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## Value-at-Risk and expected shortfall for rare events

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No. 2008/14

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A handwritten signature in black ink, appearing to read "Jan Pieter Krahenen".

Prof. Dr. Jan Pieter Krahenen

A handwritten signature in black ink, appearing to read "Volker Wieland".

Prof. Volker Wieland, Ph.D.



CFS Working Paper No. 2008/14

## **Value-at-Risk and Expected Shortfall for Rare Events**

Stefan Mittnik<sup>1</sup> and Tina Yener<sup>2</sup>

April 24, 2008

### **Abstract:**

We show that the use of correlations for modeling dependencies may lead to counterintuitive behavior of risk measures, such as Value-at-Risk (VaR) and Expected Shortfall (ES), when the risk of very rare events is assessed via Monte-Carlo techniques. The phenomenon is demonstrated for mixture models adapted from credit risk analysis as well as for common Poisson-shock models used in reliability theory.

An obvious implication of this finding pertains to the analysis of operational risk. The alleged incentive suggested by the New Basel Capital Accord (Basel II), namely decreasing minimum capital requirements by allowing for less than perfect correlation, may not necessarily be attainable.

**JEL Classification:** C52, G11, G32

**Keywords:** Operational Risk, Latent Variables, Correlated Events

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## 1. INTRODUCTION

Since the initiation of the New Basel Capital Accord (*Basel II*) in 1999 when operational risk was introduced to the regulatory landscape, the attention to this risk type has risen substantially. The Committee (Basel Committee on Banking Supervision (2006)) defines operational risk as “risk of loss resulting from inadequate or failed internal processes, people and systems or from external events.” The fact that events like bookkeeping errors and terrorist attacks are both contained in this characterization illustrates the broad range of risks, especially when compared to credit or market risk. Taking this heterogeneity of loss events into account, the Basel Committee categorizes losses into seven event types and eight business lines. Banks are supposed to calculate risk measures for each of these  $8 \times 7 = 56$  “cells”. Examples are “Internal Fraud” in “Trading and Sales” or “Damage to Physical Assets” in “Commercial Banking”.

The risk measure specified by the Committee is the *Unexpected Loss* at a confidence level of 99.9%. Generally speaking, this refers to the 99.9% quantile of the loss distribution (possibly reduced by the *Expected Loss*, referring to the mean of the distribution). This quantity is also known as *Value-at-Risk* (VaR), which measures the maximum loss that will not be exceeded with a given confidence level and is widely used in financial institutions since the 1990s.

The total risk capital under the *Advanced Measurement Approaches* (AMA) is obtained by summing over all 56 event-type/business-line VaRs, a strategy implicitly expecting the joint occurrence of all loss types involved or, in other words, perfect positive correlation between all loss processes. The Committee takes this into account by allowing a bank “...to use internally determined correlations [...] provided it can demonstrate to the satisfaction of the national supervisor that its systems for determining correlations are sound, implemented with integrity, and take into account the uncertainty surrounding any such correlation estimates (particularly in periods of stress).” (Basel Committee on Banking Supervision (2006)). As moving from the highly unrealistic assumption of perfect dependence (summing the Unexpected Losses of all cells) to an approach relying on estimated correlations should lead to a decrease in risk capital, banks have a strong interest in developing and establishing adequate approaches.

This expected decrease in estimated risk capital caused by a lower correlation of loss processes is the focus of our study. We want to find out if a general statement can be made about how risk capital estimates might be altered by such consideration of less than perfect correlation. Secondly, we want to analyze the impact of the concrete model setup on our findings.

In the following, we concentrate on rare event losses, such as natural catastrophes or terrorist attacks, rather than “everyday losses” such as typical bookkeeping errors. Furthermore, we focus on models well-known from credit risk and reliability theory, but with

broader parameter ranges than those typically considered. We confine ourselves to analyzing the frequency part of operational losses to check for the impact of dependent occurrences and disregard the severity dimension. Therefore, in our notion, “risk” measures the number of event occurrences rather than monetary units.

