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Working Paper

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Working Paper Series in Economics, No. 50

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Suggested Citation: Baumgärtner, Stefan; Jöst, Frank; Winkler, Ralph (2007) : Optimal dynamic scale and structure of a multi-pollution economy, Working Paper Series in Economics, No. 50, Leuphana Universität Lüneburg, Institut für Volkswirtschaftslehre, Lüneburg

This Version is available at:
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University of Lüneburg
Working Paper Series in Economics

No. 50

May 2007

www.uni-lueneburg.de/vwl/papers

ISSN 1860 - 5508

Optimal dynamic scale and structure of a multi-pollution economy

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Abstract: We analyze the optimal dynamic scale and structure of a two-sector-economy, where each sector produces one consumption good and one specific pollutant. Both pollutants accumulate at different rates to stocks which damage the natural environment. This acts as a dynamic driving force for the economy. Our analysis shows that along the optimal time-path (i) the overall scale of economic activity may be less than maximal; (ii) the time scale of economic dynamics (change of scale and structure) is mainly determined by the lifetime of pollutants, their harmfulness and the discount rate; and (iii) the optimal control of economic scale and structure may be non-monotonic. These results raise important questions about the optimal design of environmental policies.

Keywords: dynamic economy-environment interaction, multi-pollutant emissions, non-monotonic control, optimal scale, stock pollution, structural change, time scale

JEL-classification: Q20, O10, O41

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*We are grateful to Ulf Moslener, John Proops, Martin Quaas, Till Requate; seminar participants at Ascona (SURED 2004), Heidelberg and Budapest (EAERE 2004). We thank the Simulation and Optimization Group of the Interdisciplinary Center for Scientific Computing (IWR), University of Heidelberg, for a free license of the MUSCOD-II software package.

1 Introduction

The natural environment is being damaged by the stocks of various pollutants, which are produced in different sectors of the economy, accumulate according to different dynamic relationships, and damage different environmental goods. As an example, think of the two economic sectors 'agriculture' and 'industry'. Nitrate and pesticide run-off from agricultural cultivation accumulates in groundwater and decreases its quality as drinking water (UNEP 2002); carbon dioxide emissions from fossil fuel combustion in the industrial sector accumulate in the atmosphere and contribute to global climate change (IPCC 2001). In general, the different pollutants differ in their internal dynamics, i.e. natural degradation processes, and in their harmfulness. This has implications for the optimal dynamics of both the scale and structure of the economy. By *scale* we mean the overall level of economic activity, measured by total factor input; by *structure* we mean the composition of economic activity, measured by relative factor inputs to different sectors.

In this paper, we look into these coupled environmental-economic dynamics from a macroeconomic point of view. In particular, we are interested in the following questions: How should the macroeconomic scale and structure change over time in response to the dynamics of environmental pollution? Is this dynamic process monotonic over time, or can a trade-off between long-run and short-run considerations (e.g. lifetime versus harmfulness of pollutants) induce a non-monotonic economic dynamics? What is the time scale of economic dynamics (i.e. change of scale and structure), and how is it influenced by the different time scales and constraints of the economic and environmental systems? These questions are relevant for the current policy discussion on the sustainable biophysical scale of the aggregate economy relative to the surrounding natural environment (e.g. Arrow et al. 1995, Daly 1992, 1996, 1999), and how economic policy should promote structural economic change as a response to changing environmental pressures (e.g. de

Bruyn 1997, Winkler 2005).

We address these questions based on a model which comprises two economic sectors, each of which produces one distinct consumption good and, at the same time, gives rise to one specific pollutant. Both pollutants accumulate to stocks which display different internal dynamics, in the sense that the respective natural deterioration rates differ, and cause welfare decreasing environmental damage independently of each other. Of course, this relatively simple model cannot offer detailed policy prescriptions. However, it is detailed enough to clarify the underlying theoretical issues. In fact, we perform a *total* analysis of economy-environment interactions in a twofold manner. First, we analyze a multi-sector economy, which is fully specified in terms of resource endowment, technology, preferences and environmental quality. Second, we consider a 'disaggregate' natural environment. This goes beyond many contributions to environmental economics, where either only one (aggregate) pollutant is considered or different pollutants give rise to the same environmental problem.

Many studies in the extant literature assume that it is the *flow* of emissions which causes environmental problems. This neglects stock accumulation and, thus, an essential dynamic environmental constraint on economic action. *Stock pollution* has been taken into account by some authors (e.g. Falk and Mendelsohn 1993, Forster 1973, Luptacik and Schubert 1982, Van der Ploeg and Withagen 1991). This is usually done at a highly aggregated level, such that only one pollutant is taken into account. The case of *several stock pollutants* which all contribute to the same environmental problem (climate change) has been studied by Michaelis (1992, 1999). He is interested in finding cost-effective climate policy measures in the multi-pollution case for a given structure of the economy and does not explicitly consider the dynamics of the production side of the economy. Aaheim (1999) goes beyond Michaelis in that he analyzes numerically the dynamics of a two-sector economy which gives rise to three different stock pollutants and which

is constrained by an exogenously given policy target concerning the aggregate level of pollution. Moslener and Requate (2001) challenge the global warming potential as a useful indicator when there are many interacting greenhouse gases with different dynamic characteristics. Faber and Proops (1998: chap. 11) and Keeler et al. (1972) explicitly study the dynamics of different production sectors with pollution, assuming one single pollutant. Winkler (2005) analyzes optimal structural change of a two-sector economy characterized by two stock quantities: the capital stock and the stock of a pollutant which is emitted from the more capital-intense sector. Baumgärtner and Jöst (2000) study the optimal (static) structure of a vertically integrated two-sector economy where both sectors produce a specific by-product. The first sector's by-product can be used as a secondary resource in the second sector.

In this paper, we determine the optimal dynamic scale and structure of a multi-pollution economy within an optimal control framework. We use a linear approximation around the steady-state to obtain analytical results, and a numerical optimization of the non-approximated system to check for their robustness. The methodological innovation of our analysis is that we derive a closed form solution to the intertemporal optimization problem, which includes explicit expressions for the time scale of economic dynamics and the point in time where a non-monotonicity may occur. Our analysis shows that along the optimal time-path (i) the overall scale of economic activity may be less than maximal; (ii) the time scale of economic dynamics is mainly determined by the lifetime of pollutants, their harmfulness and the discount rate; and (iii) the control of economic scale and structure may be non-monotonic.

Although our modeling approach is inspired by Ramsey-type optimal growth models, which have previously been used to study steady state growth with environmental pollution (e.g. Gradus and Smulders 1993, 1996, Jöst et al. 2004, Keeler et al. 1972, Plourde 1972, Siebert 2004, Smith 1977, Van der Ploeg and Withagen

1991), we are essentially concerned with the issue of dynamic change in both scale and structure of economic activity. Therefore, in this paper we do not restrict the analysis to steady states but focus on the explicit time-dependence of the solution. Furthermore, we study an economy without any potential for steady state growth, as this highlights the structural-change-effect, which may be obscured by growth effects otherwise. The sole genuine generator of dynamics in our model is the accumulation of pollutant stocks in the natural environment.

The paper is organized as follows. In Section 2 we present the model. Section 3 is devoted to a formal analysis of the optimal dynamic scale and structure of the economy, based on a linear approximation around the stationary state. Section 4 confirms the analytical results thus obtained by a numerical optimization of the non-approximated system. Section 5 concludes.

2 The model

We study a two sector economy with one scarce non-accumulating factor of production, say labor, two consumption goods, and two pollutants that accumulate to stocks. Welfare is determined by the amounts consumed of both consumption goods, as well as by the environmental damage caused by the two pollutant stocks.

