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Working Paper

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Working Paper Series in Economics, No. 69

Provided in Cooperation with:
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Suggested Citation: Heinemann, Maik (2007) : E-stability and stability of adaptive learning in models of asymetric information, Working Paper Series in Economics, No. 69, Leuphana Universität Lüneburg, Institut für Volkswirtschaftslehre, Lüneburg

This Version is available at:
https://hdl.handle.net/10419/28186

WORKING PAPER

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by

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University of Lüneburg
Working Paper Series in Economics

No. 69

December, 2007

www.uni-lueneburg.de/vwl/papers
ISSN 1860–5580

E–stability and stability of adaptive learning in models with private information

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December 12, 2007

Abstract

The paper demonstrates how the E–stability principle introduced by Evans and Honkapohja [2001] can be applied to models with heterogeneous and private information in order to assess the stability of rational expectations equilibria under learning. The paper extends already known stability results for the Grossman and Stiglitz [1980] model to a more general case with many differentially informed agents and to the case where information is endogenously acquired by optimizing agents. In both cases it turns out that the rational expectations equilibrium of the model is inherently E-stable and thus locally stable under recursive least squares learning.

Keywords: Adaptive Learning, Eductive Stability, Rational Expectations
JEL–Classification: D31, E62, O41, P16

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1 Introduction

The aim of this paper is to demonstrate how the standard tools that are used to analyze the stability of adaptive learning processes, can be utilized within the context of models with private and heterogeneous information. This approach tracing back to Marcet and Sargent [1988] and comprehensively described by Evans and Honkapohja [2001] is based upon the approximation of the limiting behavior of the learning process by an ordinary differential equation. The dynamic properties of this differential equation can then be analyzed with the help of the so called T–map, which gives rise to an E–stability principle (cf. Evans and Honkapohja [2001] for a thorough discussion): According to that principle, a rational expectations equilibrium is locally stable under adaptive learning, if and only if it is E–stable. Here and in the next sections it will be demonstrated that this E–stability principle remains valid within the framework of models with private and heterogeneous information and even if the amount of private information is endogenously determined.

The validity of the E–stability principle simplifies the analysis of adaptive learning processes in models with heterogeneous and private information considerably. This will be shown by first looking at a simple linear economic model whose main purpose is to introduce the underlying concept. By the way, however, it is demonstrated that the introduction of private information into a model with adaptively learning agents is not harmful to the convergence of the learning process towards the rational expectations equilibrium. The reason is that E–stability is governed by more fundamental properties of the model, which are unaffected by the presence of private information or other sources of learning heterogeneity.

In economic situations where agents have incomplete private information regarding payoff relevant aspects, market prices besides their allocative function also fulfill an informational function. A famous model highlighting the informational role of prices is the financial market model by Grossman and Stiglitz [1980], where agents try to extract from market prices the information of others — and thus are 'learning' from prices. In order to show how the E–stability principle can be used to answer the question whether agents can learn to extract information of others correctly from market prices in an adaptive fashion, a variant of the Grossman and Stiglitz [1980] model is analyzed. While the properties of the Grossman and Stiglitz model under learning have already been studied by Bray [1982], Marcet and Sargent [1992] and — from a somewhat different perspective — Routledge [1999], the reexamination of this model demonstrates how the stability results derived in that papers can be reproduced quite easily using the E–stability principle. Furthermore, this approach allows to generalize these results in two important directions. First, it will be shown that the rational expectations equilibrium in a model of the Grossman and Stiglitz type with an exogenously given amount of

private information is always E–stable, irrespectively of the number of differentially informed agents. Second, it will be shown that E–stability continues to hold if the amount of private information is endogenously determined, i.e. if optimizing agents decide how much costly private information they want to acquire.

2 Learning and private information: A simple model

The model that is used here is a simple linear model — reminiscent of the well known cobweb model — with n economic agents where an endogenous variable y is a linear function of individual actions a_i, $i = 1,\dots,n$ and an unobserved variable x, where $\mathrm{E}[x] = 0$ and $\mathrm{Var}[x] = \sigma_x^2$:

$$y = \beta_0 + \beta_1 \left(\frac{1}{n} \sum_{i=1}^{n} a_i \right) + x + \varepsilon$$

Agents possess private information regarding x, because every agent observes a private signal $s_i = x + u_i$, where u_i represents the noise associated with the signal. Regarding this noise it is assumed that $\mathrm{E}[u_i] = 0$ and $\mathrm{Var}[u_i] = \sigma_u^2$ for all $i = 1,\dots,n$. Given this and with $\pi = \frac{\sigma_x^2}{\sigma_u^2}$ denoting the signal to noise ratio of the private signals, the conditional expectation of x given a signal s_i results as:

$$\mathrm{E}[x \,|\, s_i] = \frac{\sigma_x^2}{\sigma_x^2 + \sigma_u^2} s_i = \frac{\pi}{1+\pi} s_i \tag{1}$$

We assume that individual actions are each a linear function of the conditional expectation of the endogenous variable y given the respective private signal:

$$a_i = \delta \, \mathrm{E}[y \,|\, s_i], \quad i = 1,\dots,n \tag{2}$$

2.1 Rational expectations equilibrium and the T–map

To compute the rational expectations equilibrium (REE) of this model we assume that agents use linear decision rules, implying that the expectation of y conditional on s_i is a linear function of the private signal s_i

$$a_i = \delta \underbrace{(\gamma_{0,i} + \gamma_{1,i} s_i)}_{=\mathrm{E}[y \,|\, s_i]} \tag{3}$$

The yet undetermined coefficients $\gamma_{0,i}$ and $\gamma_{1,i}$ in (3) now have to be determined for all $i = 1,\dots,n$ such that the respective expectation coincides with the true conditional expectation of y.

Under the assumption of such linear decision rules the endogenous variable is now given as follows:

$$y = \beta_0 + \beta_1 \delta \left(\bar{\gamma}_0 + \bar{\gamma}_1 x + (1/n) \sum_{j=1}^{n} \gamma_{1,j} u_j \right) + x + \varepsilon \tag{4}$$

Here $\bar{\gamma}_0 = (1/n)\sum_n \gamma_{0,i}$ and $(1/n)\bar{\gamma}_1 = \sum_n \gamma_{1,i}$ denote the averages of all individual coefficients. With the conditional expectation of y based on equation (4), the updated decision rule for an agent i is now given by:

$$\begin{aligned} a_i &= \delta\, \mathrm{E}[y\,|\,s_i] = \delta\left(\bar{y} + \frac{\mathrm{Cov}[s_i y]}{\mathrm{Var}[s_i]}(s_i - \bar{s})\right) \\ &= \delta \underbrace{\left[\beta_0 + \delta\beta_1\bar{\gamma}_0\right]}_{\gamma'_{0,i}} + \delta \underbrace{\left[\frac{(1+\delta\beta_1\bar{\gamma}_1)\pi + \delta\beta_1\gamma_{1,i}/n}{1+\pi}\right]}_{\gamma'_{1,i}} s_i \end{aligned} \tag{5}$$

With $\gamma_i = (\gamma_{0,i}, \gamma_{1,i})'$ for $i = 1,\ldots,n$ denoting the vector of individual coefficients, equation (5) can be used to construct a mapping from the vector $\gamma = (\gamma_1,\ldots,\gamma_n)$ of individual coefficients to the updated coefficient γ'_i of an agent i:

$$\gamma'_i = T_i(\gamma) = T_i(\gamma_1,\ldots,\gamma_n), \quad i = 1,\ldots,n \tag{6}$$

This mapping is the individual T–map which can be used to construct the overall T–map $\gamma' = T(\gamma)$:

$$\gamma' = T(\gamma) = \begin{pmatrix} T_1(\gamma) \\ \vdots \\ T_n(\gamma) \end{pmatrix}$$

As usual, the REE — in the present case a limited information REE — is a fixed point of this mapping. In such a REE individual coefficients are identical, i.e. $\gamma_i = \gamma^*$ for all $i = 1,\ldots,n$. Using equation (5) we get:

$$\gamma^* = T(\gamma^*) \quad \Rightarrow \quad \begin{cases} \gamma_0^* = \frac{\beta_0}{1-\delta\beta_1} \\ \gamma_1^* = \frac{1}{1+\pi}\frac{1}{1-\delta\beta_1\frac{1+n\pi}{n(1+\pi)}} \end{cases}$$

2.2 E–stability and stability under learning

Local stability of this REE under adaptive learning — which in the present context means learning using recursive least squares or a stochastic gradient procedure — can now be quite easily analyzed using the T–map. First of all it is, however, necessary to embed the hitherto static model into a dynamic framework such it is at all possible to analyze real time learning processes. Thus, from now on it is assumed that the just described static model is repeated over a long horizon. In each period t, an unobserved random variable x_t realizes and agents observe their private signals $s_{i,t} = x_t + u_{i,t}$. Individual actions as before depend on an expectation regarding the endogenous variable y_t which is based on a linear perceived law of motion

$$y^e_{i,t} = z'_{i,t}\,\gamma_{i,t}\,,$$

where $\gamma_{i,t}$ is a 2×1 vector of individual coefficients and $z'_{i,t} = (1, s_{i,t})$. At the end of every period, agents then revise their estimate $\gamma_{i,t}$ in the light of new data, consisting of the endogenous variable y_t and their private signal $s_{i,t}$. This recursive estimation is done using either recursive least squares or a stochastic gradient procedure, the asymptotic properties of which are identical in the present context.

