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Loss aversion and mental accounting: the favorite longshot bias in parimutuel betting

Jianying Qiu*

Abstract

Parimutuel betting markets are simplified financial markets, and can thus provide a clearer view of pricing issues which are more complicated elsewhere. Though empirical studies generally conclude that the parimutuel betting markets are surprisingly efficient, it is also found that for horses with lowest odds (favorites), market estimates of winning probabilities are smaller than objective winning probabilities; for horses with highest odds (longshot), the opposite is observed. This phenomenon, called the favorite longshot bias, has many explanations such as risk seeking preference, transaction costs, and non-linear transformation of probabilities into decision weights, etc. This paper combines loss aversion with mental accounting, and provides a new explanation for the favorite longshot bias. We show that the bias exists in the absence of all above mentioned reasons, and the degree of the bias differs depending on the type of the mental accounting process that bettors apply.

Keywords: loss aversion, mental accounting, parimutuel betting, the favorite longshot bias.

JEL Classification: C72 D40 D81 G10

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1 Introduction

Parimutuel betting is a betting system in which all bets of a particular type are placed together in a pool; taxes and a house take are removed, and payoff odds are calculated by sharing the pool among all placed bets. Parimutuel betting markets are special kinds of financial markets where participants take a financial position on the outcome of a horse race. Although these markets are a tiny feature of most economies, they present significant opportunities for economic analysis. This stems from the fact that parimutuel betting markets are simplified financial markets in which the scope of pricing has been greatly reduced:

- Fundamental value of traded assets, namely bets, are both observable and exogenous;
- The time horizon of assets is well defined.

Hence, parimutuel betting markets can provide a clearer view of pricing issues which are more complicated elsewhere.

Empirical studies generally conclude that betting markets are surprisingly efficient. It has been shown that betting markets can effectively aggregate private information, and that the prices, market odds, are good estimates of horses' winning probability. But it is also found that for horses with lowest odds (favorites), market estimates of winning probabilities are smaller than objective winning probabilities; for horses with highest odds (longshot), the opposite is observed. This puzzling regularity, called the favorite longshot bias (hereafter referred as the bias), can not be reconciled with the behavior of risk-averse individuals who behave in line with expected utility theory (von-Neumann Morgenstern, 1944)[1].

[1]This is because all longshots are second degree stochastically dominated by favorites (for second degree stochastic dominance please see Hadar and Russell, 1969). Comparing

Numerous explanations for the bias exist. Assuming players with local risk preference in utility, Weitzman (1965) and Quandt (1986) show that risk loving bettors with mean and variance preference are willing to trade mean for variance, and the bias is the equilibrium outcome of their play. Hurley and McDonough (1995) attribute the bias to the existence of transaction costs, which prevent full movement back of relative frequencies of bets to objective probabilities, though this was not supported by the experiment they conducted later.

These explanations, although providing useful insights, seem to miss something. Why is there a tendency for the favorite-longshot bias to become more pronounced for the last couple of races of the day? And why do people trade in these markets despite of the negative aggregate expected returns? It is increasingly accepted that a more promising way of explaining the bias is to build models which rely on behavioral assumptions that are supported by empirical and experimental evidences. The findings in prospect theory and mental accounting are important sources. The main findings in prospect theory are: 1) people possess (inverted) S-shaped probability weighting function, i.e., they tend to overweight small probabilities and underweight large probabilities; 2) value is assigned to gains and losses rather than to final assets, and the value function is normally concave for gains (implying risk aversion), convex for losses (risk seeking), and is generally steeper for losses than for gains (loss aversion). The key assumption of mental accounting is that people adopt mental accounts and act as if the

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the prospects over two horses: Horse A: winning x with probability $1/(1+x)$, lose 1 with probability $x/(1+x)$. Horse B: winning y with probability $1/(1+y)$, lose 1 with probability $y/(1+y)$. If $x > y$, then A is the longshot relative to B. Let $F_A(t)$ (respective $F_B(t)$) denote the cumulative probability distribution of the prospect A (respectively B), since $\int_{-\infty}^{z} F_A(t)dt \geq \int_{-\infty}^{z} F_B(t)dt$ for all z, the prospect over A is second degree stochastically dominated by the prospect over B. And as shown by Hadar and Russell(1969), this implies favorites should be preferred by individuals possess any utility function with $U'(x) > 0$ and $U''(x) < 0$.

money in these accounts is not fungible.

There have been some such attempts. Koessler, Ziegelmeyer and Broihanne (2003) formulate a game theoretical model with non-expected utility players, where one of the driving forces of the bias is the (inverted) S-shaped probability weighting function. Thaler and Ziemba (1988) use mental accounting to explain the tendency of more pronounced bias in the last race of the day relative to earlier races.

This paper shares some features of Thaler and Ziemba (1988). We also rely on mental accounting. But there is one critical difference. Thaler and Ziemba (1988) aim to show that when players take part in *serial betting markets*, all the outcomes of these bets are evaluated in the same mental account, thus players show higher risk preference if there is a loss in prior bets due to the risk seeking in the loss domain. Whereas, in this paper, we show that, *in a single betting market*, the bias results from loss aversion, and depending on the specific process of mental accounting, the degree of the bias differs. Since people are more sensitive to loss than to gain, they are less willing to bet than otherwise. When market is off equilibrium, this prevents full movement back of relative frequencies of bets to objective probabilities, and results in the bias. Moreover, observe that in parimutuel betting markets bets are paid before the revelation of the outcomes and are thus essentially prior losses. As pointed out by Thaler (1985) and Thaler and Johnson (1990), prior losses can either be integrated with or segregated from currently available alternatives, and that prospect theory often predicts different choices depending on which coding process is used. This can be illustrated by following example:
Consider a player who evaluates any prospect P of winning x with probability p or y with probability $1-p$ as follows:

$$V(P) = \pi(p)v(x) + \pi(1-p)v(y)$$

Here $\pi(\cdot)$ is the probability weighting function, and $v(\cdot)$ is the prospect value function. Think of a bettor's decision of buying the following risky alternative at price c:

