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## Modelling Aggregate Consumption Growth with Time-Varying Parameters

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# BONN ECON DISCUSSION PAPERS

Discussion Paper 15/2005

## Modelling Aggregate Consumption Growth with Time-Varying Parameters

by

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July 2005



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# Modelling Aggregate Consumption Growth with Time-Varying Parameters

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# Modelling Aggregate Consumption Growth with Time-Varying Parameters

## Abstract

Using the Family Expenditure Survey (FES) data for the United Kingdom (UK), the paper specifies and estimates a 'complete' Hildenbrand Kneip (HK) model of consumption, extending earlier efforts that were 'partial' in nature. As the estimated parameters in the 'partial' HK model are time varying, the paper provides empirical evidence that their movements over time reflect a near unit root process. To estimate the 'complete' HK model, the paper specifies a simple OLS model of the 'remainder term' in the 'partial' HK model. The remainder term in the partial HK model, which as per theory should be influenced by unobservable variables like expectation formation of households, is found to be affected by housing prices. The complete model is found to explain movements in consumption better than the partial model. Results based on bootstrap suggest that given the sampling error in the FES data, the overall fit of the model should be considered as good.

**Journal of Literature Classification:** C4, C5, D1

**Keywords:** Aggregation, Consumption Function, Average Derivative

# Modelling Aggregate Consumption Growth with Time-Varying Parameters

## 1 Introduction

Consumption function is one of the most thoroughly researched topics in quantitative economics. Starting from the early contributions of Keynes, Friedman and Modigliani, numerous studies have specified and estimated consumption functions for different economies, as well as for individual households, a few benchmark studies since the late 1970s being Hall (1978), Flavin (1981), Hall and Mishkin (1982), Epstein and Zin (1989), Zeldes (1989), Cabalero (1990, 1991) etc.<sup>1</sup> As macroeconomic consumption is the aggregate of consumptions of economic agents, it is necessary that the macro specifications and their empirical counterparts should be aggregationally consistent with the micro ones. However, studies on consumption function have so far typically ignored the aggregation problem.

The 'aggregate' consumption functions that have been specified and estimated for different economies could be roughly divided into two types. In studies of the first type, the consumption function emerges as an optimal solution of an intertemporal maximization problem under constraints in an uncertain environment. In contrast, studies of the second type examine the time series relationships of consumption with income and a few other explanatory variables directly at the macro-level, the major purpose of this literature being forecasting.

So far as aggregation is concerned, the first type of studies that start with an optimization problem often run into an uncharted territory. The optimisation problem specified in the microeconomic literature is, in fact, a complicated stochastic dynamic programme. Even for a single individual or household, a closed form analytical solution of this programme exists only in special cases, not to say about the economy as a whole. The existence of a

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<sup>1</sup>The literature is so large that the above list is only indicative and not exhaustive. An informal Google search entitled 'Consumption Function', in fact, leads to about 5 million web pages. Deaton (1992) reviews this literature till the late 1980s. A comparatively recent survey is Attanasio (1999).

mythical representative agent and subsequent application of micro-theory on aggregate data and that too with other equally 'convenient' assumptions on characterising uncertainty, is, therefore, an often practiced way-out for shying away from this problem.<sup>2</sup>

Despite its popularity, the representative agent approach has been severely criticized in the literature. In a stinging criticism, Kirman (1992) calls this approach, in a more general context, being 'fatally flawed' (p. 132). In the specific context of consumption, Deaton (1992) admits the possibility of this approach being a 'convenient fiction' (p. 37). At the same time, developments in direct surveys on income expectations and laboratory experiments during the 1990s have raised uncomfortable questions on expectation formation and their ad hoc treatment in the economic literature (Dominitz and Manski, 1997; Dominitz, 1998; Das and van Soest, 1999; Dominitz, 2001). These developments support to explore the possibility that the specification of behavioral relation at the individual level may not be necessary to examine relations among aggregates (Hildenbrand, 1998).

Benchmark examples of studies of the second type, that directly model the aggregate series without specifying a behavioral relation at the individual level are Davidson et al (1978) and Hendry et al (1981) (DHSY and HUS respectively). These studies, both on UK data, are benchmarks for studies on other countries as well. Forecast performances of these models, during the late 1980s, however, were not good. Two factors that are believed to have led to poor performance of these models are (i) financial liberalisation during the 1980s (Muellbauer and Murphy, 1990), (ii) expectational factors, especially during the year 1987-88 (King, 1990). There is evidence that a generalised model with time varying parameters explain and forecast variations in consumption in a better manner (Song et al, 1998; Eliasson, 1999), though Stewart (1998) warns that such approach should be viewed with caution. In these studies, the time varying parameters are specified in an ad hoc manner. Attempts have been made to justify their use through LCH/PIH models (Song et al, 1998). However, the LCH/PIH interpretations of parameters in these models on macro-data cannot be considered appropriate unless these models become aggregationally consistent.

An alternative approach to derive the aggregate consumption function without relying on the specification of a utility function is the distributional

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<sup>2</sup>For example, the often cited study of Hall (1978) has assumed a representative agent framework.

approach. Traditional economic theory has a limited role in these models in the sense that the theory only helps to identify the set of variables (e.g., income, wealth etc.) that would explain individual or household consumption. The distributional approach focuses on a few 'invariants' of the distributions of explanatory variables and based on these invariants, derives the specification for aggregate period-to-period change or growth in consumption. The major purpose is to explain this aggregate change or growth in consumption in terms of the moments (or other summarized measures) of these explanatory variables. Though these models are aggregationally consistent, majority of the existing studies based on distributional approach have not attempted an explicit treatment of unobservable variables (e.g. expected future income). Further, empirically, the aggregate relationship derived in these models do not allow parameter variability over time.<sup>3</sup>

Recently, based on the distributional approach, Hildenbrand and Kneip (2005) (HK for short) have derived an aggregate relationship for the relative changes in consumption that allows parameter variability over time. A novelty of this approach is that some time-varying parameters in the model can be estimated from cross-section data, using non-parametric average derivative estimation techniques. The HK model allows a 'partial' analysis, i.e., to quantify the effect of a change in the observable explanatory variables *independently of each other*. It does not require a joint estimation of all the parameters as in traditional OLS, GLS or time series models or estimation by the maximum likelihood method, neither does it require an explicit treatment of the unobservable variables. An empirical application of this 'partial' model based on the consumption data in UK yields reasonable estimates of the parameters and indicate a moderate fit. Two recent applications on HK model are Chakrabarty and Schmalenbach (2002a, 2002b) (CS1 and CS2 for short respectively). CS1 have estimated the impact of current labor income on a few broad commodity groups in a 'partial' framework. Based on this framework, CS2 have proposed an empirical test of the representative agent hypothesis.

While these studies highlight innovative approaches on aggregation and estimation of parameters in a model, both suffer from a few limitations. First, the empirical findings of HK and CS1 were all based on annual UK data for a short period. Even within this limited span, lack of availability in data

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<sup>3</sup>Stoker (1993) and Blundell and Stoker (2000) provide detailed reviews on different approaches to the problem of aggregation, including the distributional approach.



in a particular year created problems in forming a continuous time series. Thus, the limited number of annual observations were serious impediments in examining parameter stabilities in these models. Second, the limited time span also meant that a systematic study of the time series properties of the estimated coefficients and the 'remainder term' were also not possible.

This paper attempts to extend the earlier findings on HK model in three different ways. First, we present the HK model with more frequent and exhaustive observations on a continuous quarterly series based on FES data starting from the first quarter of 1968 to the fourth quarter of 1993. Although the FES data in the year 1978 experienced problems, availability of alternative data sources enables us to prepare a continuous series. A 'partial' analysis based on this data set yields a time series of estimated coefficients and the 'remainder term'. Our second extension is a possible empirical characterisation of the stochastic properties of these coefficients. Our third extension is the estimation of a 'complete' HK model. In this paper, we specify a time series model of the 'remainder term'. Empirical performances of the 'partial' and the 'complete' HK model are then compared. As our empirical models are based on sample data, a bootstrap analysis is carried out as a cross-check, to assess the limits of explanatory power one could expect from the sample. This part also examines to what extent the final residuals in the estimated models are affected by period-to-period sampling errors.