As is well known, local stability of such an adaptive learning process is governed by E–stability conditions (this is the so called E–stability principle formulated by Evans and Honkapohja [2001]). According to this, local stability of the rational expectations equilibrium γ^* under learning obtains whenever γ^* is a locally stable stationary point of the $2n$ dimensional ordinary differential equation(see Appendix A.1 for a derivation of equation (7)):

$$\dot{\gamma} = T(\gamma) - \gamma \tag{7}$$

Local stability therefore requires that all eigenvalues of the $2n\times 2n$ matrix $J(\gamma^*)$ of partial derivatives of the map $T(\gamma)-\gamma$ with respect to γ evaluated at γ^* are negative. Now using (5) and (6), the respective derivatives of $T(\gamma^*)$ can be written as:

$$\frac{\partial \gamma'_i}{\partial \gamma_i} = A + B, \quad \frac{\partial \gamma'_i}{\partial \gamma_j} = A, \quad i = 1,\ldots,n, \quad j \neq i$$

Here A is a 2×2 matrix of partial derivatives of $T_i(\gamma)$ with respect to γ_j, $i \neq j$, evaluated at γ^*:

$$A = \begin{pmatrix} \frac{\delta\beta_1}{n} & 0 \\ 0 & \frac{\delta\beta_1\pi}{n(1+\pi)} \end{pmatrix},$$

and B is given by:

$$B = \begin{pmatrix} 0 & 0 \\ 0 & \frac{\delta\beta_1}{n(1+\pi)} \end{pmatrix}$$

Since A and B are diagonal matrices, $2(n-1)$ of the n eigenvalues of $T'(\gamma^*)$ are given as repetitions of the eigenvalues of B, while the remaining two eigenvalues are given as n times the eigenvalues of A plus the respective eigenvalue of B:

$$\lambda = \Bigg(\underbrace{0,\ldots,0}_{(n-1)\times}, \underbrace{\frac{\delta\beta_1}{n(1+\pi)},\ldots,\frac{\delta\beta_1}{n(1+\pi)}}_{(n-1)\times}, \delta\beta_1, \frac{(1+n\pi)\,\delta\beta_1}{n(1+\pi)} \Bigg)$$

Stability of the REE under learning requires that all eigenvalues are smaller than one. Since $n \geq 1$ and $\pi > 0$, a necessary and sufficient condition for stability therefore is that $\delta\beta_1 < 1$. This is exactly the condition that determines local stability of the REE under learning in case of homogeneous firms under full information, i.e. the case where x_t is observed.

In the cobweb model — or more generally, in all models that are characterized by strategic substitutability of individual actions —, we have $\delta\beta_1 < 0$ such that

this stability condition is always satisfied (see Guesnerie [2005] for a discussion of strategic substitutability and complementarity in the context of learning). It may also be satisfied in models, where strategic complementarity of individual actions as measured by the product $\delta\beta_1$ is sufficiently small. Irrespective of this, however, heterogeneity of information will lead to no additional conditions for the stability of adaptive learning processes. As long as the stability condition for the homogeneous case (i.e. $\delta\beta_1 < 1$) is satisfied, stability also obtains for the case where differentially informed agents learn using recursive least squares.

3 A competitive market model with learning from prices

In the model considered so far, the only information of agents regarding the unobserved variable x_t consists of privately observed signals $s_{i,t}$. This leaves open the question how this private information comes into the model in the first place, meaning that individual decisions regarding the acquisition of information are neglected. Furthermore, the very stylized model neglects the important aspect that market prices in competitive markets may comprise disparate private information and transmit this information to market participants.

In what follows, we will consider a modified model, where agents posses private information regarding a payoff relevant variable but do also observe a market price that transmits information. It will be first shown that it is quite straightforward to compute the T–map that governs the stability of adaptive learning processes even in environments where such 'learning from prices' takes place. Second, it will be demonstrated, how this learning process can be modified to allow for an endogenously determined amount of information acquisition during the learning process. The central question then is, whether or not an endogenously determined amount of information acquisition leads to stronger conditions for the stability of the learning process.

3.1 The model

The model used here is a model of a competitive commodity market with privately informed firms borrowed from Vives [1993]. Vives [1993] shows that it is possible to restate the this model such that it can be interpreted as a financial market model where agents are buyers of an asset with unknown ex–post return similar to the Grossman and Stiglitz [1980] model.

Here we stick to the commodity market interpretation of the model and assume that there is a continuum of risk neutral firms in $I = [0,1]$. Market demand X for the commodity is random, but the inverse demand function is known to the firms:

$$p = \beta - \frac{1}{\phi} X + \varepsilon$$

Here, ε is a normally distributed demand shock with zero mean and precision τ_ε. $\beta > 0$ and $\phi > 0$ are known constants. Every firms faces increasing marginal costs that are affected by the parameter θ. With $x(j)$ denoting the output of firm j, her costs are $c(j) = \theta x(j) + \frac{1}{2}\frac{1}{\psi}x(j)^2$, where $\psi > 0$. The cost parameter θ is unknown to the firms. The firms, however, know that this parameter is drawn from a normal distribution with mean $\overline{\theta}$ and precision τ.

Firms have private information regarding the unknown parameter θ. Regarding this private information, we assume that the continuum of firms is divided into n types of firms $i = 1, \ldots, n$. Each type of firms has measure m_i, where $\sum_{i=1}^{n} m_i = 1$, and all firms of the same type are homogeneous with respect to their private information. A firm j of type i observes a private signal $s(j)_i$ that reveals additional private information. The private signal is given by $s(j)_i \equiv s_i = \theta + u_i$, where the signal's noise u_i is normally distributed with mean zero and precision $\tau_{u,i}$. Thus, all firms of the same type observe signals with the same precision, but precisions are allowed to differ across firm types.

3.2 Rational expectations equilibrium

Profit maximization on the side of the firms then implies that each firm's optimal output $x(j)_i^* \equiv x_i^*$ is proportional to the difference between the market price p and the conditional expectation of the unknown cost parameter θ, where the respective conditional expectation is based on the observed market price as well as the private signal:

$$x_i^* = \psi \left(p - \mathrm{E}[\theta \,|\, p, s_i] \right)$$

Restricting attention to linear equilibria and using the fact that equilibrium decisions of all firms of the same type are identical, we posit that the conditional expectation of a firm of type i is a linear function of the market price and her private signal:

$$\mathrm{E}[\theta \,|\, p, s_i] = \gamma_{0,i} + \gamma_{1,i}\, s_i + \gamma_{2,i}\, p$$

This implies that a firm of type i uses a linear supply function according to which $x_i = \psi(p - \gamma_{0,i} + \gamma_{1,i}\, s_i + \gamma_{2,i}\, p)$ and from this the market clearing price results as (here and in what follows, we define $\alpha \equiv \psi/\phi > 0$):

$$\begin{aligned} p &= \beta - \alpha \left(p - \sum_{i=1}^{n} m_i \left(\gamma_{0,i} + \gamma_{1,i} \left[\theta + u_i \right] + \gamma_{2,i} p \right) \right) + \varepsilon \\ &= \frac{\beta + \alpha \left(\sum_{i=1}^{n} m_i \left(\gamma_{0,i} + \gamma_{1,i} \left[\theta + u_i \right] \right) \right) + \varepsilon}{1 + \alpha - \alpha \sum_{i=1}^{n} m_i \gamma_{2,i}} \end{aligned} \tag{8}$$

A rational expectations equilibrium requires that the above described conditional expectation is based on the joint equilibrium distribution of the unknown parameter, the market price and the signals. Therefore let $z_i = (s_i - \overline{\theta}, p - \overline{p})'$, $\gamma_i = (\gamma_{1,i}, \gamma_{2,i})'$ and $\gamma = (\gamma_1, \ldots, \gamma_n)'$ and define the following matrix and vector of moments:

$$M_{zz,i}(\gamma) = \mathrm{E}\left[z_i z_i'\right] = \begin{pmatrix} \frac{1}{\tau} + \frac{1}{\tau_{u,i}} & \mathrm{Cov}(s_i p) \\ \mathrm{Cov}(s_i p) & \mathrm{Var}(p) \end{pmatrix}$$

$$M_{z\theta,i}(\gamma) = \mathrm{E}\left[z_i (\theta - \overline{\theta})\right] = \begin{pmatrix} \frac{1}{\tau} & \mathrm{Cov}(\theta p) \end{pmatrix}'$$

