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Debt Maturity without Commitment

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Debt Maturity without Commitment

Abstract

I analyze how lack of commitment affects the maturity structure of sovereign debt. Governments balance benefits of default induced redistribution and costs due to income losses in the wake of a default. Their choice of short- versus long-term debt affects default and rollover decisions by subsequent policy makers. The equilibrium maturity structure is shaped by revenue losses on inframarginal units of debt that reflect the price impact of these decisions. The model predicts an interior maturity structure with positive gross positions and a shortening of the maturity structure when debt issuance is high, output low, or a cross default more likely. These predictions are consistent with empirical evidence.

JEL Code: E62, F34, H63.

Keywords: debt, maturity structure, no commitment, default.

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1 Introduction

Sovereign borrowers exert considerable effort to structure their debt maturities optimally. This is difficult to reconcile with predictions of a frictionless benchmark model in which gross financial positions and thus, the maturity structure are indeterminate and only net positions affect agents' wealth and incentives (as in Modigliani and Miller (1958) and Barro (1974)).

In this paper, I offer an explanation for borrowers' scrupulous choice of maturity, arguing that lack of commitment paired with social losses in the wake of a default undermines the neutrality of the maturity structure. Focusing on these two factors appears natural. After all, a large literature concerned with sovereign borrowing emphasizes the pervasiveness of limited contract enforceability while providing strong evidence for the presence of social losses in the aftermath of defaults.[1]

I consider a government issuing real non-contingent debt of various maturities. Successive governments (or selves of the government) decide whether, and to what extent, to honor maturing debt. They also choose the level of taxation and new debt issuance to finance contemporaneous debt repayment as well as exogenous government purchases. The government's desire to redistribute from foreign bondholders to domestic taxpayers creates an incentive to default on the maturing debt which interacts with an opposing incentive to avoid the cost of defaulting.[2] I model this cost as temporary income losses for taxpayers, serving as stand-in for various types of social losses discussed in the literature. Both bondholders and the government form rational expectations. The price of debt maturities therefore reflects their expected repayment rate, and government policy is subgame perfect.

In equilibrium, the risk-adjusted returns on short- and long-term funding are identical and the optimal maturity structure is determined on the demand side. In particular, it is critically shaped by revenue losses on inframarginal units of debt, reflecting the price impact of default and rollover choices by subsequent policy makers. Under plausible conditions, such inframarginal revenue losses normalized by the price of debt are a convex function of the quantity of a maturity issued. Optimal policy therefore amounts to "smoothing" the revenue losses on inframarginal units of maturities, in parallel with the familiar tax (distortion) smoothing prescription. As default renders debt state contingent, this smoothing prescription interacts with standard portfolio choice considerations.

In a benchmark case with exponentially distributed costs in the wake of debt repudiation, default and rollover decisions are independent of the stock of outstanding debt. As a consequence, spillover effects between different maturities are minimal and the smoothing

[1] See Eaton and Fernandez (1995) for an overview over the literature. Reinhart and Rogoff (2004) and Sturzenegger and Zettelmeyer (2006, pp. 49–52), among many other authors, provide strong evidence that sovereign defaults are costly.

[2] The incentive to default might alternatively derive from the government's desire to transfer funds from the private to the public sector, in order to avoid tax distortions. Focusing on the redistributive motive is attractive for two reasons. On the one hand, conflict between interest groups indeed appears to affect governments' default decisions, see the discussion later in the text. On the other hand, abstracting from tax distortions allows to disregard a second source of time inconsistency, related to the optimal timing of taxes (Lucas and Stokey, 1983).

prescription implies a fully balanced maturity structure. If default costs are distributed according to any other distribution with increasing hazard (guaranteeing uniqueness of the equilibrium maturity structure), then outstanding debt depresses short-term debt issuance. Long-term debt issuance therefore increases the debt maturing in the long term by less than one-to-one, reducing the associated inframarginal losses and giving rise to an optimal maturity structure that is tilted towards the long end.

Higher quantities of debt reduce this cost advantage of long-term debt, inducing a more balanced maturity structure. This has direct implications for the debt portfolio over the cycle: In periods with high marginal utility where total debt issuance increases, the maturity structure shortens. The same result holds even if default costs are distributed exponentially, provided that these costs tend to be lower in times of scarce resources.

In a second benchmark case analyzed, default costs are dependent over time such that a default on maturing debt triggers a (subsequent) default on currently outstanding debt as well. Under this assumption, default probabilities depend on the stock of maturing and outstanding debt, rendering the default decision dynamic. Long-term debt issuance is always dominated by short-term issuance in this case and the maturity structure is concentrated on the short end.

The broad picture that emerges from the model is one of an interior maturity structure with positive gross positions, in line with the empirical evidence, but in contrast with predictions from models that stress the role of the maturity structure in completing markets or avoiding rollover crises (see below). The model predicts a shortening of the maturity structure when debt issuance is high, in line with evidence summarized by Rodrik and Velasco (1999); around times of low output ("crises"), consistent with the evidence reported by Broner, Lorenzoni and Schmukler (2007); and in periods with increased risk of cross default and acceleration.[3] Since the model predictions relate to the quantities of different maturities (rather than just their ratios, say), the model offers a theory of the equilibrium *level* of debt in addition to a theory of its structure.

As mentioned before, revenue losses on inframarginal units of debt play a central role in the model. Closely related to these revenue losses, previous literature has emphasized debt dilution as a consequence of lack of commitment. In particular, it has been pointed out that debt issuance reduces the value of *outstanding* debt and that this effect may increase governments' incentives to issue new debt ex post.[4] In contrast, the revenue losses of interest in the present paper arise with respect to *contemporaneously* issued debt and are fully internalized by the government seeking funding. Ex-post benefits from diluting outstanding debt therefore contrast with ex-ante costs of issuing new debt maturities, due to the social losses associated with a default.

[3]According to Rodrik and Velasco (1999), "the overall debt burden (debt/GDP ratio) is positively correlated with short-term borrowing in the time-series (but not in the cross-section). One interpretation is that countries that go on a borrowing binge are forced to shorten the maturity of their external liabilities in the short run" (p. 21). According to Broner et al. (2007) "emerging economies issue relatively more short-term debt during periods of financial turmoil, and wait for tranquil times to issue longterm debt" (p. 3).

[4]Dilution may be present even if outstanding debt is prioritized, see Bizer and DeMarzo (1992) who analyze the case where increased borrowing leads a borrower to take actions that lower the probability of repayment.

Importantly, these findings derive from entirely standard premises. For example, the assumption that debt contracts stipulate non-contingent payments and social losses are triggered in the absence of contractually specified gross payments is standard, presumably reflecting the notion that informational constraints prevent sovereign borrowers from entering into more sophisticated financial arrangements. The present paper does not address the question of why such constraints arise, nor does it rationalize other central tenets in the sovereign debt literature, in particular lack of commitment. Instead, the paper maintains the standard set of assumptions and analyzes the determinants of sovereign debt maturity within their context.

Related Literature Lack of commitment and the associated difficulty to sustain borrowing take center stage in the sovereign debt literature. Kydland and Prescott (1977) and Fischer (1980) discuss the government's ex-post incentive to default when taxes are distorting. In Tabellini (1991), Dixit and Londregan (2000), Kremer and Mehta (2000) or Niepelt (2004), distributive motives counteract these incentives.[5]

Eaton and Gersovitz (1981) suggest that the threat of financial autarky discourages strategic default. For discussions and applications of this hypothesis as well as analyses of the role played by the available financial instruments, see Bulow and Rogoff (1989*b*), Grossman and Han (1999), Kletzer and Wright (2000), Alvarez and Jermann (2000), Kehoe and Perri (2002) and Ljungqvist and Sargent (2004, ch. 19), among many others. Cole and Kehoe (1998) and Sandleris (2006) argue that a sovereign default serves as a negative signal, inducing parties outside of the credit relationship to initiate actions that are costly for the government. More direct default costs of the type considered here are present, for example, in the models of Bulow and Rogoff (1989*a*), Bulow and Rogoff (1989*b*), Cole and Kehoe (2000), Aguiar and Gopinath (2006) and Arellano (2008).[6]

To motivate an optimal maturity structure, some authors suggest that short-term debt renders a country vulnerable to rollover crises, and that long-term debt reduces such vulnerability (Calvo, 1988; Alesina, Prati and Tabellini, 1990; Giavazzi and Pagano, 1990; Rodrik and Velasco, 1999; Cole and Kehoe, 2000). Chamon (2007) proposes a mechanism to eliminate the coordination failure associated with rollover crises. Phelan (2004) draws a distinction between the maturity of debt and the sequencing of debt rollovers which matters for crises. Broner et al. (2007) argue that supply side features induce emerging markets to borrow short-term in spite of the increased risk of a rollover crisis. In their model, lenders are risk averse and heavily exposed to the price risk of long-term emerging-markets debt. Higher quantities of long-term debt therefore drive up term premia and thus, the cost of long-term funding.

Angeletos (2002) argues that a sufficiently rich maturity structure of non-contingent bonds may serve as a substitute for state-contingent debt by completing markets for the

[5]The model of this paper is silent about the choice of maturity structure in countries whose debt is perceived to be default-risk free. Choices of debt structure in those countries appear to be affected by liquidity concerns. In the UK, for example, the Debt Management Office "argues that cost is not the only factor. There is a virtue in being predictable, and in keeping all sections of the bond market supplied with debt to trade" (*The Economist*, "Losing interest," June 14th 2008).

[6]See also Tirole (2006, p. 180) where a default might trigger a costly loss of social capital.

government (see also Gale, 1990). Faraglia, Marcet and Scott (2008) document that the quantitative implications of this "complete market approach" are at odds with the data.

Closer in spirit to the present paper, Calvo and Guidotti (1990) and Missale and Blanchard (1994) discuss the role of the maturity structure of nominal debt for the government's incentive to engineer surprise inflation. Hatchondo and Martinez (2008) analyze numerically how the duration of government debt affects debt issuance, default choices and risk premia (see also Arellano and Ramanarayanan, 2008). Finally, a large literature in corporate finance analyzes the role of asymmetric information and control rights for the financial structure of firms (for an overview, see Hart, 1995; Tirole, 2006); see Jeanne (2004) for an application in the sovereign debt context.

The remainder of the paper is structured as follows. Section 2 presents the model. The main analysis is contained in Section 3. Section 4 characterizes the maturity structure in several special cases of the model. Section 5 analyzes an extension of the basic model with cross default, and Section 6 concludes.

2 Model

Time is discrete and indexed by $n = 0, 1, 2, \ldots$. There is a government that levies taxes, t_n, and issues debt of various maturities, $\{b_{nm}\}$, at prices $\{q_{nm}\}$. Here, the first and second index of a debt maturity or its price denote the issuance and maturity dates, respectively. The debt maturity b_{nm} promises a return in period m that is independent of the state of nature in that period. Without loss of generality, exogenous government spending is normalized to zero.

2.1 Private Sector

All government debt is held by foreign investors while all taxes are paid by domestic agents. Taxpayers do not save nor borrow.[7] The assumption that the two groups of taxpayers and investors do not "overlap" is unimportant for the central results, but simplifies the analysis. Modeling a mixed rather than concentrated ownership structure of debt would require a theory of how the ownership structure is determined in equilibrium.[8] No such theory is available that would appear plausible in the current context.[9] Assuming that taxpayers do not save also rationalizes why the government issues debt in the first place, in spite of taxes being non-distorting.

[7] Mankiw (2000) or Matsen, Sveen and Torvik (2005) analyze fiscal policy in economies with "savers" and "spenders."

[8] In equilibrium, the government's default decision depends on the ownership structure of debt relative to the distribution of tax burdens across the population, see below. Changes in the ownership structure therefore affect the default decision ex post and thus, investment decisions ex ante.

[9] Tabellini (1991) and Dixit and Londregan (2000) provide theories of the ownership structure of debt. They assume that households can only save in government debt (Tabellini, 1991), or that the return on the only alternative asset is household specific (Dixit and Londregan, 2000). Both assumptions are not applicable in the current context. See also Niepelt (2004).

Taxpayers have time- and state-additive preferences over consumption and discount the future according to the discount factor $\delta \in (0,1)$. Conditional on current and anticipated state-contingent incomes y_i^u as well as taxes t_i, the objective function of taxpayers is given by

$$U_n \equiv \mathrm{E}\left[\sum_{i \geq n} \delta^{i-n} u(y_i^u - t_i) | s_n\right].$$

Here, s_n denotes the state at time n, to be specified in more detail later. I will focus throughout on Markov equilibria, excluding artificial state variables of the type sustaining trigger strategies. The utility function $u(\cdot)$ is strictly increasing and concave.

Investors are risk neutral and have access to a large international capital market with a riskfree interest rate equal to $\beta^{-1}, \beta \in (0,1)$. In equilibrium, newly issued government debt therefore pays an expected return of β^{-1} per period.

2.2 Government

The government maximizes the welfare of taxpayers.[10] Policies are chosen sequentially and political decision makers cannot commit their successors (or future selves). In each period n, the government in power therefore chooses taxes as well as the repayment rate, $r_n \in [0,1]$, on the debt maturing in the period, $b_{xn} \equiv \sum_{l=0}^{n-1} b_{ln}$. While all maturing debt is treated equally, independently of its issuance date (pari passu), a government's default decision only applies with respect to the contemporaneously maturing debt. This feature is a direct consequence of the government's lack of commitment: Being unable to tie the hands of its successors, a government cannot force its successors to pay a certain rate of return, for example zero. Nevertheless, cross default on outstanding debt may of course occur as an equilibrium outcome; see the discussion in Section 5.

A large literature on sovereign debt discusses the restrictions that lack of commitment imposes on a government's ability to issue debt. As mentioned in the introduction, this literature suggests that various default costs are responsible for a government's decision to honor its obligations ex post rather than renege on them. Following that literature, I assume that a government default—defined as a situation where the repayment rate falls short of unity—triggers income losses for taxpayers (cf. Eaton and Gersovitz, 1981; Cole and Kehoe, 2000; Aguiar and Gopinath, 2006; Arellano, 2008). Ex post, the government therefore balances the benefit to taxpayers of a default induced reduction of government spending and the cost to taxpayers of default induced income losses.

