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#### Abstract

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# Christian Bredemeier and Falko Juißen* 

## Household Labor Supply and Home Services in a General-Equilibrium Model with Heterogeneous Agents


#### Abstract

We propose a new explanation for differences and changes in labor supply by gender and marital status, and in particular for the increase in married women's labor supply over time. We argue that this increase as well as the relative constancy of other groups' hours are optimal reactions to outsourcing labor in home production becoming more attractive to households over time. To investigate this hypothesis, we incorporate heterogeneous agents into a household model of labor supply and allow agents to trade home labor. This model can generate the observed patterns in US labor supply by gender and marital status as a reaction to declining frictions on the market for home services. We provide an accounting exercise to highlight the role of alternative explanations for the rise in hours in a model where home labor is tradable.


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[^1]
## 1 Introduction

Weekly hours of market work in the US have increased steadily over the last 50 years. An aggregate view, however, hides a number of important subgroup-specific patterns in labor supply. Most striking is the substantial increase in married women's hours of market work. By contrast, married men slightly decreased their working time, while singles of both genders increased their labor supply somewhat.

The rise in aggregate hours has been explained by demographic changes, such as the fertility and marriage decline (Chiappori and Weiss 2006; Albanesi and Olivetti 2007), and overall productivity growth (Mincer 1962; Smith and Ward 1985). Concerning group-specific developments, especially the strong increase in married women's labor supply has received considerable attention. One line of argument builds on the closure of the gender productivity gap (Galor and Weil 1996; Jones, Manuelli, and McGrattan 2003; Knowles 2007; Attanasio, Low, and Sánchez-Marcos 2008). Other explanations are based on technological improvements in home production (Greenwood, Seshadri, and Yorukoglu 2005) or reductions in child care costs (Attanasio, Low, and Sánchez-Marcos 2008). Other studies attribute the increase in married women's labor supply to increasing returns to experience (Olivetti 2006) or to insurance motives (Attanasio, Low, and Sánchez-Marcos 2005). Some recent studies have pointed to social norms as additional determinants of gender-specific labor supply (Fernández, Fogli, and Olivetti 2004; Fernández 2007).

We propose a new explanation for differences in labor supply by gender and marital status, and in particular for the increase in married women's labor supply over time. The increase in labor supply of married women may be due to the fact that outsourcing labor in home production has become more attractive to households over time. We propose a theoretical model in which married women increase their market hours as an optimal reaction to rising attractiveness of outsourcing home labor. At the same time, such development also induces patterns of labor supply for married men and singles of both genders which are in accordance with empirical evidence.

Outsourcing home labor implies that home labor is tradable. The possibility to trade home labor distinguishes our model from previous studies of household labor supply. There are several reasons to believe that home labor is tradable. For instance, when we think of home production being
cleaning and washing, hiring housekeepers is an alternative to own labor. When it comes to childcare or geriatric care as a "home good", the assignment of babysitters or nannies and the use of outpatient care is not unusual. Similarly, instead of cooking on one's own, food preparation can be outsourced to service providers.

In our model, the possibility to outsource home labor induces differences in the amount of supplied labor between population groups. Group-specific labor supply is also affected by the possibility to specialize within the household. Relatively productive agents will delegate home production either to their spouse or to other agents in order to realize an efficiency gain. This delegation induces married men to work the most on average, followed by single men and single women, while married women work the fewest hours.

To make home services tradable in our model, we distinguish between two labor markets. On a "first market", labor is used for the production of usual consumption goods, whereas on the "second market", labor is used as an input for home production. Since there are two markets, agents have the possibility to specialize. Some agents may supply home services on the second market, while others find it more attractive to work on the first market solely, depending on the wages they can earn on each market. In addition to heterogeneity by gender and marital status, agents in our model differ by first-market productivity. This heterogeneity in productivities is a driving force for the allocation of time.

Our model of group-specific labor supply can be characterized by five key elements: (i) home labor is tradable, (ii) the female productivity distribution is a downward spread of the male one, (iii) couples can take advantage of intra-household specialization, (iv) couples decide collectively on labor supply ${ }^{1}$, (v) mating occurs in a perfectly assortative way ${ }^{2}$.

This structure explains differences in subgroup-specific labor supply which are in accordance with empirical evidence. On average, husbands work more hours than single men because some husbands gain time to work on the market due to intra-household specialization. Similarly, female singles work more hours on the market than married women because some wives spend more time in home production in order to provide their husbands with home consumption. Since men are assumed to be more

[^2]productive than women in our model, more male singles find it rational to outsource home labor. As a consequence, they have more time to work on the market than female singles.

These results hinge on the existence of a market for home labor. However, a frictionless market for home labor would have a counterfactual efficiency implication. Individuals whose opportunity cost of time is higher than the wage to be paid for home services should not work at home at all. Obviously, this is not how people behave. There seem to be further costs of hiring home services in addition to wage costs. For instance, hiring home services may be associated with search costs. There may also be quality differences between hired and own home labor. Another example is that outsourcing child care is associated with utility costs. In all three examples, home-services demanders have to bear additional costs to the monetary compensation a hired home worker receives.

Such costs create a wedge between the price of buying one's way out of one hour of house work and the wage for one hour of house work. In our model, this wedge corresponds to a distortionary "as-if" tax on the market for home services, i.e. frictions on this market are modeled in the spirit of Chari, Kehoe, and McGrattan (2007). We show that the model with an "as-if" tax is the reduced form to several models with structural frictions, such as quality differences, search frictions, and utility costs.

We suggest that outsourcing of home labor has become more attractive over time, for instance because search costs or utility costs have declined over time. In our model, such developments correspond to a declining "as-if" tax on the market for home services. The induced changes in labor supply by population groups are consistent with empirical evidence for the US. When the wedge decreases, more households will decide to hire labor for home production instead of doing these tasks on their own. The respective singles and wives gain time to work on the first market. Husbands, on the contrary, are not affected by the outsourcing decision since, in the respective households, they do not work in the household anyway due to assumed productivity advantages relative to their wives. Most importantly, declining costs of outsourcing labor in home production have a particularly strong effect on married women's labor supply: In all population groups, a declining wedge induces households to hire more home services. This is freeing up time to work on the market. But depending on their productivity, married women tended to work more hours in home production than singles because wives provided their husbands with home consumption as well. Being released of this type of
work therefore has a strong impact on married women's labor supply.
We use our model for a quantitative analysis in which we determine the sequence of "as-if" tax rates under which the model generates patterns of labor supply by groups that best correspond to observed patterns in the Current Population Survey (CPS). Furthermore, we provide an accounting exercise in order to assess the relative importance of different exogenous causes for changes in group-specific labor supply, such as the marriage decline, productivity growth, and women's catching-up in terms of market wages. We find that, in our model, these changes in observable factors are not sufficient to account for the substantial rise in hours worked by married women. The remaining portion can be explained by a declining "as-if" tax on the market for home services. At the same time, this decline does not generate counterfactual implications for other groups' hours.

The remainder of the paper is organized as follows. The next section briefly summarizes empirical facts on labor supply in the US over the last decades. Section 3 describes the theoretical model. A quantitative analysis is presented in Section 4. We discuss potential sources for frictions on the market for home services in Section 5. Section 6 concludes the paper and an appendix follows.

## 2 Empirical Facts

To provide the empirical background of our analysis, we recapitulate the observed patterns in US labor market data to which we will compare our model. Figure 1 shows average hours of market work by gender and marital status. The data stems from the March Supplement of the CPS, from 1962 to 2007, in the format arranged by Unicon Research. ${ }^{3}$ We define working age as 18 to 65 and restrict the sample to the civilian population of that age.

Over the last decades labor supply of married women increased substantially. In the early 1960's they worked on average just a little more than 10 hours per week. Until 2006 this number more than doubled to over 20 hours a week. At the same time, labor supply of married men slightly fell from somewhat below 40 hours per week to approximately 37.5 - a decrease of $6 \%$. Both single men and women did not change their labor supply by much. They tended to work less than married men, but

[^3]

Figure 1: Average Weekly Market Hours by Gender and Marital Status in the US (March CPS, Persons aged 18-65)
more than married women. On average, male singles worked slightly above, female singles slightly below 25 hours a week. Both time series showed a slight upward trend in the 1970's and 80's.

Overall, these developments have led to an increase in aggregate hours per capita over time, see Figure 2. During the period 1962 to 2007, aggregate hours rose by $15 \%$. Our model aims at replicating the ordering of hours worked by groups, as well as the direction and magnitude of changes in hours over time. In particular, our model should generate a decline in married men's hours in response to a declining as-if tax on the market for home services and a rise in married women's working hours which is larger in absolute value. We will also examine the implications of our model for the behavior of aggregate labor supply.

## 3 A Market for Home Services

In this section, we introduce a market for home labor into a collective model of household labor supply. We transfer the framework of Jones, Manuelli, and McGrattan (2003) into a heterogeneous-agents model where households differ with respect to their first-market pro-


Figure 2: Aggregate Weekly Per-Capita Hours in the US (March CPS, Persons aged 18-65)
ductivities, which is to say potential wages. The decision process within households is modeled using a stylized version of endogenous bargaining positions (Chiappori 1988; Chiappori 1992).

### 3.1 Environment

Population composition. The population consists of women (mass 1) and men (also mass 1). An exogenous fraction $s$ of both genders are singles, the remaining fraction $(1-s)$ is married to an individual of the other gender. For trade in home services to occur, households have to differ with respect to their productivities. We assume heterogeneity in productivities on the "first market" where consumption goods are produced. Home goods are produced within households using labor which can be traded at a second market.

We incorporate heterogeneity with respect to first-market productivities by assuming a continuum of agents with a continuous wage structure. By contrast, all agents are assumed to earn the same wage on the second market. Productivities on the first market are distributed uniformly on $[0,1]$ for men and on $[0, \alpha]$ for women, respectively, $\alpha<1 .{ }^{4}$ Empirical

[^4]evidence that productivity distributions have gender-specific supports is provided by Albanesi and Olivetti (2006), who observe that among topsalary receivers men earn more than women. More generally, there is considerable empirical evidence that, even after controlling for a number of observable characteristics, women's wages are lower than men's, see e.g. Goldin (1990), Blau and Kahn (1997), Blau (1998).

We introduce subscripts to indicate an individual's gender, marital status, and her position in the productivity distribution. Genders are coded by $F$ (or $f$ ) for female and $M$ (or $m$ ) for male. Capital letters indicate that the respective person is married, whereas lower-case letters stand for singles. For instance, the index $(M, 0.25)$ refers to a married man located at the lower . 25 quantile of the productivity distribution.

