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## Taxation in two-sided markets

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# TAXATION IN TWO-SIDED MARKETS

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# TAXATION IN TWO-SIDED MARKETS

## Abstract

Two-sided platform firms serve distinct customer groups that are connected through interdependent demand, and include major businesses such as the media industry, banking, and the software industry. A well known textbook result in one-sided markets is that a government may increase a monopolist's output and reduce the deadweight loss by subsidizing output. The present paper shows that this result need not hold in a two-sided market. On the contrary, a higher ad-valorem tax rate - rather than a subsidy - could increase output and enhance welfare.

JEL Code: D4, D43, H21, H22, L13.

Keywords: two-sided markets, ad-valorem taxes, specific taxes, imperfect competition, industrial organization.

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# 1 Introduction

A benchmark result in economics is that a higher ad valorem tax rate generally reduces output.<sup>1</sup> In this paper we show a new result. Consumers may actually buy more of a good sold by a two-sided platform firm if the tax rate increases. In particular, a higher ad valorem tax may lower the end-user price, increase sales, and improve welfare.

Two-sided platform firms cater to two distinct groups of customers that are connected through quantity spillovers, and the firms maximize profit by facilitating value-creating interactions between these groups.<sup>2</sup> Two-sided platforms operate in many economically significant industries, such as the media sector, the financial sector (payment card systems), real-estate brokerage, and the computing industry (computer operating systems, software, game consoles etc.). The pricing strategies of a platform firm must account for interactions between the demands of different customer groups and the externalities that arise in these relationships. For instance, in the media industry, advertising may be perceived as a nuisance (a negative externality) or a benefit (a positive externality) by readers/viewers, while advertisers benefit from an increase in readers/viewers of the media outlet. In the credit card industry there are positive quantity spillovers between merchants and cardholders. Merchants who accept a credit card welcome an increase in the number of households joining the credit card system, and vice versa.<sup>3</sup>

We show that the sign, size and direction of externalities in two-sided markets are decisive for the effects of changes in ad valorem tax rates. In two-sided markets, an increase in the ad valorem tax in one side of the market affects the relative profitability between the two markets, such that the firm will want to shift its earnings to the market where the tax rate is unchanged. By doing so it reduces the burden of the tax increase. Contrary to what one

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<sup>1</sup>An overview of the tax incidence literature is given by Fullerton and Metcalf (2002).

<sup>2</sup>Evans (2003a,b) provides examples and classifications of two-sided markets.

<sup>3</sup>As will become clear in the discussion below, it is important to distinguish the concept of two-sided markets from that of complementarities. See also Rochet and Tirole (2003).

might expect, this may involve increasing output on both sides of the market.

The behavior of the platform firm in response to a tax increase in one side of the market can be illustrated by a media firm. A media firm is a two-sided platform that derives income from selling a newspaper and advertisements, and where the income from advertisements depends positively on newspaper sales. An increase in the ad valorem tax rate on the newspaper may induce the media firm to rely more on income from advertisements. Thus, it may reduce the price of the newspaper in order to attract more readers. A larger readership means that the newspaper becomes more attractive for the advertisers, and the media firm may therefore end up selling more of both ads and newspapers following a tax increase. We show that this is particularly likely to be true if newspaper readers consider ads as a nuisance (rather than as a complement which increases the intrinsic value of the media product). A very high tax on newspapers could even lead a media platform to provide the newspaper free of charge and rely on income from advertising only.

Our analysis of taxation has implications for the understanding of tax incidence in two-sided markets. We identify situations in which the end-user prices charged by the platform drop when taxes rise. In such cases the tax burden is fully borne by the platform, even though the demand for the platform's output is not perfectly elastic. This is in contrast to a one-sided market, where an elastic demand implies at least a partial shifting of the tax burden. A further result relates to the welfare effects of taxation in two-sided markets. In one-sided markets, the existence of market power may imply that output is too low from a social point of view. This calls for a subsidy on costs, or a reduction in the VAT rate in order to entice a monopoly firm to produce more (see e.g. Delipalla and Keen, 1992). In contrast, a welfare-enhancing policy in a two-sided market may be to increase the ad valorem tax rate on one side of the market.

Many two-sided platform firms operate in markets that traditionally have received preferential tax treatment, often through a reduced-rate regime. Newspapers, for example, are taxed at a reduced rate or completely exempted

from value-added taxation in most countries, since governments consider such publications to be an essential channel for disseminating vital information about e.g. culture, politics, and international affairs.<sup>4</sup> The preferential tax treatment indeed increases newspaper circulation in one-sided markets. The analysis shows that the logic of one-sided markets does not necessarily extend to the newspaper industry and other industries that operate in two-sided markets. As a matter of fact, a lower VAT rate may reduce output in such industries.

Our analysis is related to a growing literature on Industrial Organization that analyzes the price-setting behavior of firms in two-sided markets. In this literature a key result is that two-sided platform firms may find it profitable to charge prices that are below marginal cost or even negative for one product (customer group).<sup>5</sup> Furthermore, an increase in marginal costs on one side of the market does not necessarily imply a higher price on that side of the market relative to the price on the other side. This is in contrast to conventional markets (one-sided) where marginal cost equal to marginal revenue pricing is well established as a guidance. In such markets the effects of taxation are well known both under perfect and imperfect competition. Under imperfect competition a tax can be overshifted onto the consumer side in certain circumstances, but in general (i) the burden of the tax is shared between producers and consumers depending on elasticities of supply and demand and (ii) taxation causes an excess burden on the economy and impairs welfare.<sup>6</sup>

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<sup>4</sup>In Germany, for instance, newspapers are subject to a rate of 7% (16% is the regular rate) while in e.g. the UK and Denmark they are exempted from value-added taxation all together (European Commission, 2004). Newspapers are also either fully or partially exempted from sales taxes in a number of U.S. states. Other examples can be found in the financial sector and in the computer industry and Internet sales business.

<sup>5</sup>See for instance Caillaud and Jullien (2003), Rochet and Tirole (2003, 2004), Anderson and Coate (2005), Armstrong (2006) and Crampes, Haritchabalet and Jullien (2005)

<sup>6</sup>See Keen and Delipalla (1992), Dierickx, Matutes and Neven (1998) and Anderson et. al. (2001a,b), and Fullerton and Metcalf (2002) for a survey.

The literature on two-sided platforms does not consider taxation issues, whilst the literature on indirect taxation, on the other hand, does not consider the effects two-sidedness may have on tax incidence and welfare. The present paper tries to bridge this gap.

The rest of the paper is organized as follows: Section 2 sets up the basic model, while Section 3 analyzes the effects of an ad valorem tax on prices and quantities. Section 4 carries out an analysis with respect to specific taxes, and section 5 discusses welfare consequences of ad valorem taxation. Section 6 illustrates the results by means of a numerical example and section 7 concludes.

