

Caporale, Guglielmo Maria; Cerrato, Mario

Working Paper

Using chebyshev polynomials to approximate partial differential equations

CESifo Working Paper, No. 2308

Provided in Cooperation with:

Ifo Institute – Leibniz Institute for Economic Research at the University of Munich

Suggested Citation: Caporale, Guglielmo Maria; Cerrato, Mario (2008) : Using chebyshev polynomials to approximate partial differential equations, CESifo Working Paper, No. 2308, Center for Economic Studies and ifo Institute (CESifo), Munich

This Version is available at:

<https://hdl.handle.net/10419/26353>

Standard-Nutzungsbedingungen:

Die Dokumente auf EconStor dürfen zu eigenen wissenschaftlichen Zwecken und zum Privatgebrauch gespeichert und kopiert werden.

Sie dürfen die Dokumente nicht für öffentliche oder kommerzielle Zwecke vervielfältigen, öffentlich ausstellen, öffentlich zugänglich machen, vertreiben oder anderweitig nutzen.

Sofern die Verfasser die Dokumente unter Open-Content-Lizenzen (insbesondere CC-Lizenzen) zur Verfügung gestellt haben sollten, gelten abweichend von diesen Nutzungsbedingungen die in der dort genannten Lizenz gewährten Nutzungsrechte.

Terms of use:

Documents in EconStor may be saved and copied for your personal and scholarly purposes.

You are not to copy documents for public or commercial purposes, to exhibit the documents publicly, to make them publicly available on the internet, or to distribute or otherwise use the documents in public.

If the documents have been made available under an Open Content Licence (especially Creative Commons Licences), you may exercise further usage rights as specified in the indicated licence.

Using Chebyshev Polynomials to Approximate Partial Differential Equations

GUGLIELMO MARIA CAPORALE
MARIO CERRATO

CESIFO WORKING PAPER NO. 2308
CATEGORY 10: EMPIRICAL AND THEORETICAL METHODS
MAY 2008

An electronic version of the paper may be downloaded

- *from the SSRN website:* www.SSRN.com
- *from the RePEc website:* www.RePEc.org
- *from the CESifo website:* www.CESifo-group.org/wp

Using Chebyshev Polynomials to Approximate Partial Differential Equations

Abstract

This paper suggests a simple method based on a Chebyshev approximation at Chebyshev nodes to approximate partial differential equations. It consists in determining the value function by using a set of nodes and basis functions. We provide two examples: pricing a European option and determining the best policy for shutting down a machine. The suggested method is flexible, easy to programme and efficient. It is also applicable in other fields, providing efficient solutions to complex systems of partial differential equations.

JEL Code: C63, G12.

Keywords: European options, Chebyshev polynomial approximation, Chebyshev nodes.

Guglielmo Maria Caporale
Centre for Empirical Finance
Brunel University
Uxbridge, Middlesex UB8 3PH
United Kingdom
Guglielmo-Maria.Caporale@brunel.ac.uk

Mario Cerrato
Department of Economics
University of Glasgow, G12 8RT
United Kingdom
m.cerrato@lbss.gla.ac.uk

April 2008

We are grateful to Karim Abadir, Paresh Date, Brian Eales, and Geoff Rodgers for useful comments and suggestions. The usual disclaimer applies.

1. Introduction

The study of partial differential equations (PDE) is a fundamental topic in applied mathematics, as well as in physics, natural science and finance. For example, in finance PDEs are used in arbitrage-based asset models. The widely cited Black and Scholes PDE, that must be satisfied by each European option within an arbitrage free market, is a canonical example. In this specific case, the PDE has an analytical solution; however, in many other interesting cases in finance, as well as in other fields, closed form solutions are very difficult to obtain and researchers rely on various numerical methods to obtain a solution. The study of these numerical methods represents the area of Computational Partial Differential Equations.

The most simple applicable algorithms to approximate PDEs rely on the concept of discretisation - that is, replacing the PDE of interest by a finite dimensional problem. However, replacing the PDE by a discrete model is not at all trivial and generally the choice of the finite dimensional model to be used depends on the properties of the mathematical model itself.

The development of high speed computers has made it easier to find accurate solutions to PDEs, even in the most extreme cases of very large systems of PDEs. In this study we show how to use polynomial methods to approximate PDEs. We shall only be focusing on second-order linear PDEs, although it would also be interesting to evaluate our methodology when dealing with non-linear types of PDEs. We leave this for future research.

Crack-Nicolson (CN) implicit schemes are amongst the most widely used approximation methods. However, their effectiveness is affected by the choice of time steps, which, very often, depends on the problem one is facing. Also, CN methods, in general, suffer from poor convergence.