The production of consumption goods in sectors 1 and 2 of the economy is described by two production functions, $y_i = P^i(l_i)$ for $i = 1, 2$, where l_i denotes the amount of labor allocated in sector i. With index l denoting derivatives with respect to the sole argument l_i, $P^i_l \equiv dP^i/dl_i$ and $P^i_{ll} \equiv d^2P^i/dl_i^2$, the production functions are assumed to exhibit the following standard properties:

$$P^i(0) = 0 \ , \quad P^i_l > 0 \ , \quad \lim_{l_i \to 0} P^i_l = +\infty \ , \quad P^i_{ll} < 0 \qquad (i = 1, 2) \ . \tag{1}$$

Since we want to analyze an economy without potential for steady state growth, we assume a fixed supply of labor, $\lambda > 0$. Consumption possibilities are described

by

$$y_i = P^i(l_i) \qquad (i = 1, 2) \ , \tag{2}$$

$$l_1 + l_2 \leq \lambda \ . \tag{3}$$

In addition to the consumption good, each sector yields a pollutant which comes as a joint output in a fixed proportion to the desired output. Without loss of generality,

$$e_i = y_i \qquad (i = 1, 2) \ . \tag{4}$$

Both flows of pollutants, e_1 and e_2, add to the respective stock of the pollutant, which deteriorates at the constant rate δ_i:[1]

$$\dot{s}_i = e_i - \delta_i s_i \qquad \text{with } \delta_i > 0 \qquad (i = 1, 2) \ . \tag{5}$$

Instantaneous social welfare V depends on consumption of both goods, y_1 and y_2, and on the damage to environmental quality which hinges upon the stocks of pollutants s_1 and s_2. We consider the following welfare function:

$$V(y_1, y_2, s_1, s_2) = U(y_1, y_2) - \left[\frac{\sigma_1}{2} s_1^2 + \frac{\sigma_2}{2} s_2^2\right] \qquad \text{with } \sigma_1, \sigma_2 > 0 \ , \tag{6}$$

where σ_i indicates the harmfulness of pollutant i $(i = 1, 2)$ and U represents welfare gains due to consumption. The function U is assumed to exhibit the usual property of positive and decreasing marginal welfare in both consumption goods. In order to have an additively separable welfare function in all four arguments (y_1, y_2, s_1, s_2), we assume that neither consumption good influences marginal welfare of the other. With index i denoting the partial derivative with respect to argument y_i, i.e. $U_i \equiv \partial U / \partial y_i$ and $U_{ij} \equiv \partial^2 U / \partial y_i \partial y_j$ with $i, j = 1, 2$, the assumptions are:

$$U_i > 0 \ , \quad \lim_{y_i \to 0} U_i = +\infty \ , \quad U_{ii} < 0 \ , \quad U_{ij} = 0 \qquad (i, j = 1, 2 \text{ and } i \neq j) \ . \tag{7}$$

[1] In general, the decay rate may depend on emissions and the stock: $\delta_i = \delta_i(e_i, s_i)$. For analytical tractability, we assume δ_i to be constant.

Both stocks of pollutants exert an increasing marginal damage, which is captured in the welfare function V, for the sake of tractability, by quadratic damage functions. Furthermore, both stocks decrease welfare independently. This is plausible if they damage different environmental goods. Thus, the welfare effect of one additional unit of one pollutant does not depend on the amount of the other. Note that the overall welfare function V is strictly concave.

Since we are interested in studying questions related to the scale as well as the structure of economic activity, and in order to simplify the analysis of corner solutions in the optimization problem, we introduce new dimensionless variables in the following way:

$$c = \frac{l_1 + l_2}{\lambda} \qquad \text{and} \qquad x = \frac{l_1}{l_1 + l_2} \ . \tag{8}$$

The variable c stands for the *scale* of economic activity. It indicates what fraction of the total available amount of labor is devoted to economic activity, and may take values between 0 and 1. The remaining fraction $1 - c$ is left idle. This can be interpreted as an implicit form of pollution abatement. By not using all available labor in the production of the consumption goods (and, consequently, emissions) but leaving part of the labor endowment idle, the variable c can be thought of as measuring the scale of economic activity *in the sectors producing consumption goods and pollution*, whereas the fraction $1 - c$ of labor may be thought of as being employed in (implicit) pollution abatement.[2] The variable x stands for the *structure* of economic activity. It indicates the fraction of the total labor employed in production, $l_1 + l_2$, that is allocated to sector 1, and may take values between 0 and 1. The remaining fraction $1 - x$ is allocated to sector 2. The variables l_1

[2]Not taking into account potential abatement activities for the scale of economic activity is in line with arguments from the 'green national product' discussion, according to which defensive and restorative activities should not be counted as augmenting the net national product (e.g. Ahmad et al. 1989, World Bank 1997).

and l_2 can then be expressed in terms of c and x:

$$l_1 = l_1(c,x) = cx\lambda \qquad \text{and} \qquad l_2 = l_2(c,x) = c(1-x)\lambda \ .$$

This allows us to replace l_1 and l_2 in the problem. For notational convenience, we introduce new production functions F^i which depend directly on c and x, and which are defined in the following way:

$$F^i(c,x) \equiv P^i(l_i(c,x)) \qquad \text{for all } c,\ x \ . \tag{9}$$

From (1) and (9) one obtains that the F^i have the following properties:

$$F^1_c = xP^1_l\lambda > 0 \ , \qquad \lim_{c\to 0} F^1_c(x \neq 0) = +\infty \ , \tag{10}$$

$$F^1_x = cP^1_l\lambda > 0 \ , \tag{11}$$

$$F^2_c = (1-x)P^2_l\lambda > 0 \ , \qquad \lim_{c\to 0} F^2_c(x \neq 1) = +\infty \ , \tag{12}$$

$$F^2_x = -cP^2_l\lambda < 0 \ . \tag{13}$$

3 Optimal scale and structure of the economy

Taking a social planner's perspective, we now determine the optimal scale and structure of the multi-pollution economy described in the previous section. The control variables are the scale (c) and the structure (x) of economic activity. In terms of pollution, the choice over c and x is a choice over (i) how much pollution to emit overall, and (ii) what particular pollutant to emit. These are the two essential macroeconomic dimensions of every multi-pollution allocation decision.

3.1 Intertemporal optimization

We maximize the discounted intertemporal welfare over c and x,

$$\int_0^\infty \left[U(y_1,y_2) - \frac{\sigma_1}{2}s_1^2 - \frac{\sigma_2}{2}s_2^2\right] e^{-\rho t}dt \ , \tag{14}$$

where ρ denotes the discount rate and $y_i = F^i(c,x)$ $(i,j = 1,2)$, subject to the dynamic constraints for the two state variables s_1 and s_2 which are given by Equations (5):

$$\dot{s}_i = F^i(c,x) - \delta_i s_i \qquad \text{with} \quad \delta_i > 0 \qquad (i = 1,2) \ . \tag{15}$$

In addition, the following restrictions for the control variables c and x hold:

$$0 \leq c \leq 1 \qquad \text{and} \qquad 0 \leq x \leq 1 \ . \tag{16}$$

Corner solutions with $x = 0$ or $x = 1$ cannot be optimal since either case would imply, due to Assumptions (1) and (7), that the marginal utility of one consumption good would go to infinity while the marginal utility of the other would remain finite. Similarly, a corner solution with $c = 0$ cannot be optimal since in that case the marginal utility of both consumption goods would go to infinity while the marginal damage from environmental pollution would remain finite. Hence, the only remaining restriction, which we have to control for explicitly, is:

$$c \leq 1 \ . \tag{17}$$

We introduce two costate variables, p_1 and p_2, and a Kuhn-Tucker parameter, p_c. The current value Hamiltonian of the problem then reads

$$\begin{aligned} \mathcal{H}(c,x,s_1,s_2;p_1,p_2,p_c) \ &= \ U(F^1(c,x),F^2(c,x)) - \frac{\sigma_1}{2}s_1^2 - \frac{\sigma_2}{2}s_2^2 \\ &\quad + p_1\left[F^1(c,x) - \delta_1 s_1\right] \\ &\quad + p_2\left[F^2(c,x) - \delta_2 s_2\right] \\ &\quad + p_c\left[1 - c\right] \ . \end{aligned} \tag{18}$$

Since both control variables, c and x, are always strictly positive, the two state variables, s_1 and s_2, are always nonnegative and the Hamiltonian $\mathcal{H}$ is continuously differentiable with respect to c and x, the first order conditions of the control

problem are:

$$U_1 F_c^1 \; + \; U_2 F_c^2 \; + \; p_1 F_c^1 \; + \; p_2 F_c^2 \; - \; p_c \;\; = \;\; 0 \; , \tag{19}$$

$$U_1 F_x^1 \; + \; U_2 F_x^2 \; + \; p_1 F_x^1 \; + \; p_2 F_x^2 \;\; = \;\; 0 \; , \tag{20}$$

$$\sigma_1 s_1 \; + \; (\delta_1 + \rho) p_1 \;\; = \;\; \dot{p}_1 \; , \tag{21}$$

$$\sigma_2 s_2 \; + \; (\delta_2 + \rho) p_2 \;\; = \;\; \dot{p}_2 \; , \tag{22}$$

$$p_c \geq 0 \; , \; p_c (1 - c) \;\; = \;\; 0 \; , \tag{23}$$

plus the dynamic constraints (15) and the restriction (17). These necessary conditions are also sufficient if, in addition, the transversality conditions

$$\lim_{t \to \infty} p_i(t) \, e^{-\rho t} \cdot s_i(t) = 0 \qquad (i = 1, 2) \; , \tag{24}$$

hold (see Appendix A.1). Note that the optimal path is also unique.

