y_h \alpha^h(p_h)}{\alpha^h(q_h)}. \quad (4.7)$$

From the definition (4.1) of G we then obtain

$$F(y, p, q) = \sum_{h=1}^H b_h \ln y_h + \ln \alpha^h(p_h) - \ln \alpha^h(q_h) \quad (4.8)$$

which clearly satisfies (3.4) and thus RPIWP. •

Lemma 4.2: G and $\{v^h\}$ satisfy LPIWP, on household specific price domains, if and only if (4.1) and (4.2).

Proof: LPIWP implies that

$$\tilde{F}(D(y, q), p, q) = G(\{e^h(v^h(y_h, p_h), q_h)\}). \quad (4.9)$$

Conditional on the vector of reference prices q , (y_h, p_h) is separable from its complement in (y, p) on the right side of (4.9) and hence must be so on the left. As the vector y is separable from its complement in (y, p) conditional on q on the left side it must also be so on the right. We have here two overlapping separable sets conditional on q , neither of which is a subset of the other, and can invoke Gorman's overlapping theorem.⁸ Let y_{-h} and p_{-h} be the income and price vectors purged of those components indexed by h . Then Gorman's overlapping theorem implies that

$$\tilde{F}(D(y, q), p, q) = A^h(a^h(y_h, q) + b^h(p_h, q) + c^h(y_{-h}, q), p_{-h}, q). \quad (4.10)$$

As we can do this for all h this yields, using Lemma 2 in Gorman(1968) or Theorem 4.8 in Blackorby, Primont, and Russell (1978),

$$\tilde{F}(D(y, q), p, q) = A \left(\sum_h a^h(y_h, q) + \sum_h b^h(p_h, q), q \right). \quad (4.11)$$

From (4.11) we have that p_h is separable from y_h in \tilde{F} and hence from (4.9) that p_h is separable from y_h in v^h . From Lemma 3.4 in Blackorby, Primont, and Russell (1978) this implies that the preferences of household h are homothetic for all h and the equivalent income of household h can be written as in (4.7). Substituting this into G yields

$$G(\{e^h(v^h(y_h, p_h), q_h)\}) = G \left(\left\{ \frac{y_h \alpha^h(p_h)}{\alpha^h(q_h)} \right\} \right). \quad (4.12)$$

Substituting this into (4.9) and using (4.11) demonstrates that each p_h and each q_h are separable from their respective complements in \tilde{F} and hence in A . Setting $\alpha^h(p_h) = t_h$ and $\alpha^h(q_h) = s_h$ allows us to write

$$A \left(\sum_h \bar{a}^h(y_h, s) + \sum_h \bar{b}^h(t_h, s), s \right) = G \left(\left\{ \frac{y_h t_h}{s_h} \right\} \right) \quad (4.13)$$

where $s = (s_1, \dots, s_H)$. Setting $s_h = t_h = 1$ for all h demonstrates that W is additively separable in its arguments and hence we can rewrite (4.13) as

$$A \left(\sum_h \bar{a}^h(y_h, s) + \sum_h \bar{b}^h(t_h, s), s \right) = \bar{G} \left(\sum_h G^h \left(\frac{y_h t_h}{s_h} \right) \right). \quad (4.14)$$

Next, setting $y_h = 1$ and $t_h = 1$ we see that the left side of (4.14) must be additive in s as well and hence can be rewritten, in an abuse of notation, as

⁸ See Theorem 1 in Gorman (1968) or Theorem 4.7 in Blackorby, Primont, and Russell (1978).

$$A\left(\sum_h a^h(y_h) + \sum_h b^h(t_h) + \sum_h c^h(s_h)\right) = \overline{G}\left(\sum_h G^h\left(\frac{y_h t_h}{s_h}\right)\right). \quad (4.15)$$

This however is a Pexider equation whose solution is (4.1); to see this, use sequentially Theorem 3.5.5 in Eichhorn (1978). This establishes the Lemma. •

Thus RPIWP and LPIWP are equivalent in this situation, and using Proposition 3.2 we obtain

Theorem 4.1: *On household-specific price domains, RPIWP, LPIWP, and GPIWP are equivalent and yield (4.1) and (4.2).*

This completes our characterization on household-specific price domains. As a practical matter, we note that the empirical results of King (1983 a and b), Colombino and Del Boca (1989), and Nichèle (1990) are based on individualized prices. In addition they assume a family of equivalent-income aggregator functions parameterised by an index of inequality ϵ . Theorem 4.1 asserts that the only acceptable value for this parameter is 1 if the prescriptions are to be free of the reference price vector, and this only if preferences are homothetic. The latter is satisfied in the studies of King and Nichèle but not in the study of Colombino and Del Boca. There, the methodology used does not warrant any safe welfare prescription.

5 RPIWP on Common Price Domains

The case of common price domains is more difficult. To date no one has provided a complete characterization of PIWP on common price domains; and we certainly have not succeeded in providing a complete characterization of RPIWP. We do however have several partial characterizations which are of some interest.