We assume that mating occurs in a perfect assortative way (Becker 1973). ${ }^{5}$ As a consequence of assortative mating, first-market productivities are perfectly correlated in marriages, i.e. a wife's wage is a constant fraction of her husband's one:

$$
\begin{equation*}
w_{F, i}=\alpha \cdot w_{M, i}, \tag{1}
\end{equation*}
$$

such that we can use the subscript $i \in[0,1]$ for the entire household, reflecting its position in the productivity distribution. With the assumption of assortative mating, the distribution parameter $\alpha$ determines intra-household productivity differentials between husbands and wives. Equation (1) implies that there are households where both spouses are highly productive, but there are also households where both partners have rather low productivities. Thus, not every woman is earning less than every man, but every wife is earning less than her husband (if both work on the first market).

With the uniform distribution of wages on $[0,1]$ and $[0, \alpha]$ for men and women, respectively, we have:

$$
\begin{equation*}
w_{M, i}=i, w_{F, i}=\alpha i, w_{m, i}=i, w_{f, i}=\alpha i \tag{2}
\end{equation*}
$$

model analytically. In the quantitative part of the paper we replace the assumption of a uniform distribution by assuming a more realistic log-normal one.
${ }^{5}$ Becker (1973) shows that such sorting is the only stable outcome of a perfect marriage market when marital surpluses are supermodular (i.e. if there are marital complementarities). Empirical evidence for marital sorting is provided by Fernández, Guner, and Knowles (2004), who find that the correlation of wife's and husband's productivity is remarkably high across Europe and both North and South America.

Preferences and technology. We assume that preferences of agents are given by the individual utility function

$$
\begin{align*}
u_{g, i} & =\mu \log c_{g, i}^{1}+\nu \log c_{g, i}^{2}+(1-\mu-\nu) \log \ell_{g, i}, \\
g & =F, M, f, m . \tag{3}
\end{align*}
$$

An individual has logarithmic preferences over her own consumption of market goods $\left(c^{1}\right)$, her consumption of home goods $\left(c^{2}\right)$, and her time spent on leisure ( $\ell$ ). The utility weights $\mu$ and $\nu$ are positive parameters, which sum up to less than one. The utility function is the same for all agents.

First, we consider the decision problem for married couples. We assume that a couple $i$ realizes an efficient intra-household bargaining solution. This is equivalent to assuming the household would maximize

$$
\begin{equation*}
U_{i}=\lambda_{F, i} \cdot u_{F, i}+\lambda_{M, i} \cdot u_{M, i}, \tag{4}
\end{equation*}
$$

which is a weighted sum of the (log) utilities of the two spouses. If the weights on individual utilities are endogenous, this is a version of the collective household model initially introduced by Chiappori (1988, 1992). In order to endogenize the weights, $\lambda_{F, i}$ and $\lambda_{M, i}$, we assume that an individual's bargaining position depends on his or her outside options. As a simple specification of this idea, we assume that a spouse's weight is equal to her relative contribution to the household's full income:

$$
\begin{equation*}
\lambda_{F, i}=\frac{W_{F, i}}{W_{F, i}+W_{M, i}}, \lambda_{M, i}=\frac{W_{M, i}}{W_{F, i}+W_{M, i}} \tag{5}
\end{equation*}
$$

Full incomes, $W_{F, i}$ and $W_{M, i}$, should be understood as the amount of earnings on both markets if the entire time endowment would be spent on paid labor. As time endowments are equal for all agents, relative full incomes correspond to relative wages. For the intra-household decision process it makes no difference whether the full incomes are actually earned or not, i.e. $W_{F, i}$ and $W_{M, i}$ are hypothetic incomes.

If utility weights were fixed, wives' leisure would decrease when the gender wage gap closes, which is counterfactual to what we observe, see Knowles (2007). Knowles argues that, when female wages rise, not only becomes her leisure more expensive to the household, but also improves her intra-household bargaining position due to better outside opportunities. Equation (5) is a stylized version of utility weights which reflect
outside options. This decision rule allows us to solve the model in closed form. ${ }^{6}$

A wife has a time endowment of one, which can be used for leisure $\ell_{F, i}$, first-market labor $h_{F, i}^{1}$, labor at home $l_{F, i}^{2}$, and labor at the second market $h_{F, i}^{2, S}$ (superscript $S$ indicating supply). Husbands face an equivalent constraint:

$$
\begin{align*}
\ell_{F, i}+h_{F, i}^{1}+l_{F, i}^{2}+h_{F, i}^{2, S} & \leq 1  \tag{6}\\
\ell_{M, i}+h_{M, i}^{1}+l_{M, i}^{2}+h_{M, i}^{2, S} & \leq 1 \tag{7}
\end{align*}
$$

Home goods have to be produced using capital $k_{i}$ and labor as inputs. Labor in home production of a partnership is the sum of husband's $\left(l_{M, i}^{2}\right)$ and wife's labor $\left(l_{F, i}^{2}\right)$ and the amount of hired home services $\left(h_{i}^{2, D}\right)$ :

$$
\begin{equation*}
\zeta \cdot\left(c_{F, i}^{2}+c_{M, i}^{2}\right) \leq A \cdot\left(k_{i}\right)^{\theta} \cdot\left(l_{F, i}^{2}+l_{M, i}^{2}+h_{i}^{2, D}\right)^{1-\theta} \tag{8}
\end{equation*}
$$

Own and hired labor are thus perfect substitutes. Olivetti (2006) has already studied the effects of external services in home production (production of "child quality" in her case), but in her model these services are complements to own labor. We model home services as substitutes to own labor, such that they can actually "free up" time to use for market labor.

The parameter $\zeta \leq 1$ measures economies of scale in consumption of the home good in a partnership. Such specification goes back to Barten (1964), who argued that the provision of home goods to two people may cost less than twice the provision to one.

The budget constraint for couple $i$ reads as:

$$
\begin{equation*}
c_{F, i}^{1}+c_{M, i}^{1}+q \cdot k_{i}+p \cdot h_{i}^{2, D} \leq w_{F, i} \cdot h_{F, i}^{1}+w_{M, i} \cdot h_{M, i}^{1}+\omega \cdot\left(h_{M, i}^{2, S}+h_{F, i}^{2, S}\right), \tag{9}
\end{equation*}
$$

where $q$ is the relative price of home capital. We assume that home capital fully depreciates each period. ${ }^{7} \quad p$ is the price for hired home services, while $\omega$ is the respective wage earned by home-services suppliers.

A key point of our analysis is that $p$ and $\omega$ are not necessarily equal. If $p>\omega$, there is a wedge between the costs of hiring home services and the compensation for supplying them. This means that an employer of such labor has to bear further (time, utility, or direct) costs in addition

[^5]to the monetary compensation a hired provider of home services receives. In a short-cut way to model this wedge between $\omega$ and $p$, we assume that there are further resource costs associated with hiring home labor. Since such wedge is equivalent to a distortionary tax on the second-market wage, we will refer to the relative wedge, $\tau$, as an "as-if" tax: ${ }^{8}$
\[

$$
\begin{equation*}
p=(1+\tau) \omega \tag{10}
\end{equation*}
$$

\]

All "tax revenues" are assumed to be used in a non-distortionary way.
Singles face a similar decision problem as couples. They maximize individual utility (3) without the possibility of intra-household specialization and economies of scale $(\zeta=1)$.' Singles face the following constraints (for $g=f, m$ ):

$$
\begin{align*}
\ell_{g, i}+h_{g, i}^{1}+l_{g, i}^{2}+h_{g, i}^{2, S} & \leq 1  \tag{11}\\
c_{g, i}^{2} & \leq A \cdot\left(k_{g, i}\right)^{\theta} \cdot\left(l_{g, i}^{2}+h_{g, i}^{2, D}\right)^{1-\theta}  \tag{12}\\
c_{g, i}^{1}+q k_{g, i}+p h_{g, i}^{2, D} & \leq w_{g, i} h_{g, i}^{1}+\omega h_{g, i}^{2, S} \tag{13}
\end{align*}
$$

### 3.2 Supply and Demand

Depending on the relation between first and second market wages, households take discrete labor market choices, i.e. they decide on which market to work. Given the composition of the economy, couples split up into four groups of households regarding their behavior on the market for home services. There is

1. a top-wage group with $i>\frac{p}{\alpha}$ that hires home services (group 1),
2. a high-wage group with $\frac{p}{\alpha}>i>\frac{\omega}{\alpha}$ that neither hires nor supplies home services on the second market (group 2),

[^6]3. a low-wage group with $\frac{\omega}{\alpha}>i>\omega$ where wives supply home services (group 3), and
4. a bottom-wage group with $\omega>i$ where both, wife and husband, supply home services on the market (group 4).

Singles do not have the possibility of intra-household specialization and earn wages $w_{f, i}=\alpha i$ and $w_{m, i}=i$, respectively. They belong either to ( $g=f, m$ )

1. a high-productivity group with $w_{g, i}>p$ which hires home services (group a),
2. a medium-productivity group with $p>w_{g, i}>\omega$ that neither demands nor supplies home services (group b), or
3. a low-productivity group with $w_{g, i}<\omega$ which supplies home services (group c).

### 3.2.1 Couples

Group 1: Top-wage couples. For a household in this group it is rational to hire home services and to supply own labor only on the first market since both their wages (husband and wife) exceed the effective price of hired home services. The full (potential) income of a household of this type is the sum of the two wages, $W_{i}=w_{F, i}+w_{M, i}=(1+\alpha) i$. The household splits this full income on the goods which provide utility according to the corresponding utility weights.

For instance, husband's leisure multiplied with its opportunity costs is a constant fraction of full income in the optimum:

$$
\begin{equation*}
\ell_{M, i} \cdot w_{M, i}=\lambda_{M, i}(1-\mu-\nu) W_{i} \tag{14}
\end{equation*}
$$

Since $\lambda_{M, i}=\frac{W_{M, i}}{W_{F, i}+W_{M, i}}=\frac{w_{M, i}}{w_{F, i}+w_{M, i}}$,

$$
\begin{equation*}
\ell_{M, i}=1-\mu-\nu \tag{15}
\end{equation*}
$$

and analogously

$$
\begin{equation*}
\ell_{F, i}=1-\mu-\nu . \tag{16}
\end{equation*}
$$

As both spouses do not work at home, they spend their remaining time at the first market:

$$
\begin{equation*}
h_{M, i}^{1}=h_{F, i}^{1}=\mu+\nu \tag{17}
\end{equation*}
$$

For the amount of hired home labor, total opportunity costs have to equal a constant fraction of total income that is determined by the corresponding utility weight and the efficient share of labor in the production of the home consumption good:

$$
\begin{align*}
h_{i}^{2, D} \cdot p & =(1-\theta) \nu W_{i} \\
\Longleftrightarrow h_{i}^{2, D} & =(1+\alpha) \cdot(1-\theta) \nu \frac{i}{p} \tag{18}
\end{align*}
$$

Hence the demand for home services of a household of this type depends positively on the ratio of its own wage to the effective price of home services. This ratio reflects the "leisure price" of home services.