Since the work of Artzner et al. (1999) it is well-known that Value-at-Risk (VaR) is not a coherent risk measure. To be precise, it lacks the subadditivity property, which would imply in the context of aggregation of operational risk capital that the joint risk measured for two event-type/business-line cells should not be higher than the sum of the individual risks measured for the two cells. This appears to be a reasonable requirement. Unfortunately, the widely used VaR in general does not fulfil the subadditivity criterion. One recommendation is to calculate the marginal contributions of each business line to the overall risk using conditional expectations and *Expected Shortfall* (ES), i.e, the expected loss given that VaR is exceeded (Glasserman (2005)). However, despite its deficiencies, VaR remains to be the dominant risk measure in practice. Therefore, we consider the two risk measures VaR and ES.

The paper is organized as follows. Section 2 defines latent-variable models and describes the relationship between latent and observed correlation. Mixture models as an alternative representation which offers greater flexibility are presented in Section 3. We introduce a simple common Poisson-shock model in Section 4 and present the results from simulating dependent event occurrences in the aforementioned modeling frameworks in Section 5. Conclusions are presented in Section 6.

## 2. EVENT OCCURRENCES IN LATENT-VARIABLE MODELS

**2.1. Latent-variable models.** The idea common to all latent-variable specifications is that there exists a second layer of – possibly observable – latent variables which drive the discrete counting process for the observed loss occurrences. Formally, a latent-variable model (LVM) can be defined as follows, cf. Embrechts et al. (2005).

**Definition** (*Latent-Variable Model*) Let  $X = (X_1, \dots, X_n)'$ ,  $i = 1, \dots, n$ , be a random vector and  $D \in \mathbb{R}^{n \times m}$  a deterministic matrix. Suppose that

$$S_i = j \Leftrightarrow d_{ij} < X_i < d_{i,j+1}, \quad i \in \{1, \dots, n\}, j \in \{0, \dots, m\},$$

where  $d_{i0} = -\infty$ ,  $d_{i,m+1} = \infty$ . Then,  $(X, D)$  is a *latent-variable model* for the state vector  $S = (S_1, \dots, S_m)'$ , where  $X_i$  are the latent variables and  $d_{ij}$  the appertaining thresholds of the latent-variable model.

For our applications, we introduce a new variable  $Y_i$  defined by

$$Y_i = 1 \Leftrightarrow S_i = 0 \quad \text{and} \quad Y_i = 0 \Leftrightarrow S_i > 0,$$

to indicate event occurrence, as we only need to distinguish between the two states of “event occurrence” and “non-occurrence”. The probability of occurrence for individual/process  $i$  is

defined by

$$P(Y_i = 1) = P(X_i \leq d_{i1}) = \pi_i .$$

In the credit risk literature,  $Y_i = 1$  indicates “default” of counterparty  $i$ , meaning that obligor  $i$  cannot make his payments. In structural credit risk models, the latent variable is interpreted as the obligor’s assets; if their value falls below some threshold (the default boundary), the obligor defaults.

This approach can be adapted to suit operational risk settings, but with  $Y_i \in \{0, 1, 2, \dots\}$  the number of loss events rather than the two outcomes “default” or “no default”. As a consequence, the Poisson distribution instead of the Bernoulli distribution is appropriate. The Poisson distribution is a natural candidate since it is an approximation for sums of Bernoulli random variables with low success probabilities. This will be realized below in the mixture model representation.

**2.2. Latent versus observed correlation.** We want to construct a setup in which the probability of the occurrence of an event can depend on events in other processes. Clearly, the probability of a flood damaging equipment will increase when that same event hits a nearby building. Similarly, a system breakdown in one corporate division may propagate to another inducing a failure there. In latent-variable models, this is modeled by allowing for dependence among the latent variables. Thus, dependencies are introduced in an indirect fashion – typically unobservable – latent variables,  $X_i$ , which affect the observed variables,  $Y_i$ .