Roberts (1980) gives a wide range of results concerning PIWP on a common price domain which are closely related to the characterizations available for RPIWP. We stress in the sequel only those that have direct empirical relevance.

Proposition 5.1: *If G and $\{v^h\}$ satisfy RPIWP, then G is homothetic.*

Proof: Given RPIWP, we obtain from (3.2) and (3.4), after setting $p = q$,

$$G(\xi) = \hat{F}(E(\xi, p), p) \quad (5.1)$$

where E is a Bergson-Samuelson indirect social welfare function. From Theorem 3.3 we know that RPIWP implies PIWP and hence we can write

$$E(\xi, p) = \overline{E}(\tilde{E}(\xi), p) \quad (5.2)$$

where \tilde{E} is linearly homogeneous. Combining (5.1) and (5.2) yields the result. •

Proposition 5.1 is the analogue of Proposition 3 in Roberts (1980). The analogue of his Proposition 4 follows from Theorem 3.3.

Proposition 5.2: *Suppose that all households have identical homothetic preferences. Then, G and $\{v^h\}$ satisfy RPIWP if and only if $\$G\$$ is homothetic.*

Finally, an analogue to Roberts' Proposition 6 in our context follows:

Proposition 5.3: *If G is linear, then G and $\{v^h\}$ satisfy RPIWP if and only if preferences are quasi-homothetic with identical slopes. Conversely, in the latter case, the member of the Kolm-Atkinson family of equivalent-income aggregators G leading to RPIWP is linear.*

Proof: If G is linear, then

$$G(\xi^a) \geq G(\xi^b) \leftrightarrow G(\xi^a - \xi^b) \geq 0. \quad (5.3)$$

By setting q equal to the status quo price vector the first part of the proposition follows from the characterization of ethical consistency for functions of individual equivalent variations by Blackorby and Donaldson (1985).

Conversely, consider quasi-homothetic preferences with identical slopes:

$$\xi^h = \alpha^h(q) + \frac{\beta(q)}{\beta(p)}(y^h - \alpha^h(p)) \quad (5.4)$$

and the symmetric Kolm-Atkinson family

$$G(\xi) = \frac{1}{1-\varepsilon} \sum_h (\xi^h)^{1-\varepsilon} \quad \text{for } \varepsilon \neq 1 \quad (5.5)$$

$$G(\xi) = \sum_h \ln \xi^h \quad (\varepsilon = 1).$$

If (y, p) is separable from q in F (see equation (3.2)), in particular p is separable from q . Given differentiability as above, this means that the ratio $\partial G/\partial p_i$ over $\partial G/\partial p_j$ must be independent of q for all pairs of goods $\{i, j\}$. But this ratio is equal to

$$\frac{\partial G/\partial p_i}{\partial G/\partial p_j} = \frac{\sum_h (\xi^h)^{-\varepsilon} [\alpha_i^h + (y^h - \alpha^h(p)) \beta_i(p)/\beta(p)]}{\sum_h (\xi^h)^{-\varepsilon} [\alpha_j^h + (y^h - \alpha^h(p)) \beta_j(p)/\beta(p)]} \quad (5.6)$$

and it will only be independent of q if $\varepsilon = 0$.

In addition, Proposition 5.3 shows that RPIWP is *not* characterized by Proposition 5.2. Proposition 5.3 is, however, of direct relevance for the study of Baccouche and Laisney (1990). They present results for quasi-homothetic individual preferences with identical slopes in terms of the Kolm-Atkinson family of equivalent-income aggregators: only their results for $\varepsilon = 0$ are robust to changes in the reference price. How much the other results would be affected by a drastic change of the reference price remains to be studied.

Patrizi and Rossi (1989) specify PIGLOG individual preferences and consider values of 0.5, 1, 2 and 5 of the parameter ϵ of the Kolm-Atkinson family of equivalent income aggregators. Proposition 6 of Roberts suggests that the only value of ϵ warranting RPIWP is $\epsilon = 1$ and indeed this is easily checked directly.

6 Conclusions

Having accumulated negative evidence against the possibility of RPIWP as we have, we must stress the fact that, in the case of differentiable v^h and G and with (y', p') in a neighbourhood of (y, p) , one can expect from definition (3.3) to find no violation of reference price independence over a reasonably large neighbourhood of a given q . This comment applies to the study of Patrizi and Rossi, for instance, where the VAT reforms considered do not alter prices dramatically and the distribution of income is left unchanged in most cases.

A potential extension of this work would be to focus on independence of welfare prescriptions with respect to the choice of reference characteristics, which becomes necessary if households differ but one still insists on anonymity of the equivalent-income aggregator. Given the negative results obtained here for a global definition of RPIWP one should probably focus attention on local results, at least as regards prices and incomes. This problem, as well as that of the choice of a reference price is also discussed by Willig (1981), in the related context of social welfare dominance. Some interesting results are given by Donaldson (1990): in particular, for a given reference household, the requirement of RPIWP is less stringent than here, as it places restrictions only on the preferences of the reference household and on the SWF.

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