Risky alternative R

Outcome	Prob.	Outcome	Prob.
x	p	0	$1-p$

When the bettor integrates the prior loss, the price c, with the risky alternative, following mental account is established:

New risky alternative R'

Outcome	Prob.	Outcome	Prob.
$x-c$	p	$-c$	$1-p$

Thus

$$V(R') = \pi(p)v(x-c) + \pi(1-p)v(-c)$$

Whereas, if the bettor segregates the prior loss from the risky alternative, his mental account can be presented as:

A sure loss of c and

The risky alternative R

which can be denoted as a compound prospect C. The segregation of sure loss from risky alternative is because that above two things appear to be cognitively different: one is sure loss and the other is a lottery. And the integration of the two needs some cognitive effort.Notice $v(0) = 0$

$$V(C) = v(-c) + \pi(p)v(x)$$

And in general $V(R') > 0$ does not imply $V(P) > 0$, vice versa. Thus depending on the mental accounting process people apply, they might make different decisions.

We analyze the implication of loss aversion and mental accounting on the bias using the sequential betting model developed by Koessler, Ziegelmeyer and Broihanne (2003). The exogenously determined sequence greatly simplifies the analysis but retains the basic dynamic feature of parimutuel betting markets, where players place bets based on the odds they observe so far and the expectation of final odds. Moreover, this structure is theoretically appealing for our purpose: it involves sequential evaluations of risky alternatives, which, due to the institutional features of parimutuel markets, changes with every new bet.

The remainder of the paper is organized as follows. Section 2 introduces the basic model. Section 3 derives the theoretical results for both mental accounting processes. We explore the effects of mental accounting and loss aversion on the favorite longshot bias in section 4. Section 5 concludes.

2 The model

The model is a multi-stage game. There are n players (called strategic bettors), who place their bets sequentially at a predefined stage, and in each stage only one player moves. Players behave in line with prospect theory, and thus maximize the decision weighted value of bets. The set of players is denoted by N. There are two horses called F (standing for favorite) and L (standing for longshot), with respective objective winning probabilities of p and $1-p$, where $p > 1/2$. Before the starting of the game, there are initial bets placed by some unmodelled noisy bettors with k units of money on each of the horses.

When a player moves in her predefined stage, she can choose to bet *one* unit of money on either of two horses, F or L, or refrain from betting. More precisely, each player chooses an action $a_i \in A = \{F, L, D\}$ in stage i, where F (respectively L) means to bet 1 unit of money on horse F (respectively L), and D means to refrain from betting. Let $h^t = (a_1, a_2, \ldots, a_t)$ denote the history up to stage t, $h^0 = \emptyset$ represents the starting of the game, and $h^n = z$ represents one terminal history. At the beginning of stage i, player i knows history h^{i-1}. Let H^t denote the set of histories up to stage t. For any non-empty history $h^t \in H^t$, we partition the players moved in $\{1, 2, \ldots, t\}$ into three sets as

$$F(h^t) = \{i \in \{1, 2, \ldots, t\} : \textit{such that } a_i = F\}$$
$$L(h^t) = \{i \in \{1, 2, \ldots, t\} : \textit{such that } a_i = L\}$$
$$D(h^t) = \{i \in \{1, 2, \ldots, t\} : \textit{such that } a_i = D\}$$

Hence, after history h^t, $F(h^t)$ (respectively $L(h^t)$) denotes the set of players who have bet on horse F (respectively horse L), and $D(h^t)$ denotes the set of bettors who have refrained from betting. After history h^t, let $n_F(h^t) = |F(h^t)|$ (respectively $n_L(h^t) = |L(h^t)|$) denote the number of players who have bet on horse F (respectively horse L), and $n_D(h^t) = |D(h^t)|$ denotes the number of bettors who have refrained from betting.

By the institutional features of parimutuel betting markets, the total money bet on all horses, net of the track take, is shared proportionally among those who bet on the winning horse. This implies when players choose to bet conditioning on the histories observed so far, they need to take into account the effect of his own bet and future bets on the payoff. It has been argued by Hurley and McDonough (1995) that track take is responsible for the favorite-longshot bias. Since here we are mainly concerned with the effects of loss aversion and mental accounting on the bias, we assume a zero track take ratio to avoid possible disturbances.

Players evaluate risky alternatives in line with prospect theory. In order to make unique numerical predictions, the weighting function, $\pi(\cdot)$, is defined to be

$$\pi(p) = p$$

This form provides a good approximation when the parameter p is not in extreme range, and this rules out the effect of non-linear probability weighting function to the bias. The prospect theory value function, $v(\cdot)$, is defined for present purposes as a segmented power function with three parameters (Tversky and Kahneman, 1992)

$$v(x) = \begin{cases} x^{\alpha} & if\ x \geq 0 \\ -\lambda(-x)^{\beta} & if\ x < 0 \end{cases}$$

where α determines the gain domain concavity and β determines loss domain convexity of the value function, and λ relates to the extent of loss aversion. Though the exact values of α, β and λ are hard to determine, experimental findings generally conclude $0 < \alpha < 1$, $0 < \beta < 1$ and $\lambda > 1$, which implies the value function is convex on the loss domain and concave on the gain domain, and is loss averse. Since players are allowed to place only one unit of bet, players's loss is always one, which implies the value of β does not influence players' behavior in this model since $v(-1) = -\lambda(1)^{\beta} = -\lambda$. It is also clear that this construction is in fact a special form of expected utility expression.