## 2 The Model

Consider a two-sided monopoly platform which sells good  $N$  at price  $p^N$  to one group of customers and good  $A$  at price  $p^A$  to another group of customers. Let  $n$  and  $a$  denote the respective quantities of the two goods. For the sake of convenience, and to emphasize the economic intuition and policy relevance of our results, we shall in what follows relate our model and results to a media firm (the platform). A media firm is a typical example of a two-sided platform firm, which derives income from two distinct customer groups (newspaper readers and advertisers), and where there are externalities (possibly positive from readers to advertisers, and negative from advertisers to readers) between the two groups. In such a setting we may interpret  $n$  as sales of newspapers, and  $a$  as sales of advertising space to firms. However, we would like to emphasize that the model is general in nature, and not restricted to the media industry.<sup>7</sup>

We assume that both customer groups are price takers. The inverse demand function for each good is downward-sloping in own quantity;  $p_n^N \equiv$

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<sup>7</sup>As a matter of fact the media industry is one of the two-sided industries where monopoly issues have been brought up in anti-trust cases (see Evans and Schmalensee, 2005)

$\partial p^N / \partial n < 0$ ;  $p_a^A \equiv \partial p^A / \partial a < 0$  (subscripts henceforth denote partial derivatives). The willingness to pay for each good, however, may also depend on how much is sold of the other good. The sale of good  $A$  imposes a positive externality on buyers of good  $N$  if the willingness to pay for  $N$  is increasing in output of good  $A$  ( $p_a^N > 0$ ) and a negative externality if  $p_a^N < 0$ .<sup>8</sup> In the same manner, good  $N$  may impose a positive ( $p_n^A > 0$ ) or negative ( $p_n^A < 0$ ) externality on the demand for good  $A$ . The inverse demand functions can thus be written as  $p^N = p^N(n, a)$  and  $p^A = p^A(n, a)$ . We resort to a partial equilibrium analysis by abstracting from other determinants of demand.

An ad valorem tax ( $t$ ) is levied on good  $N$ , which implies that the platform receives the price  $p^N / (1 + t)$  from this group of customers. The tax rate  $t$  may deviate from the general VAT rate  $\bar{t}$  which for simplicity is set to 0. Our focal point here is to examine the effects of a change in the tax rate  $t$ , holding  $\bar{t}$  fixed.

The platform has the following profit level:

$$\pi = \max_{n,a} \left[ ap^A(a, n) + \frac{np^N(n, a)}{1 + t} - k(n, a) \right], \quad (1)$$

where  $k(n, a)$  is the cost function, with  $k_i \geq 0$  ( $i = a, n$ ) and  $k_{na} \gtrless 0$ .

The first-order condition for good  $A$  ( $\pi_a = 0$ ) implies

$$[p^A + ap_a^A] - k_a = -\frac{np_a^N}{1 + t}. \quad (2)$$

The squared bracket in equation (2) measures marginal revenue on the advertising side of the market of selling more ads. In the profit maximizing optimum in a one-sided market this term is equal to marginal cost ( $k_a$ ) so that the left hand side would be zero. However, in a two-sided market there is an additional term (right hand side) that captures the fact that the sales of advertising (good  $A$ ) may influence the sales of newspapers (good  $N$ ). This term is positive if the demand for newspapers is decreasing in the level of

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<sup>8</sup>This is an externality since producers and consumers are price takers. Thus, they do not take into account the effect of their actions on the demand in either side of the market.

advertising (that is,  $p_a^N < 0$ ), while it is negative if advertising imposes a positive externality on demand for newspapers. In the former case, the level of advertising should be set lower than the level that maximizes profit in the advertising market in isolation (i.e., in a one-sided market), while the opposite is true if a larger advertising volume increases the demand for newspapers. Thus, it is clear from (2) that profit maximizing prices (and quantities) may be below the marginal cost of supplying good  $A$ .

From the first-order condition for good  $N$  ( $\pi_n = 0$ ), we likewise find that

$$\left[ \frac{p^N + np_n^N}{1+t} \right] - k_n = -ap_n^A. \quad (3)$$

The squared bracket is marginal revenue from selling the newspaper (good  $N$ ) to consumers, and would be equal to  $k_n$  in a one-sided market (i.e., when  $p_n^A = 0$ ). However, if demand for ads is higher the larger the number of readers ( $p_n^A > 0$ ), profit is maximized by raising the sale of newspapers beyond the volume that maximizes profit of good  $A$  in isolation (and vice versa for  $p_n^A < 0$ ).

From the first-order conditions we see that equilibrium prices and quantities on both sides of the market depend on the tax rate. Since  $p^A = p^A(a, n)$  and  $p^N = p^N(n, a)$ , the price changes subsequent to a tax increase are given by

$$\frac{dp^A}{dt} = p_a^A \frac{da}{dt} + p_n^A \frac{dn}{dt}, \quad \text{and} \quad \frac{dp^N}{dt} = p_n^N \frac{dn}{dt} + p_a^N \frac{da}{dt}. \quad (4)$$

The second-order conditions for profit maximum require that  $\pi_{aa} < 0$ ,  $\pi_{nn} < 0$ , and  $H \equiv \pi_{aa}\pi_{nn} - \pi_{an}^2 > 0$ .

In order to have a two-sided market, there must be positive externalities from at least one side of the market to the other.<sup>9</sup> The implication is that  $p_n^A > 0$  and/or  $p_a^N > 0$ , but whether both terms are positive depends on the particularities of the industry in question. Related to our media example, we

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<sup>9</sup>Evans (2003b) defines a two-sided market as one where we have (a) two distinct groups of customers, (b) positive network externalities (at least from one of the customer groups to the other), and (c) an intermediary that internalizes the externalities between the groups. See Rochet and Tirole (2004) for a more formal definition.

cannot predetermine the sign of  $p_a^N$ , since empirical evidence does not give a clear answer as to whether consumers consider advertising to be a good or a bad.<sup>10</sup> However, it seems reasonable to assume that the willingness to pay for advertising is increasing in the number of readers. We shall therefore assume that

**Assumption 1:** *The willingness to pay for good A ( $p^A$ ) is increasing in sales of good N, that is:  $p_n^A > 0$ .*

It should be emphasized that the model is applicable to two-sided markets in general and that our mathematical derivations and results also hold for  $p_n^A \leq 0$  (in which case two-sidedness requires  $p_a^N > 0$ ).

For the analysis to follow, the sign of  $\pi_{an}$  is of particular relevance. Differentiating equation (2) or (3) we find

$$\pi_{an} = p_n^A [1 + \varepsilon_{p^A}] + p_a^N [1 + \varepsilon_{p^N}] (1 + t)^{-1} - k_{an}, \quad (5)$$

where  $\varepsilon_{p^A} \equiv \frac{a}{p_n^A} \frac{\partial p_n^A}{\partial a}$  and  $\varepsilon_{p^N} \equiv \frac{n}{p_a^N} \frac{\partial p_a^N}{\partial n}$ .

The cross derivative  $\pi_{an}$  measures how the marginal profitability of selling advertising space,  $\pi_a$ , changes if the number of readers increases. One might think that  $\pi_{an}$  is positive, given the assumption that the willingness to pay for advertising is increasing in the number of readers, that is,  $p_n^A > 0$ . However, this is not necessarily true. To see why, note that  $\partial p_n^A / \partial a < 0$  if the marginal value of a larger readership for the advertisers is decreasing in the advertising volume. Thus, the first term in (5) may be negative; this is the case when the elasticity of  $p_n^A$  with respect to  $a$  is smaller than minus one ( $\varepsilon_{p^A} < -1$ ). The interpretation of the second term in (5) is similar; this term is negative if consumers are ad-lovers ( $p_a^N > 0$ ) and  $\varepsilon_{p^N} < -1$ , or if consumers dislike ads ( $p_a^N < 0$ ) and  $\varepsilon_{p^N} > -1$ . Summing up, it is thus clear that the sign of

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<sup>10</sup>Readers in European countries seem to be averse to advertising ( see Ferguson 1983, p. 637; Blair and Romano 1993, and Sonnac 2000) For retail advertising there is some evidence showing that American readers like advertising.