Restricting ourselves to linear dependence, we distinguish between *latent correlation* among the  $X_i$  and *observed correlation* among the  $Y_i$ , the latter being given by

$$(1) \quad \rho_Y = \frac{\text{Cov}[Y_i, Y_j]}{\sqrt{\text{Var}[Y_i] \cdot \text{Var}[Y_j]}} = \frac{\text{E}[Y_i Y_j] - \pi_i \pi_j}{\sqrt{\pi_i(1 - \pi_i)\pi_j(1 - \pi_j)}} ,$$

where  $\text{E}[Y_i Y_j] = P(Y_i = 1, Y_j = 1) = P(X_i \leq d_{i1}, X_j \leq d_{j1})$  denotes the joint cumulative distribution function of the latent variables associated with processes  $i$  and  $j$ . The observed correlation,  $\rho_Y$ , is often called “default correlation” in the credit risk literature, as opposed to (latent) “asset correlation”,  $\rho_X$ , that refers to the linear dependence between latent variables. From (1) it follows that observed correlations depend on marginal occurrence probabilities,  $\pi_i$  and  $\pi_j$ , and on latent correlation,  $\rho_X$ , the latter entering via  $\text{E}[Y_i Y_j]$ .

**2.3. The distribution of latent variables.** Normal variance mixtures are obvious and widely used candidates for the distribution of latent variables. In normal variance mixtures latent variables can be written as

$$X = \mu + \sqrt{W}Z ,$$

where  $Z \sim N_n(0, \Sigma)$ ,  $W$  is a scalar random variable independent of  $Z$  and  $\mu$  is a constant. An event occurs in process  $i$  when  $X_i \leq d_{i1}$ , or

$$Z_i \leq \frac{d_{i1} - \mu}{\sqrt{W}}.$$

The case of multivariate normally distributed latent variables is achieved by setting  $\mu = 0$  and  $W = 1$ . Alternatively, a joint Student-t distribution can be obtained by letting  $\nu/W \sim \chi_\nu^2$ , where  $\nu$  is the degrees of freedom parameter of the t distribution and  $\chi^2$  denotes the chi-square distribution with  $\nu$  degrees of freedom. This latter model is often cited in the credit risk literature, e.g. Frey et al. (2001), because it has the appealing feature of treating the KMV and the CreditMetrics model as special cases for which  $\nu \rightarrow \infty$ , but admits lower tail dependence and greater flexibility due to the additional parameter. Other types of latent-variable distributions will be discussed below after having introduced the mixture model representation.

### 3. EVENT OCCURRENCES IN MIXTURE MODELS

**3.1. Mixture models.** Mixture models can arise when distributional parameters do not remain constant. For example, it appears to be natural that in times of tectonic plate movements, the probability of an earthquake occurrence rises, that storms are more likely to happen in one season than in others, or that a management change in a global company can affect the probability of fraud. Therefore, in an operational risk context, it seems to be a realistic assumption that the parameters of the assumed distributions might be subject to changes, i.e., be random variables themselves.

A formal definition of a special mixture model in the spirit of Embrechts et al. (2005) is as follows.

**Definition** (*Bernoulli Mixture Model*) Let  $Y = (Y_1, \dots, Y_n)'$ ,  $i = 1, \dots, n$ , be a random vector in  $\{0, 1\}^n$  and  $\Psi = (\Psi_1, \dots, \Psi_p)'$ ,  $p < n$ , be a factor vector. Then,  $Y$  follows a *Bernoulli mixture model* with factor vector  $\Psi$  if there exist functions  $p_i : \mathbb{R}^p \rightarrow [0, 1]$  such that conditional on  $\Psi$  the elements of  $Y$  are independent Bernoulli random variables with  $P(Y_i = 1 | \Psi = \psi) = p_i(\psi)$ .

It is also possible to define  $Y$  as being conditionally Poisson distributed. Then,  $Y$  is a count variable rather than a binary variable, and we obtain a *Poisson mixture model*. Both models can be mapped into each other by setting  $Y = \mathcal{I}_{\tilde{Y} > 0}$  where  $\tilde{Y} \sim Poi(\lambda)$ . The parameters are related via  $p_i = 1 - e^{-\lambda_i}$ , a property we will use to simulate from both models in a comparable way.

