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## Taxation of risky investment and paradoxical investor behavior

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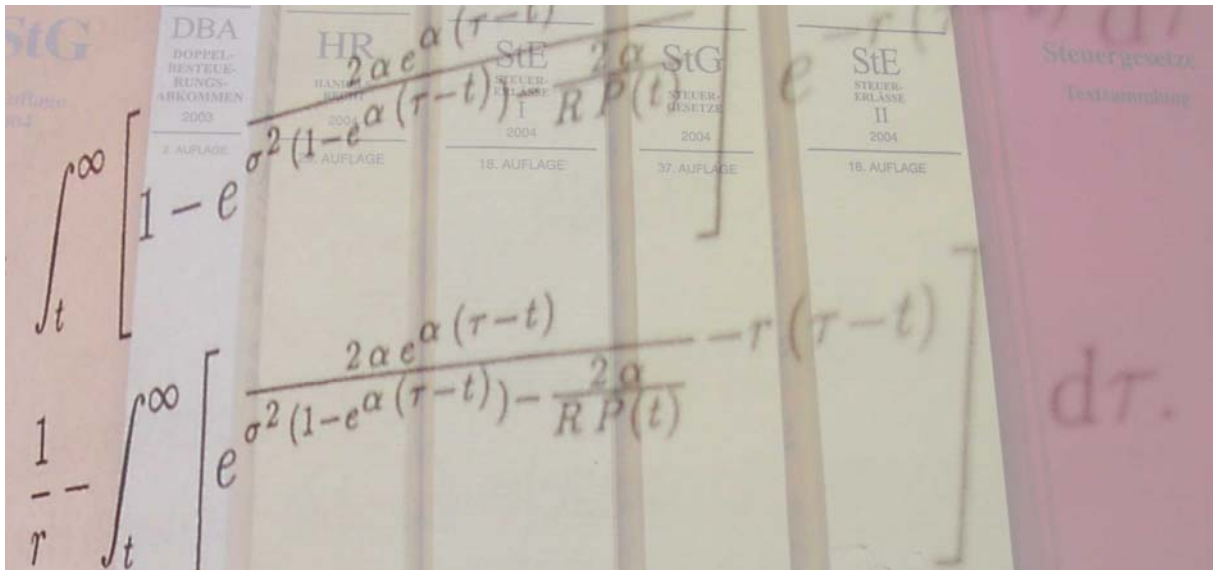
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Taxation of Risky Investment and Paradoxical Investor Behavior

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# Taxation of Risky Investment and Paradoxical Investor Behavior

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## Abstract

Under uncertainty and irreversibility, real option-based models are widely accepted for assessing investment projects. So far the existing post-tax analyses do not provide a general analytical description of investor reactions towards profit tax rate changes. This paper sets out to fill part of the void. We implement a simple tax system and focus on risky capital market investment and an option to wait. Taxes affect risk-free and risky capital market investment asymmetrically and hence cause distortions. We analytically identify a set of neutral tax rates (tax regimes) that preserve the critical post-tax investment threshold in case of tax rate changes as well as general normal and paradoxical settings. Unlike for other tax paradoxa neither depreciation rules nor loss offset restrictions are responsible for the observed paradoxical reaction. Identifying normal and paradoxical tax regimes can be regarded as a first step to a generalized description of tax effects under uncertainty, both for individual project evaluation as well as for understanding tax effects on an aggregate level.

JEL classification: H25; H21

Keywords: investment decisions, real options, tax paradox, uncertainty

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# Taxation of Risky Investment and Paradoxical Investor Behavior

## 1 Introduction

The influence of taxes on investment decisions has been analyzed by public economics for many years. So-called neutral tax systems that do not affect investment decisions are often considered desirable from a tax policy perspective. Neutral tax systems may serve as a benchmark for identifying normal and paradoxical effects of tax changes on investment decisions and thus are helpful for individual tax planning activities and tax policy discussions. Deterministic examples of neutral tax systems are the cash flow tax<sup>1</sup> and the taxation of true economic profit.<sup>2</sup>

Economists have been especially interested in tax effects under uncertainty. Conditions for a neutral business taxation under uncertainty have been addressed by Bond and Devereux (1995). Under uncertainty and irreversibility, real option-based models<sup>3</sup> are widely accepted for assessing investment projects. Enriching the real option literature by integrating taxation<sup>4</sup> leads to investment rules that consider managerial flexibility, irreversibility and tax effects. Further, under specific assumptions it is possible to identify tax systems that are neutral with respect to investment decisions. For risk neutral investors, neutral tax systems have already been proved in the real option context by Niemann (1999) and Sureth (2002). First results for neutral taxation under risk aversion have been presented by Niemann and Sureth (2004, 2005). As the discussion on tax

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<sup>1</sup>Cf. Brown (1948).

<sup>2</sup>Cf. Samuelson (1964) and Johansson (1969).

<sup>3</sup>Cf. Dixit/Pindyck (1994); Trigeorgis (1996).

<sup>4</sup>E.g., Harchaoui/Lasserre (1996); Jou (2000); Pennings (2000); Agliardi (2001); Panteghini (2001, 2004), Niemann/Sureth (2004), Schneider, Dirk (2005).

systems and tax reforms is an on-going process<sup>5</sup> it is important to understand the effects of tax rate changes on investment decisions as well as distortions which might occur. So far the existing post-tax analyses do not provide a general analytical description of investor reactions to profit tax rate changes.

There are several theoretical and empirical studies examining the economic impact of taxation on risky investment decisions. Domar and Musgrave (1944) and later Schneider, Dieter (1980) and Konrad (1991) investigate the influence of proportional income taxes on risk-taking depending on loss offsetting rules. E.g., Stiglitz (1969) investigates the effects of capital gains taxes on the demand for risky assets.

Furthermore, there is a body of empirical papers on investor reactions to tax rate changes. Lang and Shackelford (2000) empirically document the extent to which stock prices react to cuts in the capital gains tax rate. Shackelford and Verrecchia (2002) and Blouin, Raedy and Shackelford (2003) show that capital gains taxes lead investors to defer selling appreciated stock. Keuschnigg and Nielsen (2004) empirically analyze the influence of capital gains tax on start-up finance with double moral hazard. Corresponding to the findings of Poterba (1989a, 1989b), they point out that capital gains tax particularly discourages entrepreneurial efforts. Blouin, Hail, and Yetman (2005), Cook (2006) and Dai, Maydew, Shackelford, and Zhang (2006) examine empirically the response of equity values to the announcement of a decrease in the capital gains tax rate. Edmiston (2004) estimates tax volatility in a cross-country investigation and provides a panel regression suggesting that the volatility of effective tax rates on capital income has a significant negative impact on investment.

MacKie-Mason (1990) models nonlinear tax effects under uncertainty and demon-

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<sup>5</sup>Cf. Auerbach/Hines (1988); Kaplow (1986), p. 607; Hammond (1990), p. 26.

strates that policy may subsidize or discourage individual investment depending on the tax system. Altug, Demers and Demers (2001) examine the implications of tax risk and persistence on irreversible investment decisions theoretically. Panteghini and Scarpa (2003) show that regulatory risk may or may not affect negatively investment decisions. Gamba, Sick and León (2005) analyze the effect of uncertainty and debt financing on the real option value of an investment. Pawlina and Kort (2005) find that policy changes under uncertainty may have a non-monotonous impact on the investment threshold. Bloom, Bond and Van Reenen (2007) point out that companies' responsiveness to any given policy is much lower in periods of high uncertainty.

Beyond the identification of neutral tax systems, the existing real option-oriented analyses that take account of tax effects are rather limited and do not provide a general analytical description of so-called normal and paradoxical investor reactions to profit tax rate changes in this context. Either they fail to focus on this issue or they are limited to numerical investigations (e.g., Pawlina and Kort (2005, p. 1204)).

Besides the well-known tax paradoxa under certainty caused either by depreciation allowances that exceed economic depreciation in present value terms (see Samuelson (1964) and Schneider, Dieter (1969, 1992, p. 246)) or by loss carry forwards, minimum taxation or wealth taxation (see e.g. Auerbach and Poterba (1987, p. 319, 336), Niemann (2004), Kiesewetter and Niemann (2004) and Sureth and Maiterth (2005)), we provide an analytical approach to identify tax paradoxa under uncertainty even by looking at nothing more than the uncertain stream of cash flows.

We implement a simple tax system and focus on risky capital market investment decisions applying the Dixit-Pindyck (1994) paradigm. An investor faces

the opportunity to acquire a risky project with stochastic cash flow. This opportunity comprises an option to wait. Assuming irreversible investment, the investor compares the costs and benefits of investing immediately. If the investor observes a sufficiently high realization of the cash flow process, the project will be carried out.

Taxation may cause distortions as taxes asymmetrically affect risk-free and risky capital market investment. If cash flows are stochastic and an investor faces an option to invest rising tax rates may be neutral for the investment decision, or may even cause unexpected, paradoxical investment reactions. Finally, we identify analytically general paradoxical settings and furthermore, describe tax rates for investment projects with specific characteristics (growth rate, market rate and volatility) that preserve the critical post-tax investment threshold in case of deterministic tax rate changes. We determine a whole set of neutral tax rates describing tax regimes under which specific risky investments are not distorted when tax rates change and enables us to distinguish between normal and paradoxical investment reactions.

Thus, we are able to determine under which circumstances a marginal tax rate change discriminates or rather subsidizes a risky project in comparison to a risk-free alternative or even leaves the investment decision unaffected. Identifying normal and paradoxical tax regimes can be regarded as a first step to a generalized description of tax effects under uncertainty. The results are useful for tax rate discussions as they help to forecast the impact of tax rate changes on investment activities of specific types of investment projects. This is interesting information for a tax planning individual investor as well as for discussing the economic impact of tax reforms.

The remainder of this paper begins with a description of the model and a brief

deduction of the critical investment threshold in section 2. In section 3 we introduce neutral tax regimes and distinguish analytically between normal, neutral and paradoxical tax effects in section 4. We summarize and draw some conclusions in section 5.

## 2 The model

**General setting:** In this partial analytic framework we analyze a risky investment opportunity including an option to invest. The investor may either realize the investment project and earn stochastic cash flow or postpone the investment, holding the option to invest while sacrificing cash flows and thereby avoiding unexpectedly low cash flows. The initial investment cost  $I_0$  is given and constant. Cash flow uncertainty is summarized in an exogenously given single continuous-time stochastic process,  $P$ , following a geometric Brownian motion

$$\frac{dP}{P} = \alpha dt + \sigma dz \quad (1)$$

with a constant drift  $\alpha$  and a constant volatility  $\sigma$ , where  $\alpha, \sigma > 0$  and  $dz$  denotes the increment of a standard Wiener process.

Further, we assume the investment to be irreversible once it is accomplished, which implies that it is impossible to abandon a project during its economic life ending at time  $T$ .  $T$  is supposed to be infinite. Thus, the return from the project is given by the expected cash flow. The project's cash flow  $\pi$  is a function of the stochastic process  $P$  and time  $t$ :  $\pi \equiv \pi(P, t)$ . To simplify we set the pre-tax cash flow  $\pi(P, t)$  equal to the geometric Brownian motion  $P$ :  $\pi(P, t) = P(t, \alpha, \sigma)$ .

There are two approaches to derive the optimal investment rule under uncertainty and to assess the value of the option to invest: dynamic programming and contingent claims analysis. Without taxes both approaches are extensively dis-



cussed in real option theory. However, even considering that taxes have already been included in these analyses, the discussion is far from complete.<sup>6</sup>

In this model we would like to focus on effects arising from irreversibility and flexibility only, so we concentrate on the case of an investment into risky non-depreciable investment projects like listed shares. We therefore exclude periodical tax-deductible depreciation allowances from our analysis.<sup>7</sup> Hence, an investor faces the opportunity to invest in a risky project or alternatively a risk-free bond. Furthermore, we will assume a simple tax system with a proportional profit tax only. The investor's income consists of the post-tax cash flow from the risky investment that is a dividend payout. Taxable capital gains may not arise, as the investment is assumed to be irreversible and  $T \rightarrow \infty$ .

The tax base equals the cash flow  $\pi = P$ . The tax rate  $\tau$  is assumed to be deterministic. The post-tax cash flow  $P_\tau$  is defined as:

$$P_\tau = (1 - \tau) P. \quad (2)$$

If the investor does not realize the investment project funds may alternatively be invested into bonds and yield the risk-free capital market rate  $r$  that is assumed to be constant. The debit and credit rates are identical and the risk-free after-tax interest rate  $r_\tau$  can be written as  $r_\tau = (1 - \tau) r$ . As the underlying risk-free financial investment is just a special case of a real investment project, whose return always equals true economic profit and herewith implies a neutral depreciation of zero, it may serve as yardstick.

**Investment decisions and critical threshold:** In order to derive a rule for optimal investment, we have to determine the value of the underlying risky

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<sup>6</sup>See Dixit/Pindyck (1994). For a post-tax comparison of the two approaches see Niemann/Sureth (2002).