Let Z denote the set of terminal histories. Since a terminal history represents an entire sequence of a play, i.e. the outcome of a play, and recall the k units of initial bets on each of horses, the decision weighted value of *one unit* bet based on the integration process and the segregation process is, respectively,

- The integration process

$$V_i : Z \rightarrow \Re$$
$$V_i(z) = \begin{cases} p(\frac{n_F(z)+n_L(z)+2k}{n_F(z)+k} - 1)^{\alpha} + (1-p)(-\lambda) \ if \ i \in F(z) \\ (1-p)(\frac{n_F(z)+n_L(z)+2k}{n_L(z)+k} - 1)^{\alpha} + p(-\lambda) \ if \ i \in L(z) \\ 0 \ if \ i \in D(z) \end{cases}$$

- The segregation process

$$V_i : Z \rightarrow \Re$$
$$V_i(z) = \begin{cases} p(\frac{n_F(z)+n_L(z)+2k}{n_F(z)+k})^{\alpha} - \lambda \ if \ i \in F(z) \\ (1-p)(\frac{n_F(z)+n_L(z)+2k}{n_L(z)+k})^{\alpha} - \lambda \ if \ i \in L(z) \\ 0 \ if \ i \in D(z) \end{cases}$$

As a terminal history z is uniquely defined by a strategy profile s, players' decision weighted value of one unit bet can be written more explicitly as $V_i(z(s))$.

Bettor i's behavioral strategy is denoted by

$$s_i : h^{i-1} \longrightarrow A = \{F, L, D\}$$

And a profile of behavioral strategies is denoted by $s = (s_1, s_2, \ldots, s_n)$. To break the tie, we assume player refrains from betting should she expects zero decision weighted value from risky alternatives. Let $z(s|h^t)$ be the final history reached according to the strategy profile s, given the history $h^t \in H^t$, and thus $z(s)$ is simply the final history generated by strategy profile s. A strategy profile s^* is subgame perfect equilibrium if for $\forall h^{i-1} \in H_i^{i-1}$, and for $\forall i \in N$

$$V_i(z(s_i^*, s_{-i}^*|h^{i-1})) \geq V_i(z'(s_i', s_{-i}^*|h^{i-1}))\ for\ \forall s_i' \in S_i$$

3 General results

In this section we derive the equilibrium outcome using the subgame perfect equilibrium as solution concept.

Proposition 1 *There is no equilibrium outcome in which some players bet on the longshot and some players bet on the favorite.*

Proof: Assume by way of contradiction that z is an equilibrium outcome, where exists some i and j such that $a_i = F$ and $a_j = L$. We first prove for the case where players apply the integration process. That z is an equilibrium outcome implies

$$V_i(z) = p(\frac{n_F(z) + n_L(z) + 2k}{n_F(z) + k} - 1)^\alpha + (1-p)(-\lambda) > 0$$

and

$$V_j(z) = (1-p)(\frac{n_F(z) + n_L(z) + 2k}{n_L(z) + k} - 1)^\alpha + p(-\lambda) > 0$$

which is equivalent to

$$\frac{n_L(z) + k}{n_F(z) + k} > \frac{(1-p)^{1/\alpha}\lambda^{1/\alpha}}{p^{1/\alpha}}$$

and

$$\frac{n_L(z) + k}{n_F(z) + k} < \frac{(1-p)^{1/\alpha}}{p^{1/\alpha}\lambda^{1/\alpha}}$$

Since $\lambda > 1$, this yields a contradiction.

Similarly when players apply segregation process. Similarly, that z is an equilibrium outcome implies

$$V_i(z) = -\lambda + p(\frac{n_F(z) + n_L(z) + 2k}{n_F(z) + k})^\alpha > 0$$

and

$$V_j(z) = -\lambda + (1 - p)(\frac{n_F(z) + n_L(z) + 2k}{n_L(z) + k})^\alpha > 0$$

after some algebraic manipulation, we get

$$\frac{n_F(z) + k}{n_F(z) + n_L(z) + 2k} < (\frac{p}{\lambda})^{1/\alpha}$$

and

$$\frac{n_L(z) + k}{n_F(z) + n_L(z) + 2k} < (\frac{1 - p}{\lambda})^{1/\alpha}$$

which implies

$$(\frac{p}{\lambda})^{1/\alpha} + (\frac{1 - p}{\lambda})^{1/\alpha} > 1$$

A contradiction to $\lambda > 1$. Thus in equilibrium players should bet only on the favorite or on the longshot.

Proposition 2 *Let s^* be an subgame perfect equilibrium, then in any subgame perfect equilibrium outcome $z(s^*)$, there is no bet on the longshot.*

Proof: Assume by way of contradiction that $n_L(z(s^*)) > 0$, then by proposition 1, we must have $n_F(z(s^*)) = 0$. We first consider the integration process. $n_L(z(s^*)) > 0$ and $n_F(z(s^*)) = 0$ implies that for players $i \in L(z(s^*))$

$$V_i(z(s^*)) = (1 - p)(\frac{n_L(z(s^*)) + 2k}{n_L(z(s^*)) + k} - 1)^\alpha + p(-\lambda) > 0$$

which implies that

$$p < \frac{\left(\frac{k}{n_L(z(s^*))+k}\right)^{\alpha}}{\left(\frac{k}{n_L(z(s^*))+k}\right)^{\alpha} + \lambda} < \frac{1}{1+\lambda} < \frac{1}{2}$$

in contradiction with the assumption of $p > 1/2$.

