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## Multi-Bank Loan Pool Contracts

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### Abstract

We show that multi-bank loan pools improve the risk-return profile of banks' loan business. Banks write simple contracts on the proceeds from pooled loan portfolios, taking into account the free-rider problems in joint loan production. Thus, banks benefit greatly from diversifying credit risk while limiting the efficiency loss due to adverse incentives. We present calibration results that the formation of loan pools reduce the volatility in default rates, proxying for credit risk, of participating banks' loan portfolios by roughly 70% in our sample. Under reasonable assumptions, the gain in return on equity (in certainty equivalent terms) is around 20 basis points annually.

EFM classification: 330, 350

## **Introduction**

With the Basle Accord on capital adequacy and increasing default rates, academics and practitioners alike are turning attention to credit risk. Discussions on measuring and managing credit risk often focus on the benefits and disadvantages of credit sales, asset-backed securities, and credit derivatives. Multi-bank loan pools, which we define as an instrument that allows participating banks to share profits and losses from a joint loan pool according to a multilateral contract, have been largely ignored. We show that loan pools cater especially to the needs of smaller, regional banks.

Loan pools exploit the diversification benefits available to large banks, spreading risk over many investments, while using local knowledge available to small banks, focusing on a specific market segment. However, inducing the pool of banks to apply the same care in loan production as an independent bank would is difficult. In a pool, a bank spending extra resources on screening loan applicants and monitoring debtors recoups only a fraction in increased returns; other pool participants capture the rest without any effort of their own. Optimal loan pool contracts take this problem into account trading off the benefits of diversification against diminished incentives to induce effort.

For the class of contracts affine in a bank's own loan portfolio returns and the pooled loan portfolio returns, we derive optimal contracts. These optimal contracts are convex combinations of those returns, the weights depending on the parameters of the problem. We further study the features of the optimal contracts as parameters vary. We also calculate the benefits, measured as certainty equivalent returns, of loan pools. Calibrating an example to data on the regional dispersion of bankruptcy rates, we find that loan pools can almost eliminate idiosyncratic credit risk, reducing total credit risk (measured as volatility of portfolio returns) by as much as 70%. The reduction in credit risk translates into a 20 basis point gain in certainty equivalent returns, after subtracting the efficiency loss to reduced effort.

Loan pools have potentially great advantages for the institutions participating and also the economy as a whole. The participating institutions can reduce significantly the risk of loans, which lowers cost of capital. This, in turn, implies lower interest rate for loans, which gives rise to two macro-economic effects. As a direct effect, lower interest rates will increase investment. As an indirect effect, credit rationing (see Stiglitz and Weiss, 1981) will occur at a lower level, inducing additional investment.

Compared to competing, market-based solutions, such as credit sales or issuing collateralized loan obligations (CLO's), loan pools have probably lower transaction costs. Information asymmetries are likely to be greater between an originating bank and outside market investors than among loan pool participants, who are both originators and investors. Pool participants can employ their own screening technologies combined with local market knowledge to assess the credit risk in the pool. Also, participating banks are probably more willing to share relevant information on loan quality and productivity levels with non-competing peers than with a rating agency or a large number of outside investors. Since information asymmetries have been identified to be a major impediment to transferring credit risk by market transactions, we suggest that multi-bank loan pools provide a cost-efficient instrument to diversify credit risk. Moreover, we argue that the cost of administering a multi-bank loan pool is lower than the combined costs for trust managers, investment banks, lawyers etc that originator banks face in the context of single-seller CLO transactions.

Taking advantage of diversification and exploiting local knowledge, loan pools are also an alternative to bank mergers. Small banks retain their independence, which provides an ideal setting for the generation and exploitation of local knowledge, while achieving diversification usually available only to large banks. Of course, various outsourcing activities may augment loan pools; e.g., in back office operations, information technology, securities trading, and other departments not vital to the core business. In this way, small banks can efficiently focus on the core competencies in retail banking and lending to small and medium-sized business.

The paper is structured as follows. After briefly reviewing the related literature in section 1, we present our model of credit risk in joint loan production and derive optimal contracts in section 2. In section 3, we provide a numerical example based on an empirical analysis of cross-sectional variation of bankruptcy rates. We show what multi-bank loan contracts might look like in the real world and gauge the benefit to participating banks. Section 4 concludes.

## **1. Related Literature**

The theoretical literature on the transfer of credit risks and on pooling, repacking and selling of bank assets in particular, focuses on single-seller settings. Most papers consider

one value-maximizing financial institution and derive optimal contracts between the financial institution and outside investors. More recent work also devises pricing schemes on securitized asset pools.<sup>1</sup> We are, however, not aware of work addressing contracts between several originating banks which reduce asset volatility while controlling adverse incentives.

Most papers address hidden information; how can investors avoid the lemon problem of purchasing a portfolio of bad loans? Greenbaum and Thakor (1987) analyze the effect of adverse selection problems on the balance sheet structure of financial institutions that possess private information on the risk of the assets. These financial institutions securitize and sell assets with a low risk profile and retain the remaining high-risk loans on their books. Agency problems hence preclude an optimal amount of credit risk transfer.

In a hidden action setting similar to ours, Pennacchi (1988) and Gorton and Pennacchi (1995) devise optimal contracts for credit sale transactions given that the seller's ex post monitoring efforts are not observable to the buyer. They analyze how sellers can mitigate the agency costs that emerge from this hidden action problem by retaining a portion of the loan. They conclude that under the optimal contract, this portion is a positive function of the loan's risk and that both, the level of monitoring. Again, the amount of credit risk to be transferred, is lower than under a first-best scenario.

A substantial strand of literature analyzes optimal design of asset-backed securities providing an economic rationale for real-world deal structures. Important contributions are Boot and Thakor (1993), Glaeser and Kallal (1997), Riddiough (1997), DeMarzo and Duffie (1999), and DeMarzo (1999). Assuming asymmetric information between originator and investors, they show how bundling assets, repackaging the proceeds from these assets into different claims, and selling these claims in capital markets can reduce agency costs and thus maximize the benefits from securitization. Usually, pay-through securities constitute the optimal security design, with a senior claim sold to less-informed outside investors and a first-loss or equity claim retained by the financial institution or sold to better-informed investors (Boot and Thakor 1993).<sup>2</sup> For example, in DeMarzo and Duffie (1999) agency costs from adverse selection are a positive function of the sensitivity of an asset-backed security's cash flows to the private information of the originating

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<sup>1</sup> We do not deal with pricing issues explicitly in our paper, but refer to Childs, Ott, and Riddiough (1996) and Duffie and Garleanu (2001) as representative contributions.

<sup>2</sup> Skarabot (2002) contains a model in which the optimal mix of three types of claims, namely senior debt, junior debt and equity, maximizes the value of the securitization entity.

financial institution. As a consequence, financial institutions find it optimal to package and sell-off a claim that bears less credit risk than the overall pool and to retain a higher-risk first-loss claim. Pass-through securities, where outside investors obtain a pro-rate share of the proceeds from an asset pool, rarely emerge as optimal. DeMarzo (1999) and Glaeser and Kallal (1997) find that they are dominated by the sale of portions of single loans, because single-entity pooling deprives the originator of the possibility to use the retained portion of each single loan as a signal for its true value.<sup>3</sup> Most models hence imply that asset pool securitization primarily serves the purpose of reducing refinancing cost and reinvesting the proceeds from securitization into higher-return assets, respectively. Only in Boot and Thakor (1993) does optimal security design allow for the transfer of a significant amount of credit risk.

By shifting the focus from single-originator settings with outside investors to multi-originator settings with inside investors, our model captures credit risk diversification effects and hence ascribes a more vital role to active credit risk management rather than liquidity management. For the sake of tractability we restrict the analysis in this paper to contract structures affine in the returns of loan portfolios. As the literature cited above suggests that convex structures that call for a first-loss claim offer efficiency gains over affine contracts with pro-rate shares in the proceeds, we interpret the contractual arrangements presented as yielding lower-bound efficiency gains over the case of independent loan production.