Notice that $M_{zz,i}$ and $M_{z\theta,i}$ are identical for all firms of the same type. Given this, the conditional expectation of θ for a firm of type i can be computed as follows:

$$\mathrm{E}[\theta \,|\, s_i, p] = \underbrace{\overline{\theta} - M_{zz,i}(\gamma)^{-1} M_{z\theta,i}(\gamma)\, \overline{z}}_{\gamma_{0,i}} + \underbrace{M_{zz,i}(\gamma)^{-1} M_{z\theta,i}(\gamma)}_{(\gamma_{1,i}, \gamma_{2,i})}\, z_i'$$

This expression again defines a T-map for all firm types $i = 1, \ldots, n$ ($\gamma_{0,i} = \overline{\theta} - T_i(\gamma)\overline{z}$):

$$\gamma_i = T_i(\gamma) = M_{zz,i}(\gamma)^{-1} M_{z\theta,i}(\gamma)$$

Restricting attention to the parameters $\gamma_{1,i}$ and $\gamma_{2,i}$ of all type specific supply schedules the overall T-map can be constructed as follows:[1]

$$\gamma = \begin{pmatrix} \gamma_1 \\ \vdots \\ \gamma_n \end{pmatrix} = \begin{pmatrix} T_1(\gamma) \\ \vdots \\ T_n(\gamma) \end{pmatrix} = T(\gamma)$$

As usual, a rational expectations equilibrium defined as a fixed point of this mapping. The coefficients of the equilibrium supply schedules γ^* therefore solve $\gamma^* = T(\gamma^*)$. Within the linear framework used here, existence and uniqueness of such an equilibrium is guaranteed.

3.3 Stability under learning

An analysis of real time adaptive learning again requires to embed the static model considered so far into a dynamic context. This is done here in the same way as in the model considered in Section 2. Thus, we assume that the static model is repeated over a long horizon, where in each period t an unobserved random variable θ_t realizes. Firms observe their private signals $s_{i,t}$ and decide on their optimal output $x_{i,t}$, where this decision is based on the expectation $\theta^e_{i,t}$ regarding the unknown θ_t conditional on the actual market price p_t:

$$x_{i,t} = \psi\left[p_t - \theta^e_t\right]$$

[1] In Appendix A.4 we show that the dynamics of $\gamma_{0,i}$ under learning give rise to no additional stability conditions. Thus, it is of no harm to disregard this parameter in the subsequent analysis.

The expectation $\theta_{i,t}^e$ in turn is based on an auxiliary model according to which $\theta_t = \gamma_{0,i} + \gamma_{1,i}\, s_{i,t} + \gamma_{2,i}\, p_t$. At the end of each period t, firms observe the true value of θ_t and then re–estimate the parameters of their auxiliary model using e.g. recursive least squares.

As before, local stability of the rational expectations equilibrium γ^* under such an adaptive learning scheme then requires that the rational expectations equilibrium is E–stable, i.e. that γ^* is a locally stable stationary point of the ordinary differential equation $\dot{\gamma} = T(\gamma) - \gamma$. This in turn requires that all $2n$ eigenvalues of the $2n \times 2n$ matrix $J(\gamma)$,

$$J(\gamma) = \frac{dT(\gamma)}{d\gamma} - \mathrm{I}_{2n},$$

evaluated at the rational expectations equilibrium γ^* are negative.

There are some general properties of the matrix $T'(\gamma) = \frac{dT(\gamma)}{d\gamma}$ that prove to be useful in the subsequent analysis. Let for all $i = 1, \ldots, n$ denote A_i the 2×2 matrix of derivatives of $T_i(\gamma)$ with respect to γ_i. Furthermore, let B_{ij} denote the 2×2 matrix of derivatives of $T_i(\gamma)$ with respect to γ_j for $j \neq i$. Then $T'(\gamma)$ is given by:

$$T'(\gamma) = \begin{pmatrix} A_1 & B_{12} & \cdots & B_{1n} \\ B_{21} & A_2 & \cdots & B_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ B_{n1} & B_{n2} & \cdots & A_n \end{pmatrix}$$

While, it is possible to draw some conclusions regarding the properties of the matrices A_i and B_{ij}, even this doesn't facilitate the computation of the eigenvalues of $T'(\gamma^*)$ in the fully heterogeneous case (n firm types of different measure that differ with respect to the precision of their private information). For this reason, we now look at two interesting special cases of this model, where it is possible to derive a closed form solution for the eigenvalues of $T'(\gamma^*)$.

3.3.1 The Grossman–Stiglitz model

One important special case of the above described model is the case where there are only two types of firms. Firms of the first type are informed regarding θ, whereas firms of the second type have no private information at all. In this case, the model becomes analytically identical to the model used by Grossman and Stiglitz [1980] in their famous paper on the impossibility of informationally efficient markets. The question whether or not the rational expectations equilibrium of this model is stable under learning has already been answered by Bray [1982] and Marcet and Sargent [1992]. Here we briefly reproduce the respective results demonstrating that these results can be easily derived from the respective T–map.

Thus, assume that $n = 2$ and that there is a mass $0 < m_1 < 1$ of informed firms that observe a signal with precision $\tau_{u,1} > 0$, while the remaining firms (with mass $1 - m_1$) are uninformed, i.e. their private signals have precision zero ($\tau_{u,2} = 0$).

The T–maps for the two types of firms are then given by the following equations:[2]

$$\gamma'_{1,1} = \frac{\tau_{u,1}}{\tau + \tau_{u,1}}, \quad \gamma'_{2,1} = 0 \tag{9a}$$

$$\gamma'_{1,2} = 0, \qquad \gamma'_{2,2} = \frac{\alpha m_1 \gamma_{1,1} \tau_\varepsilon \tau_{u,1} (1 + \alpha - \alpha(1 - m_1)\gamma_{2,2})}{\tau\tau_{u,1} + \alpha^2 m_1^2 \gamma_{1,1}^2 \tau_\varepsilon (\tau + \tau_{u,1})} \tag{9b}$$

The economic meaning of these equations is quite obvious: Since informed firms cannot learn anything new from the observation of the market price p, their conditional expectation regarding θ is solely based on their private signals and unaffected by uninformed firms' actions. Uninformed firms, however, observe no private signals. So the best they can do is to extract some information regarding θ from the market price. The informational content of the market price in turn depends on the weight $\gamma_{1,1}$ informed firms give to their private information and, thus, the weight $\gamma_{2,2}$ they give to the market price depends on the informed firms' actions.

Looking at the respective T-map, we then get that the matrix $T'(\gamma^*)$ takes a very simple form in this case: Since uniformed firms' actions cannot reveal any private information, informed traders will not respond to the decisions of uninformed firms. This implies that A_1 and B_{12} are identical to zero. Thus, two of the 4 eigenvalues of $T'(\gamma^*)$ are equal to zero while the other two are given by the eigenvalues of A_2. From (9b) we now get that one of the remaining two eigenvalues is also identical to zero and while the last one is given by the derivative $\partial\gamma'_{2,2}/\partial\gamma_{2,2}$:

$$\lambda = -\frac{\alpha^2 \tau_\varepsilon \gamma^*_{1,1} (1 - m_1) m_1 \tau_{u,1}}{\tau \tau_{u,1} + \alpha^2 \tau_\varepsilon \gamma^*_{1,1} m_1^2 (\tau + \tau_{u,1})}$$

Thus, because $\gamma^*_{1,1} > 0$ all eigenvalues of $T'(\gamma^*)$ are always negative and so are the eigenvalues of $J(\gamma^*)$, leading to the conclusion that the rational expectations equilibrium of the Grossman and Stiglitz [1980] model is always locally stable under adaptive learning. This reproduces the respective stability result already derived by Bray [1982] and Marcet and Sargent [1992] according to which recursive least squares learning in the Grossman and Stiglitz model converges (locally) to the rational expectations equilibrium of this model:[3]

[2] Because $\tau_{u,2} = 0$ the T–map for the uninformed firms cannot be derived in the above described way, since the moment matrix $M_{zz,i}(\gamma)$ is not well defined in this case. For the derivation of equations (9a) and (9b) see appendix A.2.

[3] Both papers also present instability results, according to which the rational expectations equilibrium of the Grossman and Stiglitz model might be unstable under learning. The potential instability results from allowing for a correlation between the asset return and the random asset supply. With

3.3.2 Homogeneous firms

The above analysis has shown that the model with two types of firms where firms of one type have no private information at all is in fact a special case as there is no feedback from the learning process of uninformed firms to the actions of informed firms. This poses the question, whether the above described stability result carries over to a more general case of many partially informed firm types where firms of each type try to learn to extract others' information from prices and where such feedback effects are present.