I restrict attention to temporary income losses in the wake of a default. In particular, I assume that a default in period n on debt maturing in period n triggers an income loss $L_n \geq 0$; L_n is the realization of an i.i.d. random variable with cumulative distribution function $F(\cdot)$ and associated probability density function $f(\cdot)$. (The i.i.d. assumption

[10] If the government maximized a weighted average of taxpayers' and investors' welfare and attached a sufficiently large weight to the welfare of investors, interior repayment rates might result, in contrast to what follows. If the government attached a strictly positive weight to the welfare of investors and if investors were risk averse, the wealth of investors would enter the government's program, in contrast to what follows.

simplifies notation; it is relaxed at a later stage.) I assume that $f(L) > 0$ for all $L > 0$. The government learns about the realization of L_n at the beginning of the period, before choosing its policy instruments.

The assumption of temporary income losses is motivated by two considerations. First, temporary costs constitute a natural benchmark. Second, and more importantly, they appear plausible. While permanent exclusion from trade or credit markets and other forms of long-term punishment may serve as threat points, such permanent costs are unlikely to be realized in equilibrium if a defaulting sovereign and its lenders can renegotiate.[11] Suppose, for example, that the sovereign chooses between either repaying, or not repaying and entering into a bargaining process with lenders. This process takes one period, generating income losses L_n, and results in a settlement where lenders secure a strictly positive repayment rate, $\bar{r}_n > 0$. The analysis in this paper is consistent with this interpretation although it abstracts from any safe return component on sovereign debt (as implied by $\bar{r}_n > 0$).

As a consequence of the default induced income losses, the income of taxpayers in period n is given by

$$y_n^u = y_n - \mathbf{1}_{[r_n<1]} L_n,$$

where y_n denotes the exogenous income component and $\mathbf{1}_{[x]}$ denotes the indicator function for event x.

Section 5 analyzes the implications of an alternative assumption about default costs according to which a default in period n on debt maturing in period n renders subsequent defaults on debt outstanding in period n costless.

2.3 Equilibrium

Conditional on an inherited maturity structure and the exogenous income process, an *equilibrium* as of period n consists of a state-contingent sequence of policies (tax rates, repayment rates, issuance of debt maturities) and debt prices such that

i. the policy sequence maximizes U_n;

ii. the dynamic government budget constraint is satisfied in all periods,

$$t_i + \sum_{j>i} q_{ij} b_{ij} = b_{xi} r_i \text{ for all } i \geq n;$$

iii. the intertemporal budget constraint of the government is satisfied; and

iv. investors are willing to buy newly issued debt,

$$q_{ij} = \beta^{j-i} \mathrm{E}[r_j | s_i] \text{ for all } i, j \geq n.$$

Lack of commitment imposes additional equilibrium conditions. A *time-consistent equilibrium* as of period n consists of a sequence of policies and debt prices such that

[11] Empirically, defaulting countries are not excluded from credit markets indefinitely.

i. policies and prices constitute an equilibrium; and

ii. anticipated policies coincide with the actual policies that successive governments optimally choose to implement.

The objective of the government in period n to select the "best" time-consistent equilibrium can now be stated as follows:[12]

$$\begin{aligned} \max_{t_n, r_n \in [0,1], \{b_{ni}\}_{i>n}} \quad & \mathrm{E}\left[\sum_{i \geq n} \delta^{i-n} u(y_i - \mathbf{1}_{[r_i<1]} L_i - t_i) | s_n\right] \\ \text{s.t.} \quad & (t_i, r_i \in [0,1], \{b_{ij}\}_{j>i}) \text{ optimal, conditional on } (y_i, L_i, \{b_{lk}\}), \text{ for all } i > n, \\ & t_i + \sum_{j>i} \beta^{j-i} \mathrm{E}[r_j | s_i] b_{ij} = b_{xi} r_i \text{ for all } i \geq n, \text{ NPG condition.} \end{aligned}$$

Outstanding debt as of period n that was issued before period n and will mature in period i is denoted by b_{xni}: $b_{xni} \equiv \sum_{l=0}^{n-1} b_{li},\ i \geq n$. Accordingly, $b_{xnn} = b_{xn}$.

Since no debt is outstanding in the initial period, $b_{x0i} = 0$ for all $i \geq 0$, investors are insulated against the effects of policy ex ante. Due to rational expectations, all debt is priced at its fundamental value and no default-induced redistribution occurs in equilibrium.[13] In contrast, the ex-ante welfare of taxpayers does depend on policy. First, because the timing of tax collections determines the smoothness of taxpayers' consumption and thus, utility. Second, because default reduces taxpayers' income.

3 Analysis

I focus on the case with two maturities, short- and long-term debt. The former matures after one period, the latter after two. Accordingly, the state of the economy in period n is given by the tuple $s_n = (y_n, L_n, b_{x,n}, b_{x,n,n+1})$. (If the horizon is finite, time constitutes an additional state variable. The notation adopted in the following reflects this case.) The government's budget deficit in period n is given by

$$d_n \equiv b_{n,n+1} \beta \mathrm{E}[r_{n+1} | s_n] + b_{n,n+2} \beta^2 \mathrm{E}[r_{n+2} | s_n].$$

3.1 Optimal Debt Repayment

Consider first the government's choice of repayment rate, r_n. Due to the temporary nature of the default costs, this choice is static in nature. Substituting the government's budget

[12] I abstract from assets other than government debt. This assumption is not restrictive if the discount factor is sufficiently low. Alfaro and Kanczuk (2007) simulate a calibrated model with short term debt and a riskfree asset for savings. They find that an optimizing sovereign does not accumulate assets since doing so (rather than reducing the stock of debt) would unnecessarily undermine credibility.

[13] Ex post, investors are of course "vulnerable" because they are directly affected by the government's choice of repayment rate.

constraint in the expression for taxpayers' disposable income in period n, the choice of r_n maximizes

$$y_n - \mathbf{1}_{[r_n<1]}L_n - b_{x,n}r_n + d_n.$$

Since the marginal cost of reducing r_n equals zero for $r_n < 1$, the optimal repayment rate equals either zero or unity, depending on the realization of L_n. In particular,

$$r_n^\star(s_n) = \begin{cases} 1 & \text{if } L_n \geq b_{x,n} \\ 0 & \text{if } L_n < b_{x,n} \end{cases} \tag{1}$$

and the government defaults whenever maturing debt exceeds the income losses in the wake of a default. This implication of the model is consistent with the notion that governments tend to default when the associated political cost—i.e., income losses of pivotal pressure groups—is low.[14] Governments also tend to default when economic activity is depressed (Borensztein, Levy Yegati and Panizza, 2006; Tomz and Wright, 2007). The model is consistent with this fact as well if it is slightly extended to include a direct default cost for the government in addition to the income losses for taxpayers.[15]

Corner solutions for the optimal repayment rate follow under more general assumptions about default costs than those invoked here, see the discussion in Appendix A. Interior repayment rates would only arise if income losses in the wake of a default were a convex function of the default rate (implausible, as argued in Appendix A) or the government attached sufficiently strong weight to the welfare of foreign investors (implausible as well).

Equation (1) pins down debt prices: The price of short-term debt, issued in period n, is given by

$$q_{n,n+1} = \beta \mathrm{E}[r_{n+1}|s_n] = \beta(1 - F(b_{x,n,n+1} + b_{n,n+1})), \tag{2}$$

while the price of long-term debt satisfies

$$q_{n,n+2} = \beta^2 \mathrm{E}[r_{n+2}|s_n] = \beta^2 \mathrm{E}[1 - F(b_{n,n+2} + b_{n+1,n+2})|s_n]. \tag{3}$$

The price of each maturity is decreasing in its quantity. This negative dependence arises because higher debt issuance reduces the probability of repayment. For the same reason, higher inherited, outstanding debt reduces the price of short-term debt while higher expected short-term debt issuance by the subsequent government ($b_{n+1,n+2}$) reduces the price of long-term debt.

[14]Tomz (2002) documents that domestic audiences opposed Argentina to suspend debt payments in 1999 but supported such action two years later. Kohlscheen (2004) documents that parliamentary democracies rarely resort to rescheduling (despite shorter office terms of their executives), presumably because domestic constituencies opposed to default are more likely to be politically influential in representative democracies. MacDonald (2003) suggests that it is precisely in countries where a default does not generate clearly identifiable winners and losers among politically influential groups where sovereign defaults have been avoided.

[15]If default triggers a cost K to the government in addition to the income losses for taxpayers, the default decision reduces to $r_n = 1$ iff $u(y_n - b_{x,n} + d_n) \geq u(y_n - L_n + \bar{d}_n) - K$ where the deficit following repayment may differ from the deficit following a default. Concavity of $u(\cdot)$ implies that low income levels render a default more likely.

3.2 Optimal Debt Issuance

From (2) and (3), the deficit in period n can be expressed as

$$d_n = b_{n,n+1}\beta(1 - F(b_{x,n,n+1} + b_{n,n+1})) + b_{n,n+2}\beta^2 \mathrm{E}[1 - F(b_{n,n+2} + b_{n+1,n+2})|s_n].$$

Let $b^\star_{n,n+1}(s_n)$ and $b^\star_{n,n+2}(s_n)$ denote the optimal short- and long-term debt issuance of the government in period n, respectively, and let $d^\star_n$ denote the deficit along the equilibrium path, conditional on s_n:

$$\begin{aligned} d^\star_n \equiv\ & b^\star_{n,n+1}(s_n)\beta(1 - F(b_{x,n,n+1} + b^\star_{n,n+1}(s_n))) + \\ & b^\star_{n,n+2}(s_n)\beta^2 \mathrm{E}[1 - F(b^\star_{n,n+2}(s_n) + b^\star_{n+1,n+2}(s_{n+1}))|s_n]. \end{aligned}$$

From the perspective of the government in period n that chooses $b_{n,n+1}$ and $b_{n,n+2}$, issuing a particular maturity has two types of effects. On the one hand, it raises revenue, in proportion to the price of the maturity. On the other hand, it affects the revenue raised from inframarginal units of debt, by changing the repayment probability and thus, price of these units. This second effect is a direct consequence of the government's lack of commitment.

Formally, taking the rollover policy functions of the subsequent government as given, the effect of a marginal increase in $b_{n,n+1}$ and $b_{n,n+2}$ on the deficit is

$$\begin{aligned} \frac{\mathrm{d}\, d_n}{\mathrm{d}\, b_{n,n+1}} &= \beta \left(1 - F(b_{x,n+1}) \underbrace{-b_{n,n+1} f(b_{x,n+1})}_{\mathcal{I}_{n,ss}} \underbrace{-b_{n,n+2}\beta \mathrm{E}\left[f(b_{x,n+2}) \frac{\partial b^\star_{n+1,n+2}(s_{n+1})}{\partial b_{x,n+1}} |s_n \right]}_{\mathcal{I}_{n,sl}} \right), \\ \frac{\mathrm{d}\, d_n}{\mathrm{d}\, b_{n,n+2}} &= \beta^2 \left(\mathrm{E}[1 - F(b_{x,n+2})|s_n] \underbrace{-b_{n,n+2}\mathrm{E}\left[f(b_{x,n+2}) \left(1 + \frac{\partial b^\star_{n+1,n+2}(s_{n+1})}{\partial b_{x,n+1,n+2}} \right) |s_n \right]}_{\mathcal{I}_{n,ll}} \right), \end{aligned}$$

respectively. According to the first equation, short-term debt increases the deficit in proportion to the expected repayment rate, net of the above mentioned revenue effect on inframarginal units. This revenue effect is composed of two parts since short-term debt issuance does not only (directly) depress the price of short-term debt, but also (indirectly) the price of long-term debt if the increase of $b_{n,n+1}$ triggers responses by the subsequent government; I refer to these two parts as $\mathcal{I}_{n,ss}$ and $\mathcal{I}_{n,sl}$, respectively. According to the second equation, long-term debt issuance has direct and indirect revenue effects on inframarginal units as well, but only on newly-issued long-term debt; I refer to these as $\mathcal{I}_{n,ll}$. Revenue effects on inframarginal units along the equilibrium path are denoted by a star, $\mathcal{I}^\star_{n,ss}$ etc. Note that, conditional on the rollover policy $b^\star_{n+1,n+2}(s_{n+1})$ and the level of outstanding debt, $b_{x,n,n+1}$, the above marginal effects define the levels of short- and long-term debt issuance that attain the maximum of the "debt-Laffer surface."

Using (1) and substituting the government's budget constraint in the expression for taxpayers' disposable income, the value function of the government in period n, $G_n(s_n)$,

satisfies

$$\begin{aligned} G_n(s_n) &= u(y_n - \min[b_{x,n}, L_n] + d_n^\star) + \delta \mathrm{E}[G_{n+1}(s_{n+1})|s_n] \\ \text{s.t.} \quad & b_{x,n+1} = b_{x,n,n+1} + b_{n,n+1}^\star(s_n), \\ & b_{x,n+1,n+2} = b_{n,n+2}^\star(s_n). \end{aligned} \tag{4}$$

The value function defines the maximal payoff to the government in period n conditional on the state variables s_n. This maximal payoff is attained if the government defaults optimally (as reflected by the $\min[\cdot]$ operator) and issues the optimal amount of short- and long-term debt (as reflected by the functions $b_{n,n+1}^\star(s_n)$ and $b_{n,n+2}^\star(s_n)$ implicit in $d_n^\star$). Debt issuance in turn determines the stock of the two maturities in the subsequent period, as reflected in the two constraints.