3.2 Stationary state

Setting $\dot{p}_1 = 0$, $\dot{p}_2 = 0$, $\dot{s}_1 = 0$ and $\dot{s}_2 = 0$ in the system of first order conditions (15), (17) and (19)–(23) yields the necessary and sufficient conditions for an optimal stationary state $(c^\star, x^\star, s_1^\star, s_2^\star)$, in which neither the scale nor the structure of economic activity nor the stocks of pollution accumulated in the environment change over time. From conditions (21) and (22) one obtains for the costate variables p_i $(i = 1, 2)$:

$$p_i = -\frac{\sigma_i s_i^\star}{\delta_i + \rho} \qquad (i = 1, 2) \; . \tag{25}$$

Inserting (25) in (19) and (20), and rearranging terms, yields the following necessary and sufficient conditions for an optimal stationary state:

$$U_1^\star - \frac{\sigma_1 s_1^\star}{\delta_1 + \rho} \;\; = \;\; \frac{p_c F_x^{2\star}}{F_c^{1\star} F_x^{2\star} - F_x^{1\star} F_c^{2\star}} \; , \tag{26}$$

$$U_2^\star - \frac{\sigma_2 s_2^\star}{\delta_2 + \rho} \;\; = \;\; \frac{-p_c F_x^{1\star}}{F_c^{1\star} F_x^{2\star} - F_x^{1\star} F_c^{2\star}} \; , \tag{27}$$

where $U_i^\star$ and $F_j^{i\star}$ $(i = 1, 2\,;\; j = c, x)$ denote functions evaluated at stationary state values of the argument. From the signs of the F_j^i and p_c stated in (10)–(13) and (23), it follows that:

$$U_i^\star \geq \frac{\sigma_i s_i^\star}{\delta_i + \rho} \qquad (i = 1, 2)\ , \tag{28}$$

where the ">" sign indicates a corner solution ($c^\star = 1$). Furthermore, from the equations of motion (15) one obtains

$$s_i^\star = \frac{F^{i\star}}{\delta_i} = \text{const.} \qquad (i = 1, 2)\ . \tag{29}$$

The interpretation of the two conditions (28) is that in an interior (corner) optimal stationary state the scale and structure of economic activity are such that for each sector the marginal welfare gain due to consumption of that sector's output equals (is greater than) the aggregate future marginal damage from that sector's current emission which comes as an inevitable by-product with the consumption good.[3]

An optimal stationary state exists if the system (23), (26), (27) and (29) of five equations for the five unknowns $(c^\star, x^\star, s_1^\star, s_2^\star)$ and $p_c^\star$ has a solution with $0 < c^\star \leq 1$ and $0 < x^\star < 1$. With the properties of the utility and production functions assumed here, a unique optimal stationary state always exists.

Proposition 1:

(i) There exists a unique stationary state $(c^\star, x^\star, s_1^\star, s_2^\star)$, which is given as the solution to (23), (26), (27) and (29).

(ii) The optimal stationary state of the economy is an interior solution with $c^\star < 1$, if the total available amount of labor λ in the economy is strictly greater than some threshold value $\bar{\lambda} = \bar{l}_1 + \bar{l}_2$, where the $\bar{l}_i$ are specified by

[3]Note that taking account of discounting and the natural degradation of the respective pollution stock, the net present value of the accumulated damage of one marginal unit of pollution sums up to the right-hand-side of (28), as $\int_0^\infty \sigma_i s_i^\star e^{-(\rho+\delta_i)t} dt = \sigma_i s_i^\star/(\rho + \delta_i)$ $(i = 1, 2)$.

the following implicit equations:

$$U_i(P^1(\bar{l}_1), P^2(\bar{l}_2)) = \frac{\sigma_i P^i(\bar{l}_i)}{\delta_i^2 + \delta_i \rho} \qquad (i = 1, 2) \ .$$

Proof: *see Appendix A.2.*

In the following, we shall concentrate on the case of an interior stationary state with $c^\star < 1$. Hence, we assume that the total labor amount λ exceeds $\bar{\lambda}$ as specified in Proposition 1. In order to study the properties of the interior optimal stationary state $(c^\star, x^\star)$ some comparative statics can be done with Conditions (26), (27) and (29). The results are stated in the following proposition.

Proposition 2:

An interior optimal stationary state, if it exists, has the following properties:

$$\begin{array}{llll} \dfrac{dc^\star}{d\delta_1} > 0 \ , & \dfrac{dx^\star}{d\delta_1} > 0 \ , & \dfrac{dc^\star}{d\delta_2} > 0 \ , & \dfrac{dx^\star}{d\delta_2} < 0 \ , \\ \dfrac{dc^\star}{d\sigma_1} < 0 \ , & \dfrac{dx^\star}{d\sigma_1} < 0 \ , & \dfrac{dc^\star}{d\sigma_2} < 0 \ , & \dfrac{dx^\star}{d\sigma_2} > 0 \ , \\ \dfrac{dc^\star}{d\rho} > 0 \ , & \dfrac{dx^\star}{d\rho} \gtreqless 0 \quad \text{for} & \dfrac{[U_{22}^\star \delta_2(\delta_2+\rho) - \sigma_2](\delta_2+\rho)}{[U_{11}^\star \delta_1(\delta_1+\rho) - \sigma_1](\delta_1+\rho)} \gtreqless \dfrac{\sigma_2 F^{2\star} F_c^{1\star}}{\sigma_1 F^{1\star} F_c^{2\star}} \ . & \end{array}$$

Proof: *see Appendix A.3.*

These results can be interpreted as follows. For both pollutants i $(i = 1, 2)$, the lower is the natural deterioration rate δ_i and the higher is the harmfulness σ_i, the lower is the relative weight of the emitting sector in the total economy and the lower is the overall scale of economic activity in the stationary state. An increase in the discount rate ρ increases the optimal stationary scale of economic activity, $c^\star$, while its effect on the optimal stationary structure of economic activity, $x^\star$, is ambiguous.

3.3 Optimal dynamic path and local stability analysis

In the following we solve the optimization problem by linearizing the resulting system of differential equations around the stationary state. Since our model is characterized by only mild non-linearities,[4] we expect the linear approximation to yield insights which should also hold for the exact problem. In Section 4 below, we shall numerically optimize the exact problem, and confirm this expectation.

As we have assumed an interior stationary state, the optimal path will also be an interior optimal path at least in a neighborhood of the interior stationary state. Hence, we restrict the analysis to the case of an interior solution, i.e. $c^\star < 1$. As shown in Appendix A.4, the optimal dynamics of the two control variables c, x and the two state variables s_1, s_2 can be described by a system of four coupled first order autonomous differential equations:

$$\dot{c} = \frac{[U_1(\delta_1+\rho)-\sigma_1 s_1]U_{22}F_x^2 - [U_2(\delta_2+\rho)-\sigma_2 s_2]U_{11}F_x^1}{U_{11}U_{22}df} , \tag{30}$$

$$\dot{x} = \frac{[U_2(\delta_2+\rho)-\sigma_2 s_2]U_{11}F_c^1 - [U_1(\delta_1+\rho)-\sigma_1 s_1]U_{22}F_c^2}{U_{11}U_{22}df} , \tag{31}$$

$$\dot{s}_1 = F^1 - \delta_1 s_1 , \tag{32}$$

$$\dot{s}_2 = F^2 - \delta_2 s_2 , \tag{33}$$

with $df \equiv F_c^1 F_x^2 - F_x^1 F_c^2 < 0$. Linearizing around the stationary state $(c^\star, x^\star, s_1^\star, s_2^\star)$ yields the following approximated dynamic system (see Appendix A.5):

$$\begin{pmatrix} \dot{c} \\ \dot{x} \\ \dot{s_1} \\ \dot{s_2} \end{pmatrix} \approx J^\star \begin{pmatrix} c-c^\star \\ x-x^\star \\ s_1-s_1^\star \\ s_2-s_2^\star \end{pmatrix} \quad \text{with} \tag{34}$$

[4] Remember that the welfare function V is additively separable in all four arguments.

$$J^\star = \begin{pmatrix} \rho + \frac{\delta_1 F_c^{1\star} F_x^{2\star} - \delta_2 F_x^{1\star} F_c^{2\star}}{df^\star} & \frac{(\delta_1 - \delta_2) F_x^{1\star} F_x^{2\star}}{df^\star} & -\frac{\sigma_1 F_x^{2\star}}{U_{11}^\star df^\star} & \frac{\sigma_2 F_x^{1\star}}{U_{22}^\star df^\star} \\ \frac{(\delta_2 - \delta_1) F_c^{1\star} F_c^{2\star}}{df^\star} & \rho + \frac{\delta_2 F_c^{1\star} F_x^{2\star} - \delta_1 F_x^{1\star} F_c^{2\star}}{df^\star} & \frac{\sigma_1 F_c^{2\star}}{U_{11}^\star df^\star} & -\frac{\sigma_2 F_c^{1\star}}{U_{22}^\star df^\star} \\ F_c^{1\star} & F_x^{1\star} & -\delta_1 & 0 \\ F_c^{2\star} & F_x^{2\star} & 0 & -\delta_2 \end{pmatrix} .$$