We stress that in the context of HK model, the last two tasks could be important. So far as the time-varying parameters are concerned, discovery of a specific stochastic structure in them could simplify the estimation procedure in HK model. As estimation of the 'partial' HK model is carried out on independent cross-sections for several years, data requirement is huge. With a structured set of coefficients, only a few summary measures from these cross-section might turn out to be adequate for an estimation of the 'partial' HK model in a time series framework.<sup>4</sup> Although CS2 have examined whether the time series of the estimated coefficients in the 'partial' HK model have unit root properties, due to limited number of observations and data gaps in their studies, further tests are needed. So far as the 'remainder term' is concerned, we stress that estimates of successive such terms in the HK model are obtained from independent cross-sections. Therefore, a

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<sup>4</sup>For example, if the estimated coefficients follow AR(1) processes with drift, the 'partial' HK model reduces to a random coefficient model and may be estimated by time series techniques.

time series obtained from them by combining results from each period will not satisfy the common properties of standard time series residuals. Any predictable pattern detected in these residuals then raises the possibility of extending the explanatory power of the 'partial' HK model. We also stress that the modelling of the remainder term would be like a macro-model of consumption *sans* the effect of commonly used household specific explanatory variables like income or wealth. As the effects of these key explanatory variables are filtered out using innovative techniques, it is likely that the true impact of the other variables could be better assessed. In particular, the relationship of the remainder term with the variables that act like proxies of the so called 'unobservables' could be worth investigation. Examples of such variables could be changes in unemployment rate, real returns from the asset market and inflation in housing prices.

The plan of the paper is as follows: Section 2 briefly recapitulates the HK approach and Section 3 elaborates the estimation technique. Section 4 describes the data used in this study in detail, providing cross-verification of different series from alternative sources. Section 5 presents the partial HK model based on quarterly data. In this section, we also examine the time series properties of the estimated coefficients in the partial HK model. Section 6 augments the partial HK model further by specifying a time series model for the remainder term. In particular, we examine the relationship of the remainder term with some macroeconomic variables that could influence expectation formation of the economic agents. The empirical performance of this augmented model is then compared with the partial model. Section 7 carries out a bootstrap analysis to know the limits of explanatory power one could expect from the FES sample. This section, thus provides a cross-check to the estimated models. Finally, Section 8 summarizes the major findings and concludes the paper with a few comments and observations.

## 2 The Analytical Framework

The details of the analytical framework adopted in this paper are available in HK. CS1, in their study, have also reviewed the approach. However, for the sake of completeness, we repeat the specification of HK in brief.

The major focus in the distributional approach adopted by HK is to explicitly model the aggregation process rather than taking the convenient assumption of representative agent. Let  $C_t$  be the aggregate consumption

expenditure in time  $t$ . The HK model attempts to explain the variations in  $\Delta_s C_t$  or  $\Delta_s(\log(C_t))$ , where time  $(t - s)$  is close to time  $t$ .

Let  $c_t^h$  be the consumption expenditure of household  $h$  at time  $t$ . HK organizes all the variables that could explain  $c_t^h$  into three groups, viz., (i) micro-specific observable variables (e.g., income, wealth and household specific attributes), (ii) micro-specific unobservable variables (e.g., expected future income) and (iii) observable, but not micro-specific variables (e.g., prices, interest rates etc.). Let these three sets of variables be denoted by the vectors  $y_t^h$ ,  $v_t^h$  and  $p_t$  respectively. Each of  $y_t^h$ ,  $v_t^h$  and  $p_t$  may contain components which refer to periods  $(t - 1)$ ,  $(t - 2)$  etc. Thus,

$$c_t^h = c(y_t^h, v_t^h, p_t) \quad (1)$$

where  $c(\cdot)$  is a continuously differentiable function in all variables.

It is relevant to highlight the difference between the HK model and traditional models of consumption that begin with the specification of a utility function. In the latter, the problem of optimisation of consumption over life cycle becomes a complicated stochastic dynamic program whose closed-form analytical solutions, even for a single household, are available only for specific cases. Economic theory, however, identifies a set of variables that are used to explain movements in consumption, both at micro and the macro level. In principle, all these variables may be considered as explanatory variables in the right hand side of equation (1). Thus the specification of the consumption function of the households in the HK model is general enough to accommodate a broad set of traditional models on consumption. However, a point to note is that this functional form is specified by HK *without* any assumption on individual or household preferences.

Aggregate consumption expenditure  $C_t$  in period  $t$  is defined as

$$C_t = \frac{1}{\#H_t} \sum_{h \in H_t} c_t^h \quad (2)$$

where  $\#H_t$  denotes the number of households in the population  $H_t$ .

In their distributional approach of aggregation, HK express the period-to-period change or relative change in  $C_t$  in terms of the joint distribution of the explanatory variables  $y_t^h$  and  $v_t^h$ . To derive estimable functional forms of the aggregates, they model the evolution of this joint distribution over time. The heterogeneity of the population in income, consumption expenditure, and household attributes is essential in this approach.

Let the "standardized values" of  $y_t^h$  be denoted by  $z_t^h$ .<sup>5</sup> HK hypothesized that the distribution of  $z_t^h$  would be time-invariant for two periods ( $t - s$ ) and  $t$  that are sufficiently close to each other. Further, it is hypothesized that for two periods ( $t - s$ ) and  $t$  that are sufficiently close to each other, the distributions of the attribute profiles for same values of  $z_t^h$  also remain time invariant. It may be noted that these hypotheses can be tested from data and empirical findings based on income distributions seem to support them (Hildenbrand et al, 1998). In addition, a few sufficiently general restrictions on the distributional structure of  $v_t^h$  are assumed.

With the hypotheses and the assumptions, HK showed that the knowledge of the functional form of  $c(\cdot)$  is not essential to derive estimable functional forms of the period-to-period change or growth in aggregate consumption, as Taylor series approximations for the aggregates could be obtained as long as the function  $c(\cdot)$  is continuously differentiable in all its arguments.

In this paper, we simplify the aggregate relationship by assuming  $y_t^h$  being two dimensional, consisting of current income and wealth only. We also attempt to explain the relative change in consumption, i.e.,  $\frac{C_t - C_{t-s}}{C_{t-s}}$  for any two periods  $s$  and  $t$  that are sufficiently close to each other. The aggregate relationship could then be written as:

$$\frac{C_t - C_{t-s}}{C_{t-s}} = \sum_{i=1,2} \left[ \beta_{(t-s)i} (m_{ti} - m_{(t-s)i}) + \gamma_{(t-s)i} \frac{(\sigma_{ti} - \sigma_{(t-s)i})}{\sigma_{(t-s)i}} \right] + \Psi_t \quad (3)$$

The major innovative appeal in equation (3) is the use of independent cross-section data for different time periods to estimate  $\beta_{(t-s)i}$  and  $\gamma_{(t-s)i}$  ( $i = 1, 2$ ). Here  $\beta_{(t-s)i}$  and  $\gamma_{(t-s)i}$  ( $i = 1, 2$ ) are coefficients that are by definition average derivatives of observable regression functions. One may use non-parametric techniques on the cross-section data involving consumption and the respective micro-specific observable variables to estimate them. Therefore, these coefficients are directly related to the micro-data in period ( $t - s$ ) and they do not depend on the postulated micro-relation. A major implication of this approach is that estimations of  $\beta_{(t-s)i}$  and  $\gamma_{(t-s)i}$  may be carried out *independently of each other and independent of  $\Psi_t$* . HK showed that both  $\beta_{(t-s)i}$  and  $\gamma_{(t-s)i}$  ( $i = 1, 2$ ) could also be interpreted as elasticities.

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<sup>5</sup>Note that in a multivariate set-up, standardization may be carried out in alternative manners. HK defined  $z_t^h$  in the traditional way involving the mean vector and the dispersion matrix of  $y_t^h$

For example,  $\beta_{(t-s)1}$  is an income elasticity measure and  $\gamma_{(t-s)1}$  is an income dispersion elasticity measure.