<sup>7</sup>Concerning distortions caused by depreciation allowances see Sureth (1999, pp. 278-287) who identifies tax paradoxa caused by non-neutral depreciation allowances in a real option model with contingent claims analysis assuming a setting with temporary suspension and operating costs.

asset, the investment project. Once the project is realized, i.e. the investment object is acquired, the project does not involve any flexibility, and its economic value consists solely of its future cash flows. For a risk neutral investor the post-tax project value  $V_\tau$  is given by its expected present value computed with the after-tax cash flow from the project  $P_\tau$  and the risk-free after-tax market rate of return  $r_\tau$ .

$$V_\tau \equiv V_\tau(P) = \mathbb{E} \left[ \int_s^\infty [(1 - \tau) P(t)] e^{-r_\tau(t-s)} dt \right]. \quad (3)$$

which finally is:<sup>8</sup>

$$V_\tau(P) = \frac{(1 - \tau) P}{r_\tau - \alpha}; \quad r_\tau > \alpha. \quad (4)$$

Given the value of the underlying asset (4), the post-tax value of the option to invest  $F_\tau$  can be determined applying dynamic programming. The investor wants to maximize

$$\max \{V_\tau^T - I_0, F_\tau^T\}$$

with

$$F_\tau(V_\tau) = \max \left\{ \max_T \mathbb{E} \left[ (V_\tau^T - I_0) e^{-r_\tau(T-t)} \right], 0 \right\}.$$

thus he will compare at every point in time the difference of the expected present value of the risky project and the initial outlay with the option value. The investor will give up the option to invest at an optimal time  $T$  and realize the project as soon as this difference is at least identical to the option value. Focussing on a non-depreciable option to invest we can determine the post-tax option value  $F_\tau$  which requires the continuous-time Hamilton-Jacobi-Bellman equation<sup>9</sup>

$$r_\tau F_\tau dt \stackrel{!}{=} \mathbb{E} [dF_\tau].$$

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<sup>8</sup>See appendix 1.

<sup>9</sup>For a more detailed description of deriving the value of the option see appendix 2a. For the properties of  $\lambda$  see appendix 2b.

Applying Itô's lemma to the stochastic differential  $dF_\tau$  we have to use the well-known boundary conditions<sup>10</sup>

$$F_\tau(0) = 0 \quad (5)$$

$$F_\tau(P_\tau^*) = V_\tau(P_\tau^*) - I_0 \quad (6)$$

$$\frac{dF_\tau(P_\tau^*)}{dP} = \frac{dV_\tau(P_\tau^*)}{dP}. \quad (7)$$

Equation (5) implies that a call on a worthless underlying is itself worthless. The free boundary conditions equations (6) and (7) determine the transition from the continuation region to the exercise region at the critical investment threshold  $P_\tau^*$ . The so-called value-matching condition (6) ensures that the benefit from the project is equal to its costs at the point of transition. Equation (7) is called smooth-pasting condition requiring identity of marginal benefits and marginal costs at the critical threshold. Finally we obtain the value of the option

$$F_\tau(P) = A_\tau P^{\lambda_\tau}, \quad A_\tau > 0, \quad \lambda_\tau = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r_\tau}{\sigma^2}} > 1, \quad (8)$$

where  $A_\tau$  is a constant factor to be determined. Solving for  $P_\tau^*$  leads to the post-tax critical investment threshold for an investment into a risky project:<sup>11</sup>

$$P_\tau^* = \frac{\lambda_\tau}{\lambda_\tau - 1} \frac{r_\tau - \alpha}{1 - \tau} I_0. \quad (9)$$

$P_\tau^*$  indicates whether or not the investment should be postponed. If the actually observed realization  $P_\tau$  is higher than the critical value  $P_\tau^*$ , the investment should be carried out immediately, otherwise it should be delayed until  $P_\tau^*$  is reached.<sup>12</sup>

<sup>10</sup>For the pre-tax model cf. Dixit/Pindyck (1994), p. 141.

<sup>11</sup>See appendix 3a. For the properties of  $P_\tau^*$  see appendix 3b.

<sup>12</sup>To illustrate the impact of taxes on the threshold it is interesting to have a look at the pre-tax threshold which is:  $P^* = \frac{\lambda}{\lambda-1} (r - \alpha) I_0$ . Cf. Dixit/Pindyck (1994), p. 143.

### 3 Distortion-free tax rate changes

Since neutral tax systems are well-known under certainty and have already been derived under risk neutrality in real option literature,<sup>13</sup> we will not discuss their properties in detail.

Here we look at investment rules for risky investment projects (e.g. investments in stocks on the capital market or other non-depreciable investment objects) compared to risk-free investments (e.g. bonds) when tax rates change. The investment decision depends on the expected growth rate  $\alpha$  of cash flows generated by the risky project and the inherent volatility of cash flows captured by  $\sigma$  as well as the rate of return of the alternative risk-free investment  $r$  and the investor's individual tax rate  $\tau$ . For all potential combinations of  $\alpha$ ,  $r$ ,  $\sigma$  and  $\tau$  we identify those tax rates where a change in  $\tau$  does not affect the threshold ( $dP^*/d\tau = 0$ ). In other words, for certain settings of  $\alpha$ ,  $r$ ,  $\sigma$  we determine the tax rates which do not generate a distortion of the investment decision if tax rates change. Moreover, given certain conditions of the growth rate  $\alpha$ , the interest rate  $r$  and  $\sigma$  we can state whether a deterministic change in the tax rate will foster future investment, make it less likely that an investment project will be realized or even leave the investment decision unchanged (neutral tax rate). As this tax rate is neutral only for a specific investment project with the attributes given by the required combination of values for  $\alpha$ ,  $r$ ,  $\sigma$ , we refer to such a tax rate as a (parameter-specific) neutral tax rate  $\tau^N$ .

Having determined the critical investment thresholds, it is possible to derive parameter-specific neutral tax rates as just described above. On this basis, we can identify a whole set of neutral tax rates that we will refer to as *neutral tax regime* in the following. Such a neutral tax regime describes scenarios under

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<sup>13</sup>E.g. Bond/Devereux (1995); Panteghini (2001); Sureth (2002); Niemann/Sureth (2004).

which risky investments are not distorted when tax rates change. Given the environment with the parameters  $\alpha$ ,  $r$ ,  $\sigma$  and assuming  $I_0 = 1$  a tax regime can be described by these coefficients and a tax rate  $\tau$ .

**Definition 1** A *tax regime* is a set of points  $(\alpha, r, \tau, \sigma) \subsetneq \mathbb{R}^4$ . A tax regime is called *neutral* if  $\frac{dP_\tau^*}{d\tau} = 0 \quad \forall \tau \in [0, 1)$ .

$$\frac{dP_\tau^*}{d\tau} = -\frac{d\lambda_\tau}{d\tau} \frac{r_\tau - \alpha}{1 - \tau} \frac{1}{(\lambda_\tau - 1)^2} - \frac{\lambda_\tau}{\lambda_\tau - 1} \frac{\alpha}{(1 - \tau)^2} = 0, \quad (10)$$

I.e., a marginal tax rate change has no effect on the critical threshold if the tax rate belongs to the neutral tax regime.

After having defined a neutral tax regime we would now like to look at the major properties of this tax regime. In other words, identifying a neutral regime enables us to describe the conditions for risky investment projects not suffering from distortions caused by tax rate changes. We show that there is a set of points that solves for the above condition  $\frac{dP_\tau^*}{d\tau} = 0$ . In order to capture all neutral combinations of  $\alpha$ ,  $r$ ,  $\tau$  and  $\sigma$  we first show that the *neutral tax regime* is a three-dimensional manifold. Second, we use the implicit function theorem to define neutral tax rates  $\tau^N$  as a function of  $(\alpha, r, \sigma)$ .  $\tau^N = \tau^N(\alpha, r, \sigma)$  covers all possible neutral tax rates for variations in  $\alpha$ ,  $\sigma$  and  $r$  and thereby describes different possible neutral settings of various risky investment projects.

**Proposition 1:** *Let the cash flow of our investment project with cash flow  $P$  follow a geometric Brownian motion (1) and let the profit be taxed at the tax rate  $\tau$ . Then the neutral tax regime (set of points  $(\alpha, r, \tau, \sigma)$  with  $\frac{dP_\tau^*}{d\tau} = 0$ ) with the growth rate  $\alpha$ , the volatility  $\sigma$  and the risk-free market rate  $r$  forms a three-dimensional submanifold of the  $\mathbb{R}^4$ .*

Using proposition 1<sup>14</sup> we can show that there is an implicit function for neutral tax rates  $\tau^N$  which depends on  $\alpha$ ,  $r$  and  $\sigma$ .  $\tau^N$  defines neutral tax rates, i.e. all tax rates, which do not change the investment decision for a marginal change in  $\tau$  given a set of  $\alpha$ ,  $r$  and  $\sigma$ .

**Proposition 2:** For each vector  $(\alpha_0, r_0, \tau_0, \sigma_0)$  that fulfills condition (10) there is a marginal environment around this vector, such that  $\tau^N$  is an implicit function of  $\alpha$ ,  $r$  and  $\sigma$ .

$$\tau^N = \tau^N(\alpha, r, \sigma). \quad (11)$$

With the implicit function (11) we are able to describe neutral tax rates for the possible parameter settings.<sup>15</sup>

## 4 Tax regimes: normal - neutral - paradox

As shown in the previous section it is possible to identify settings for risky financial investments, where changes in the tax rate would not distort the investment decision. These settings may serve as a reference point. They enable a distinction to be made between settings with distortions and those without distortions if tax rates change.

As we are able to describe a neutral tax regime under uncertainty, it must be possible to identify tax regimes that are non-neutral. Thus, we can determine under which conditions a marginal tax rate change discriminates or rather subsidizes a risky project in comparison to a risk-free alternative or even leaves the investment decision unaffected. Among these regimes there will be tax regimes invoking a "normal" influence of taxation on the investment decision, i.e. an increase in the tax rate will lead to an increase in the critical threshold and thus

<sup>14</sup>For a proof of proposition 1 see appendix 4.

<sup>15</sup>For a proof of proposition 2 see appendix 5.

to a postponement of the underlying investment. An investor who wants to invest immediately would then prefer to realize the risk-free investment instead. Furthermore, there will be other tax regimes invoking a paradoxical effect on the investment decision. I.e., an investor who integrates taxes in his decision calculus will be more likely to realize the risky project for a higher tax rate. Identifying normal and paradoxical tax regimes can be regarded as a first step to a general description of tax effects under uncertainty.

**Neutral tax regime** Before we turn to other than neutral regimes we would like to have a closer look at the characteristics of the neutral tax regime. With the help of the implicit function  $\tau^N = \tau^N(\alpha, r, \sigma)$  we can discuss the shape and location of the manifold in different dimensions by looking at the relevant partial derivatives. As it is not possible to identify conditions for neutral tax regimes that hold for all possible parameter settings analytically we focus on settings with a sufficiently small difference between  $r_\tau$  and  $\alpha$ , i.e.

$$\varepsilon = r_\tau - \alpha \tag{12}$$

is small.<sup>16</sup> For sufficiently small  $\varepsilon$  we are able to distinguish exactly between the different types of tax regimes.

Numerical examples like e.g. the parameter combination  $r = 0.05, \alpha = 0.02, \sigma = 0.25$  and  $\tau = 0.35$  lead to a sufficiently small  $\varepsilon$ . Figure 1 illustrates a selection of  $\sigma$  and  $\tau$  combinations that fulfill this condition for given  $r = 0.05$  and  $\alpha = 0.02$ . These examples represent feasible combinations of parameters that allow us to identify a neutral tax regime. The example suggests that for relatively high volatilities, many typically observable combinations of  $\alpha$  and  $r$  fulfill this condition.

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<sup>16</sup>An analytical description of sufficiently small  $\varepsilon$  is given in appendix 3b.

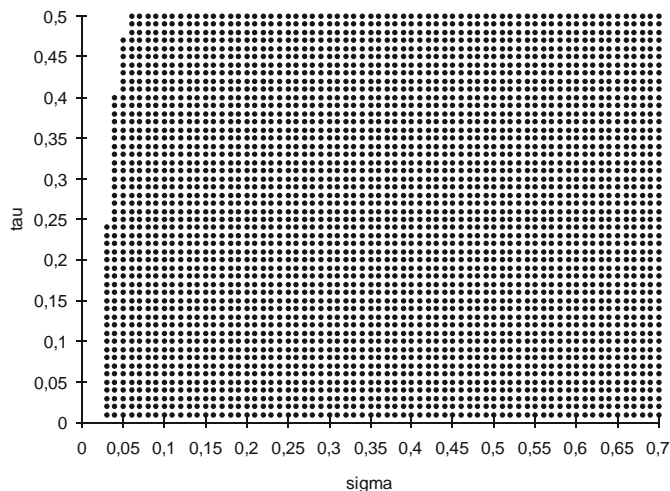


Figure 1: Sufficiently small  $\varepsilon$

Up to now effects of tax changes on investment decision under uncertainty have been mostly discussed as numerical examples. In this analysis we try to obtain general analytical results. If we assume small  $\varepsilon$  this will enable us to identify unambiguous normal, neutral and even paradoxical effects under more general conditions. Restricting the analysis to small  $\varepsilon$  does not mean that these effects do not exist for larger  $\varepsilon$ . It just means that we do not have general conditions for these regimes.