The Jacobian evaluated at the stationary state, $J^\star$, has four real eigenvalues (see Appendix A.5), two of which are strictly negative (ν_1, ν_2) and two of which are strictly positive (ν_3, ν_4). Hence, the system dynamics exhibits saddlepoint stability, i.e. for all initial stocks of pollutants, s_1^0 and s_2^0, there exists a unique optimal path which asymptotically converges towards the stationary state. Because of the transversality conditions (24) the optimal path is restricted to the stable hyperplane, which is spanned by the eigenvectors associated with the negative eigenvalues. Given the eigenvalues and the eigenvectors, which are calculated in Appendix A.5, the explicit system dynamics in a neighborhood around the stationary state is given by:

$$\begin{aligned} c(t) &= c^\star + (s_1^0 - s_1^\star) \frac{F_x^{2\star}(\nu_1 + \delta_1)}{F_c^{1\star} F_x^{2\star} - F_x^{1\star} F_c^{2\star}} e^{\nu_1 t} - \\ &\quad (s_2^0 - s_2^\star) \frac{F_x^{1\star}(\nu_2 + \delta_2)}{F_c^{1\star} F_x^{2\star} - F_x^{1\star} F_c^{2\star}} e^{\nu_2 t} , \qquad (35) \\ x(t) &= x^\star - (s_1^0 - s_1^\star) \frac{F_c^{2\star}(\nu_1 + \delta_1)}{F_c^{1\star} F_x^{2\star} - F_x^{1\star} F_c^{2\star}} e^{\nu_1 t} + \\ &\quad (s_2^0 - s_2^\star) \frac{F_c^{1\star}(\nu_2 + \delta_2)}{F_c^{1\star} F_x^{2\star} - F_x^{1\star} F_c^{2\star}} e^{\nu_2 t} , \qquad (36) \\ s_1(t) &= s_1^\star + (s_1^0 - s_1^\star) e^{\nu_1 t} , \qquad (37) \\ s_2(t) &= s_2^\star + (s_2^0 - s_2^\star) e^{\nu_2 t} , \qquad (38) \end{aligned}$$

where $s_i^0 = s_i(0)$ $(i = 1, 2)$ denote the initial pollutant stocks.

As a measure of the overall rate of convergence of a process $z(t)$ which asymptotically approaches $z^\star$, we define the *characteristic time scale of convergence* τ_z by

$$\tau_z{}^{-1} \equiv \overline{\left| \frac{\dot{z}(t)}{z(t) - z^\star} \right|} , \qquad (39)$$

where the horizontal bar denotes the average over time. The greater is the time scale τ_z, the slower is the convergence towards $z^\star$. With this definition, it is obvious from Equations (37) and (38) that the pollutant stock s_i $(i = 1, 2)$ converges towards its stationary state value $s_i^\star$ with a characteristic time scale $\tau_{s_i} = 1/|\nu_i|$. As the system approaches the stationary state for $t \to \infty$, the scale c and structure x (Equations 35 and 36) converge towards their stationary state values $c^\star$ and $x^\star$ with a characteristic time scale which is determined by the eigenvalue with the smaller absolute value, $\tau_c = \tau_x = 1/\min\{|\nu_1|, |\nu_2|\}$ (see Appendix A.6). Proposition 3 summarizes these results.

Proposition 3:

For the linear approximation (34) around the stationary state $(c^\star, x^\star, s_1^\star, s_2^\star)$ *the following statements hold:*

(i) The stationary state is saddlepoint-stable.

(ii) The explicit system dynamics is given by Equations (35)–(38).

(iii) The characteristic time scale of convergence towards the stationary state is given by

- $\tau_c = \tau_x = 1/\min\{|\nu_1|, |\nu_2|\}$ *for the control variables c and x, and by*
- $\tau_{s_i} = 1/|\nu_i|$ *for stock variable* s_i *(*$i = 1, 2$*).*

As shown in Appendix A.5 the eigenvalues ν_1 and ν_2 are given by

$$\nu_1 = \frac{1}{2}\left[\rho - \sqrt{(\rho + 2\delta_1)^2 - \frac{4\sigma_1}{U_{11}^\star}}\right] < 0 \,, \tag{40}$$

$$\nu_2 = \frac{1}{2}\left[\rho - \sqrt{(\rho + 2\delta_2)^2 - \frac{4\sigma_2}{U_{22}^\star}}\right] < 0 \,. \tag{41}$$

Hence, the absolute value of ν_i (time scale of convergence) decreases (increases) with the discount rate ρ and the curvature of consumption welfare in the stationary state $|U_{ii}^\star|$ $(i = 1, 2)$. It increases (decreases) with the harmfulness σ_i and the deterioration rate δ_i of the pollutant stock.

We now turn to the question of the (non-)monotonicity of the optimal path. According to Equations (37) and (38), the stocks of the two pollutants converge monotonically towards their stationary state values $s_1^\star$ and $s_2^\star$. In order to show that the optimal paths for the control variables c and x may be non-monotonic, we differentiate Equations (35) and (36) with respect to t:

$$\begin{aligned}\dot{c}(t) &= \nu_1(s_1^0 - s_1^\star)\frac{F_x^{2\star}(\nu_1+\delta_1)}{F_c^{1\star}F_x^{2\star} - F_x^{1\star}F_c^{2\star}}\, e^{\nu_1 t} - \\ &\quad \nu_2(s_2^0 - s_2^\star)\frac{F_x^{1\star}(\nu_2+\delta_2)}{F_c^{1\star}F_x^{2\star} - F_x^{1\star}F_c^{2\star}}\, e^{\nu_2 t}\ , \qquad (42)\\ \dot{x}(t) &= -\nu_1(s_1^0 - s_1^\star)\frac{F_c^{2\star}(\nu_1+\delta_1)}{F_c^{1\star}F_x^{2\star} - F_x^{1\star}F_c^{2\star}}\, e^{\nu_1 t} + \\ &\quad \nu_2(s_2^0 - s_2^\star)\frac{F_c^{1\star}(\nu_2+\delta_2)}{F_c^{1\star}F_x^{2\star} - F_x^{1\star}F_c^{2\star}}\, e^{\nu_2 t}\ . \qquad (43)\end{aligned}$$

The optimal path is non-monotonic if $\dot{c}$ or $\dot{x}$ change their sign, i.e. if the paths $c(t)$ or $x(t)$ exhibit a local extremum for positive times t. According to the signs of the ν_i and F_j^i ($i = 1, 2$ and $j = c, x$) and given that $\nu_1 \neq \nu_2$, $c(t)$ exhibits a unique local extremum if $\mathrm{sgn}(s_1^0 - s_1^\star) \neq \mathrm{sgn}(s_2^0 - s_2^\star)$, and $x(t)$ exhibits a unique local extremum if $\mathrm{sgn}(s_1^0 - s_1^\star) = \mathrm{sgn}(s_2^0 - s_2^\star)$.[5] Solving $\dot{c}(t) = 0$ and $\dot{x}(t) = 0$ for t, using expressions (42) and (43) for $\dot{c}$ and $\dot{x}$, yields:

$$\hat{t} = \begin{cases} \ln\left[\frac{\nu_2(s_2^0 - s_2^\star)F_x^{1\star}(\nu_2+\delta_2)}{\nu_1(s_1^0 - s_1^\star)F_x^{2\star}(\nu_1+\delta_1)}\right](\nu_1-\nu_2)^{-1}\ , & \text{if} \quad \mathrm{sgn}(s_1^0 - s_1^\star) \neq \mathrm{sgn}(s_2^0 - s_2^\star) \\ \ln\left[\frac{\nu_2(s_2^0 - s_2^\star)F_c^{1\star}(\nu_2+\delta_2)}{\nu_1(s_1^0 - s_1^\star)F_c^{2\star}(\nu_1+\delta_1)}\right](\nu_1-\nu_2)^{-1}\ , & \text{if} \quad \mathrm{sgn}(s_1^0 - s_1^\star) = \mathrm{sgn}(s_2^0 - s_2^\star) \end{cases} . \qquad (44)$$

According to this equation, it is possible that $\hat{t}$ may be negative or infinite, which is meaningless in the context of this analysis. In this case we would observe monotonic optimal paths for both control variables c and x for times $0 < t < +\infty$. For instance, $\hat{t}$ is negative if $\left|s_2^0 - s_2^\star\right|$ is sufficiently small, that is, the second pollutant stock is initially already close to its stationary state level. Furthermore, $\hat{t}$ equals (plus or minus) infinity if either $\left|s_1^0 - s_1^\star\right| = 0$ or $|\nu_1 - \nu_2| = 0$, that

[5] Note that $\nu_i + \delta_i < 0$, which can easily be verified from Equations (A.26) and (A.27).

is, the first pollutant stock is initially already at its stationary state level or the eigenvalues are identical. The following proposition summarizes the behavior of the optimal control path.