Let B denote the aggregate betting volume, with $B = B^I$ if players apply integration process and $B = B^S$ if players apply segregation process. Here $B^I (> 0)$ is the integer defined by

$$p\left(\frac{B^I + 2k}{B^I + k} - 1\right)^{\alpha} - (1-p)\,\lambda > 0 \tag{1}$$

and

$$p\left(\frac{B^I + 2k + 1}{B^I + k + 1} - 1\right)^{\alpha} - (1-p)\,\lambda \leq 0 \tag{2}$$

$B^S (> 0)$ is the integer defined by

$$-\lambda + p\left(\frac{B^S + 2k}{B^S + k}\right)^{\alpha} > 0 \tag{3}$$

and

$$-\lambda + p\left(\frac{B^S + 2k + 1}{B^S + k + 1}\right)^{\alpha} \leq 0 \tag{4}$$

We have following proposition:

Proposition 3 *In any Nash equilibrium s^*,*

(i) if $B \leq 0$, then $n_F(z(s^*)) = 0$;

(ii) if $B > 0$ and $n \leq B$, then $n_F(z(s^*)) = n$;

(iii)if $n > B > 0$, then $n_F(z(s^*)) = B$.

Proof: by proposition 1 and 2, we have $n_L(z(s^*)) = 0$. Following we prove for the integration process. The segregation process follows similarly.

(i) Notice that, from inequation (**??**), $B^I \leq 0$ implies $(\frac{p}{(1-p)\lambda})^{1/\alpha}k - k - 1 \leq 0$. Suppose we have $n_F(z(s^*)) > 0$, then it follows that for any player $i \in F(z(s^*))$

$$p\left(\frac{k}{n_F(z(s^*)) + k}\right)^{\alpha} - (1-p)\lambda > 0$$

which yields

$$1 \leq n_F(z(s^*)) < (\frac{p}{(1-p)\lambda})^{1/\alpha}k - k$$

In contradiction with $(\frac{p}{(1-p)\lambda})^{1/\alpha}k - k - 1 \leq 0$.

(ii) When $B^I > 0$ and $n \leq B^I$, it follows that

$$p\left(\frac{k}{n + k}\right)^{\alpha} - (1-p)\lambda \geq 0$$

Since it is impossible to have $n_F(z(s^*)) > n$, suppose by way of contradiction that $n_F(z(s^*)) < n$. This implies $D(z(s^*))$ is not empty. Suppose player $i \in D(z(s^*))$ unilaterally deviates and chooses to bet on the favorite. Let z' denote the outcome after deviation, then this player i's decision weighted value is

$$V_i(z') = p\left(\frac{n_L(z') + k}{n_F(z') + k}\right)^{\alpha} - (1-p)\lambda$$

Notice $n_F(z') \leq n$ and $\frac{n_L(z')+k}{n_F(z')+k} \geq \frac{k}{n+k}$, combined with above inequation, we have

$$V_i(z') \geq p\left(\frac{k}{n + k}\right)^{\alpha} - (1-p)\lambda > 0$$

which contradicts to s^* being a subgame perfect equilibrium.

(iii) By the construction of B^I, obviously $n_F(z(s^*)) > B^I$ can not be supported as an equilibrium outcome. Suppose $n_F(z(s^*)) + 1 \leq B^I$, then by the construction of B^I, we get

$$p\left(\frac{k}{n_F(z(s^*)) + 1 + k}\right)^{\alpha} - (1-p)\lambda > 0$$

Since $n > B^I$ and $n_L(z(s^*)) = 0$, it follows that $D(z(s^*))$ is not empty. Consider the *last* player who refrains from betting, player i. It is then clear that the $n - i$ players after player i all choose to bet on the favorite, and that there are $n_F(z(s^*)) - n + i$ bets before stage i.

Now suppose player i unilaterally deviates and chooses to bet on the favorite. This deviation does not affect actions of players who move earlier than player i, thus at the end of stage i, there are $n_F(z(s^*)) - n + i + 1$ bets on the favorite and zero bet on the longshot. Let z' denote the terminal history after i's deviation, and let $n_L^{-i}(z')$ and $n_F^{-i}(z')$ respectively denote the bets on the longshot and the favorite *after* stage i. Player i's decision weighted value is then

$$V_i(z') = p\left(\frac{n_L^{-i} + k}{n_F(z(s^*)) + i + 1 - n + n_F^{-i} + k}\right)^{\alpha} - (1-p)\,\lambda$$

Notice $n_F^{-i} \leq n - i$ and $n_L^{-i} \geq 0$

$$\frac{n_L^{-i} + k}{n_F(z(s^*)) + i + 1 - n + n_F^{-i} + k} > \frac{k}{n_F(z(s^*)) + 1 + k}$$

It follows then

$$V_i(z') > 0$$

in contradiction with s^* is subgame perfect equilibrium.

The equilibrium outcome is clear in case (i) and (ii), therefore now we focus on (iii) and characterize the sequence of play for it.

Proposition 4 *If $n > B > 0$ and s^* is a subgame perfect equilibrium, then in the subgame following any h^t with $n_F(h^t) \leq B$ and $n_L(h^t) = 0$, bets realized according to s^* is such that zero bet on the longshot and at most*

$B - n_F(h^t)$ *bets on the favorite.*

Proof: The proof is similar to proposition 3, case (iii).