Group sizes are endogenous in our model. All households with $i>\frac{p}{\alpha}$ belong to the top-wage group, thus its size decreases in the price $p$ for home services.

With Cobb-Douglas preferences and technology, time decisions neither depend on the price for home capital, $q$, nor on total factor productivity, A, see Jones, Manuelli, and McGrattan (2003) for a detailed discussion. The same holds for the parameter $\zeta$, measuring economies of scale in home production. These properties are an implication of Cobb-Douglas technology and preferences and they do not necessarily hold under different assumptions. However, Jones, Manuelli, and McGrattan (2003) show that for these parameters to have a substantial impact on hours worked, one would need to assume rather extreme forms of complementarities or substitutabilities, which come along with other, counterfactual, implications.

Group 2: High-wage couples. This group does not act on the second market since it is neither rational to hire nor to supply home services. Labor supply decisions are given by:

$$
\begin{align*}
h_{M, i}^{1} & =\mu+\nu  \tag{19}\\
h_{F, i}^{1} & =\mu+\theta \nu-\alpha^{-1}(1-\theta) \nu \tag{20}
\end{align*}
$$

Due to husband-wife wage differentials it is efficient to specialize. Married women in this group supply less labor on the first market than their husbands but spend more time on home production. Thus these wives provide home consumption not only to themselves but also to their husbands.

The high-wage group contains households with $\frac{p}{\alpha}>i>\frac{\omega}{\alpha}$, thus its size increases in the wedge between $p$ and $\omega$. If there is no such wedge, this second group disappears and every household either supplies or demands home services. This is not what we observe in reality, which is giving support to our assumption that there exists a wedge as measured by the parameter $\tau$, see equation (10).

Group 3: Low-wage couples. In this group, husbands work solely on the first labor market, whereas wives work on the second market and at home. The full income of a household in this group is therefore given by $W_{i}=i+\omega$.

As husbands in this group do not work at home they spend their nonleisure time on the first market,

$$
\begin{equation*}
h_{M, i}^{1}=\mu+\nu . \tag{21}
\end{equation*}
$$

Wives' total non-leisure time is $\mu+\nu$ as for their husbands. For time spent on home production optimization requires

$$
\begin{align*}
l_{F, i}^{2} \cdot \omega & =(1-\theta) \nu W_{i} \\
\Longleftrightarrow l_{F, i}^{2} & =(1-\theta) \nu\left(1+\frac{i}{\omega}\right) . \tag{22}
\end{align*}
$$

The wife supplies her remaining time at the second market:

$$
\begin{equation*}
h_{F, i}^{2, S}=\mu+\theta \nu-(1-\theta) \nu \frac{i}{\omega} \tag{23}
\end{equation*}
$$

Labor supply on the second market hence increases in the wage for home services and decreases in first-market wages. The latter effect is an income effect. When the husband receives a salary rise, the household becomes richer and hence wishes to consume more home goods. This increase has to be accompanied by a rise in the wife's labor input into this production. Thus remaining time to be supplied on the second market decreases.

The size of this group is $\left(\alpha^{-1}-1\right) \omega$ and therefore depends on malefemale productivity differentials.

Group 4: Bottom-wage couples. Households in this group would earn that low wages at the first market that both, husband and wife, offer their labor at the second market. Full income of these households is

| (1) | (2) | (3) | (4) | (5) | (6) |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | male market 1 $h_{M, i}^{1}$ | hours market 2 $h_{M, i}^{2, S}$ | female market 1 $h_{F, i}^{1}$ | $\begin{aligned} & \text { iours } \\ & \text { market } 2 \\ & h_{F, i}^{2, S} \end{aligned}$ | labor 2 demand $h_{i}^{2, D}$ |
| $\begin{aligned} & \text { group } 1 \\ & i \in\left[\frac{p}{\alpha}, 1\right] \end{aligned}$ | $\mu+\nu$ | 0 | $\mu+\nu$ | 0 | $\begin{gathered} (1+\alpha) \\ (1-\theta) \nu \frac{i}{p} \end{gathered}$ |
| $\begin{aligned} & \text { group } 2 \\ & i \in\left[\frac{\omega}{\alpha}, \frac{p}{\alpha}\right] \end{aligned}$ | $\mu+\nu$ | 0 | $\begin{gathered} \mu+\theta \nu \\ -\alpha^{-1}(1-\theta) \nu \end{gathered}$ | 0 | 0 |
| $\begin{gathered} \text { group 3 } \\ i \in\left[\omega, \frac{\omega}{\alpha}\right] \end{gathered}$ | $\mu+\nu$ | 0 | 0 | $\begin{gathered} \mu+\theta \nu \\ -(1-\theta) \nu \frac{i}{\omega} \end{gathered}$ | 0 |
| $\begin{aligned} & \text { group 4 } \\ & i \in[0, \omega] \\ & \hline \end{aligned}$ | 0 | $\mu+\theta \nu$ | 0 | $\mu+\theta \nu$ | 0 |

Table 1: Summary of Couples' Labor Supply and Demand Decisions
therefore $W_{i}=2 \omega$. For both household members, optimization requires $\ell_{F, i}=\ell_{M, i}=1-\mu-\nu$. They work at home according to

$$
\begin{align*}
\left(l_{M, i}^{2}+l_{F, i}^{2}\right) \cdot \omega & =(1-\theta) \nu W_{i} \\
\Longleftrightarrow\left(l_{M, i}^{2}+l_{F, i}^{2}\right) & =2(1-\theta) \nu \tag{24}
\end{align*}
$$

and spend the rest of their time on the second market:

$$
\begin{equation*}
\left(h_{M, i}^{2, S}+h_{F, i}^{2, S}\right)=2-2(1-\theta) \nu-2(1-\mu-\nu)=2 \mu+2 \theta \nu \tag{25}
\end{equation*}
$$

As wages on the second market do not differ by productivity or gender, the allocation of these working times across spouses is indetermined. We can only state how much they will supply together. In the following, we will assume that households in the bottom-wage group split both types of labor equally among wife and husband. ${ }^{9}$ Total home-services supply of a household is a constant in this group, but the group size, which is $\omega$, increases in the wage for home services.

Average labor supply of husbands and wives. Table 1 summarizes supply and demand decisions of the four groups of couples. Group sizes can be read from the first column. Aggregating individual decisions

[^7]provides average market hours by gender for couples. Market hours refer to total compensated work and consist of first- and second-market labor supply. Average market hours of married men are derived by integrating columns (2) and (3):
\[

$$
\begin{equation*}
H_{M}=\mu+\nu-(1-\theta) \nu \cdot \omega \tag{26}
\end{equation*}
$$

\]

When the wage for home-services increases, compensated male labor decreases since group 4 grows and in this group, men do also work in the household and less on the market than men in other groups.

Average compensated hours of wives are calculated by integrating labor supply as given in columns (4) and (5):

$$
\begin{align*}
H_{F}= & \mu+\left(1-\frac{\omega}{\alpha}\right) \nu+\frac{\omega}{\alpha} \theta \nu-\left(\frac{p}{\alpha}-\frac{\omega}{\alpha}\right) \frac{1+\alpha}{\alpha}(1-\theta) \nu \\
& -\int_{\omega}^{\omega / \alpha}\left[(1-\theta) \nu \frac{i}{\omega}\right] \mathrm{di}  \tag{27}\\
= & \mu+\nu-(1-\theta) \nu\left[\alpha^{-2}+\alpha^{-1}\right] p+(1-\theta) \nu\left[\frac{\alpha^{-2}}{2}+\frac{1}{2}\right] \omega
\end{align*}
$$

Wives' hours increase in the home-services wage $\omega$ but decrease in the price $p$. When $\omega$ rises, more women receive male help in the household (group 4 grows) and some women (those in group 3) face rising opportunity costs of non-market time. If $p$ rises, group 1 becomes smaller and in this group, wives work the most.

### 3.2.2 Singles

Singles of both genders form a continuum each, which can be split up into three groups with respect to their behavior on the second labor market. High-productivity singles find it rational to hire home services in order to gain time to work on the first market (group a). Singles with a medium productivity neither demand nor supply home services (group b). Low-productivity singles supply home services (group c).

Within the groups, decisions are the same for both men and women. The only difference is that for men, more individuals belong to groups a and b , due to the gender-specific productivity distributions. For men $w_{m, i}=i$ holds and thus individuals $i \in[p, 1]$ belong to group a, $i \in[\omega, p]$ are in group b , and only those men with $i \in[0, \omega]$ find themselves in group c. Women's productivity is described by $w_{f, i}=\alpha \cdot i$, thus only those with $i \in[p / \alpha, 1]$ are in group a. Female singles $i \in[\omega / \alpha, p / \alpha]$ belong to group b, and those with $i \in[0, \omega / \alpha]$ to group c.

Group a: Highly productive singles. In this group, single women have a full income of $W_{i}=\alpha \cdot i$, while single men's full income is $W_{i}=$ $i$. Equivalent to the decisions of couples in the previous section they consume the amount $1-\mu-\nu$ of leisure time. Since they do not work at home themselves, they spend their remaining time on the first market ( $g=f, m$ ):

$$
\begin{equation*}
h_{g, i}^{1}=\mu+\nu \tag{28}
\end{equation*}
$$

Optimization with respect to hired home labor requires that total opportunity costs equal a constant fraction of full income that is determined by the corresponding utility weight and the efficient share of labor in home production:

$$
\begin{equation*}
h_{g, i}^{2, D}=(1-\theta) \nu \cdot \frac{w_{g, i}}{p} \tag{29}
\end{equation*}
$$

Group b: Medium-productivity Singles. Singles in this group work on the first market and at home. They allocate their time in the following way:

$$
\begin{align*}
\ell_{g, i} & =1-\mu-\nu  \tag{30}\\
h_{g, i}^{1} & =\mu+\theta \nu  \tag{31}\\
l_{g, i}^{2} & =(1-\theta) \nu \tag{32}
\end{align*}
$$

Group c: Low-productivity Singles. Full income of singles in this group is given by $W_{g, i}=\omega$, independent of gender. As any single, they consume $1-\mu-\nu$ of leisure time. They allocate the remaining time on supplied home services and labor at home. The opportunity cost of working at home is the second-market wage $\omega$, therefore:

$$
\begin{align*}
l_{g, i}^{2} \cdot \omega & =(1-\theta) \nu \cdot W_{g, i} \\
\Longleftrightarrow l_{g, i}^{2} & =(1-\theta) \nu \tag{33}
\end{align*}
$$

and

$$
\begin{equation*}
h_{g, i}^{2, S}=\mu+\theta \nu \tag{34}
\end{equation*}
$$

Average labor supply of singles. Table 2 summarizes the time allocations of singles. Once again, we consider average market hours by gender. Integrating total labor of types 1 and 2 , one can see that average

$\left.$| $(1)$ | $(2)$ | $(3)$ | $(4)$ | $(5)$ | $(6)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | range <br> for women | range <br> for men | hours <br> market 1 <br> $h_{g, i}^{1}$ |  | market 2 <br> $h_{g, i}^{2, S}$ | | labor 2 |
| :---: |
| demand |
| $h_{g, i}^{2, D}$ | \right\rvert\,

Table 2: Decisions of Singles $(g=f, m) ; w_{f, i}=\alpha \cdot i, w_{m, i}=i$
compensated labor of female singles decreases in the second-market price $p$ and is independent of the second-market wage $\omega$ :

$$
\begin{equation*}
H_{f}=\mu+\nu-(1-\theta) \nu \cdot \frac{p}{\alpha} \tag{35}
\end{equation*}
$$

The home-services wage $\omega$ does not affect hours of single women because changes in $\omega$ only induce some women to change the market (women changing from group b to group c). But these women still keep working the same amount of time. However, decreases in $p$ motivate some women to hire someone for doing house work and to increase their activity on the first market (women changing from group b to group a).