If  $\varepsilon$  in condition (12) is sufficiently small the signs of the partial derivatives of  $\tau^N$  with respect to  $\alpha$ ,  $r$  and  $\sigma$  will be unambiguous for each project-specific setting.<sup>17</sup>

$$\frac{d\tau^N}{d\alpha} < 0, \quad \frac{d\tau^N}{d\sigma} < 0, \quad \frac{d\tau^N}{dr} > 0 \quad (13)$$

From the viewpoint of an investor, investors can anticipate whether a risky project is discriminated, subsidized or treated neutrally by taxation if they

<sup>17</sup>See appendix 5, lemma 2 for determining the marginal reactions of the implicit function  $\tau^N$  with respect to different parameters.



know the type of tax regime for each investment project that complies with the required condition. Hence, facing tax rate changes tax planning will be easier, i.e. it is easier for an investor to forecast the tax effects. Furthermore, from the viewpoint of the government, it will be easy to identify the direction of distortion of tax rate changes and to control for tax policy effects at least for some types of investment project.

**Normal and paradox tax regimes** With the help of the neutral tax regime we can distinguish between regions with normal reactions of the critical threshold and paradoxical reactions when tax rates rise.

**Definition 2** *A tax reaction is called normal if an increase in the tax rate increases the required threshold  $P_{\tau}^*$ ,  $\frac{dP_{\tau}^*}{d\tau} > 0$ .*

**Definition 3** *A tax reaction is called paradox if an increase in the tax rate decreases the required threshold  $P_{\tau}^*$ ,  $\frac{dP_{\tau}^*}{d\tau} < 0$ .*

**Proposition 3:** *If tax rates are higher/lower than the rates of the neutral tax regime, the reactions of the threshold are normal/paradox.*

Figure 2 illustrates the linearized partial shape of the function for the neutral tax rate  $\tau^N$  and the growth rate  $\alpha$  and risk  $\sigma$ . We can identify the corresponding regions for normal and paradoxical tax effects. The graphs indicate the location of the different regimes in each dimension. The described different regimes reflect the general effect of taxes on investment decisions under uncertainty depending on the characteristics of the underlying investment project given by  $\alpha$  and  $r$  and  $\sigma$ . If c.p. former deterministic cash flows become stochastic and an investor faces an option to invest rising tax rates may be neutral for the

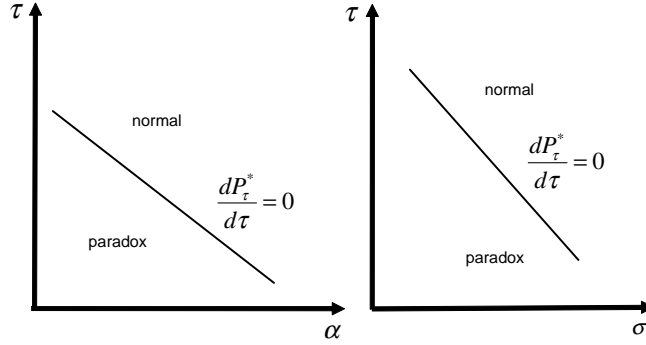


Figure 2: Neutral tax regime, partial derivatives

investment decision, or may even switch the sign of the reaction under certainty.

What is the economics of switching the sign of the threshold caused by uncertainty? Under uncertainty the option to invest has an own economic value. Hence the net present value of an investment which is the objective value of the investor includes this component. Consequently, the present value of the option affects the decision. However, taxes affect the benefit from waiting (value of holding period) differently than they do the other components of the investment decision. The contribution of the expected cash flow from the investment and of the option to the net present value of the whole project are treated asymmetrically by taxation. We observe two major effects:

One arises from the tax treatment of the option and thereby is induced directly by uncertainty. As the increase in the option value during the holding period is not subject to tax and a corresponding economic appreciation for tax purposes

is missing, the option enjoys a tax privilege.<sup>18</sup>

The second effect is caused by taxation in a continuous-time growth model and thus is an effect that can be identified under certainty as well. At time  $t$  all realized cash flows are subject to tax. In contrast, the growth of cash flows that will be realized during the infinitesimal small period  $t$  will become tax-liable at  $t + dt$ . Consequently, this marginal return and growth will be temporarily tax-exempt invoking asymmetric treatment of the underlying riskless and risky investments. Under certainty (perfect foresight) an investor and the public sector would know about this marginal return and would be able to burden it with taxes. Under uncertainty both agents have no more than expectations about this marginal return. Thus, exact taxation is not possible. This effect until now has not been treated and analyzed in the literature.

This effect from continuous-time modelling may exert a different influence on the threshold than the one from the tax privilege of the option. We can show that, depending on the type of tax regime, the direct effect from uncertainty may be stronger or weaker than the reaction from the after-tax growth process and hence, the effect from the component addressing uncertainty may or may not overcompensate the second effect and overall change the sign of the reaction of the threshold.

Looking at the reaction in figure 2 we see the following mechanics from taxation and option pricing. Assume an investment project with a given growth rate  $\alpha$  is just taxed at a neutral tax rate  $\tau^N$ . Neutral tax rates  $\tau^N$  are drawn as a decreasing function of  $\alpha$  and  $\sigma$ . Now, we assume the tax rate to rise and future cash flows from the investment to be taxed at this higher rate  $\tau$  with  $\tau > \tau^N$ . As the option is part of the value of the opportunity to invest in a

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<sup>18</sup>Cf. Niemann (1999).

risky project and further the option is tax-favored a rise in  $\tau$  implies an increase of this tax privilege. The relative advantage from holding the option grows. Consequently, an investor will be willing to abandon the option and carry out the risky project only for relative higher values of  $P$ . Thus, increasing the tax rate will increase the critical investment threshold which is a normal reaction. The relative advantage from holding the option increases and the critical threshold will be higher. Exploring the environmental conditions under which the original threshold would be preserved in case of a tax rate rise, we discover that a simultaneous decrease in  $\alpha$  that leads to a decrease in  $P_\tau^*$  as described above may compensate for the tax rate effect. Then, under the resulting new setting with decreasing  $\alpha$  and given  $r$  and  $\sigma$  we would fall back to a neutral regime (negative slope of the  $\tau^N$ -curve).

The reaction below the  $\tau^N$ -function is quite different. Again, with rising tax rates when  $\tau < \tau^N$  the component of the threshold covering the option value increases. C.p. this effect from the option pushes up the critical threshold. However, in the paradox regime we realize that the tax-benefit from the option is now overcompensated by an opposing effect. This second effect arises from the temporal tax-exemption of continuous growth in the present tax period. Whereas a realized cash flow from either the risky investment project or the risk-less investment into bonds is cut proportionally by the tax rate  $\tau$ , the investor's benefit from simultaneous growth of revenues ( $\alpha$ ) during each period is tax-exempt as it does not become an instantaneously realized cash flow during the same period. Therefore, an asymmetric effect of taxes favours the risky investment project. C.p from this asymmetry we obtain a partial decrease in the threshold when the tax rate increases. In the paradox regime this second effect is overcompensating the first effect. Hence, the higher the tax rate the more attractive becomes the risky project. If  $\tau < \tau^N$  this effect from asymmetric

taxation of projects with continuously growing cash flow and an investment into a bond overcompensates the tax impact on the option values arising from uncertainty. Overall, the increase in the tax rate causes a reduction of the investment threshold. The investor faces a paradox situation.

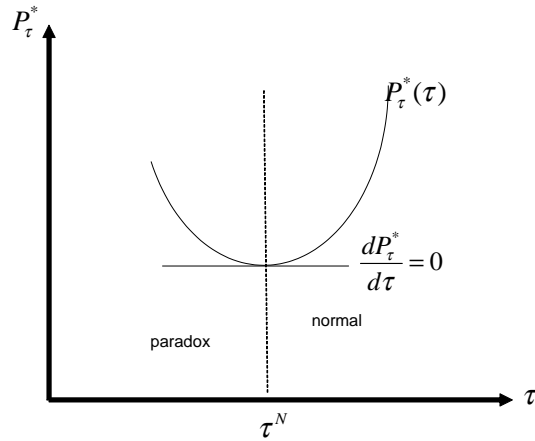


Figure 3: Tax rate variations and tax regimes

These reactions for the different regimes are also depicted in figure 3. For given external conditions the reaction of the threshold to an increase in the tax rate is described. To the right of  $\tau^N$  increasing taxes will cause the expected increase in the threshold (*normal reaction*). The increase in the threshold may lead to reject the project that was favorable before. To the left of  $\tau^N$  increasing taxes will decrease the threshold and improve the evaluation of the uncertain investment project (*paradoxical reaction*).

In figure 4 we draw the shape and location of the neutral tax regime applying a three-dimensional illustration of the neutral tax rates depending on  $\alpha$  and  $\sigma$ . The plane separates the different regimes and shows that even in the underlying

simple case of an investment in a risky financial project, i.e. in a non-depreciable project, uncertainty may change the sign of the investment reaction on tax rate changes.

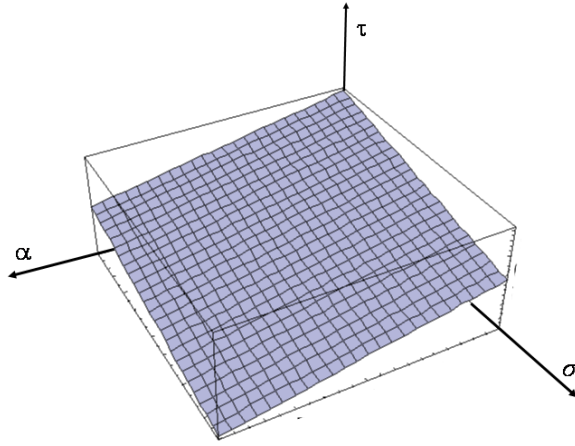


Figure 4: Neutral tax regime

This figure illustrates the relation between  $\alpha$ ,  $\sigma$  and  $\tau$ . Obviously, neutral tax rates need to decrease with increasing growth rates, whereas they need to rise to compensate for decreasing risk.

We see in figure 4 that an increasing risk (rising  $\sigma$ ) will increase the value of the waiting time. Being able to wait and not having to start immediately has an increasing economic value. The relatively high value of the waiting time is pushing up the threshold, as the investor wants to be compensated for higher risk. If we are looking for tax rates that preserve the threshold in case of higher risk there needs to be a compensation in the tax parameters for bearing more risk. This compensation can be achieved by a decrease in the tax  $\tau$ . A simultaneously decreasing tax rate would adjust the threshold and lead back to the neutral tax condition.

The discussion on a change in the growth rate  $\alpha$  is similar. If  $\alpha$  increases this will push up the critical threshold as holding the option implies rather rapid growth and thus a high value of the waiting time. An investor will therefore ask for a relatively high compensation if he gives up the option to invest. As the increase in the value of the cash flow during waiting time is not tax-liable, high tax rates amplify the option's benefit. Hence, if  $\alpha$  increases lower tax rates are necessary to provide neutrality in the above defined sense.

Having identified the two asymmetries when taxing risky investment projects it seems beneficial to introduce a tax system under certainty that treats projects with continuously growing cash flows and bonds in the same way. This would require a depreciation term that is a function of  $\alpha$ ,  $r$  and  $\sigma$  eliminating this asymmetry. If we implement additionally an ex-post adjustment mechanism to balance out the deviation between expected cash flow and realized cash flows, then just the tax effect from the option would occur under uncertainty. Under these conditions we would observe exclusively the tax effect from the option and hence the investor will always face a normal reaction if  $r_\tau > \alpha$  holds. However, the adjustment procedure dissolves continuous-time modelling.

Furthermore, to neutralize all the asymmetries from taxation either an economic appreciation or depreciation (corresponding to economic depreciation known from taxing true economic profit) could be introduced into tax law. As such an adjustment rule is not included in the underlying and in real-world tax systems, an asymmetry remains. Speaking more generally, it seems beneficial to introduce a tax system that treats projects with continuously growing cash flows, bonds and options in the same way. This would require a adjustment term that is a function of  $\alpha$ ,  $r$  and  $\sigma$  eliminating this asymmetry.