Theorem 1 *If $n > B > 0$, then in the outcome z^* supported by subgame perfect equilibrium s^*, the first B players bet on the favorite, and the others refrain from betting.*

Proof: Let h^{*i} denote the history up to stage i and a_i denote the action of player i in z^*. Suppose by way of contradiction that in z^* there exists a player $j > B$ such that $a_j = F$. By proposition 3, we know that in any Nash equilibrium outcome, there are B bets on the favorite and zero bet on the longshot. Thus $n_F(h^{*B}) < B$, and there exist at least one player $i \leq B$ such that $a_i = D$. Consider the unilateral deviation of player i such that she choose to bet on the favorite now. This deviation does not affect the actions of players who move earlier than i, so after stage i, there are $n_F(h^{*i}) + 1 \leq B$ bets on the favorite and zero bets on the longshot. Since others are still playing subgame perfect strategy, by proposition 4, bets realize in the subsequent periods should be zero bet on the longshot and at most $B - n_F(h^{*i}) - 1$ bets on the favorite. Thus in the outcome after deviation, z', there are at most B bets on the favorite and zero bet on the longshot, and player i receives positive decision weighted value. A contradiction to z^* is an equilibrium outcome.

4 The Bias: Integration vs. Segregation

Solving the inequations defining the equilibrium betting volume under the integration process B^I and the segregation process B^S, (**??**), (**??**), (**??**), and

(**??**), we have B^I to be the integer defined by

$$Max\left\{0, Int^I\right\}$$

where Int^I is the integer in the interval of $\left[\frac{p^{1/\alpha}}{(1-p)^{1/\alpha}\lambda^{1/\alpha}}k - k - 1, \frac{p^{1/\alpha}}{(1-p)^{1/\alpha}\lambda^{1/\alpha}}k - k\right)$. And B^S is the integer defined by

$$Max\left\{0, Int^B\right\}$$

where Int^B is the integer in the interval of $\left[\frac{p^{1/\alpha}}{\lambda^{1/\alpha}-p^{1/\alpha}}k - k - 1, \frac{p^{1/\alpha}}{\lambda^{1/\alpha}-p^{1/\alpha}}k - k\right)$.

As shown in figure 1, if bettors apply integration process, the price they pay is combined with the risky alternative. This limits the loss that individuals perceive and thus encourage betting. Whereas in the segregation process, the price is segregated from the risky alternative and thus framed as prior loss. Due to the heightened sensitivity to losses as compared to equivalent gains, bettors are less willing to buy the risky alternative comparing to the integration process.

Let ρ_i denote the ratio of aggregate bets on horse i ($i \in \{F, L\}$) divided by the total bets on two horses. With zero track take, rational players who maximize expected payoff evaluate one unit of bet on horse i with winning probability p_i, $i \in \{F, L\}$, as

$$p_i(\frac{1}{\rho_i} - 1) + (1 - p_i)(-1)$$

where $p_F = p$ and $p_L = 1 - p$. Rational decision making theory suggests that in equilibrium we should have

$$p_F(\frac{1}{\rho_F} - 1) + (1 - p_F)(-1) = p_L(\frac{1}{\rho_L} - 1) + (1 - p_L)(-1)$$

which implies

$$\frac{\rho_F}{p_F} = \frac{\rho_L}{p_L} = \kappa \tag{5}$$

where κ is a constant. Since $\rho_F+\rho_L = p_F+p_L = 1$, we have $\kappa = 1$. It follows that in markets where all players maximize expected payoff, in equilibrium we have

$$\rho_i = p_i \text{ for } i \in \{F, L\} \tag{6}$$

hence ρ_i can be interpreted as the market estimate of horse i's objective winning probability. Obviously, when markets are efficient, odds should perfectly reveal horses' objective winning probability. And we say there exists the bias if $p_F > p_L$ and

$$\frac{\rho_F}{p_F} = k_F < \frac{\rho_L}{p_L} = k_L$$

This occurs when there are not sufficient bets on the favorite or too many bets on the longshot. In our model, the market estimate of the favorite's winning probability is $\rho_F = \frac{B+k}{B+2k}$. Notice that ρ_F is increasing in B. Define ρ_F^I(respectively ρ_F^S) be the equilibrium market estimate of the favorite's winning probability when players apply integration (segregation) process. Using the inequations defining B^I and B^S, when $B^I > 0$ and $B^S > 0$, we get

$$\frac{kp^{1/\alpha} - (1-p)^{1/\alpha}\lambda^{1/\alpha}}{kp^{1/\alpha} + (k-1)(1-p)^{1/\alpha}\lambda^{1/\alpha}} \leq \rho_F^I < \frac{p^{1/\alpha}}{p^{1/\alpha} + (1-p)^{1/\alpha}\lambda^{1/\alpha}} \tag{7}$$

and

$$\frac{kp^{1/\alpha} + p^{1/\alpha} - \lambda^{1/\alpha}}{kb^{1/\alpha} + p^{1/\alpha} - \lambda^{1/\alpha}} \leq \rho_F^S < \frac{p^{1/\alpha}}{\lambda^{1/\alpha}} \tag{8}$$

We measure the degree of the bias by the ratio of market estimates divided by objective winning probabilities

$$\tau_i = \frac{\rho_i}{p_i} \; i \in \{F, L\}$$

Again, since $\rho_F + \rho_L = p_F + p_L = 1$, $\tau_F < 1$ implies $\tau_L > 1 > \tau_F$, which implies the existence of the bias. The smaller the τ_F, the worse are markets in estimating horses' objective winning probability, and thus the severer the bias. Let $\tau_F = \tau_F^I$ when players apply integration process, and let $\tau_F = \tau_F^S$ when players apply segregation process.

Consider first the bias when the segregation process is used. Since $\lambda > 1$, from **??** we know that $\tau_F^S = \frac{\rho_F^S}{p} < \frac{p^{1/\alpha - 1}}{\lambda^{1/\alpha}} < 1$. Thus, if players apply segregation process, the bias always emerges.