I assume in the following that conditions are satisfied that render the government's program well behaved, implying that the policy functions are smooth. Later, when considering special cases of the model, I verify that this is indeed the case.[16]

Consider the effect of a marginal increase in the stock of maturing debt, given by

$$\frac{\partial G_n(s_n)}{\partial b_{x,n}} = \begin{cases} -u'(y_n - b_{x,n} + d_n^\star) & \text{if } L_n \geq b_{x,n} \\ 0 & \text{if } L_n < b_{x,n} \end{cases} . \tag{5}$$

According to (5), changes in the amount of maturing debt do not have an effect on $G_n(s_n)$ if the debt is defaulted upon anyway. In those states where the government does repay, in contrast, higher maturing debt reduces the government's value in proportion to taxpayers' marginal utility of consumption. This negative effect arises because debt repayment translates into higher taxes, notwithstanding the fact that a change of $b_{x,n}$ may also lead to adjustments of short- and long-term debt issuance. Such adjustments do not have a first-order effect on $G_n(s_n)$ and thus, are not reflected in the above condition since debt issuance is chosen optimally from the perspective of period n.

The derivative of the *expected* continuation value function is given by

$$\begin{aligned} &\frac{\partial \mathrm{E}[G_{n+1}(s_{n+1})|s_n]}{\partial b_{x,n+1}} = \\ &= \frac{\partial}{\partial b_{x,n+1}} \Bigg\{ \mathrm{E}_y[\int_0^{b_{x,n+1}} u(y_{n+1} - L_{n+1} + d_{n+1}^\star) \mathrm{d}\, F(L_{n+1})|s_n] + \\ &\qquad\qquad \mathrm{E}_y[\int_{b_{x,n+1}}^{\infty} u(y_{n+1} - b_{x,n+1} + d_{n+1}^\star) \mathrm{d}\, F(L_{n+1})|s_n] \Bigg\} \\ &= -(1 - F(b_{x,n+1}))\mathrm{E}[u'(y_{n+1} - b_{x,n+1} + d_{n+1}^\star)|s_n], \end{aligned}$$

where the last equality uses the independence of y_{n+1} and L_{n+1}. (The deficit $d_{n+1}^\star$ in the last line is a function of $b_{x,n+1}, b_{x,n+1,n+2}$ and the realization of y_{n+1}, given that $L_{n+1} \geq b_{x,n+1}$.)

[16] In general, the objective function is not concave in the amounts of debt issued, due to the option to default. In particular, two features might undermine concavity. First, the fact that higher debt issuance reduces the probability of repayment in the future. Second, if the price function is convex, the fact that higher debt issuance implies increasingly smaller revenue losses on inframarginal units of debt.

Consider next the effect of a marginal increase in the stock of outstanding debt. Differentiating (4) with respect to $b_{x,n,n+1}$ yields

$$\begin{aligned}\frac{\partial G_n(s_n)}{\partial b_{x,n,n+1}} &= u'(y_n - \min[b_{x,n}, L_n] + d_n^\star)\beta\left(\mathcal{I}_{n,ss}^\star + \mathcal{I}_{n,sl}^\star\right) + \delta\frac{\partial \mathrm{E}[G_{n+1}(s_{n+1})|s_n]}{\partial b_{x,n+1}} \\ &= u'(y_n - \min[b_{x,n}, L_n] + d_n^\star)\beta\left(\mathcal{I}_{n,ss}^\star + \mathcal{I}_{n,sl}^\star\right) \\ &\qquad - \delta(1 - F(b_{x,n+1}))\mathrm{E}[u'(y_{n+1} - b_{x,n+1} + d_{n+1}^\star)|s_n], \qquad (6)\end{aligned}$$

where it is understood that the debt maturing in periods $n+1$ and $n+2$ depends on $b_{n,n+1}^\star(s_n)$ and $b_{n,n+2}^\star(s_n)$, respectively. Again, indirect effects on $G_n(s_n)$ due to induced adjustments of short- and long-term debt issuance in period n are not of first order.

According to (6), higher outstanding debt has two effects on $G_n(s_n)$. On the one hand, it affects contemporaneous felicity by changing the prices of short- and long-term debt issued in period n and thus, taxes. This is reflected in the $\mathcal{I}_n^\star$ terms in (6). On the other hand, higher outstanding debt reduces taxpayers' felicity in those states in the subsequent period where the outstanding debt is repaid. This effect is reflected in the second term on the right-hand side of (6).

With these results at hand, I now turn to a characterization of the debt issuance choice. Consider first the choice of short-term debt. From (4), the effect of a marginal increase in $b_{n,n+1}$ in equilibrium is given by

$$u'(y_n - \min[b_{x,n}, L_n] + d_n^\star)\beta\left(1 - F(b_{x,n+1}) + \mathcal{I}_{n,ss}^\star + \mathcal{I}_{n,sl}^\star\right) + \delta\frac{\partial \mathrm{E}[G_{n+1}(s_{n+1})|s_n]}{\partial b_{x,n+1}},$$

which can be expressed as

$$\begin{aligned}&u'(y_n - \min[b_{x,n}, L_n] + d_n^\star)\beta\left(\mathcal{I}_{n,ss}^\star + \mathcal{I}_{n,sl}^\star\right) + \qquad (7) \\ &\quad (1 - F(b_{x,n+1}))\left(\beta u'(y_n - \min[b_{x,n}, L_n] + d_n^\star) - \delta\mathrm{E}[u'(y_{n+1} - b_{x,n+1} + d_{n+1}^\star)|s_n]\right).\end{aligned}$$

Condition (7) identifies two distinct effects of short-term debt issuance on the government's objective. On the one hand, a consumption smoothing effect, represented by the second line of (7) and reflecting the fact that debt issuance allows to shift consumption to periods when taxpayers' marginal utility of consumption is high. By issuing one unit of short-term debt at price $\beta(1 - F(b_{x,n+1}))$, taxpayers increase the deficit by the corresponding amount and gain marginal utility. At the same time, however, taxpayers face lower future consumption in those states where the debt is repaid. This negative effect is discounted at the discount factor δ.

On the other hand, the expression in the first line of condition (7) represents the revenue effect on inframarginal units of debt, reflecting lack of commitment and the fact that the choice of $b_{n,n+1}$ triggers responses by the subsequent government. These responses partly run counter to the interests of the government issuing the debt. To see this, consider the term $\mathcal{I}_{n,ss}^\star$ and suppose that the government wishes to raise revenue and issues short-term debt, $b_{n,n+1} > 0$. The higher stock of maturing debt in the following period induces the subsequent government to default more often, thereby depressing the issuance price of the debt. From the perspective of the government in period $n+1$, the increased default

probability does not have welfare effects since the government in period $n+1$ is indifferent at the margin between repaying the debt or defaulting on it. From the perspective of the government in period n, in contrast, the increased default probability is suboptimal as it reduces the revenue raised through debt issuance without a corresponding gain. The fact that the government in period $n+1$ does not internalize the consequences of its choice of repayment rate on its predecessor's revenue from debt issuance is at the source of the time inconsistency problem analyzed in this paper.

In the alternative case where the government prematurely redeems outstanding long-term debt ($b_{n,n+1} < 0$), the induced behavioral response in the subsequent period again runs counter to the interests of the government in period n. For debt redemption increases the expected repayment rate in the following period and therefore raises the price at which the government buys back its bonds. Both positive and negative choices of $b_{n,n+1}$ therefore are associated with a negative expression for $\mathcal{I}^{\star}_{n,ss}$, indicating that such choices contribute negatively to the government's objective. In contrast, the sign of the revenue effect on inframarginal long-term debt, $\mathcal{I}^{\star}_{n,sl}$, is ambiguous and depends on the sign of the induced change of rollover policy in the subsequent period. If increased short-term debt issuance induces the subsequent government to issue more short-term debt as well, then the default probability on long-term debt increases, implying revenue losses on inframarginal units of long-term debt. If the debt issuance induces reduced short-term debt issuance in the following period, however, then such losses may be averted. Appendix B further discusses the revenue effects on inframarginal units of debt, focusing on the role played by social (rather than private) losses in the wake of a default in shaping these effects.

Consider next the choice of long-term debt. From (4), the effect of a marginal increase in $b_{n,n+2}$ in equilibrium is given by

$$u'(y_n - \min[b_{x,n}, L_n] + d^{\star}_n)\beta^2 \left(\mathrm{E}[1 - F(b_{x,n+2})|s_n] + \mathcal{I}^{\star}_{n,ll}\right) + \delta \frac{\partial \mathrm{E}[G_{n+1}(s_{n+1})|s_n]}{\partial b_{x,n+1,n+2}},$$

which can be expressed as

$$\begin{aligned} &u'(y_n - \min[b_{x,n}, L_n] + d^{\star}_n)\beta^2 \mathcal{I}^{\star}_{n,ll} + \\ &\quad \delta \mathrm{E}[u'(y_{n+1} - \min[b_{x,n+1}, L_{n+1}] + d^{\star}_{n+1})\beta \left(\mathcal{I}^{\star}_{n+1,ss} + \mathcal{I}^{\star}_{n+1,sl}\right)|s_n] + \\ &\quad \mathrm{E}[(1 - F(b_{x,n+2}))\left(\beta^2 u'(y_n - \min[b_{x,n}, L_n] + d^{\star}_n) - \delta^2 u'(y_{n+2} - b_{x,n+2} + d^{\star}_{n+2})\right)|s_n]. \end{aligned} \tag{8}$$

Parallel to (7), expression (8) contains a consumption-smoothing effect (represented by the expression in the last line) and a revenue effect on inframarginal units. In contrast to (7), the revenue effect in (8) arises with respect to both contemporaneous and subsequent debt issuance. This is a direct consequence of the fact that long-term debt issuance affects subsequent short-term debt issuance, and that the effect of the latter on revenue raised in period n remains unaccounted for by the government in period $n+1$.

If short-term debt issuance is interior, combining the marginal effects (7) and (8) and using the definition of prices yields an alternative, instructive representation of (8):

$$\begin{aligned} &u'(y_n - \min[b_{x,n}, L_n] + d^{\star}_n)q_{n,n+2} - \delta \mathrm{E}[u'(y_{n+1} - \min[b_{x,n+1}, L_{n+1}] + d^{\star}_{n+1})q_{n+1,n+2}|s_n] \\ &\quad + u'(y_n - \min[b_{x,n}, L_n] + d^{\star}_n)\beta^2 \mathcal{I}^{\star}_{n,ll}. \end{aligned} \tag{9}$$

Condition (9) displays in the first line the consumption-smoothing effect from long-term debt that is prematurely redeemed after one period. The second line reflects the revenue effect on inframarginal units of long-term debt due to long-term debt issuance. A comparison of conditions (7) and (9) reveals that short-term debt smoothes consumption between period n and the repayment states in period $n+1$, while long-term debt smoothes consumption between period n and all states in period $n+1$. Furthermore, short- and long-term debt generate different revenue effects on inframarginal units of debt.

To put these results into perspective, it is useful to recall the benchmark case with commitment, distinguishing between an environment with safe debt on the one hand and state-contingent debt on the other. If the government could commit its successors to honor maturing debt at face value, all $\mathcal{I}^\star$ terms in the expressions above would be absent; all $\min[b_{x,n}, L_n]$ terms would be replaced by $b_{x,n}$; and all repayment probabilities would equal unity. In an interior optimum, both (7) and (9) then would reduce to the *same* condition,

$$\beta u'(y_n - b_{x,n} + d_n^\star) - \delta \mathrm{E}[u'(y_{n+1} - b_{x,n+1} + d_{n+1}^\star)|s_n] = 0,$$

indicating that the government's portfolio choice would be indeterminate. This result hinges on the fact that, due to the exogenous asset pricing kernel of investors, the price of outstanding debt does not respond to the realization of shocks. If, in contrast, the price of outstanding debt were state contingent because of an endogenous asset pricing kernel, then the government's choice of maturity structure would be determinate (see Gale, 1990; Angeletos, 2002).

If the government could commit its successors to honor maturing debt at state-contingent repayment rates, all $\mathcal{I}^\star$ terms in the expressions above would again be absent; all $\min[b_{x,n}, L_n]$ terms would be replaced by $b_{x,n}r_n$; and all repayment probabilities would correspond to the respective averages of state-contingent repayment rates chosen ex ante. The optimal maturity structure then would be determinate if the returns to maturities correlated differently with taxpayers' marginal utility, as in a standard portfolio choice problem. Absent such differences in the correlation structure (for example because of risk neutrality on the part of taxpayers), the choice of maturity structure would again be indeterminate.

In the model of this paper, determinacy of the optimal maturity structure does not rely on any of these features. In fact, the optimal maturity structure is pinned down although the asset pricing kernel is exogenous and even if tax payers are risk neutral (see below).

4 Special Cases

I now turn to several special cases of the model. In all of these cases, the marginal utility of consumption is assumed to be exogenous. (As will become clear, this assumption can sometimes be relaxed.) The level of disposable income and thus, $b_{x,n}$, y_n and L_n therefore do not affect the government's rollover decision in period n. This simplifies the analysis and allows to characterize the optimal maturity structure in closed form.

I consider three settings, distinguished by the underlying rationale for debt policy. First, I focus on the role of impatience (small δ), and second, on the role of cyclicality of output (reflected in cyclical marginal utility). Finally, I briefly discuss the implications of risk.

4.1 Impatience

Suppose that $u(c) = c$ for all $n \geq 0$ and $\delta \leq \beta$. The marginal effects from issuing short- and long-term debt, (7) and (8) respectively, then reduce to

$$\begin{aligned} \beta\left(\mathcal{I}_{n,ss} + \mathcal{I}_{n,sl}\right) &+ (1 - F(b_{x,n+1}))(\beta - \delta), & (10) \\ \beta^2\mathcal{I}_{n,ll} + \delta\beta\mathrm{E}[\mathcal{I}_{n+1,ss} + \mathcal{I}_{n+1,sl}|s_n] &+ \mathrm{E}[1 - F(b_{x,n+2})|s_n](\beta^2 - \delta^2). & (11) \end{aligned}$$

The $\mathcal{I}$-terms on the left-hand side of these expressions reflect the revenue losses on inframarginal units of debt; the terms on the right-hand side reflect the consumption-smoothing benefits of a marginal unit of debt.