Analogously to female singles, we can derive average market hours of single men by integrating total labor of both types, as given in columns (4) and (5) of Table 2:

$$
\begin{equation*}
H_{m}=\mu+\nu-(1-\theta) \nu \cdot p \tag{36}
\end{equation*}
$$

Average hours of single men also decrease in $p$ and are independent of $\omega$. The economic reasons are the same as for women. Men's response to a change in the second-market price $p$ is weaker than that of women.

### 3.3 Equilibrium

We now analyze the equilibrium of the market for home services and its dependency on the relative wedge, $\tau$, between the price paid and the wage received for home labor, $p=(1+\tau) \omega$. Having solved for equilibrium prices, $p$ and $\omega$, we can analyze average market hours by gender and marital status as given by equations (26), (27), (35), and (36). In particular, we can examine how hours worked respond to changes in $\tau$.

Integrating the home-services supply and demand decisions of couples reported in Table 1 and those of singles reported in Table 2, and accounting for their respective masses ( $1-s$ for couples, $s$ for singles, respectively), yields total demand for and supply of home labor in the economy:

$$
\begin{align*}
H^{2, D} & =\frac{1}{2}(1-\theta) \nu \frac{1}{p} \cdot\left(1+\alpha-\left(\alpha^{-1}+\alpha^{-2}+s-\alpha^{-2} s\right) p^{2}\right)  \tag{37}\\
H^{2, S} & =\left[\left(\alpha^{-1}+1\right)(\mu+\theta \nu)-\frac{1}{2}(1-s)\left(\alpha^{-2}-1\right)(1-\theta) \nu\right] \omega \tag{38}
\end{align*}
$$

Defining $D_{1}:=\frac{1}{2}(1-\theta) \nu>0, D_{2}:=\alpha^{-1}+\alpha^{-2}+s-\alpha^{-2} s>0$, and $S_{1}:=\left[\left(\alpha^{-1}+1\right)(\mu+\theta \nu)-\frac{1}{2}(1-s)\left(\alpha^{-2}-1\right)(1-\theta) \nu\right]$, equalizing supply and demand for home services results in: ${ }^{10}$

$$
\begin{align*}
& p=(1+\tau)^{1 / 2} \cdot\left[\frac{(1+\alpha) D_{1}}{S_{1}+(1+\tau) D_{1} D_{2}}\right]^{1 / 2}  \tag{39}\\
& \omega=(1+\tau)^{-1 / 2} \cdot\left[\frac{(1+\alpha) D_{1}}{S_{1}+(1+\tau) D_{1} D_{2}}\right]^{1 / 2} \tag{40}
\end{align*}
$$

The price $p$ for home labor is increasing in the "as-if" $\operatorname{tax} \tau$ and the second-market wage $\omega$ is decreasing in $\tau$ :

$$
\begin{align*}
& \frac{\partial p}{\partial \tau}=\frac{1}{2} \cdot(1+\tau)^{-1 / 2} \cdot S_{1} \cdot \frac{\left((1+\alpha) D_{1}\right)^{1 / 2}}{\left(S_{1}+(1+\tau) D_{1} D_{2}\right)^{3 / 2}}>0  \tag{41}\\
& \frac{\partial \omega}{\partial \tau}=-\frac{1}{2} \cdot(1+\tau)^{-3 / 2} \cdot\left(S_{1}+2(1+\tau) D_{1} D_{2}\right) \cdot \frac{\left((1+\alpha) D_{1}\right)^{1 / 2}}{\left(S_{1}+(1+\tau) D_{1} D_{2}\right)^{3 / 2}}<0 \tag{42}
\end{align*}
$$

### 3.4 Hours Worked in Equilibrium

In this section we derive labor-supply patterns suggested by our model. The data suggests the following three observations with respect to market hours by gender and marital status (see Section 2): (i) Husbands work the most, followed by male singles and female singles. Married women

[^8]

Figure 3: Market Hours of Population Groups
work the fewest hours. (ii) Over time, husbands' hours decreased and all other groups' hours increased. (iii) Comparing the magnitudes of the changes, wives' change was by far the strongest. We now illustrate that the model outlined in Section 3 is able to generate the ordering of hours, i.e. $H_{M}>H_{m}>H_{f}>H_{F}$. Moreover, the model generates the direction and relative magnitude of the changes as a (comparative-static) response to a decrease in the "as-if" tax on the market for home services, $\tau$.

Ordering of average labor supply. Figure 3 summarizes labor supply decisions of singles and couples. Labor supply consists of both first and second-market hours. The vertical axis refers to a household's position in the productivity distribution, $i$. We can see that the ordering of aggregate labor supply by groups is as in the data. We consider men first. For any $i$, comparing the husband's labor supply to the corresponding labor supply of a single man (associated with the same productivity), yields the following: No husband works less but some work more than the corresponding single (those with $p>i>\omega$ ). Husbands and singles in this range do not outsource home labor. For singles, this means that they have to work in home production on their own, while husbands can take advantage of intra-household specialization.

A similar reasoning applies to male and female singles, as well as for female singles and wives. Since men are more productive than women in our model, more male singles find it rational to outsource home labor. As a consequence, they have more time to work on the market. Female singles work more hours on the market than married women because some wives (those with $\omega<i<p / \alpha$ ) spend more time in home production to provide their husbands with home consumption. Since each interval in Figure 3 has the same impact on all four group-specific averages, the ordering of average labor supply is $H_{M}>H_{m}>H_{f}>H_{F}$, as in the data. This result is shown analytically in Appendix A.2.

Changes in group-specific labor supply. We suggest that changes in group-specific labor supply over time are the result of a declining as-if tax on the market for home services. We already know from the previous section that, when the tax rate $\tau$ falls, the home-services wage $\omega$ increases and the price for these services $p$ decreases. To illustrate the effects on hours worked, we consider a situation where, due to a declining $\tau, \omega$ increases from $\omega_{0}$ to $\omega_{1}$ and $p$ declines from $p_{0}$ to $p_{1}$.
' Figure 4 presents an overview of how market labor changes in such experiment. All couples with $i>p_{0} / \alpha$ or $i<\omega_{0}$ will not change their labor supply decisions as they stay in groups 1 or 4 , respectively. Furthermore, no husband in a household with $i>\omega_{1}$ will change his labor supply, he might change between groups 1 to 3 , but that does not have an impact on his market hours. Only husbands in households $i \in\left[\omega_{0}, \omega_{1}\right]$ change their market hours. These households move from group 3 to 4 and hence husbands begin to work in the household because there are no longer intra-household market-wage differentials. Overall, average market hours of married men decrease.

Wives in households with $i \in\left[\omega_{0}, \omega_{1} / \alpha\right]$ all increase their market hours. Some of them move into group 4 and get male help in the household. Others just face higher opportunity costs of not working on the market due to the increase in $\omega$. Households $i \in\left[p_{1} / \alpha, p_{0} / \alpha\right]$ move from group 2 to group 1 and decide to hire home services, thereby saving time for the wife which she uses for market labor. No wife works less but some work more, that is to say average hours of married women increase.

Comparing the magnitudes of the changes, the female effect dominates the male one. Considering households $i \in\left[\omega_{0}, \omega_{1}\right]$ in Figure 4, it is apparent that all wives in this range at least compensate their husbands' market-hours decrease by own increases since $i / \omega_{0}>1$ for all

|  | (1) husbands | (2) <br> wives | (3) <br> fem. singles | (4) <br> male singles |
| :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & p_{0} / \alpha \\ & p_{1} / \alpha \end{aligned}$ | no change | no change | no change | no change |
|  | no change | $+(1-\theta) \nu \alpha^{-1}$ | $+(1-\theta) \nu$ |  |
|  | no change | no change | no change | $+(1-\theta) \nu$ |
| $\begin{aligned} & \omega_{1} / \alpha \\ & \hat{\omega_{0}} / \alpha \end{aligned}$ | no change | $+(1-\theta) \nu\left(i / \omega_{1}^{2}-\alpha^{-1}\right)$ | no change | no change |
|  | no change | $+(1-\theta) \nu i / \omega_{o}^{2} \cdot \Delta \omega$ | no change |  |
| $\begin{aligned} & \omega_{1} \\ & \stackrel{1}{\omega}_{0} \end{aligned}$ | $-(1-\theta) \nu$ | $+(1-\theta) \nu i / \omega_{0}$ |  | no change |
|  | no change | no change |  | no change |

Figure 4: Market-Labor Changes in Response to a Decreasing As-If Tax, $\tau$
these households. Overcompensation occurs because opportunity costs of non-market time increase due to the wage rise on the second market. Total market-labor supply of these households thus increases. All other husbands record constant hours in our hypothetical situation, whereas there are some other wives who also increase their market time. Thus, in reaction to a fall in $\tau$, the rise in hours of married women is stronger than the decline in male hours. This comparative-statics result is shown analytically in Appendix A.2.

Considering singles, the changes in $p$ and $\omega$ only affect the amount of supplied labor of those singles who change from group b to group a, as illustrated in the right part of Figure 4. These singles decide to hire home services and use the freed-up time to work on the first labor market. Thus they increase their labor supply. Therefore, total labor supply of both groups of singles rises since no single decreases his or her labor supply. Since there are more female than male singles changing groups ( $\Delta p / \alpha>\Delta p$ ), the female effect is larger than the male one.