$\pi_{an}$  is ambiguous.<sup>11</sup> In order to simplify the discussion in the main text, we nonetheless assume that :

**Assumption 2:** *The marginal profitability of selling good A ( $\pi_a$ ) is increasing in the output of good N, that is:  $\pi_{an} > 0$ .*

In the Appendix we discuss how to interpret our results if  $\pi_{an} < 0$ .

### 3 Profit-maximizing platform responses to a tax increase

It is evident from our discussion above that the effect of a change in the ad valorem tax depends on assumptions linked to the externalities between the two customer groups. We would like to emphasize that our analysis should not be confused with the standard theory of complements. Complements are used to describe a situation where an increase in the price of one good causes a decline in consumption of both goods, measured by the change in the compensated demand by a single consumer (see e.g., Kreps 1990, p. 61). This is different from a two-sided market, where there are two distinct groups of customers that may respond differently to changes in prices (see Rochet and Tirole (2003) for a general discussion). Also, the main results of our analysis do not hinge on the goods being complementary in demand by the two groups of customers. In order to see this as simply as possible, we start out by considering a situation where buyers of good  $N$  are *indifferent* about the output of good  $A$ .

#### 3.1 Zero externalities from good A ( $p_a^N = 0$ )

For the sake of intuitive convenience we continue to relate our analysis to a media platform that is partly financed by advertising revenue. If readers

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<sup>11</sup>Note also that with a sufficiently high value of  $k_{an}$ ,  $\pi_{an}$  may be negative even if the first two terms in (5) are positive.

are indifferent to the advertising level, there is no externality from good  $A$  to good  $N$ . Therefore the quantity sold of good  $A$  does not affect the willingness to pay for good  $N$ . In this case we have that  $p_a^N = 0$ . The effect of a higher value-added tax can be found by using (4) and totally differentiating first order conditions (2) and (3). We then obtain<sup>12</sup>

$$\left. \frac{dp^N}{dt} \right|_{p_a^N=0} = p_n^N \left. \frac{dn}{dt} \right|_{p_a^N=0}; \quad \left. \frac{dn}{dt} \right|_{p_a^N=0} = \frac{-\pi_{aa} (ap_n^A - k_n)}{H(1+t)} \quad (6)$$

and

$$\left. \frac{da}{dt} \right|_{p_a^N=0} = \frac{\pi_{an} (ap_n^A - k_n)}{H(1+t)} \quad (7)$$

Equations (6) and (7) show that we may get the seemingly paradoxical result that a higher VAT on newspapers reduces the end-user price of that good and increases sales on both sides of the market. This happens if  $(ap_n^A - k_n) > 0$ . To see why, recall that the willingness to pay for advertising increases by  $p_n^A$  units if the newspaper attracts one more reader. With a total advertising volume equal to  $a$ , the value for the newspaper of attracting one extra reader equals  $ap_n^A$ . If the size of this indirect network effect is greater than the marginal cost  $k_n$  of serving one extra reader, it is profitable for the media firm to charge a lower price for the newspaper subsequent to the tax increase.<sup>13</sup> Thereby the readership increases, allowing the media firm to sell more advertising and make a higher profit than if it increased the price and reduced the output of newspapers.<sup>14</sup>

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<sup>12</sup>The full derivation is stated in the Appendix.

<sup>13</sup>Differentiating the equilibrium value of equation (1) with respect to  $t$ , and using the envelope theorem, we find  $d\pi/dt = -p^N(n, a)n(1+t)^{-2} < 0$  so the profit level is strictly decreasing in the tax rate. However, the marginal change in profits earned in the ad market is  $(p_a^A a + p^A) da/dt + p_n^A dn/dt$  which, by (2) and  $p_n^A > 0$ , is positive if quantity responses are positive (i.e.,  $ap_n^A - k_n > 0$ ).

<sup>14</sup>To see the intuition for this result as clearly as possible, assume that  $t$  approaches infinity. Obviously, the newspaper would then have no reason to charge a positive consumer price. However, it can still raise revenue through the advertising market and give the

Whether  $ap_n^A - k_n > 0$  holds depends on the industry in question. For platforms in the software industry, there are typically large fixed costs of developing e.g. a new data program, but very low marginal production costs. In our media example there are high fixed cost of creating the first copy of a newspaper, but relatively low marginal cost of reproducing it (and on the Internet  $k_n$  is approximately equal to zero even for pay-to-view sites). It should further be noted that advertising is the primary or only source of income for some media outlets, indicating that  $ap_n^A$  is relatively high.<sup>15</sup> This is presumably one reason why we see an increasingly large number of free newspapers.

The results in equations (6) and (7) are in stark contrast to benchmark results in one-sided markets, from which it is well known that (i) consumers buy less of a taxed good if marginal costs are positive ( $k_n > 0$ ), and that (ii) an ad valorem tax is effectively a tax on pure profit with no effect on output if marginal costs are zero ( $k_n = 0$ ).<sup>16</sup> Contrary to a firm operating in a one-sided market a two-sided platform firm can reduce its tax burden by shifting revenue to the side of the market where the tax rate is unchanged. This is particularly profitable if the marginal costs of the more heavily taxed good are smaller than the size of the indirect network effect. In such a case our results demonstrate that the output response to a tax increase is positive. A crucial implication of the insights above is that it is no longer necessarily true that the introduction of a tax causes consumer welfare to be reduced. This is a topic for analysis in later sections.

The effect of the tax increase on the price of ads is from equation (4) given by

$$\left. \frac{dp^A}{dt} \right|_{p_a^N=0} = p_a^A \left. \frac{da}{dt} \right|_{p_a^N=0} + p_n^A \left. \frac{dn}{dt} \right|_{p_a^N=0} \geq 0.$$

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newspaper away for free.

<sup>15</sup>See Kind, Nilssen and Sørsgard (2006) for a discussion of why so many Internet newspapers rely only on advertising income.

<sup>16</sup>See e.g. Rosen 1995, ch. 13.

Since  $p^A(n, a)$  is downward-sloping in own quantity, an increase in the advertising volume tends to reduce  $p^A$  ( $p_a^A < 0$ ). At the same time, the firm can charge a higher advertising price if the size of the readership increases (since  $p_n^A > 0$ ). Consequently, it is uncertain whether the price of advertising will go up or down.