To keep the setup simple, we examine only exchangeable mixture models, where conditional probabilities of event occurrence are identical, i.e.,  $p_i(\psi) = p(\psi)$ . Defining the new random variable  $Q = p(\Psi)$ , the observed correlation between indicator variables can then be



obtained from

$$\rho_Y = \frac{\pi_2 - \pi^2}{\pi - \pi^2},$$

where  $\pi = E[Q]$  and  $\pi_k = E[Q^k]$ .

**3.2. Latent-variable models as mixture models.** In fact, latent-variable models and Bernoulli mixture models can be viewed as two different representations of the same underlying mechanism. The following lemma is based on Frey and McNeil (2003).

**Lemma 1.** *Let  $(X, D)$  be a latent-variable model with  $n$ -dimensional random vector  $X$ . If  $X$  has a  $p$ -dimensional conditional independence structure with conditioning variable  $\Psi$ , the default indicators  $Y_i = \mathcal{I}_{X_i \leq d_{i1}}$  follow a Bernoulli mixture model with conditional event probabilities  $p_i(\psi) = P(X_i \leq d_{i1} | \Psi = \psi)$ .*

In case of the latent-variable model  $(X, D)$  where  $X$  is a normal variance mixture and we assume a one-factor structure for  $Z$ , we can write

$$\begin{aligned} X &= \mu + \sqrt{W}Z, \\ Z_i &= \sqrt{\rho_X} \Psi + \sqrt{1 - \rho_X} \varepsilon_i, \end{aligned}$$

where  $\rho_X$  is the latent correlation,  $\varepsilon_i \sim \text{iid } N(0, 1)$ , and  $\Psi \sim N(0, 1)$  is the only factor and conditioning variable. We thus obtain a conditional independence structure for  $X$ , which allows us to proceed using the equivalent mixture model representation. For multivariate normal latent variables with  $\mu = 0$  and  $W = 1$ , the observed conditional default probability is

$$p(\psi) = P(X_i \leq d_{i1} | \Psi = \psi) = \Phi \left( \frac{\Phi^{-1}(\pi) - \sqrt{\rho_X} \psi}{\sqrt{1 - \rho_X}} \right).$$

For a multivariate Student-t distribution the analogous result is

$$p(\psi) = P(X_i \leq d_{i1} | \Psi = \psi) = \Phi \left( \frac{t_\nu^{-1}(\pi) W^{-1/2} - \sqrt{\rho_X} \psi}{\sqrt{1 - \rho_X}} \right).$$

We see that we can easily map the latent-variable models of Section 2.3 into the mixture model setup; at the same time, simulation is much easier, because we do not have to draw from the multivariate normal or multivariate Student-t probability density function.

**3.3. The mixing distribution.** Within the mixture model framework, one can easily allow for different distributional assumptions with respect to latent variables. In our analyses, we consider several examples which are often suggested in risk-management and actuarial applications. In each case the model was calibrated to the multivariate normal latent-variable model, to assess to what extent the choice of mixing distribution affects the number of event occurrences, with the multivariate normal model serving as benchmark.

3.3.1. *Beta Mixing Distribution.* In case of a Beta mixing distribution we assume a mixing variable  $Q = p(\Psi) \sim \text{Beta}(a, b)$ . As the moments of a Beta distribution can be directly calculated from the distributional parameters,  $a$  and  $b$ , we can easily derive unconditional occurrence probabilities from

$$\pi_k = \frac{\beta(a+k, b)}{\beta(a, b)} = \prod_{j=0}^{k-1} \frac{a+j}{a+b+j} ,$$

from which we obtain the observed correlation

$$\rho_Y = \frac{1}{a+b+1} .$$

3.3.2. *Probit Model.* We assume a standard normally distributed factor  $\Psi \sim N(0, 1)$ . Conditional event probabilities have to be determined using the fact that  $Q = \Phi(\mu + \sigma\Psi)$ . Marginal occurrence probabilities are not as easily obtained as in the Beta case, since this involves the integration

$$\pi_k = \mathbb{E}[Q^k] = \int_{-\infty}^{\infty} (\Phi(\mu + \sigma z))^k \phi(z) dz ,$$

making simulations of event occurrences rather complicated. Matters become much easier when recalling that the Probit model is equivalent to a latent-variable model with multivariate normally distributed latent variables. Hence, this model is already covered by the benchmark model described in Section 3.2.