Depending on the value of the government's discount factor, δ, the marginal effects (10) and (11) encompass three interesting scenarios. First, the case of $\delta = 0$ where the government exclusively cares about taxpayers' current consumption. The consumption smoothing motive then reduces to the motive of raising revenue and the government aims at attaining the maximum of the debt-Laffer surface. As a consequence, all terms in the government's objective function are proportional to taxpayers' utility in the current period, implying that the assumption of risk neutrality is without loss of generality. Formally, if $\delta = 0$, the marginal effects (10) and (11) reduce to

$$\begin{aligned} \mathcal{I}_{n,ss} + \mathcal{I}_{n,sl} &+ 1 - F(b_{x,n+1}), \\ \mathcal{I}_{n,ll} &+ \mathrm{E}[1 - F(b_{x,n+2})|s_n]. \end{aligned}$$

Second, the case of $\delta = \beta$. In this case, consumption smoothing considerations are absent from the government's program and the government exclusively aims at minimizing the revenue losses on inframarginal units of debt. Formally, if $\delta = \beta$, the marginal effects (10) and (11) reduce to

$$\begin{aligned} \mathcal{I}_{n,ss} + \mathcal{I}_{n,sl} &\quad , \\ \mathcal{I}_{n,ll} + \mathrm{E}[\mathcal{I}_{n+1,ss} + \mathcal{I}_{n+1,sl}|s_n] &\quad . \end{aligned}$$

Note that, absent a consumption-smoothing motive, there is no reason for the government to issue debt in the first place. When characterizing the ex-ante optimal maturity structure in the case $\delta = \beta$, I will therefore posit an exogenous revenue requirement in the initial period.

Finally, the case of $0 < \delta < \beta$. In this intermediate case, the government's objective is dynamic and the low discount factor generates a motive for the government to front load consumption. In the quantitative sovereign debt literature, the assumption $0 < \delta < \beta$ is typically adopted because it is considered necessary to match high debt quotas in the data (see, for example, Aguiar and Gopinath, 2006; Arellano, 2008).

As noted earlier, the linear utility assumption renders optimal debt issuance in period n independent of $b_{x,n}$, y_n, and L_n. To see this, suppose the government in period n expects subsequent rollover decisions to be unaffected by these variables, in particular $\partial b^\star_{n+1,n+2}(s_{n+1})/\partial b_{x,n+1} = 0$. This implies $\mathcal{I}_{n,sl} = 0$ such that the marginal effect of short-term debt issuance, (10), reduces to $\beta\mathcal{I}_{n,ss} + (1 - F(b_{x,n+1}))(\beta - \delta)$ which is independent of $b_{x,n}$, y_n, or L_n.

Scaling this marginal effect by the price of short-term debt yields $-b_{n,n+1}H(b_{x,n+1}) + (\beta - \delta)/\beta$ where $H(L) \equiv f(L)/(1 - F(L))$ denotes the hazard function. In what follows, I assume that this hazard function is differentiable and weakly increasing—a rather weak assumption.[17] In this case, the marginal effect of short-term debt issuance is strictly decreasing in $b_{n,n+1}$ and the condition

$$b_{n,n+1}H(b_{x,n+1}) = \frac{\beta - \delta}{\beta} \tag{12}$$

defines a smooth positive function $b^\star_{n,n+1}(b_{n-1,n+1}) \geq 0$. Conditional on the amount of outstanding debt, $b_{n-1,n+1}$, equation (12) therefore pins down a unique, positive level of short-term debt issuance.

For later reference, note that the function $b_{n-1,n+1} + b^\star_{n,n+1}(b_{n-1,n+1})$ is strictly increasing in $b_{n-1,n+1}$. For if $H(L)$ is constant, the function $b^\star_{n,n+1}(b_{n-1,n+1})$ is constant as well; and if $H(L)$ is strictly increasing, then $-1 < \partial b^\star_{n,n+1}(b_{n-1,n+1})/\partial b_{x,n,n+1} < 0$. Note also that the same function is convex if the hazard function satisfies a second-order criterion. In particular, $\partial^2 b^\star_{n,n+1}(b_{n-1,n+1})/(\partial b_{x,n,n+1})^2 \geq 0$ requires that the following condition be satisfied:

(C) The function $H'(L)^2 - H(L)H''(L)$ is weakly positive, for example because the hazard function is concave.

(The exponential and Weibull distribution functions, among others, satisfy condition (C), see footnote 17.)

Turning to the marginal effect of long-term debt issuance, (11), and maintaining the assumption $H'(L) \geq 0$, we again have $\mathcal{I}_{n+1,sl} = 0$. Since $b^\star_{n+1,n+2}(b_{n,n+2})$ is deterministic, all expectation operators in (11) can be dropped. Moreover, since the expression in (10) equals zero, the marginal effect simplifies to $\beta^2\mathcal{I}_{n,ll} + (1 - F(b_{x,n+2}))(\beta^2 - \delta\beta)$. Scaled by the price of long-term debt, this yields $-b_{n,n+2}H(b_{x,n+2})\left(1 + \frac{\partial b^\star_{n+1,n+2}(b_{n,n+2})}{\partial b_{x,n+1,n+2}}\right) + (\beta^2 - \delta\beta)/\beta^2$. Since $H'(L) \geq 0$ and $\partial b^\star_{n+1,n+2}(b_{n,n+2})/\partial b_{x,n+1,n+2} > -1$, the condition

$$b_{n,n+2}H(b_{n,n+2} + b^\star_{n+1,n+2}(b_{n,n+2}))\left(1 + \frac{\partial b^\star_{n+1,n+2}(b_{n,n+2})}{\partial b_{x,n+1,n+2}}\right) = \frac{\beta - \delta}{\beta} \tag{13}$$

[17] Examples of distribution functions with increasing hazard functions include uniform, normal, exponential, logistic, extreme value, Laplace, power, Weibull, gamma, chi-squared, chi, or beta distributions (see, e.g., Bagnoli and Bergstrom, 2005).

If L_n is distributed according to an exponential distribution, $F(L) = 1 - \exp(-\lambda L)$, then the hazard function is constant, $H(L) = \lambda$.

If L_n is distributed according to a Weibull distribution, $F(L) = 1 - \exp(-L^\lambda), \lambda > 1$, then the hazard function is strictly increasing, $H(L) = \lambda L^{\lambda-1}$; moreover, for $1 \leq \lambda \leq 2$, the hazard function is concave, and for all $\lambda > 1$, $H'(L)^2 - H(L)H''(L) > 0$.

therefore pins down a unique, positive level of long-term debt issuance, $b^\star_{n,n+2} \geq 0$, if the partial derivative on the left-hand side of the equation does not decline too quickly as a function of $b_{n,n+2}$. Under condition (C), for example, this is the case and the solution therefore is unique. Note that $b^\star_{n,n+2}$ is independent of $b_{x,n}$, $b_{x,n,n+1}$, y_n, or L_n.

Summarizing, we have the following Lemma:

Lemma 1. Suppose that the utility function is linear, the hazard function weakly increasing and $\delta \leq \beta$. There exists an equilibrium in which the policy functions $b^\star_{n,n+1}(s_n)$ and $b^\star_{n,n+2}(s_n)$ do not depend on $b_{x,n}$, y_n, or L_n, for all $n \geq 0$. If the left-hand side of equation (13) is increasing in $b_{n,n+2}$, for example because condition (C) is satisfied, then the maturity structure in this equilibrium is unique.

The equilibrium characterized in the Lemma is the only equilibrium that arises in a finite horizon economy, including the limiting case where the number of periods approaches infinity. This follows from a straightforward backward induction argument. In the subsequent discussion, I focus on this type of equilibrium.

According to the first-order conditions (12) and (13), the government issues short- and long-term debt as long as the marginal cost falls short of the marginal benefit.[18] The marginal cost, given on the left-hand side of the two conditions, is given by the revenue losses on inframarginal debt (normalized by the price of the respective maturity). The marginal benefit, on the right-hand sides, is given by the net utility gain from the revenue and expected repayment of a marginal unit of debt (also normalized by the price of the respective maturity). This gain varies with the value of δ/β. If $\delta = 0$, the gain equals unity, the marginal utility of current consumption; if $\delta = \beta$, the gain equals zero because consumption smoothing considerations are absent from the government's program; finally, if $0 < \delta < \beta$, the gain lies between zero and one.

The optimal maturity structure depends on the value of δ/β as well as the shape of the hazard function. Three constellations may arise. First, the case with a constant hazard function and an arbitrary value for δ, $0 \leq \delta \leq \beta$. Second, the case of $\delta = \beta$ and an arbitrary, weakly increasing hazard function. Finally, the case of a strictly increasing hazard function paired with a "small" value for δ, $0 \leq \delta < \beta$. I consider these cases in turn.

If the hazard function is constant, condition (12) implies that $b^\star_{n,n+1}(b_{n-1,n+1})$ is a constant function and thus, that the partial derivative in (13) equals zero. As a consequence, the two conditions determine an interior and *fully balanced* maturity structure. Intuitively, with a constant hazard function, both the revenue losses on inframarginal debt and the revenue or smoothing gain from a marginal unit of debt only depend on the respective maturity. The optimal amount of each maturity therefore is determined independently of the other. Constancy of the hazard function also implies that the revenue losses on inframarginal debt relative to the marginal revenue gain are a convex function of the amount of debt issued. The optimal policy therefore smoothes maturities (or better, the inframarginal losses associated with them), for parallel reasons as those driving Barro's (1979) tax-smoothing prescription.

[18] If the hazard function is not weakly increasing, the optimal maturity structure might be concentrated.

Since the quantities of short- and long-term debt issuance coincide in every period and are constant across periods, the default risk is time invariant as well. The market value of long-term debt issuance therefore constitutes a fraction β of the market value of short-term debt issuance. In this sense, the maturity structure is *tilted towards short-term debt*. Moreover, in terms of the market value of stocks rather than flows, the maturity structure equals

$$\frac{(b_{n,n+1} + b_{n-1,n+1})\beta(1 - F(\cdot))}{b_{n,n+2}\beta^2(1 - F(\cdot))} = 2\beta^{-1}.$$

Summarizing, we have the following result:

Proposition 1. Suppose that the utility function is linear, the hazard function constant, and $0 \leq \delta \leq \beta$. The unique optimal maturity structure is fully balanced in terms of quantities, tilted towards short-term debt in terms of market values, and equal to $2\beta^{-1}$ in terms of market value of the stock of maturities.

Consider next the case where $\delta = \beta$. Conditions (12) and (13) then imply $b^{\star}_{n,n+1} = b^{\star}_{n,n+2} = 0$. Intuitively, absent a consumption-smoothing motive, the government solely aims to avoid losses on inframarginal units of debt. Such losses arise if new debt is issued because debt issuance depresses the price of inframarginal units. They also arise if outstanding debt is prematurely redeemed (that is, if $b_{x,n,n+1} > 0$ and $b_{n,n+1} < 0$) because debt redemption increases the price of inframarginal units, rendering debt issuance rather than redemption beneficial. As a consequence, the optimal policy abstains both from issuing and redeeming debt.

Suppose the government faces some exogenous revenue requirement in the initial period, $\bar{d}_0 > 0$. The government's program then runs over just three periods, $n = 0, 1, 2$. In particular, in period 0, the government issues short- and/or long-term debt, and in periods 1 and 2, this debt may or may not be repaid at maturity. Since no new debt is issued or prematurely redeemed, $b_{x,1} = b_{0,1}$ and $b_{x,2} = b_{0,2}$. Letting μ denote the multiplier on the revenue requirement in the initial period, the first-order conditions characterizing debt issuance in period $n = 0$ then read

$$\begin{aligned} b_{0,1}H(b_{0,1}) &= \mu, \\ b_{0,2}H(b_{0,2}) &= \mu \end{aligned}$$

and the optimal maturity structure is *fully balanced*, for the same smoothing reasons as before. Clearly, this result generalizes to settings with an arbitrary finite number of maturities. In summary:

Proposition 2. Suppose that the utility function is linear, the hazard function weakly increasing, and $\delta = \beta$. The unique optimal maturity structure in the initial period is fully balanced in terms of quantities, and tilted towards short-term debt in terms of market values.

Finally, consider the case of a strictly increasing hazard function paired with a "small" value for δ, $0 \leq \delta < \beta$. The optimality condition (12) then implies that $b^{\star}_{n,n+1}(b_{n-1,n+1})$ is

strictly decreasing and thus, from (13), that the optimal maturity structure generally is not balanced. (Condition (C) guarantees that the optimal maturity structure is unique.)

More specifically, consider a stationary equilibrium to which the economy has converged (since $\delta < \beta$) after a sequence of sufficiently high realizations of L. In such a stationary equilibrium, $b^\star_{n-1,n+1} = b^\star_{n,n+2} \equiv b^\star_{\text{long}}$ and $b^\star_{n,n+1} = b^\star_{n+1,n+2} \equiv b^\star_{\text{shrt}}$ and the two conditions read

$$b^\star_{\text{shrt}} H(b^\star_{\text{shrt}} + b^\star_{\text{long}}) = 1 - \frac{\delta}{\beta},$$
$$b^\star_{\text{long}} H(b^\star_{\text{shrt}} + b^\star_{\text{long}}) \left(1 + \frac{\partial b^\star_{\text{shrt}}(b^\star_{\text{long}})}{\partial b_{x,n+1,n+2}}\right) = 1 - \frac{\delta}{\beta}.$$

Since the partial derivative is negative, the optimal maturity structure is *tilted towards long-term* debt, $b^\star_{\text{shrt}} < b^\star_{\text{long}}$. Intuitively, with an increasing hazard function, higher outstanding debt in period $n+1$ drives up the revenue losses on inframarginal short-term debt, discouraging short-term debt issuance in period $n+1$. This, in turn, reduces the revenue losses on inframarginal long-term debt in period n since long-term debt issuance increases the amount of debt maturing in the long run by less than one-to-one. Ceteris paribus, the government therefore finds it "cheaper" to issue long-term debt.

If the partial derivative in the second equation is strictly increasing (as is the case, for example, if L_n is distributed according to a Weibull distribution) then the tilt towards long-term debt becomes smaller as the total amount of debt issued increases. *Higher debt quotas then go hand in hand with a shortening* of the optimal maturity structure, in line with the evidence cited earlier (Rodrik and Velasco, 1999).