The method suggested in this paper relies on polynomial interpolation to approximate the PDE characterising the option pricing problem. In our two applications, we use Monte Carlo methods to solve the boundary condition for the PDE, and fit the

functional at Chebyshev nodes to estimate the coefficients¹. The advantage of our approach is its flexibility, and the fact that it is easily implementable and since the functional, at least in the first empirical example, is approximated using deterministic nodes, the estimates of the coefficients are less scattered. In addition, our method is applicable in other fields, providing efficient solutions to complex systems of partial differential equations. These features make our approach very attractive. One reason why polynomial approximations of this type are underutilised (in comparison to direct *ad hoc* approximation methods) by applied researchers might be lack of familiarity. Therefore, in the following section, we provide some guidance on how to use them to solve systems of differential equations.

The layout of the paper is the following. Section 2 outlines the approximation method we advocate. Section 3 describes an option pricing valuation model. In Section 4 we apply our methodology to obtain the solution to the option pricing problem, as well as to an Investment Under Uncertainty problem. Section 5 summarises the main findings of this study and offers some concluding remarks.

2. Polynomial Approximation

In this section we describe in greater detail the approximation method adopted in this paper. Let $V \in \mathfrak{R}^{n+1}$ be a function defined on the interval $[a, b]$, which may well not be tractable analytically, and assume that P is a polynomial that interpolates V at the distinct $n+1$ points $s_i \in [a, b]$, with $P(s) = \sum_{i=0}^n c_i \phi_i(s)$. In order to solve the problem by approximation we need to define: (a) the family of basis functions to approximate the function V , (b) the interpolation nodes, s_i . In this section we show that Chebyshev polynomials in conjunction with Chebyshev nodes offer the best solution to our problem.

¹ Tzavalis and Wang (2003) use a similar approach based on Chebyshev approximation to approximate the optimal exercise boundary in the context of a stochastic volatility model. Their method also relies on extrapolation procedures.

Theorem 1: if $V \in \mathfrak{R}[a,b]$, then for all $\varepsilon > 0$ there exists a polynomial $P(s)$ such that $\forall s \in [a,b] |V(s) - P(s)| \leq \varepsilon$.

Remark 1. The above theorem is known as the Weierstrass theorem. It states that any continuous function can be approximated with a certain degree of accuracy by using a polynomial. Although very important theoretically, this theorem is of little practical use since it does not give any indication of what polynomial is the most appropriate to use, or even what order polynomial is needed to achieve a certain degree of accuracy.

The error made by using a polynomial of order n to approximate the function given in Theorem 1 can be easily calculated as:

$$V(s) - P(s) = \frac{1}{n+1} V^{(n+1)}(\xi) \prod_{i=0}^n (s - s_i)$$

The objective of using such an *efficient* polynomial consists in choosing a set of nodes s_i so as to make the term $\prod_{i=0}^n (s - s_i)$ as small as possible (Judd, 1998). One possibility is to approximate the function V at the n -evenly spaced nodes. However, it is well known that in general, even for smooth functions, polynomials of this type do not produce very good approximations.² Therefore, we suggest approximating the function over the interval $[a,b]$, at the Chebyshev nodes defined as:

$$s_i = \cos\left(\frac{2i+1}{2n+2}\pi\right), i = 0, 1, \dots, n$$

Our approach can be justified by appealing to Rivlin's theorem, stating that Chebyshev node polynomial interpolants are nearly optimal polynomial approximants (Rivlin, 1990),

² A classic example is Runge's function (Rivlin, 1990).

and has been shown to perform well empirically (Rivlin, 1990). Chebyshev nodes are also known to possess a further convenient property, i.e. equi-oscillation ³(Judd, 1998).

As important as the choice of the nodes interpolants is that of a family of functions from which the approximant P will be drawn. We suggest using a Chebyshev polynomial. This is defined as⁴:

$$\Gamma_i(s) = \cos(ia \cos(s)) \quad i = 0, 1, \dots, n$$

with $\Gamma_0(s) = 1$, $\Gamma_1(s) = s$, and $\Gamma_{n+1}(s) = 2s\Gamma_n - \Gamma_{n-1}(s)$

Therefore:

$$V(s) = \sum_{i=0}^n c_i \Gamma_i(s) \quad (1)$$

$$\text{where } c_0 = \frac{1}{n+1} \sum_{i=0}^n V(s_k) \text{ and } c_i = \frac{2}{n+1} \sum_{i=0}^n V(s_k) \cos(ia \cos(s_k)), \quad i = 1, \dots, n$$

A Chebyshev basis polynomial, in conjunction with Chebyshev interpolation nodes, produces an efficient interpolation equation which is very accurate and stable over n . However, in our case, to solve the problem given by (1), the polynomial we choose should be able to replicate, not just the function V at s_1, s_2, \dots, s_n , but also its derivatives s'_1, s'_2, \dots, s'_n . Therefore the approximant that solves our problem can be defined as follows⁵:

$$\begin{aligned} \sum_{i=1}^n c_i \Gamma_i(s) &= V(s_i), \forall_i = 1, \dots, n_1 \\ \sum_{i=1}^n c_i \Gamma'_i(s') &= V'(s_i), \forall_i = 1, \dots, n_2 \end{aligned}$$

³ This property states that the maximum error of a cubic function, for example, shall be reached at least five times, and the sign of this error should alternate between the interpolation points.