## 2. Contracts for multi-bank loan pools

We provide a model of credit risk in joint loan production. Focusing on the incentive effects of pooling loans, we derive closed-form solutions in a highly stylized setting

### 2.1. Setup

There is a set of  $N \geq 2$  agents. We interpret agents as individual savings institutions or banks, endowed with preferences and a loan production technology. We describe loans by their stochastic returns. For convenience, we model the loan portfolio of an agent instead of the set of an agent's individual loans. The value of a loan portfolio is the amount of cash  $W_n$  that agent  $n$  has handed to debtors.

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<sup>3</sup> Subramanyam (1991) and Gorton and Penacchi (1993) hold an opposing view.



The return on agent  $n$ 's loan portfolio is  $r_n = m + c_n$ , where the random variable  $m$  is distributed normally with mean  $\mu_m$  and standard deviation  $\sigma_m$ , and the random variables  $c_n$  are distributed normally with mean  $\bar{c}_n$  and standard deviation  $\sigma_n$ . The random variables  $c_n$  are mutually uncorrelated, and are uncorrelated with the random variable  $m$ .

The macro component  $m$  captures credit risk common to the loan portfolios of all agents. Major macro-level risk factors are aggregate default risk, common components in the valuation of collateral, and so forth. In general, the macro component represents credit risk that cannot be altered or diversified by any agent or group of agents. The cluster component  $c_n$  captures credit risk common to all loans on the book of an individual bank. For example, the cluster component represents risk due to debtors being concentrated geographically or in a few industries. In general, the cluster component is risk that cannot be diversified at the level of individual agents, but may be reduced by pooling loans across agents.

We assume that the agent controls the mean  $\bar{c}_n$  of the distribution of  $c_n$  by choosing  $e_n$  such that  $\bar{c}_n = \mu_n + \theta_n e_n$ , where  $\mu_n$  is a real number,  $\theta_n$  is a positive real number, and  $e_n$  is a non-negative real number.<sup>4</sup> We think of  $e_n$  as representing credit risk that the agent can eliminate by applying sufficient care in the loan production process. For example, appropriate screening and efficient monitoring, using specific knowledge available only to the agent, may improve credit quality. Since the return distribution of loans is capped at the face value plus interest payments, reductions in credit risk translate into improved profitability.

The constant  $\theta_n$  captures the agent's productivity, i.e. the effectiveness in controlling loan profitability. The larger the coefficient, the more sensitive is the loan portfolio's profitability to the agent's actions. Productivity  $\theta_n$  may be different across agents; some agents presumably find it easier to control credit quality than others. For example, a loan portfolio consisting largely of established manufacturing firms (having substantial assets in place) is probably more effectively managed than a loan portfolio of growth firms. A similar argument applies comparing home mortgages and consumer loans.

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<sup>4</sup> For now, we assume that the agent controls directly the profitability of the loan portfolio rather than the risk of the portfolio. The agent being able to control only the first moment, and not the second moment of the return distribution, is a restrictive assumption, and we hope to analyze the more general case in the future.

Agents are risk-averse and dislike imposing high levels of control  $e_n$ . To keep the exposition simple and facilitate closed-form solutions, we model agents' preferences by mean-variance utility functions

$$(1) \quad Eu(R_n) = ER_n - \frac{1}{2} A_n Var[R_n] - \frac{1}{2} e_n^2,$$

where  $E$  and  $Var$  denote the expectation and variance operator,  $R_n$  is agent  $n$ 's profit from loan operations, and  $A_n$  is agent  $n$ 's coefficient of risk aversion. If the agent is operating independently,  $R_n = r_n$ . If the agent joins a credit pool,  $R_n$  may be different from  $r_n$ .

This functional form for preferences is standard for individuals. For organizations some discussion is necessary. Ascribing risk aversion to an organization is a reduced-form model of the preferences of actual people within the organization. For example, the managers of savings institutions dislike risk in loan portfolios because bad outcomes may tarnish their reputation, and thus diminish their human capital, may increase the probability of the board or regulators intervening, and so forth. Alternatively, an organization exhibits risk averse behavior, if the payoff function of the equity holders are convex, e.g. due to progressive tax schemes or bankruptcy costs.

The quadratic<sup>5</sup> cost component for  $e_n$  is usually interpreted as aversion to effort. In the case of loan portfolios, the activities of screening and monitoring require resources such as loan officers' time, maintaining data bases, processing loan and debtor specific information, prosecuting debtors in default, and so forth. These activities are associated with costs that are directly measurable. There are costs, however, that are less easily assessed. For example, being more stringent towards debtors in default, which improves loan profitability, might cause psychological stress to the agent or reduce her social status. Finally, it is worth noting that effort enters the utility function in a way similar to expected returns on the loan amount. Thus,  $e_n$  should be interpreted as rate of effort per loan amount rather than a level of effort.

## 2.2. Independent Operations

If banks operate in isolation, optimal strategies maximize the utility function (1) with respect to the effort level  $e_n$ . The optimal effort level is  $e_n^{IO} = \theta_n$ ; marginal benefit  $\theta_n$  equals marginal cost  $e_n$ . The expected profitability of the loan portfolio is  $\mu_m + \mu_n + \theta_n^2$ , while the

risk is  $\sigma_m^2 + \sigma_n^2$ , and the costs of the loan production are  $\frac{1}{2}\theta_n^2$ , and, therefore, the expected utility is  $Eu_n^{IO} = \mu_m + \mu_n + \frac{1}{2}\theta_n^2 - \frac{1}{2}A_n(\sigma_m^2 + \sigma_n^2)$ . This expected utility is the benchmark against which to measure the benefits of loan pools. Only if joint production improves upon this benchmark expected utility, agents are willing to participate in the pool.

### 2.3. Loan Pools

The value of the loan pool is  $W = \sum W_n$ . Thus, agent  $n$ 's percentage contribution to the pool is  $w_n = W_n / W$ . The return on the pool is  $r = \sum w_n r_n$ . The principal advantage of a loan pool is the potential to diversify imperfectly correlated risks. The variance of the pool's return is  $Var(r) = \sigma_m^2 + \sum w_n^2 \sigma_n^2$ . As the pool grows larger, the individual share  $w_n$  of agent  $n$  decreases. Thus, the pool risk decreases under reasonable conditions. If, for all  $n$  and some finite numbers  $u$  and  $v$ ,  $\sigma_n^2 < u$  and  $W_n < v$ , then  $Var(r) \rightarrow \sigma_m^2$  as  $N \rightarrow \infty$ . This is a direct consequence of the law of large numbers. Thus, the pool's risk reduces to the macro component for a large pool of banks that are small relative to each other. This captures the intuition that, as the pool grows, risk is diversified across more members. However, this intuition is not always true. For example, adding a very large individual loan portfolio to a small pool might increase the risk of the pool. Similarly, adding a very large risk to a relatively safe pool might increase pool risk.

A naïvely constructed loan pool, ignoring the incentive effects of joint production, might stipulate that all members of the pool share in the returns according to  $w_n$ , the share of the loan value contributed. In this case, the return of agent  $n$ 's portfolio is simply  $r$ . Collecting only terms in the expected utility that depend on  $e_n$ ,

$$(2) \quad Eu(R_n) \propto w_n \theta_n e_n - \frac{1}{2} e_n^2.$$

Thus, the marginal benefit from employing unit of  $e_n$  is no longer  $\theta_n$  but  $w_n$  times  $\theta_n$ . By the first order condition, agent  $n$  chooses effort level

$$e_n = w_n \theta_n = w_n e_n^{IO} < e_n^{IO}.$$

Since  $w_n$  is a number less than one, and considerably less than one for large pools, agents significantly reduce effort in a pool operating under such a sharing rule. Agents' incentive

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<sup>5</sup> The form of the cost function is not meant to be too descriptive. A quadratic cost term captures increasing marginal costs, yet delivers tractable solutions. As with all our simplifying assumptions, more realistic choices can be easily implemented in applied work.

to shirk, i.e., free-riding on other agents' effort, is the key problem in joint production. Benefits of diversification are bought at the price of greatly reduced production efficiency. Depending on the relative magnitude of diversification and reduction in efficiency, a pool with such a simple sharing rule might, or might not, be a viable alternative to independent operations. However, this simple sharing rule is rather arbitrary.