Outside of a stationary equilibrium, closed form solutions obtain if L_n is distributed according to a Weibull distribution with parameter $\lambda = 2$ such that the hazard function equals $H(L) = 2L$ and a unique interior optimum is guaranteed. Equation (12) can then be solved to yield an expression for short-term debt issuance as a function of the stock of outstanding debt,

$$b^\star_{n,n+1}(b_{n-1,n+1}) = -\frac{b_{n-1,n+1}}{2} + \sqrt{\frac{b^2_{n-1,n+1} + 2(1 - \frac{\delta}{\beta})}{4}}.$$

Using this relation, the condition characterizing long-term debt issuance, equation (13), reduces to

$$b^\star_{n,n+2} = \sqrt{\sqrt{2}(1 - \frac{\delta}{\beta}) - (1 - \frac{\delta}{\beta})}.$$

For small values of $b_{n-1,n+1}$, the optimal maturity structure therefore is *tilted towards short-term* debt; for larger values, it is *tilted towards long-term* debt. The maturity structure converges after one period. In the stationary equilibrium, $b^\star_{n,n+1}/b^\star_{n,n+2} \approx 0.7$, independently of the ratio δ/β and in line with the general finding discussed earlier. Summarizing, we have the following result:

Proposition 3. Suppose that the utility function is linear, the hazard function strictly increasing, and $0 \leq \delta < \beta$. Condition (C) guarantees that the optimal maturity structure

is unique. In a stationary equilibrium, the unique optimal maturity structure is tilted towards long-term debt; moreover, if the function in condition (C) is strictly positive, higher debt quotas go hand in hand with a shortening of the maturity structure. During the transition, the optimal maturity structure may be tilted towards long- or short-term debt.

4.2 Cyclicality

I now turn to a setting with cyclical marginal utility of consumption. In particular, I assume that in even periods, $u_e(c) = c$, while in odd periods, $u_o(c) = u_o \cdot c$, with $u_o \geq 1$, $\delta < \beta$, $\delta u_o \leq \beta$. This setting offers a useful approximation to an environment with cyclical variation in exogenous output—high output in even periods and low output in odd periods. The approximation is exact if the effects of debt management on disposable income are negligible compared with the effect of output variation.

Consider an equilibrium cycle with debt issuance $(b^\star_{\text{short},e}, b^\star_{\text{long},e})$ in even periods and $(b^\star_{\text{short},o}, b^\star_{\text{long},o})$ in odd periods to which the economy has converged after a sequence of sufficiently high realizations of L. In even periods, the marginal effects of short- and long-term debt, (7) and (8) respectively, are given by

$$\begin{aligned} \beta\left(\mathcal{I}_{e,ss} + \mathcal{I}_{e,sl}\right) &+ (1 - F(b_{x,o}))(\beta - \delta u_o), \\ \beta^2 \mathcal{I}_{e,ll} + \delta\beta u_o(\mathcal{I}_{o,ss} + \mathcal{I}_{o,sl}) &+ (1 - F(b_{x,e})(\beta^2 - \delta^2), \end{aligned}$$

while in odd periods, they are given by

$$\begin{aligned} \beta\left(\mathcal{I}_{o,ss} + \mathcal{I}_{o,sl}\right) &+ (1 - F(b_{x,e}))\left(\beta - \frac{\delta}{u_o}\right), \\ \beta^2 \mathcal{I}_{o,ll} + \frac{\delta\beta}{u_o}(\mathcal{I}_{e,ss} + \mathcal{I}_{e,sl}) &+ (1 - F(b_{x,o})(\beta^2 - \delta^2). \end{aligned}$$

Constancy of marginal utility within a period implies as before that $\partial b^\star_{n,n+1}(s_n)/\partial b_{x,n} = 0$ and thus, $\mathcal{I}_{e,sl} = \mathcal{I}_{o,sl} = 0$. Suppose that the hazard function $H(L)$ is weakly increasing, and condition (C) is satisfied. The first-order conditions

$$\begin{aligned} b^\star_{\text{short},e} H(b^\star_{\text{short},e} + b_{\text{long},o}) &= 1 - \frac{\delta u_o}{\beta}, \\ b^\star_{\text{short},o} H(b^\star_{\text{short},o} + b_{\text{long},e}) &= 1 - \frac{\delta}{\beta u_o} \end{aligned}$$

then define smooth positive functions, $b^\star_{\text{short},e}(b_{\text{long},o}) \geq 0$ and $b^\star_{\text{short},o}(b_{\text{long},e}) \geq 0$. Using these first-order conditions to simplify the marginal effects of long-term debt issuance and following steps parallel to those leading to Lemma 1 yields the first-order conditions

$$\begin{aligned} b^\star_{\text{long},e} H(b^\star_{\text{short},o} + b^\star_{\text{long},e}) \left(1 + \frac{\partial b^\star_{\text{short},o}(b^\star_{\text{long},e})}{\partial b_{x,n,n+1}}\right) &= 1 - \frac{\delta u_o}{\beta}, \\ b^\star_{\text{long},o} H(b^\star_{\text{short},e} + b^\star_{\text{long},o}) \left(1 + \frac{\partial b^\star_{\text{short},e}(b^\star_{\text{long},o})}{\partial b_{x,n,n+1}}\right) &= 1 - \frac{\delta}{\beta u_o}, \end{aligned}$$

which pin down unique, positive long-term debt levels, $b^\star_{\text{long},e} \geq 0$ and $b^\star_{\text{long},o} \geq 0$. Jointly, the four conditions determine the maturity structure over the two-period cycle.

Consider the case with a constant hazard function (exponentially distributed income losses in the wake of a default). The first-order conditions then imply $b^\star_{\text{short},e} = b^\star_{\text{long},e} \leq b^\star_{\text{short},o} = b^\star_{\text{long},o}$, such that the maturity structure is *fully balanced* with *more debt* being issued *in periods of high marginal utility*. Since $b^\star_{\text{short},e} + b^\star_{\text{long},o} = b^\star_{\text{short},o} + b^\star_{\text{long},e}$, the default risk is constant over time. As a consequence, the *revenue raised* and the *cash flow* generated under the equilibrium debt policy are *countercyclical*—the market value of debt issuance in periods with high marginal utility exceeds the market value of debt issuance in periods with low marginal utility, and the same holds true net of debt repayment. The maturity structure in terms of market values is *tilted towards short-term debt* (by a factor of β^{-1}). The relative market value of the stock of maturities equals

$$\frac{(b^\star_{\text{short},e} + b^\star_{\text{long},o})\beta(1 - F(\cdot))}{b^\star_{\text{long},e}\beta^2(1 - F(\cdot))} \geq 2\beta^{-1}$$

in even periods while it is smaller than $2\beta^{-1}$ in odd periods. Measured by market value of the stocks, the maturity structure therefore *shortens* in periods *preceding times of high marginal utility*. Summarizing:

Proposition 4. Suppose that marginal utility follows a two-period cycle, the hazard function is constant, and $0 \leq \delta u_o \leq \beta, \delta < \beta$. The unique optimal maturity structure is fully balanced in terms of quantities and tilted towards short-term debt in terms of market values. Revenue raised and cash flow are countercyclical.

Figure 1 displays an example, illustrating how different debt statistics depend on the ratio of marginal utilities, u_o.[19] Solid lines in the figure correspond with periods of low marginal utility, dashed lines with periods of high marginal utility. The top two panels show that the quantities of short- and long-term debt issued within a period coincide while more debt is issued in periods of high marginal utility. According to the panels in the second row, the equilibrium prices of short- and long-term debt do not vary over the cycle (since the stock of maturing debt is constant over time). However, equilibrium prices in more cyclical environments are higher, reflecting lower quantities of maturing debt as u_o increases. The panels in the third row display the maturity structure in market values, both in terms of flows (on the left-hand side) and stocks (on the right-hand side).

Equilibrium quantities and prices imply that debt policy raises more revenue in periods of high marginal utility. The effect of debt policy on disposable household incomes (in periods where the government does not default) is negative, but less so in periods where resources are scarce, reflecting front loading of consumption on the one hand and consumption smoothing over the cycle on the other. With a smaller discount factor, the equilibrium policy involves more debt, issued less cyclically, and generating more negative cash flows for households in "good" and "bad" times.

[19] Unless stated differently, the following examples are computed under the assumption that $\beta = 0.9$ and $\delta = 0.5$. The parameter of the exponential distribution equals $\lambda = 2/\sqrt{\pi}$, the parameter of the Weibull distribution equals $\lambda = 2$. The two distributions therefore have the same mean.

If the government could only issue short-term debt, these effects would be smaller. Since the hazard function is constant, having access to one rather than two maturities would lead the government to issue just half the quantity of debt in each period, raising less revenue, and raising revenue less cyclically.

Relaxing the assumption of a constant and time invariant hazard function in Proposition 4 generates implications for the flow maturity structure over the cycle. Suppose first that the distribution of income losses in the wake of a default varies over time and maintain the assumption that this distribution is exponential. The previous derivations then go through, except for the fact that the relevant hazard functions vary over time as well. Letting $\lambda_o = \lambda_e\phi = \lambda\phi$, the conditions characterizing the equilibrium cycle generalize to

$$\begin{aligned}
b^\star_{\text{short},e} &= \left(1 - \frac{\delta u_o}{\beta}\right)(\lambda\phi)^{-1},\ b^\star_{\text{short},o} = \left(1 - \frac{\delta}{\beta u_o}\right)\lambda^{-1},\\
b^\star_{\text{long},e} &= \left(1 - \frac{\delta u_o}{\beta}\right)\lambda^{-1}, \qquad b^\star_{\text{long},o} = \left(1 - \frac{\delta}{\beta u_o}\right)(\lambda\phi)^{-1},\\
q^\star_{\text{short},e} &= q^\star_{\text{short},o} = \beta \exp\left(-2 + \frac{\delta(1+u_o^2)}{\beta u_o}\right),\\
q^\star_{\text{long},e} &= q^\star_{\text{long},o} = \beta^2 \exp\left(-2 + \frac{\delta(1+u_o^2)}{\beta u_o}\right).
\end{aligned}$$

Plausibly, low realizations of L and thus (ceteris paribus), defaults are more likely in times of scarce resources. This implies that the hazard rate in odd periods exceeds the one in even periods, $\phi \geq 1$. As a consequence, the *flow maturity structure* (in quantities or market values) *shortens in periods of scarce resources*, in line with the evidence cited earlier (Broner et al., 2007). Formally, defining the flow maturity structure in period n as

$$\mu^q_n \equiv \frac{b^\star_{\text{short},n}}{b^\star_{\text{long},n}}, \quad \mu^v_n \equiv \frac{q^\star_{\text{short},n} b^\star_{\text{short},n}}{q^\star_{\text{long},n} b^\star_{\text{long},n}},$$

$\phi > 1$ implies $\mu^q_e < \mu^q_o$ and $\mu^v_e < \mu^v_o$. Figure 2 displays an example with $\phi = 1.1$.

Proposition 5. Suppose that marginal utility follows a two-period cycle, the hazard function is constant and the hazard rate correlates positively with marginal utility, and $0 \leq \delta u_o \leq \beta, \delta < \beta$. The unique optimal maturity structure is cyclical in terms of quantities and market values, shortening in periods of high marginal utility.

The result of a cyclical flow maturity structure also follows, second, if $F(\cdot)$ has a strictly increasing hazard function rather than a time-varying one. In that case, the effects summarized in Proposition 3 come into play. Due to the strictly negative partial derivatives in the first-order conditions characterizing long-term debt issuance, the flow maturity structure is *tilted towards long-term debt*. This effect is relatively weaker in times of *high marginal utility* where *higher debt issuance* is associated with a *shortening of the maturity structure*, in line with the evidence (Broner et al., 2007). Formally, a result along these lines can be proved if income losses in the wake of a default are assumed to be distributed according to a Weibull distribution with parameter $\lambda = 2$. Under this

assumption, (lengthy) closed-form solutions establish that $\frac{\partial \mu_e^q}{\partial u_o}|_{u_o=1} < \frac{\partial \mu_o^q}{\partial u_o}|_{u_o=1}$ as well as $\frac{\partial \mu_e^v}{\partial u_o}|_{u_o=1} < \frac{\partial \mu_o^v}{\partial u_o}|_{u_o=1}$. Figure 3 displays an example.

Proposition 6. Suppose that marginal utility follows a two-period cycle, L_n is distributed according to a Weibull distribution with parameter $\lambda = 2$, and $0 \leq \delta u_o \leq \beta, \delta < \beta$. The unique optimal maturity structure is cyclical in terms of quantities and market values, shortening in periods of high marginal utility.

These findings do not hinge on the assumption that the length of the cycle and the maturity of long-term debt coincide. In fact, Proposition 6 can be modified to cover the case of a three-period cycle with marginal utility of consumption equal to unity during the first two periods (periods 1 and 2) and $u_o \geq 1$ during the last, "odd" period. In such a setting, the quantity of maturing debt and thus, debt prices vary over the cycle, see Figure 4 for an example. Nevertheless, (lengthy) closed-form solutions again establish that the maturity structure shortens in odd periods where resources are scarcest; in period 1, the maturity structure is most strongly biased towards the long end. Formally, $\frac{\partial \mu_1^q}{\partial u_o}|_{u_o=1} < \frac{\partial \mu_2^q}{\partial u_o}|_{u_o=1} < \frac{\partial \mu_o^q}{\partial u_o}|_{u_o=1}$ as well as $\frac{\partial \mu_1^v}{\partial u_o}|_{u_o=1} < \frac{\partial \mu_2^v}{\partial u_o}|_{u_o=1} < \frac{\partial \mu_o^v}{\partial u_o}|_{u_o=1}$.

Proposition 7. Suppose that marginal utility follows a three-period cycle, L_n is distributed according to a Weibull distribution with parameter $\lambda = 2$, and $0 \leq \delta u_o \leq \beta, \delta < \beta$. The unique optimal maturity structure is cyclical in terms of quantities and market values, shortening towards periods of high marginal utility.