⁴ Note that in this application we use the general formula for the Chebyshev basis,; however, there exists also a recursive formula.

⁵ Note that, although one can also use Hermite polynomials to approximate the functional and the slopes, the latter are inefficient (Judd, 1998).

with $n_1 + n_2 = n$.

Once the basis functions (approximants) have been chosen and the approximant nodes defined, the basis coefficients c_i can be obtained. If we define the following Chebyshev-Vandermode type matrix T :

$$T = \begin{bmatrix} \Gamma_0(s_1) & \Gamma_1(s_1) & \dots & \Gamma_{n-1}(s_1) \\ \Gamma_0(s_2) & \Gamma_1(s_2) & \dots & \Gamma_{n-1}(s_2) \\ \vdots & \vdots & \ddots & \vdots \\ \Gamma_0(s_n) & \Gamma_1(s_n) & \dots & \Gamma_{n-1}(s_n) \end{bmatrix}$$

then the coefficients c_i ; $c = (c_0, c_1, \dots, c_{n-1})'$ of $V(s)$ solve $Tc = V$, with $\Gamma_{ij} = \Gamma_i(s_j)$ being the j basis function evaluated at the i -th interpolation node. When s is allowed to vary over some other interval, say $[t, T] \neq [-1, 1]$, we rescale the value of s to s^* where

$$s^* = \frac{1}{2}((T-t)s + (T+t)) \text{ }^6.$$

As an example of using different basis functions, we anticipate some of the empirical results presented in the next section and after pricing an European option we calculate the approximation error. We use two different basis functions (i.e. Chebyshev basis and spline basis). The approximation error is shown in Figures (1-2).

Insert Figures (1-2) here

As can be seen, when the approximation is calculated using Chebyshev basis functions the error is of the order of 1×10^{-15} for a polynomial of order 20. Spline functions do not

⁶ An interesting issue here is the non-singularity of the Vandermode matrix over Chebyshev basis as above. In theory, there is no guarantee that the matrix is non-singular. However, in practice, in general applications such as ours, we can conjecture that as long as the number of indeterminates exceeds the sparsity with respect to T , non-singularity should hold. Alternatively, we suggest two ways to overcome the problem: (a) simply use the singular value decomposition of T ; (b) use the generalised Vandermode matrix over Chebyshev. In fact, for this type of matrix Werther (1993) proves that, as long as the indeterminates take a value $[1, \infty]$, the generalised Vandermode matrix over Chebyshev basis is non-singular.

achieve a comparable degree of accuracy even increasing the order of the polynomial to 30. Furthermore, Chebyshev polynomials exhibit their usual oscillation which appears fairly evenly over the interval we have considered. On the other hand, spline polynomials exhibit larger oscillations at the edges of the interval.

3. The Valuation Model

This section outlines an option pricing problem to which our method will be applied. Suppose that the price of a non-dividend-paying asset in period 0 is S_0 , and denote with K the strike price of a put option written on that asset.

Assumption 1: We assume that the option value depends on the stock price at expiry of the option and time, $V_t(S_t, t)$.

Suppose also that the process for S is described by the following geometric Brownian motion:

$$dS_t = S_t r dt + S_t \sigma dZ_t \quad (2)$$

where dZ is a standard increment of a Wiener process, and σ the variance parameter.

We can expand $E(dV) \frac{1}{dt}$, using Ito's Lemma and the stochastic process above to obtain:

$$rV = rSV_s + V_t + \frac{1}{2} S^2 \sigma^2 V_{ss} \quad (3)$$

where $V_{(.)}$ represents the derivative with respect to the argument in the subscript.

All European options, in absence of arbitrage, must satisfy Equation (3). A call option will have at expiry a payoff given by $S - K$, if $S > K$, while for a put option at expiry the payoff is $K - S$, if $K > S$. Therefore, in our specific case, the boundary condition is given by

$$V(S, t) = \max(0, K - S) \quad (4)$$

If we set the value function above $V(s, t) \approx \phi(s)c(t)$, where ϕ is a suitable basis for an n -dimensional family of approximating functions and $c(t)$ is an n -vector of time-varying coefficients, equation (3) can be re-written as follows:

$$\phi(s)c'(t) \approx [rs\phi'(s) + \frac{1}{2}\sigma^2 s^2 \phi''(s) - r\phi(s)]c(t) \approx \psi(s)c(t) \quad (5)$$

To determine $c(t)$, we select n -values (nodes) of s , s_i , and solve (4) for that particular set of values. Given the n -dimensional family of basis functions chosen, (5) can now be re-written in the form of a system as follows:

$$\Phi c'(t) = \Psi c(t) \quad (6)$$

where Φ and Ψ are two $n \times n$ matrices.