3.3.3. *Latent Variables with Clayton Copula.* The Clayton Copula is a subtype of an Archimedean Copula

$$C(u_1, \dots, u_d) = \phi^{-1}(\phi(u_1) + \dots + \phi(u_d))$$

with generator  $\phi(t) = t^{-\theta} - 1$  being the inverse of the Laplace transform of cumulative distribution function  $G$  on  $\mathbb{R}$ .

Using  $\Psi \sim \text{Ga}(1/\theta, 1)$ , conditional occurrence probabilities can be calculated from  $Q = p(\Psi)$  with

$$Q = p(\psi) = P(U_i \leq \pi | \Psi = \psi) = \exp(-\psi\phi(\pi)) ,$$

where  $U_i \sim \text{Unif}(0, 1)$ . The bivariate occurrence probability is

$$\pi_2 = \phi^{-1}(\phi(\pi) + \phi(\pi)) = (2\pi^{-\theta} - 1)^{-1/\theta} .$$

#### 4. EVENT OCCURRENCES IN COMMON POISSON-SHOCK MODELS

4.1. **A simple common Poisson-shock model.** Adapting the frameworks of Powojowski et al. (2002) and Lindskog and McNeil (2003), one can assume the presence of both idiosyncratic and common Poisson processes. Altogether we assume  $m = n + n_c$  underlying

processes. The number of loss events for the observed loss process  $i$  can be written as

$$Y_i = \sum_{j=1}^m \delta_{ij} M_j \quad i = 1, \dots, n, \quad j = 1, \dots, m,$$

where  $\delta_{ij}$  is an indicator variable which is equal to one if underlying process  $j$  can lead to loss events of observed process  $i$ , and  $M_j$  represents the number of occurrences of underlying process  $j$  with intensity  $\lambda_j$ . Among the  $m$  underlying processes there are  $n_c$  common ones, which affect more than one observed process and are characterized by equal intensities,  $\lambda_c$ . The remaining  $n$  underlying processes with intensities  $\lambda_i^*$  are idiosyncratic in the sense that they only affect the observed process  $i$ .

The correlation between two observed loss event processes,  $k$  and  $l$ , can be written as

$$(2) \quad \rho_{kl} = \frac{\sum_{j=1}^m \delta_{jk} \lambda_j \delta_{jl}}{\sqrt{\sum_{j=1}^m \delta_{jk} \lambda_j \sum_{j=1}^m \delta_{jl} \lambda_j}}.$$

We use a simplified setup comparable to the exchangeable mixture model where idiosyncratic intensities  $\lambda_i^* = \lambda^*$  are identical as well and where all  $n_c$  common processes cause events in all  $n$  observed loss processes. Equation (2) can then be written as

$$\rho_{kl} = \frac{n_c \lambda_c}{\lambda^* + n_c \lambda_c}.$$

For a given  $\lambda = \lambda^* + n_c \lambda_c$  and observed correlation  $\rho_{kl} = \rho$ , we can calculate idiosyncratic and common parts from

$$\begin{aligned} \lambda^* &= \lambda(1 - \rho), \\ n_c \lambda_c &= \lambda - \lambda^*. \end{aligned}$$