Considering columns (2) and (3) of Figure 4, we can compare the change in labor supply of female singles to that of married women and see that the effect of wives is the stronger one. Consider those women with $i \in\left[p_{1} / \alpha, p_{0} / \alpha\right]$ : Married women in this range increase their labor supply by more than the respective single women. The reason for both groups' increase in market labor is that hiring home services is freeing up time. But married women in this range did work more at home than singles as they provided their husbands with home consumption as well. Hence, being released of this type of work has a stronger impact on married women's labor supply than it has on single women's hours. Outside of this range, no female single alters her labor supply, but some married women decide to work more. Thus the effect on married women is larger than that on singles and therefore wives experience the strongest change of all groups.

To illustrate these effects with a quantitative example, Figure 5 shows how the model reacts to linear decreases in the "as-if" $\operatorname{tax} \tau$ for the parameter constellation $\mu=0.4, \nu=0.2, \theta=0.33, \alpha=0.75$, and $s=0.5$. The figure resembles Figure 1 qualitatively, underlining the model's capability to replicate the observed patterns in labor supply.


Figure 5: Market Hours Response to a Linear Decrease in the "As-If" $\operatorname{Tax}, \tau(\mu=0.4, \nu=0.2, \theta=0.33, \alpha=0.75, s=0.5)$

## 4 Quantitative Analysis

We have shown that the model presented in the previous section is able to replicate the observed labor market trends qualitatively as a response to a declining "as-if" tax on the second market. The "as-if" tax is a reduced-form representation capturing several frictions on the market for home services, which may hamper outsourcing of household-related activities, see Section 5 for a discussion. In this sense, the tax rate $\tau$ is a deep parameter itself. Our next aim is to use our model to back out a sequence of as-if tax rates that best enables our model to match real-world patterns in hours by gender and marital status as displayed in Figure 1.

### 4.1 Productivity Distribution

So far we have assumed a uniform distribution of productivities, which allowed us to solve our model in closed form and to derive comparativestatic properties analytically. When we want to quantify deep parameters of our model, we should make the specification of the productivity distribution more realistic. Therefore, in the quantitative analysis, we replace


Figure 6: Relation between Male and Female Productivity Distributions
the uniform distribution by a log-normal one. Analogously to our previous specification with a uniform distribution, we capture male-female wage differentials by assuming that the female productivity distribution is a downward spread of the male one in the sense that

$$
\begin{equation*}
G(w)=F\left(\frac{w}{\alpha}\right), \tag{43}
\end{equation*}
$$

where $G$ is the female cumulative log-normal density and $F$ the male one. Figure 6 illustrates the relation between $F$ and $G$. For any particular wage $w_{0}$, more women than men fall short of this wage, which implies that $G \geq F \forall w$. Thus men are earning more than women in the sense of firstorder dominance. Assumption (43) implies that the entire wage structure in a specific year is described by three parameters, mean and variance of the male productivity distribution as well as the gender difference $\alpha$. Under assortative mating, a husband with first-market wage $w$ is then paired with a woman earning $\alpha \cdot w$.

The choice of productivity distribution affects aggregation and equilibrium prices. By contrast, it does not affect individual decisions for given prices as discussed in Section 3.2. For instance, total second-market labor
supply of married women is given by:

$$
\begin{align*}
H_{F}^{2, S} & =\int_{0}^{\omega}(\mu+\theta \nu) f(i) \mathrm{di}+\int_{\omega}^{\omega / \alpha}\left(\mu+\theta \nu-(1-\theta) \nu \frac{i}{\omega}\right) f(i) \mathrm{di} \\
& =(\mu+\theta \nu) \cdot F\left(\frac{\omega}{\alpha}\right)-(1-\theta) \nu \omega^{-1} \int_{\omega}^{\omega / \alpha} i \cdot f(i) \mathrm{di} \tag{44}
\end{align*}
$$

where $f(\cdot)$ is the density function and $F(\cdot)$ denotes the cumulative distribution function. As before, this quantity is derived by integrating the fifth column of Table 1, taking into account the density of the wage distribution. Analogously, we calculate second-market supply and demand of all other population groups. Aggregating over groups results in the following excess demand function:

$$
\begin{align*}
X= & (1-\theta) \nu p^{-1} \cdot\binom{[(1-s)(1+\alpha)+s \alpha] \int_{p / \alpha}^{\infty} i \cdot f(i) \mathrm{di}}{+s \int_{p}^{\infty} i \cdot f(i) \mathrm{di}+(1-s) \omega^{-1} \int_{\omega}^{\omega / \alpha} i \cdot f(i) \mathrm{di}} \\
& -(\mu+\theta \nu)\left(F(\omega)+F\left(\frac{\omega}{\alpha}\right)\right) \tag{45}
\end{align*}
$$

We use numerical techniques to determine the equilibrium $\omega$ for which excess demand for a given $\tau$ is zero.

We then use the equilibrium values for $\omega$ and $p=(1+\tau) \omega$ to calculate market hours by gender and marital status, which are given by:

$$
\begin{align*}
H_{M} & =\mu+\nu-(1-\theta) \nu F(\omega)  \tag{46}\\
H_{F} & =\mu+\nu-(1-\theta) \nu\binom{\left(1+\alpha^{-1}\right) F\left(\frac{p}{\alpha}\right)-\alpha^{-1} F\left(\frac{\omega}{\alpha}\right)}{\quad+\omega^{-1} \int_{\omega}^{\omega / \alpha} i \cdot f(i) \mathrm{di}}  \tag{47}\\
H_{m} & =\mu+\nu-(1-\theta) \nu F(p)  \tag{48}\\
H_{f} & =\mu+\nu-(1-\theta) \nu F\left(\frac{p}{\alpha}\right) \tag{49}
\end{align*}
$$

These moments are the analogs to equations (26), (27), (36), and (35), which measured hours worked under uniformly distributed productivities. We will match the moments $H_{M}, H_{F}, H_{m}$, and $H_{f}$ to their empirical counterparts by choosing values for $\tau$ and thereby implicitly for $p$ and $\omega$.

### 4.2 Parameter Choices

We take preference parameters and home production elasticities as constant over time. The equipment share of output in home production, $\theta$, is
taken from Knowles (2007), who calibrates this number to 0.08 in order to match equipment spending as a share of total consumption. Jones, Manuelli, and McGrattan (2003) choose with 0.22 a higher value for this elasticity. Similarly to Knowles (2007), we find that the exact number chosen for $\theta$ within this magnitude has no substantial influence on the quantitative results.

To characterize preferences, the relation between the weights on market and home consumption is crucial. The absolute values of the weights and thus the weight on leisure stand in direct relation to the chosen weekly time endowment. ${ }^{11}$ Knowles (2007) chooses 118 hours as weekly time endowment and a relative weight on leisure of 0.66. Since Knowles (2007) works with log utility as we do, we use these numbers. To derive the ratio of the taste for market consumption, $\mu$, and for home consumption, $\nu$, we build on Jones, Manuelli, and McGrattan (2003), who assume CES utility. We translate the CES weights into Cobb-Douglas weights by setting the CES substitution parameter to zero. This yields $\mu / \nu \approx 2 / 1$, thus we set $\mu=0.22$ and $\nu=0.12$.

In the quantitative analysis, we allow the composition of the economy to change over time. To quantify the fraction of singles in the economy, $s$, we extract a time series of the share of non-married individuals from CPS data. This time series is depicted in Figure 7.

We implicitly assume complete depreciation of home capital each period. This is an extreme assumption. It allows us to abstract from the capital accumulation decision and to concentrate on the decision whether to outsource home labor. Taking incomplete depreciation into account would not affect the outsourcing decisions since capital - once it is accumulated - does affect the productivity of own labor in home production and that of hired home services in the same way. Incomplete capital depreciation would give rise to several complications, such as the predictability of future frictions on the market for home services, i.e. whether changes in $\tau$ are predictable to agents or come as a surprise. With incomplete depreciation, the capital stock can not be reduced instantaneously after a decrease in $\tau$. This could give rise to overshooting phenomena or muted responses, which we exclude by abstracting from accumulation. Doing so bears the technical advantage that we can solve

[^9]

Figure 7: Parameter Choices: Share of Singles $s$ (Non-Married Individuals, March CPS, Persons aged 18-65)
for each period's decisions separately.
The remaining technological parameters of our model are the Barten scale, $\zeta$, total factor productivity in home production, $A$, and the relative price of home equipment, $q$. As shown in Section 3.2, these variables have no impact on the allocation of time given that decisions are not linked intertemporally. Therefore, we can ignore them in the quantitative analysis.

### 4.3 Methodology

In order to solve the model, we need to choose a set of parameters characterizing the distribution of first-market productivities. In order to ensure that our quantitative exercise is consistent with the underlying structural model, we have to take into account the model's predictions on labor market choices. Labor market choices determine whether first-market productivities of agents are actually observable or not. According to our model, agents having lower first-market productivities than $\omega_{t}$ decide to work on the market for home services and consequently, their first-market productivities should not be observable in year $t$. Since the equilibrium value for $\omega_{t}$ depends on the as-if tax $\tau_{t}$, labor market choices
and the observability of first-market productivities depend on this deep parameter as well. For this reason, our quantitative exercise needs to determine combinations of $\tau$ and the parameters characterizing the productivity distribution that are consistent with each other. In order to achieve this, we determine $\tau$ and the productivity parameters in an iterative way. The iterative procedure is performed separately in each year and can be described as follows.

1. To obtain a first guess for the parameters of the productivity distribution in a specific year $t$ we fit log-normal distributions to all strictly positive hourly wages in our selected sample in this year using Maximum Likelihood. We do so for both genders separately in order to obtain the means, $m_{\text {female,t }}$ and $m_{\text {male }, t}$, and the standard deviations, $\sigma_{\text {female }, t}$ and $\sigma_{\text {male }, t}$, of the gender-specific normal distributions of $\log$ wages in year $t$. Under the assumption that the female wage distribution is a downward spread of the male one in the sense that $g(w)=f\left(\frac{w}{\alpha}\right)$, it must hold that

$$
\begin{align*}
\sigma_{\text {female }, t} & =\sigma_{\text {male }, t}  \tag{50}\\
\text { and } m_{\text {female }, t} & =m_{\text {male }, t}+\ln \alpha_{t} \tag{51}
\end{align*}
$$

Relation (50) is shown by Browning, Chiappori, and Weiss (2009) as a stylized empirical result and allows us to describe the wage structure in our model by just one variance. To extract a value for $\alpha_{t}$, we use the estimated parameters for $m_{\text {male }, t}$ and $m_{\text {female }, t}$ and calculate $\alpha_{t}$ as

$$
\begin{equation*}
\alpha_{t}=\exp \left(m_{\text {female }, t}-m_{\text {male }, t}\right) . \tag{52}
\end{equation*}
$$

Then, we use the parameters of the fitted male distribution and approximate the female one by $g(w)=f\left(\frac{w}{\alpha}\right)$. Since the entire wage structure in our model economy is described by the parameters referring to the male productivity distribution along with $\alpha$, we drop the index male and summarize the productivity parameters by $m_{t}, \sigma_{t}$, and $\alpha_{t}$.