### 3.2 Negative externalities from good $A$ ( $p_a^N < 0$ )

When  $p_a^N < 0$ , the demand for good  $N$  (newspaper) depends negatively on the level sold of good  $A$  (the advertising level). One might think that higher value-added taxes are more likely to reduce the sales of newspapers the more consumers dislike ads (since tax-motivated increased sales of  $A$  would reduce demand for  $N$ ). However, total differentiation of equations (2) and (3) makes it clear that the opposite is true:

$$\left. \frac{da}{dt} \right|_{p_a^N < 0} = \left. \frac{da}{dt} \right|_{p_a^N = 0} + \left( \frac{1}{1+t} \right)^2 \overbrace{\frac{\pi_{nn} n p_a^N}{H}}^{+} \quad (8)$$

$$\left. \frac{dn}{dt} \right|_{p_a^N < 0} = \left. \frac{dn}{dt} \right|_{p_a^N = 0} + \left( \frac{1}{1+t} \right)^2 \overbrace{\frac{(-\pi_{an} n p_a^N)}{H}}^{+}. \quad (9)$$

The first term in (8) and (9) shows how advertising and newspaper sales respond to a tax increase if consumers are indifferent about ads ( $p_a^N = 0$ ). As argued above, this term may be positive or negative. The second term, though, is unambiguously positive and increasing in the consumers' disutility of ads. The reason is that if sales in the newspaper market are adversely affected by advertising ( $p_a^N < 0$ ) the media firm has incentives to set a smaller advertising level than the volume which maximizes profit in the advertising market (c.f. equation (2)). This incentive becomes weaker with a heavier taxation of newspaper sales, making it optimal to increase the volume of ads by enlarging the size of the readership. The latter requires a reduction in the price charged by the media firm, and more so the stronger the consumers'

distaste for advertising. In particular, this implies that the tendency for the consumer price to fall subsequent to a tax increase is even more pronounced when  $p_a^N < 0$  than when  $p_a^N = 0$ .<sup>17</sup> It should be noted, though, that we still cannot sign the change in the price of advertising if both the advertising level and the size of the readership increase.

Summing up the discussion so far, we can state:

**Proposition 1:** *If  $p_a^N \leq 0$ , a sufficient condition for a higher value-added tax on good  $N$  to increase equilibrium quantities of both goods is that  $ap_n^A > k_n$ . The price of good  $N$  (inclusive of VAT) is lowered, while the sign of the change in the price of the untaxed good ( $A$ ) is ambiguous.*

### 3.3 Positive externalities from good A ( $p_a^N > 0$ )

When  $p_a^N > 0$ , the demand for good  $N$  depends positively on the output of good  $A$ . An example of where this constellation may occur is specialized magazines, where  $p_a^N > 0$  reflects a taste for commercials (ad-lovers). Car ads in automobile magazines and perfume ads in beauty magazines are examples of magazines whose readers appreciate ads (see Depken II and Wilson, 2004). Another example is the financial sector where cardholders have a higher willingness to pay for holding a credit card the larger the number of merchants that accept it. In order to be consistent, however, we shall continue to relate the model to the media market.

Equations (8) and (9) still hold when consumers are ad lovers, but with the potentially important difference that the last terms in both equations turn from positive to negative, that is,

$$\left. \frac{da}{dt} \right|_{p_a^N > 0} = \left. \frac{da}{dt} \right|_{p_a^N = 0} + \left( \frac{1}{1+t} \right)^2 \overbrace{\frac{\pi_{nn} n p_a^N}{H}}^{\bar{\quad}} \quad (10)$$

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<sup>17</sup>With  $p_n^N < 0$  and  $p_a^N < 0$  it follows immediately from equation (4) that  $dp^N/dt < 0$  if  $da/dt > 0$  and  $dn/dt > 0$ , and that the price reduction is larger the more consumers dislike ads.

$$\frac{dn}{dt}\Big|_{p_a^N > 0} = \frac{dn}{dt}\Big|_{p_a^N = 0} + \left(\frac{1}{1+t}\right)^2 \frac{\overbrace{(-\pi_{an} n p_a^N)}^{\bar{\quad}}}{H}. \quad (11)$$

If  $p_a^N > 0$  is small, the last term is insignificant relative to the first term and our results in the previous sections are reproduced. If  $p_a^N$  is sufficiently high, it follows from equations (10) and (11) that the sales of newspapers and advertising are decreasing in taxes. To see why, notice that the newspaper when consumers love ads (c.f. equation (2)) has more commercials than the quantity which maximizes profit on the advertising side. An increase in VAT, though, implies that it becomes less profitable for the media firm to attract readers by having a large advertising volume. Instead, the media firm will have incentives to reduce the level of advertising, and approach the volume that maximizes profit on the advertising side. If  $p_a^N$  is sufficiently high, both the level of advertising and the demand for the media product will therefore fall and the signs of  $dp^A/dt$  and  $dp^N/dt$  will be ambiguous (c.f. equation 4).

To summarize:

**Proposition 2:** *Suppose  $p_a^N > 0$ .*

- (a) *If  $p_a^N$  is not too high, a higher value-added tax on good  $N$  increases sales on both sides of the market and lowers the price of good  $N$  if  $ap_n^A > k_n$ .*
- (b) *If  $p_a^N$  is sufficiently high, a higher tax on good  $N$  reduces sales on both sides of the market, while the effect on prices is ambiguous.*

In the sections above we have shown that an ad valorem tax levied on a product sold by a platform firm has effects on prices and quantities not previously found in the literature that examines ad valorem taxation in one-sided markets. The purpose of the next section is therefore to analyze if a specific tax also has surprising effects on firm behavior.

## 4 Specific Taxation

Under a specific tax the profit of the platform is

$$\pi = \max_{n,a} \left[ ap^A(n,a) + \left( \frac{p^N(n,a)}{1+t} - \tau \right) n - k(n,a) \right],$$

where  $\tau$  is the specific tax that falls on good  $N$  (newspapers). From the first order conditions  $\pi_a = 0$  and  $\pi_n = 0$ , we can characterize the profit maximizing behavior of the platform as follows

$$p^A + ap_a^A - k_a = -\frac{np_a^N}{1+t} \quad (12)$$

$$\left[ \frac{p^N + np_n^N}{1+t} \right] - k_n = -ap_n^A + \tau. \quad (13)$$

The first-order conditions for the platform are the same as before (c.f. equations (2) and (3)), except that the specific tax imposes an additional cost on the production of good  $N$  as is evident from the right hand side of (13).

Totally differentiating (12) and (13), holding  $t$  fixed, we find

$$\frac{dn}{d\tau} = \frac{\pi_{aa}}{H} < 0 \text{ and } \frac{da}{d\tau} = -\frac{\pi_{na}}{H} < 0. \quad (14)$$

Equation (14) makes it clear that specific taxes unambiguously have a negative impact on output in both markets, independently of consumer preferences for ads. The reason is that higher specific taxes are equivalent to increased unit costs, as shown by equation (13). Since higher unit costs lower the marginal profitability for any given output, it is optimal to reduce sales of newspapers ( $dn/d\tau < 0$ ). As a result, the advertising level falls ( $da/d\tau < 0$ ).

The change in the newspaper price is

$$\frac{dp^N}{d\tau} = \overbrace{p_n^N \frac{dn}{d\tau}}^+ + \overbrace{p_a^N \frac{da}{d\tau}}^{? -}. \quad (15)$$

Equation (15) is unambiguously positive if consumers dislike ads ( $p_a^N < 0$ ). However, with ad-lovers ( $p_a^N > 0$ ) the second term is negative, reflecting

that the consumers' willingness to pay for the newspaper falls when the level of advertising decreases. If this effect is sufficiently strong, we obtain  $dp^N/d\tau < 0$ .

We likewise find that

$$\frac{dp^A}{d\tau} = \overbrace{p_a^A \frac{da}{d\tau}}^+ + \overbrace{p_n^A \frac{dn}{d\tau}}^- \quad (16)$$

is negative if the fall in readership,  $p_n^A (dn/d\tau)$ , dominates the increase in ads, that is  $p_a^A (da/d\tau)$ . Equations (14) - (16) thus show that an increase in  $\tau$  may reduce output and prices of both goods.