## 5 Conclusions

The effect of tax rate changes on investments will change substantially if uncertainty and irreversibility is included in the investment decision. Using a real option model with dynamic programming for risky non-depreciable irreversible investments, a simple tax system with profit tax only and a cash flow that follows a geometric Brownian motion, we can identify three regimes of tax effects on investment decisions. In contrast to the existing literature that usually falls back on numerical analyses we succeed in identifying analytically sets of tax rates for which an increase in tax rates will lead to the expected increase in the threshold and hence a decrease in investments. Our findings are general whenever the differential between the growth rate and the market rate of return is sufficiently small. This set of tax rates is called a normal tax regime. There is also a set of tax rates, where an increase in tax rates will not cause any effects on the threshold and hence investment decision. This set of tax rates is referred to as a parameter-specific neutral tax regime. However, there is a set of tax rates where an increase in tax rates will even decrease the threshold and favour the risky investment. These unexpected reactions are called paradox. Unlike for other tax paradoxa neither depreciation rules<sup>19</sup> nor loss offset restrictions are responsible for the observed paradoxical reaction.

What is the economics of these paradoxical reactions? Under uncertainty the option to invest has a positive economic value. Taxes affect the benefit from waiting (value of holding period) differently than they do the other components of the investment decision. The contribution of the expected cash flow from the investment and of the option to the net present value of the whole project are treated asymmetrically by taxation. Whereas realized cash flow from the risk-

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<sup>19</sup>In fact there is a depreciation rule that is equal to zero. This rule is not identical to economic depreciation.



free investment into bonds is cut proportionally by the tax rate, the investor's benefit from the option during the holding period is completely tax-exempt as it does not become a realized cash flow. The non realized increase in stochastic cash flows during the potential period of waiting is not taxed. Furthermore, the marginal return and growth from the risky project is temporarily tax-exempt.

Identifying these regimes is interesting from two perspectives: From the viewpoint of an investor, investors can anticipate whether a risky project is discriminated, subsidized or treated neutrally by tax rate changes knowing the type of tax regime. From the viewpoint of the government, it will be easier to identify the direction of distortion of tax rate reforms. Further, as the analysis is looking at a single project with its environment and the environment is described by the growth rate and volatility of the cash flow as well as the return of the risk-free investment, tax rate changes may have opposite effects on the different investment projects. Depending on the external conditions the same change in tax rates may have normal, neutral as well as paradoxical effects on different projects. The tax effect on aggregate investment becomes generally ambiguous.

# Extended Appendix

## Appendix 1: Expected present value of the investment project

$dP/dt = \alpha P + \sigma P dz/dt$  geometric Brownian motion

$$\begin{aligned} V_\tau &= E \left( \int_s^\infty P_\tau e^{-r_\tau(t-s)} dt \right) \\ &= E \left( \int_s^\infty [(1-\tau)P] e^{-r_\tau(t-s)} dt \right) \\ &= E \left[ \frac{1}{\alpha - r_\tau} (1-\tau) P e^{\alpha(t-s)} e^{-r_\tau(t-s)} \right]_s^\infty \\ &= 0 - \frac{1-\tau}{\alpha - r_\tau} P e^{(\alpha - r_\tau)(s-s)} \\ &= \frac{1-\tau}{r_\tau - \alpha} P \end{aligned}$$

## Appendix 2a: Determining the solution for $\lambda_\tau$ : The Hamilton-Bellman

equation for the option of waiting  $F_\tau$  yields

$$r_\tau F_\tau = E(dF_\tau).$$

The owner of the option expects an instantaneous return that in equilibrium equals the post-tax risk-free rate.

From Ito's Lemma we know:

$$E(dF_\tau) = \frac{dF_\tau}{dt} + \alpha P \frac{dF_\tau}{dP} + \frac{1}{2} \sigma^2 P^2 \frac{d^2 F_\tau}{dP^2}.$$

Now we will use the last two equations in order to get the following differential equation:

$$\frac{dF_\tau}{dt} + \alpha P \frac{dF_\tau}{dP} + \frac{1}{2} \sigma^2 P^2 \frac{d^2 F_\tau}{dP^2} - r_\tau F_\tau = 0.$$

If the option is perpetual we have  $\frac{dF_\tau}{dt} = 0$  and for that reason

$$\alpha P \frac{dF_\tau}{dP} + \frac{1}{2} \sigma^2 P^2 \frac{d^2 F_\tau}{dP^2} - r_\tau F_\tau = 0.$$

We are searching for solutions to the differential equation of the form:  $F_\tau(P) = A_\tau P^{\lambda_\tau}$ . We will plug in this term into the differential equation to get:

$$\begin{aligned} \alpha P \frac{dF_\tau}{dP} + \frac{1}{2} \sigma^2 P^2 \frac{d^2 F_\tau}{dP^2} - r_\tau A_\tau P^{\lambda_\tau} &= 0 \\ \alpha \lambda_\tau A_\tau P P^{\lambda_\tau - 1} + \frac{1}{2} (\lambda_\tau - 1) \sigma^2 P^2 \lambda_\tau A_\tau P^{\lambda_\tau - 2} - r_\tau A_\tau P^{\lambda_\tau} &= 0 \\ \alpha \lambda_\tau A_\tau P^{\lambda_\tau} + \frac{1}{2} (\lambda_\tau - 1) \sigma^2 \lambda_\tau A_\tau P^{\lambda_\tau} - r_\tau A_\tau P^{\lambda_\tau} &= 0 \\ \alpha \lambda_\tau + \frac{1}{2} (\lambda_\tau - 1) \sigma^2 \lambda_\tau - r_\tau &= 0. \end{aligned}$$

Now we use the p-q formula to determine  $\lambda_\tau$  :

$$\begin{aligned} \alpha \lambda_\tau + \frac{1}{2} \lambda_\tau^2 \sigma^2 - \frac{1}{2} \sigma^2 \lambda_\tau - r_\tau &= 0 \\ \lambda_\tau^2 \sigma^2 + 2\alpha \lambda_\tau - \sigma^2 \lambda_\tau - 2r_\tau &= 0 \end{aligned}$$

We call the last term  $Q(\lambda_\tau)$  and finally receive:

$$\lambda_\tau = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \left[ \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r_\tau}{\sigma^2} \right]^{\frac{1}{2}} \quad \text{q.e.d.}$$

**Appendix 2b: Properties of  $\lambda_\tau$  :**

$$\lambda_\tau > 1 \quad \text{because} \quad Q(1) < 0 \quad (14)$$

$$\text{we assume small } \varepsilon \text{ with } \varepsilon = r_\tau - \alpha = \left(\frac{r_\tau}{\alpha} - 1\right) \alpha > 0$$

$$\begin{aligned} \text{hence } \lambda_\tau &= \frac{1}{2} - \frac{r_\tau - \varepsilon}{\sigma^2} + \left[ \left( \frac{1}{2} - \frac{r_\tau - \varepsilon}{\sigma^2} \right)^2 + \frac{2r_\tau}{\sigma^2} \right]^{\frac{1}{2}} \\ \text{and further } \lim_{\varepsilon \rightarrow 0} \lambda_\tau &= 1, \quad \lim_{\sigma \rightarrow \infty} \lambda_\tau = 1. \end{aligned} \quad (15)$$

**Corollary 1: The assumption  $r_\tau > \alpha$  is equivalent to**

$$\lambda_\tau - \frac{r_\tau}{\alpha} < 0 \quad (16)$$

**Proof of corollary 1:**

$$\begin{aligned} \text{It applies } \frac{r_\tau}{\alpha} &< \left(\frac{r_\tau}{\alpha}\right)^2 \quad \text{as } r_\tau > \alpha \\ &\Leftrightarrow \left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)^2 + \frac{2r_\tau}{\sigma^2} < \left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)^2 - \frac{r_\tau}{\alpha} + \frac{2r_\tau}{\sigma^2} + \left(\frac{r_\tau}{\alpha}\right)^2 \\ &\Leftrightarrow \left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)^2 + \frac{2r_\tau}{\sigma^2} < \left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)^2 - \frac{2r_\tau}{\alpha} \left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right) + \left(\frac{r_\tau}{\alpha}\right)^2 \\ &\Leftrightarrow \left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)^2 + \frac{2r_\tau}{\sigma^2} < \left[ -\left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right) + \frac{r_\tau}{\alpha} \right]^2 \\ &\Leftrightarrow \left[ \left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)^2 + \frac{2r_\tau}{\sigma^2} \right]^{\frac{1}{2}} < -\left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right) + \frac{r_\tau}{\alpha} \\ &\Leftrightarrow \overbrace{\left[ \left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)^2 + \frac{2r_\tau}{\sigma^2} \right]^{\frac{1}{2}}} + \frac{1}{2} - \frac{\alpha}{\sigma^2} < \frac{r_\tau}{\alpha} \\ &\Leftrightarrow \lambda_\tau - \frac{r_\tau}{\alpha} < 0 \end{aligned}$$

**Corollary 2:**  $0 < \delta < 1$  and finite

**Proof of corollary 2:**

$$\begin{aligned}\lambda_\tau - \frac{r_\tau}{\alpha} &< 0 \Leftrightarrow \lambda_\tau - 1 < \frac{r_\tau}{\alpha} - 1 \\ 0 &< 1 - \frac{\lambda_\tau - 1}{\frac{r_\tau}{\alpha} - 1} \\ \delta &: = 1 - \frac{\lambda_\tau - 1}{\frac{r_\tau}{\alpha} - 1} \quad \text{with } 0 < \delta < 1\end{aligned}$$

$$\begin{aligned}\frac{\lambda_\tau - 1}{\frac{r_\tau}{\alpha} - 1} + \delta &= \frac{(\lambda_\tau - 1)\alpha}{\varepsilon} + \delta = 1 \\ \lim_{\varepsilon \rightarrow 0} \frac{(\lambda_\tau - 1)\alpha}{\varepsilon} &: \text{ using L'Hopital's rule:} \\ \lim_{\varepsilon \rightarrow 0} \frac{\alpha \frac{d\lambda_\tau}{d\varepsilon}}{1} &= \frac{\alpha \left[ \left( \frac{1}{2} - \frac{r_\tau - \varepsilon}{\sigma^2} \right)^2 + \frac{2r_\tau}{\sigma^2} \right]^{-\frac{1}{2}}}{\sigma^2} \lambda_\tau > 0 \text{ and finite} \\ \text{hence } &: \quad \delta > 0 \text{ and finite}\end{aligned}$$

$$\varepsilon := \frac{(\lambda_\tau - 1)\alpha}{1 - \delta} \tag{17}$$

Derivatives for  $\lambda_\tau$  :

$$\begin{aligned}\frac{d\lambda_\tau}{d\tau} &= - \left[ \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r(1-\tau)}{\sigma^2} \right]^{-1/2} \frac{r}{\sigma^2} < 0 \\ \frac{d^2\lambda_\tau}{d\tau^2} &= - \left[ \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2(r-r\tau)}{\sigma^2} \right]^{-1/2-1} \left( \frac{r}{\sigma^2} \right)^2 < 0\end{aligned}$$

$$\begin{aligned}
\frac{d\lambda_\tau}{d\alpha} &= -\frac{1}{\sigma^2} - \frac{2}{2} \left[ \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2(r-r\tau)}{\sigma^2} \right]^{\frac{1}{2}-1} \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right) \frac{1}{\sigma^2} \\
&= -\frac{1}{\sigma^2} \left[ 1 + \left[ \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2(r-r\tau)}{\sigma^2} \right]^{-\frac{1}{2}} \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right) \right] < 0 \\
&= -\frac{\left[ \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2(r-r\tau)}{\sigma^2} \right]^{-\frac{1}{2}}}{\sigma^2} \left[ \left[ \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2(r-r\tau)}{\sigma^2} \right]^{\frac{1}{2}} + \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right) \right] < 0 \\
&= -\frac{\left[ \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2(r-r\tau)}{\sigma^2} \right]^{-\frac{1}{2}}}{\sigma^2} \lambda_\tau < 0
\end{aligned}$$

$$\begin{aligned}
\frac{d\lambda_\tau}{d\varepsilon} &= \frac{1}{\sigma^2} + \frac{2}{2} \left[ \left( \frac{1}{2} - \frac{r_\tau - \varepsilon}{\sigma^2} \right)^2 + \frac{2r_\tau}{\sigma^2} \right]^{-\frac{1}{2}} \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right) \frac{1}{\sigma^2} \\
&= \frac{\left[ \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2(r-r\tau)}{\sigma^2} \right]^{-\frac{1}{2}}}{\sigma^2} \lambda
\end{aligned}$$

$$\frac{d\lambda_\tau}{dr} = \left[ \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r(1-\tau)}{\sigma^2} \right]^{-\frac{1}{2}} \frac{(1-\tau)}{\sigma^2} > 0$$