Proposition 4:

In the linear approximation (34) around the stationary state $(c^\star, x^\star, s_1^\star, s_2^\star)$*, the following statements hold for the optimal path:*

(i) The stocks of pollutants $s_1(t)$ *and* $s_2(t)$ *converge exponentially, and hence monotonically, towards their stationary state values* $s_1^\star$ *and* $s_2^\star$*.*

(ii) If and only if $\hat{t}$ *as given by Equation (44) is strictly positive and finite, then the optimal control is non-monotonic over time and* $\hat{t}$ *denotes the time at which the optimal control has a unique local extremum. In particular, if* $sgn(s_1^0 - s_1^\star) \neq sgn(s_2^0 - s_2^\star)$*,* $c(t)$ *is non-monotonic and* $x(t)$ *is monotonic. If* $sgn(s_1^0 - s_1^\star) = sgn(s_2^0 - s_2^\star)$*,* $x(t)$ *is non-monotonic and* $c(t)$ *is monotonic.*

4 Numerical optimization

In this section we illustrate the results derived in Section 3 by numerical optimizations of the original, non linearized optimization problem (14)–(16). The results thus obtained confirm that the insights from analyzing the linearized system also hold for the exact solution. All numerical optimizations were carried out with the advanced optimal control software package MUSCOD-II (Diehl et al. 2001), which exploits the multiple shooting state discretization (Leineweber et al. 2003).

There are four different qualitative scenarios which have to be examined. (i) Both stocks of pollutants exhibit the same harmfulness but differ in their deterioration rates, i.e. $\sigma_1 = \sigma_2$, $\delta_1 < \delta_2$. (ii) The two pollutants differ in their harmfulness but have equal deterioration rates, i.e. $\sigma_1 < \sigma_2$, $\delta_1 = \delta_2$. (iii) The pollutants differ in both harmfulness and deterioration rates and the more harmful pollutant has

the higher deterioration rate, i.e. $\sigma_1 < \sigma_2$, $\delta_1 < \delta_2$. (iv) Both harmfulness and deterioration rates are different, and the more harmful pollutant has a lower deterioration rate, i.e. $\sigma_1 < \sigma_2$, $\delta_1 > \delta_2$. Furthermore, each of the four scenarios splits into four subcases, depending on the initial stocks of pollutants (both initial stocks below, only first stock above, only second stock above and both stocks above the stationary state levels).

In the following we discuss these four different scenarios. The parameter values used for the numerical optimization have been chosen so as to illustrate clearly the different effects, and do not necessarily reflect the characteristics of real environmental pollution problems. For all numerical examples, the total labor supply λ has been chosen so as to guarantee an interior stationary state scale $c^\star < 1$. As it is not possible to optimize numerically over an infinite time horizon, the time horizon has been set to 250 years and all parameters have been chosen in such a way that the system at time $t = 250$ is very close to the stationary state. For a more convenient exposition, the figures show the time paths up to $t = 125$ only. The parameter values for the numerical optimization are listed in Appendix A.7.

In the first scenario ($\sigma_1 = \sigma_2$), both stocks of pollutants exhibit the same harmfulness but the deterioration rate is smaller for the first pollutant than for the second. Figure 1 shows the result of a numerical optimization of this case. In this example the initial stocks for both pollutants are above their stationary state levels ($s_1^0 = 30, s_2^0 = 30$). The optimal path for the structure exhibits non-monotonic behavior as expected from Proposition 4. Further, we expect that the optimal stationary state structure $x^\star$ is clearly below 0.5, indicating that relatively more labor is employed in the second sector, because as the second stock of pollutant deteriorates at a higher rate the aggregate intertemporal damage of one unit of emissions is smaller for the second pollutant.[6] This expectation is confirmed by

[6]Note that both consumption goods are equally valued by the representative consumer, i.e. $\mu_1 = \mu_2$ (see Appendix A.7).

the numerical optimization.

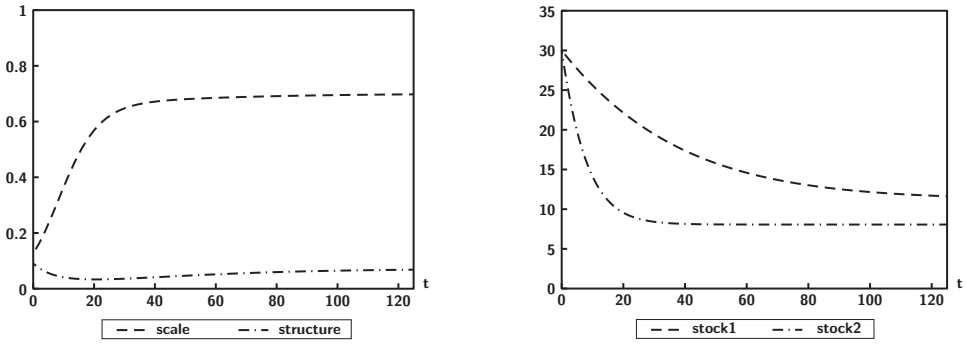

Figure 1: Optimal paths for scale and structure (left) and the two pollutant stocks (right) for the case $\sigma_1 = \sigma_2, \delta_1 < \delta_2$. Parameter values used for the numerical optimization are given in Appendix A.7.

In the second scenario ($\sigma_1 < \sigma_2, \delta_1 = \delta_2$), the two stocks of pollutants are of different harmfulness but the deterioration rate for the two pollutants are equal. The result of a numerical optimization of this case is presented in Figure 2. In this example the initial stock for the first (second) pollutant is above (below) their stationary state levels ($s_1 = 40, s_2 = 0$). Now, the optimal path for the scale exhibits a non monotonic behavior as expected from Proposition 4. Further, we expect that the optimal stationary state structure $x^\star$ is clearly above 0.5, indicating that relatively more labor is employed by the second sector, because as the second stock of pollutant is less harmful the aggregate intertemporal damage of one unit of emissions is smaller for the second pollutant. This expectation is confirmed by the numerical optimization.

The third scenario ($\sigma_1 < \sigma_2, \delta_1 < \delta_2$) – both harmfulness and deterioration rates are different and the more harmful pollutant has the higher deterioration rate – is the most interesting as neither of the two pollutants exhibits a priori more favorable dynamic characteristics for the economy. Hence, we are not able to predict which production sector will be used to a greater extent in the station-

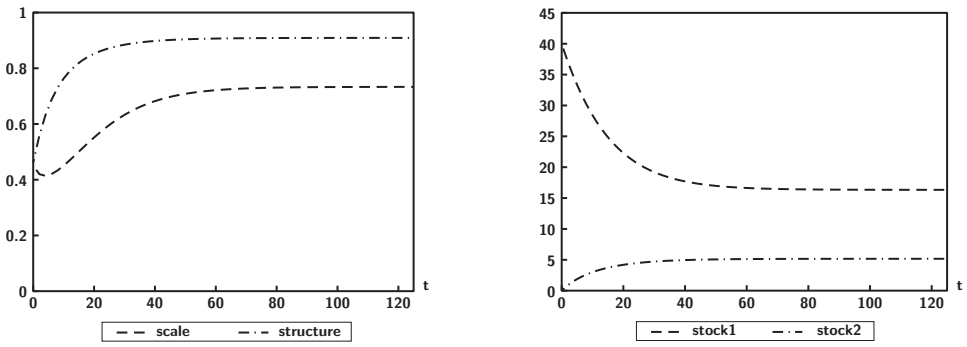

Figure 2: Optimal paths for scale and structure (left) and the two pollutant stocks (right) for the case $\sigma_1 < \sigma_2, \delta_1 = \delta_2$. Parameter values used for the numerical optimization are given in Appendix A.7.

ary state. Furthermore, non monotonic paths – if they occur – are likely to be more pronounced than in the other cases. Figure 3 shows the optimal paths for a numerical example for all four subcases (initial pollutant stocks above or below stationary state levels for one and both pollutants). Of course, the long run stationary state to which the economy converges, is the same in all four subscenarios, as all parameters are identical except for the initial stocks of the two pollutants. Nevertheless, the optimal paths and especially their convergence towards the stationary state is quite different for the four subcases. As expected from Proposition 4, we observe that – if at all – the optimal path for the structure is non-monotonic if both stocks start above or below their stationary state levels (subcases a and d) and the optimal path for the scale is non-monotonic if one initial stock is higher and one is lower than their stationary state levels (subcase b). We also see that both, structure and scale, may exhibit monotonic optimal paths (subcase c).