Also, in equation (3),  $m_{ti}$  and  $\sigma_{ti}$  denote the means and the standard deviations in period  $t$ , respectively for log-income ( $i = 1$ ) and log-wealth ( $i = 2$ ) distribution.  $\Psi_t$  is the 'remainder term'.

In this paper, we shall call models that ignore  $\Psi_t$  and attempt to explain  $\frac{C_t - C_{t-s}}{C_{t-s}}$  by

$$\sum_{i=1,2} \left[ \hat{\beta}_{(t-s)i} (m_{ti} - m_{(t-s)i}) + \hat{\gamma}_{(t-s)i} \frac{(\sigma_{ti} - \sigma_{(t-s)i})}{\sigma_{(t-s)i}} \right]$$

as the partial or the incomplete HK model. The empirical results on HK model obtained so far are partial and incomplete in nature.<sup>6</sup>

In contrast to  $\beta_{ti}$  and  $\gamma_{ti}$ , an estimate of  $\Psi_t$  would, however, require time series modelling. To model  $\Psi_t$ , we, therefore, need to understand the factors that could affect its movements over time. The  $\Psi_t$  in any period  $t$  is influenced by three factors. First, its movements over time depend on the observable, but not micro-specific variables. Second, it carries the impact of all other variables that are household specific, but not observable. Third, as the aggregate relationship in the HK model is based on a Taylor series approximation of first order,  $\Psi_t$  is also affected by the higher order terms in that Taylor series.

Thus a simple linear specification of the remainder term would be

$$\Psi_t = \sum_{i=1}^K \alpha_i Z_{it} + \epsilon_t \quad (4)$$

where  $\{Z_{it}\}$  is a set of explanatory variables for  $\Psi_t$ . Some of these explanatory variables could be components of  $p_t$  or their transformations. The other explanatory variables could be observable proxies on expectation formation of households. The  $\epsilon_t$  in equation (4) is the residual. In the simplest possible specification, equation (4) may be estimated by OLS.

In this paper, a HK model that also attempts to explain  $\Psi_t$  rigorously would be referred as a "complete" HK model. Note that a complete model

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<sup>6</sup>CS1 included the inflation rate as an additional explanatory variable and examined its impact on the 'remainder term' by specifying an OLS model without a constant term.

would attempt to explain  $\frac{C_t - C_{t-s}}{C_{t-s}}$  by

$$\sum_{i=1,2} \left[ \hat{\beta}_{(t-s)i} (m_{ti} - m_{(t-s)i}) + \hat{\gamma}_{(t-s)i} \frac{(\sigma_{ti} - \sigma_{(t-s)i})}{\sigma_{(t-s)i}} \right] + \hat{\Psi}_t$$

The complete model would thus have a better fit, with traditional statistical residuals or error terms.

### 3 Estimation Techniques

So far as estimation is concerned, the HK approach is flexible enough to accommodate the modelling of both  $\Delta_s C_t$  and  $\Delta_s(\log(C_t))$  where  $s$  and  $t$  are close. As transformations to logarithmic scale is a common practice, we prefer the second option. So far as a choice for  $s$  is concerned, as we use quarterly data in this paper, to avoid the vexing problem of seasonality, we choose  $s = 4$ . Thus, in this paper, we attempt to explain variations in  $\Delta_4(\log(C_t))$ .

Estimates of  $\beta_{ti}$  and  $\gamma_{ti}$  may be obtained through alternative techniques. It is hard to obtain reasonable estimates of average derivative in ranges where data points are sparse. Even in large-scale sample surveys, a direct estimate of average derivatives using data for quarter  $t$  and  $(t - 4)$  is unlikely to have sufficient number of observations in the boundary regions of income and wealth. In this paper, we adopt an adjustment that allows data for an entire year to be utilized in average derivative estimation to explain variations in  $\Delta_4(\log(C_t))$ .

Let the sample size in quarter  $t$  be  $n_t$  and the total sample size between quarters  $t$  to  $(t - 3)$  (i.e., for the past one year at quarter  $t$ ) be  $n_t^*$ . Let  $C_t^*$  be the average consumption for the quarters  $t$  to  $(t - 3)$ . Then, it is easy to show that  $\Delta_4(\log(C_t))$  can be approximated as:

$$\Delta_4(\log(C_t)) = A \cdot \frac{C_t^* - C_{t-1}^*}{C_{t-1}^*} + B \quad (5)$$

with  $A$  and  $B$  defined as

$$A = \frac{n_t^*}{n_t} \cdot \frac{C_{t-1}^*}{C_{t-4}^*}, \quad B = \frac{(n_t^* - n_{t-1}^*)}{n_t} \frac{C_{t-1}^*}{C_{t-4}^*} - \left(1 - \frac{n_{t-4}}{n_t}\right)$$

Note that both  $A$  and  $B$  contain the term  $\frac{C_{t-1}^*}{C_{t-4}^*}$ . This term is likely to be affected by seasonality. However, since both the numerator and the denominators are average per-household consumption, the value of this term is likely to be not too different from unity. Also note that if the sample sizes for each quarter are equal, then  $B = 0$  and  $A = 4 \frac{C_{t-1}^*}{C_{t-4}^*}$ .

We can write

$$\frac{C_t^* - C_{t-1}^*}{C_{t-1}^*} = \sum_{i=1,2} \left[ \beta_{(t-1)i}^* (m_{ti}^* - m_{(t-1)i}^*) + \gamma_{(t-1)i}^* \frac{(\sigma_{ti}^* - \sigma_{(t-1)i}^*)}{\sigma_{(t-1)i}^*} \right] + \Psi_t^* \quad (6)$$

where  $m_{ti}^*$ ,  $\sigma_{ti}^*$  are defined in a similar manner as in equation (3),  $\Psi_t^*$  being the remainder term. In our paper, the parameters  $\beta_{(t-s)i}^*$  and  $\gamma_{(t-s)i}^*$  are estimated by average derivative estimation techniques using the data for one whole year.

We define  $\beta_{ti}^{Adj} = A \beta_{ti}^*$ ,  $\gamma_{ti}^{Adj} = A \gamma_{ti}^*$  and  $\Psi_t^{Adj} = A \Psi_t^* + B$ .

While the micro consumption function  $c_t^h$  can be seen to be influenced in a smooth fashion by variables like income or wealth, there are other household-specific variables that are either integers or are categorical (e.g., age of head of household, economic status of the household, region of domicile, number of children in the household etc.).

This suggests a semiparametric formulation for the estimation of  $c_t^h$ .

$$c_t^h = f_1(y_{t1}^h) + f_2(y_{t2}^h) + \sum_j \vartheta_{tj} a_{tj}^h + \epsilon_t^h \quad (7)$$

Here,  $f_1(\cdot)$  and  $f_2(\cdot)$  represent the smooth dependence of consumption on income ( $y_{t1}$ ) and wealth ( $y_{t2}$ ) respectively, while  $a_{tj}^h$  are household specific attributes with  $\vartheta_{tj}$  as the respective coefficients. Similar in spirit to the approach taken in HK, we used smoothing splines for the estimation of the functions  $f_1(\cdot)$  and  $f_2(\cdot)$ , where the amount of smoothing for each function is determined by an appropriate choice of a single parameter  $\lambda_i$  ( $i = 1, 2$ ) with respect to generalized cross-validation (GCV). For the estimation of the parameters, we resorted to a backfitting algorithm. The algorithm as well as the choice of a GCV optimal smoothing parameter  $\lambda$  are described in the book of Green and Silverman (1994) and Hastie and Tibshirani (1997).

Apart from the choice of smoothing parameter, control for outlying observations near the boundary is of great importance. Although by taking logs of income and wealth this problem is somewhat damped down, it is

still important as a few outliers (especially in the boundary regions of income or wealth) might influence the estimated curve severely. This is more problematic as we are primarily interested in the average derivative of  $c$ . A mechanical choice of either a smoothing parameter or a cutoff value at the boundary is thus difficult.