We then define a vector $\psi_{t}$ containing the parameter values of our model for year $t$. This vector comprises $\mu, \nu$, and $\theta$, which are constant across years, as well as $s_{t}, m_{t}, \sigma_{t}$, and $\alpha_{t}$ :

$$
\psi_{t}=\left(\begin{array}{lllllll}
\mu & \nu & \theta & s_{t} & m_{t} & \sigma_{t} & \alpha_{t} \tag{53}
\end{array}\right)^{\prime}
$$

2. Taking the values in $\psi_{t}$ as given, we determine the value for $\tau_{t}$ that best enables the underlying structural model to match mean hours worked by gender and marital status in year $t$. We choose that $\tau_{t}$ that minimizes a measure of the distance between the model and empirical moments, whereas the vector of moments comprises mean hours worked by gender and marital status, see equations (46) to (49).

Formally, let $\mathbf{H}_{\psi_{t}}\left(\tau_{t}\right)$ denote the vector of model moments for a given $\tau_{t}$ and a given parameter vector $\psi_{t}$, and let $\hat{\mathbf{H}}_{t}$ denote the corresponding empirical moments. Our estimate $\hat{\tau}_{t}$ is the solution to the quadratic minimization problem

$$
\begin{equation*}
\min _{\tau_{t}} \Gamma\left(\tau_{t}\right)=\left[\hat{\mathbf{H}}_{t}-\mathbf{H}_{\psi_{t}}\left(\tau_{t}\right)\right]^{\prime} \times \Omega_{t}^{-1} \times\left[\hat{\mathbf{H}}_{t}-\mathbf{H}_{\psi_{t}}\left(\tau_{t}\right)\right] \tag{54}
\end{equation*}
$$

where $\Omega_{t}^{-1}$ is a weighting matrix, which determines the relative importance of the individual moments in determining $\hat{\tau}_{t}$. Thus, in each year, we have an overidentified system with four moments to be matched and one parameter to be estimated. Minimization is carried out using a direct search method. As a weighting matrix $\Omega_{t}^{-1}$, we use the inverse of a diagonal matrix with the sample variances of mean hours worked by gender and marital status along the diagonal.
3. With the value for $\hat{\tau}_{t}$ resulting from step 2 and the current parameter vector $\psi_{t}$, we identify those agents who decide to work on the second market. Observed wages for those agents should not influence the identification of the distribution of first-market productivities. For men, the fraction of individuals working on the second market evaluates as $F(\omega)$, for women as $F\left(\frac{\omega}{\alpha}\right)$. To re-estimate the parameters of the productivity distribution we use a censoring routine. This censoring routine fits log-normal distributions to the upper $1-F(\omega)$ quantile of male wages and to the upper $1-F\left(\frac{\omega}{\alpha}\right)$ quantile of female wages. The MaximumLikelihood estimation thereby takes into account that it deals with left-censored data. As in step 1, the gender difference $\alpha_{t}$ is calculated as $\alpha_{t}=\exp \left(m_{\text {female }, t}-m_{\text {male }, t}\right)$. The parameters $m_{t}$ and $\sigma_{t}$ are again taken from the fitted male distribution (the one obtained using censored data).


Figure 8: Estimated Gender Productivity Difference $\alpha$

As long as the resulting values for $\alpha_{t}, m_{t}$ and $\sigma_{t}$ are not sufficiently close to those in $\psi_{t}$, we update $\psi_{t}$ with the new productivity parameters. ${ }^{12}$ We then proceed with step 2 using the updated parameter vector $\psi_{t}$.

The final parameterization reflects a combination of productivity parameters, $\alpha_{t}, m_{t}, \sigma_{t}$, and the as-if tax, $\tau_{t}$, for which the resulting labor market choices are consistent with the selection criterion to determine the parameters of the productivity distribution. When the iterative procedure has converged, we proceed with the next year.

### 4.4 Results

Figures 8 to 10 summarize the results of the moments-matching exercise described in the previous section.

The solid line in Figure 8 shows the estimated time series for the gender productivity difference $\alpha_{t}$. The series indicates that women are catching up to men in terms of first-market productivities. The first guess for

[^10]

Figure 9: Backed-Out Sequence for $\tau$
this parameter, which we derived from the entire data set without taking into account labor market choices, is depicted by the thin dashed line in Figure 8. Compared to this first guess, our final parameterization shows similar gender differences in productivity.

Figure 9 displays the backed-out sequence of "as-if" taxes. The sequence indicates that outsourcing of household-related activities has become easier over time, in the sense that $\tau$ declines steadily. However, there still exists a wedge between the wage received for home services and their associated price in recent years. This implies that there are agents who neither supply nor demand labor on the market for home services (groups 2 and b in the model).

The backed-out sequence for $\tau$ also shows fluctuations at business-cycle frequency. The pronounced downward trend in $\tau$ makes us confident that the results are not driven by these fluctuations.

We insert the estimated values for $\tau$ into the model in order to illustrate the extent to which our model is able to replicate the observed patterns in hours worked. Figure 10 displays the actual time series of hours worked by gender and marital status, together with the series of hours worked implied by our model under the estimated sequence $\hat{\tau}_{t}$. Actual data moments from the CPS are solid lines, while the dashed lines are their model counterparts.


Figure 10: Per-Capita Hours under the Backed-Out Sequence for $\tau$ versus Actual Per-Capita Hours in the CPS

Figure 10 suggests that our model provides a reasonable description of hours worked by married individuals, while it has difficulties in matching the level of hours worked by singles. In particular the increase in married women's labor supply is well captured. For husbands, the model replicates the downward trend in hours worked. According to our model, singles should work more hours than they actually do.

### 4.5 Accounting for the Changes in Hours

To shed some light on further issues regarding our model, we present an accounting exercise for the rise in aggregate market hours and for the changes in market hours by groups. In the literature, there is a number of explanations for these developments. Some studies emphasize demographic changes, such as the fertility and marriage decline, see Chiappori and Weiss (2006) and Albanesi and Olivetti (2007). The marriage decline potentially raises aggregate hours because female singles tend to work more hours than married women. Another explanation attributes the rise in hours to overall productivity growth (Mincer 1962; Smith and Ward 1985). Rising wages increase the opportunity costs of not working. Two further explanations arise from the study of household models
of labor supply: the closure of the gender productivity gap (Galor and Weil 1996; Jones, Manuelli, and McGrattan 2003; Knowles 2007; Attanasio, Low, and Sánchez-Marcos 2008) and technological improvements in home production (Greenwood, Seshadri, and Yorukoglu 2005). A closure of the productivity gap leads to less intra-household specialization and thus to more female market labor. Improvements in home technology, that is to say cheaper access to home capital, such as vacuum cleaners or microwaves, is freeing up time that married women can nowadays work on the market. ${ }^{13}$

If trading home services has become easier over time, this is an additional explanation for rising hours. As shown in Section 3.4, a declining "as-if" tax on the market for home services increases activity on this market. This leads to rising hours of married women as well as in the aggregate.

We now use our model and try to assess the relative importance of the aforementioned explanations for the rise in hours. In our model, the respective causes for the changes in hours can be represented by changes in exogenous parameters. Specifically, the marriage decline can be captured by an exogenous increase in $s$. An exogenous increase in the mean of the productivity distribution, $m$, represents overall productivity growth. The parameter $\alpha$ measures the gender productivity gap. ${ }^{14}$

[^11]| (1) | (2) | (3) | (4) | (5) | (6) |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | CPS <br> data | marriage decline | + productivity growth | + productivity gap closure | + estimated changes in $\tau$ |
|  |  | $s_{t}, \bar{m}, \bar{\sigma}, \bar{\alpha}, \bar{\tau}$ | $s_{t}, m_{t}, \sigma_{t}, \bar{\alpha}, \bar{\tau}$ | $s_{t}, m_{t}, \sigma_{t}, \alpha_{t}, \bar{\tau}$ | $s_{t}, m_{t}, \sigma_{t}, \alpha_{t}, \hat{\tau}_{t}$ |
| aggregate hours | 15.09\% | 3.17\% | 3.17\% | 6.38\% (8.31\%) | 17.36\% |
| married women | 113.98\% | -0.03\% | -0.03\% | 28.55\% (45.57\%) | 109.30\% |
| married men | -5.95\% | -0.01\% | -0.01\% | -0.19\% (-0.32\%) | -6.25\% |
| single women | 11.88\% | -0.01\% | -0.01\% | 0.45\% (0.69\%) | 7.90\% |
| single men | 2.98\% | -0.02\% | -0.02\% | -1.43\% (-1.83\%) | 7.50\% |

Table 3: Accounting Exercise for the Rise in Hours (Overall Increase between 1961 and 2006; Parameter Changes Incorporated in a Cumulative Way)

To isolate the effects, we insert the time series for $s_{t}, m_{t}$, and $\alpha_{t}$ discussed in Section 4.2 into our model one after another, holding the other parameters at their 1961 level. This way, we first shut down all but the marriage-decline channel and then include the other channels in a cumulative fashion. The aim of this accounting exercise is to highlight the role of the explanations for the rise in hours in a model where home labor is tradable. Specifically, we ask the question whether allowing for time-varying frictions on the market for home services, as measured by the parameter $\tau$, provides additional insight with respect to changes in subgroup-specific labor supply over time.

The results of the accounting exercise are summarized in Table 3. Over the period 1961 to 2006, aggregate hours worked per capita increased by $15 \%$. The disaggregation by gender and marital status displayed in column (2) illustrates that the substantial increase in married women's labor supply (increase by $114 \%$ ) is the driving force for the rise in aggregate hours.