An example that yields the result that both prices fall subsequent to a tax increase is the following. Let  $p^A = -a/10 + n$ ,  $p^N = z - n/10 + a$  and  $\pi = ap^A + (p^N - \tau)n - a^2 - n^2$ . Then we have that  $\partial^2\pi/\partial n\partial a = 2 > 0$ . It is easily verified that all second-order conditions are satisfied. Solving  $\partial\pi/\partial n = \partial\pi/\partial a = 0$  we find  $p^A = a = 50(z - \tau)/21$ ,  $p^N = 131z/42 - 89\tau/42$  and  $n = 55(z - \tau)/21$ , from which it is immediately clear that a higher tax rate reduces all prices and quantities. Related to the media market, we may intuitively regard the reduction in readership (respectively advertising) as a quality reduction of the newspaper from the advertisers' (respectively readers) point of view. Other things equal, this leads to a lower willingness to pay for the newspaper and ad inserts.

Our result above can be summarized as follows:

**Proposition 3:** *A higher specific tax on good N reduces output of both goods. If  $p_a^A$  and  $p_n^N$  are positive and sufficiently large, end-user prices fall.*

The analysis in Sections 3 and 4 makes it clear that raising ad valorem taxes and specific taxes may have opposite quantity effects. The reason for this is that with specific taxes, there is a one-to-one relationship between tax payments and quantity, while there is no direct link between output and the burden of taxation under ad valorem taxation. In fact, subsequent to a higher ad valorem tax the firm can in principle both reduce tax payments and increase the quantity by lowering the price.

## 5 Welfare-Improving Ad Valorem Taxation

In this section we discuss in more detail the impact of taxation in two-sided markets from a welfare perspective. Since a higher specific tax has a negative effect on output both in one-sided and two-sided markets, we only consider the ad valorem tax.

Let  $a^*$  and  $n^*$  denote equilibrium output of goods  $A$  and  $N$ . In general the surplus enjoyed by the buyers of goods  $A$  and  $N$  is given by  $W^A \equiv \int_0^{a^*} p^A(n^*, \tilde{a}) d\tilde{a}$  and  $W^N \equiv \int_0^{n^*} p^N(\tilde{n}, a^*) d\tilde{n}$ , respectively. We define aggregate welfare ( $W$ ) as the sum of surplus for the two buyer groups, platform profit and tax revenue,

$$W = W^A + W^N + \pi + T, \quad (17)$$

where  $T = \frac{t}{1+t} p^N n$  is tax revenue.

By using the envelope theorem we find that  $\frac{d\pi}{dt}|_{t=0} = -p^N n$ , while  $\frac{dT}{dt}|_{t=0} = p^N n$ . In the neighborhood of  $t = 0$  a small tax increase thus reduces the platform profit by the same amount as tax revenue increases ( $\frac{d\pi}{dt}|_{t=0} + \frac{dT}{dt}|_{t=0} = 0$ ), so that the two last terms in equation (17) cancel each other. Therefore it suffices to look at welfare changes for the two buyer groups to analyze the effect of introducing VAT on good  $N$ .

From Propositions 1 and 2 we know that an increase in the tax rate of good  $N$  reduces the end-user price and increases the sales of that good if  $(ap_n^A - k_n) > 0$  and  $p_a^N \approx 0$ . In this case a higher tax on good  $N$  has positive welfare effects for buyers of good  $N$  (see Appendix). This turns benchmark results from one-sided markets upside-down.

From Propositions 1 and 2 we also know that output of good  $A$  increases following a higher tax on good  $N$  if  $(ap_n^A - k_n) > 0$  and  $p_a^N \approx 0$ . This has a positive welfare effect on buyers of good  $A$ . However, it may also be that the price of good  $A$  increases, and this will have a negative welfare effect. With general demand and cost functions we cannot dismiss the possibility that the negative effect of a higher price dominates over the positive effect of a larger quantity. If the price of good  $A$  falls or does not increase too much, though,

it is clear that introduction of an ad valorem tax on good  $N$  has a positive welfare effect both for buyers of good  $A$  and in aggregate:

To sum up, from Propositions 1 and 2 we have that

**Proposition 4:** *If  $p_a^N$  is in the neighborhood of 0 and  $ap_n^A - k_n > 0$ , introduction of an ad valorem tax on good  $N$ :*

- (a) *Improves welfare for buyers of that good.*
- (b) *Increases welfare in general if  $dp^A/dt$  is negative or not too positive.*

The qualification that  $p_a^N \approx 0$  and that the advertising price  $p^A$  does not rise too much subject to a tax increase on good  $N$  can be ignored if the demand functions are linear. Then the change in surplus enjoyed by buyers of good  $A$  as well as good  $N$  is strictly positive if output increases (this is illustrated in a numerical example below). Furthermore, whenever Proposition 4 (b) is satisfied, the government could tax good  $N$  and achieve a Pareto-improvement by granting a lump-sum rebate to the platform.

The results in Proposition 4 are in stark contrast to benchmark results in one-sided markets. Standard analysis has shown that a negative ad valorem tax (subsidy) will bring the monopoly solution closer to the social optimum, while a positive ad valorem tax will increase the deadweight loss. As demonstrated here, a welfare enhancing policy in a two-sided market may be to impose a positive tax instead of a VAT subsidy.

## 6 A Numerical Example

In this section we illustrate our findings by considering a simple example with linear demand curves, where the inverse demand curves for goods  $A$  and  $N$  are given by

$$p^A = 1 - a + \frac{n}{2} \text{ and } p^N = 1 - n + \beta a. \quad (18)$$

With this specification we have positive externalities from good  $N$  to good  $A$ , since  $p_n^A = \frac{1}{2} > 0$  (in our media example, this means that advertisers prefer a large audience). There are also positive externalities from good  $A$

to good  $N$  if  $p_a^N = \beta > 0$  (readers are ad-lovers), while the externalities are negative if  $p_a^N = \beta < 0$  (readers are ad-haters).

In order to clearly distinguish our analysis from that of a multiproduct monopolist with cost synergies between the goods, we assume that the platform's marginal cost of producing good  $N$  is independent of output of good  $A$ , and vice versa. This assumption corresponds to setting  $k_{na} = 0$  in equation (1). We now let the platform profit be given by

$$\pi = p^A a + \frac{p^N n}{1+t} - k(a+n), \quad (19)$$

where  $k \geq 0$  is the marginal cost of producing each of the goods.

To obtain algebraic solutions which are as simple as possible, we further set  $k = 0$ . However, it can be shown that a sufficient condition for the qualitative results we obtain to hold is that  $k < 1/4$ .<sup>18</sup>

Maximizing (19) subject to (18) we find that the first-order condition for the equilibrium price and output of goods  $A$  and  $N$  equals

$$\begin{aligned} p^A &= 2 \frac{5(1+t) - \beta(3+t) - 2\beta^2}{D} \text{ and } a = 2 \frac{5(1+t) + 2\beta}{D}, \\ p^N &= \frac{(1+t)(5-3t+2\beta)}{D} \text{ and } n = \frac{2(1+t)(5+t+2\beta)}{D}, \end{aligned} \quad (20)$$

where  $D \equiv (1+t)(15-t) - 4\beta(\beta+1+t)$ .