To measure goodness of fit of the partial HK model, we use two criteria:

- The average absolute error (AAE)

$$AAE = \frac{1}{T} \sum_t | \Delta_4(\log(C_t)) - \Delta_4(\log(\widehat{C}_t)) | \quad (8)$$

- The relative residual sum of squares (RRSS)

$$RRSS = \frac{\sum_t (\Delta_4(\log(C_t)) - \Delta_4(\log(\widehat{C}_t)))^2}{\sum_t (\Delta_4(\log(C_t)))^2} \quad (9)$$

*RRSS* measures the sum of squared residuals relative to the original squared differences  $(\Delta_4(\log(C_t)))^2$ . In a standard parametric model,  $RRSS = 1 - R^2$ . Obviously, a well fitted model shall have low values of *AAE* and *RRSS*.

## 4 Data Description and Validation

For empirical estimation of the models, we use the UK Family Expenditure Survey (FES) data. The FES is carried out in the UK on annual basis. Every year, about 7,000 households are covered from all over Great Britain and Northern Ireland. For each household, the different forms of incomes and expenditures on a large variety of consumption items are recorded, along with a large set of household attributes such as household size and composition, age and employment status of the head of household etc.<sup>7</sup>

FES's for different years yield a set of cross-section samples. Due to changes in the economic environment, the underlying concepts in the survey also change occasionally. Hence, to build a consistent series of variables from

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<sup>7</sup>For a precise definition of the sampling design, sample units and the variables, one may consult the respective yearly FES manuals. These manuals also discuss in detail the techniques relating to interviewing and field work, confidentiality and reliability of the data. Such aspects are also discussed in detail in the Family Survey Handbook by Kemsley et al (1980).



these surveys is a challenge in itself and in this endeavour, we consistently maintain the HBAI standard, which is a benchmark for income surveys in the UK (DSS, 1993). We include in our analysis data for all quarters made available to us for all years between 1968 and 1993. Earlier studies like HK and CS1 have omitted the year 1978 because of problems within the FES database that rendered an appropriate construction of the income variable difficult. In this study, the difference was sorted out by calculating the income data from alternative sources. For 1978, we used the data on income from the 'Households Below Average Income Dataset, 1961-1991' published by the Institute for Fiscal Studies (IFS) and available through the DataArchive. Unlike HK, we also include Northern Ireland in the domain of our analysis.

In this paper, we use the definition of consumption, income and wealth as used by HK. HK defined consumption as the total consumption expenditure on all goods and services minus housing costs and durable goods. Thus, following the tradition in the literature on consumption, HK primarily focused on non-durable goods and services. So far as estimates of household income are concerned, CS1 has considered disposable labor income only. However, the approach of HK had been more general. In this study, we follow the HK approach and obtain household income by extracting the relevant items from the FES database. We distinguish between current non-property as well as asset income for each household. The definition of asset income corresponds to the aggregate "investment income" used in the HBAI. It includes all sources of income that are due to private investments or property.

It may be noted that the FES does not contain direct information on wealth, but "property income", i.e., income which is due to private investments or property, is recorded for each household. Following HK, an approximation of household financial assets is then obtained by using this quotient of property income and a representative average quarterly interest rate.

So far as the conversion of a series in real terms is concerned, one should ideally consider regional price indices. However, due to their lack of availability, we used the RPI series published by the Central Statistical Office (CSO) in the UK. Consumption, income and wealth were transformed to their real values by simply dividing them by the price index of the corresponding month in which the household was included in the survey. The notations  $C$ ,  $Y$  and  $W$  henceforth stand for real consumption, income and wealth per household.

It may be noted that the price level could be different for different regions. As the HK model utilizes independent cross-sections data for estimation of average derivatives, ideally for meaningful comparison in cross-section, con-

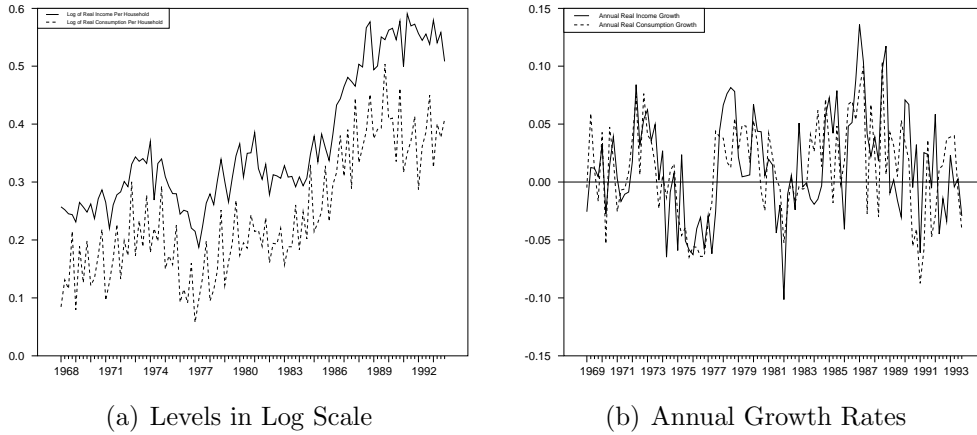


Figure 1: Quarterly Real Income and Real Consumption Per Household and Their Annual Growth Rates

sumption, income and wealth should be deflated by a price scale. Availability of such price scale for a country on continuous basis is, however, rare. In case of UK, Baran and O'Donoghue (2002) compared the relative level of prices of goods and services in different regions to the national average and to the price level in London. Unfortunately, the comparison was restricted to the year 2000 only, a year outside the domain of our analysis. A comparison with the previous periods were difficult because of differences in the purpose as well as coverage. In this study, no attempt of price scaling is, therefore, made.

Figure 1(a) presents the movements of the logarithm of real income and consumption per household. Figure 1(a) reveals that there is a strong seasonal pattern in both the series. The seasonal pattern appears to be more pronounced in case of consumption. This is not surprising because of the sharp peaks in consumption expenditure in the fourth quarter before Christmas. Besides seasonality, Figure 1(a) reveals an important feature. From the late 1980s, the divergence between the two series increased. Figure 1(b) presents the annual growth rates of income and consumption. In contrast to the movements in levels, Figure 1(b) reveals that the seasonal movements in both  $\Delta_4 C_t$  and  $\Delta_4 Y_t$  are negligible.<sup>8</sup> However, it may be observed that both

<sup>8</sup>This is also verified by simple OLS regressions of both series, with a constant term and dummies corresponding to quarters 2 to 4 as explanatory variables. In the estimated equations, none of the quarter dummies turn out to be significant.

Table 1: STATISTICAL FEATURES OF  $\Delta_4(\log(C_t))$  AND  $\Delta_4(\log(Y_t))$ 

		Mean	S.D.	Skewness	Kurtosis
First Half	$\Delta_4(\log(Y_t))$	0.0075	0.0415	-0.02	-0.77
	$\Delta_4(\log(C_t))$	0.0067	0.0377	-0.28	-0.75
Second Half	$\Delta_4(\log(Y_t))$	0.0188	0.0484	0.27	0.10
	$\Delta_4(\log(C_t))$	0.0146	0.0436	-0.12	-0.42
Entire Period	$\Delta_4(\log(Y_t))$	0.0129	0.0451	0.20	-0.11
	$\Delta_4(\log(C_t))$	0.0105	0.0406	-0.14	-0.50

Note: First Half is from 1969:1 to 1981:4, Second Half is from 1982:1 to 1993:4.

the series display a pronounced cyclical pattern.

Table 1 presents the first four moments of  $\Delta_4(\log(C_t))$  and  $\Delta_4(\log(Y_t))$ . In each half, as well as for the entire period, income is more variable than consumption. This is consistent with theoretical and empirical evidence of consumption smoothing of households. To explore the statistical properties of both series further, the first ten autocorrelations and partial autocorrelations of  $\Delta_4(\log(C_t))$  and  $\Delta_4(\log(Y_t))$  are presented in Table 2.