We now turn to the bias when players using integration process. A closer examination of τ_F^I shows that, in principle, both $\tau_F^I > 1$ and $\tau_F^I < 1$ can be possible. To see this, notice that

$$\tau_F^I = \frac{\rho_F^I}{p} < \frac{p^{1/\alpha - 1}}{(1-p)^{1/\alpha} \lambda^{1/\alpha} + p^{1/\alpha}} = \frac{1}{(1/p - 1)^{1/\alpha} \lambda^{1/\alpha} p + p}$$

thus when

$$(1/p - 1)^{1/\alpha} \lambda^{1/\alpha} p + p > 1$$

we have $\tau_F^I < 1$, which is equivalent to

$$(1/p - 1)^{1/\alpha} \lambda^{1/\alpha} > \frac{1}{p} - 1$$

This inequation holds when both α and λ are sufficiently large. Whereas, when α and λ are sufficiently small, we may have $\tau_F^I > 1$, which implies the *inverse* bias. Think of following numerical example:

$$p = 4/5, \lambda = 2, k = 40, a1 = 1/4, a2 = 1/2, a3 = 4/5$$

where $a1$, $a2$ and $a3$ are values for the parameter α. It can easily be shown that, for different values of α, τ_F^I are respectively

$$a1 = 1/4, \tau_F^I = 1.18 \tag{9}$$

$$a2 = 1/2, \tau_F^I = 1 \tag{10}$$

$$a3 = 4/5, \tau_F^I = 0.88 \tag{11}$$

This can be more intuitively shown by figure 2. Suppose the market is already at $\rho_F^I = 4/5$, and players are free to withdraw or increase bets. We want to see whether $\rho_F^I = 4/5$ can be supported as an equilibrium outcome, and if not, how ρ_F^I will change in order to arrive at a new equilibrium.

As shown in figure 2, the decision weighted value of one unit of bet on the favorite, depending on the value of α, is

$$\frac{4}{5}v(\frac{1}{\rho_F^I} - 1) + \frac{1}{5}v(-1) = \frac{4}{5}v(0.25) + \frac{1}{5}v(-1)$$

which can be represented respectively by $V1$, $V2$ and $V3$. Thus when $\alpha = 4/5$, players refrain from betting well before $\rho_F = 4/5$ is reached, which results in the bias. Whereas when $\alpha = 1/4$, after reaching $\rho_F = 4/5$, players still find betting on the favorite attractive and thus continue to bet on the favorite, which results in the inverse bias. This effect can also been seen in figure 3, where the inverse bias occurs when p is sufficiently large and λ is small.

Intuitively, this inverse bias is due to the combined effects of loss aversion and concavity (convexity) of the value function. Though we have little evidence about the exact shape of individuals' value function, we tend to believe that rarely individuals simultaneously possess small α and λ, which implies strong risk aversion (risk seeking) in the gain (loss) domain and low loss aversion. And based on experimental evidence, Tversky and Kahneman (1992) suggest that the median value of α and λ are respectively 0.88 and 2.25, which is unlikely to result in the inverse bias.

Since ρ increases with B and $B^I \geq B^S$, we always have $\tau_F^I \geq \tau_F^S$, thus the bias is severer in the segregation process (figure 4 and 5).

As shown in figure 5, when p approaches to one, τ_F^I converges to one:

$$\underset{p \to 1}{Lim}\ \tau_F^I \geq \underset{p \to 1}{Lim} \left(\frac{kp^{1/\alpha} - (1-p)^{1/\alpha}\lambda^{1/\alpha}}{\left(kp^{1/\alpha} + (k-1)(1-p)^{1/\alpha}\lambda^{1/\alpha}\right)p} \right) = 1$$

This is because when p increases, players put less weight on the loss, this encourages betting and decreases the degree of bias.

But interestingly, τ_F^S seems rather insensitive to the increase of p:

$$\underset{p \to 1}{Lim}\ \tau_F^S \leq \underset{p \to 1}{Lim} \left(\frac{p^{1/\alpha}}{\lambda^{1/\alpha}} \right) \approx (\frac{1}{\lambda})^{1/\alpha}$$

This can also be seen from players' decision weighted value function. Since the one unit bet is perceived as an ex ante loss, it is uncorrelated with p. Of course, it would be inappropriate to predict the existence of severe bias even when $p = 1$, since, as prospect theory suggests, people perceive rather differently when one thing is certain and almost certain.

When $\alpha = 1$, since there is no transaction cost in our model, traditional theory usually suggests the absence of the bias. But in our model, the range of ρ_F^I and ρ_F^S becomes:

$$\frac{kp - (1-p)\lambda}{kp + k(1-p)\lambda - (1-p)\lambda} \leq \rho_F^I < \frac{p}{p + (1-p)\lambda}$$

and

$$\frac{kp + p - \lambda}{kb + p - \lambda} \leq \rho_F^S < \frac{p}{\lambda}$$

Since $\lambda > 1$, it follows that ρ_F^I and ρ_F^S are strictly smaller than p. Thus, as figure 6 shows, the bias occurs even then.

5 Conclusion

Behavioral assumptions motivated by empirical and experimental evidence such as the non-linear transformation of probability into decision weights, mental accounting, etc., have been adopted to shed new lights to the bias. In this paper, we combine loss aversion and mental accounting, and offer a new explanation for the bias. In this model, the bias exists in the absence of risk seeking preference, transaction costs, and non-linear decision weighting function. The degree of the bias is stronger if players apply the segregation process instead of the integration process.

As illustrated in the debate of market efficiency, not all agents need to be informed for a market to be efficient, and a finding of market efficiency does not imply the absence of irrational behavior. What's most important, it is the marginal agent that is crucial to efficient pricing. The presented model suggests that the bias is more severe if players apply the segregation process. But in parimutuel betting markets, it is unlikely that only one type of players exists. The co-existence of both types of players implies the players using the integration process will play a more important role in determining the degree of bias. To see this, notice that these players are more tolerate to the loss, they will continue to bet even if the players applying segregation process find betting unattractive. Thus, in equilibrium, if there are enough integration type of players, only they will be active in the market. In another words, integraton type of players become the marginal players.