$$\begin{aligned}
\frac{d\lambda_\tau}{d\sigma} &= \frac{2\alpha}{\sigma^3} + \frac{1}{2} \left( \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r_\tau}{\sigma^2} \right)^{-\frac{1}{2}} \left( 2 \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right) \cdot \frac{2\alpha}{\sigma^3} - \frac{4r_\tau}{\sigma^3} \right) < 0 \\
&= \frac{2\alpha}{\sigma^3} + \left( \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r_\tau}{\sigma^2} \right)^{-\frac{1}{2}} \left( \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right) \cdot \frac{2\alpha}{\sigma^3} - \frac{2r_\tau}{\sigma^3} \right) < 0 \\
&= \frac{2\alpha}{\sigma^3} \left[ 1 + \left( \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r_\tau}{\sigma^2} \right)^{-\frac{1}{2}} \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} - \frac{r_\tau}{\alpha} \right) \right] < 0 \\
&= \frac{2\alpha \left( \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r_\tau}{\sigma^2} \right)^{-\frac{1}{2}}}{\sigma^3} \left[ \left( \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r_\tau}{\sigma^2} \right)^{\frac{1}{2}} + \frac{1}{2} - \frac{\alpha}{\sigma^2} - \frac{r_\tau}{\alpha} \right] < 0
\end{aligned}$$

$$\frac{d^2 \lambda_\tau}{d\alpha d\tau} = - \left[ \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r(1-\tau)}{\sigma^2} \right]^{-1/2} \frac{r}{\sigma^4} \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right) \begin{matrix} \leq 0 \\ > 0 \end{matrix} \text{ for } \frac{\alpha}{\sigma^2} \begin{matrix} \leq \frac{1}{2} \\ > \frac{1}{2} \end{matrix}$$

$$\begin{aligned} \frac{d^2 \lambda_\tau}{dr d\tau} &= \frac{1}{2} \left[ \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r(1-\tau)}{\sigma^2} \right]^{-1/2-1} \frac{r}{\sigma^2} \frac{2(1-\tau)}{\sigma^2} - \left[ \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r(1-\tau)}{\sigma^2} \right]^{-1/2} \frac{1}{\sigma^2} \\ &= \left[ \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r(1-\tau)}{\sigma^2} \right]^{-1/2-1} \frac{r}{\sigma^2} \frac{(1-\tau)}{\sigma^2} - \frac{1}{\sigma^2} \left[ \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r(1-\tau)}{\sigma^2} \right]^{-1/2} \\ &= \left[ \left[ \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + 2 \frac{r(1-\tau)}{\sigma^2} \right]^{-1} \frac{r(1-\tau)}{\sigma^2} - 1 \right] \frac{1}{\sigma^2} \left[ \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r(1-\tau)}{\sigma^2} \right]^{-1/2} \\ &= \left[ \frac{r(1-\tau)}{\sigma^2} - \left[ \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + 2 \frac{r(1-\tau)}{\sigma^2} \right] \right] \frac{1}{\sigma^2} \left[ \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r(1-\tau)}{\sigma^2} \right]^{-1/2-1} \\ &= \left[ \frac{r(1-\tau)}{\sigma^2} - \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 - 2 \frac{r(1-\tau)}{\sigma^2} \right] \frac{1}{\sigma^2} \left[ \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r(1-\tau)}{\sigma^2} \right]^{-1/2-1} \\ &= - \left[ \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{r(1-\tau)}{\sigma^2} \right] \frac{1}{\sigma^2} \left[ \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r(1-\tau)}{\sigma^2} \right]^{-1/2-1} < 0 \quad ok \\ \frac{d^2 \lambda_\tau}{d\sigma d\tau} &= \frac{2r}{\sigma^3} \cdot \frac{1}{\left( \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + 2 \frac{r_\tau}{\sigma^2} \right)^{\frac{1}{2}}} + \frac{1}{2} \cdot \frac{r}{\sigma^2} \left( \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + 2 \frac{r_\tau}{\sigma^2} \right)^{-\frac{3}{2}} \cdot \left( 2 \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right) 2 \frac{\alpha}{\sigma^3} - \frac{4r_\tau}{\sigma^3} \right) \end{aligned}$$

is bounded for a constant  $\sigma$ .

### Appendix 3a: Deriving the threshold price $P_\tau^*$

$$F_\tau(0) = 0$$

$$F_\tau(P_\tau^*) = V_\tau(P_\tau^*) - I_0$$

$$\frac{dF_\tau(P_\tau^*)}{dP} = \frac{dV_\tau(P_\tau^*)}{dP}.$$

$$F_\tau(P_\tau^*) = V_\tau(P_\tau^*) - I_0$$

$$A_\tau(P_\tau^*)^{\lambda_\tau} = V_\tau - I_0 = \frac{(1-\tau)}{r_\tau - \alpha} P_\tau^* - I_0$$

$$\begin{aligned}
\frac{dF_\tau(P_\tau^*)}{dP} &= \frac{dV_\tau(P_\tau^*)}{dP} \\
\lambda_\tau A_\tau(P_\tau^*)^{\lambda_\tau-1} &= \frac{1-\tau}{r_\tau-\alpha} \\
A_\tau(P_\tau^*)^{\lambda_\tau} &= \frac{1-\tau}{r_\tau-\alpha} \frac{P_\tau^*}{\lambda_\tau}
\end{aligned}$$

$$\begin{aligned}
\frac{1-\tau}{r_\tau-\alpha} \frac{P_\tau^*}{\lambda_\tau} &= V_\tau - I_0 = \frac{1-\tau}{r_\tau-\alpha} P_\tau^* - I_0 \\
\frac{1-\tau}{r_\tau-\alpha} \left(1 - \frac{1}{\lambda_\tau}\right) P_\tau^* - I_0 &= 0 \\
\frac{1-\tau}{r_\tau-\alpha} (\lambda_\tau - 1) \frac{1}{\lambda_\tau} P_\tau^* &= I_0 \\
\frac{1-\tau}{r_\tau-\alpha} \frac{\lambda_\tau - 1}{\lambda_\tau} P_\tau^* &= I_0 \\
P_\tau^* &= \frac{\lambda_\tau}{\lambda_\tau - 1} \frac{r_\tau - \alpha}{1 - \tau} I_0
\end{aligned}$$

**Appendix 3b: Properties of the investment threshold** Derivatives of  $P_\tau^*$  with respect to the tax rate:  $:\frac{dP_\tau^*}{d\tau}$

$$P_\tau^* = \frac{\lambda_\tau}{\lambda_\tau - 1} \frac{r_\tau - \alpha}{1 - \tau} I_0 \quad (18)$$

$$= \left[ \frac{\lambda'_\tau (\lambda_\tau - 1)}{(\lambda_\tau - 1)^2} - \frac{\lambda'_\tau \lambda_\tau}{(\lambda_\tau - 1)^2} \right] \frac{r_\tau - \alpha}{1 - \tau} I_0 \quad (19)$$

$$= \left[ \frac{\lambda'_\tau \lambda_\tau - \lambda'_\tau}{(\lambda_\tau - 1)^2} - \frac{\lambda'_\tau \lambda_\tau}{(\lambda_\tau - 1)^2} \right] \frac{r_\tau - \alpha}{1 - \tau} I_0 \quad (20)$$

$$= -\lambda'_\tau \frac{1}{(\lambda_\tau - 1)^2} \frac{r_\tau - \alpha}{1 - \tau} I_0 \quad (21)$$

for  $I_0 = 1$  we find

$$\frac{dP_\tau^*}{d\tau} = -\frac{d\lambda_\tau}{d\tau} \frac{r_\tau - \alpha}{1 - \tau} \frac{1}{(\lambda_\tau - 1)^2} - \frac{\lambda_\tau}{\lambda_\tau - 1} \frac{\alpha}{(1 - \tau)^2} \quad (22)$$



**Lemma 1:** Let  $G = \frac{dP_\tau}{d\tau} = -\frac{d\lambda_\tau}{d\tau} \frac{r_\tau - \alpha}{1 - \tau} \frac{1}{(\lambda_\tau - 1)^2} - \frac{\lambda_\tau}{\lambda_\tau - 1} \frac{\alpha}{(1 - \tau)^2}$  and  $\varepsilon \equiv r_\tau - \alpha > 0$

then

$$\frac{d^2 P_\tau}{d\tau^2} = \frac{dG}{d\tau} > 0 \quad \text{for sufficiently small } \varepsilon$$

**Proof of Lemma 1:** Since  $G = \frac{dP_\tau}{d\tau} = -\frac{d\lambda_\tau}{d\tau} \frac{r_\tau - \alpha}{1 - \tau} \frac{1}{(\lambda_\tau - 1)^2} - \frac{\lambda_\tau}{\lambda_\tau - 1} \frac{\alpha}{(1 - \tau)^2}$  and

$\varepsilon \equiv r_\tau - \alpha > 0$  then

$$\begin{aligned} \frac{d^2 P_\tau^*}{d\tau^2} &= \frac{dG}{d\tau} = -\frac{d^2 \lambda_\tau}{d\tau^2} \frac{r_\tau - \alpha}{1 - \tau} \frac{1}{(\lambda_\tau - 1)^2} + \frac{d\lambda_\tau}{d\tau} \frac{r_\tau}{1 - \tau} \frac{1}{(\lambda_\tau - 1)^2} - \frac{d\lambda_\tau}{d\tau} \frac{r_\tau - \alpha}{(1 - \tau)^2} \frac{1}{(\lambda_\tau - 1)^2} \\ &\quad + \frac{d\lambda_\tau}{d\tau} \frac{r_\tau (1 - \tau) - \alpha}{1 - \tau} \frac{2}{(\lambda_\tau - 1)^3} \frac{d\lambda_\tau}{d\tau} - \frac{\frac{d\lambda_\tau}{d\tau}}{\lambda_\tau - 1} \frac{\alpha}{(1 - \tau)^2} \\ &\quad + \frac{\lambda_\tau \frac{d\lambda_\tau}{d\tau}}{(\lambda_\tau - 1)^2} \frac{\alpha}{(1 - \tau)^2} - \frac{\lambda_\tau}{\lambda_\tau - 1} \frac{\alpha 2(1 - \tau)}{(1 - \tau)^4} \\ &= -\frac{d^2 \lambda_\tau}{d\tau^2} \frac{r_\tau - \alpha}{1 - \tau} \frac{1}{(\lambda_\tau - 1)^2} + \frac{d\lambda_\tau}{d\tau} \frac{r_\tau}{(1 - \tau)^2} \frac{1}{(\lambda_\tau - 1)^2} - \frac{d\lambda_\tau}{d\tau} \frac{r_\tau - \alpha}{(1 - \tau)^2} \frac{1}{(\lambda_\tau - 1)^2} \\ &\quad + \frac{d\lambda_\tau}{d\tau} \frac{r_\tau (1 - \tau) - \alpha}{1 - \tau} \frac{2}{(\lambda_\tau - 1)^3} \frac{d\lambda_\tau}{d\tau} - \frac{\frac{d\lambda_\tau}{d\tau}}{(\lambda_\tau - 1)^2} \frac{(\lambda_\tau - 1) \alpha}{(1 - \tau)^2} \\ &\quad + \frac{\lambda_\tau \frac{d\lambda_\tau}{d\tau}}{(\lambda_\tau - 1)^2} \frac{\alpha}{(1 - \tau)^2} - \frac{\lambda_\tau}{\lambda_\tau - 1} \frac{\alpha 2(1 - \tau)}{(1 - \tau)^4} \\ &= -\frac{d^2 \lambda_\tau}{d\tau^2} \frac{r_\tau - \alpha}{1 - \tau} \frac{1}{(\lambda_\tau - 1)^2} + \frac{d\lambda_\tau}{d\tau} \frac{r_\tau}{(1 - \tau)^2} \frac{1}{(\lambda_\tau - 1)^2} - \frac{d\lambda_\tau}{d\tau} \frac{r_\tau - \alpha}{(1 - \tau)^2} \frac{1}{(\lambda_\tau - 1)^2} \\ &\quad + \frac{d\lambda_\tau}{d\tau} \frac{r_\tau (1 - \tau) - \alpha}{1 - \tau} \frac{2}{(\lambda_\tau - 1)^3} \frac{d\lambda_\tau}{d\tau} + \frac{d\lambda_\tau}{d\tau} \frac{1}{(\lambda_\tau - 1)^2} \frac{\alpha}{(1 - \tau)^2} - \frac{\lambda_\tau}{\lambda_\tau - 1} \frac{\alpha 2}{(1 - \tau)^3} \\ &= -\frac{d^2 \lambda_\tau}{d\tau^2} \frac{\varepsilon (1 - \tau)}{(1 - \tau)^2} \frac{1}{(\lambda_\tau - 1)^2} + \frac{d\lambda_\tau}{d\tau} \frac{\varepsilon (1 - \tau)}{(1 - \tau)^2} \frac{2}{(\lambda_\tau - 1)^3} \frac{d\lambda_\tau}{d\tau} \\ &\quad + 2 \frac{d\lambda_\tau}{d\tau} \frac{1}{(\lambda_\tau - 1)^2} \frac{\alpha}{(1 - \tau)^2} - \frac{\lambda_\tau (\lambda_\tau - 1)}{(\lambda_\tau - 1)^2} \frac{\alpha 2}{(1 - \tau)^3} \end{aligned}$$

$$\begin{aligned}
&= -\frac{d^2\lambda_\tau}{d\tau^2} \frac{\varepsilon(1-\tau)}{(1-\tau)^2} \frac{1}{(\lambda_\tau-1)^2} + 2\frac{\varepsilon(1-\tau)}{(1-\tau)^2} \frac{1}{(\lambda_\tau-1)^3} \left(\frac{d\lambda_\tau}{d\tau}\right)^2 \\
&\quad + \frac{d\lambda_\tau}{d\tau} \frac{2}{(\lambda_\tau-1)^2} \frac{\alpha}{(1-\tau)^2} - \frac{\lambda_\tau(\lambda_\tau-1)}{(\lambda_\tau-1)^2} \frac{\alpha 2}{(1-\tau)^3} \\
&= \frac{1}{(\lambda_\tau-1)^2} \frac{1}{(1-\tau)^2} \left[ \varepsilon(1-\tau) \left[ -\frac{d^2\lambda_\tau}{d\tau^2} + \frac{2}{(\lambda_\tau-1)} \left(\frac{d\lambda_\tau}{d\tau}\right)^2 \right] \right. \\
&\quad \left. + \frac{d\lambda_\tau}{d\tau} 2\alpha - \lambda_\tau(\lambda_\tau-1) \frac{\alpha 2}{(1-\tau)} \right]
\end{aligned}$$