Columns (3) to (6) display the results of our model when we incorporate information one by one. The third column refers to the case where we set $m, \alpha$, and $\tau$ to their 1961 levels, but use the full time series for the parameter $s$. Under this scenario, aggregate labor supply per capita in 2006 exceeds that of 1961 by $3.17 \%$ in our model. Average market hours did not increase within groups. This reveals that the increase in aggregate hours is a compositional effect in the first place. A similar pattern emerges if we additionally allow mean productivity to rise over time (at the same time allowing for changes in $\sigma$ ), see column (4). ${ }^{15}$

Column (5) displays the changes in hours when we allow $\alpha$ to change over time, in addition to $s, m$, and $\sigma$. The first entry refers to the case in which we used the time series for $\alpha$ which is consistent with labor market choices, the values in parentheses are obtained with the first-guess series for $\alpha$ (see Section 4.3). Qualitatively, conclusions do not depend on which series we use. In line with Jones, Manuelli, and McGrattan (2003) we find that the closure of the productivity gap plays an important role when accounting for the rise in hours. This factor explains an additional 3 to 5 percentage-point increase in aggregate hours, depending on the specification. Even more important so, if women are catching up in terms of productivity, this has substantial effects at the subgroup level. Women increase their labor supply due to higher opportunity costs and in the

[^12]

Figure 11: Hours of Married Women; Parameter Changes Incorporated in a Cumulative Way
case of married women also due to improved intra-household bargaining positions. This effect is more pronounced when productivity gaps are calculated from the entire sample. ${ }^{16}$ Concerning men, the increase in $\alpha$ leads to reduced labor supply. The negative sign for the change in single men's hours is counterfactual to what is observed in the data.

The numbers displayed in column (5) illustrate that a considerable portion of the changes in hours remains unexplained by the factors marriage decline, productivity growth, and productivity gap closure. If we want to use our model to generate changes in hours in the magnitude as those reported in column (2), we need to allow for changes in $\tau$. Column (6) shows that the calibrated sequence for $\tau$ brings the growth rates in hours closer to their real-world counterparts. ${ }^{17}$ It is noteworthy that accounting for changes in $\tau$ yields improvements in all five moments reported in Table 3 (the change in hours of single men is now positive, though too large).

While the numbers reported in Table 3 refer to a beginning to end-

[^13]of-period comparison, Figure 11 provides a year-by-year view on average hours worked by married women. The figure once more illustrates that exogenous changes in observable variables, such as $s, m$, and $\alpha$, are not sufficient to account for the entire rise in labor supply of married women. Our accounting exercise shows that the inclusion of an unobservable factor measured by parameter $\tau$ can fill this gap.

## 5 Discussion

In our model, the parameter $\tau$ measures a wedge between the cost of outsourcing one hour of labor in home production and the compensation for one hour of supplied home services. This wedge is a measure of how easy one can outsource home labor. It has been documented empirically that the possibility to outsource home labor has an influence on female labor supply. For instance, the survey article of Del Boca and Locatelli (2006) isolates three important determinants of cross-country differences in female labor supply: the availability of child care, the quality of such services, and the cultural perception of women. We understand $\tau$ as a proxy for these determinants and argue that several developments have made outsourcing home labor more attractive, corresponding to reductions in $\tau$.

The unobservable factor $\tau$ can be understood as a reduced-from representation of several structural frictions on the market for home services. We now consider examples for such frictions that lead to equivalent time allocations as if there was an "as-if" tax $\tau .{ }^{18}$ Chari, Kehoe, and McGrattan (2007) point out that models with short-cut frictions can lead to equivalent results as models with structural frictions. We pick up this idea and provide structural interpretations for $\tau$.

For instance, suppose that there is no "as-if" tax, i.e. $p=\omega$, but there are quality differences between hired and own home services. Specifically, consider a production function of the type

$$
\begin{equation*}
f\left(l_{F, i}^{2}, l_{M, i}^{2}, h_{i}^{2, D}\right)=A \cdot\left(k_{i}\right)^{\theta} \cdot\left(l_{F, i}^{2}+l_{M, i}^{2}+\frac{1}{1+\tilde{\tau}_{q}} h_{i}^{2, D}\right)^{1-\theta} \tag{55}
\end{equation*}
$$

which reflects that one unit of hired time does not replace one unit of own

[^14]time completely, given $\tilde{\tau}_{q}>0 .{ }^{19}$ An otherwise identical model with such production function yields the same time-use and time-demand decisions for any household and thus the same time allocations as the model with the tax $\tau$ presented in Section 3.

Technological progress which makes hired home services a better substitute to own labor corresponds to a decrease in $\tilde{\tau}_{q}$ in the production function above. Albanesi and Olivetti (2007) provide empirical evidence that such progress has indeed occurred over the last century. Furthermore, there have been governmental programs in order to increase the quality in the child care sector, see Blau and Mocan (2002). A decrease in $\tilde{\tau}_{q}$ is equivalent to reductions in $\tau$ in our benchmark model.

An alternative structural interpretation of the wedge between $p$ and $\omega$ are search costs. For instance, a lack of transparency on the market for home services may create search costs that demanders have to bear in addition to the actual wage costs. Consider a case where the time $\xi$ that a household $j$ searches for a home service provider is proportional to the fraction of income it wishes to spend on these services, i.e. ${ }^{20}$

$$
\begin{equation*}
\xi_{j}\left(h^{2, D}\right)=\tilde{\tau}_{s} \cdot \frac{\omega \cdot h_{j}^{2, D}}{W_{j}} . \tag{56}
\end{equation*}
$$

With this specification, time allocations are equivalent to the case in which there are true resource costs in the form of a distortionary $\operatorname{tax} \tau$.

Over the last decades, the trade of home services shifted from the shadow economy onto formalized markets. For instance, Attanasio, Low, and Sánchez-Marcos (2008) report an increase in "organized child care" by more than $100 \%$ from 1977 to 1994, while other, informal, forms of child care remained rather constant. This increases in market transparency may have reduced search frictions, as measured by the parameter $\tilde{\tau}_{s}$. A decline in $\tilde{\tau}_{s}$ affects time allocations in an equivalent way as a declining as-if tax $\tau$.

A third interpretation of our model is related to utility costs and social norms. Parents who outsource child care may suffer from being apart from their children. In the past, mothers who outsourced child care may have suffered from discrimination. Similar discrimination may affect the decision whether to outsource geriatric care. It seems plausible that these

[^15]issues have prevented in particular women from supplying market labor. This point is emphasized by Fernández (2007), who argues that female labor supply "may depend on how a woman conceives of her role in the household, $[\ldots]$ or how she is treated as a result of her choice" (p. 6).

One can formalize this idea in a model in which hired home services enter the utility function negatively. Specifically, consider the utility function

$$
\begin{equation*}
u_{G, i}=\mu \log c_{G, i}^{1}+\nu \log c_{G, i}^{2}+(1-\mu-\nu) \log \ell_{G, i}-\tilde{\tau}_{d} \cdot \frac{\omega \cdot h_{i}^{2, D}}{W_{i}} \tag{57}
\end{equation*}
$$

In this utility function, the pain an individual suffers from discrimination is proportional to the share of income he or she spends for hiring home services. ${ }^{21}$ Assuming no wedge between price and wage for hiring home services, $p=\omega$, but replacing the utility function (3) by (57) yields equivalent time allocations as if there was a $\operatorname{tax} \tau$ on the second market wage.

Over the last 50 years, the perception of working mothers in western societies has changed. From the World Values Survey there is evidence for significant cohort effects in the answers given to questions related to the acceptance of working women during their kids' childhood. Younger cohorts seem to be less reluctant to outsourcing home labor than older ones. ${ }^{22}$ These developments are potentially leading to less disutility when outsourcing home labor, i.e. they lead to a decrease in $\tilde{\tau}_{d}$. In consequence, non-wage costs of home services decrease, which is again equivalent to a declining "as-if" tax $\tau$ in our model.

## 6 Conclusion

This paper has provided a new explanation for the observed genderspecific patterns in labor supply in the US. Our key argument was that differences in labor supply by population groups can be understood as optimal reactions to the possibility to outsource household-related activities. A main aspect of our model therefore was that home services are tradable. The difficulties associated with trading home labor have an important influence on the allocation of time.

[^16]To investigate the role of tradable home labor, we have introduced a market for home labor into a household model of labor supply. Agents can thus decide whether to work in home production on their own or to hire someone for doing it. A prerequisite for such specialization is the presence of heterogeneity in productivities. On the basis of individual households' labor supply and specialization decisions, we solved for an aggregate equilibrium.

We argued that the market for home labor is subject to several frictions and modeled market frictions as a wedge between the price of buying one's way out of one hour of house work and the wage for one hour of house work. In our model, this wedge corresponds to a distortionary "as-if" tax on the market for home services. We have illustrated that a declining "as-if" tax is an explanation for changes in subgroup-specific labor supply over time. The model displays comparative-static properties that resemble the real-world patterns qualitatively. In particular, it generates an increase in married women's labor supply which is the largest one of all groups.

In a quantitative analysis, we have backed out a sequence of "as-if" tax rates which best enables the underlying structural model to match mean hours worked by gender and marital status on a year-by-year basis. The backed-out sequence indicates that outsourcing of household-related activities has become easier over time, in a sense that the wedge declines steadily.

We have also presented an accounting exercise in order to highlight the role of alternative explanations for the rise in hours in a model where home labor is tradable. We found that exogenous changes in observable factors, such as the marriage decline, productivity growth, and the productivity gap closure, are not sufficient to account for the entire rise in labor supply of married women. The accounting exercise has revealed that allowing for an unobservable factor capturing frictions on the market for home services can fill this gap.

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## A Appendix

## A. 1 CPS Data

Our data stems from the Current Population Survey (CPS) in the format arranged by Unicon Research. ${ }^{23}$ The CPS is a monthly household survey conducted by the Bureau of the Census. In the CPS, respondents are interviewed to obtain information about the employment status of each member of the household 16 years of age and older. Survey questions covering hours of work, earnings, gender, and marital status are covered in the Annual Social Economic Supplement, the so-called March Supplement Files.

The sample of the CPS is representative of the civilian noninstitutional population. Our selected sample comprises civilians aged 18 to 65 , which is a standard definition of working-age population. The time period spanned by our data is 1962-2007. Data on hours and earnings is retrospective and refers to the previous year. In all our calculations, we use weights.

Our quantitative analysis is based on cross-sectional first moments measuring average hours per capita by gender and marital status. Figure 1 in the paper displays time series for these variables. The weekly hours variable is "hours worked last week at all jobs", which is the only information on hours worked which is available for all years and comparable across years. Other studies have documented that this variable yields similar results as the variable "usual weekly hours by number of weeks worked", which is not available for all years, see e.g. Knowles (2007) or Heathcote, Storesletten, and Violante (2008).

[^17]We define a person as being married if the respondent answered the relevant question with "married, spouse present" (after Unicon recode). All other individuals are defined as being singles. The share of singles in our data is depicted in Figure 7.

We compute wages as annual earnings divided by annualized hours worked. In CPS, gross annual earnings are defined as income from wages and salaries including pay for overtime. Nominal earnings are deflated with the CPI and expressed in 1992 dollars. Annual hours worked are calculated as the product of weeks worked last year and hours worked last week. The sample for which wages are calculated is restricted to people who worked at least 10 hours a week.