#### 2.4. Contracts

Trading off the benefits from diversification against the efficiency loss is the task when designing contracts for loan pools. Before agents set up the pool, they bargain over a sharing rule for the pool's profits at the end of the contracting period. After having reached an agreement over the sharing rule, each agent engages in loan production. At the end of the period contracted for, agents share profits and losses according to the sharing rule agreed upon. Agents cannot re-negotiate the sharing rule.

In general, contracts may depend only on variables that a court of law can observe and verify. We assume that returns on individual portfolios are generally observable and verifiable, or 'contractible', since elaborate accounting, auditing, and regulatory systems are in place. Hence, the returns of the grand portfolio are also contractible.

In this paper, we also assume that productivity  $\theta_n$  and risk aversion  $A_n$  are observable. More generally, the agreement upon these values is part of the negotiation stage. However, solving the negotiation game is complicated enough in its most simple form. Therefore, we do not pursue this issue further for the time being.

Effort might also be contractible although we think it rather unlikely. Without the special knowledge residing with the agent, judging the effort of the agent is difficult. For an outsider it is very costly to assess, for example, the results of screening processes; have loans been granted on grounds economically sound, or on the basis of political considerations?

We further require a sharing rule to be feasible in the sense that the revenues of the pool are completely distributed among the participants. Thus, the credit pool is a closed system; payments may not flow to, or from, the pool. The system is closed if  $r = \sum w_n R_n$  for every realization of the normally distributed random variables  $r_n$ . This condition is also known as budget balancing. While the assumption of a closed system is very natural, it is difficult to handle mathematically for arbitrary contracts. Thus, we restrict attention to contracts that are simple in structure and guarantee a closed system.

We consider only contracts affine in agents' own returns and pool returns, i.e. contracts of the form  $R_n = \alpha_n + \beta_n r_n + \gamma_n r$ . Such contracts enable agents to benefit from diversification (through the third component) and provide incentives (through the second component). Affine contracts in a one-shot game can be justified as arising from a continuous-time game with arbitrary contracts (Holmström and Milgrom, 1987). If the agent can observe the result of her actions continuously, and adapt actions accordingly, an affine contract is pareto-efficient. The closed system-condition has strong implications for the contract parameters  $\alpha_n$ ,  $\beta_n$ , and  $\gamma_n$ .

Proposition 1. If the pool is a closed system, then  $\sum w_n \alpha_n = 0$  and  $\beta_n = 1 - \sum w_n \gamma_n = \beta$  for all agents  $n$ .

In other words, the fixed components of the contract serve as pure side payments. This means that some prospective members might pay to join the pool, while others might need compensation to participate. The closed system condition also has strong implications for the relation between  $\beta_n$  and  $\gamma_n$ . In particular, the coefficient on the individual return component is the same across all agents. Thus, contracts admissible under the closed system condition have the form  $R_n = \alpha_n + (1 - \sum_i w_i \gamma_i) r_n + \gamma_n r$ . In other words, agents need only bargain over  $\alpha_n$  and  $\gamma_n$ . The coefficient  $\beta$  is completely determined by the choice of  $\gamma_n$ . For example, if the agents were to choose  $\gamma_n = 0.8$  for all  $n$ , then  $\beta = 0.2$ .

A loan pool is viable if all agents are better off participating in the pool than not,

$$Eu(R_n) \geq Eu_n^{IO}.$$

There may be loan pools that are viable for a strict subset of agents but not for all agents. In this case, we redefine the set of agents as one of the subsets such that the loan pool is viable for all members of this subset. It is possible that there is more than one such subset. It is also possible that there is a strict subset of agents preferring a viable loan pool amongst themselves over a viable loan pool comprising all agents. In other words, some members might wish to exclude other potential members. We sidestep such issues at the present. In the remainder, we focus on viable loan pools comprising all agents.

### 2.5. Operating the Pool

Collecting only terms that depend on  $e_n$ , for an agent that takes part in a loan pool operating under an affine admissible contract,

$$(3) \quad Eu(R_n) \propto \left(1 - \sum_{i \neq n} w_i \gamma_i\right) \theta_n e_n - \frac{1}{2} e_n^2.$$

Thus, under such a contract agent  $n$  chooses effort level

$$e_n^P = \max\left[0, \left(1 - \sum_{i \neq n} w_i \gamma_i\right) \theta_n\right] = \max\left[0, \left(1 - \sum_{i \neq n} w_i \gamma_i\right) e_n^{IO}\right].$$

Using the same numerical example as above, if agents agree on  $\gamma_n = 0.8$  and if the pool is large such that  $w_n$  goes to zero, the factor  $\left(1 - \sum_{i \neq n} w_i \gamma_i\right)$  approaches 0.2. In other words, under such a contract the level of effort in the pool is 20% of that under independent operation.

In game theory parlance,  $e_n^P$  is the reaction function for the production stage, given a specific contract. The reaction function does not depend on the other agents' actions at this stage of the game. It does depend only on the contract, and, therefore, on the actions of all agents at the negotiation stage.

### 2.6. Negotiating Contracts

Equipped with the reaction function, we analyze the negotiation over loan pool contracts. The reaction function enables each participant, at the negotiation stage, to forecast the future behavior of all other participants at the subsequent, production stage of the game. Taking future behavior into account, potential participants decide – jointly – on the contract to be used at the production stage.

For simplicity, we consider only the symmetric case in which all agents are identical, i.e. we set  $A_n = A$ ,  $\mu_n = \mu$ ,  $\theta_n = \theta$ ,  $w_n = w$ , and  $\sigma_n = \sigma$  for all  $n$ . In this case, symmetric, pareto-efficient contracts are natural equilibrium outcomes. More precisely, if agents are identical symmetric, pareto-efficient contracts are in the set of equilibria at the negotiation stage.

**Proposition 2.** For all  $N$ ,  $\theta$ ,  $\sigma$  and  $A$ , there is a viable symmetric, pareto-efficient contract that stipulates for all  $n$

$$\alpha_n = 0 \text{ and } \gamma_n = \gamma = \frac{A\sigma^2}{\theta^2\left(\frac{N-1}{N}\right) + A\sigma^2}, \text{ where } 0 < \gamma < 1.$$

Notably, there exists a *viable* pareto-efficient contract for *all* parameter values. In other words, we can always devise a contract that makes pool participants weakly better off than operating independently. Second, the symmetric, pareto-efficient contract depends on the productive efficiency, risk aversion, and the risk of individual return component. The contract does not depend on the means of the common component or the individual components. Neither does the risk of the common component affect the contract. Third, under the symmetric, pareto-efficient contract the return to a pool member is a strictly convex combination of the individual return and the pool return. In particular, the member's return has always some exposure to both individual returns and pool returns.

Proposition 3. The symmetric, pareto-efficient contract exhibits the following properties

$$(1) \frac{\partial \gamma}{\partial N} < 0, \quad (2) \frac{\partial \gamma}{\partial \theta} < 0, \quad (3) \frac{\partial \gamma}{\partial \sigma^2} > 0, \quad \text{and} \quad (4) \frac{\partial \gamma}{\partial A} > 0.$$

Property (1) states that, as the pool grows larger ( $N$  increases), the symmetric, pareto-efficient contract applies less weight to the common component ( $\gamma$  is smaller). Since  $\gamma$  and  $\beta$  are inversely related, this implies a greater weight on the individual return component ( $\beta$  is larger). Thus, agents share less of the common return and retain more of their individual returns in large pools. Intuitively, a larger pool provides inherently more benefits of diversification in the common return, so that individual incentives can be strengthened. A complementary intuition is that, as pools grow larger, shirking becomes more pervasive inducing contracts with stronger incentives. Property (2) of the symmetric, pareto-efficient contract is that, as the productive efficiency of individual members increases ( $\theta$  increases), members receive a smaller fraction ( $\gamma$  decreases) of the pool's returns and retain more individual risk. More productive agents can control risk at lower cost, and thus receive strong incentives to do so. Properties (3) and (4) are related to risk and risk-aversion, and have straight-forward interpretations; more risk in individual return components induces more risk-sharing, as does larger risk-aversion.

Under the symmetric contract, all agents provide effort  $e^P = \left(1 - \frac{N-1}{N}\gamma\right)\theta = \left(1 - \frac{N-1}{N}\gamma\right)e^{IO}$ .

The efficiency factor  $\phi = 1 - \frac{N-1}{N}\gamma$  captures the difference between the effort agents provide in a pool and the effort agents exert when operating independently.