In order to answer this question, we now look at another special case of the model, where firms are homogeneous with respect to the precision of their private information but observe — dependent on their type — different private signals. We assume that the precision of private information is identical for all firms, i.e. $\tau_{u,i} = \tau_u$, and furthermore that $m_i = 1/n$ for all $i = 1, \ldots, n$.

Due to symmetry, the matrix $T'(\gamma^*)$ again takes a very simple form in this case. For all $i = 1, \ldots, n$, we have $A_i = A$ and $B_{ij} = B$ for all $j \neq i$. In order to investigate the properties of the model under adaptive learning, it is thus sufficient to look at the T–map $T_i(\gamma)$ for a firm of a representative type i. Some tedious algebra (see Appendix A.3 for details) shows that the map $T_i(\gamma)$ is given as follows:

$$\gamma'_{1,i} = \frac{n^2\,\tau_u + \alpha^2\,\tau_\varepsilon\,\tau_u\left(\sum_{j\neq i}^n \gamma_{1,j}^2 - \gamma_{1,i}\sum_{j\neq i}^n \gamma_{1,j}\right)}{n^2\,\tau_u[\tau+\tau_u] + \alpha^2\,\tau_\varepsilon\left[(\tau+\tau_u)\sum_{j\neq i}^n \gamma_{1,j}^2 + \tau_u\left(\sum_{j\neq i}^n \gamma_{1,j}\right)^2\right]} \tag{10a}$$

$$\gamma'_{2,i} = -\frac{\alpha\tau_\varepsilon\tau_u\sum_{j\neq i}^n \gamma_{1,j}\left(\alpha\sum_{j=1}^n \gamma_{2,j} - n\,(1+\alpha)\right)}{n^2\,\tau_u[\tau+\tau_u] + \alpha^2\,\tau_\varepsilon\left[(\tau+\tau_u)\sum_{j\neq i}^n \gamma_{1,j}^2 + \tau_u\left(\sum_{j\neq i}^n \gamma_{1,j}\right)^2\right]} \tag{10b}$$

Based on equations (10a) and (10b) the following steps lead to the final conclusion that the rational expectations equilibrium γ^* is always E–stable:

1) For all $i = 1, \ldots, n$, the matrix A of derivatives of $T_i(\gamma)$ with respect to γ_i is equal to $a(\gamma)$ times the 2×2 identity matrix, i.e. $A = a(\gamma)\,\mathrm{I}_2$, where from (10a) it follows that $a(\gamma)$ is given by:

$$a(\gamma) = \frac{-\alpha^2\,\tau_\varepsilon\,\tau_u\sum_{j\neq i}^n \gamma_{1,j}}{n^2\,\tau_u[\tau+\tau_u] + \alpha^2\,\tau_\varepsilon\left[(\tau+\tau_u)\sum_{j\neq i}^n \gamma_{1,j}^2 + \tau_u\left(\sum_{j\neq i}^n \gamma_{1,j}\right)^2\right]}$$

respect to the model considered here, this would mean to allow for a positive correlation between the unknown cost parameter θ and the noise term affecting market demand ε. Such a positive correlation is ruled out in the present analysis.

2) The matrix B of derivatives of $T_i(\gamma)$ with respect to γ_j for all $j \neq i$ is lower triangular since $\frac{\partial \gamma'_{1,i}}{\partial \gamma_{2,j}} = 0$ for all $j \neq j$. Thus, $B = \begin{pmatrix} b(\gamma)_{11} & 0 \\ b(\gamma)_{12} & b(\gamma)_{22} \end{pmatrix}$, where furthermore from (10b) it follows that $b(\gamma)_{22} = a(\gamma)$.

3) Now, step 1) and step 2) imply that the eigenvalues λ of $T'(\gamma)$ are given as follows:

$$\lambda = \Big(\underbrace{0, \ldots, 0}_{(n-1)\times}, \underbrace{a(\gamma) - b(\gamma)_{11}, \ldots, a(\gamma) - b(\gamma)_{11}}_{(n-1)\times}, n\,a(\gamma), a(\gamma) + (n-1)\,b(\gamma)_{11} \Big) \quad (11)$$

4) In a rational expectations equilibrium with $\tau_u > 0$ we have $a(\gamma^*) < 0$. Moreover, it can be shown (cf. Appendix A.5 for a proof) that $a(\gamma^*) - b(\gamma^*)_{11} < 1$ as well as $a(\gamma^*) + (n-1)b(\gamma^*)_{11} < 0$. So, all eigenvalues of $T'(\gamma^*)$ are smaller than 1 and consequently, all eigenvalues of $J(\gamma^*)$ are always negative.

Thus, as in the Grossman and Stiglitz [1980] model we again conclude that the rational expectations equilibrium γ^* is always locally stable under adaptive learning. A continuum of firms divided into n types that observe different signals regarding an unknown cost parameter θ is therefore able to learn (using e.g. least squares) how much information of others is revealed through prices and will thus form rational expectations in the limit.[4] The stability result derived by Bray [1982] and Marcet and Sargent [1992] for the special case of the model with two types therefore in fact carries over to the more general setting with an arbitrary number of partially informed firm types.

4 Endogenous acquisition of information

Up to now, the amount of private information was fixed exogenously, leaving open the question how this, perhaps costly information comes into the market in the first place. In order to answer this question, some additional assumptions regarding the individual decision to acquire costly private information are necessary.

First of all, since it is easier to analyze smooth decisions, we disregard the model of the Grossman and Stiglitz type here, because there each firm's decision to acquire information is dichotomous: They simply decide to acquire information with a given precision or no information at all.[5] Instead, it is assumed that each firm j is able to

[4] Since our stability concept is a local one, this is strictly speaking only true if the learning process is in addition inhibited to leave the neighborhood of the REE.

[5] See Grossman and Stiglitz [1980] for an analysis of the decision to acquire information in this case.

acquire a signal $s(j)$ with any desired precision $\tau(j)_u \geq 0$, where the cost of acquiring that signal depends on its precision according to a cost function $K(\tau(j)_u)$. Regarding this cost function it is assumed that $K'(\tau(j)_u) > 0$ and $K''(\tau(j)_u) \geq 0$, i.e. a more precise signal induces higher information costs. To be able to refer to the analysis of the preceding section it is also assumed that all firms of a given type behave perfectly identical such that every firm j of type i acquires a signal s_i with precision $\tau_{u,i}$. As before, however, precisions of acquired signals are allowed to differ across firms of different types.

Let us begin, again, with a static version of the model. Given linear decision rules $x_i = \psi(p - \gamma_{0,i} - \gamma_{1,i} s_i - \gamma_{2,i} p)$ for all firm types $i = 1, \ldots, n$, every firm has to decide ex ante on the profit maximizing amount of information. The optimal precision $\tau_{u,i}$, thus, balances the marginal costs of information acquisition for firm of type i with the ex ante expected marginal revenue $MR(\tau_{u,i})$. Computing ex ante expected profit of a type i firm, this ex ante expected marginal revenue turns out to be $MR(\tau_{u,i}) = \frac{\psi}{2}\frac{\gamma_{1,i}^2}{\tau_{u,i}^2}$ (cf. Appendix A.6 for details). Thus, the optimal precision acquired by a type i firm is the solution to the equation

$$\frac{\psi}{2}\frac{\gamma_{1,i}^2}{\tau_{u,i}^2} = K'(\tau_{u,i}) \tag{12}$$

It is neither guaranteed that the individually optimal amount of information acquisition is positive nor that the rational expectations equilibrium entails a positive amount of information acquisition of all firms.[6] Among other things, this depends on the nature of the marginal cost function at zero (i.e. $K'(0)$). For obvious reasons, the following analysis is restricted to equilibria with a strictly positive amount of information acquisition $\tau_u^* > 0$.

Given this optimal decision regarding information acquisition, we can now proceed with the analysis of learning. According to equation (12), each firm's acquired level of precision is a function $h(\gamma_{1,i})$ of the weight the firm will give to her private information. If — as we have assumed — marginal costs of information acquisition are nondecreasing, we have $h'(\gamma_{1,i}) > 0$, i.e. a firm that is going to put more weight to her private information will also acquire a more precise signal.