## 5. SIMULATION RESULTS

For each of the models discussed above, we simulated event occurrences and estimated risk capital for different levels of latent correlation,  $\rho_X$ . In doing this, we used the multivariate normal latent-variable model as benchmark model to which we calibrated the other models. Throughout the simulations, we assumed  $n = 1\,000$  loss processes, to match with the studies in Frey et al. (2001) and Frey and McNeil (2001). For the mixture models, we simulated a factor realization  $\psi$  and calculated conditional occurrence probabilities  $p(\psi)$  which were then used to conduct  $n$  Bernoulli or Poisson trials. After summing up the number of event occurrences, we repeated 100 000 times and calculated VaR and ES of the resulting empirical distribution. For the common Poisson-shock model, we assumed  $n_c = 1$  and calculated  $\lambda$ ,  $\lambda^*$  and  $\lambda_c^c$  from  $\pi$  and  $\rho_Y$ . These quantities were then used to conduct Poisson trials and proceed further as in the mixture model setup.

For all models and low occurrence probabilities ( $\pi \leq 0.01$ ), we observe a counterintuitive behavior of VaR: it decreases for increasing correlations, this effect being the more

pronounced the lower the confidence level. An illustration of this phenomenon is given in Figure 1, which plots the logarithm of the 99% VaR depending on the level of latent correlation and occurrence probability  $\pi$ . While for  $\pi = 0.01$ , VaR behaves as intuitively expected, i.e., increasing in  $\rho_X$  over the entire range of latent correlations, it clearly declines for lower levels of latent correlation beyond a certain threshold of  $\rho_X$ , with lower thresholds for lower values of  $\pi$ .

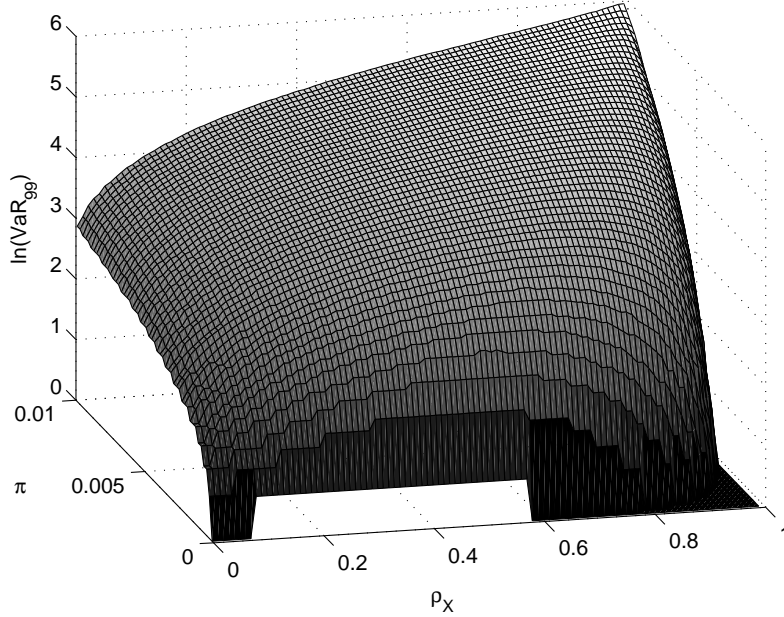


Figure 1: Simulated 99%-VaR (in logs) in the multivariate normal LVM,  $\pi \in [0.0001, 0.01]$

This effect is the more pronounced, the fatter the tails of the distribution of latent variables, as is shown in Figure 2. For  $\nu = 100$ , we observe an increase in VaR up to a latent correlation of  $\rho_X \approx 0.5$  and a decrease for higher levels; the lower  $\nu$ , the broader the range of  $\rho_X$  for which this peculiar behavior occurs. For  $\nu = 4$ , VaR decreases over the entire range of latent correlations. The results for Poisson mixture models are qualitatively the same, as was to be expected from the low level of occurrence probabilities involved. Also, scenarios for the common Poisson-shock model setup can be established which lead to this counterintuitive behavior.

For ES, using 100 000 replications leads to ambiguous results. For very low occurrence probability levels ( $\pi \leq 0.00001$ ), decreases in ES can be observed for increasing  $\rho_X$ . But in contrast to the risk capital estimates based on VaR, this effect vanishes when the number of replications increases to up to 10 million. Figure 3 illustrates that ES behaves as intuitively expected, i.e., it rises over the entire range as  $\rho_X$  grows. Therefore, the counterintuitive behavior has to be taken into account when designing the Monte-Carlo simulation.