Table 2: Correlation Structures of Different Series

	Autocorrelations									
$\Delta_4(\log(Y))$	0.48	0.33	0.16	-0.09	0.14	0.17	0.19	0.00	-0.05	-0.12
$\Delta_4(\log(C))$	0.42	0.41	0.39	0.02	0.15	0.05	-0.03	-0.14	-0.00	-0.06
	Partial Autocorrelations									
$\Delta_4(\log(Y))$	0.48	0.13	-0.05	-0.24	0.34	0.13	-0.00	-0.39	0.15	0.02
$\Delta_4(\log(C))$	0.42	0.29	0.19	-0.34	0.10	0.01	0.02	-0.35	0.30	-0.00

It may be noted that DHSY examined the aggregate time-series relationship between consumer expenditure and income in the UK at quarterly frequency between 1957:1 to 1976:4. DHSY used the macroeconomic data from the *Economic Trends* (1976 Annual Supplement). Though the definitions, coverage and period of our study do not match with DHSY results reveal that statistical properties of both the consumption and the income series are generally similar to those obtained by them. Interestingly, the nature of the relationship between consumption and income appears to have changed. The changing nature of this relationship can be observed by a plot of time-varying correlation coefficient between  $\Delta_4(\log(Y))$  and  $\Delta_4(\log(C))$

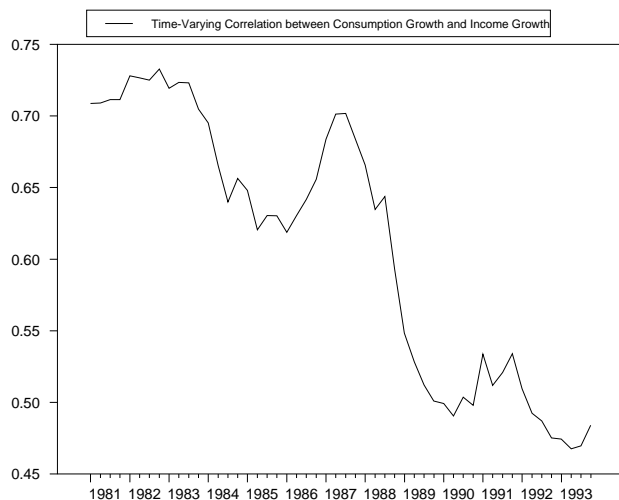
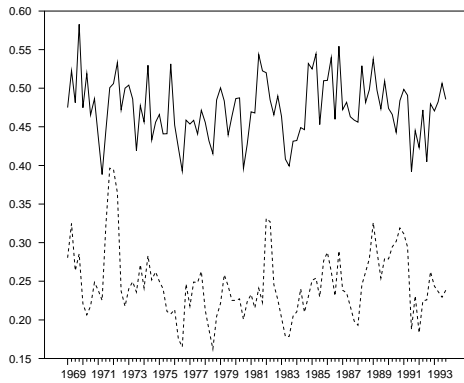


Figure 2: Time-Varying Correlation Coefficient between Annual Growth Rates of Income and Consumption

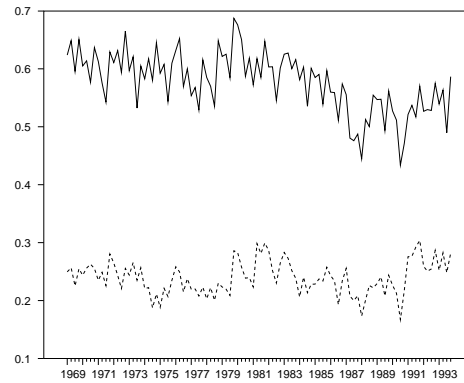
with a moving window of 48 quarters. Figure 2 presents this plot. Figure 2 reveals that there is a very sharp drop of correlation between 1987 and 1988. The fall in correlation, however, has been noted by earlier researchers and is interpreted either as changes due to financial liberalisation or as changes in expectation formation (Song et al, 1998).

## 5 Results on Partial HK Model

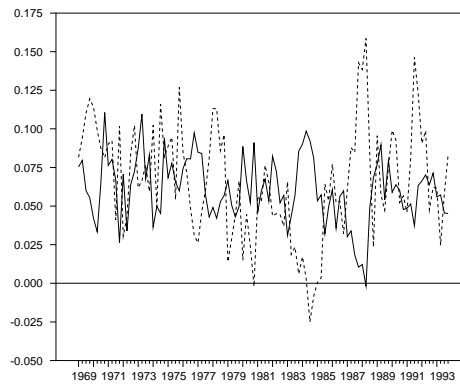
Figures 3(a) to 3(c) present the estimated values of the coefficients  $\beta_{ti}^*$  and  $\gamma_{ti}^*$ , ( $i = 1, 2$ ) respectively. The interpretations of the estimated values for these coefficients have been discussed in detail by both HK and CS1. In this section, however, we examine the statistical properties of  $\hat{\beta}_{ti}^*$  and  $\hat{\gamma}_{ti}^*$ .



(a)  $\hat{\beta}^*$  and  $\hat{\gamma}^*$  for income (wealth group)



(b)  $\hat{\beta}^*$  and  $\hat{\gamma}^*$  for income (no wealth group)



(c)  $\hat{\beta}^*$  and  $\hat{\gamma}^*$  for wealth

Figure 3:  $\hat{\beta}^*$  (—) and  $\hat{\gamma}^*$  (· · ·) for income and wealth

Results on ADF tests for  $\hat{\beta}_{ti}^*$  and  $\hat{\gamma}_{ti}^*$  are presented in Table 3. Table 3 presents the joint test statistic corresponding to a unit root and no linear trend. To examine the robustness of the results, separate tests have been conducted for the first half (1969:1 to 1981:4), the second half (1982:1 to 1993:4) and the entire period using both AIC and BIC criteria. The integers within the third bracket are the optimal number of lags in the model in each case.

Table 3: Results of Unit Root Tests on  $\hat{\beta}^*$  and  $\hat{\gamma}^*$ 

	AIC			BIC		
	First Half	Second Half	Entire Period	First Half	Second Half	Entire Period
<b>wealth group :: income coefficients</b>						
$\hat{\beta}_1^*$	15.10 (#)[0]	6.28 (\$) [9]	11.23 (#)[4]	15.10 (#)[0]	11.42 (#)[0]	26.25 (#)[0]
$\hat{\gamma}_1^*$	5.80 (\$) [0]	5.73 (\$) [0]	13.91 (#)[2]	5.80 (\$) [0]	5.73 (\$) [0]	10.32 (#)[0]
<b>wealth group :: wealth coefficients</b>						
$\hat{\beta}_2^*$	13.96 (#)[3]	9.33 (#)[3]	20.238 (#)[3]	16.94 (#)[0]	4.567 [0]	20.238 (#)[3]
$\hat{\gamma}_2^*$	14.12 (#)[0]	5.647 (\$) [0]	12.236 (#)[0]	14.12 (#)[0]	5.647 (\$) [0]	12.236 (#)[0]
<b>non-wealth group :: income coefficients</b>						
$\hat{\beta}^*$	3.676 [5]	1.213 [12]	2.417 [16]	8.136 (@)[4]	2.851 [4]	5.517 (\$) [4]
$\hat{\gamma}^*$	1.8488 [7]	2.237 [6]	6.451 (\$) [17]	7.037 (@)[0]	6.819 (@)[0]	13.38 (#)[0]

Note:

1. First Half is from 1969:1 to 1981:4, Second Half is from 1982:1 to 1993:4
2. The terms within third brackets denote the optimum lags.
3. #, @ and \$ denote significance at 1%, 5% and 10% level.

The unit root tests are generally supportive of the stationarity of the coefficients, although for non-wealth groups the results generally point to the contrary when the AIC is used to determine optimal lags. As the tests are not unambiguous, we conclude that estimated coefficients reflect near-unit-root situations, in which a clear identification of the underlying stochastic structure is difficult.

It may be noted that the econometric issue of near unit root has received extensive attention in the consumption literature in a different context. The entire debate on 'excess sensitivity' versus 'excess smoothness' of consumption during the 1990s revolved around the issue of whether the income process is trend stationary or difference stationary. While reviewing the problem Deaton (1992), mentioned that econometric tests for unit root may not have sufficient power to satisfactorily resolve the issue. Further, Deaton (1992) showed with examples that real life situations may lead to either of these processes.<sup>9</sup> Given that these tests have limited power, Deaton's example thus urges one not to take these tests mechanically in near unit root

<sup>9</sup>Deaton(1992, pp.110–112) gave the example of the stochastic process of salaries of professors. While age-wage norm is likely to lead to a trend stationary process, the possibility

situations. Rather, in near unit root situations, the stochastic properties of a series should be cross-validated from alternative sources to the extent possible.