In the fourth scenario ($\sigma_1 < \sigma_2, \delta_1 > \delta_2$), where both pollutants exhibit different harmfulness and deterioration rates but the second pollutant is more harmful and has the lower deterioration rate, the first pollutant exhibits clearly more favorable dynamic properties than the second pollutant. In this case the economy will

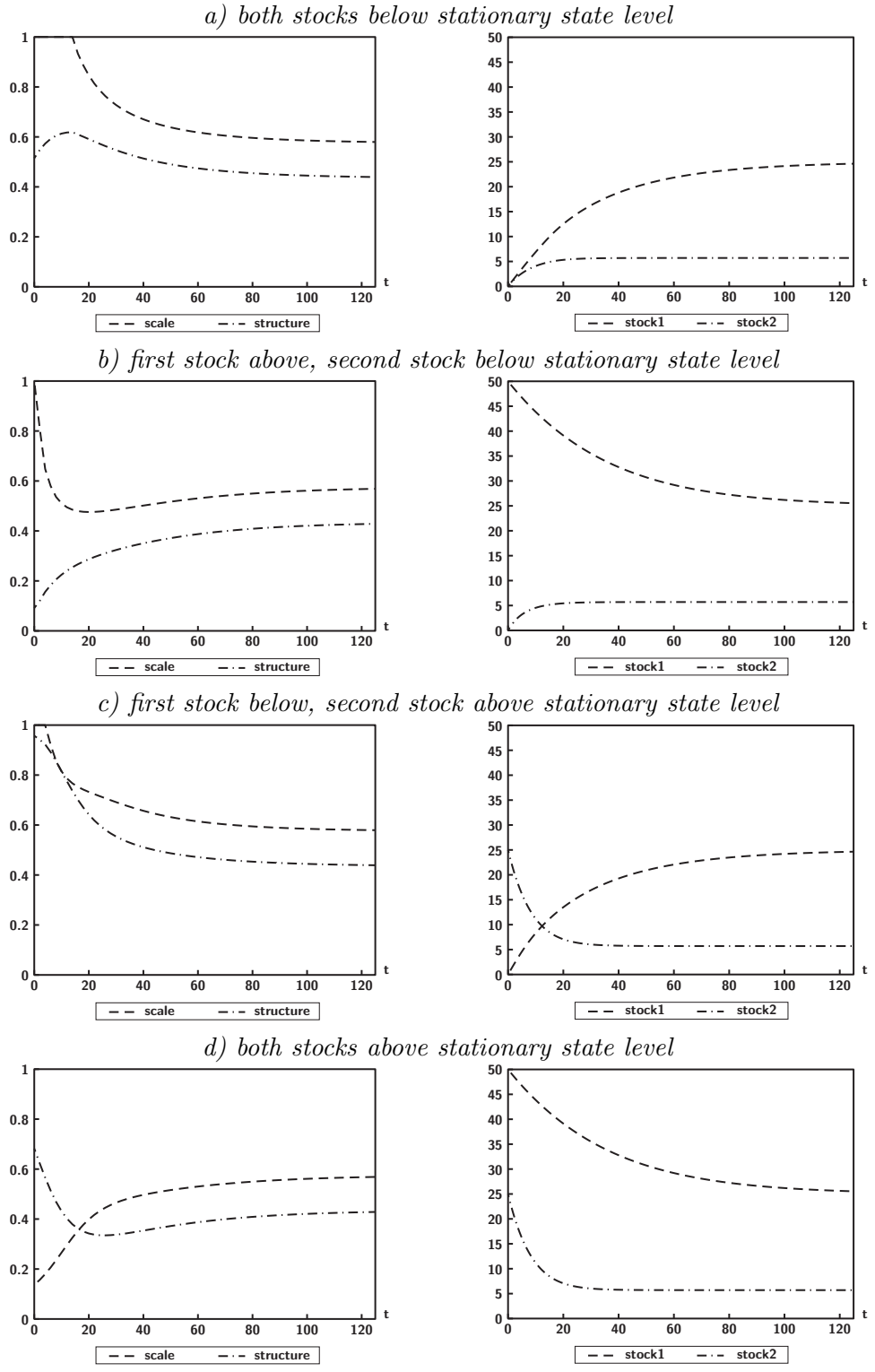

Figure 3: Optimal paths for scale and structure (left) and the two pollutant stocks (right) for the case $\sigma_1 < \sigma_2$, $\delta_1 < \delta_2$ and all four subscenarios. Parameter values used for the numerical optimization are given in Appendix A.7.

nearly exclusively use the first production sector. Although non-monotonicities in the optimal paths for scale and structure can occur according to Proposition 4, they are not pronounced. As nothing new can be learned from this case, we do not show a numerical optimization example.

5 Conclusion

In this paper, we have studied the mutual interaction over time between the scale and structure of economic activity on the one hand, and the dynamics of multiple environmental pollution stocks on the other hand. We have carried out a total analysis of a two-sector-economy, in which each sector produces one distinct consumption good and one specific pollutant. The pollutants of both sectors were assumed to differ in their environmental impact in two ways: (i) with respect to their harmfulness and (ii) with respect to their natural deterioration rates in the environment.

Most of the results are intuitive. First, it may be optimal not to use all available labor endowment in the production of consumption goods in order to avoid excessive environmental damage. Second, under very general conditions a change in scale and structure of economic activity over time is optimal. Thus, the optimal economic dynamics is driven by the dynamics of the environmental pollution stocks. The less harmful is a pollutant, the higher are the relative importance of the emitting sector and the overall scale of economic activity in the stationary state. The shorter lived is a pollutant, the higher are the relative importance of the emitting sector and the overall scale of economic activity in the stationary state. If emissions differ either in their environmental harmfulness or in their deterioration rates, we should have structural change towards the sector emitting the less harmful or the shorter-lived pollutant. However, if the harmfulness and deterioration rates differ and if the environmentally less harmful emission is also

the longer-lived pollutant, no general conclusion concerning the direction of structural change can be drawn. Third, the characteristic time scale of convergence of scale and structure towards the stationary state is given by (the inverse of) the eigenvalue with the smaller absolute value. It increases with the discount rate and the curvature of consumption welfare in the stationary state; it decreases with the harmfulness and the deterioration rate of the respective pollutant stock.

Most importantly, our formal analysis as well as the numerical optimizations, show that it is likely that the optimal control paths, i.e. the change in the scale and structure of the economy, are non-monotonic over time.[7] If a non-monotonic control is optimal, our numerical optimizations suggest that the local extremum of the control path may be pronounced and that it occurs at the beginning of the control path.

These results have implications for the design of environmental indicators and policies. First, the traditional view is that different environmental problems – such as e.g. acidification of soils and surface waters, groundwater contamination by nitrates or pesticides, and climate change due to anthropogenic greenhouse gas emission – can be regulated by independent environmental policies. In contrast, our total analysis of a multi-sector economy with several independent environmental pollutants, shows that these problems – even without any direct physical interaction – interact indirectly because they all affect social welfare, and the mitigation of all of them is constrained by the available economic resources. As a result, even for non-interacting environmental pollutants the optimal regulation has to take an encompassing view, taking into account all of the environmental problems together.

[7] Non-monotonic optimal control paths, in particular limit-cycles, are known to exist for control problems with two or more state variables, and for time-lagged and adaptive control problems, even with one single state variable (e.g. Benhabib and Nishimura 1979, Feichtinger et al. 1994, Wirl 2000, 2002, Winkler 2004).

Second, indicators and policies which are solely based on the harmfulness of environmental pollutants – which is predominant in current environmental politics – fall short of optimally controlling environmental problems. In a dynamic setting, the lifetime of pollutants is an equally important determinant of the optimal environmental policy.

Third, the non-monotonicity-result challenges common intuition which suggests that policies should achieve optimal change in a monotonic way. In contrast to this simple intuition, our analysis shows that if pollutants accumulate on different time scales and if they differ in environmental harmfulness, the optimal policies may be non-monotonic. In particular, the optimal time-path of structural change towards the stationary state structure may be characterized by 'optimal overshooting'; that is, the optimal relative importance of a sector starts below (above) the stationary state level, increases (decreases) to a point above (below) the stationary state level, and finally decreases (increases) again. The same goes for the optimal dynamics of the overall economic scale.

Summing up, in order to develop sustainable solutions to the multiple environmental problems that we face in reality – such as climate change, depletion of the ozone layer, groundwater contamination, acidification of soil and surface water, biodiversity loss, etc. – we should adopt an encompassing view and base policy advice on a total analysis of economy-environment interactions. As our analysis shows, the resulting optimal policies need to take account of the history, the empirical parameter values and the dynamic relationships of all of the problems, and these policies might be non-monotonic.

Appendix

A.1 Concavity of the optimized Hamiltonian

We show that the Hamiltonian $\mathcal{H}$, without taking into account the restriction $c \leq 1$, i.e. $p_c = 0$, is strictly concave whenever the necessary conditions are satisfied. Thus, the unique optimal solution is the local extremum of $\mathcal{H}$ if we have an interior solution; it is a corner solution with $c = 1$ if the local extremum of $\mathcal{H}$ is reached for unfeasible $c > 1$.