In what follows we confine ourselves to analyzing the effects of a small increase in the ad valorem tax rate from  $t = 0$ , even though equation (20) apply as long as the tax rate is not so high as to yield negative output or profits.<sup>19</sup>

In Section 2 we made the assumption that the marginal profitability of selling good  $A$  is increasing in the output of the other good  $N$ , and vice versa

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<sup>18</sup>With demand functions that have intercept equal to 1,  $k = 1/4$  is a rather high number, showing that these results hold even if marginal costs are relatively high.

<sup>19</sup>It can be shown that all non-negativity constraints and second-order conditions hold for  $\beta \in (-5/2, 1)$  in the neighborhood of  $t = 0$ .

(confer Assumption 2 where we assumed  $\pi_{an} > 0$ ). In our example we have that

$$\pi_{an}|_{t=0} = \frac{\partial^2 \pi}{\partial a \partial n} \Big|_{t=0} = \frac{1 + 2\beta}{2} > 0 \text{ if } \beta > -1/2$$

In order for our example to be in line with assumption 2 and thus comparable to our previous analysis we shall assume that  $\beta \in (-1/2, 1)$ . However, it will be clear from the analysis to follow that there are no qualitative changes in the effects of taxation in the neighborhood of  $\beta = -1/2$ . The assumption  $\pi_{an} > 0$  is thus not critical.

## 6.1 Tax incidence and profit-shifting

The effect of an ad valorem tax on prices (tax incidence) can be found by differentiating (20) with respect to  $t$  in the neighborhood of  $t = 0$ . This yields

$$\begin{aligned} \frac{dp^A}{dt} \Big|_{t=0} &= -2 \frac{2\beta^2 - 5\beta - 1}{(3 - 2\beta)^2 (5 + 2\beta)} < 0 \text{ for } \beta < \frac{1}{4} (5 - \sqrt{33}) \approx -0.1021 \\ \frac{dp^N}{dt} \Big|_{t=0} &= -2 \frac{2\beta^2 - 3\beta + 4}{(3 - 2\beta)^2 (5 + 2\beta)} < 0 \quad \forall \beta. \end{aligned} \quad (22)$$

We see from (22) that a higher tax rate on good  $N$  reduces the end-user price ( $dp^N/dt < 0$ ). The left-hand side panel of Figure 1 illustrates this tax incidence result. Recall that  $\beta$  measures the externality from good  $A$  to good  $N$ , where  $\beta > 0$  indicates ad-lovers and  $\beta < 0$  is ad-haters. The left panel shows that the platform will bear the entire tax burden for  $p_a^N = \beta \leq -0.19$ . However, the burden of the tax is partly shifted onto buyers of good  $A$  - on which the tax rate is unchanged - if  $p_a^N \in (-0.19, 1.0]$ .<sup>20</sup>

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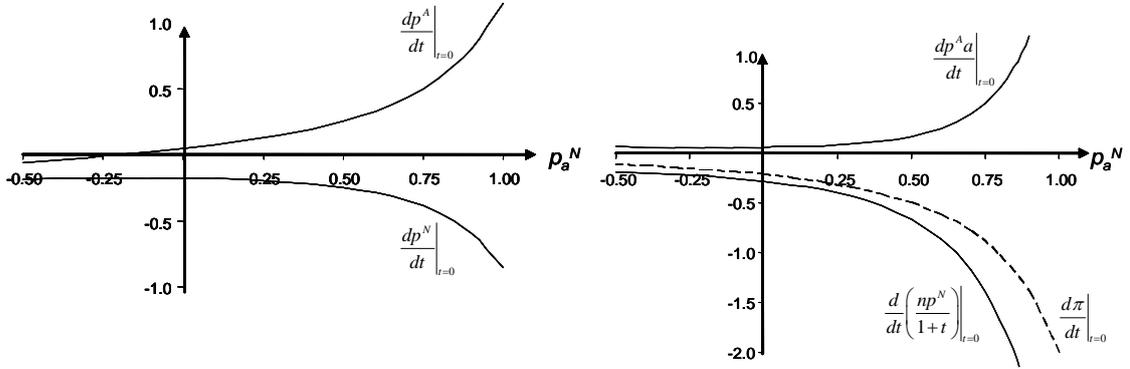
<sup>20</sup>From the analysis in Section 2.1 we know that if good  $A$  imposes a strong negative externality on buyers of good  $N$ , the platform sells a smaller quantity and sets a higher price than what maximizes profit on the  $A$ -side of the market. However, the incentive to set a high price on good  $A$  at the expense of a low output of that good is less pronounced the more heavily good  $N$  is taxed. If the externalities from good  $A$  are positive, we have the opposite result. This explains why  $dp^A/dt$  is upward-sloping and eventually becomes positive for sufficiently high values of  $\beta$ .

From Propositions 1 and 2 we know that the platform will reduce its tax burden by shifting profits from sales of good  $N$  to good  $A$  independent of the size of  $\beta$ . For our linear demand example this is illustrated by the curves  $\frac{d(ap^A)}{dt}$  and  $\frac{d}{dt} \left( \frac{np^N}{1+t} \right)$  in the right-hand side panel of Figure 1, where

$$\frac{d}{dt} (ap^A) \Big|_{t=0} = 8 \frac{1 + 2\beta^2 - \beta}{(3 - 2\beta)^3 (5 + 2\beta)} > 0$$

and

$$\frac{d}{dt} \left( \frac{np^N}{1+t} \right) \Big|_{t=0} = -2 \frac{19 + 4\beta^2 - 8\beta}{(3 - 2\beta)^3 (5 + 2\beta)} < 0.$$



**Figure 1:** *Tax shifting vs profit shifting.*

It should be pointed out that a tax levied on good  $N$  affects the profit of the platform negatively even if it shifts sales and revenue to the  $A$ -good side of the market. The dotted curve  $d\pi/dt|_{t=0} = -\frac{2}{(3-2\beta)^2} < 0$  shows the total loss in profit for the platform of introducing a value-added tax on good  $N$ .

## 6.2 Welfare Analysis

With linear demand functions it is straight forward to show that  $\int_0^{n^*} p^N(\tilde{n}, a^*) d\tilde{n} = \frac{1}{2} (n^*)^2$  and  $\int_0^{a^*} p^A(n^*, a) da = \frac{1}{2} (a^*)^2$ . We therefore have<sup>21</sup>

$$\frac{dW}{dt} \Big|_{t=0} = n^* \frac{dn}{dt} \Big|_{t=0} + a^* \frac{da}{dt} \Big|_{t=0}. \quad (23)$$

<sup>21</sup>Recall that  $\frac{d\pi}{dt} \Big|_{t=0} + \frac{dT}{dt} \Big|_{t=0} = 0$  by the envelope theorem.