Alternatively, in a simple OLS regression framework, the estimated coefficients corresponding to the constant terms and the respective first lags are, however, high, indicating slow speeds of convergence to the respective steady state coefficients. The fits of simple first order autoregressive equations with drift, in both cases, appear to be good. The residual of these estimated equations, however, indicate the presence of significant autocorrelations and partial autocorrelations in a few lags. However, generally these figures fall between -0.40 to 0.40. These findings, therefore, support the possibility that all  $\beta^*$  and  $\gamma^*$  series could be approximated by AR(1) processes with drifts. Note that an AR(1) structure with drift for both  $\beta^*$  and  $\gamma^*$  series imply that in time series, the partial HK model reduces to a random coefficient model.

It may be noted that it is possible to estimate the contribution of each variable to the overall fit of the model separately. Tables 4 and 5 present *AAE* and *RRSS* for the partial HK model. In Table 4, the no-wealth group is modelled with  $\beta_t$  only, while in Table 5 the no-wealth group is modelled also with the  $\gamma_t$  parameter. It may be noted that the results corresponding to the wealth group, would be same in both the tables. Results indicate that addition of wealth as a household-specific variable adds only little explanatory power. This, however, could be the case due to the limitation in the wealth data and, in particular, its construction based on the available information in the FES data and other sources. So far as the contribution of income is concerned, the contributions to the overall explanation by the  $\gamma$  coefficients was also found to be limited. The empirical findings on contributions of different variables are, however, consistent with the earlier findings of HK, CS1 and CS2.

Incidentally, the simple time series regression of  $\Delta_4(\log(C))$  on  $\Delta_4(\log(Y))$  yields the following equation

$$\Delta_4(\log(C)) = \underset{(0.64)}{0.0021} + \underset{(6.55)}{0.4838}\Delta_4(\log(Y)) \quad (10)$$

$R^2 = 0.29$ ,  $\bar{R}^2 = 0.28$ , DW Statistic = 1.42

Note: The bracketed terms are estimated  $t$ -values

When the constant term is omitted, the coefficient corresponding to of random 'raises' is likely to lead to a difference stationary one.

$\Delta_4(\log(Y))$  increased to 0.4975, with no changes in other features (except in  $R^2$ , which falls to 0.28). We stress that the HK model does not impose any minimization norm in the time series like the OLS model does. The estimated *AAE* and *RRSS* in the partial HK model should, therefore, be considered as good.

Table 4: GOODNESS OF FIT OF THE PARTIAL HK-MODEL  
(NO WEALTH GROUP IS MODELLED WITH  $\beta_t$  ONLY)

	wealth group		total	
	<i>AAE</i>	<i>RRSS</i>	<i>AAE</i>	<i>RRSS</i>
$\Delta_4(\log(C_t))$	5.1236		3.3512	
$\beta_{t1}$	4.3309	0.6677	2.7448	0.7042
$\beta_{t1}, \gamma_{t1}$	4.3006	0.6541	2.7170	0.6816
$\beta_{t1}, \gamma_{t1}, \beta_{t2}, \gamma_{t2}$	4.3467	0.6427	2.6885	0.6677

Note: For no Wealth group,  $\frac{1}{T}\Sigma | \Delta_4(\log(C_t)) | = 3.1647$ ,  
AAE = 2.3732, RRSS = 0.6076

Table 5: GOODNESS OF FIT OF THE PARTIAL HK-MODEL  
(NO WEALTH GROUP IS MODELLED WITH BOTH  $\beta_t$  AND  $\gamma_t$ )

	wealth group		total	
	<i>AAE</i>	<i>RRSS</i>	<i>AAE</i>	<i>RRSS</i>
$\Delta_4(\log(C_t))$	5.1236		3.3512	
$\beta_{t1}$	4.3309	0.6677	2.7195	0.6728
$\beta_{t1}, \gamma_{t1}$	4.3006	0.6541	2.7005	0.6557
$\beta_{t1}, \gamma_{t1}, \beta_{t2}, \gamma_{t2}$	4.3467	0.6427	2.6771	0.6453

Note: For no Wealth group,  $\frac{1}{T}\Sigma | \Delta_4(\log(C_t)) | = 3.1647$ ,  
AAE = 2.4018, RRSS = 0.5927

## 6 Modelling of the Remainder Term

To build a complete model of consumption based on HK approach, we need to model the remainder term,  $\Psi_t^{Adj}$ . The  $\Psi_t^{Adj}$ 's for different periods in the HK model form a time series whose stochastic properties have not been examined



in detail so far.<sup>10</sup> In this section, we attempt to construct time series models of  $\Psi_t^{Adj}$ . A good time series model that explains variations in  $\Psi_t^{Adj}$  would be of additional benefit because the explained part of  $\Psi_t^{Adj}$  could then be used to explain variations in  $\Delta_4(\log(C_t))$  further. An examination of the properties of  $\Psi_t^{Adj}$  is also important because the coefficients in the partial HK model are estimated using cross-section data for different periods. Therefore, the vector  $\Psi_t^{Adj}$  may not share the statistical properties of typical residual vectors obtained by OLS, GLS or standard time series techniques.<sup>11</sup>

The  $\Psi_t^{Adj}$  vector could be an amalgamation of four factors. First, it includes the higher order terms of the Taylor series approximation of the functional form for aggregate consumption growth. Second, it is affected by the movements of variables that are observable, but not household-specific. Third, it also captures the effect of micro-specific unobservable variables. Finally, it may also be affected by seasonality because of the seasonality in  $A$  and  $B$  in the affine transformation  $\Psi_t^{Adj} = A \Psi_t^* + B$ . So far as the first is concerned, it is not clear to what extent these second and higher order terms contribute to  $\Psi_t^{Adj}$ . In this paper, we therefore, attempt to see the impact of other factors on  $\Psi_t^{Adj}$ .

The statistical properties of  $\Psi_t^{Adj}$  is presented in Table 6. From Table 6 it is seen that the mean of  $\Psi_t^{Adj}$  is not significantly different from zero in both halves as well as the entire period. The higher moments of  $\Psi_t^{Adj}$  do not change much across periods. In fact, for both the sub-periods as well as the entire period, skewness and kurtosis of  $\Psi_t^{Adj}$  are not significantly different from zero. Further, the Jarque-Bera statistic in Table 6 reveals that the marginal distribution of  $\Psi_t^{Adj}$  is not significantly different from normal distribution.

Table 7 presents the ADF joint test statistic corresponding to a unit root and no linear trend. Table 7 reveals that the results on ADF tests are mixed. While the BIC criterion strongly rejects the presence of unit root in  $\Psi_t^{Adj}$  for both the sub-periods as well as for the entire period, when AIC criterion is applied the results are mixed and for the First Half as well as for the entire period point to the contrary.

To examine the stochastic structure of  $\Psi_t^{Adj}$  in more detail, the first thirty

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<sup>10</sup>As the studies of HK and SC were both based on limited number of annual observations and that too with gaps in between, a rigorous specification of a time series model of  $\Psi_t^{Adj}$  was not possible.

<sup>11</sup>For example,  $\Psi_t^{Adj}$  in the HK model may not be orthogonal to the design matrix.

Table 6: Statistical Properties of  $\Psi_t^{Adj}$

	First Half	Second Half	Entire Period
Mean	0.0056	0.0047	0.0052
Standard Deviation	0.0280	0.0370	0.0324
Skewness	-0.1049	-0.1820	-0.1710
Kurtosis	-0.7343	-0.6801	-0.5386
Jarque-Bera	1.2635	1.1901	1.6959

Note: First Half is from 1969:1 to 1981:4, Second Half is from 1982:1 to 1993:4.