The players in parimutuel betting markets consist of not only individuals but also institutional bettors. It is normally believed that institutional bettors are less subject to both biased mental accounting and loss aversion. This fact, by above argument, renders institutional bettors the marginal players, and tends to beat down the degree of the bias further. Consequently, the loss aversion and mental accounting process might play only a minor role in the bias.

But the fact that players applying the segregation process might not affect the market outcome does not imply these players are irrelevant. Rather by understanding how different types of market participants behave, we gain insights about how markets really operate. Moreover, we have little knowledge about the proportion of the segregation type of players. It might be surprisingly large such that market can not afford to lose them. If we understand why they behave, we could induce them to become the integration type by appropriate mechanisms. But the topic, the mechanism design, is of its own interest and shall thus be treated independently.

Figures

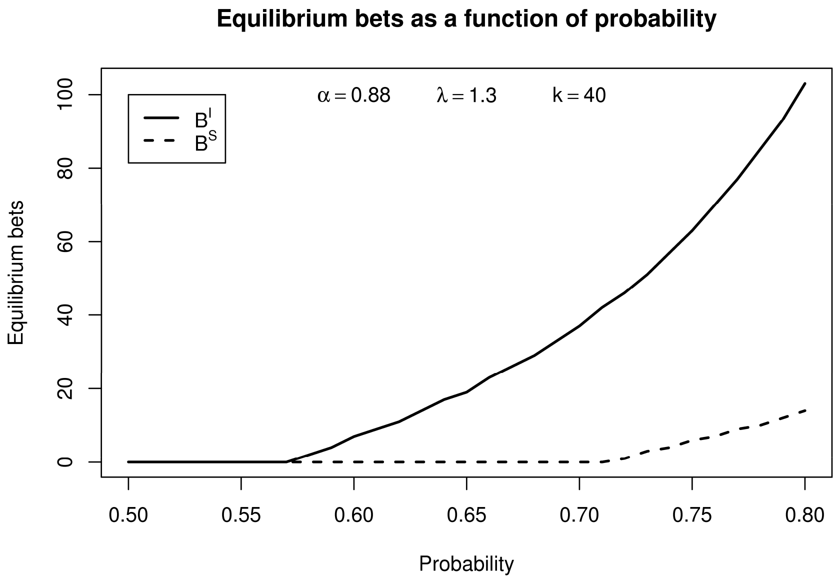

Figure 1: Aggregate equilibrium bets under the integration and the segregation process