Proposition 4. The efficiency factor  $\phi$  of the symmetric, pareto-efficient contract satisfies

$$(1) \ 0 < \phi < 1, \ (2) \ \frac{\partial \phi}{\partial N} < 0, \ (3) \ \frac{\partial \phi}{\partial \theta} > 0, \ (4) \ \frac{\partial \phi}{\partial \sigma^2} < 0, \ (5) \ \frac{\partial \phi}{\partial A} < 0.$$

Property (1) shows that there is always some loss in productive efficiency, since the effort contributed to the pool is never as large as when operating independently. On the other hand, incentives are never so weak as to prevent any effort contribution at all. Property (2) states that less effort is contributed to larger pools. This result is surprising given that contracts of larger pools provide stronger incentives [Proposition 3(1)]. However, the definition of the efficiency factor  $\phi$  shows that the pool size has a direct effect through  $N$  and an indirect effect through  $\gamma$ . It so happens that explicitly strengthening incentives in the contract does not compensate for the implicit reduction in incentives due to enlarging the pool. Property (3) indicates that more efficient producers are affected less by incentive problems, whereas properties (4) and (5) show that more risk or higher risk aversion render pool production less efficient.

Under a symmetric contract, credit risk, measured as the variance, is

$Var[R] = \sigma^2 \left( 1 - \frac{N-1}{N} \gamma(2-\gamma) \right)$ . Thus, the risk factor  $\zeta = \left( 1 - \frac{N-1}{N} \gamma(2-\gamma) \right)$  captures the difference between credit risk when pooling loans and independent operations. The risk factor captures the pool's diversification potential. A risk factor of one indicates no diversification, a risk factor of zero indicates complete diversification, i.e., elimination of idiosyncratic risk.

Proposition 5. The risk factor  $\zeta$  of the symmetric, pareto-efficient contract satisfies

$$(1) \ 0 < \zeta < 1, \ (2) \ \frac{\partial \zeta}{\partial N} < 0, \ (3) \ \frac{\partial \zeta}{\partial \theta} > 0, \ (4) \ \frac{\partial \zeta}{\partial \sigma^2} < 0, \ (5) \ \frac{\partial \zeta}{\partial A} < 0.$$

Property (1) shows that optimal contracts diversify, but some idiosyncratic risk remains. Not surprisingly, property (2) shows that diversification increases in the number of participants. Property (3) indicates that diversification is lower if the pool member are more productive. This result might be surprising at first sight, but is consistent with stronger incentives for more productive agents. Intuitively, more productive agents face a steeper trade-off between incentives and risk. As expected, properties (4) and (5) indicate



that more risk or higher risk aversion induce contracts with a larger extent of diversification.

Proposition 6. Under a symmetric, pareto-efficient contracts agents are strictly better off than under independent operations, i.e.,  $Eu(R) > Eu^{IO}$  for all  $N$ ,  $\theta$ ,  $\sigma$  and  $A$ . More specifically, the utility gain from entering a pool is

$$\Delta Eu = Eu(R) - Eu^{IO} = \frac{A\sigma^2}{2} \left( \frac{N-1}{N} \right) \gamma.$$

This result states that the benefits of diversification strictly outweigh the incentive problems of joint production under optimally designed contracts. In other words, if running a pool carries no costs other than reduced efficiency, optimally designed pools are always profitable! In practice, there are set-up costs and administrative costs. The quantity  $\Delta Eu$ , which is equal to the certainty equivalent return (CER), indicates the upper bound on such costs (in percent per annum) for profitable pools. Proposition 6 shows further that the benefits of forming a pool are a function of the pool size  $N$ , productive efficiency  $\theta$ , idiosyncratic risk  $\sigma^2$ , and risk aversion  $A$  only.

Proposition 7. The gain from adding a new member to the pool is  $\frac{\partial \Delta Eu}{\partial N} = \frac{A\sigma^2}{2} \frac{\gamma^2}{N^2} > 0$ .

Thus, the largest pool possible is best. However, as is clear from Proposition 6, the marginal benefit of adding new members levels off quickly; the marginal benefit decreases at a rate of  $\gamma^2 / N^2$ , an upper bound on which is  $1 / N^2$ ! In other words, pools of relatively modest size achieve already large gains of diversification.

### 3. Economic Relevance of Loan Pools: A Numerical Example

To understand better the behavior of contracts and to evaluate loan pools' economic relevance, we calibrate an numerical example. We discuss a base case, using reasonable estimates of the parameters. In addition, we provide sensitivity analyses for a range of parameters.

### 3.1 Empirical Evidence on the Regional Variation in Bankruptcy Rates

For our calculations, we require an estimate of loan pools' diversification potential. One source of diversification is the regional variation in credit risk. As an illustration, we study the cross-sectional dependence and variation in bankruptcy across the 96 counties in the state of Bavaria. At the end of 2001, 506 thousand or 17.4% of a total 2.9 million German enterprises were located in Bavaria.<sup>6</sup> 3,943 Bavarian enterprises went bankrupt in 2001, resulting in a total bankruptcy rate of 0.78%<sup>7</sup>.

Figure 1 shows the time-series average of annual bankruptcy rates for each of the 96 counties.<sup>8</sup> Average bankruptcy rates range from 0.27% to 0.87% suggesting a large differential in expected default rates for each county.<sup>9</sup> Figure 2 documents the time-series variation of annual bankruptcy rates across the 96 counties. For each of the 14 years, we sort counties into deciles according to bankruptcy rates, and plot the average bankruptcy rates for the deciles against the bankruptcy rates for the entire sample. Average bankruptcy rates for the tenth decile exceed bankruptcy rates for the first decile by a factor ranging from 3.6 (1992) to 13.0 (1980), indicating - not surprisingly - considerable regional differences in bankruptcy rates also for any given year. Notably, the membership of a county to a particular decile or to a small number of adjacent deciles, is not stable over time. In fact, not a single county belonged to the same or to the same pair of deciles over our observation period. Instead, 90% of the counties switched between five or more deciles and 50% switched between 7 and more deciles.

Figure 3 and Table 1 provide additional evidence on the variation of bankruptcy rates across counties and time. In Figure 2, we rank the 96 counties according to the standard deviation of bankruptcy rates over the 14 years. For 84 out of the 96 counties standard deviations are higher than the corresponding value (0.15%) for the entire state of Bavaria. Regressing annual bankruptcy rates for each county on the annual bankruptcy rate

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<sup>6</sup> Data on the number of enterprises and bankruptcies are from the Federal German Statistical Office ([www.destatis.de](http://www.destatis.de)) and the Bavarian Statistical Office ([www.statistik.bayern.de](http://www.statistik.bayern.de))

<sup>7</sup> In 2001, the bankruptcy rate for Germany was considerably higher at 1.11%, although the Bavarian industry mix was broadly in line with the German average: 48% of Bavarian enterprises (46% of German enterprises) belonged to the services sector, 25% (24%) to the retail/wholesale sector, 11% (10%) to the construction sector, 11% (10%) to the manufacturing sector and 7% (8%) to all other sectors.

<sup>8</sup> Due to data unavailability we were only able to compile bankruptcy rates for 14 selected years between 1980 and 2001.

<sup>9</sup> Industry mix does not explain the differential in bankruptcy rates between counties. In a panel study, we regressed the difference between a county's bankruptcy rate and Bavaria's total bankruptcy rate on the share of four industries in the total number of enterprises for this county. We found none of the four coefficients to be statistically significant.

for Bavaria, we decompose the variation in county bankruptcy rates into a systematic component (the  $R^2$  from each regression) and a county-specific component (one minus  $R^2$ ). The dark bars in Figure 3 show the systematic variation in “bankruptcy risk” whereas the light colored bars indicate the component that is due to county-specific effects. The ratio of state-wide effects and county-specific effects varies widely across counties but – as indicated by a correlation coefficient below 1% - is not related to the total standard deviation for a given county. On average, 50% of the time-series variation in county bankruptcy rates is due to specific effects. Table 1 shows the distribution of the prevalence of county-specific effects across the 96 counties. Not a single county has a well diversified portfolio of firms in the sense that county-specific effects play no role in explaining bankruptcy rates. Rather, specific effects dominate systematic effects for half of the counties. All in all, the evidence supports our claim that most banks with a large credit exposure in a single county could significantly reduce volatility in default rates if they participated in a multi-bank loan pool.