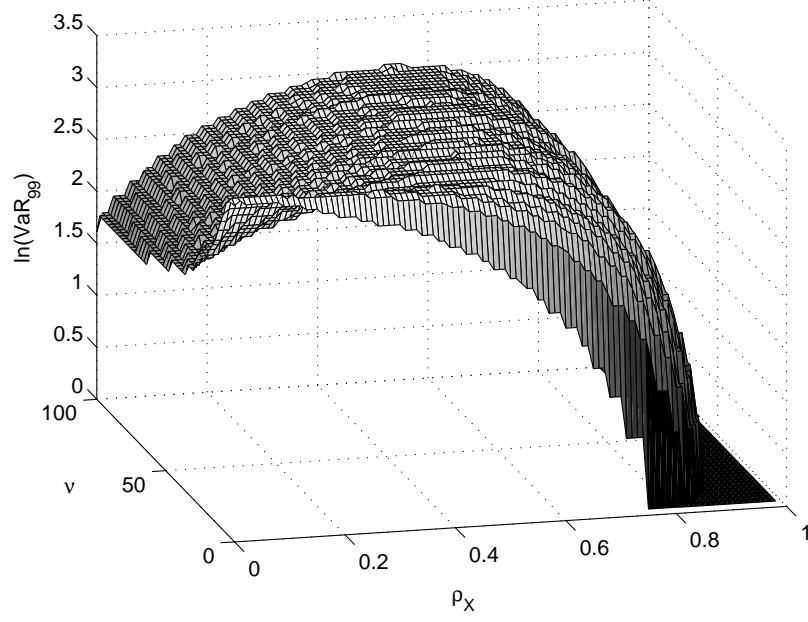


Figure 2: Simulated 99%-VaR (in logs) in the multivariate Student-t LVM,  $\pi = 0.001$ ,  $\nu \in [4, 100]$

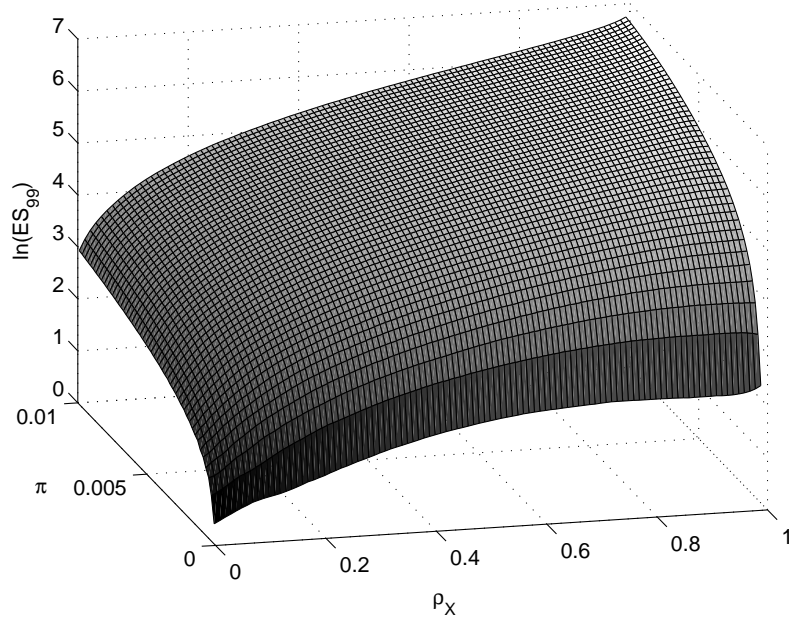


Figure 3: Simulated 99%-ES (in logs) in the multivariate normal LVM,  $\pi \in [0.0001, 0.01]$

Otherwise, simulated ES figures may seem to decrease as  $\rho_X$  rises – just as is in the VaR case.