4.3 Risk

Risk introduces three new elements in the analysis, even if marginal utility is exogenous. To see this, recall the marginal effects of short- and long-term debt issuance in period n, (7) and (9) respectively. Stochastic marginal utility renders short-term debt issuance in the subsequent period and thus, the quantity of debt maturing in the long term stochastic. As a consequence, the condition characterizing long-term debt issuance involves the ratio of the *expected* probability density and cumulative density. Stochastic marginal utility also implies that the marginal effect of short-term debt issuance features the expected marginal utility in the subsequent period. Finally, it implies that the marginal effect of long-term debt issuance includes the covariance between marginal utility in the subsequent period and the repayment probability in the long term as well as between the rollover policy in the subsequent period and the marginal default probability in the long term. The implications of these new elements for the optimal maturity structure can be analyzed numerically.

5 Cross Default

Sovereign defaults often involve repudiation of maturing *and outstanding* debt. In an environment where governments cannot commit, such cross defaults on outstanding debt cannot be interpreted as choices by the government in power since the final decision on

the repayment of currently outstanding debt will be taken by a subsequent government. Instead, cross defaults can be interpreted as debt buybacks at very low prices, where the price drop reflects equilibrium expectations about the subsequent government's default decision.

In the following, I analyze a setting that generates such equilibrium expectations and accordingly, cross default. Slightly modifying the structure of income losses in the wake of a default, I make the extreme assumption that a default on maturing debt in period n (carrying income losses L_n) reduces the cost for the subsequent government of defaulting on debt *outstanding* in period n to zero. A default on maturing debt therefore triggers a complete devaluation of the outstanding debt in the period as well.

With this modified structure of income losses, the government's program only changes as far as the law of motion for debt maturing in the subsequent period is concerned. This law of motion now reads

$$b_{x,n+1} = \mathbf{1}_{[r_n=1]} b_{x,n,n+1} + b^{\star}_{n,n+1}(s_n),$$

where the indicator function multiplying $b_{x,n,n+1}$ is the novel feature.

Since a default on maturing debt triggers a cross default on outstanding debt, the repayment decision now is dynamic. In particular, condition (1) changes to

$$r^{\star}_n(s_n) = \begin{cases} 1 & \text{if } L_n \geq b_{x,n} + \alpha^{\star}_n(s_n) \\ 0 & \text{if } L_n < b_{x,n} + \alpha^{\star}_n(s_n) \end{cases}, \tag{14}$$

where the positive function $\alpha^{\star}_n(s_n)$ is defined by the condition that the government be indifferent between repaying and defaulting,

$$\begin{aligned} & u(y_n - b_{x,n} + d^{\star}_n(s_n)) && + \delta \mathrm{E}[G_{n+1}(s_{n+1})|s_n] \mid \text{no default in period } n \\ \equiv\ & u(y_n - b_{x,n} - \alpha^{\star}_n(s_n) + d^{\star}_n(s_n)) && + \delta \mathrm{E}[G_{n+1}(s_{n+1})|s_n] \mid \text{default in period } n. \end{aligned}$$

To analyze the government's modified program it is useful to introduce some notation. Let $1 - F_{n+1} \equiv 1 - F(b_{x,n+1} + \alpha^{\star}_{n+1}(s_{n+1})|y_{n+1})$ denote the probability of repayment in period $n+1$, conditional on y_{n+1}. Similarly, let $1 - F_{n+2} \equiv 1 - F(b_{x,n+2} + \alpha^{\star}_{n+2}(s_{n+2})|y_{n+1}, y_{n+2}, r_{n+1} = 1)$ denote the probability of repayment in period $n+2$, conditional on y_{n+1}, y_{n+2} and no default in period $n+1$. Finally, let f_{n+1} and f_{n+2} denote the corresponding probability density functions. Condition (14) implies that the equilibrium prices of debt maturities are given by

$$q_{n,n+1} = \beta \mathrm{E}[r_{n+1}|s_n] = \beta \mathrm{E}_y[1 - F_{n+1}|s_n], \tag{15}$$

$$q_{n,n+2} = \beta^2 \mathrm{E}[r_{n+2}|s_n] = \beta^2 \mathrm{E}_y[(1 - F_{n+1})(1 - F_{n+2})|s_n], \tag{16}$$

where E_y indicates that expectations are taken with respect to y_{n+1} or y_{n+1}, y_{n+2}. The central difference to the main model relates to the price of long-term debt: Since long-term debt is only repaid if short-term debt is repaid (and L_{n+2} is sufficiently high), the price of long-term debt is bounded above by β times the price of short-term debt.

The deficit can then be expressed as

$$d_n = b_{n,n+1}\beta \mathrm{E}_y[1 - F_{n+1}|s_n] + b_{n,n+2}\beta^2 \mathrm{E}_y[(1 - F_{n+1})(1 - F_{n+2})|s_n]$$

and the effect of marginal increases in $b_{n,n+1}$ or $b_{n,n+2}$ on the deficit is given by

$$\frac{\mathrm{d}\, d_n}{\mathrm{d}\, b_{n,n+1}} = \beta \left(\mathrm{E}_y[1-F_{n+1}|s_n] \underbrace{-b_{n,n+1}\mathrm{E}_y[f_{n+1}|s_n]}_{\mathcal{I}_{n,ss}} \underbrace{+b_{n,n+2}\beta\mathrm{E}_y\left[\frac{\partial(1-F_{n+1})(1-F_{n+2})}{\partial b_{x,n+1}}|s_n\right]}_{\mathcal{I}_{n,sl}} \right),$$

$$\frac{\mathrm{d}\, d_n}{\mathrm{d}\, b_{n,n+2}} = \beta^2 \left(\mathrm{E}_y\left[(1-F_{n+1})(1-F_{n+2})|s_n\right] \underbrace{+b_{n,n+2}\mathrm{E}_y\left[\frac{\partial(1-F_{n+1})(1-F_{n+2})}{\partial b_{n,n+2}}|s_n\right]}_{\mathcal{I}_{n,ll}} \right.$$

$$\left. \underbrace{+b_{n,n+1}\beta^{-1}\mathrm{E}_y\left[\frac{\partial(1-F_{n+1})}{\partial b_{n,n+2}}|s_n\right]}_{\mathcal{I}_{n,ls}} \right),$$

respectively.

Relative to the main model, debt issuance affects the deficit through several new channels. On the one hand, long-term debt issuance reduces the value of short-term debt because the former affects the default decision in the short run (this gives rise to the new term $\mathcal{I}_{n,ls}$). On the other hand, short- or long-term debt issuance reduces the value of long-term debt, also by increasing the risk of default in the short run.

Turning to the envelope conditions, consider first the effect of a marginal increase of maturing debt. Condition (5) is replaced by

$$\frac{\partial G_n(s_n)}{\partial b_{x,n}} = \begin{cases} -u'(y_n - b_{x,n} + d_n^\star) & \text{if } L_n \geq b_{x,n} + \alpha_n^\star(s_n) \\ 0 & \text{if } L_n < b_{x,n} + \alpha_n^\star(s_n) \end{cases}, \tag{17}$$

implying

$$\frac{\partial \mathrm{E}[G_{n+1}(s_{n+1})|s_n]}{\partial b_{x,n+1}} = -\mathrm{E}_y[(1-F_{n+1})u'(y_{n+1} - b_{x,n+1} + d_{n+1}^\star)|s_n].$$

The marginal effect of outstanding debt depends on the default decision in the period, in contrast to the situation in the main model. The expression in (6) therefore is replaced by

$$\frac{\partial G_n(s_n)}{\partial b_{x,n,n+1}} =$$
$$\begin{cases} u'(y_n - b_{x,n} + d_n^\star)\beta\left(\mathcal{I}_{n,ss}^\star + \mathcal{I}_{n,sl}^\star\right) & \text{if } L_n \geq b_{x,n} + \alpha_n^\star(s_n) \\ -\delta\mathrm{E}_y[(1-F_{n+1})u'(y_{n+1} - b_{x,n+1} + d_{n+1}^\star)|s_n] & \\ 0 & \text{if } L_n < b_{x,n} + \alpha_n^\star(s_n) \end{cases}, \tag{18}$$

implying

$$\frac{\partial \mathrm{E}[G_{n+1}(s_{n+1})|s_n]}{\partial b_{x,n+1,n+2}} =$$
$$\mathrm{E}_y[(1-F_{n+1})u'(y_{n+1}-b_{x,n+1}+d^\star_{n+1})\beta\left(\mathcal{I}^\star_{n+1,ss}+\mathcal{I}^\star_{n+1,sl}\right)-\delta(1-F_{n+1})(1-F_{n+2})\times$$
$$u'(y_{n+2}-b_{x,n+2}+d^\star_{n+2})|s_n].$$

Using these results, the effect of a marginal increase in $b_{n,n+1}$ in equilibrium is given by

$$u'(c^\star_n)\beta\left(\mathcal{I}^\star_{n,ss}+\mathcal{I}^\star_{n,sl}\right)+\mathrm{E}_y\left[(1-F_{n+1})\left(\beta u'(c^\star_n)-\delta u'(y_{n+1}-b_{x,n+1}+d^\star_{n+1})\right)|s_n\right] \quad (19)$$

where $c^\star_n = y_n - \mathbf{1}_{[r_n=1]}b_{x,n} - (1-\mathbf{1}_{[r_n=1]})L_n + d^\star_n$. Similarly, the effect of a marginal increase in $b_{n,n+2}$ can be expressed as

$$u'(c^\star_n)\beta^2(\mathcal{I}^\star_{n,ll}+\mathcal{I}^\star_{n,ls})+\delta\mathrm{E}_y[(1-F_{n+1})u'(y_{n+1}-b_{x,n+1}+d^\star_{n+1})\beta\left(\mathcal{I}^\star_{n+1,ss}+\mathcal{I}^\star_{n+1,sl}\right)|s_n] \quad (20)$$
$$+\mathrm{E}_y[(1-F_{n+1})(1-F_{n+2})\left(\beta^2u'(c^\star_n)-\delta^2u'(y_{n+2}-b_{x,n+2}+d^\star_{n+2})\right)|s_n].$$

If short-term debt issuance is interior, the latter marginal effect can be rewritten as

$$u'(c^\star_n)q_{n,n+2}-\delta\mathrm{E}_y[(1-F_{n+1})u'(y_{n+1}-b_{x,n+1}+d^\star_{n+1})q_{n+1,n+2}|s_n]$$
$$+u'(c^\star_n)\beta^2(\mathcal{I}^\star_{n,ll}+\mathcal{I}^\star_{n,ls}), \quad (21)$$

where the price $q_{n+1,n+2}$ refers to states without default.

There are two main differences between the marginal effects (19)–(21) and the corresponding effects in the main model, (7)–(9). First, the risk of cross default modifies the revenue losses on inframarginal debt and introduces new types of such losses, as discussed earlier. Second, since debt repayment in the short run is a precondition for debt repayment in the long run, the burden of long-term debt in (20) and (21) is evaluated in repayment states rather than all states of nature, and multiplied by the factor $1-F_{n+1}$.

In parallel with the strategy pursued earlier, I assume from now on that marginal utility of consumption is exogenous. The realization of y_n therefore does not affect the choice of repayment rate or debt issuance. This renders $\alpha^\star_n$ a function of outstanding debt only and implies that the conditional distribution functions F_{n+1} and F_{n+2} can be replaced by their unconditional counterparts (F_{n+2} continues to be conditioned on $r_{n+1}=1$). Accordingly,

the effects of marginal increases in short- or long-term debt on the deficit simplify to

$$\begin{aligned}
\frac{\mathrm{d}\, d_n}{\mathrm{d}\, b_{n,n+1}} &= \beta \left(1 - F_{n+1} \underbrace{-b_{n,n+1} f_{n+1}}_{\mathcal{I}_{n,ss}} \underbrace{-b_{n,n+2} \beta \left(f_{n+1}(1 - F_{n+2}) \right)}_{\mathcal{I}_{n,sl}} \right) \\
&= \beta(1 - F_{n+1})(1 - H_{n+1}[b_{n,n+1} + b_{n,n+2}\beta(1 - F_{n+2})]), \\
\frac{\mathrm{d}\, d_n}{\mathrm{d}\, b_{n,n+2}} &= \beta^2 \Bigg((1 - F_{n+1})(1 - F_{n+2}) \underbrace{-b_{n,n+2}(1 - F_{n+1}) f_{n+2} \left(1 + \frac{\mathrm{d}\, \nu_{n+1}}{\mathrm{d}\, b_{n,n+2}} | r_{n+1} = 1 \right)}_{\mathcal{I}_{n,ll}} \\
&\qquad \underbrace{-f_{n+1} \alpha_{n+1}^{\star\prime}(b_{n,n+2})(b_{n,n+1}\beta^{-1} + b_{n,n+2}(1 - F_{n+2}))}_{\mathcal{I}_{n,ls} \text{ and } \mathcal{I}_{n,ll}} \Bigg) \\
&= \beta^2 (1 - F_{n+1})(1 - F_{n+2}) \Bigg(1 - b_{n,n+2} H_{n+2} \left(1 + \frac{\mathrm{d}\, \nu_{n+1}}{\mathrm{d}\, b_{n,n+2}} | r_{n+1} = 1 \right) \\
&\qquad - H_{n+1} \frac{\alpha_{n+1}^{\star\prime}(b_{n,n+2})}{\beta(1 - F_{n+2})} [b_{n,n+1} + b_{n,n+2}\beta(1 - F_{n+2})] \Bigg),
\end{aligned}$$

respectively, where

$$\nu_{t+1} \equiv b^{\star}_{n+1,n+2}(s_{n+1}) + \alpha^{\star}_{n+2}(b_{n+1,n+3}).$$

Let $G_{n+1}(s_{n+1})^{\text{default}}$ denote the value for the government in period $n+1$ of defaulting, net of the income loss L_{n+1}, and let $G_{n+1}(s_{n+1})^{\text{no default}}$ denote the value for the government in period $n+1$ of not defaulting. Finally, let u'_n denote the exogenous marginal utility in period n. Since

$$\alpha^{\star}_{n+1}(b_{n,n+2}) = \frac{G_{n+1}(s_{n+1})^{\text{default}} - u'_{n+1} b_{x,n+1} - G_{n+1}(s_{n+1})^{\text{no default}}}{u'_{n+1}}$$

and $G_{n+1}(s_{n+1})^{\text{default}}$ is independent of $b_{x,n+1,n+2}$, we have

$$\begin{aligned}
\frac{\partial \alpha^{\star}_{n+1}(b_{n,n+2})}{\partial b_{n,n+2}} &= \delta(1 - F_{n+2}) \frac{u'_{n+2}}{u'_{n+1}} - \beta(\mathcal{I}_{n+1,ss} + \mathcal{I}_{n+1,sl}) \\
&= \beta(1 - F_{n+2})
\end{aligned}$$

where the last equality uses (19). Intuitively, an increase in the amount of outstanding debt increases the critical income loss $\hat{L}_{n+1}$ at which the government in period $n+1$ is indifferent between defaulting and not defaulting twofold. First, because a default wipes out outstanding debt with market value $\beta(1-F_{n+2})$, increasing $\hat{L}_{n+1}$ by $\beta(1-F_{n+2})\frac{\delta u'_{n+2}}{\beta u'_{n+1}}$. Second, because a default improves the conditions subject to which the government issues new debt, by reducing the default probability in period $n+2$. Optimality of short-term debt issuance in period $n+1$ (condition (19)) assures that the sum of these two effects reduces to the market price of outstanding debt.