Statistics from the Bavarian association of savings banks reveal that this insight can be directly applied to the situation of its member institutions and arguably also to most other German savings banks and to most German co-operative banks. At the end of 2001, 84 savings banks operated in Bavaria. This number came down from 91 at the end of 2000 and from over 100 during most of the 1980s and 1990s. Bavarian savings banks are owned by local (county or city) governments. To avoid intra-group competition, savings banks may not operate outside their local area or encroach upon their neighbors’ territory.<sup>10</sup> Hence, the local markets of Bavarian savings banks roughly coincide with the 96 counties in our sample. Moreover, the loan portfolios of Bavarian savings banks, which comprise roughly 20% of total bank loans to all Bavarian enterprises and roughly 40% of total bank loans to Bavaria’s small and medium sized enterprises, can be safely assumed to be representative of the local industry structure.<sup>11</sup> Thus, bankruptcy rates per county should be good approximations for the default rates in the loan portfolios of local savings banks. In this case, pooling loans would indeed reduce the variance in the loan losses of most Bavarian savings banks, allowing them to afford smaller economic equity cushions.

### 3.2. Calibrating an Example

First, we estimate of idiosyncratic credit risk, productive efficiency, risk aversion, and the number of pool participants. The average time-series standard deviation of county bankruptcy rates is 0.25%, of which approximately 0.7 ( $R^2 = 50\%$ ) is idiosyncratic to each county. We assume that the bankruptcy rate is a good proxy for the default rate of county banks. Assuming a stable recovery rate, we arrive at idiosyncratic credit risk (defined as annual standard deviation of returns due to credit defaults) of  $\sigma^2 = 0.125\%$  of an average bank's loan portfolio.<sup>12</sup> This number might appear too small to bother. We stress, however, that total default risk, which banks and regulators clearly consider substantial, is of similar magnitude. The reason is, of course, the considerable leverage in bank balance sheets. While credit risk is small compared to total bank assets, it is more substantial when compared to banks' regulatory equity. Assuming a leverage of 20, total credit risk contributes 500 basis points to volatility on equity. Accordingly, idiosyncratic, diversifiable risk is 350 basis points volatility on equity. From this perspective, credit risk is substantial, consistent with the amount of resources devoted to risk management.

To obtain an estimate of productivity, we suggest that 1% p.a. in costs should generate 1% p.a. in added return from fewer defaults and higher recovery rates. These figures imply a productivity estimate  $\theta = 0.05$ . This number appears plausible, but is not based on any data. Finally, we require an estimate of risk aversion. Commonly numbers between 1 and 10 are regarded reasonable. We opt for the middle,  $A = 5$ . For the number of pool members, we choose the number of counties,  $N = 96$ . These numbers constitute the base case.

The optimal contract parameters are  $\gamma = 71\%$  and  $\beta = 29\%$ . Thus, the return accruing to a pool member is an average of the pool return weighted by 0.71 and the individual member's loan performance weighted by 0.29. The effort factor is approximately 0.29, i.e., pool members provide only roughly one third of the effort they would provide if operating independently. Idiosyncratic credit risk is almost entirely eliminated at 0.2% (compared to 3.5% under independent operations). The net effect of

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<sup>10</sup> Hackethal (2003) provides more detail on the German savings banking sector and its legal ramifications.

<sup>11</sup> A comparison of the Bavarian industry mix (by number) with the industry mix of the aggregated loan portfolio of all Bavarian savings banks (by outstanding loan amounts) for 2001 shows notable deviations only for the retail/wholesale and the construction sectors. These deviations can, however, be explained by the smaller balance sheet of the typical retailer as compared to the typical construction enterprise.

<sup>12</sup> From the Bavarian data, the expected loss from credit defaults is 0.55% per annum.

lower effort and lower credit risk induces a certainty equivalent return (CER) gain of 0.23%. In other words, participating in a loan pool has the same value to equity holders as receiving an additional 23 basis points with certainty.

Next, we study the effect of varying the pool size  $N$ . In Figure 3, we depict the effect on the contract parameter  $\gamma$ , the efficiency factor  $\phi$ , and the CER from pooling loans. Except for the pool size  $N$ , all parameters are the same as in the base case. Figure 3 shows that the contract parameter, efficiency factor, and the CER all stabilize very quickly as  $N$  grows. Above 20 members, changes are barely perceivable.

Figure 4 shows the dependence of the contract variables on productive efficiency  $\theta$ . Here the patterns are more interesting. For values of  $\theta$  between 0 and 0.25, both the contract parameter  $\gamma$  and the efficiency factor  $\phi$  vary widely between 0% and 100%. Even in the smaller  $\theta$  region between 0 and 0.1,  $\gamma$  varies between 100% and 40%. Accordingly, CER's also vary greatly between 30 bp and 0 bp. Thus, obtaining a precise estimate of the productivity parameter  $\theta$  is very important when designing optimal contracts.

Figure 5 depicts the relation between idiosyncratic credit risk  $\sigma$  and the contract variables. Over the range between 0% and 7.5% volatility, the contract parameter  $\gamma$  increases from 0% to roughly 90%. In a narrower range of 1% around the base case volatility of 3.5%,  $\gamma$  varies between approximately 60% and 80%, and the CER varies between 10 bp and 50 bp.

In general, we find that obtaining precise estimates for the parameters of the problem, namely productive efficiency  $\theta$ , idiosyncratic credit risk  $\sigma$ , and risk aversion  $A$  is very important for determining optimal contracts. In contrast, the number of pool participants, as long as it is above reasonable minimum (say ten), is not crucial.

#### 4. Conclusion

In this paper, we devise contracts, which are optimal within a class of simple (affine) contracts, for multi-bank loan pools. Affine contracts are pareto-efficient in a continuous control setting (Holmström and Milgrom, 1987), and, thus, cover a wide range of practical cases. Optimal contracts strike the ideal balance between diversifying credit risk and inducing incentives to manage loan portfolios effectively. Loan pools enable small and medium-sized banks to benefit from diversifying credit risk, while remaining independent

and exploiting local knowledge. We argue that loan pools are a cost-efficient alternative to market-based solutions such as credit sales and collateralized loan obligations. Information asymmetries are presumably lower between banks of similar types than between banks and general investors.

To gauge the benefits of loan pools, we provide empirical evidence on the cross-sectional and time-series variation in bankruptcy rates, which proxy for default risk, within the German state of Bavaria. The variation in the state-wide bankruptcy rate explains, on average, 50% of the time-series variation in bankruptcy rates of individual counties, typically corresponding to the area covered by individual savings banks. In other words, 50% of credit risk of individual institutions is idiosyncratic and, thus, can be diversified in principle. For reasonable estimates of productive efficiency and risk aversion, we find that optimal contracts specify pool members' returns as a combination of 70% pool returns and 30% returns on the performance of the member's loan portfolio. Such a contract reduces idiosyncratic risk by roughly 90% and effort by roughly 70%. The net benefit, which we measure as the gain in certainty equivalent return, of forming a loan pool is 23 basis points in this example.

In this paper, we restrict our attention to the simplest problem. In an straightforward extension, we may allow for heterogeneous participants in terms of size, productivity, credit risk in the loan portfolio, and risk aversion. We expect differences in efficiency and credit risk across banks to generate patterns in contract parameters. In particular, larger or riskier banks, contributing more risk to the pool risk, will receive less exposure to individual contribution. More productive or less risk averse banks, being able to deal more efficiently with risk, will be exposed more to individual performance. Furthermore, considering also non-affine contract structures, i.e., more debt-like structures, could provide interesting insights. In particular, in the presence of asymmetric information (about efficiency and credit risk) more complex structures probably offer efficiency gains over simple, affine contracts.

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## Appendix

Proof of Proposition 1.

Since the component  $\alpha_n$  does not depend on the realization of returns, and the closed system condition must hold for every realization of returns,  $\sum w_n \alpha_n = 0$ .