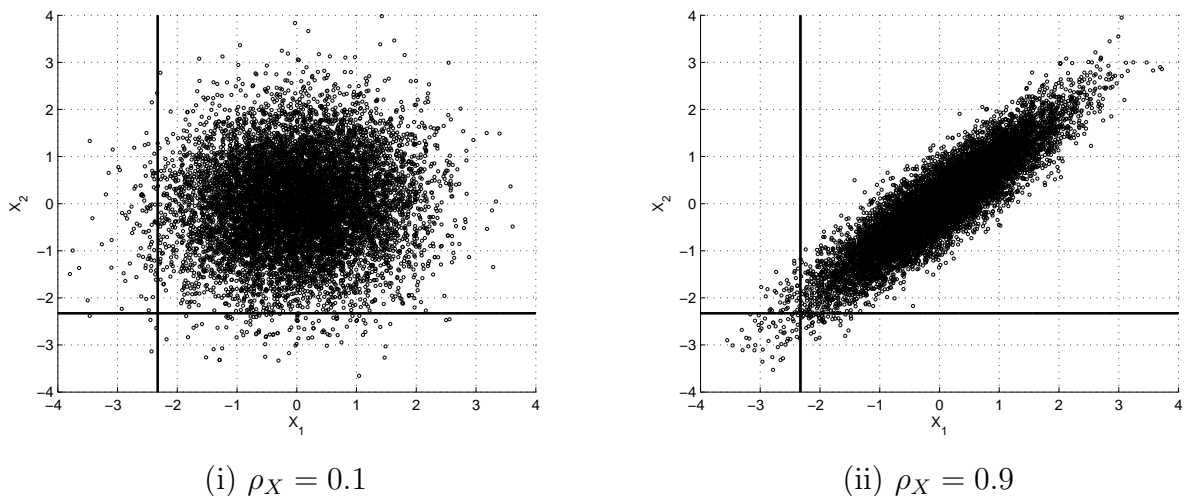


Figure 4: Scatterplots for a bivariate normal distribution

The explanation of this effect is illustrated in Figure 4. It shows 10 000 draws from a bivariate normal distribution for two different correlation assumptions. The solid line represents the thresholds implied by an occurrence probability of  $\pi = 0.01$ . In the left plot, where the latent correlation is  $\rho_X = 0.1$ , this threshold leads to 4 joint “occurrences” (in the southwestern quadrant) and 9,798 joint “non-occurrences”. In the right plot with a higher correlation of  $\rho_X = 0.9$ , the concentration on extremes leads to 94 joint “occurrences” and 9,854 joint “non-occurrences”. As it turns out, high correlation not only leads to more events, but also to more joint “non-events”. It is this phenomenon which moves Value-at-Risk towards zero as correlation levels rise.

## 6. CONCLUSION

Introducing less than perfect dependencies should lead to a more realistic description of loss event occurrences. Our results show that it is very important to assess the impact of correlations within the chosen modeling framework. Be it mixture models, common Poisson-shock models or a different setup, in the case of rare events, simulated values for risk measures, such as Value-at-Risk and Expected Shortfall, can decrease as the level of correlation increases. The parameter ranges for which this phenomenon occurs may not be so relevant for credit risk applications, but may arise in operational risk applications where several business lines at close locations could, for example, be affected by some catastrophic event.

While this effect can be eliminated in the case of Expected Shortfall by an appropriate design of the Monte-Carlo setup, this is unfortunately not so for the widely used Value-at-Risk which systematically declines above certain levels of latent correlations. The extent to which this arises depends on the observed occurrence probabilities, the confidence level and the fat-tailedness of the distribution of the latent variables.

If the clustering of realizations at zero (“joint non-occurrences”) that causes this behavior is a misleading feature of the model which contradicts the true risk-generation mechanisms, risk capital can severely be underestimated, and other dependence concepts should be considered for calculating risk capital.

A practical implication of our analysis is that the inclusion of non-perfect correlations in models used for assessing minimum capital requirements for operational risk may, in fact, lead to an increase of the assessed amount.

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