Once the coefficients have been obtained as in (6), to price the financial option, first we use the process in (2) to obtain estimates of (4). Then, we multiply it by the estimated coefficients. Averaging gives the price of the option.

4. Two Empirical Examples: Option Pricing and Investment Under Uncertainty

In order to show how to use in practice the proposed method we now provide two examples. The first is the valuation problem described in section 2. We use the proposed methodology to value an European put option written on a stock. In this case, we can compare our empirical results with the Black and Scholes closed-form solution. We use the absolute error (ASE) as a measure of accuracy.

Table 1 shows the results for the entire set of options considered. We also report the results using the Black and Scholes method (1973 - B&S henceforth).

Insert Table 1 here

We fit (4) using the first twenty Chebyshev basis to estimate the parameters c_i in (4)⁷. The basis number has been chosen using Theorem 6.4.2 in Judd (1998). Once the coefficients have been estimated, we estimate the boundary condition in (3) by simulating 200,000 paths for the stock. We can see that our method produces rather accurate option prices. This is confirmed by the absolute error reported at the bottom of Table 1.

The second empirical example we consider is taken from the theory of Investment Under Uncertainty. The problem is to decide when it is optimal to shutdown a machine, assuming that there is no maintenance cost for maintaining it alive. Suppose that $\pi(t)$ is the profit generated by the machine at time t , and suppose that it follows the Brownian motion process below

$$d\pi(t) = a dt + \sigma dz, \quad \pi(0) = \pi_0 \quad (7)$$

where a is the rate of depreciation of the investment, σ is the volatility of profit, and dz is an increment of a Wiener process.

We report three paths of the process in Figure 3. We consider the parameters reported at the bottom of Table 3, and the time $T = 10$ has been divided in one-hundred time steps. It appears that the investment will, in general, produce a loss before 3 years. However, the decision to shutdown the machine cannot be only taken by looking at these dynamic paths. In fact, this is a more complex problem since once the machine has been shutdown it cannot be re-started again, that is, the investment is irreversible. Therefore, one has to consider the optimal policy to decide when it is convenient to shut it down.

Following Dixit and Pindyck (1994), suppose that $F(\pi, t)$ is a claim on the profit flow, π , and suppose it is determined as

$$F(\pi, t) = \max_{\tau \in \Gamma} E \int_0^T e^{-rt} \pi(t) dt \quad (8)$$

where r is the risk free rate of interest, T is the time and $\tau \in \Gamma$ is a random stopping time.

Suppose that the machine can be shut down only up to time \bar{T} . After that time it has to run forever. We assume that if it is shut down it cannot be re-installed, which highlights the irreversibility of the investment.

The Bellman Equation for the optimal stopping problem can be written as

$$F(\pi, t) = \max(0, \pi dt + (1 - rdt)^{-1} E[F(\pi + d\pi, t + dt)]) \quad (9)$$

Therefore, if the machine is shut down the profit will be zero. If it is kept alive, then profit is given by the conditional expectation in (9). One can show that, for this case, the value function F , in the continuation region, satisfies the following Bellman equation

$$rF = \pi + aF_{\pi} + 1/2\sigma^2 F_{\pi\pi} \quad (10)$$

where F_{π} is the derivative with respect to the sub-script.

Under the assumption that, if not abandoned by \bar{T} , the machine has to run forever, the terminal condition can be written as $F(\pi, \bar{T}) = \max(0, a/r^2 + r/\pi)$.

⁷ Note that we estimate these coefficients using Chebyshev nodes and Chebyshev basis.

We can still use the same approach as in Section (2) to solve this optimal stopping problem. However, now solving (10) at each stopping times require more effort. One possibility would be to use finite difference methods. However, this approach turns out to be very time consuming. On the other hand, using Richardson extrapolation methods would increase the speed but at a cost of, sometimes, poor convergence. The method described in Section (2) can be adapted to this specific case. In fact, it is very similar in spirit to the Least-Squares algorithm introduced in Longstaff and Schwartz (2001). The coefficient c in Section 2 are still estimated using Chebyshev polynomials and the same approach as before but now the interpolant nodes are projected using (7)⁸. We assume that there is a threshold at \$0.87 and once it has been reached one has to decide if it is convenient to shut down the machine. Using 200 time steps, four Chebyshev basis functions and one hundred thousands replications, we estimate that it is optimal to shut down the machine soon after its second year of life. This empirical result is in line with the three paths shown in Figure (3).