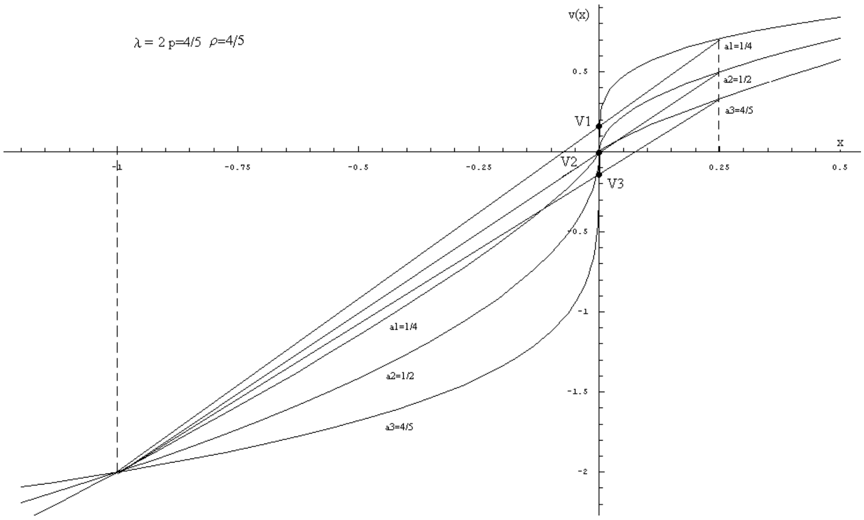

Figure 2: The standard and inverse the bias under the integration process

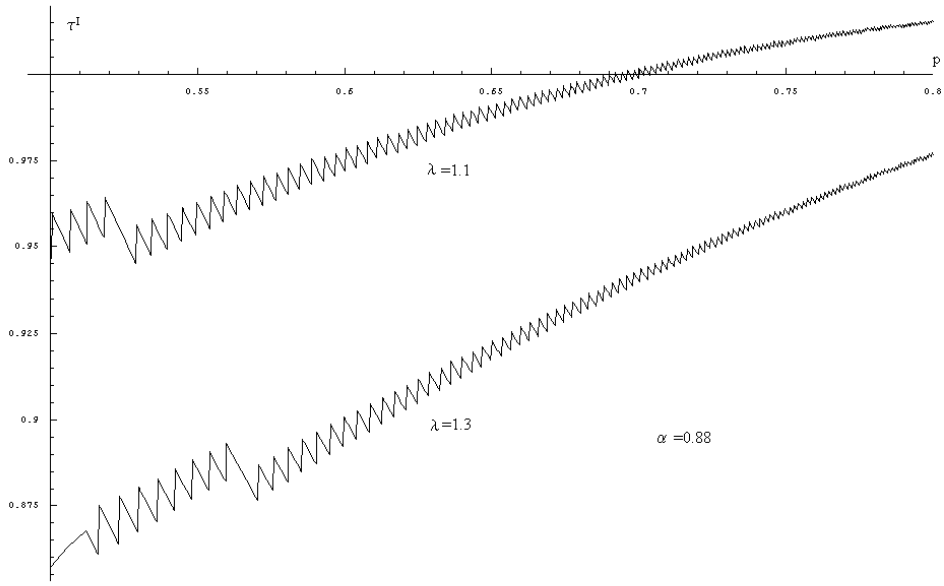

Figure 3: The standard or inverse of the bias: the integration process

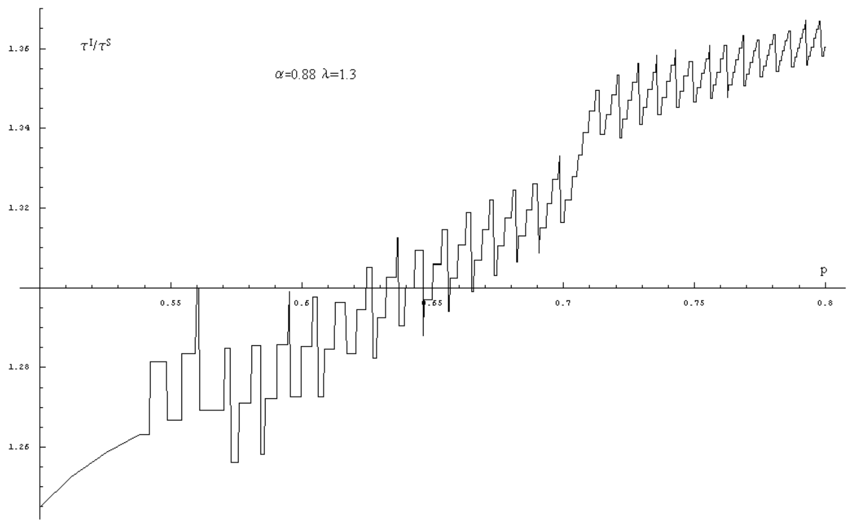

Figure 4: The ratio of the bias under the integration and the segregation process

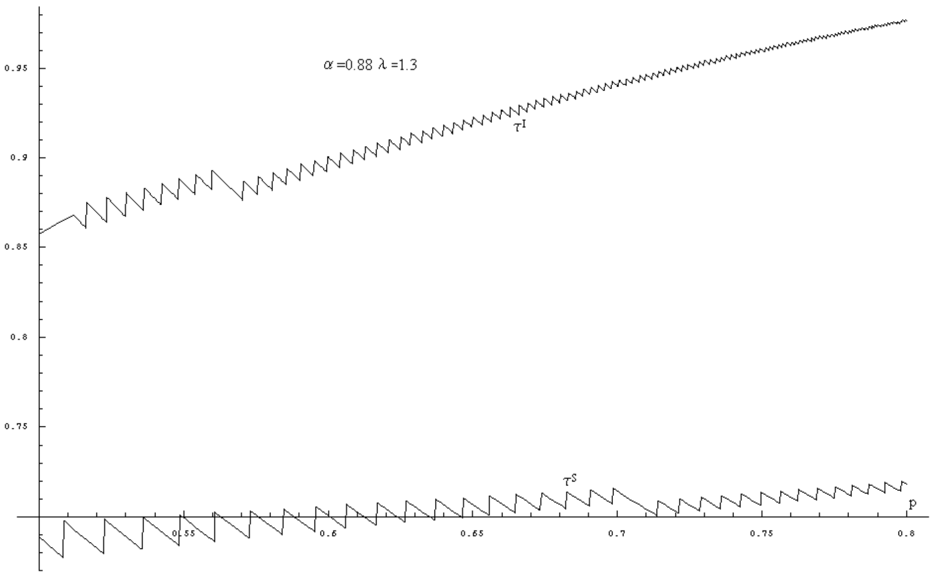

Figure 5: The bias under the integration and the segregation process

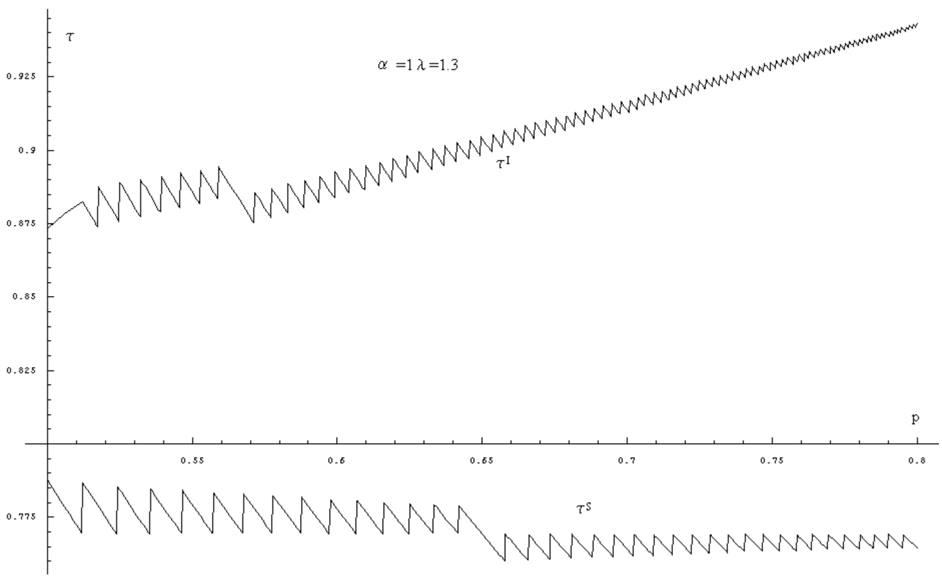

Figure 6: The bias under the integration and the segregation process: when $\alpha = 1$

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