The just described endogeneity of the precisions acquired by firms of different types must be taken into account when we go on to analyze the T–map. Let $\bar{\tau}_u = (\tau_{u,1}, \ldots, \tau_{u,n})$ denote the vector of precisions acquired by firms of different types and rewrite the T-map of the model with exogenously given precisions as $\gamma' = T(\gamma, \bar{\tau}_u)$. The T–map of the model with endogenous acquisition of information is then given by:

[6] See Verrecchia [1982] for a discussion of this issue in the context of the original Grossman and Stiglitz model.

$$\gamma' = T\left(\gamma, h(\gamma_{1,1}), \ldots, h(\gamma_{1,n})\right) \tag{13}$$

As usual, E–stability requires that all $2n$ eigenvalues of the matrix $J(\gamma) = dT/d\gamma - \mathrm{I}_{2n}$ evaluated at the REE γ^* are negative. Since firms are homogeneous with respect to costs of information acquisition, the rational expectations equilibrium is symmetric. Thus, as before in the case with exogenously given private information, it is sufficient to look at the T–map for a firm of a representative type i. The differentiation of this T–map with respect to γ_i and γ_j for $j \neq i$ results in the following two matrices:

$$\frac{\partial T_i\left(\gamma, h(\gamma_{1,1}, \ldots, h(\gamma_{1,n}))\right)}{\partial \gamma_i} = A + h'(\gamma_{1,i})\hat{A} \tag{14a}$$

$$\frac{\partial T_i\left(\gamma, h(\gamma_{1,1}), \ldots, h(\gamma_{1,n})\right)}{\partial \gamma_j} = B + h'(\gamma_{1,j})\hat{B} \tag{14b}$$

In equations (14a) and (14b) the matrices A and B are the same as in case of exogenously given private information and $\hat{A}$ and $\hat{B}$ are matrices that capture the now appearing additional effect of endogenous information. Since $\hat{A}$ and $\hat{B}$ are matrices whose second columns are made up of zeros, the eigenvalues of $T'(\gamma)$ can be expressed as follows: Let $\lambda_1 = a(\gamma^*) - b(\gamma^*)_{11}$, $\lambda_2 = n a(\gamma^*)$ and $\lambda_3 = a(\gamma^*) + (n-1)b(\gamma^*)_{11}$ denote the three eigenvalues from the model with exogenously given information which are in general different from zero (cf. equation (11)). Furthermore, let $\hat{a}^*_{11}$ and $\hat{b}^*_{11}$ denote the elements in the first row and first column of $\hat{A}$ and $\hat{B}$, respectively, evaluated at the rational expectations equilibrium γ^* and let finally h'^* denote the derivative of $h(\gamma_1)$ evaluated at the rational expectations equilibrium. The eigenvalues $\tilde{\lambda}$ of $T'(\gamma^*, \overline{\tau}^*_u)$ are:

$$\tilde{\lambda} = \Bigg(\underbrace{0, \ldots, 0}_{(n-1)\times}, \underbrace{\lambda_1 + h'^*(\hat{a}^*_{11} - \hat{b}^*_{11}), \ldots, \lambda_1 + h'^*(\hat{a}^*_{11} - \hat{b}^*_{11})}_{(n-1)\times}, \lambda_2, \lambda_3 + h'^*\left(\hat{a}^*_{11} + (n-1)\hat{b}^*_{11}\right) \Bigg) \tag{15}$$

It is by no means obvious that the REE with endogenously acquired informed is always stable under learning. In fact, while a formal proof that all eigenvalues are smaller than 1 is possible, it is rather cumbersome. For this reason, we here present only a proof for the special case of a large number of possible firm types, i.e. for the case $n \to \infty$. In the following subsection we then present a numerical example which at least provides some evidence that stability under learning also obtains in case of a finite number of firm types.

In appendix A.7 it is shown that for $n \to \infty$ the three nonzero eigenvalues λ_1, λ_2 and λ_3 of the model with exogenous information are given by

$$\lambda_1 = 0\,, \quad \lambda_2 = -\frac{\alpha^2(\gamma_1^*)^2\tau_\varepsilon}{\tau_u}\,, \quad \lambda_3 = -2\,\frac{\alpha^2(\gamma_1^*)^2\tau_\varepsilon}{\tau+\tau_u+\alpha^2(\gamma_1^*)^2\tau_\varepsilon}\,,$$

while the eigenvalues of the map $T'(\gamma^*, \overline{\tau}_u^*)$ are given as follows:

$$\tilde{\lambda}_1 = \frac{2}{2+\kappa}\,\frac{\tau+\alpha^2(\gamma_1^*)^2\tau_\varepsilon}{\tau+\tau_u^*+\alpha^2(\gamma_1^*)^2\tau_\varepsilon}\,, \quad \tilde{\lambda}_2 = \lambda_2\,, \quad \tilde{\lambda}_3 = \lambda_3 + \tilde{\lambda}_1$$

From the above analysis of the model with exogenous information we already know that $\lambda_2 < 0$ and $\lambda_3 < 0$. Thus, all eigenvalues are less than one if $\tilde{\lambda}_1 < 1$. This, however, is always the case since $\frac{2}{2+\kappa} \leq 1$ and $\frac{\tau+\alpha^2(\gamma_1^*)^2\tau_\varepsilon}{\tau+\tau_u^*+\alpha^2(\gamma_1^*)^2\tau_\varepsilon} < 1$. Hence, the rational expectations equilibrium is always E-stable if $n \to \infty$. Notice that because the eigenvalues are continuous functions of the number n of different firm types, this implies that stability under learning also results whenever there is a large enough number of different firm types.

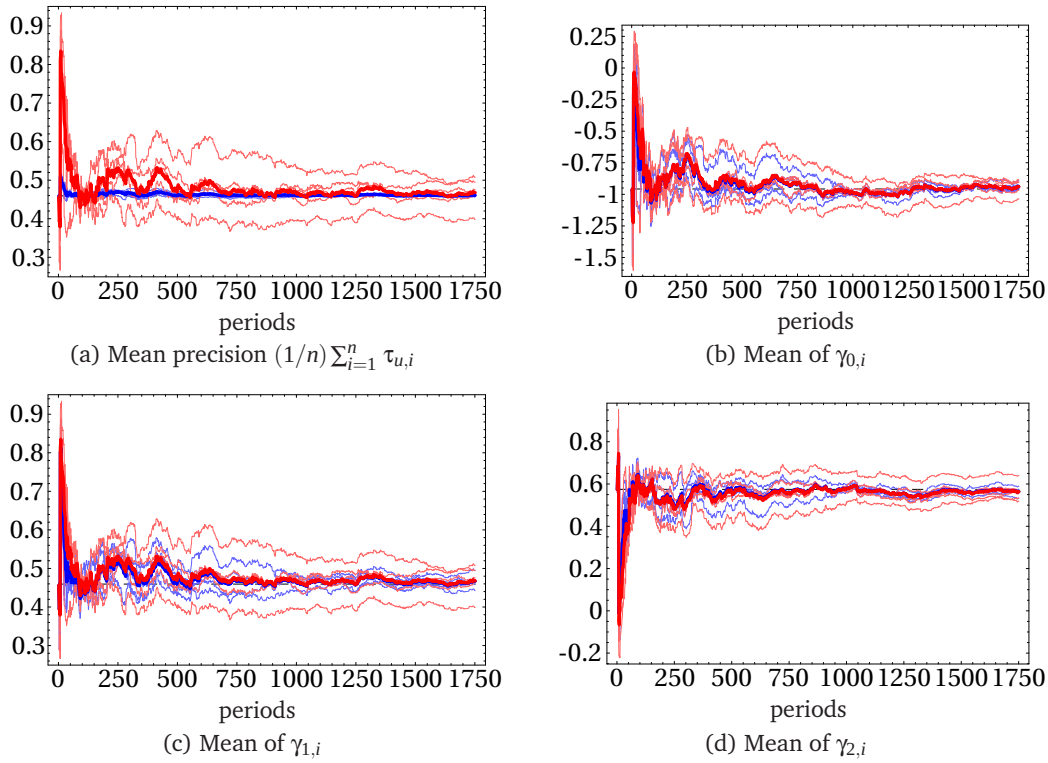
Thus, the rational expectations equilibrium of this generalized version of the Grossman and Stiglitz model where firms learn to extract others' information from market prices is not only always stable if private information is exogenous. It is moreover also always stable under learning if information is endogenously acquired and if marginal costs of information acquisition are non decreasing.

An illustrative example

In this subsection we present a numerical example of the above analyzed model with a finite number of firm types in order to illustrate some properties of the adaptive learning process. By the way the respective simulation results provide at least some weak evidence that stability under learning obtains irrespectively of the number of firm types.

In this numerical example it is assumed that the costs of information acquisition are given by the function $K(\tau_u) = \delta\tau_u^{\kappa+1}$, with $\kappa > 0$ and $\delta > 0$ such that the elasticity of marginal costs of information acquisition with respect to τ_u is given by $\kappa > 0$.