A sufficient condition for strict concavity of the Hamiltonian is that its Hessian $H = \frac{\partial^2 \mathcal{H}}{\partial i \partial j}$ $(i, j = c, x, s_1, s_2)$ is negative definite. The Hessian H reads:

$$H = \begin{pmatrix} \mathcal{H}_{cc} & \mathcal{H}_{cx} & 0 & 0 \\ \mathcal{H}_{xc} & \mathcal{H}_{xx} & 0 & 0 \\ 0 & 0 & -\sigma_1 & 0 \\ 0 & 0 & 0 & -\sigma_2 \end{pmatrix} \tag{A.1}$$

Due to its diagonal form, H is negative definite if the reduced Hessian $H' = \frac{\partial^2 \mathcal{H}}{\partial i \partial j}$ $(i, j = c, x)$ is negative definite, i.e. $\mathcal{H}_{cc}, \mathcal{H}_{xx} < 0$ and $\det H' > 0$.

$$\mathcal{H}_{cc} = U_{11}(F_c^1)^2 + (U_1 + p_1)F_{cc}^1 + U_{22}(F_c^2)^2 + (U_2 + p_2)F_{cc}^2 \ , \tag{A.2}$$

$$\mathcal{H}_{xx} = U_{11}(F_x^1)^2 + (U_1 + p_1)F_{xx}^1 + U_{22}(F_x^2)^2 + (U_2 + p_2)F_{xx}^2 \ , \tag{A.3}$$

$$\mathcal{H}_{cx} = U_{11}F_c^1F_x^1 + (U_1 + p_1)F_{cx}^1 + U_{22}F_c^2F_x^2 + (U_2 + p_2)F_{cx}^2 \ . \tag{A.4}$$

Along the optimal path, the necessary conditions have to be satisfied. In particular, for an interior solution, i.e. $c^\star < 1$, the necessary and sufficient conditions (19) and (20) become:

$$(U_1 + p_1)F_c^1 + (U_2 + p_2)F_c^2 = 0 \ , \tag{A.5}$$

$$(U_1 + p_1)F_x^1 + (U_2 + p_2)F_x^2 = 0 \ . \tag{A.6}$$

Thus, for an interior optimal path the following equations hold:

$$p_i = -U_i \qquad (i = 1, 2) \ . \tag{A.7}$$

With this, one obtains:

$$\mathcal{H}_{cc} = U_{11}(F_c^1)^2 + U_{22}(F_c^2)^2 < 0 \ , \tag{A.8}$$

$$\mathcal{H}_{xx} = U_{11}(F_x^1)^2 + U_{22}(F_x^2)^2 < 0 \ , \tag{A.9}$$

$$\begin{aligned} \det H' &= \mathcal{H}_{cc}\mathcal{H}_{xx} - \mathcal{H}_{cx}^2 \\ &= U_{11}U_{22}\left[(F_c^1)^2(F_x^2)^2 + (F_x^1)^2(F_c^2)^2 - 2F_c^1F_x^1F_c^2F_x^2\right] > 0 \ . \end{aligned} \tag{A.10}$$

Hence, whenever $\mathcal{H}$ has an extremum it is a maximum. As a consequence, the necessary conditions (plus the transversality condition 24) are also sufficient.

A.2 Proof of Proposition 1

(i) Inserting Equations (29) into Equations (26) and (27), and using the relationship between F^i and P^i, as given from Equation (9), one obtains:

$$U_i^\star = \frac{\sigma_i P^{i\star}}{\delta_i(\delta_i+\rho)} + \frac{p_c}{\lambda P_l^{i\star}} \qquad (i=1,2) \ . \tag{A.11}$$

With the properties for P^i, as given by (1), and the properties for U_i, as given by (7), the left-hand-side of Equation (A.11) is strictly decreasing while the right-hand-side is strictly increasing in l_i. Thus, there exists at most one $l_i^\star$ which satisfies Equation (A.11). The existence of such a solution is guaranteed by the properties $\lim_{l_i\to 0} P_l^i = +\infty$ and $\lim_{y_i\to 0} U_i = +\infty$.

(ii) We derive $\bar{l}_i$ by solving (A.11) for $l_i^\star$ assuming $p_c = 0$. Thus, $\bar{l}_i$ is the maximal amount of labor which will be assigned to production process i in an optimal stationary state without taking account for the restriction $c \le 1$. If $\bar{l}_1 + \bar{l}_2 \ge \lambda$ the labor supply is short of the optimal labor demand and thus the stationary state is a corner solution. If, on the other hand, the total labor supply λ exceeds the sum $\bar{l}_1 + \bar{l}_2$, then not all labor will be used for economic activity and the optimal stationary state will be an interior solution.

A.3 Proof of Proposition 2

Setting $p_c = 0$ in Equation (A.11) yields for an interior stationary path:

$$U_i^\star = \frac{\sigma_i F^{i\star}}{\delta_i(\delta_i+\rho)} \qquad (i=1,2)\ . \tag{A.12}$$

By implicit differentiation of (A.12) with respect to δ_j $(j=1,2)$ one obtains:

$$\begin{aligned}
\left(F_c^{j\star}\frac{\partial c^\star}{\partial \delta_j} + F_x^{j\star}\frac{\partial x^\star}{\partial \delta_j}\right)\left(U_{jj}^\star - \frac{\sigma_j}{\delta_j(\delta_j+\rho)}\right) &= -\frac{\sigma_j F^{j\star}(2\delta_j+\rho)}{\delta_j^2(\delta_j+\rho)^2} \qquad (j=i)\ ,\\
\left(F_c^{i\star}\frac{\partial c^\star}{\partial \delta_j} + F_x^{i\star}\frac{\partial x^\star}{\partial \delta_j}\right)\left(U_{ii}^\star - \frac{\sigma_i}{\delta_i(\delta_i+\rho)}\right) &= 0 \qquad (j\neq i)\ .
\end{aligned}$$

Solving for $\partial c^\star/\partial\delta_j$ and $\partial x^\star/\partial\delta_j$ yields:

$$\frac{\partial c^\star}{\partial \delta_j} = \frac{\sigma_j F^{j\star}F_x^{i\star}(2\delta_j+\rho)}{(F_c^{i\star}F_x^{j\star}-F_c^{j\star}F_x^{i\star})(U_{jj}^\star\delta_j(\delta_j+\rho)-\sigma_j)\delta_j(\delta_j+\rho)}\ , \tag{A.13}$$

$$\frac{\partial x^\star}{\partial \delta_j} = \frac{\sigma_j F^{j\star}F_c^{i\star}(2\delta_j+\rho)}{(F_c^{j\star}F_x^{i\star}-F_c^{i\star}F_x^{j\star})(U_{jj}^\star\delta_j(\delta_j+\rho)-\sigma_j)\delta_j(\delta_j+\rho)}\ . \tag{A.14}$$

From the signs of the F_j^i $(i=1,2; j=c,x)$ it follows that

$$\frac{\partial c^\star}{\partial \delta_1} > 0\ , \qquad \frac{\partial c^\star}{\partial \delta_2} > 0, \qquad \frac{\partial x^\star}{\partial \delta_1} > 0, \qquad \frac{\partial x^\star}{\partial \delta_2} < 0\ . \tag{A.15}$$

By implicit differentiation of (A.12) with respect to σ_j $(j=1,2)$ one obtains:

$$\begin{aligned}
\left(F_c^{j\star}\frac{\partial c^\star}{\partial \sigma_j} + F_x^{j\star}\frac{\partial x^\star}{\partial \sigma_j}\right)\left(U_{jj}^\star - \frac{\sigma_j}{\delta_j(\delta_j+\rho)}\right) &= \frac{F^{j\star}}{\delta_j(\delta_j+\rho)} \qquad (j=i)\ ,\\
\left(F_c^{i\star}\frac{\partial c^\star}{\partial \sigma_j} + F_x^{i\star}\frac{\partial x^\star}{\partial \sigma_j}\right)\left(U_{ii}^\star - \frac{\sigma_i}{\delta_i(\delta_i+\rho)}\right) &= 0 \qquad (j\neq i)\ .
\end{aligned}$$

Solving for $\partial c^\star/\partial\sigma_j$ and $\partial x^\star/\partial\sigma_j$ yields:

$$\frac{\partial c^\star}{\partial \sigma_j} = \frac{F^{j\star}F_x^{i\star}}{(F_c^{j\star}F_x^{i\star}-F_c^{i\star}F_x^{j\star})(U_{jj}^\star\delta_j(\delta_j+\rho)-\sigma_j)}\ , \tag{A.16}$$

$$\frac{\partial x^\star}{\partial \sigma_j} = \frac{F^{j\star}F_c^{i\star}}{(F_c^{i\star}F_x^{j\star}-F_c^{j\star}F_x^{i\star})(U_{jj}^\star\delta_j(\delta_j+\rho)-\sigma_j)}\ . \tag{A.17}$$

From the signs of the F_j^i $(i=1,2; j=c,x)$ it follows that

$$\frac{\partial c^\star}{\partial \sigma_1} < 0\ , \qquad \frac{\partial c^\star}{\partial \sigma_2} < 0, \qquad \frac{\partial x^\star}{\partial \sigma_1} < 0, \qquad \frac{\partial x^\star}{\partial \sigma_2} > 0\ . \tag{A.18}$$