Differentiation of equation (20) with respect to  $t$  yields

$$\left. \frac{da^*}{dt} \right|_{t=0} = 2 \frac{1 - 6\beta}{(3 - 2\beta)^2 (5 + 2\beta)} > 0 \text{ if } \beta < \frac{1}{6} \quad (24)$$

and

$$\left. \frac{dn^*}{dt} \right|_{t=0} = 4 \frac{2 - \beta - 2\beta^2}{(3 - 2\beta)^2 (5 + 2\beta)} > 0 \text{ if } \beta < \frac{1}{4} (\sqrt{17} - 1) \approx 0.78. \quad (25)$$

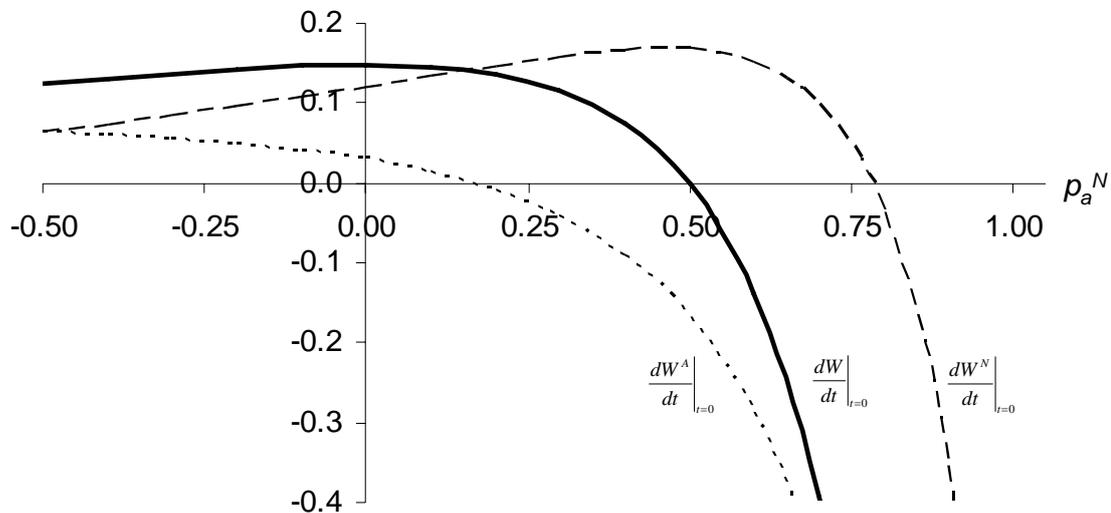
Using equations (23) - (25) we find

$$\left. \frac{dW}{dt} \right|_{t=0} = 4 \frac{1 - 2\beta}{(3 - 2\beta)^3} > 0 \text{ if } \beta < 1/2.$$

A small increase in the tax rate of good  $N$  from  $t = 0$  raise surplus for buyers of this good if  $\beta < 0.78$ . The buyers of good  $A$  will lose out if  $\beta < 1/6$ , but the change in aggregate welfare is nonetheless positive if  $\beta < 1/2$ . This is illustrated in Figure 2, where the change in total welfare ( $W$ ) and welfare to buyers of good  $A$  and  $N$  is depicted.<sup>22</sup>

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<sup>22</sup>In figure 2 it is seen that  $W^N$  is an inverted U-shaped function of  $p_a^N$ . The reason is that there are two opposing effects of increasing the tax rate on good  $N$ . On the one hand, it reduces the price of good  $N$ . This tends to make  $dW^N/dt > 0$ . On the other hand, we also know that output of good  $A$  increases if  $\beta < 1/6$  in our example. This tends to make  $dW^N/dt < 0$  if there is a negative externality from good  $A$  to good  $N$  ( $\beta < 0$ ). However, this effect is less important the weaker the negative externalities. This explains the upward-sloping part of the  $dW^N/dt$ -curve. The downward-sloping part follows from the fact that the output of both goods falls if  $\beta$  is positive and sufficiently large (c.f. Proposition 2), and this is unambiguously negative for buyers of good  $N$  if there are positive externalities from good  $A$  to good  $N$ .



**Figure 2:** *Welfare improving ad-valorem taxation*

To sum up, the linear example has demonstrated that welfare may rise if a tax is levied on a good produced by a two-sided platform firm. This result is in contrast to standard findings and indicates that caution should be taken when assessing the impact of policy in markets where two-sided platform firms operate.

## 7 Conclusion

Traditional analysis of tax incidence has focused on conventional (one-sided) markets. In such markets a general insight is that indirect taxes are partly shifted (or even overshifted) onto consumers, resulting in lower sales of the taxed good. Our analysis has shown that this result is challenged in a two-sided market. If demand for the taxed good matters for the quantity sold to a different group of customers, the incidence of taxation changes. In a two-sided market an increase in an ad valorem tax may, under certain conditions, lead to lower prices for both goods as well as to higher sales. The results

obtained under ad valorem taxation are in sharp contrast to our findings under specific taxation, where a higher tax unambiguously has a negative effect on output.

The existence of positive quantity responses to higher ad valorem taxation straightforwardly leads into the question of whether ad valorem taxation is an appropriate policy to improve welfare. In a one-sided market a welfare improving policy would be to provide a subsidy which increases output toward the level at which prices equal marginal cost (Delipalla and Keen, 1992). In two-sided markets a welfare enhancing policy may be to introduce an ad valorem tax.

Our study has been carried out in a monopoly setting. An interesting path for future research would be to check the robustness of our results under different market structures. However, we believe that the main results in this paper would survive under oligopoly as well. As long as firms have some market power, a tax increase on one side of the market implies that the firms will have incentives to shift profit to the other side of the market. In an appendix, available from the authors upon request, we show that this conjecture holds in a simple duopoly model with linear demand functions.

Even though our discussion is related to the media market, we have not incorporated any of the particularities of the media market or the advertising market into the model. The reason is that we have used a model sufficiently general in structure to highlight the most common mechanisms in two-sided markets. This said, we believe that there is also a need for industry-specific analysis in both theoretical and empirical terms to identify peculiarities of the respective industries for tax policy design.

It is worth stressing once more that the notion of two-sided markets should be distinguished from that of complementarity. If a price reduction of good  $A$  leads to higher sales of both goods, then we may consider them as complements. This is the case if there are positive externalities from good  $A$  to good  $N$  and vice versa. However, if there is a negative externality from good  $A$  to good  $N$ , then a lower price and higher output of good  $A$  reduce sales of

good  $N$ , other things equal. In our numerical example, it is precisely in the latter case that a higher tax of good  $N$  is likely to reduce the price of both goods and increase welfare. It should further be noted that only the sum of prices matters for complements; it is irrelevant for a consumer whether a shop sets different prices for right and left shoes. Only the total price matters. In two-sided markets, on the other hand, the price structure is decisive. Indeed, this is one of the distinguishing features of two-sided markets, as stressed by Rochet and Tirole (2003, 2006). If the VAT is increased for a good in a two-sided market, it will be optimal for the platform to change the price structure in order to make a relatively higher revenue from the buyers of the other good. This is precisely the reason why a higher tax on one good may reduce prices and increase output on both sides of the market.