As  $\lambda_\tau = \frac{1}{2} - \frac{r_\tau - \varepsilon}{\sigma^2} + \left[ \left( \frac{1}{2} - \frac{r_\tau - \varepsilon}{\sigma^2} \right)^2 + \frac{2r_\tau}{\sigma^2} \right]^{\frac{1}{2}}$  tends to 1 for sufficiently decreasing  $\varepsilon$ , the term  $-\frac{(1-\tau)\alpha(\lambda_\tau-1)}{(1-\delta)} \frac{d^2\lambda_\tau}{d\tau^2} - \lambda_\tau(\lambda_\tau-1) \frac{2\alpha}{(1-\tau)}$  tends to become sufficiently close to zero. Further, as  $0 < \delta < 1$  we must check the sign of  $-\frac{(1-\tau)\alpha(\lambda_\tau-1)}{(1-\delta)} \frac{2}{(\lambda_\tau-1)} \left(\frac{d\lambda_\tau}{d\tau}\right)^2 - 2\frac{d\lambda_\tau}{d\tau}\alpha$ : for sufficiently small  $\varepsilon$ :

$$\begin{aligned}
\frac{d^2P_\tau^*}{d\tau^2} &= \frac{dG}{d\tau} > 0 \quad \text{if} \quad \frac{(\lambda_\tau-1)\alpha 2(1-\tau)}{1-\delta} \left(\frac{d\lambda_\tau}{d\tau}\right)^2 + \frac{d\lambda_\tau}{d\tau} 2\alpha > 0 \\
2\frac{d\lambda_\tau}{d\tau} \left[ \frac{(1-\tau)\alpha}{1-\delta} \frac{d\lambda_\tau}{d\tau} + \alpha \right] &> 0
\end{aligned}$$

$$\begin{aligned}
0 &> -\frac{(1-\tau)}{1-\delta} \left[ \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r(1-\tau)}{\sigma^2} \right]^{-1/2} \frac{r}{\sigma^2} + 1 \\
0 &< \frac{r_\tau}{1-\delta} \left[ \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r(1-\tau)}{\sigma^2} \right]^{-1/2} \frac{1}{\sigma^2} - 1
\end{aligned}$$

$$\begin{aligned}
1 &< \frac{r_\tau \varepsilon}{(\lambda_\tau - 1) \alpha \sigma^2 \left[ \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r(1-\tau)}{\sigma^2} \right]^{1/2}} \quad \text{using } \varepsilon = \frac{(\lambda_\tau - 1) \alpha}{1 - \delta} \\
A_1 &: = \frac{\varepsilon r_\tau}{(\lambda_\tau - 1) \alpha \sigma^2 \left[ \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r(1-\tau)}{\sigma^2} \right]^{1/2}} \\
\lim_{\varepsilon \rightarrow 0} A_1 &= \frac{r_\tau}{\frac{d\lambda_\tau}{d\varepsilon} \alpha \sigma^2 \left[ \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r(1-\tau)}{\sigma^2} \right]^{1/2}} \quad \text{using L'Hopital's rule} \\
&= \frac{r_\tau}{\frac{\left[ \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2(r-r\tau)}{\sigma^2} \right]^{-\frac{1}{2}}}{\sigma^2} \lambda_\tau \alpha \sigma^2 \left[ \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r(1-\tau)}{\sigma^2} \right]^{1/2}} \\
&= \frac{r_\tau}{\lambda_\tau \alpha} \\
1 &< \frac{r_\tau}{\lambda_\tau \alpha} \Leftrightarrow \lambda_\tau < \frac{r_\tau}{\alpha} \quad \text{see (16)}
\end{aligned}$$

Hence, if  $\varepsilon$  decreases there will be a sufficiently small  $\varepsilon$  so that

$$\frac{d^2 P_\tau^*}{d\tau^2} = \frac{dG}{d\tau} > 0$$

#### Appendix 4: Proof of proposition 1 and 2:

**Proof of Proposition 1:** Our investment threshold  $P_\tau^*$  is given by

$$P_\tau^* = \frac{\lambda_\tau}{\lambda_\tau - 1} \frac{r_\tau - \alpha}{1 - \tau} \quad \text{see (9)}$$

for  $I_0 = 1$ .

For a neutral tax regime condition (10) must hold. I.e.

$$\frac{dP_\tau^*}{d\tau} = -\frac{d\lambda_\tau}{d\tau} \frac{r_\tau - \alpha}{1 - \tau} \frac{1}{(\lambda_\tau - 1)^2} - \frac{\lambda_\tau}{\lambda_\tau - 1} \frac{\alpha}{(1 - \tau)^2} = 0 \quad \text{see (10)}.$$

Now we will need the notion "regular value". A differentiable function  $f$  has the regular value  $y$  if for all  $x \in f^{-1}(y)$  the derivative  $Df(x)$  has a full rank.

As the derivative of  $G$  with respect to  $\tau$  is  $\frac{dG}{d\tau} = \frac{d^2 P_\tau}{d\tau^2} > 0$  (see Lemma 1, appendix 3b), 0 is a regular value of  $G : R^4 \rightarrow R$  and the set of points  $G^{-1}(0)$  is a manifold of dimension  $4 - 1 = 3$  (see Milnor (1997) , p. 11 ) .

### Appendix 5: implicit function

**Proof of Proposition 2:** As  $G^{-1}(0)$  is a manifold and as for each vector  $(\alpha_0, r_0, \tau_0, \sigma_0)$  the derivative  $\frac{dG}{d\tau}(\alpha_0, r_0, \tau_0, \sigma_0)$  is positive and as the partial derivatives of  $G$  by  $\tau, \alpha, r$  and  $\sigma$  are continuous , we can apply the implicit function theorem. Hence for a marginal environment of any vector  $(\alpha_0, r_0, \tau_0, \sigma_0)$ ,  $\tau^N$  is an implicit function of  $\alpha, r$  and  $\sigma$ .

q.e.d.

**Lemma 2:** Let  $\frac{dP_\tau^*}{d\tau} = G = -\frac{d\lambda_\tau}{d\tau} \frac{r_\tau - \alpha}{1 - \tau} \frac{1}{(\lambda_\tau - 1)^2} - \frac{\lambda_\tau}{\lambda_\tau - 1} \frac{\alpha}{(1 - \tau)^2} = 0$  and  $\varepsilon \equiv$

$r_\tau - \alpha > 0$  then

$$\text{a) } \frac{dG}{d\alpha} > 0, \quad \text{b) } \frac{dG}{dr} < 0 \quad \text{and} \quad \text{c) } \frac{dG}{d\sigma} > 0 \quad \text{for sufficiently small } \varepsilon$$

**Proof of Lemma 2:**

$$\begin{aligned} \text{a) } \frac{dG}{d\alpha} &= -\frac{d^2 \lambda_\tau}{d\tau d\alpha} \frac{r_\tau - \alpha}{1 - \tau} \frac{1}{(\lambda_\tau - 1)^2} + \frac{d\lambda_\tau}{d\tau} \frac{1}{1 - \tau} \frac{1}{(\lambda_\tau - 1)^2} \\ &\quad + \frac{d\lambda_\tau}{d\tau} \frac{r_\tau - \alpha}{1 - \tau} \frac{2 \frac{d\lambda_\tau}{d\alpha}}{(\lambda_\tau - 1)^3} + \frac{\frac{d\lambda_\tau}{d\alpha} \alpha}{(1 - \tau)^2} - \frac{\lambda_\tau}{\lambda_\tau - 1} \frac{1}{(1 - \tau)^2} \\ &= \frac{1}{(\lambda_\tau - 1)^2 (1 - \tau)^2} \left[ \begin{array}{l} -\frac{d^2 \lambda_\tau}{d\tau d\alpha} \varepsilon (1 - \tau) + \frac{d\lambda_\tau}{d\tau} (1 - \tau) \\ + \frac{d\lambda_\tau}{d\tau} \frac{2\varepsilon(1 - \tau) \frac{d\lambda_\tau}{d\alpha}}{(\lambda_\tau - 1)} + \frac{d\lambda_\tau}{d\alpha} \alpha - \lambda_\tau (\lambda_\tau - 1) \end{array} \right] > 0 \end{aligned}$$

Given  $\varepsilon = \frac{(\lambda_\tau - 1)\alpha}{1 - \delta}$ , we must show that [...]  $< 0$  :

$$0 < -\frac{d^2\lambda_\tau}{d\tau d\alpha}\varepsilon(1-\tau) + \frac{d\lambda_\tau}{d\tau}(1-\tau) + \frac{d\lambda_\tau}{d\tau} \frac{2\varepsilon(1-\tau)}{(\lambda_\tau-1)} \frac{\frac{d\lambda_\tau}{d\alpha}}{(\lambda_\tau-1)} + \frac{d\lambda_\tau}{d\alpha}\alpha - \lambda_\tau(\lambda_\tau-1)$$

with  $\varepsilon$  sufficiently small,  $(\lambda_\tau - 1)$  tends to become sufficiently close to zero.

Hence we obtain

$$\begin{aligned} 0 &< \frac{d\lambda_\tau}{d\tau}(1-\tau) + \frac{d\lambda_\tau}{d\tau} \frac{(\lambda-1)\alpha}{(1-\delta)} \frac{2(1-\tau)}{(\lambda_\tau-1)} \frac{\frac{d\lambda_\tau}{d\alpha}}{(\lambda_\tau-1)} + \frac{d\lambda_\tau}{d\alpha}\alpha \\ 0 &< \frac{d\lambda_\tau}{d\tau}(1-\tau) + \frac{d\lambda_\tau}{d\tau} \frac{2(1-\tau)\alpha}{(1-\delta)} \frac{\frac{d\lambda_\tau}{d\alpha}}{d\alpha} + \frac{d\lambda_\tau}{d\alpha}\alpha \\ &< \frac{d\lambda_\tau}{d\tau}(1-\tau) + \frac{d\lambda_\tau}{d\tau} \frac{2(1-\tau)\alpha}{(1-\delta)} \frac{\frac{d\lambda_\tau}{d\alpha}}{d\alpha} + \frac{d\lambda_\tau}{d\alpha}\alpha \end{aligned}$$

$$0 < \frac{\left[\left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)^2 + \frac{2r_\tau}{\sigma^2}\right]^{-\frac{1}{2}}}{\sigma^2} \left[ -r_\tau + 2 \frac{r_\tau\alpha}{(1-\delta)} \frac{\left[\left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)^2 + \frac{2r_\tau}{\sigma^2}\right]^{-\frac{1}{2}}}{\sigma^2} \lambda - \lambda\alpha \right]$$

$$0 < \left[ -r_\tau + 2 \frac{r_\tau\alpha}{(1-\delta)} \frac{\left[\left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)^2 + \frac{2r_\tau}{\sigma^2}\right]^{-\frac{1}{2}}}{\sigma^2} \lambda - \lambda\alpha \right]$$

$$0 < -r_\tau + 2 \frac{r_\tau\alpha}{(1-\delta)} \frac{\left[\left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)^2 + \frac{2r_\tau}{\sigma^2}\right]^{-\frac{1}{2}}}{\sigma^2} \lambda - \lambda\alpha$$

$$r_\tau + \lambda\alpha < 2 \frac{r_\tau\alpha}{(1-\delta)} \frac{\left[\left(\frac{1}{2} - \frac{\alpha}{\sigma^2}\right)^2 + \frac{2r_\tau}{\sigma^2}\right]^{-\frac{1}{2}}}{\sigma^2} \lambda$$