Agent  $n$ 's return under the contract is  $R_n = \alpha_n + \beta_n r_n + \gamma_n \sum_i w_i r_i$ . The closed system condition requires that  $\sum_n w_n (\beta_n r_n + \gamma_n \sum_i w_i r_i) = \sum w_n r_n$  for every realization (of the vector of random variables  $r_n$ ). Rewriting,  $\sum_n [\beta_n + \sum_i w_i \gamma_i - 1] w_n r_n = 0$  for every realization (of the vector of  $r_n$ 's). Given any number of unique realizations (of the vector), this forms a system of linear equations with as many equations as realizations. The set of unique realizations is uncountable. However, uncountably many unique realizations are linear transformations of other unique realizations (an example is "all returns are 1%" and the linear transform "all returns are 2%"). Nonetheless, the number of linearly independent unique realizations is uncountable, while the number  $n$  of variables is finite. Therefore, the solution of this system of equations is trivial. Thus, for all  $n$   $\beta_n + \sum w_i \gamma_i - 1 = 0$ .

Proof of Proposition 2.

(1) From proposition 1 and  $\alpha_n = \alpha$  for all  $n$ ,  $\alpha = 0$ .

(2) If optimal effort (as prescribed by the reaction function) is strictly positive (we show below that this is indeed the case for optimal contracts), the expected utility, given optimal effort, under the contract  $\gamma$  is

$$Eu(R) = \mu + \mu_n + \theta^2 \left(1 - \frac{N-1}{N} \gamma\right) - \frac{\theta^2}{2} \left(1 - \frac{N-1}{N} \gamma\right)^2 - \frac{A}{2} \left[ \sigma_m^2 + \sigma^2 \left( (1-\gamma)^2 + \frac{2\gamma}{N} - \frac{\gamma^2}{N} \right) \right].$$

Differentiating with respect to  $\gamma$ , applying the first-order condition, and solving for  $\gamma$  yields the symmetric, pareto-efficient contract.

(3) Clearly,  $\theta^2 \frac{N-1}{N} + A\sigma^2 > 0$ . Thus,  $\gamma > 0$ . We have further

$$\beta = 1 - \gamma = \frac{\theta^2 \frac{N-1}{N}}{\theta^2 \frac{N-1}{N} + A\sigma^2} > 0.$$



Since  $1 = \beta + \gamma$  and  $\gamma > 0$  and  $\beta > 0$ , it follows that  $\gamma < 1$ . This also shows that, under the optimal contract, effort is strictly positive.

(4) Calculating the difference in expected utility under the pool and independent operations,

$$\begin{aligned}\Delta Eu &= Eu(R) - Eu^{IO} \\ &= \frac{\gamma^2}{2} \frac{N-1}{N} \left[ -A\sigma^2 - \theta^2 \frac{N-1}{N} \right] + \gamma \frac{N-1}{N} A\sigma^2.\end{aligned}$$

Substituting the symmetric, pareto-efficient contract

$$\Delta Eu = \gamma \frac{A\sigma^2}{2} \frac{N-1}{N} > 0.$$

Thus, the pool is (strictly) viable under the symmetric, pareto-efficient contract.

Proof of Proposition 3.

$$(1) \quad \frac{\partial \gamma}{\partial N} = \frac{-1}{N(N-1)} \gamma(1-\gamma) < 0.$$

$$(2) \quad \frac{\partial \gamma}{\partial \theta} = \frac{-2}{\theta} \gamma(1-\gamma) < 0.$$

$$(3) \quad \frac{\partial \gamma}{\partial \sigma^2} = \frac{1}{\sigma^2} \gamma(1-\gamma) > 0.$$

$$(4) \quad \frac{\partial \gamma}{\partial A} = \frac{1}{A} \gamma(1-\gamma) > 0.$$

Proof of Proposition 4.

$$(1) \quad \text{Both, } 0 < \frac{N-1}{N} < 1 \text{ and } 0 < \gamma < 1. \text{ Thus, } 0 < \frac{N-1}{N} \gamma < 1.$$

$$(2) \quad \frac{\partial \phi}{\partial N} = -\frac{\gamma^2}{N^2} < 0.$$

The other properties follow immediately from Proposition 5.

Proof of Proposition 5.

(1) The risk factor is a positive parabola in  $\gamma$

$$\zeta = \frac{N-1}{N} \gamma^2 - 2 \frac{N-1}{N} \gamma + 1,$$

which has no roots since  $4 \left( \frac{N-1}{N} \right)^2 - 4 \frac{N-1}{N} = -\frac{4(N-1)}{N^2} < 0$ . Thus, for all  $\gamma, \zeta > 0$ .

Further,  $\frac{N-1}{N} \gamma(2-\gamma) > 0 \Leftrightarrow 1 > 1 - \frac{N-1}{N} \gamma(2-\gamma) = \xi$ .

$$(2) \quad \frac{\partial \zeta}{\partial N} = -\frac{\gamma^2}{N^2} (3-2\gamma) < 0.$$

$$(3) \quad \frac{\partial \zeta}{\partial \theta} = -2 \frac{N-1}{N} \left[ \frac{\partial \gamma}{\partial \theta} (1-\gamma) \right] > 0.$$

$$(4) \quad \frac{\partial \zeta}{\partial \sigma^2} = -2 \frac{N-1}{N} \left[ \frac{\partial \gamma}{\partial \sigma^2} (1-\gamma) \right] < 0.$$

$$(5) \quad \frac{\partial \zeta}{\partial A} = -2 \frac{N-1}{N} \left[ \frac{\partial \gamma}{\partial A} (1-\gamma) \right] < 0.$$

Proof of Proposition 6.

See (4) in the Proof of Proposition 2.

Proof of Proposition 7.

Straightforward.

Table 1: Fraction of county-specific effects (frequencies for 96 counties)

| Range             | 0-<br>0.1 | 0.1-<br>0.2 | 0.2-<br>0.3 | 0.3-<br>0.4 | 0.4-<br>0.5 | 0.5-<br>0.6 | 0.6-<br>0.7 | 0.7-<br>0.8 | 0.8-<br>0.9 | 0.9-1 |
|-------------------|-----------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------|
| Frequency         | 0         | 2           | 12          | 17          | 17          | 23          | 11          | 8           | 4           | 2     |
| Cum.<br>Frequency | 0%        | 2%          | 15%         | 32%         | 50%         | 74%         | 85%         | 94%         | 98%         | 100%  |

Figure 1. Average bankruptcy rates in Bavarian counties (1980-2001)