Two specifications of the model which differ with respect to κ are considered: The first one assumes constant marginal costs of information acquisition, i.e. $\kappa = 0$, and $\delta = 1$ while the second one assumes $\kappa = 10$, i.e. a large elasticity of marginal costs of information acquisition. The parameter δ in the second specification is then chosen in such a way that given all other parameters of the model the resulting rational expectations equilibria in both specifications are identical. With respect to these remaining parameters, we assume $n = 5$, $\tau_\varepsilon = 0.1$, $\tau = 0.1$, $\beta = 10$, $\overline{\theta} = 1$, $\psi = 2$ and $\phi = 0.4$. The corresponding rational expectations equilibrium is then characterized by $\gamma_0^* = 0.9577$, $\gamma_1^* = 0.4594$, $\gamma_2^* = 0.5746$ and $\tau_u^* = 0.4594$.



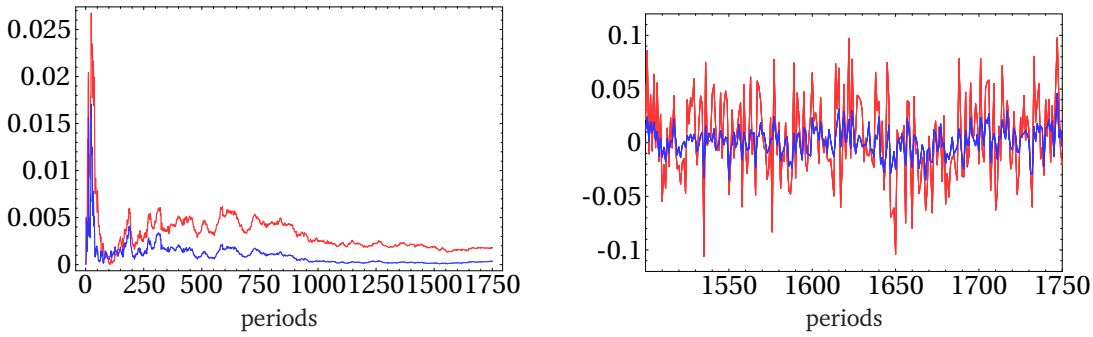
(a) Mean precision $(1/n)\sum_{i=1}^{n}\tau_{u,i}$ (b) Mean of $\gamma_{0,i}$

(c) Mean of $\gamma_{1,i}$ (d) Mean of $\gamma_{2,i}$

Figure 1: Simulations of a learning process with $\kappa = 0$ (red) and $\kappa = 10$ (blue).

The simulations of the learning process are performed as follows: The process starts with $\gamma_{i,0}$ for $i = 1,\ldots,n$ in a neighborhood of the rational expectations equilibrium γ^*. In a pre–learning period with a duration of 50 periods these initial parameter vectors $\gamma_{i,0}$ are used to generate a data set that is used to initialize the learning process. After that, firms learn from period to period, i.e. they estimate the parameters of their auxiliary model using recursive least squares and decide on the amount of information to be acquired based on these estimates.

The simulation results are depicted in figure 1. Each subfigure shows the time paths of the respective variables for the different firm types (thin lines) and averages across all firm types (thick lines). The dotted lines in each subfigure indicate the respective rational expectations equilibrium value. The main message of these figures is that learning indeed seems to converge toward the rational expectations equilibrium. Not surprisingly, the average of the acquired precision across firm types fluctuates more during the learning process if marginal costs of information acquisition are constant. Moreover, the variance of the estimated parameters across firm types is larger in this case. The simple reason for this is that lower marginal costs of information acquisition induce stronger reactions of firms to the time varying ex-



(a) Variance of $\gamma_{1,i}$ (b) Deviation of p_t from REE price

Figure 2: Variance of $\gamma_{1,i}$ and deviation of the market price from the REE price with $\kappa = 0$ (red) and $\kappa = 25$ (blue).

pected marginal revenue of information acquisition. It is interesting to see, however, that the average values of the estimated coefficients of the firms' auxiliary model are more or less the same for both simulations during the learning process. Thus, with respect to these average estimates it doesn't matter much for the properties of the learning process whether marginal costs of information acquisition are high or low.

Even though the means of the estimates across firm types differ not much in both simulations, it turns out that the dispersion of these estimates is larger in case of lower marginal costs of information acquisition. Thus, we should expect that this greater dispersion also results in greater fluctuations of the market price around its respectice rational expectations equilibrium value during the learning process. This is confirmed by the evidence presented in figure 2.(a) which shows the variance of the estimate of the parameter γ_1 across firm types for both specifications of the model (the respective figures for the other two parameters give a quite similar picture). As can be seen, this variance decreases quite slowly and it is always larger in the case where marginal costs of information acquisition are low. As a consequence of this greater dispersion of the firms' estimates the fluctuations of the market price are larger. This is shown in figure 2.(b) where for both specifications the deviations of the market price from its respective rational expectations equilibrium value are shown. As can be seen, even after a long period of learning price fluctuations are larger in case of lower marginal costs of information acquisition. Clearly, as the learning process converges toward the rational expectations equilibrium, these fluctuations will become smaller and smaller. However, even from this only illustrative example one gets the impression that the costs associated with the acquisition of private information are relevant for the transient properties of a learning process and that these costs will affect the duration of such a learning process.

5 Conclusion

The aim of the paper was to demonstrate how the E–stability principle can be applied to models with heterogeneous and private information in order to assess the stability of rational expectations equilibria under learning. As was shown, it is possible to derive the T–map that governs the properties of such learning processes in a quite straightforward way from economic models with private information. With regard to the linear model of the cobweb type considered in the first part of the paper, the analysis revealed that the presence of private information leads no further stability conditions beyond those known from the case where private information is absent.

The analysis of a model of the Grossman and Stiglitz [1980] type, where agents try to extract from the market price private information of others, has shown that it is not only possible to reproduce the stability results of Bray [1982] and Marcet and Sargent [1992] in a quite simple way. Moreover, the rational expectations equilibrium in this kind of model turns out to be stable under learning even in a more general setting with an arbitrary number of differentially informed firms. Furthermore, stability under learning is conserved if the amount of private information is endogenously determined by optimizing firms that decide on the amount of privately acquired information. Thus, the fact that agents decide on the amount of privately acquired information and are able to react to the information revealed by market prices, doesn't harm the stability properties of adaptive learning processes. In this sense, the kind of rational expectations equilibria considered in models of the Grossman and Stiglitz [1980] type appear to be quite robust.

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A Appendix

A.1 Asymptotic properties of the learning process

Here we show in a more detailed way that the T-map of the model described in Section 2 can in fact be used to analyze the asymptotic properties of an adaptive learning process based on a recursive least squares procedure.

Thus, assume that firms use recursive least squares to estimate the parameters γ_i of their auxiliary model $y_t^e = \gamma_{0,i} + \gamma_{1,i} s_{i,t}$. With $\gamma_i = (\gamma_{0,i}, \gamma_{1,i})'$ for all $i = 1, \ldots, n$, the learning algorithm of a representative agent i can then be written as:

$$\gamma_{i,t+1} = \gamma_{i,t} + \frac{1}{t} R_{i,t}^{-1} z_{i,t} \left[y_t - z_{i,t}' \gamma_{i,t} \right] \tag{16}$$

$$R_{i,t+1} = R_{i,t} + \frac{1}{t} \left(z_{i,t} z_{i,t}' - R_{i,t} \right) \tag{17}$$

The fundamental step is that the stochastic approximation tools described by Evans and Honkapohja [2001], can be used to show that the asymptotic dynamics of the learning algorithm are governed by an ODE, which is given as follows:

$$\dot{\gamma}_i = \mathrm{E}\left[R_i^{-1} z_i \left(y - z_i' \gamma_i \right) \right] = \left(\mathrm{E}[z_i z_i'] \right)^{-1} \mathrm{E}[z_i y] - \gamma_i \tag{18}$$

Now, since $z_{i,t}' = (1, s_{i,t})$, the moments that appear in equation (18) depend on $\gamma = (\gamma_1, \ldots, \gamma_n)'$ and coincide with the moments stated in equation (5). Thus, $\left(\mathrm{E}[z_i z_i'] \right)^{-1} \mathrm{E}[z_i y] = T_i(\gamma)$ and the ODE (18) for a single agent i becomes:

$$\dot{\gamma}_i = T_i(\gamma) - \gamma_i$$

Therefore, the whole dynamical system can be written as follows:

$$\begin{pmatrix} \dot{\gamma}_1 \\ \vdots \\ \dot{\gamma}_n \end{pmatrix} = \begin{pmatrix} T_1(\gamma) - \gamma_1 \\ \vdots \\ T_n(\gamma) - \gamma_n \end{pmatrix} \quad \Rightarrow \dot{\gamma} = T(\gamma) - \gamma$$