5. Conclusions

In this study we suggest a simple method to approximate partial differential equations based on a Chebyshev approximation at Chebyshev nodes. We provide two empirical examples, in both cases obtaining highly satisfactory results. The first example consisted in pricing an European put option, the second in solving an optimal stopping problem. In the latter case stochastic rather than determinist nodes were used to approximate the functional. Our method is simple to apply and to extend and provides a reliable framework which can be used either for pricing more complex derivative instruments or in many interesting cases in economics and in other fields. Our future research will carry out further applications to provide additional evidence of the advantages of our approach.

⁸

We have also tried to estimates the parameters using Chebyshev nodes but in this specifi case we obtained a poor fit. Therefore we suggest, when dealing with optimal stopping problems, not to use Chebyshev nodes.

References

Abadir, K., and M., Rockinger, 2003, “Density Functionals, With An Option-Pricing Application”, *Econometric Theory*, 4.

Black, F. and M. Scholes, 1973, “The Pricing of Options and Corporate Liabilities”, *Journal of Political Economy*, 81, 637-654.

Chang, C., C.S. Chung and R.C. Stapleton, 2002, “Richardson Extrapolation Techniques for the Pricing of American-Style Options”, Department of Finance, The Management School, National Central University, Taiwan, Working Paper No. 3.

Dixit, A., 1992, “Investment and Hysteresis”, *Journal of Economic Perspectives*, 6, 107-132.

Dixit, A., and S. Pindyck, 1994, “Investment Under Uncertainty”, Princeton University Press.

Judd, K., 1998, “Numerical Methods in Economics”, Massachusetts Institute of Technology Press.

Longstaff, F.A. and E.S. Schwartz, 2001, “Valuing American Options by Simulation: A Simple Least-Squares Approach”, *Review of Financial Studies*, 14, 1, 113-147.

Pindyck, R.S., 1991, “Irreversibility and the Explanation of the Investment Behaviour”, in D. Lund and B., Oksendal (eds.), *Stochastic Models and Option Values*, Amsterdam, North Holland.

Rivlin, Theodore J., 1990, “Chebyshev Polynomials: From Approximation Theory to Algebra and Number Theory”, 2nd edition, New York: John Wiley and Sons.

Tzavalis, E., and S. Wang, 2003, “Pricing American Options Under Stochastic Volatility: A New Method Using Chebyshev Polynomials to Approximate the Early Exercise Boundary”, Working Paper No. 488, Queen Mary, University of London, Department of Economics.

Werther, T., 1993, “Generalized Vandermode Determinants over the Chebyshev Basis”, International Computer Science Institute, 1947, Center Street, Berkeley, California.

Tables and Figures

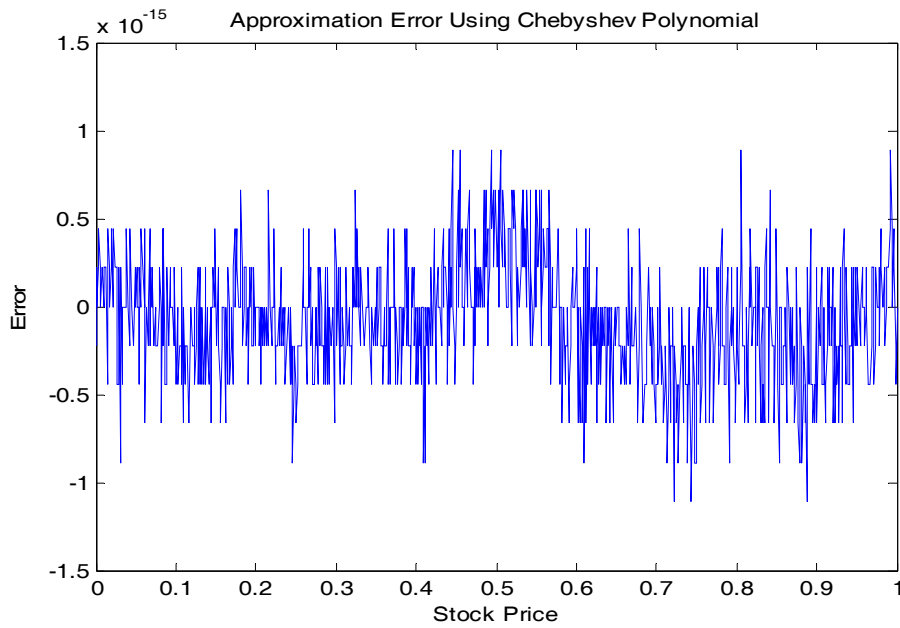


Figure 1

Approximation error using (8) and Chebyshev polynomial when volatility is equal to 0.2 and the interest rate is 0.048.