Using these results, the effects on the government's objective of marginal increases in short- and long-term debt, respectively, are given by

$$\beta(1-F_{n+1})\left\{u'_n-\frac{\delta}{\beta}u'_{n+1}-u'_nH_{n+1}[b_{n,n+1}+b_{n,n+2}\beta(1-F_{n+2})]\right\},$$

$$\beta^2(1-F_{n+1})(1-F_{n+2})\left\{u'_n-\frac{\delta}{\beta}u'_{n+1}-u'_nH_{n+1}[b_{n,n+1}+b_{n,n+2}\beta(1-F_{n+2})]-u'_nB_n\right\},$$

$$B_n\equiv b_{n,n+2}H_{n+2}\left(1+\frac{\mathrm{d}\,\nu_{n+1}}{\mathrm{d}\,b_{n,n+2}}|r_{n+1}=1\right)$$

where the second expression uses condition (21). It follows that $b^\star_{n,n+2}=0$ constitutes an equilibrium with $b^\star_{n,n+1}$ implicitly determined by the condition $1-\frac{\delta}{\beta}\frac{u'_{n+1}}{u'_n}=H_{n+1}b^\star_{n,n+1}$.

Intuitively, issuing short-term debt generates consumption smoothing benefits on the one hand and revenue losses on inframarginal short- and long-term debt on the other. These revenue losses arise because debt issuance drives up the probability of default in the subsequent period. Long-term debt issuance generates the same ratio of inframarginal revenue losses due to higher default risk in the short term and consumption smoothing benefits. In addition, however, long-term debt issuance also generates revenue losses due to higher default risk in the long term, conditional on no default in the short term.

Proposition 8. Suppose that marginal utility is exogenous and the hazard function weakly increasing. In the model with cross default, only short-term debt is issued.

Figures 5 and 6 illustrate the effect of cross default on default choices and expected repayment rates and prices.[20] The first figure illustrates the case without cross default (the main model) while the second figure illustrates the case with cross default. In both figures, the top left panel displays the values of L_n at which the government is indifferent between defaulting and repaying, conditional on the stock of maturing and outstanding debt $(b_{x,n},b_{x,n,n+1})$. The top right panel displays the expected repayment rate conditional on this stock, $\int_0^\infty r^\star_n(b_{x,n},b_{x,n,n+1},L_n)dF(L_n)$. The bottom left panel displays the expected price of outstanding debt, namely $\int_0^\infty q^\star_{n,n+1}dF(L_n)$ in the case without cross default and $\int_0^\infty \mathbf{1}_{[r^\star_n=1]}q^\star_{n,n+1}dF(L_n)$ in the case with cross default. Finally, the bottom right panel displays the expected price of newly-issued short-term debt, $\int_0^\infty q^\star_{n,n+1}dF(L_n)$.

Several points are worth stressing. First, the threshold value for L_n increases one-to-one with the level of maturing debt. In the case with cross default, it also increases with the level of outstanding debt. Intuitively, outstanding debt depresses the price of newly-issued debt. A higher stock of outstanding debt therefore renders a default more attractive—even if $\delta=0$. Second, this positive effect of outstanding debt on the default threshold in the model with cross default results in a negative effect of outstanding debt on the expected repayment rate which is not present in the model without cross default. Third, in the model without cross default, the expected price of outstanding debt on the one hand and of newly-issued short-term debt on the other is equal to each other.

[20]The examples are computed under the assumption that $\beta=0.9$ and $\delta=0$. Income losses are distributed exponentially with parameter $\lambda=1$.

In the model with cross default, this is not the case. There, the price of outstanding debt reflects the fact that both maturing and outstanding debt determine the effective default probability. Since new short-term debt can be issued at better conditions after an effective default on outstanding debt, the price of the equilibrium quantity of newly-issued short-term debt *increases* in the stock of maturing debt.

6 Conclusion

Lack of commitment to debt repayment paired with social losses in the wake of a default gives rise to a determinate optimal maturity structure. This structure is critically shaped by revenue losses on inframarginal units of debt, reflecting the price impact of default and rollover choices by subsequent policy makers. Under plausible conditions, such inframarginal revenue losses normalized by the price of debt are a convex function of the quantity of a maturity issued. Optimal policy therefore amounts to "smoothing" the revenue losses on inframarginal units of maturities, in parallel with the familiar tax (distortion) smoothing prescription.

The cost of a default has important implications for the optimal maturity structure. I have considered two extreme cases: First, the situation where default costs do not directly depend on default choices in the past, and second, the situation with such direct dependence. If default costs are independent, default probabilities are a function of the stock of maturing debt only. Moreover, if default costs are distributed exponentially, then future rollover decisions are independent of the stock of outstanding debt. As a consequence, spillover effects between different maturities are minimal and the smoothing prescription implies a fully balanced maturity structure. If default costs are distributed according to any other distribution with increasing hazard, then outstanding debt depresses short-term debt issuance. Long-term debt issuance therefore increases the debt maturing in the long term by less than one-to-one, reducing the associated inframarginal losses. Accordingly, the optimal maturity structure is tilted towards the long end. Higher quantities of debt reduce this cost advantage of long-term debt, giving rise to a more balanced maturity structure.

Over the cycle, total debt issuance increases in times of high marginal utility. If the hazard function is strictly increasing, the maturity structure therefore shortens during such times as well. A parallel result holds if default costs are distributed exponentially, provided that these costs tend to be lower in times of scarce resources.

If default costs are dependent over time such that a default on maturing debt triggers a subsequent default on currently outstanding debt, then default probabilities depend on the stock of maturing and outstanding debt. This renders the default decision dynamic and implies that long-term debt issuance is always dominated by short-term issuance such that the maturity structure is concentrated on the short end.

The broad picture that emerges from the model is one of an interior and roughly balanced maturity structure that shortens in periods of high debt issuance, high marginal utility ("crises"), and increased risk of cross default and acceleration. While the empirical evidence is broadly consistent with these predictions, future research should aim at

improving our understanding of default induced losses. As this paper has shown, the characteristics of these losses crucially affect the maturity structure of countries with credibility problems.

A Alternative Specifications of Social Losses

Corner solutions for the optimal repayment rate follow under more general assumptions about the losses in the wake of a default. Consider for example the case where income losses are proportional to L_n and the *default rate*,

$$\text{losses}_n = (1 - r_n)L_n.$$

The optimal repayment choice then is identical to the one given in the text.

Consider next the situation where income losses are proportional to L_n and the total *amount* defaulted upon,

$$\text{losses}_n = (1 - r_n)b_{x,n}L_n.$$

The optimal repayment rate then varies with L_n but does not depend on the amount of maturing debt, rendering such a specification unattractive.

Consider next the situation where income losses are a *concave* function of the amount defaulted upon, for example

$$\text{losses}_n = [(1 - r_n)b_{x,n}]^{1/2}L_n$$

or

$$\text{losses}_n = \mathbf{1}_{[r_n<1]}L_n + k(1 - r_n)b_{x,n},\ 0 < k < 1.$$

Again, the optimal repayment rate then equals either unity or zero since the total cost from debt repayment and income losses is a concave function of the default rate.

If income losses are a *convex* function of the amount defaulted upon, for example

$$\text{losses}_n = [(1 - r_n)b_{x,n}]^2L_n,$$

then the equilibrium repayment rate is no longer discrete. However, convexity of income losses appears less plausible than the previously discussed specifications, for at least two reasons. First, most notions of income losses are consistent with concave costs: The marginal cost of defaulting on the first 5 percent of debt exceeds the one from defaulting on the following 5 percent. Second, convex income losses would lead governments to *always* default at least partially, in contrast with the empirical evidence.

B Social Losses and the Incentive to Dilute

In this section, I analyze how the assumption of social losses in the wake of a default shapes the government's rollover decision. I focus on the case where the government only

issues short-term debt. Recall from the text that, in this case,

$$\frac{\mathrm{d}\, d_n}{\mathrm{d}\, b_{n,n+1}} = \beta \left(1 - F(b_{x,n+1}) \underbrace{- b_{n,n+1} f(b_{x,n+1})}_{\mathcal{I}_{n,ss}} \right)$$

while the marginal effect of short-term debt issuance on the government's objective is given by

$$\begin{aligned} & u'(y_n - \min[b_{x,n}, L_n] + d_n^\star)\beta \mathcal{I}_{n,ss}^\star + \\ & \quad (1 - F(b_{x,n+1})) \left(\beta u'(y_n - \min[b_{x,n}, L_n] + d_n^\star) - \delta \mathrm{E}[u'(y_{n+1} - b_{x,n+1} + d_{n+1}^\star) | s_n] \right). \end{aligned}$$

To understand the role played by social losses, consider an alternative setup without such losses. Assume as before that the government either fully repays the maturing debt or suffers a cost L_n. In contrast to the main model, however, suppose now that this cost corresponds to a *transfer* to bondholders rather than a social loss. One can interpret this modified setting as a situation where the realization of L_n determines the bargaining power of bondholders vis-a-vis the government. According to this interpretation, bondholders can successfully press for full repayment if the realization of L_n is high. If the realization of L_n falls short of the maturing debt, however, bondholders must concede and settle for a reduced repayment equal to L_n.

In this modified setup, the repayment rate in period n is given by

$$r_n^\star(s_n) = \begin{cases} 1 & \text{if } L_n \geq b_{x,n} \\ \frac{L_n}{b_{x,n}} & \text{if } L_n < b_{x,n} \end{cases}$$

and the expected repayment rate therefore features a new component that accounts for payments in the partial default case:

$$\mathrm{E}\left[r_{n+1} | s_n \right] = 1 - F(b_{x,n+1}) + \underbrace{\frac{1}{b_{x,n+1}} \int_0^{b_{x,n+1}} L_{n+1} \mathrm{d}\, F(L_{n+1})}_{\text{new term}}.$$

Accordingly, the marginal effect of debt issuance in period n on the deficit in that period changes to

$$\begin{aligned} \frac{\mathrm{d}\, d_n}{\mathrm{d}\, b_{n,n+1}} &= \beta \left(1 - F(b_{x,n+1}) - b_{n,n+1} f(b_{x,n+1}) \right) \\ &\quad + \underbrace{\beta \left(b_{n,n+1} f(b_{x,n+1}) + \frac{1}{b_{x,n+1}} \int_0^{b_{x,n+1}} L_{n+1} \mathrm{d}\, F(L_{n+1}) \left(1 - \frac{b_{n,n+1}}{b_{x,n+1}} \right) \right)}_{\text{new terms}}. \end{aligned}$$

The presence of transfers rather than social losses introduces three marginal effects in addition to those present in the main model. First, the increase in $b_{n,n+1}$ raises more revenue because newly-issued debt is *partially* repaid in some states, as reflected in the

term $\frac{1}{b_{x,n+1}}\int_0^{b_{x,n+1}} L_{n+1}\mathrm{d}\,F(L_{n+1})$. Second, as reflected in the term $b_{n,n+1}f(b_{x,n+1})$, an increase in $b_{n,n+1}$ raises the probability of *partial* repayment of the newly-issued debt at the critical income loss, $b_{x,n+1}$. Finally, the increase in $b_{n,n+1}$ causes revenue losses on newly-issued inframarginal debt, $-\frac{b_{n,n+1}}{b_{x,n+1}^2}\int_0^{b_{x,n+1}} L_{n+1}\mathrm{d}\,F(L_{n+1})$, because it reduces the repayment rate in case of *partial* default.

The second of these additional effects cancels with the loss on inframarginal debt that is already present in the main model. Intuitively, the revenue gain due to more likely, partial repayment exactly compensates for the revenue loss due to less likely, full repayment. On net, the marginal effect on the deficit therefore amounts to $\beta(1-F(b_{x,n+1}))+\beta\frac{1}{b_{x,n+1}}\int_0^{b_{x,n+1}} L_{n+1}\mathrm{d}\,F(L_{n+1})\left(1-\frac{b_{n,n+1}}{b_{x,n+1}}\right)$. If $0<b_{x,n,n+1}<b_{x,n+1}$ such that debt is outstanding and the government issues additional debt, then this marginal effect exceeds $\beta(1-F(b_{x,n+1}))$ because debt issuance effectively redistributes collateral from outstanding to newly-issued debt, in contrast with the situation in the main model.