A.2 The T–map of the Grossman–Stiglitz model

The weight $\gamma'_{1,1}$ informed firms will give to their private information is simply given by the covariance between the signal s_i and the unknown cost parameter θ divided by the variance of the signal. Thus:

$$\gamma_{1,1} = \frac{\tau_{u,1}}{\tau + \tau_{u,1}}$$

The weight $\gamma'_{2,2}$ uninformed firms give to the market price is given by the covariance between the market price p and the unknown cost parameter θ divided by the variance of the market price. Since $\gamma_{2,1} = 0$ and $\gamma_{1,2} = 0$ it now follows from (8) that:

$$\begin{aligned}
\mathrm{Cov}(\theta\, p) &= \frac{\alpha\, m_1\, \gamma_{1,1} \frac{1}{\tau}}{1 + \alpha - \alpha(1 - m_1)\gamma_{2,2}} \\
\mathrm{Var}(p) &= \frac{\alpha^2\, m_1^2\, \gamma_{1,1}^2 \left(\frac{1}{\tau} + \frac{1}{\tau_{u,1}}\right) + \frac{1}{\tau_\varepsilon}}{[1 + \alpha - \alpha(1 - m_1)\, \gamma_{2,2}]^2}
\end{aligned}$$

It therefore follows:

$$\gamma'_{2,2} = \frac{\mathrm{Cov}(\theta\, p)}{\mathrm{Var}(p)} = \frac{\alpha\, m_1\, \gamma_{1,1} \tau_\varepsilon \tau_{u,1}\, (1 + \alpha - \alpha(1 - m_1)\gamma_{2,2})}{\tau\tau_{u,1} + \alpha^2\, m_1^2\, \gamma_{1,1}^2 \tau_\varepsilon (\tau + \tau_{u,1})}$$

A.3 The T–map with homogenous firms

With the market price p given by equation (8), the elements of $M_{zz,i}(\gamma)$ and $M_{z\theta,i}(\gamma)$ are given by:

$$\begin{aligned}
\mathrm{Cov}(s_i\, p) &= \frac{\alpha\, (1/n)\, \sum_{j=1}^n \gamma_{1,j}/\tau + \alpha(1/n)\, \gamma_{1,i}/\tau_{u,i}}{1 + \alpha\left(1 - (1/n) \sum_{j=1}^n \gamma_{2,j}\right)} \\
\mathrm{Cov}(\theta p) &= \frac{\alpha\, (1/n)\, \sum_{j=1}^n \gamma_{1,j}/\tau}{1 + \alpha\left(1 - (1/n) \sum_{j=1}^n \gamma_{2,j}\right)} \\
\mathrm{Var}(p) &= \frac{\alpha^2\, (1/n^2)\, \sum_{j=1}^n \gamma_{1,j}^2/\tau + \alpha^2\, (1/n)\, \sum_{j=1}^n \gamma_{1,j}^2/\tau_{u,j} + \tau_\varepsilon}{\left(1 + \alpha\left(1 - (1/n) \sum_{j=1}^n \gamma_{2,j}\right)\right)^2}
\end{aligned}$$

With respect to $\gamma'_{1,i}$ and $\gamma'_{2,i}$ we therefore get:

$$\begin{aligned}
\gamma'_{1,i} &= \frac{\mathrm{Var}(p)/\tau + \mathrm{Cov}(s_i p)\, \mathrm{Cov}(\theta p)}{\mathrm{Var}(p)\, \mathrm{Var}(s_i) - \mathrm{Cov}(s_i p)^2} \\
\gamma'_{2,i} &= \frac{\mathrm{Cov}(s_i p)/\tau + (\tau + \tau_{u,i})\, \mathrm{Cov}(\theta p)/(\tau\tau_{u,i})}{\mathrm{Var}(p)\, \mathrm{Var}(s_i) - \mathrm{Cov}(s_i p)^2}
\end{aligned}$$

Substitution of the above stated expressions then results in equations (10a) and (10b).

A.4 The dynamics of the constant γ_0

In order to simplify the exposition, it is assumed that $\bar{\theta} = 0$. Using the equations (10a) and (10b), the respective equation $\gamma'_{0,i} = \bar{\theta} - T_i(\gamma)\,\bar{z}$ for the parameter $\gamma'_{0,i}$ results as:

$$\gamma'_{0,i} = -\frac{\alpha\tau_u\tau_\varepsilon\left(n\beta + \alpha\sum_{j=1}^n \gamma_{0,j}\right)\sum_{j\neq i}^n \gamma_{0,j}}{n^2\tau_u[\tau+\tau_u] + \alpha^2\tau_\varepsilon\left[(\tau+\tau_u)\sum_{j\neq i}^n \gamma_{1,j}^2 + \tau_u\left(\sum_{j\neq i}\gamma_{1,j}\right)^2\right]}$$

Evaluated at the REE γ^*, the derivative of $\gamma'_{0,i}$ with respect to $\gamma_{0,i}$ equals $a(\gamma^*)$, while the derivatives with respect to $\gamma_{1,i}$ and $\gamma_{2,i}$ are zero. Moreover, the derivatives of $\gamma'_{0,i}$ with respect to $\gamma_{0,j}$ also equal $a(\gamma^*)$ while the derivatives with respect to $\gamma_{2,j}$ equal zero. Taking into account the special structure of the respective Jacobian, it then follows that the eigenvalues of the overall T–map including the constant $\gamma_{0,i}$ for all $i = 1,\ldots,n$ are the same as the eigenvalues of $T(\gamma^*)$.

A.5 Eigenvalues of the T–map at the REE

In a REE with identical precisions τ_u for all firm types $i = 1,\ldots,n$ we have $\gamma_{1,i} = \gamma_1^*$ for all $i = 1,\ldots,n$, where from (10a) it follows that:

$$\gamma_1^* = \frac{n^2\tau_u^2}{n^2\tau_u[\tau+\tau_u] + (n-1)\alpha^2(\gamma_1^*)^2\tau_\varepsilon(\tau+n\tau_u)} \tag{19}$$

Thus, $a(\gamma^*)$ and $b(\gamma^*)_{11}$ are given by:

$$a(\gamma^*) = \frac{-\alpha^2\tau_\varepsilon\tau_u(n-1)\gamma_1^*}{n^2\tau_u[\tau+\tau_u] + (n-1)\alpha^2(\gamma_1^*)^2\tau_\varepsilon(\tau+n\tau_u)} \tag{20}$$

$$b(\gamma^*)_{11} = \frac{\alpha^2\gamma_1^*\tau_\varepsilon\tau_u\left((n-1)\alpha^2(\gamma_1^*)^2\tau_\varepsilon(\tau+n\tau_u) - n^2\tau_u(\tau+(2n-1)\tau_u)\right)}{\left[n^2\tau_u[\tau+\tau_u] + (n-1)\alpha^2(\gamma_1^*)^2\tau_\varepsilon(\tau+n\tau_u)\right]^2} \tag{21}$$

Since $a(\gamma^*)$ is always negative, the eigenvalue equal to $n\,a(\gamma^*)$ is also always negative.

Thus, we proceed with the remaining eigenvalues equal to $a(\gamma^*) - b(\gamma^*)_{11}$ and $a(\gamma^*) + (n-1)\,b(\gamma^*)_{11}$. First, using (19), $b(\gamma^*)_{11}$ simplifies to:

$$b(\gamma^*)_{11} = -a(\gamma^*)\frac{1}{n-1}\left(1 - 2\gamma_1^*\frac{\tau+n\tau_u}{\tau_u}\right) \tag{22}$$

We next show that $b(\gamma^*)_{11}$ is always negative such that the eigenvalue $a(\gamma^*) + (n-1)\,b(\gamma^*)_{11}$ is negative too. The proof is a little bit awkward since it requires to assess the REE value γ_1^* of the coefficient γ_1. From (19) it is quite obvious that $\gamma_1^* < \frac{\tau_u}{\tau+\tau_u}$. A lower bound for γ_1^* can be constructed as follows: Rewrite (19) as:

$$f(\gamma_1) \equiv \gamma_1\,n^2\tau_u(\tau+\tau_u) + (n-1)\alpha^2\gamma_1^2\tau_\varepsilon(\tau+n\tau_u) = n^2\tau_u^2$$

The function $f(\gamma_1)$ increases monotonically with $f'' > 0$. Therefore the root $\tilde{\gamma}_1$ of the equation $\gamma_1 f'(\gamma_1^*) = n^2\tau_u^2$ is positive and smaller than the unique positive root γ_1^* of (19), i.e.:

$$0 < \tilde{\gamma}_1 = \frac{n^2\tau_u}{n^2\tau_u(\tau+\tau_u) + 3(n-1)\alpha^2(\gamma_1^*)^2\tau_\varepsilon(\tau+n\tau_u)} < \gamma_1^*$$

Using (19) this is equivalent to:

$$\tilde{\gamma}_1 = \frac{\tau_u}{3\frac{\tau_u}{\gamma_1^*} - 2(\tau+\tau_u)} < \gamma_1^* \tag{23}$$

Now, $b(\gamma^*)_{11} < 0$ requires $1 - 2\gamma_1^*\frac{\tau+n\tau_u}{\tau_u} < 0$ and since $\tilde{\gamma}_1 < \gamma_1^*$ this is always the case if

$$\begin{aligned} 1 - 2\tilde{\gamma}_1\frac{\tau+n\tau_u}{\tau_u} < 0 \qquad &\Leftrightarrow \quad 1 - 2\frac{\tau_u}{3\frac{\tau_u}{\gamma_1^*} - 2(\tau+\tau_u)}\frac{\tau+n\tau_u}{\tau_u} < 0 \\ &\Leftrightarrow \quad \frac{3\frac{\tau_u}{\gamma_1^*} - 4\tau - 2(n+1)\tau_u}{3\frac{\tau_u}{\gamma_1^*} - 2(\tau+\tau_u)} < 0 \end{aligned} \tag{24}$$

The last inequality is always satisfied because $\gamma_1^* < \frac{\tau_u}{\tau+\tau_u}$. Thus, we have $b(\gamma^*)_{11} < 0$ and the eigenvalue equal to $a(\gamma^*) + (n-1)b(\gamma^*)_{11}$ is necessarily negative.