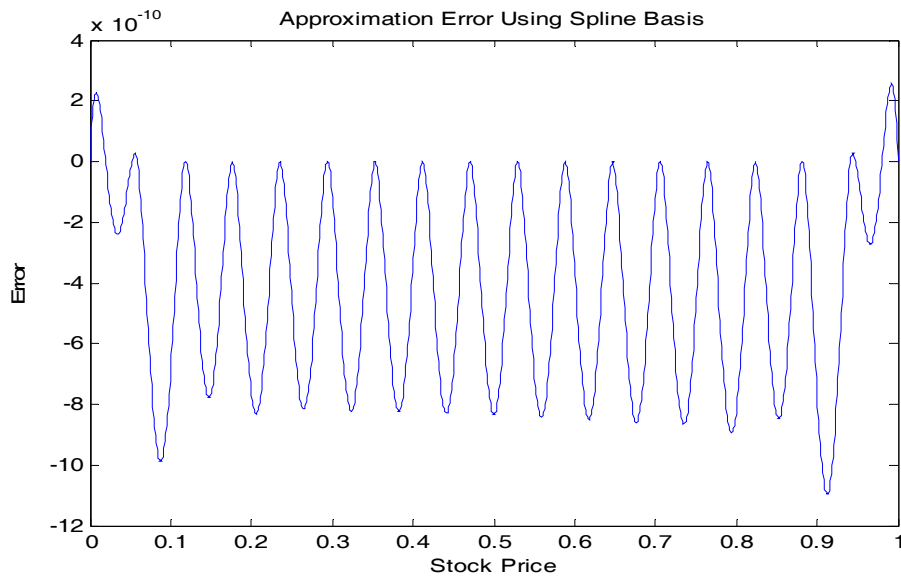


Figure 2

Approximation error using (8) and splines basis when volatility is equal to 0.2 and the interest rate is 0.048

K	Sigma	t (yrs)	B&S	CC (2008)
40	0.2	0.0833	0.8403	0.838
40	0.2	0.3333	1.5221	1.472
40	0.2	0.5833	1.8812	1.802
45	0.2	0.0833	4.8399	4.837
45	0.2	0.3333	4.7804	4.771
45	0.2	0.5833	4.84	4.813
40	0.3	0.0833	1.2988	1.297
40	0.3	0.3333	2.4275	2.426
40	0.3	0.5833	3.0634	3.062
45	0.3	0.0833	4.9796	4.976
45	0.3	0.3333	5.5288	5.517
45	0.3	0.5833	5.9723	5.946
40	0.4	0.0833	1.7575	1.758
40	0.4	0.3333	3.3336	3.3335
40	0.4	0.5833	4.2473	4.246
45	0.4	0.0833	5.236	5.235
45	0.4	0.3333	6.3767	6.359
45	0.4	0.5833	7.1654	7.131
ABE				0.0151

Table 1

Column 4 shows the results using the Black and Scholes (1973) method.

Column 5 shows the results using our suggested method (Caporale and Cerrato – CC).

RMSE at the bottom of the table is the root mean square error.

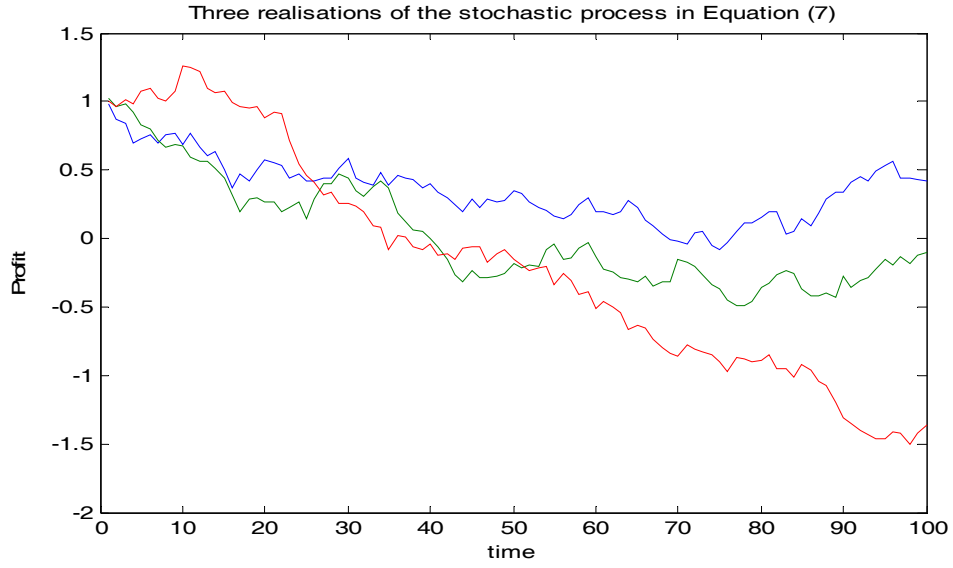


Figure 3

The stochastic process in Figure 3 has been simulated using $a = -0.1$, $\sigma = 0.2$, and $\pi_0 = 1$.