Implicit differentiation of (A.12) with respect to ρ yields:

$$\begin{aligned} F_c^{1\star}\frac{\partial c^\star}{\partial \rho} + F_x^{1\star}\frac{\partial x^\star}{\partial \rho} &= -\frac{\sigma_1 F^{1\star}}{[U_{11}^\star \delta_1(\delta_1+\rho)-\sigma_1](\delta_1+\rho)} , \\ F_c^{2\star}\frac{\partial c^\star}{\partial \rho} + F_x^{2\star}\frac{\partial x^\star}{\partial \rho} &= -\frac{\sigma_2 F^{2\star}}{[U_{22}^\star \delta_2(\delta_2+\rho)-\sigma_2](\delta_2+\rho)} , \end{aligned}$$

Solving for $\partial c^\star/\partial\rho$ and $\partial x^\star/\partial\rho$ yields:

$$\begin{aligned} &\frac{\partial c^\star}{\partial \rho}(F_c^{2\star}F_x^{1\star} - F_c^{1\star}F_x^{2\star}) = \\ &= \frac{\sigma_1 F^{1\star}F_x^{2\star}}{[U_{11}^\star \delta_1(\delta_1+\rho)-\sigma_1](\delta_1+\rho)} - \frac{\sigma_2 F^{2\star}F_x^{1\star}}{[U_{22}^\star \delta_2(\delta_2+\rho)-\sigma_2](\delta_2+\rho)} , \qquad \text{(A.19)} \\ &\frac{\partial x^\star}{\partial \rho}(F_c^{2\star}F_x^{1\star} - F_c^{1\star}F_x^{2\star}) = \\ &= \frac{\sigma_2 F^{2\star}F_c^{1\star}}{[U_{22}^\star \delta_2(\delta_2+\rho)-\sigma_2](\delta_2+\rho)} - \frac{\sigma_1 F^{1\star}F_c^{2\star}}{[U_{11}^\star \delta_1(\delta_1+\rho)-\sigma_1](\delta_1+\rho)} , \qquad \text{(A.20)} \end{aligned}$$

From the signs of the F_j^i $(i = 1, 2; j = c, x)$ it follows that

$$\frac{\partial c^\star}{\partial \rho} > 0 , \qquad \frac{\partial x^\star}{\partial \rho} \gtreqless 0 \Leftrightarrow \frac{[U_{22}^\star \delta_2(\delta_2+\rho)-\sigma_2](\delta_2+\rho)}{[U_{11}^\star \delta_1(\delta_1+\rho)-\sigma_1](\delta_1+\rho)} \gtreqless \frac{\sigma_2 F^{2\star}F_c^{1\star}}{\sigma_1 F^{1\star}F_c^{2\star}} . \qquad \text{(A.21)}$$

A.4 Derivation of the differential equation system

Differentiation of $p_i = -U_i$ (Equation A.7) with respect to time and inserting into Equations (21) and (22) yields, together with the equations of motion (15), a system of four differential equations in the four unknowns c, x, s_1 and s_2:

$$\begin{aligned} \sigma_1 s_1 - U_1(\delta_1+\rho) + U_{11}(F_c^1\dot{c} + F_x^1\dot{x}) &= 0 , \qquad \text{(A.22)} \\ \sigma_2 s_2 - U_2(\delta_2+\rho) + U_{22}(F_c^2\dot{c} + F_x^2\dot{x}) &= 0 , \qquad \text{(A.23)} \\ \dot{s_1} - F^1 + \delta_1 s_1 &= 0 , \qquad \text{(A.24)} \\ \dot{s_2} - F^2 + \delta_2 s_2 &= 0 . \qquad \text{(A.25)} \end{aligned}$$

The conditions (A.22)–(A.25) for an interior optimal solution can be rearranged to yield the system (30)–(33) of four coupled autonomous differential equations.

A.5 Eigenvalues and eigenvectors of the Jacobian

We obtain the Jacobian J^* by differentiating the right-hand-sides of Equations (30)–(33) with respect to c, x, s_1 and s_2 and evaluating them at the stationary

state. Taking into account that in the interior stationary state (28) holds with equality, $U_i = \sigma_i s_i^\star/(\delta_i+\rho)$, one obtains for the Jacobian $J^\star$:

$$J^* = \begin{pmatrix} \rho + \frac{\delta_1 F_c^{1\star} F_x^{2\star} - \delta_2 F_x^{1\star} F_c^{2\star}}{df^\star} & \frac{(\delta_1-\delta_2) F_x^{1\star} F_x^{2\star}}{df^\star} & -\frac{\sigma_1 F_x^{2\star}}{U_{11}^\star df^\star} & \frac{\sigma_2 F_x^{1\star}}{U_{22}^\star df^\star} \\ \frac{(\delta_2-\delta_1) F_c^{1\star} F_c^{2\star}}{df^\star} & \rho + \frac{\delta_2 F_c^{1\star} F_x^{2\star} - \delta_1 F_x^{1\star} F_c^{2\star}}{df^\star} & \frac{\sigma_1 F_c^{2\star}}{U_{11}^\star df^\star} & -\frac{\sigma_2 F_c^{1\star}}{U_{22}^\star df^\star} \\ F_c^{1\star} & F_x^{1\star} & -\delta_1 & 0 \\ F_c^{2\star} & F_x^{2\star} & 0 & -\delta_2 \end{pmatrix} .$$

The eigenvalues ν_i and eigenvectors ξ_i are the solutions of the equation $J^\star \cdot \xi = \nu \cdot \xi$. The four eigenvalues are:

$$\nu_1 = \frac{1}{2}\left[\rho - \sqrt{(\rho+2\delta_1)^2 - \frac{4\sigma_1}{U_{11}^\star}}\right] < 0 \ , \tag{A.26}$$

$$\nu_2 = \frac{1}{2}\left[\rho - \sqrt{(\rho+2\delta_2)^2 - \frac{4\sigma_2}{U_{22}^\star}}\right] < 0 \ , \tag{A.27}$$

$$\nu_3 = \frac{1}{2}\left[\rho + \sqrt{(\rho+2\delta_1)^2 - \frac{4\sigma_1}{U_{11}^\star}}\right] > 0 \ , \tag{A.28}$$

$$\nu_4 = \frac{1}{2}\left[\rho + \sqrt{(\rho+2\delta_2)^2 - \frac{4\sigma_2}{U_{22}^\star}}\right] > 0 \ . \tag{A.29}$$

The eigenvectors associated with the negative eigenvalues ν_1 and ν_2 are:

$$\xi_1 = \left(\frac{F_x^{2\star}(\nu_1+\delta_1)}{df^\star}, -\frac{F_c^{2\star}(\nu_1+\delta_1)}{df^\star}, 1, 0\right) \ , \tag{A.30}$$

$$\xi_2 = \left(-\frac{F_x^{1\star}(\nu_2+\delta_2)}{df^\star}, \frac{F_c^{1\star}(\nu_2+\delta_2)}{df^\star}, 0, 1\right) \ . \tag{A.31}$$

A.6 Time scale of convergence

Equations (35) and (36) are of the following type:

$$z(t) = z^\star + Ae^{\nu_1 t} + Be^{\nu_2 t} \qquad (\nu_1, \nu_2 < 0) \ , \tag{A.32}$$

with real constants A and B. Without loss of generality assume that $|\nu_1| < |\nu_2|$. Since we are interested in the system dynamics in a neighborhood of the stationary state, we calculate the characteristic time scale of convergence for z as

$t \to \infty$. According to (39), the characteristic time scale of convergence of z in a neighborhood of the stationary state $z^\star$ is given by:

$$\tau_z^{-1} = \overline{\left| \lim_{t\to\infty} \frac{A\nu_1 e^{\nu_1 t} + B\nu_2 e^{\nu_2 t}}{Ae^{\nu_1 t} + Be^{\nu_2 t}} \right|} = \overline{\left| \lim_{t\to\infty} \frac{A\nu_1 + B\nu_2 e^{(\nu_2-\nu_1)t}}{A + Be^{(\nu_2-\nu_1)t}} \right|} = |\nu_1| \ . \quad \text{(A.33)}$$

Hence, for $t \to \infty$ the characteristic time scale of convergence is constant and given by $1/\min\{|\nu_1|, |\nu_2|\}$.

A.7 Parameter values for the numerical optimization

We used a Cobb-Douglas welfare function for the numerical optimizations,

$$U(y_1, y_2) = 0.5\ln(y_1) + 0.5\ln(y_2) \ , \quad \text{(A.34)}$$

and the following production functions:

$$P^1(l_1) = \sqrt{l_1} \ , \qquad P^2(l_2) = \sqrt{l_2} \ . \quad \text{(A.35)}$$

For all numerical optimizations we set $\lambda = 1$ and $\rho = 0.03$. In addition, we used the following parameter values for the different scenarios:

Figure	σ_1	σ_2	δ_1	δ_2	s_1	s_2
1	0.01	0.01	0.02	0.1	30	30
2	0.003	0.03	0.05	0.05	40	0
3a	0.002	0.02	0.02	0.1	0	0
3b	0.002	0.02	0.02	0.1	50	0
3c	0.002	0.02	0.02	0.1	0	25
3d	0.002	0.02	0.02	0.1	50	25

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