$$\begin{aligned}
1 &< 2 \frac{r_\tau \alpha}{(1-\delta)(r_\tau + \lambda_\tau \alpha)} \frac{\left[ \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r_\tau}{\sigma^2} \right]^{-\frac{1}{2}}}{\sigma^2} \lambda \quad \text{with } \varepsilon = \frac{(\lambda_\tau - 1)\alpha}{1-\delta} \\
A_2 &: = \frac{2\varepsilon r_\tau}{(\lambda_\tau - 1)(r_\tau + \lambda_\tau(r_\tau - \varepsilon))} \frac{\lambda}{\sigma^2 \left[ \left( \frac{1}{2} - \frac{(r_\tau - \varepsilon)}{\sigma^2} \right)^2 + \frac{2r_\tau}{\sigma^2} \right]^{\frac{1}{2}}} \quad \text{using L'Hopital's rule:} \\
\lim_{\varepsilon \rightarrow 0} A_2 &= \lim_{\varepsilon \rightarrow 0} \frac{2r_\tau \lambda}{(r_\tau + \lambda_\tau \alpha) \frac{d\lambda_\tau}{d\varepsilon} \sigma^2 \left[ \left( \frac{1}{2} - \frac{(r_\tau - \varepsilon)}{\sigma^2} \right)^2 + \frac{2r_\tau}{\sigma^2} \right]^{\frac{1}{2}}} \\
&= \frac{2r_\tau \lambda}{(r_\tau + \lambda_\tau \alpha) \frac{\left[ \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2(r_\tau - r_\tau)}{\sigma^2} \right]^{-\frac{1}{2}}}{\sigma^2} \lambda \sigma^2 \left[ \left( \frac{1}{2} - \frac{(r_\tau - \varepsilon)}{\sigma^2} \right)^2 + \frac{2r_\tau}{\sigma^2} \right]^{\frac{1}{2}}} \\
\lim_{\varepsilon \rightarrow 0} A_2 &= \frac{2r_\tau}{(r_\tau + \lambda_\tau \alpha)} \\
1 &< \frac{2r_\tau}{(r_\tau + \lambda_\tau \alpha)} \\
r_\tau + \lambda_\tau \alpha &< 2r_\tau \\
\frac{r_\tau}{\alpha} + \lambda_\tau &< 2 \frac{r_\tau}{\alpha} \quad \lambda_\tau < \frac{r_\tau}{\alpha}
\end{aligned}$$

$$\frac{dG}{d\alpha} > 0 \text{ for sufficiently small } \varepsilon.$$

$$\frac{dP_\tau^*}{d\tau} = G = -\frac{d\lambda_\tau}{d\tau} \frac{r_\tau - \alpha}{1-\tau} \frac{1}{(\lambda_\tau - 1)^2} - \frac{\lambda_\tau}{\lambda_\tau - 1} \frac{\alpha}{(1-\tau)^2}$$

$$\begin{aligned}
\text{b) } \frac{dG}{dr} &= -\frac{\overset{(-)}{d^2\lambda_\tau} r_\tau - \alpha}{d\tau dr} \frac{1}{1-\tau} \frac{1}{(\lambda_\tau - 1)^2} - \frac{d\lambda_\tau}{d\tau} \frac{1}{(\lambda_\tau - 1)^2} \\
&+ \frac{\overset{(-)}{d\lambda_\tau} r_\tau - \alpha}{d\tau} \frac{\overset{(+)}{2\frac{d\lambda_\tau}{dr}}}{(1-\tau)(\lambda_\tau - 1)^3} + \frac{\overset{(+)}{\frac{d\lambda_\tau}{dr}\alpha}}{(1-\tau)^2} \\
&- \frac{\frac{d\lambda_\tau}{dr} (\lambda_\tau - 1)\alpha}{(\lambda_\tau - 1)^2 (1-\tau)^2} + \frac{\lambda_\tau \frac{d\lambda_\tau}{dr} \alpha}{(\lambda_\tau - 1)^2 (1-\tau)^2} \\
&= -\frac{\overset{(-)}{d^2\lambda_\tau} r_\tau - \alpha}{d\tau dr} \frac{(1-\tau)}{(1-\tau)^2 (\lambda_\tau - 1)^2} - \frac{d\lambda_\tau}{d\tau} \frac{(1-\tau)^2}{(1-\tau)^2 (\lambda_\tau - 1)^2} \frac{1}{(\lambda_\tau - 1)^2} \\
&+ \frac{\overset{(-)}{d\lambda_\tau} r_\tau - \alpha}{d\tau} \frac{\overset{(+)}{2(1-\tau)} \frac{d\lambda_\tau}{dr}}{(1-\tau)^2 (\lambda_\tau - 1)^3} + \frac{\overset{(+)}{\frac{d\lambda_\tau}{dr}\alpha} (\lambda_\tau - 1)^2}{(1-\tau)^2 (\lambda_\tau - 1)^2} \\
&= \frac{1}{(\lambda_\tau - 1)^2 (1-\tau)^2} \left[ \begin{array}{l} \overset{(-)}{-\frac{d^2\lambda_\tau}{d\tau dr} \varepsilon (1-\tau) - \frac{d\lambda_\tau}{d\tau} (1-\tau)^2} \\ \overset{(+)}{+\frac{d\lambda_\tau}{d\tau} \frac{2\varepsilon(1-\tau) \frac{d\lambda_\tau}{dr}}{(\lambda_\tau - 1)} + \frac{d\lambda_\tau}{d\tau} \alpha} \end{array} \right] < 0
\end{aligned}$$

$$\begin{aligned}
0 &> -\frac{\overset{(-)}{d^2\lambda_\tau} \varepsilon (1-\tau) - \frac{d\lambda_\tau}{d\tau} (1-\tau)^2}{d\tau dr} + \frac{d\lambda_\tau}{d\tau} \frac{2\varepsilon(1-\tau) \frac{d\lambda_\tau}{dr}}{(\lambda_\tau - 1)} + \frac{\overset{(+)}{d\lambda_\tau} \alpha}{d\tau} \\
0 &> -\frac{\overset{(-)}{d^2\lambda_\tau} \varepsilon (1-\tau) - \frac{d\lambda_\tau}{d\tau} (1-\tau)^2}{d\tau dr} + \frac{d\lambda_\tau}{d\tau} \frac{(\lambda_\tau - 1)\alpha}{1-\delta} \frac{2(1-\tau) \frac{d\lambda_\tau}{dr}}{(\lambda_\tau - 1)} + \frac{\overset{(+)}{d\lambda_\tau} \alpha}{d\tau} \\
0 &> -\frac{d\lambda_\tau}{d\tau} (1-\tau)^2 + \frac{d\lambda_\tau}{d\tau} \frac{2(1-\tau)\alpha \frac{d\lambda_\tau}{dr}}{1-\delta} + \frac{\overset{(+)}{d\lambda_\tau} \alpha}{d\tau}
\end{aligned}$$

$$\begin{aligned}
0 &> \left[ \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r(1-\tau)}{\sigma^2} \right]^{-1/2} \frac{r}{\sigma^2} (1-\tau)^2 \\
&- \left[ \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r(1-\tau)}{\sigma^2} \right]^{-1/2} \frac{r}{\sigma^2} \frac{\alpha(1-\tau)}{1-\delta} \left[ \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r(1-\tau)}{\sigma^2} \right]^{-\frac{1}{2}} \frac{(1-\tau)}{\sigma^2} \\
&+ \left[ \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r(1-\tau)}{\sigma^2} \right]^{-\frac{1}{2}} \frac{(1-\tau)}{\sigma^2} \alpha
\end{aligned}$$

$$\begin{aligned}
0 &> \left[ \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r(1-\tau)}{\sigma^2} \right]^{-1/2} \frac{1}{\sigma^2} \left[ r_\tau(1-\tau) - \frac{2r_\tau\alpha(1-\tau)}{1-\delta} \left[ \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r(1-\tau)}{\sigma^2} \right]^{-\frac{1}{2}} \frac{1}{\sigma^2} \right. \\
&\quad \left. + (1-\tau)\alpha \right] \\
0 &> r_\tau(1-\tau) - \frac{2r_\tau\alpha(1-\tau)}{1-\delta} \left[ \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r(1-\tau)}{\sigma^2} \right]^{-\frac{1}{2}} \frac{1}{\sigma^2} + (1-\tau)\alpha \\
0 &< -r_\tau + \frac{2r_\tau\alpha}{1-\delta} \left[ \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r(1-\tau)}{\sigma^2} \right]^{-\frac{1}{2}} \frac{1}{\sigma^2} - \alpha \\
(r_\tau + \alpha) &< + \frac{2\alpha r_\tau}{(1-\delta)\sigma^2} \left[ \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r(1-\tau)}{\sigma^2} \right]^{-\frac{1}{2}} \\
1 &< \frac{2\alpha r_\tau}{(1-\delta)(r_\tau + \alpha)\sigma^2} \left[ \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r(1-\tau)}{\sigma^2} \right]^{-\frac{1}{2}} \quad \text{with } \varepsilon = \frac{(\lambda_\tau - 1)\alpha}{1-\delta}
\end{aligned}$$

$$\begin{aligned}
A_3 &: = \frac{2\varepsilon r_\tau}{(\lambda_\tau - 1)(r_\tau + \alpha)\sigma^2 \left[ \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r(1-\tau)}{\sigma^2} \right]^{\frac{1}{2}}} \\
\lim_{\varepsilon \rightarrow 0} A_3 &= \lim_{\varepsilon \rightarrow 0} \frac{2r_\tau}{\frac{d\lambda_\tau}{d\varepsilon} (r_\tau + \alpha)\sigma^2 \left[ \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r(1-\tau)}{\sigma^2} \right]^{\frac{1}{2}}} \quad \text{using L'Hopital's rule} \\
&= \frac{2r_\tau}{\frac{\left[ \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2(r-r_\tau)}{\sigma^2} \right]^{-\frac{1}{2}}}{\sigma^2} \lambda (r_\tau + \alpha)\sigma^2 \left[ \left( \frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r(1-\tau)}{\sigma^2} \right]^{\frac{1}{2}}} \\
&1 < \frac{2r_\tau}{\lambda(r_\tau + \alpha)}
\end{aligned}$$

$$\lambda(r_\tau + \alpha) < 2r_\tau$$

$$\lambda\varepsilon + \lambda 2\alpha < 2\varepsilon + 2\alpha$$

$$(\lambda - 1)2\alpha < \varepsilon(2 - \lambda)$$

$$\frac{(\lambda - 1)2\alpha}{(2 - \lambda)} < \varepsilon$$



$$\frac{(\lambda - 1)\alpha}{\varepsilon} < \frac{1}{2}(2 - \lambda)$$

$$\frac{dG}{dr} > 0 \text{ for sufficiently small } \varepsilon.$$

$$\begin{aligned} \text{c) } \frac{dG}{d\sigma} &= -\frac{\overset{(-)}{d^2\lambda_\tau} r_\tau - \alpha}{d\sigma d\tau} \frac{1}{1 - \tau} \frac{1}{(\lambda_\tau - 1)^2} + \frac{\overset{(-)}{d\lambda_\tau} r_\tau - \alpha}{d\tau} \frac{2\frac{\overset{(-)}{d\lambda_\tau}}{d\sigma}}{(1 - \tau)(\lambda_\tau - 1)^3} + \frac{\overset{(-)}{d\lambda_\tau} \alpha}{(1 - \tau)^2} > 0 \\ &= \frac{1}{(\lambda_\tau - 1)^2 (1 - \tau)^2} \left[ -\frac{\overset{(-)}{d^2\lambda_\tau} (r_\tau - \alpha)(1 - \tau)}{d\sigma d\tau} + \frac{\overset{(-)}{d\lambda_\tau} 2(r_\tau - \alpha)(1 - \tau)\frac{\overset{(-)}{d\lambda_\tau}}{d\sigma}}{(\lambda_\tau - 1)} \right. \\ &\quad \left. + \frac{\overset{(-)}{d\lambda_\tau} \alpha (\lambda_\tau - 1)}{d\sigma} \right] > 0 \end{aligned}$$

for sufficiently small  $\varepsilon$ ,  $\lambda$  tends to one and hence  $\frac{\overset{(-)}{d\lambda_\tau}}{d\tau} \frac{\alpha}{1 - \delta} \frac{2\frac{\overset{(-)}{d\lambda_\tau}}{d\sigma}}{1 - \tau}$  dominates.