CESifo Working Paper Series

for full list see www.cesifo-group.org/wp

(address: Poschingerstr. 5, 81679 Munich, Germany, office@cesifo.de)

- 2245 Anne van Aaken, Lars P. Feld and Stefan Voigt, Power over Prosecutors Corrupts Politicians: Cross Country Evidence Using a New Indicator, March 2008
- 2246 Hans-Christian Heinemeyer, Max-Stephan Schulze and Nikolaus Wolf, Endogenous Borders? The Effects of New Borders on Trade in Central Europe 1885-1933, March 2008
- 2247 Johannes Becker and Clemens Fuest, Tax Competition – Greenfield Investment versus Mergers and Acquisitions, March 2008
- 2248 Giorgio Bellettini and Hubert Kempf, Why not in your Backyard? On the Location and Size of a Public Facility, March 2008
- 2249 Jose Luis Evia, Roberto Laserna and Stergios Skaperdas, Socio-Political Conflict and Economic Performance in Bolivia, March 2008
- 2250 Bas Jacobs and A. Lans Bovenberg, Optimal Taxation of Human Capital and the Earnings Function, March 2008
- 2251 Jan-Egbert Sturm and Timo Wollmershäuser, The Stress of Having a Single Monetary Policy in Europe, March 2008
- 2252 Guido Schwerdt, Labor Turnover before Plant Closure: ‘Leaving the Sinking Ship’ vs. ‘Captain Throwing Ballast Overboard’, March 2008
- 2253 Keith E. Maskus and Shuichiro Nishioka, Development-Related Biases in Factor Productivities and the HOV Model of Trade, March 2008
- 2254 Jeremy Edwards and Sheilagh Ogilvie, Contract Enforcement, Institutions and Social Capital: the Maghribi Traders Reappraised, March 2008
- 2255 Imed Drine and Christophe Rault, Purchasing Power Parity for Developing and Developed Countries. What can we Learn from Non-Stationary Panel Data Models?, March 2008
- 2256 Scott Alan Carson, Health, Wealth and Inequality: a Contribution to the Debate about the Relationship between Inequality and Health, March 2008
- 2257 C.A.E. Goodhart, The Regulatory Response to the Financial Crisis, March 2008
- 2258 Stefan Bauernschuster, Oliver Falck and Stephan Heblich, The Impact of Continuous Training on a Firm’s Innovations, March 2008
- 2259 Michael Grimm and Stephan Klasen, Geography vs. Institutions at the Village Level, March 2008

- 2260 Fwu-Ranq Chang, Property Insurance, Portfolio Selection and their Interdependence, March 2008
- 2261 J. Atsu Amegashie and Marco Runkel, The Paradoxes of Revenge in Conflicts, March 2008
- 2262 Hans Jarle Kind, Marko Koethenbuerger and Guttorm Schjelderup, Efficiency Enhancing Taxation in Two-sided Markets, March 2008
- 2263 M. Hashem Pesaran, Til Schuermann and L. Vanessa Smith, Forecasting Economic and Financial Variables with Global VARs, March 2008
- 2264 Volker Grossmann, Entrepreneurial Innovation and Sustained Long-run Growth without Weak or Strong Scale Effects, March 2008
- 2265 Robert S. Chirinko and Huntley Schaller, The Irreversibility Premium, March 2008
- 2266 Andrea Galeotti and José Luis Moraga-González, Platform Intermediation in a Market for Differentiated Products, April 2008
- 2267 Torben M. Andersen and Michael Svarer, The Role of Workfare in Striking a Balance between Incentives and Insurance in the Labour Market, April 2008
- 2268 Harald Badinger, Cyclical Fiscal Policy, Output Volatility, and Economic Growth, April 2008
- 2269 Thomas Aronsson and Erkki Koskela, Outsourcing and Optimal Nonlinear Taxation: A Note, April 2008
- 2270 Gary E. Bolton, Claudia Loebbecke and Axel Ockenfels, How Social Reputation Networks Interact with Competition in Anonymous Online Trading: An Experimental Study, April 2008
- 2271 Nikolaus Wolf, Scylla and Charybdis. Explaining Europe's Exit from Gold, January 1928 – December 1936, April 2008
- 2272 Michael Funke and Marc Gronwald, The Undisclosed Renminbi Basket: Are the Markets Telling us something about where the Renminbi – US Dollar Exchange Rate is Going?, April 2008
- 2273 Thor Olav Thoresen and Annette Alstadsæter, Shifts in Organizational Form under a Dual Income Tax System, April 2008
- 2274 Helge Berger and Volker Nitsch, Too many Cooks? Committees in Monetary Policy, April 2008
- 2275 Yin-Wong Cheung and Eiji Fujii, Deviations from the Law of One Price in Japan, April 2008