Notice that $b(\gamma^*)_{11} < 0$ doesn't rule out that the remaining $n-1$ eigenvalues that equal $a(\gamma^*) - b(\gamma^*)_{11}$ are positive. However, E–stability obtains, whenever $a(\gamma^*) - b(\gamma^*)_{11} < 1$, which is always the case: Using (20) and (22), $a(\gamma^*) - b(\gamma^*)_{11} < 1$ is equivalent to

$$a(\gamma^*)\left(\frac{n}{n-1} - 2\gamma_1^*\frac{\tau+n\tau_u}{\tau_u}\right) < 1$$

Using again (20), this inequality can be rewritten as:

$$\frac{n}{n-1} + \gamma_1^*\frac{\tau+n\tau_u}{\tau_u}\left(\frac{n-3}{n-1}\right) > -\frac{n^2\tau_u(\tau+\tau_u)}{(n-1)\alpha^2\gamma_1^*\tau_\varepsilon\tau_u}$$

The right hand side of this inequality is negative. The left hand side is always positive, which is obvious if $n \geq 3$ but also holds in case $n = 2$, since $\gamma^* < \frac{\tau_u}{\tau+\tau_u}$.

Thus, all eigenvalues of $T'(\gamma^*)$ are always either negative or smaller than one such that the REE is always E–stable.

A.6 Optimal information acquisition

To compute the optimal precision, consider the expected profit of a representative type i firm:

$$E[\pi_i] = E\left[[p-\theta]x_i - \frac{1}{2}\frac{1}{\psi}x_i^2\right] - K(\tau_{u,i}).$$

The partial derivative with respect to $\tau_{u,i}$ is then:

$$\frac{\partial E[\pi_i]}{\partial \tau_{u,i}} = \frac{\partial}{\partial \tau_{u,i}} E\left[[p-\theta]x_i - \frac{1}{2}\frac{1}{\psi}x_i^2\right] - K'(\tau_{u,i}),$$

where $x_i = \psi(1-\gamma_{2,i})p - \gamma_{0,0} - \gamma_{1,i}s_i$. Notice, that we consider here decisions of a single firm which is of measure zero. Thus, the decision of a single firm will not alter the variance of the market price or the covariance between the market price and a single firms signal. Therefore, $E((p-\theta)x_i))$ does not depend on $\tau_{u,i}$ and some computations show that:

$$\begin{aligned}\frac{\partial E[\pi_i]}{\partial \tau_{u,i}} &= -\frac{\psi}{2}\gamma(i)_1^2\frac{\partial E\left[s_i^2\right]}{\partial \tau_{u,i}} - K'(\tau_{u,i}),\\ &= \frac{\psi}{2}\left(\frac{\gamma_{1,i}}{\tau_{u,i}}\right)^2 - K'(\tau_{u,i}).\end{aligned}$$

A.7 Stability under learning with endogenous information and $n \to \infty$

The stability analysis simplifies considerably, if we look at the special case $n \to \infty$. From (20) and (22) we get that $\lim_{n\to\infty} a(\gamma^*) = 0$, $\lim_{n\to\infty} b(\gamma^*)_{11} = 0$ and:

$$\lim_{n\to\infty} n\,a(\gamma^*) = -\frac{\alpha^2(\gamma_1^*)^2\tau_\varepsilon}{\tau_u^*} \tag{25}$$

$$\lim_{n\to\infty} (n-1)\,b(\gamma^*) = -2\frac{\alpha^2(\gamma_1^*)^2\tau_\varepsilon}{\tau+\tau_u^*+\alpha^2(\gamma_1^*)^2\tau_\varepsilon} \tag{26}$$

Thus, in the model with exogenous information we have $\lambda_1 = 0$ while λ_2 and λ_3 are given by equations (25) and (26), respectively.

The relevant elements $\hat{a}_{11}^*$ and $\hat{b}_{11}^*$ from the matrices $\hat{A}$ and $\hat{B}$ can be computed from (10a) and (10b) as follows:

$$\hat{a}_{11}^* = \frac{((n-1)\alpha^2(\gamma_1^*)^2\tau_\varepsilon + n^2\tau_u)(n^2\tau_u\tau + (n-1)\alpha^2(\gamma_1^*)^2\tau_\varepsilon(\tau+n\tau_u))}{\left[n^2\tau_u(\tau+\tau_u) + (n-1)\alpha^2(\gamma_1^*)^2\tau_\varepsilon(\tau+n\tau_u)\right]^2} \tag{27}$$

$$\hat{b}_{11}^* = -\frac{(n-1)n\alpha^3(\gamma_1^*)^3\tau_\varepsilon(\alpha(\gamma_2^*-1)-1)(\tau+\tau_u)}{\left[n^2\tau_u(\tau+\tau_u) + (n-1)\alpha^2(\gamma_1^*)^2\tau_\varepsilon(\tau+n\tau_u)\right]^2} \tag{28}$$

Equations (27) and (28) imply that $\lim_{n\to\infty}\hat{b}_{11}^* = 0$ as well as $\lim_{n\to\infty}(n-1)\hat{b}_{11}^* = 0$, while

$$\lim_{n\to\infty}\hat{a}_{11}^* = \frac{\tau+\alpha^2(\gamma_1^*)^2\tau_\varepsilon}{\left(\tau+\tau_u+\alpha^2(\gamma_1^*)^2\tau_\varepsilon\right)^2} \tag{29}$$

Taking into account the vector of the eigenvalues of $T'(\gamma^*)$ as stated in (15), we can therefore conclude that the nonzero eigenvalues are given by:

$$\tilde{\lambda}_1 = h'^{*}\left(\lim_{n\to\infty}\hat{a}^{*}_{11}\right), \quad \tilde{\lambda}_2 = \lambda_2, \quad \tilde{\lambda}_3 = \lambda_3 + \tilde{\lambda}_1 \tag{30}$$

From the first order condition (12) for optimal information acquisition we get by differentiation:

$$\left(\frac{\psi}{2}2\gamma_{1,i}\right)d\gamma_{1,i} = \left(\tau^2_{u,i}K''(\tau_{u,i}) + 2\,\tau_{u,i}K'(\tau_{u,i})\right)d\tau_{u,i} \tag{31}$$

We have to look at the REE with an identical optimal precision τ^*_u for firms of all types and identical weights γ^*_1. With $\kappa = \frac{K''(\tau^*_u)\tau^*_u}{K'(\tau^*_u)} \geq 0$ denoting the elasticity of marginal costs of information acquisition with respect to the precision τ_u evaluated at the REE, (31) becomes:

$$\frac{d\tau_u}{d\gamma_1}_{|\gamma^*_1} \equiv h'^{*} = \frac{2}{2+\kappa}\frac{\tau^*_u}{\gamma^*_1} \tag{32}$$

Thus, from (29) and (32) the eigenvalue $\tilde{\lambda}_1 = h'^{*}(\lim_{n\to\infty}\hat{a}^{*}_{11})$ results as:

$$\tilde{\lambda}_1 = \frac{2}{2+\kappa}\frac{\tau^*_u}{\gamma^*_1}\frac{\tau+\alpha^2(\gamma^*_1)^2\tau_\varepsilon}{\left(\tau+\tau^*_u+\alpha^2(\gamma^*_1)^2\tau_\varepsilon\right)^2} \tag{33}$$

Using (19) with $n\to\infty$ this finally becomes:

$$\tilde{\lambda}_1 = \frac{2}{2+\kappa}\frac{\tau+\alpha^2(\gamma^*_1)^2\tau_\varepsilon}{\tau+\tau^*_u+\alpha^2(\gamma^*_1)^2\tau_\varepsilon} \tag{34}$$

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