## A. 2 Proofs of Static and Comparative-Static Results

## A.2.1 Average Hours Worked

Average market hours by gender and marital status are given by equations (26), (27), (35), and (36):

$$
\begin{aligned}
H_{M} & =\mu+\nu-(1-\theta) \nu \cdot \omega \\
H_{F} & =\mu+\nu-(1-\theta) \nu\left[\alpha^{-2}+\alpha^{-1}\right] p+(1-\theta) \nu\left[\frac{\alpha^{-2}}{2}+\frac{1}{2}\right] \omega \\
H_{f} & =\mu+\nu-(1-\theta) \nu \cdot \frac{p}{\alpha} \\
H_{m} & =\mu+\nu-(1-\theta) \nu \cdot p
\end{aligned}
$$

Since $p>\omega$, it holds that $H_{M}>H_{m}$. One can also state directly that $H_{m}>H_{f}$ because $\frac{p}{\alpha}>p$. It remains to show that $H_{f}>H_{F}$, which can be simplified to:

$$
\begin{aligned}
& 0>-(1-\theta) \nu \alpha^{-2} p+(1-\theta) \nu\left[\frac{\alpha^{-2}}{2}+\frac{1}{2}\right] \omega \\
\Longleftrightarrow & 0>-\frac{p}{\alpha^{2}}+\frac{\omega}{2 \alpha^{2}}+\frac{\omega}{2} \\
\Longleftrightarrow & 0>\frac{1+\tau}{\alpha^{2}}+\frac{1}{2 \alpha^{2}}+\frac{1}{2} \\
\Longleftrightarrow & 0>-\frac{1}{2}-\tau+\frac{\alpha^{2}}{2},
\end{aligned}
$$

which is true for any non-negative $\tau$ since $\alpha<1$. Therefore, the ordering of average market hours is $H_{M}>H_{m}>H_{f}>H_{F}$.

## A.2.2 Changes in Hours Worked

In Section 3 it has been shown that $\partial p / \partial \tau>0$ and $\partial \omega / \partial \tau<0$. Taking derivatives of average hours by groups with respect to $\tau$ yields:

$$
\begin{aligned}
\frac{\partial H_{M}}{\partial \tau} & =-(1-\theta) \nu \frac{\partial \omega}{\partial \tau}>0 \\
\frac{\partial H_{F}}{\partial \tau} & =(1-\theta) \nu \cdot \frac{\alpha^{-2}+1}{2} \cdot \frac{\partial \omega}{\partial \tau}-(1-\theta) \nu \cdot\left[\alpha^{-2}+\alpha^{-1}\right] \cdot \frac{\partial p}{\partial \tau}<0 \\
\frac{\partial H_{m}}{\partial \tau} & =-(1-\theta) \nu \frac{\partial p}{\partial \tau}<0 \\
\frac{\partial H_{f}}{\partial \tau} & =-(1-\theta) \nu \cdot \alpha^{-1} \cdot \frac{\partial p}{\partial \tau}<0
\end{aligned}
$$

Moreover, since $\alpha^{-1}>1$, we can state that $\left|\frac{\partial H_{f}}{\partial \tau}\right|>\left|\frac{\partial H_{m}}{\partial \tau}\right|$. Comparing the absolute value of the derivatives for husbands and wives reveals that $\left|\frac{\partial H_{F}}{\partial \tau}\right|>\left|\frac{\partial H_{M}}{\partial \tau}\right|$ because $\frac{\alpha^{-2}+1}{2}>1$. Finally, since $\alpha^{-2}+\alpha^{-1}>\alpha^{-1}$, it is also apparent that $\left|\frac{\partial H_{F}}{\partial \tau}\right|>\left|\frac{\partial H_{f}}{\partial \tau}\right|$. Thus the change in married women's market hours is the strongest one of all groups.

## A. 3 Values Survey

In this Appendix, we illustrate that the perception of family and the importance of social norms have changed over time. We do so by analyzing related questions from the World Values Survey. In the World Values Survey, there is a number of questions related to the acceptance of working women during their kids' childhood. ${ }^{24}$ A rising acceptance of working mothers may be an implication of declining discrimination of outsourcing household-related activities.

In the 1990 US survey, there is a statement "A pre-school child suffers if his or her mother works", on which subjects were asked whether they agree or not. We construct an average response by assigning the value of 2 to "strongly agree", 1 to "agree", 0 to "don't know" or "no answer", -1 to "disagree", and -2 to "strongly disagree". The upper left panel in

[^18]Figure 12 illustrates the average answer of different birth cohorts to this statement. ${ }^{25}$ The statement received an average reply of 0.40 from those born before 1940, those born between 1940 and 1960 answered with 0.18 , whereas younger respondents answered 0.10 on average.

Similarly, average agreement with the statement "A working mother can establish just as warm and secure a relationship with her children as a mother who does not work" rose from 0.64 among those born before 1940 to 0.90 in the young generation born after 1960 (see the upper right panel in Figure 12). Both findings indicate that working during childhood of one's kids has become more accepted.

The World Values Survey also suggests that a traditional perception of family with working husband and house wife seems to receive growing rejection. In the 1990 survey, subjects born after 1960 agreed to a measure of 0.83 with the statement "Both the husband and the wife should contribute to household income", but those born before 1940 only gave an average answer of 0.46 . These observations suggest that disapproval of working women has decreased, which is possibly an implication of increased acceptance of outsourcing home production, e.g. child care.

At the same time, people take it less and less serious what others think about themselves. In the 1995 survey, the measure of agreement with the statement "I make a lot of effort to live up to what my friends expect" is 0.34 for those born before 1945, whereas it is only 0.11 for people born after 1965. This allows the interpretation that a potential discrimination has less impact on decisions. Overall, we regard these results from the World Values Survey as supportive for our hypothesis that utility costs when hiring external home services have decreased over time.

[^19]

Figure 12: Average Agreement with Statements regarding Family and Child Care; Data Source: World Values Survey


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[^2]:    ${ }^{1}$ Collective decision making explicitly incorporates that a household consists of individuals with individual utility functions (Chiappori 1988; Chiappori 1992).
    ${ }^{2}$ Perfect assortative mating implies that spouses' productivities are perfectly correlated.

[^3]:    ${ }^{3}$ Details are provided in Appendix A.1.

[^4]:    ${ }^{4}$ The assumption of uniformly distributed productivities allows us to solve the

[^5]:    ${ }^{6}$ Other decision rules such as Nash Bargaining or Equal Surplus Splitting (Knowles 2007) lead to analytically non-tractable solutions of our model.
    ${ }^{7}$ We address this issue in more detail in Section 4.2.

[^6]:    ${ }^{8}$ In Section 5 we explicitly model several frictions on the market for home services which lead to equivalent time allocations as the model in which the wedge reflects true resource costs. The model can therefore be understood as the reduced form to several models with structural frictions, such as quality differences between hired and own home labor, search frictions on the market for home services, and utility costs of outsourcing household-related activities. In the quality example, one hour of hired home services does not replace one hour of own work completely. In the presence of search frictions, a demander of home services has to bear time costs in addition to the actual wage costs. If parents dislike outsourcing child care, this can be understood as utility costs. In all three examples, home-services demanders have to bear additional costs just as in the short-cut specification where the additional costs are true resource costs. See Section 5 for details.

[^7]:    ${ }^{9}$ For the following analysis, it is sufficient that households in this group split labor equally on average.

[^8]:    ${ }^{10} S_{1}$ is surely positive if $\theta>0.2$ or $\nu<4 \mu$ or any linear combination of these two restrictions holds, which we assume. If labor is exceedingly important in home production and home consumption provides much utility, the income effect of a rise in $\omega$ potentially dominates the substitution effect. This would lead to a supply curve for home services with a negative slope. We exclude this case.

[^9]:    ${ }^{11}$ A large time endowment and a high valuation of leisure can result in the same choice of non-leisure time as a small time endowment and a low utility weight on leisure. Thus, the question is how much of an individual's non-labor time one actually defines as leisure.

[^10]:    ${ }^{12}$ We stop the iteration either if the distance between $\psi_{t}$ in the current and the previous iteration does not exceed a pre-specified (small) value or if we observe an oscillating, non-converging behavior of $\psi_{t}$.

[^11]:    ${ }^{13}$ A number of recent studies has pointed to social norms as additional determinants of gender-specific labor supply. Fernández, Fogli, and Olivetti (2004) and Fernández (2007) provide cross-sectional evidence that culture and social norms have a substantial influence on working decisions. Another argument for the increase in married women's labor supply is that increasing returns to experience make maternal leaves more expensive, see Olivetti (2006). In the presence of uncertainty, labor supply of married women also has an insurance motive, see e.g. Attanasio, Low, and Sánchez-Marcos (2005) for the case of idiosyncratic earnings risk and Peters (1986) and Parkman (1992) for the case of divorce risk.
    ${ }^{14}$ Improvements in home technology can be expressed by increases in home total factor productivity $A$. Capital enhancing improvements are equivalent to reductions in the relative price for home capital, $q$. Both parameters, however, do not influence the allocation of time in our model.

[^12]:    ${ }^{15}$ These results suggest that general-equilibrium effects of changes in $s, m$, and $\sigma$ (via $\omega$ and $p$ ) on hours are negligible.

[^13]:    ${ }^{16}$ The relatively large difference between both specifications is due to the fact that in 1961 the difference between the final parameterization for $\alpha$ and the first guess is above average, see Figure 8.
    ${ }^{17}$ Here we used the series for $\alpha$ which is consistent with labor market choices.

[^14]:    ${ }^{18}$ Detailed proofs of the equivalences are presented in an appendix, which is available on request.

[^15]:    ${ }^{19}$ If home services are less productive in home production than own labor, this is equivalent to the case where home services are simply more expensive.
    ${ }^{20}$ These time costs have to be born by all household members.

[^16]:    ${ }^{21}$ This relation can be motivated by the idea that for very rich households, employing personnel has always been accepted.
    ${ }^{22}$ A detailed discussion of our results from the World Values Survey is presented in Appendix A.3.

[^17]:    ${ }^{23}$ See http://www.unicon.com/.

[^18]:    ${ }^{24}$ The World Values Survey is a worldwide investigation of sociocultural and political change. Up until today (2009), four waves of the survey have been conducted. The survey is based on interviews with nationally representative samples in 80 countries. Most questions ask for agreement to statements in a multiple-choice fashion. Available country-specific data on given answers is disaggregated by various characteristics of the respondents, including birth-cohort intervals.

[^19]:    ${ }^{25}$ Since the surveys are not consistent over time, we have to use answers disaggregated by birth cohorts to indicate changes in values over time. Doing so assumes that values are imparted during childhood.