Table 7: Results of ADF tests on  $\Psi_t^{Adj}$  Series

Criterion	First Half	Second Half	Entire Period
AIC	2.74 [11]	7.18 (@)[3]	2.90 [19]
BIC	13.91 (#)[0]	13.84 (#)[0]	28.24 (#)[0]

Note:

1. First Half is from 1969:1 to 1981:4, Second Half is from 1982:1 to 1993:4
2. The terms within third brackets denote the optimum lags.
3. @ and # denote significance at 5% and 1% level respectively.

Table 8: Correlation Structure of  $\Psi_t^{Adj}$

Autocorrelations									
0.26	0.26	0.13	-0.23	0.02	-0.13	-0.01	-0.10	0.01	0.08
0.02	0.03	-0.13	-0.07	-0.13	-0.21	-0.11	-0.10	0.07	0.01
0.08	0.07	-0.21	0.04	-0.02	-0.14	0.12	0.07	-0.01	0.13
Partial Autocorrelations									
0.26	0.20	0.03	-0.35	0.12	-0.03	0.08	-0.23	0.16	0.06
0.05	-0.21	-0.08	0.04	-0.01	-0.27	-0.05	0.13	0.23	-0.30
-0.07	0.18	-0.12	-0.14	0.10	-0.03	0.15	0.07	-0.30	0.06

autocorrelations and partial autocorrelations of  $\hat{\Psi}_t$  are presented in Table 8. Table 8, however, do not indicate any clear parsimonious time series model. We note the presence of significant correlations at higher lags, especially at multiples of four. This is not surprising because of the seasonality induced in  $\Psi_t^{Adj}$ .

It may be noted that  $\Psi_t^{Adj}$  is likely to be strongly influenced by expectation formation of households. How strong the impact would be would depend on several other factors. However, it is reasonable to assume that anticipated future income would be strongly correlated with current income across the subpopulation.<sup>12</sup> Extending this argument, Both HK and CS1 observed that the estimated coefficients  $\beta_t$  and  $\gamma_t$  may contain parts of expectation that is dependent on current income  $y_t^h$ . If expectation formation is myopic and is a random multiple of current income and the random stochastic term does not depend on  $y_t^h$ , the remainder term, in fact, may not depend on expectation formation on income.

So far as the choice of explanatory variables for  $\Psi_t^{Adj}$  is concerned, a major limitation is non-availability of direct measures of household income expectations.<sup>13</sup> An appropriate consumer confidence measure could perhaps been an alternative, as there is evidence that these measures reflect expectations of income and non-stock market wealth growth and contain information on the future path of aggregate consumer expenditure growth as well (Ludvigson, 2004). However, it is also observed that much of that information can be found in other popular economic and financial indicators. In fact, Ludvigson (2004) argues that the independent information provided by consumer confidence measures explain only a modest amount of additional variation in consumer spending. In our case, consumer confidence measures in the UK was available only from 1985 onwards, and therefore, was not of much use.

To arrive at a set of explanatory variables for  $\Psi_t^{Adj}$ , we focus on observable variables that are not household specific, and yet could be strongly correlated with income expectations. As there are several factors that could affect expectation formation on income, there could be several such variables. In this paper, we consider three such factors e.g., conditions in the labor market and in the markets for financial and physical assets. So far as the first is concerned, the signalling role of aggregate unemployment rate has long been

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<sup>12</sup>One of the early studies that provide strong evidence in favor of this hypothesis is Freeman(1971).

<sup>13</sup>In the UK, HBAI data contains a polychotomous variable on income expectations, however the data are not available for major part of our study.

recognised as a proxy for unemployment uncertainty. Dynarski and Sheffrin (1987) have found that the effect of unemployment on consumption depends on occupational characteristics, with white-collar workers reacting more to unemployment spells than blue-collar workers. Despite occupational differences, Malley and Moutos (1996) have argued that the aggregate unemployment rate could still be a valuable measure of aggregate income uncertainty. So far as the financial assets are concerned, in this study the expectation formation is assumed to be guided by the real yield from the capital market. For expectational measures relating to physical assets, we restrict our attention on housing prices. Housing being a major component of wealth, rising housing prices may stimulate consumption by increasing households' perceived wealth, or by removing the liquidity constraints. As the population ages and becomes more concentrated in the homeowners group, aggregate consumption may become more responsive to house prices (Campbell and Cocco, 2004). Studies have also confirmed that the link between changes in house prices and consumer spending has been close in the UK. Between 1971 and 2001, the simple correlation between annual household consumption growth and real house price inflation in the UK had been found to be 0.85, stronger than other major economies in Europe like France or Germany (HM Treasury, 2003). Using the FES data on consumption and regional housing prices, the micro-level study of Cambell and Cocco (2004) has also found that in the UK, predictable changes in house prices are correlated with predictable changes in consumption, particularly for households that are more likely to be borrowing constrained.

It may be noted that it is not clear what functional form of these variables should be used to explain the remainder term and also to what extent past movements of these variables would affect so. So far as the choice of functional form is concerned, to explain annual consumption growth, we have focused on either annual growth or annual changes in the corresponding indicators. For example, the data on unemployment rate in the UK for each quarter have been collected from the website <http://www.statistics.gov.uk>. The variable `DUNEMP` reflects the annual change in this variable. So far as expectations on income from financial assets is concerned, we define a variable `RFTSE` as the annual real rate of return. The variable is defined as the annual percentage change the London FTSE index (collected from the website <http://www.ftse.com>) minus the annual rate of inflation. To capture the impact of physical assets, we include the annual rate of inflation in housing prices in real terms (`INFHPI`). The UK ODPM (Office of the

Deputy Prime Minister) house price index (available online in the website <http://www.odpm.gov.uk>) has been used for this purpose

So far as the effects of past movements of these variables on consumption growth is concerned, we assume that their impacts would be myopic. Our stand is based on some of the latest findings on expectation formation of economic agents through direct surveys and laboratory experiments. A major finding of this literature is that both extrapolative and rational expectation hypotheses - whether at macro or micro level, or whether for wage or income expectations - do not hold (Leonard, 1982; Dominitz and Manski, 1997; Das and van Soest, 1999), though bounded rationality seems to be more acceptable (Pesaran, 1987). Expectations on income appeared to be biased and the forecast errors had been found to be correlated with demographic characteristics in these studies. Results also suggest that despite heterogeneity in expectation formation across the population, for a single agent, expectation formation may not go too far back in the past (Freeman, 1971) and may change only on important turning points (Schmalensee, 1976).

The estimated equation is as follows:

$$\Psi_t^{Adj} = \underset{(-0.50) [0.15]}{-0.0027} + \underset{(0.78) [0.46]}{0.0113} \text{RFTSE} - \underset{(-0.80) [0.21]}{0.0022} \text{DUNEMP} + \underset{(2.33) [0.27]}{0.0001} \text{INFHPI} \quad (11)$$

$$R^2 = 0.10, \bar{R}^2 = 0.07$$

Note: The terms in first bracket are estimated  $t$ -values. The terms in third bracket are Hansen (1992)'s stability statistics for the respective parameters.