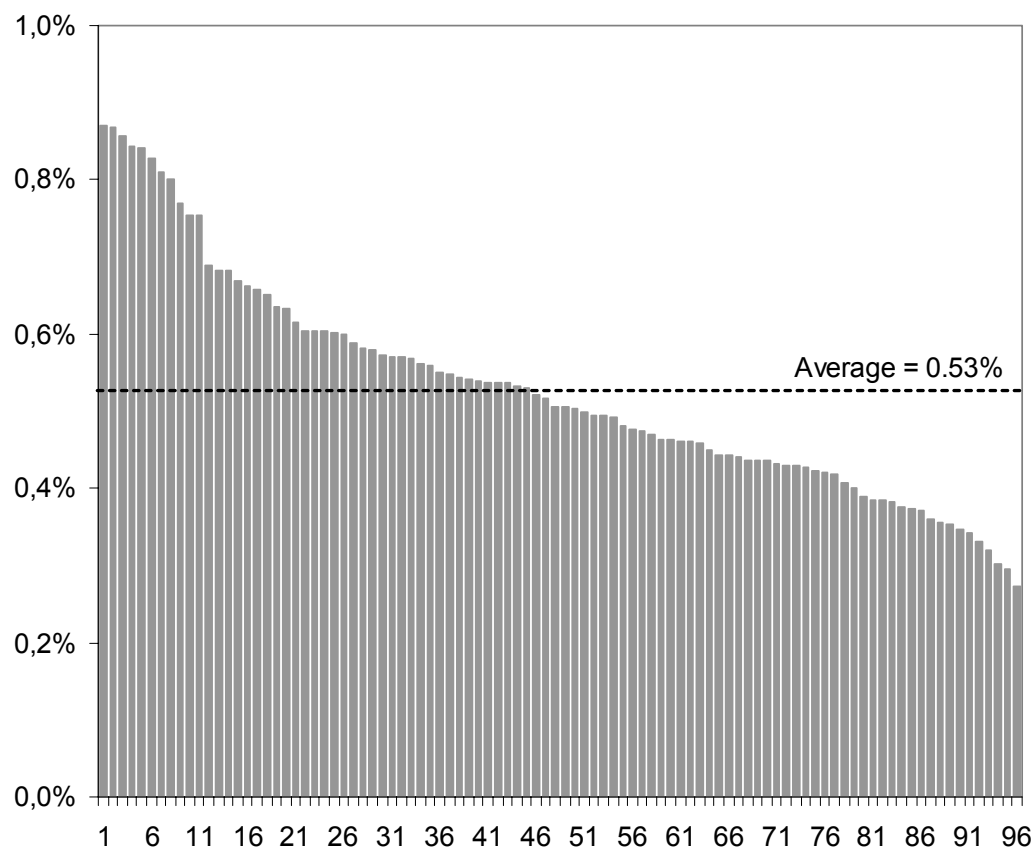


Figure 2. Average bankruptcy rates by deciles and year

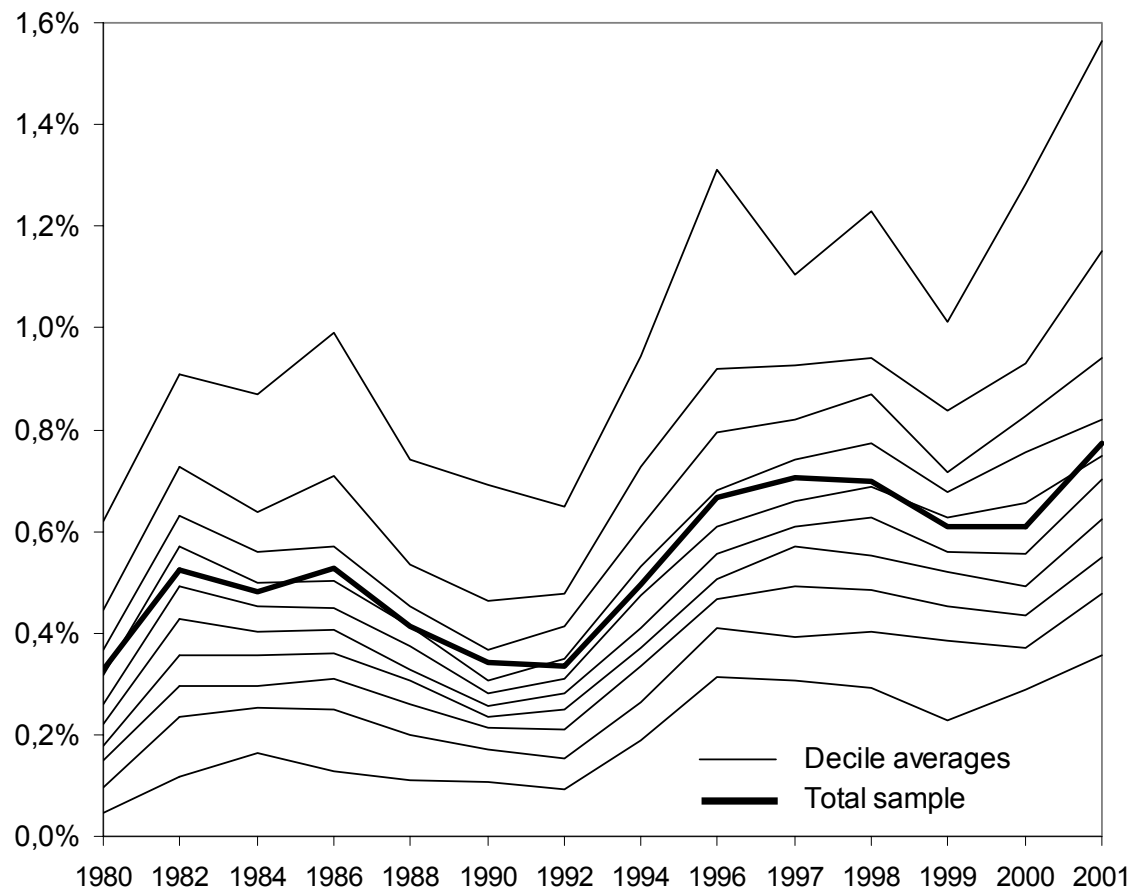


Figure 3. Time-series standard deviation of bankruptcy rates by county (1980-2001)