- 2276 Michael S. Michael, Sajal Lahiri and Panos Hatzipanayotou, Integrated Reforms of Indirect Taxes in the Presence of Pollution, April 2008
- 2277 Bas Jacobs, Is Prescott Right? Welfare State Policies and the Incentives to Work, Learn and Retire, April 2008
- 2278 Burkhard Heer and Alfred Maußner, Value Function Iteration as a Solution Method for the Ramsey Model, April 2008
- 2279 Jarko Fidrmuc and Christa Hainz, Integrating with their Feet: Cross-Border Lending at the German-Austrian Border, April 2008
- 2280 Kristof Dascher and Alexander Haupt, The Political Economy of Regional Integration Projects at Borders where Rich and Poor Meet: The Role of Cross-Border Shopping and Community Sorting, April 2008
- 2281 Katrin Assenmacher-Wesche and M. Hashem Pesaran, A VECX* Model of the Swiss Economy, April 2008
- 2282 Christophe Rault, Robert Sova and Ana Maria Sova, Modeling International Trade Flows between CEEC and OECD Countries, April 2008
- 2283 Timo Boppart, Josef Falkinger, Volker Grossmann, Ulrich Woitek and Gabriela Wüthrich, Qualifying Religion: The Role of Plural Identities for Educational Production, April 2008
- 2284 Armin Falk, David Huffman and W. Bentley MacLeod, Institutions and Contract Enforcement, April 2008
- 2285 Axel Dreher and Stefan Voigt, Does Membership in International Organizations Increase Governments' Credibility? Testing the Effects of Delegating Powers, April 2008
- 2286 Xavier Freixas and Bruno M. Parigi, Lender of Last Resort and Bank Closure Policy, April 2008
- 2287 Regina Dionisius, Samuel Muehlemann, Harald Pfeifer, Günter Walden, Felix Wenzelmann and Stefan C. Wolter, Cost and Benefit of Apprenticeship Training – A Comparison of Germany and Switzerland, April 2008
- 2288 Francesco Daveri and Cecilia Jona-Lasinio, Off-Shoring and Productivity Growth in the Italian Manufacturing Industries, April 2008
- 2289 Mikael Priks, Do Surveillance Cameras Affect Unruly Behavior? A Close Look at Grandstands, April 2008
- 2290 Marianna Belloc and Daniela Federici, A Two-Country NATREX Model for the Euro/Dollar, April 2008
- 2291 Nicolas Treich, The Value of a Statistical Life under Ambiguity Aversion, April 2008

- 2292 J. Atsu Amegashie, Socially-Tolerable Discrimination, April 2008
- 2293 M. Hashem Pesaran and Andreas Pick, Forecasting Random Walks Under Drift Instability, April 2008
- 2294 Steven Brakman, Gus Garita, Harry Garretsen and Charles van Marrewijk, Unlocking the Value of Cross-Border Mergers and Acquisitions, May 2008
- 2295 Eric O’N. Fisher and Kathryn G. Marshall, The Structure of the American Economy, May 2008
- 2296 Claudia M. Buch and Martin Schlotter, Regional Origins of Employment Volatility: Evidence from German States, May 2008
- 2297 Helmuth Cremer, Philippe De Donder, Dario Maldonado and Pierre Pestieau, Taxing Sin Goods and Subsidizing Health Care, May 2008
- 2298 Reinhilde Veugelers and Frederick van der Ploeg, Reforming European Universities: Scope for an Evidence-Based Process, May 2008
- 2299 Jon H. Fiva and Lars J. Kirkebøen, Does the Housing Market React to New Information on School Quality?, May 2008
- 2300 Tina Klautke and Alfons J. Weichenrieder, Interest Income Tax Evasion, the EU Savings Directive, and Capital Market Effects, May 2008
- 2301 Harald Badinger and Peter Egger, GM Estimation of Higher Order Spatial Autoregressive Processes in Panel Data Error Component Models, May 2008
- 2302 Jan K. Brueckner, Slot-Based Approaches to Airport Congestion Management, May 2008
- 2303 Sören Blomquist, Vidar Christiansen and Luca Micheletto, Public Provision of Private Goods and Nondistortionary Marginal Tax Rates, May 2008
- 2304 Dan Anderberg and Alessandro Balestrino, The Political Economy of Post-Compulsory Education Policy with Endogenous Credit Constraints, May 2008
- 2305 Tomer Blumkin, Yoram Margalioth and Efraim Sadka, The Role of Stigma in the Design of Welfare Programs, May 2008
- 2306 Vesa Kannianen and Paolo M. Panteghini, Tax Neutrality: Illusion or Reality? The Case of Entrepreneurship, May 2008
- 2307 Thomas Dohmen, Armin Falk, David Huffman and Uwe Sunde, The Intergenerational Transmission of Risk and Trust Attitudes, May 2008
- 2308 Guglielmo Maria Caporale and Mario Cerrato, Using Chebyshev Polynomials to Approximate Partial Differential Equations, May 2008