The government's program in period n is unchanged relative to the original setup, except for the modified expression characterizing the deficit. (From the government's point of view, it is irrelevant whether income losses in period $n+1$ correspond to transfers to bond holders rather than social losses.) The effect of a marginal increase in $b_{n,n+1}$ therefore equals

$$u'(y_n-\min[b_{x,n},L_n]+d_n^\star)\beta\frac{1}{b_{x,n+1}}\int_0^{b_{x,n+1}} L_{n+1}\mathrm{d}\,F(L_{n+1})\frac{b_{x,n,n+1}}{b_{x,n+1}}+$$
$$(1-F(b_{x,n+1}))\left(\beta u'(y_n-\min[b_{x,n},L_n]+d_n^\star)-\delta\mathrm{E}[u'(y_{n+1}-b_{x,n+1}+d_{n+1}^\star)|s_n]\right),$$

reflecting the same consumption-smoothing effect as in the main model (in the second line), but a modified revenue effect on inframarginal units of debt (in the first line). Without social losses in the wake of a default as they are present in the original setup, the government therefore has an incentive to dilute outstanding debt.

C Figures

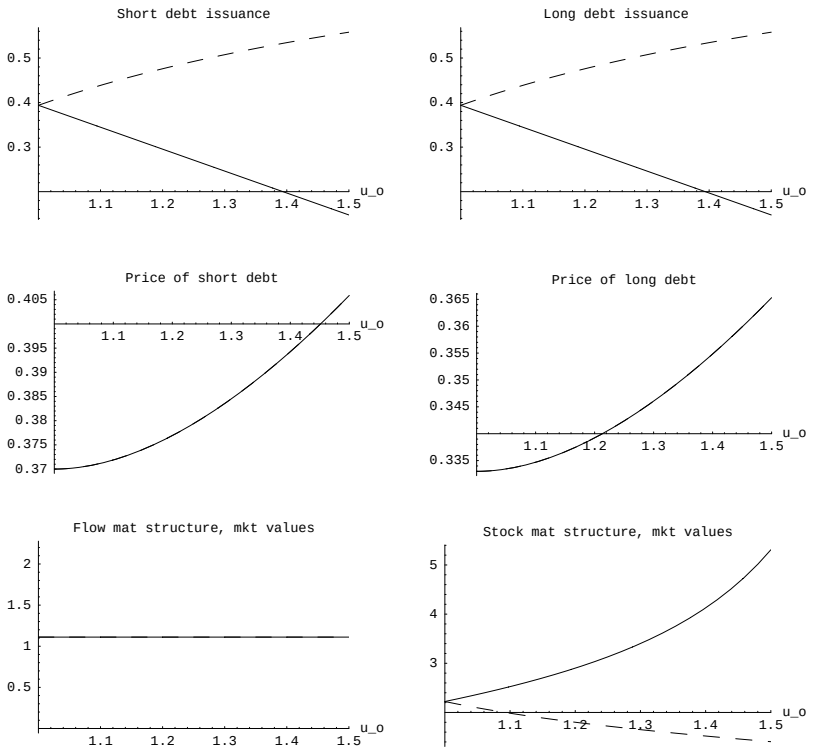

Figure 1: Debt policies if marginal utility follows a two-period cycle and income losses in the wake of a default are distributed according to an exponential distribution. The figure plots debt statistics as functions of the ratio of marginal utilities, u_o; solid lines correspond with periods of low marginal utility, dashed lines with periods of high marginal utility.

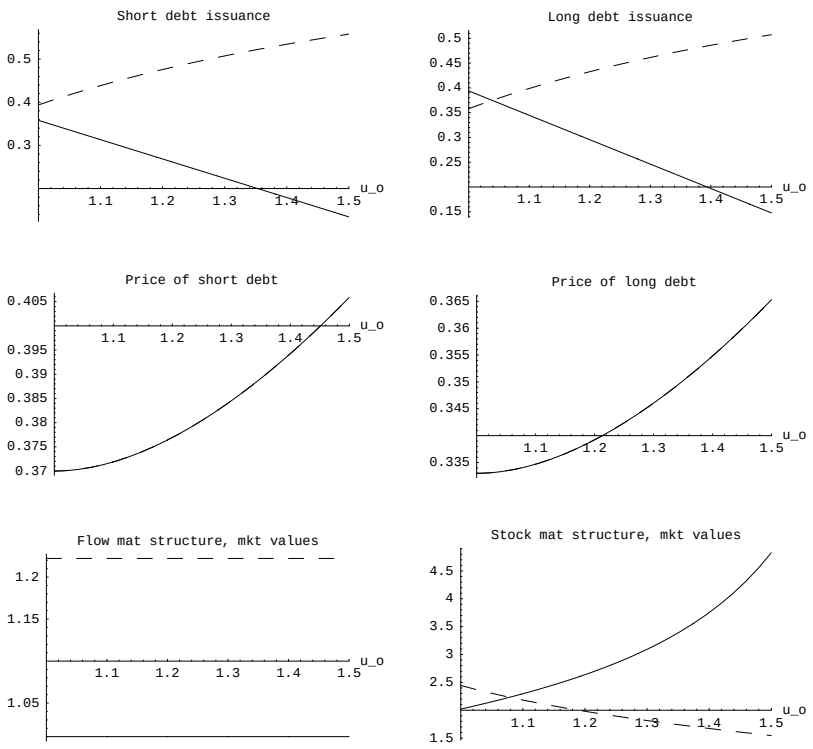

Figure 2: Debt policies if marginal utility follows a two-period cycle, income losses in the wake of a default are distributed according to an exponential distribution, and the hazard rate in periods with high marginal utility exceeds the rate in periods with low marginal utility. The figure plots debt statistics as functions of the ratio of marginal utilities, u_o; solid lines correspond with periods of low marginal utility, dashed lines with periods of high marginal utility.

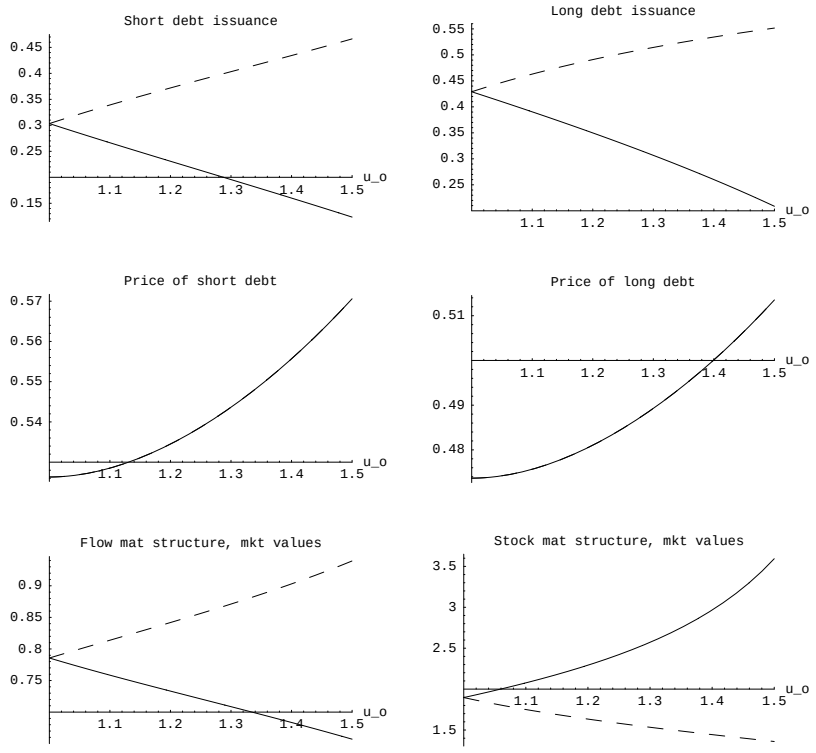

Figure 3: Debt policies if marginal utility follows a two-period cycle and income losses in the wake of a default are distributed according to a Weibull distribution. The figure plots debt statistics as functions of the ratio of marginal utilities, u_o; solid lines correspond with periods of low marginal utility, dashed lines with periods of high marginal utility.

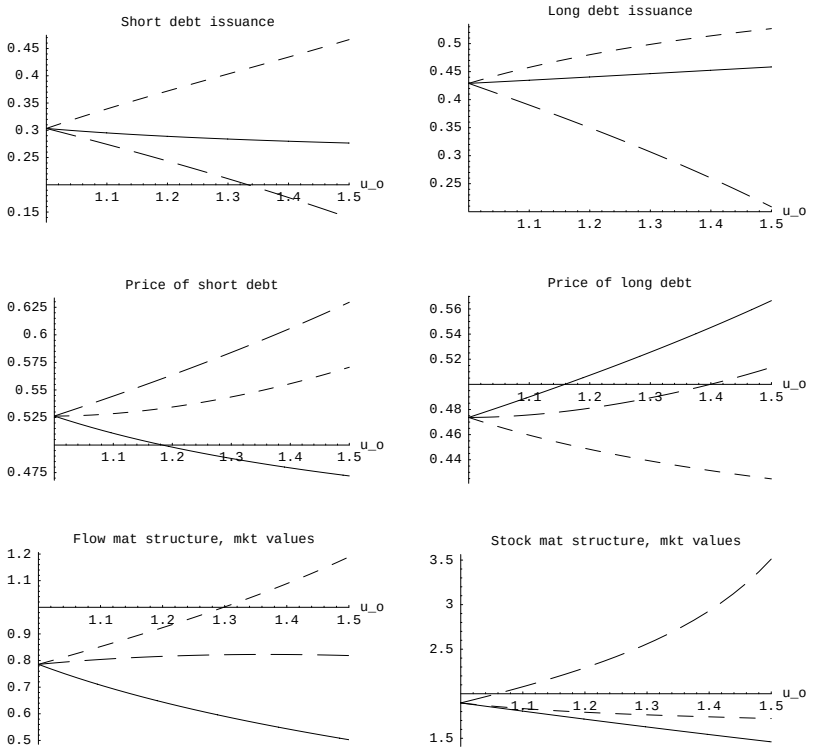

Figure 4: Debt policies if marginal utility follows a three-period cycle and income losses in the wake of a default are distributed according to a Weibull distribution. The figure plots debt statistics as functions of the ratio of marginal utilities, u_o; solid and long-dashed lines correspond with periods of low marginal utility (periods 1 and 2, respectively), short-dashed lines with periods of high marginal utility.

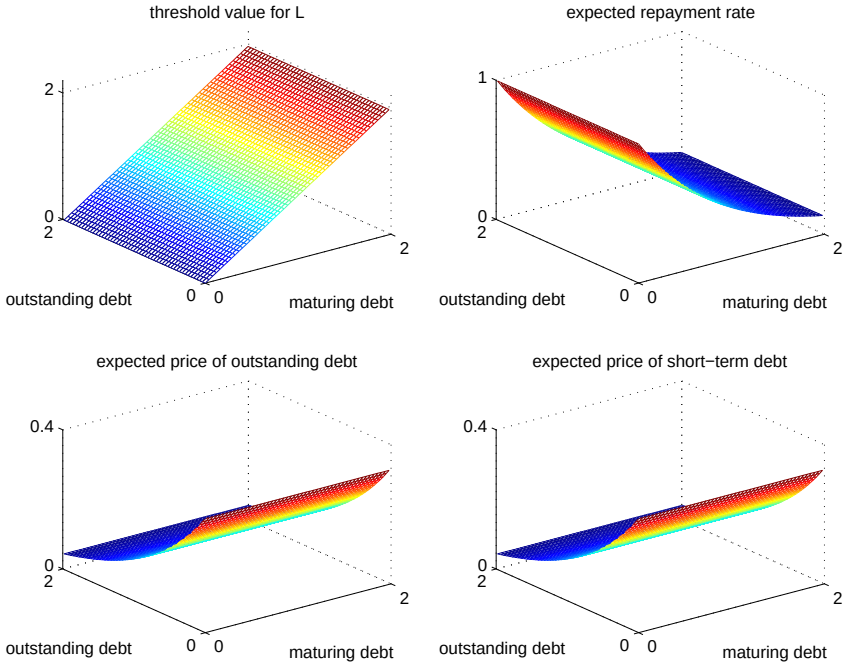

Figure 5: Default choices and expected repayment rates and prices in the model *without* cross default if income losses in the wake of a default are distributed according to an exponential distribution and $\delta = 0$.

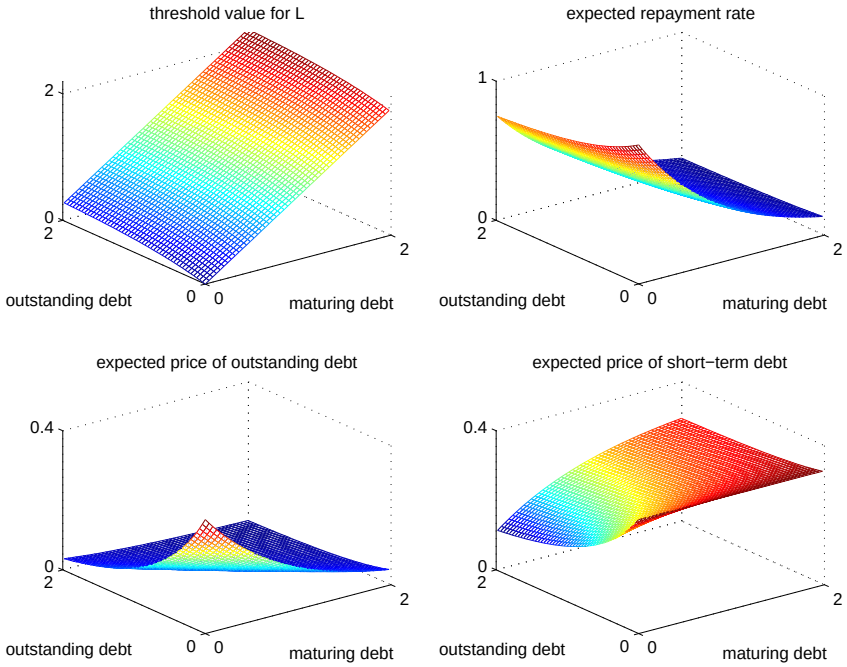

Figure 6: Default choices and expected repayment rates and prices in the model *with* cross default if income losses in the wake of a default are distributed according to an exponential distribution and $\delta = 0$.

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