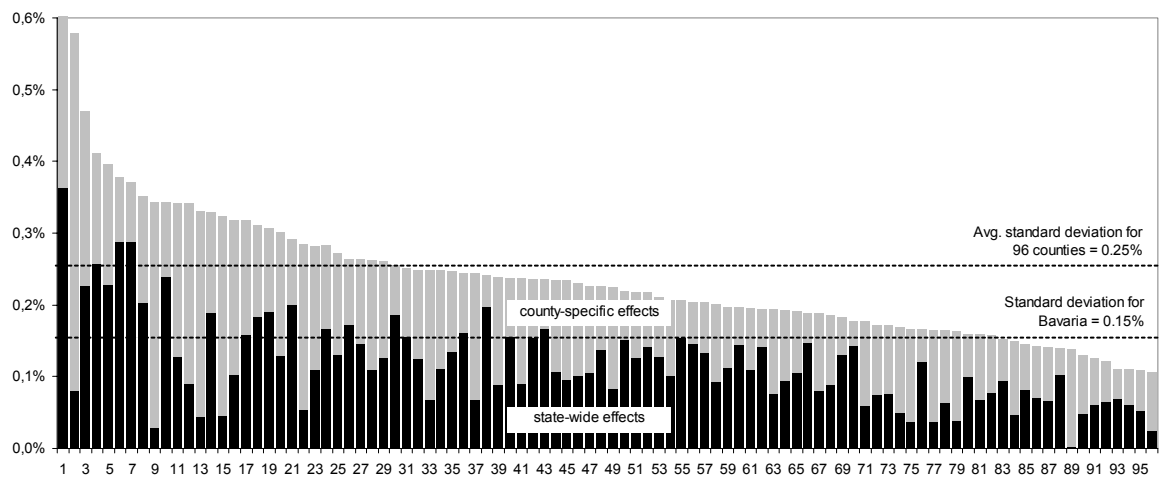


Figure 4. Contracts and number of pool members

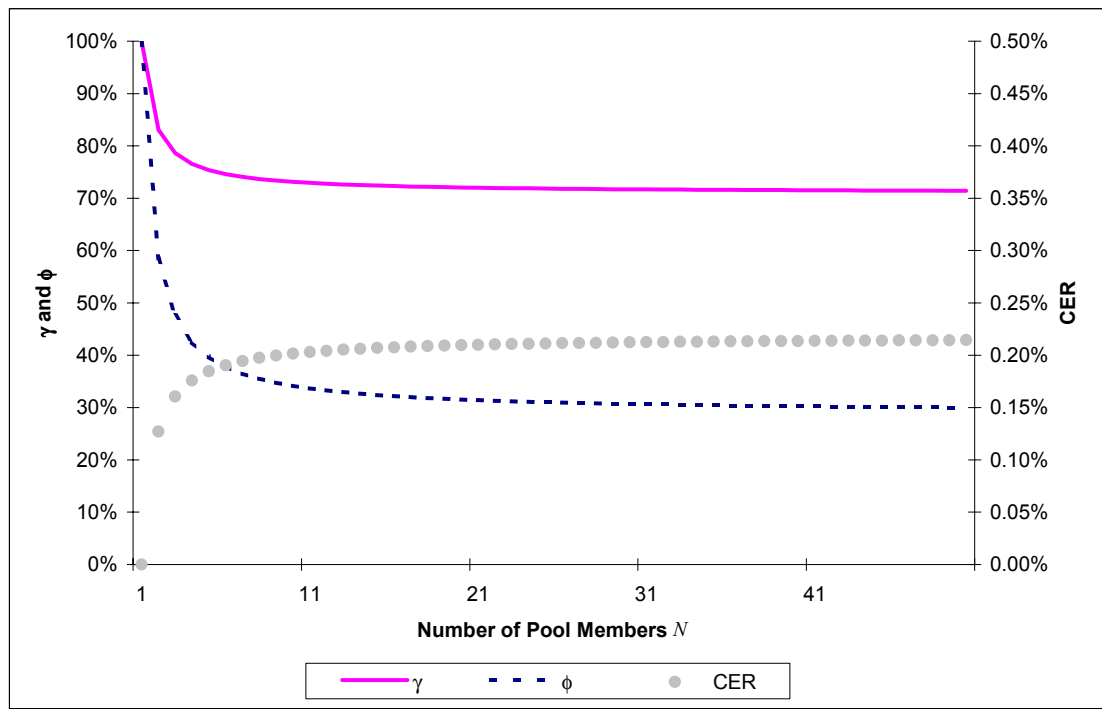


Figure 5. Contracts and productivity

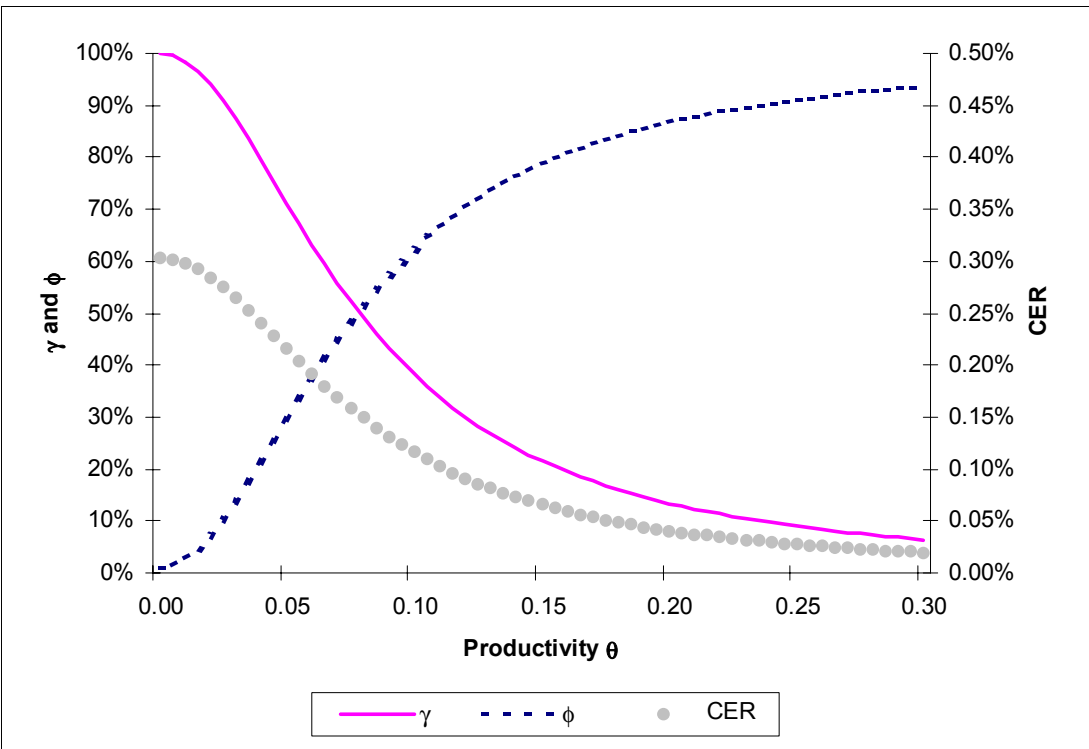


Figure 6. Contracts and idiosyncratic credit risk

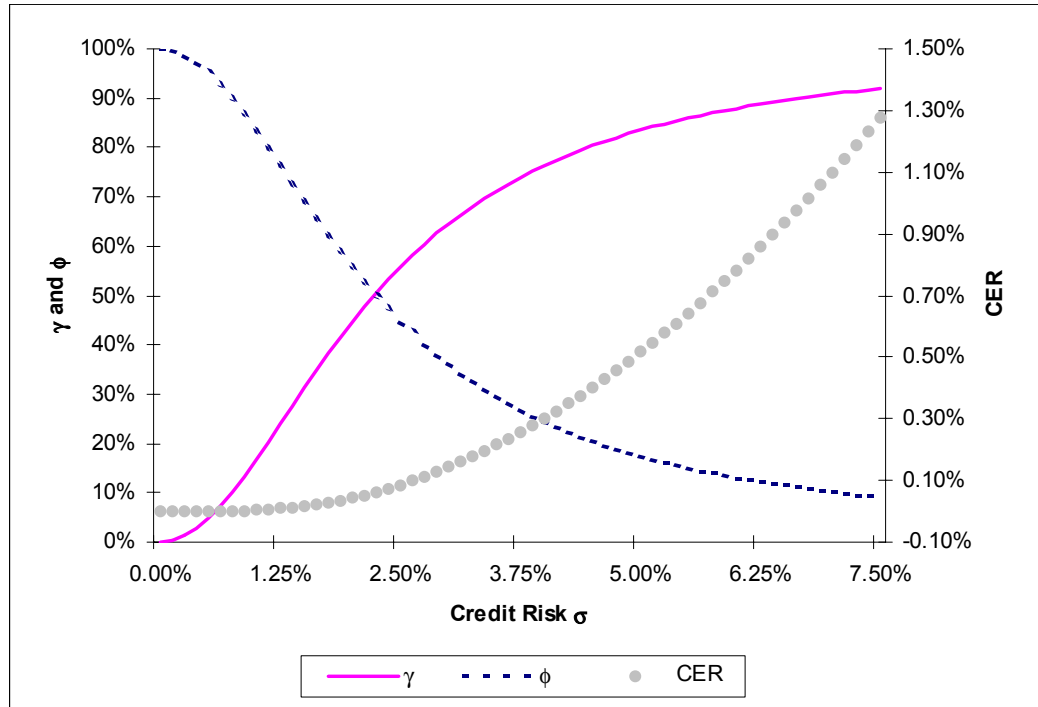
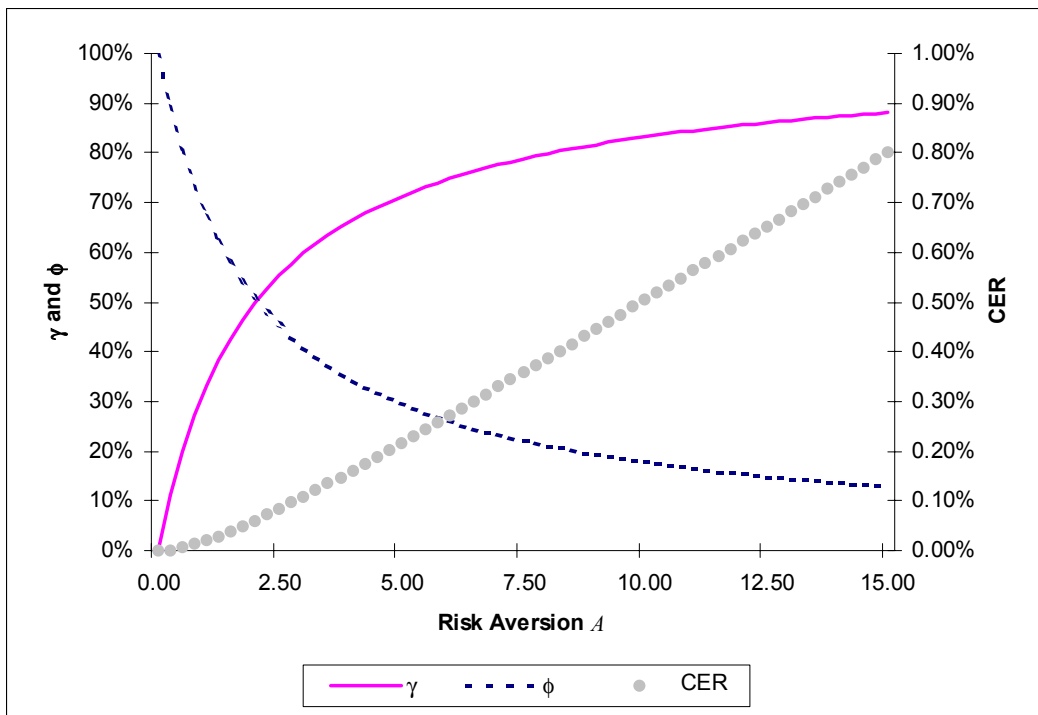


Figure 7. Contracts and risk aversion



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