For the implicit function:  $\tau^N$  we can take the derivative with respect to  $\alpha$ ,  $\frac{d\tau^N}{d\alpha}$ ,  $r$ ,  $\frac{d\tau^N}{dr}$  and  $\sigma$ ,  $\frac{d\tau^N}{d\sigma}$ . Using the condition for a neutral tax regime (10) and assuming sufficiently small  $\varepsilon$  we obtain:

$$\begin{aligned} \frac{d\tau^N}{d\alpha} &= -\frac{\frac{dG}{d\alpha}}{\frac{dG}{d\tau}} \\ &= -\frac{-\frac{\overset{(-)}{d^2\lambda_\tau}}{d\tau d\alpha} \varepsilon (1 - \tau) + \frac{\overset{(-)}{d\lambda_\tau}}{d\tau} (1 - \tau) + \frac{\overset{(-)}{d\lambda_\tau} 2\varepsilon(1 - \tau)\frac{\overset{(-)}{d\lambda_\tau}}{d\alpha}}{(\lambda_\tau - 1)} + \frac{\overset{(-)}{d\lambda_\tau} \alpha - \lambda_\tau (\lambda_\tau - 1)}{d\alpha}}{-\varepsilon (1 - \tau) \left[ \frac{\overset{(-)}{d^2\lambda_\tau}}{d\tau^2} - \frac{2\left(\frac{\overset{(-)}{d\lambda_\tau}}{d\tau}\right)^2}{(\lambda_\tau - 1)} \right] + 2\frac{\overset{(-)}{d\lambda_\tau}}{d\tau} \alpha - \lambda_\tau (\lambda_\tau - 1) \frac{\alpha 2}{(1 - \tau)}} < 0 \end{aligned}$$

$$\begin{aligned} \frac{d\tau^N}{dr} &= -\frac{\frac{dG}{dr}}{\frac{dG}{d\tau}} \\ &= -\frac{-\frac{\overset{(-)}{d^2\lambda_\tau}}{d\tau dr} (r_\tau - \alpha)(1 - \tau) - \frac{\overset{(-)}{d\lambda_\tau}}{d\tau} (1 - \tau) + \frac{\overset{(-)}{d\lambda_\tau} 2(r_\tau - \alpha)(1 - \tau)\frac{\overset{(+)}{d\lambda_\tau}}{dr}}{(\lambda_\tau - 1)} + \frac{\overset{(+)}{d\lambda_\tau} \alpha}{dr}}{-\varepsilon (1 - \tau) \left[ \frac{\overset{(-)}{d^2\lambda_\tau}}{d\tau^2} - \frac{2\left(\frac{\overset{(-)}{d\lambda_\tau}}{d\tau}\right)^2}{(\lambda_\tau - 1)} \right] + 2\frac{\overset{(-)}{d\lambda_\tau}}{d\tau} \alpha - \lambda_\tau (\lambda_\tau - 1) \frac{\alpha 2}{(1 - \tau)}} > 0 \end{aligned}$$

$$\begin{aligned}
\frac{d\tau^N}{d\sigma} &= -\frac{\frac{dG}{d\sigma}}{\frac{dG}{d\tau}} \\
&= -\frac{\overset{(-)}{\frac{d^2\lambda_\tau}{d\sigma d\tau}}(r_\tau - \alpha)(1 - \tau) + \frac{\overset{(-)}{d\lambda_\tau}}{d\tau} \frac{2(r_\tau - \alpha)(1 - \tau) \frac{\overset{(-)}{d\lambda_\tau}}{d\sigma}}{(\lambda_\tau - 1)} + \frac{\overset{(-)}{d\lambda_\tau}}{d\sigma} \alpha(\lambda_\tau - 1)}{-\varepsilon(1 - \tau) \left[ \frac{d^2\lambda_\tau}{d\tau^2} - \frac{2\left(\frac{d\lambda_\tau}{d\tau}\right)^2}{(\lambda_\tau - 1)} \right] + 2\frac{d\lambda_\tau}{d\tau} \alpha - \lambda_\tau(\lambda_\tau - 1) \frac{\alpha^2}{(1 - \tau)}} < 0
\end{aligned}$$

We therefore know that there is a marginal environment around  $\tau^N$  where the described reaction can be observed.

**Proof of Proposition 3:** In a marginal environment of the neutral tax regime there is the function that defines the neutral tax rates  $\tau^N = \tau^N(\alpha, r, \tau, \sigma)$ . We know from lemma 1 that  $\frac{d^2 P_\tau}{d\tau^2} > 0$ , hence

$$\frac{dP_\tau^*}{d\tau} \left\{ \begin{array}{l} < \\ > \end{array} \right\} 0 \quad \text{for} \quad \tau \left\{ \begin{array}{l} < \\ > \end{array} \right\} \tau^N(\alpha, r, \tau, \sigma).$$

## References

*Agliardi, Elettra* (2001), Taxation and Investment Decisions: A Real Option Approach. *Australian Economic Papers* 40, 44-55.

*Altug, Sumru/Demers, Fanny S./Demers, Michel* (2001), The Impact of Tax Risk and Persistence on Investment Decisions. *Economic Bulletin* 5, 1-5.

*Auerbach, Alan J./Hines, James R.* (1988), Investment Tax Incentives and Frequent Tax Reforms. *American Economic Review*. 78, 211-216.

*Auerbach, Alan J./Poterba, James M.* (1987), Tax-Loss Carryforwards and Corporate Tax Incentives, in: Feldstein, Martin (ed.), *The Effects of Taxation on Capital Accumulation*, Chicago, London, 305-342.

*Bloom, Nick/Bond, Stephen/Van Reenen, John* (2007), Uncertainty and Investment Dynamics. *Review of Economic Studies* 74, 391-415.

*Blouin, Jennifer/Hail, Luzi/Yetman, Michelle* (2005), Capital Gains Taxes, Pricing Spreads and Arbitrage: Evidence from Cross-Listed Firms in the U.S. SSRN Working Paper, January 2005, [http://papers.ssrn.com/sol3/papers.cfm?abstract\\_id=677040](http://papers.ssrn.com/sol3/papers.cfm?abstract_id=677040).

*Blouin, Jennifer L./Raedy, Jana Smith/ Shackelford, Douglas A.* (2003): Capital Gains Taxes and Equity Trading: Empirical Evidence. *Journal of Accounting Research* 41, 611-651.

*Bond, Stephen R./Devereux, Michael P.* (1995), On the design of a neutral business tax under uncertainty. *Journal of Public Economics* 58, 57-71.

*Brown, E. Cary* (1948), Business-Income Taxation and Investment Incentives. In: Metzler, L.A. et al. (eds.), *Income, Employment and Public Policy, Essays in Honor of A.H. Hansen*. New York. 300-316.

*Cook, Kirsten A.* (2006), Stock Price Reaction to a Reduction in the Capital Gains Tax Rate: Evidence from the Taxpayer Relief Act of 1997. *Journal of the American Taxation Association, Twelfth Annual Tax Research Conference*, San Diego, February 24, 2006, <http://aaahq.org/ata/conferences/jata-conference/2006/pdf/Cook.pdf>.

*Dai, Zhonglan/Maydew, Edward/Shackelford, Douglas A./Zhang, Harold H.* (2006),: Capital Gains Taxes and Asset Prices: Capitalization or Lock-in? NBER Working Paper No. W12342, February 2006, [http://papers.ssrn.com/sol3/papers.cfm?abstract\\_id=885907](http://papers.ssrn.com/sol3/papers.cfm?abstract_id=885907).

*Dixit, Avinash K./Pindyck, Robert S.* (1994), Investment under Uncertainty. Princeton.

*Domar, Evsey D., Musgrave, Richard A.* (1944), Proportional Income Taxation and Risk-Taking. Quarterly Journal of Economics 56, 388-422.

*Edmiston, Kelly D.* (2004), Tax Uncertainty and Investment: A Cross-Country Empirical Investigation. Economic Inquiry 42, 425-439.

*Gamba, Andrea/Sick, Gordon/León, Carmen Aranda* (2005), Investment under Uncertainty, Debt and Taxes, SSRN Working Paper, [http://papers.ssrn.com/sol3/papers.cfm?abstract\\_id=676572](http://papers.ssrn.com/sol3/papers.cfm?abstract_id=676572).

*Hammond, Peter J.* (1990), Theoretical Progress in Public Economics: A Provocative Assessment. Oxford Economic Papers 42, 6-33.

*Harchaoui, Tarek M./Lasserre, Pierre* (1996), Le choix de capacité comme l'exercice d'une option d'achat financière. Canadian Journal of Economics 29, 271-288.

*Johansson, Sven-Erik* (1969), Income Taxes and Investment Decisions. Swedish Journal of Economics 71, 104-110.

*Jou, Jyh-Bang* (2000), Irreversible Investment Decisions under Uncertainty with Tax Holidays. Public Finance Review 28, 66-81.

*Kaplow, Louis* (1986), An Economic Analysis of Legal Transitions. Harvard Law Review 99, 509-617.

*Keuschnigg, Christian/Nielsen, Soren B.* (2004), Start-ups, venture capitalists, and the capital gains tax: Journal of Public Economics 88, 1011-1042.

*Kiesewetter, Dirk/Niemann, Rainer* (2004), Steuerparadoxa durch Endbesteuerung, Mindestbesteuerung und Begünstigung einbehaltener Gewinne. *Journal für Betriebswirtschaft* 54, 129-139.

*Konrad, Kai A.* (1991), The Domar-Musgrave phenomenon and adverse selection. *European Journal of Political Economy* 7, 41-53.

*Lang, Mark H./Shackelford, Douglas A.* (2000), Capitalization of capital gains taxes: evidence from stock price reaction to the 1997 rate reduction. *Journal of Public Economics* 76, 69-85.

*MacKie-Mason, Jeffrey K.* (1990), Some Nonlinear Tax Effects on Asset Values and Investment Decisions under Uncertainty. *Journal of Public Economics* 42, 301-327.

*Milnor, John W.* (1997), *Topology from the differentiable viewpoint*, Princeton.

*Niemann, Rainer* (1999), Neutral Taxation under Uncertainty - a Real Options Approach. *FinanzArchiv* 56, 51-66

*Niemann, Rainer* (2004), Investitionswirkungen steuerlicher Verlustvorträge – Wie schädlich ist die Mindestbesteuerung. *Zeitschrift für Betriebswirtschaft* 74, 359-384.

*Niemann, Rainer/Sureth, Caren* (2002), Taxation under uncertainty – problems of dynamic programming and contingent claims analysis in real option theory, in: CESifo Working Paper No. 709, Munich.

*Niemann, Rainer/Sureth, Caren* (2004), Tax Neutrality under Irreversibility and Risk Aversion. *Economics Letters* 84, 43-47.

*Niemann, Rainer/Sureth, Caren* (2005), Capital Budgeting with Taxes under Uncertainty and Irreversibility. *Journal of Economics and Statistics* 225, 77-95.

*Panteghini, Paolo M.* (2001), Dual Income Taxation: The Choice of the Imputed Rate of Return. *Finnish Economic Papers* 14, 5-13.

*Panteghini, Paolo M.* (2004), Wide versus Narrow Tax Bases under Optimal Investment Timing. *FinanzArchiv* 60, 482-493.

*Panteghini, Paolo M./Scarpa, Carlo* (2003), Irreversible investment and regulatory risk. CESifo Working Paper No. 934, Munich.

*Pawlina, Grzegorz/Kort, Peter M.* (2005), Investment under uncertainty and policy change. *Journal of Economic Dynamics and Control* 29, 1193-1209.

*Pennings, Enrico* (2000), Taxes and stimuli of investment under uncertainty. *European Economic Review* 44, 383-391.

*Poterba, James M.* (1989a), Capital gains tax policy toward entrepreneurship. *National Tax Journal* 42, 375-389.

*Poterba, James M.* (1989b), Venture capital and capital gains taxation, in: Summers, Lawrence H. (ed.), *Tax Policy and the Economy* 3, Cambridge: MIT Press, 47-67.

*Samuelson, Paul A.* (1964), Tax Deductibility of Economic Depreciation to Insure Invariant Valuations. *Journal of Political Economy* 72, 604-606.

*Schneider, Dieter* (1969), Korrekturen zum Einfluß der Besteuerung auf die Investition, in: *Zeitschrift für betriebswirtschaftliche Forschung* 21, 297-325.

*Schneider, Dieter* (1980), The effects of progressive and proportional income taxation on risk-taking. *National Tax Journal* 33, 67-75.

*Schneider, Dieter* (1992), *Investition, Finanzierung und Besteuerung*, 7th edition, Wiesbaden.

*Schneider, Dirk* (2005), *Robustheit der Investitionsneutralität bedeutender theoretischer Steuersysteme*, Lohmar, Köln.

*Shackelford, Douglas A./Verrecchia, Robert E.* (2002), Intertemporal Tax Discontinuities. *Journal of Accounting Research* 40, 205-222.

*Stiglitz, Joseph E.* (1969), The Effects of Income, Wealth, and Capital Gains Taxation on Risk-Taking. *Quarterly Journal of Economics* 83, 263-283.

*Sureth, Caren* (1999), Der Einfluss von Steuern auf Investitionsentscheidungen bei Unsicherheit, Wiesbaden.

*Sureth, Caren* (2002), Partially irreversible investment decisions and taxation under uncertainty - a real option approach. *German Economic Review* 3, 185-221.

*Sureth, Caren/Maiterth, Ralf* (2005), Wealth Tax as Alternative Minimum Tax? - The Impact of a Wealth Tax on Business Structure and Strategy. arqus, Discussion Papers on Quantitative Tax Research, Discussion Paper No. 3, <http://www.arqus.info>.

*Trigeorgis, Lenos* (1996), Real Options – Managerial Flexibility and Strategy in Resource Allocation. Cambridge, Mass.

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