## 8 Appendix

*Derivation of the relationship between quantities and ad valorem taxes*

We assume that the second order conditions hold with non-negative prices and quantities, so that the equilibrium is characterized by first order conditions (2) and (3). To find how a higher value-added tax affects prices on the two sides of the market, we totally differentiate (2) and (3). We then find

$$\begin{aligned}\pi_{aa}\frac{da}{dt} + \pi_{an}\frac{dn}{dt} &= \left(\frac{1}{1+t}\right)^2 np_a^N \\ \pi_{an}\frac{da}{dt} + \pi_{nn}\frac{dn}{dt} &= \left(\frac{1}{1+t}\right)^2 (p^N + np_n^N).\end{aligned}$$

Making use of the first-order condition (3), the effect of the tax on quantities is now given by

$$\frac{da}{dt} = \left(\frac{1}{1+t}\right)^2 \frac{\pi_{an}(1+t)(ap_n^A - k_n) + \pi_{nn}np_a^N}{H} \quad (26)$$

and

$$\frac{dn}{dt} = - \left( \frac{1}{1+t} \right)^2 \frac{\pi_{aa} (1+t) (ap_n^A - k_n) + \pi_{an} np_a^N}{H}. \quad (27)$$

*Consequences of relaxing the assumption that  $\pi_{na} > 0$*

Suppose that  $\pi_{an} < 0$  and  $p_a^N = 0$ . From equation (6) we see that a higher ad valorem tax still increases sales of the newspaper and reduces its price if  $ap_n^A - k_n > 0$ : thus the media firm's incentive to sell a larger number of newspapers in order to shift revenue to the advertising side is unaltered. However, from equation (7) we find that  $da/dt < 0$  if  $\pi_{an} < 0$ .

If  $p_a^N < 0$ , we know that there will be less advertising than the volume which maximizes profit on the advertising side of the market. If the ad valorem tax rate on sales of newspapers increases, the media firm will care less about the revenue it captures directly from the readers. This is true independent of whether  $\pi_{an} > 0$  or  $\pi_{an} < 0$ . The second term in equation (8) shows that this effect makes the media firm sell more advertising space if  $t$  increases. However, the second term in equation (9) makes it clear that this tends to reduce the sales of newspapers.

To grasp the intuition for this result, assume that  $\pi_{an} < 0$  because  $k_{an}$  is large. In order to save costs, the media firm will then have incentives to reduce the circulation of the newspaper when the advertising volume increases.<sup>23</sup>

The case where  $p_a^N > 0$  has a similar interpretation. If the consumers are ad lovers, the newspaper has more ads than the level that maximizes profit on the advertising side of the market. Independent of the sign on  $\pi_{an}$ , the newspaper will therefore reduce the advertising level if  $t$  increases ( $da/dt < 0$ ). However, a lower advertising level means that the marginal profit of selling newspapers increases if  $\pi_{an} < 0$ , which induces the newspaper to sell more newspapers ( $dn/dt > 0$ ).

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<sup>23</sup>For the same reason, we see from equation (14) that a higher specific tax on newspapers - which always reduces sales of newspapers - increases the advertising volume if  $\pi_{an} < 0$ .

The effects of assuming  $\pi_{an} < 0$  when we consider specific taxes are analogous, and seen from equations (14) - (16).

*Discussion of Proposition 4*

From Propositions 1 and 2 we know that an increase in the tax rate of good  $N$  may reduce the price and increase the output of that good for any sign of the externalities from  $A$  to  $N$  (this happens if  $(ap_n^A - k_n) > 0$  and  $p_a^N$  is not too large). In such cases it may be tempting to use insight from one-sided markets and conclude that the surplus enjoyed by buyers of good  $N$  must increase. Figure 3 makes it clear that this is not necessarily true. Suppose that the inverse demand curve for good  $N$  initially is given by the curve  $p^N(t_0)$ , with  $p_0^N$  and  $n_0$  as the equilibrium price and quantity, respectively. At this equilibrium point ( $E_0$ ) the buyers of good  $N$  achieve a surplus given by  $W_0^N$ . Suppose further that there is a negative externality from good  $A$  to good  $N$  (newspaper readers dislike ads;  $p_a^N < 0$ ), and that the tax rate increases to  $t_1 > t_0$ . From the analysis above we know that the platform will respond by reducing the price and increasing the output of good  $N$  in order to sell more of good  $A$ . However, larger sales of good  $A$  generate a negative shift in the demand curve for good  $N$  when  $p_a^N < 0$ , illustrated by the curve  $p^N(t_1)$ . We therefore move from equilibrium point  $E_0$  to  $E_1$ . Since  $W_1^N < W_0^N$ , buyers of good  $N$  clearly have a lower surplus with this higher tax rate, despite the fact that they buy more and pay a lower price for the good.

We can nonetheless conclude that a tax increase on good  $N$  has positive welfare effects for buyers of that good if  $|p_a^N|$  is not too large, and  $(ap_n^A - k_n) > 0$ . To see why, suppose first that there are no externalities from  $A$  to  $N$  ( $p_a^N = 0$ ). From Proposition 1 we then know that the price of good  $N$  falls subsequent to a tax increase, and that output increases. With  $p_a^N = 0$  there will be no shift in the demand curve for good  $N$  even if output of good  $A$  changes. This means that we move from  $E_1$  to a point like  $E_2$  in Figure 3, and this generates a non-marginal positive increase in  $W^N$ . With any well-behaved demand and cost functions, continuity implies that

we must also have a positive welfare gain for this buyer group even if  $p_a^N$  is slightly positive or negative.

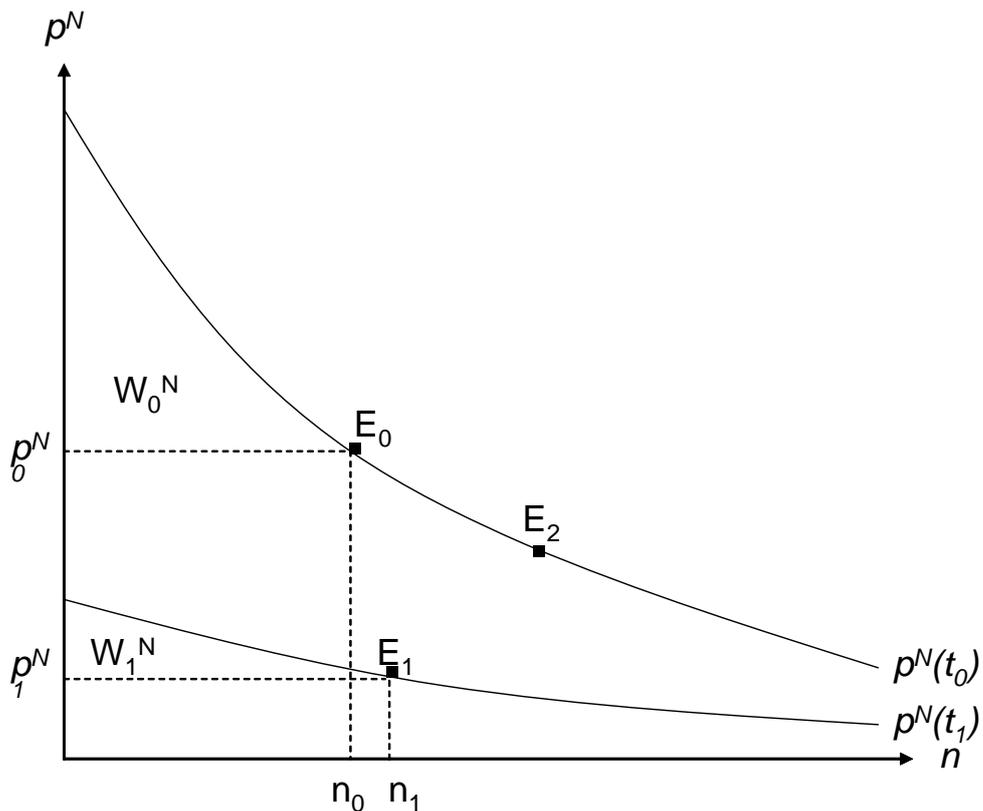


Figure 3: Demand shift and buyer surplus.

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