In equation (11), the variables RFTSE, DUNEMP and INFHPI appear with correct sign, though RFTSE and DUNEMP do not appear to be significant. An examination of the residuals indicated the possibility of further improvements in the model with the inclusions of lags 1 and 4 of  $\Psi_t^{Adj}$  as explanatory variables. Both these lags were natural choice because of the affine transformation  $\Psi_t^{Adj} = A \Psi_t^* + B$ . The estimated equation is:

$$\begin{aligned} \Psi_t^{Adj} = & \underset{(-0.38) [0.16]}{-0.0019} + \underset{(0.49) [0.31]}{0.067} \text{RFTSE} - \underset{(-0.72) [0.22]}{0.0019} \text{DUNEMP} + \underset{(2.13) [0.23]}{0.0006} \text{INFHPI} \\ & + \underset{(2.90) [0.08]}{0.2882} \Psi_t^{Adj}(-1) - \underset{(-2.81) [0.03]}{0.2700} \Psi_t^{Adj}(-4) \quad (12) \end{aligned}$$

$$R^2 = 0.23, \bar{R}^2 = 0.18$$

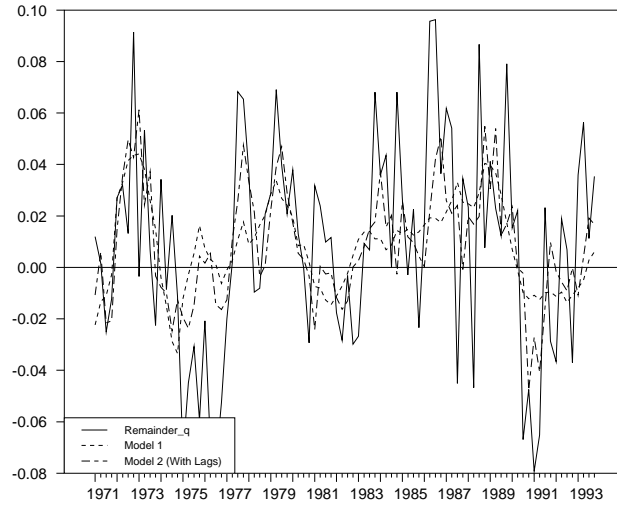


Figure 4: Fit of the Remainder Term

Note: The terms in first bracket are estimated  $t$ -values. The terms in third bracket are Hansen (1992)'s stability statistics for the respective parameters.

In equation (12), all the variables included in equation (11) retain their respective signs, once again with only  $INFHPI$  being significant at 5% level. Explanatory power of this model, however, improves substantially, as indicated in Figure 4. We call these two models Model 1 and Model 2 respectively.

For stability tests of the estimated parameters in these two models we follow Hansen (1992). All the estimated parameters reported in equations (11) and (12) appear to be stable. In each case, however, we note instability in the error variance parameter. For Model 1 and Model 2, the values of the Hansen's test statistics for error variance are 0.83 and 1.19 respectively, significant at 1% level. The instability in the error variance parameter could occur due to two possibilities. First, the population variance itself might be changing over time. Second, the FES consumption data itself might have an unstable sampling error over time. In Section 7, this aspect is examined further.

Finally, we note that the fits of both the complete models are better than the partial models. In fact, for Model 1, both the  $AAE$  and  $RRSS$  is higher (at 2.53 and 0.55 respectively) than Model 2 (at 2.32 and 0.47 respectively).

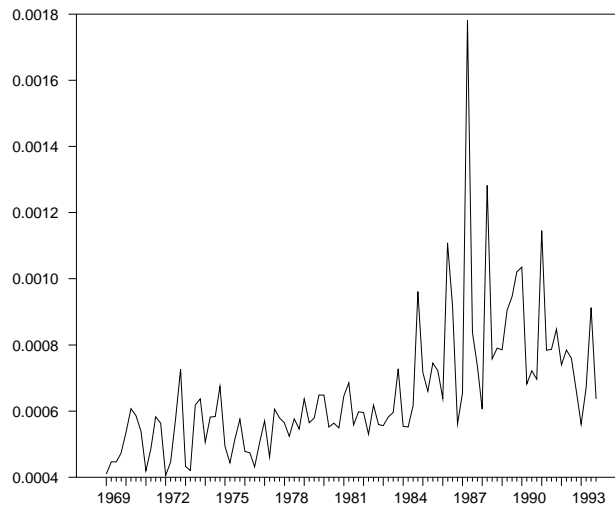


Figure 5: Estimated Bootstrap Variance of the Annual Growth Rate of Aggregate Consumption

## 7 Cross-verification of Results through Bootstrap

So far as goodness of fit of the model is concerned, it should be remembered that we are working with sample data. Among other sources of error, the estimated relationship would, therefore, be affected by sampling error too. Given this fact, if an estimated model fits too well to this data, chances that it is "explaining" at least parts of the error term cannot be ruled out. It is, therefore, necessary to assess to what extent the sampling error could contribute to the overall goodness of fit. It is also necessary to assess whether the time-varying sampling error could induce an unstable error variance in the equations corresponding to the remainder term.

To check these, we do the following:

- Treating the FES sample of household level consumer expenditure for quarter  $t$  like a population, we draw a sample of equal size with replacement. A similar sample is drawn for quarter  $(t - 4)$  as well. Using these two bootstrap samples, a typical bootstrap aggregate annual

growth rate of consumption is obtained. For each quarter, the process is repeated 10,000 times to have 10,000 bootstrap annual growth rates of consumption.

- For each quarter, the root mean squared deviation of these bootstrap growth rates from the actual FES growth rates are calculated. A plot of these deviations in Figure 5 indicates more variation in consumption during the late 1980s. Thus, it is expected that the more variability at the micro-level in consumption during the late 1980s would manifest at the macro level too.
- To get an idea about how much the sampling error contributes to the overall goodness of fit, we also computed the ratio of error sum of squares to the total sum of squares. The computation reveals that any model on aggregate consumption that would have a RRSS less than 0.43 could actually be a suspect, as it would probably attempt to model the "error" term. Given this, the fit achieved by our model could be perceived as good.

Finally, we postulate that if the model is correctly specified then the goodness of fit of our model for each quarter would be positively related to the mean squared deviation of the bootstrap growth rates from the actual FES growth rates. In a linear framework, if the mean parameters of these equations are stable, it would also explain the instability in the estimated variance in equations (11) and (12).

To examine these aspects, we carry out regressions of the quarterly bootstrap variances (BOOTVAR) on the final squared residuals SQRADJ1 and SQRADJ2 (pertaining to Model 1 and Model 2 respectively). The estimated equations are placed below:

$$\text{SQRADJ1} = - \underset{(-1.36)}{0.0005} + \underset{(4.15)}{2.1528} \text{BOOTVAR} \quad (13)$$

$$R^2 = 0.16, \bar{R}^2 = 0.15$$

$$\text{SQRADJ2} = - \underset{(-2.23)}{0.0007} + \underset{(5.21)}{2.1837} \text{BOOTVAR} \quad (14)$$

$$R^2 = 0.23, \bar{R}^2 = 0.22$$

Note: The terms in first bracket are estimated  $t$ -values. The terms in third bracket are Hansen (1992)'s stability statistics for the respective parameters.



The estimated equations (13) and (14) confirm the presence of a stable positive relationship, indicating that a part of the variation in the residuals could possibly be due to sampling error. It is likely that this variation leads to instability of the estimated variance in equations (11) and (12).

## 8 Conclusion

Using the Family Expenditure Survey (FES) data for the United Kingdom (UK) from the first quarter of 1968 to the fourth quarter of 1993, the paper specified and estimated a 'complete' Hildenbrand-Kneip (HK) model of consumption, in contrast to earlier efforts that were 'partial' in nature. As the estimated coefficients in the 'partial' HK model could be time-varying, the paper examined the statistical properties of these coefficients and provided empirical evidence that they displayed near unit root properties. To estimate the 'complete' HK model, the paper also specified a simple OLS model of the 'remainder term'. The 'remainder term', which as per theory should be influenced by unobservable variables like expectation formation of households, was found to be affected by real returns from the capital market and housing prices. The 'complete' model was found to explain movements in consumption better than the 'partial' model. Results based on bootstrap also confirmed that given the sampling error in the data, the overall fit of the model was good.

HK model is a nascent development and, so far, rigorous empirical work on it is rare. However, given that it sheds off some of the vexing assumptions like the specification of a utility function or the existence of a mythical representative agent – with no apparent cost in terms of empirical performance – it's potential need to be examined rigorously. In this context, we highlight two more possibilities. First, our analysis of the statistical properties of the estimated coefficients in the 'partial' HK model, revealed that the model might be empirically equivalent to a random coefficient model in time series. At this juncture, however, the empirical evidence, at best, applies to the UK and unless support for this comes from other data sets, should not be treated as a general phenomenon. Second, further evidence on the properties of the 'remainder term' in the 'partial' HK model – especially with richer data sets that contain typical 'unobservable variables' (e.g., income expectations) – would be of use. As yet, the stability of the distributional structures of these variables over time – a key assumption in the HK model – is not known.

Any further work in this direction would enrich our understanding of the evolution of aggregate consumption over time.

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